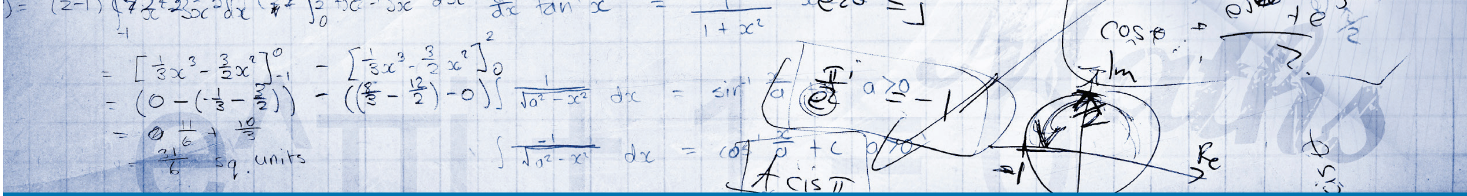

Discussion

Experimental investigation of full-scale post-tensioned composite AASHTO beams prestressed with carbon-fiber-reinforced-polymer cables

The following comments relate to “Experimental Investigation of Full-Scale Post-Tensioned Composite AASHTO Beams Prestressed with Carbon-Fiber-Reinforced-Polymer Cables,” by Mahmoud R. Manaa, Abdeldjelil Belarbi, Bora Gencturk, and Mina Dawood, which appeared in the January–February 2024 issue of *PCI Journal*.[1]

The authors are commended for addressing a difficult topic, generating critically needed data for full-scale beams prestressed with unbonded fiber-reinforced-polymer (FRP) tendons, and advancing the state of the art. This discussion aims to point out a correction and clarification to Eq. (1) and to complement the paper with important new information on Eq. (3) to (5) for total stress in the unbonded tendon f_{pf}, which was likely not available to them at the time their project started.

Ductility and Eq. (1)

The energy-based ductility index initially defined by Naaman and Jeong[2] is given (using same notation as in the paper) as follows:

$$\mu_{en} = \frac{1}{2}\left(\frac{E_{tot}}{E_{ela}} + 1\right) = \frac{1}{2}\frac{E_{tot}}{E_{ela}} + \frac{1}{2} \tag{8}$$

where

μ_{en} = energy ductility index

E_{tot} = total energy under the load-deflection curve

E_{ela} = elastic energy, which is part of the total energy

For a reinforced concrete beam, total energy under the load-deflection curve E_{ela} is assumed equal to the energy at onset of yielding of the reinforcing bars. Thus, Eq. (1) in the paper is missing the second term, 0.5.

$$\mu_{en} = 0.5\left(\frac{E_{tot}}{E_{ela}}\right) \tag{1}$$

Typically for a reinforced concrete beam with steel reinforcement, ductility is evaluated by a curvature, rotation, or deflection index μ_Δ such as ϕ_u/ϕ_y or Δ_u/Δ_y, where ϕ_u is curvature at ultimate; ϕ_y is curvature at first yield of tension reinforcement; Δ_u is deflection at ultimate, and Δ_y is deflection at first yield of tension steel. Using an energy ratio for a beam with FRP tendons, such as $\mu_{en} = E_{tot}/E_{ela}$, may seem evident at first but does not offer a way of comparison with an equivalent beam using steel. Eq. (8) was especially developed to give exactly the same numerical answer for both energy and deflection indices for a reinforced concrete beam with an elastic perfectly plastic response. Thus, if a code recommends a target ductility index for a reinforced concrete beam in a seismic zone to be, for example, $\mu_\Delta \geq 5$, Eq. (8) allows a direct correlation with that requirement should FRP reinforcement be used instead of steel.

If, in the paper, Eq. (8) is used instead of Eq. (1), then the numerical results in the last column of Table 2 of the paper should be each increased by 0.5. The new numbers do not change the related conclusion.

A final remark on using any ductility index is warranted. This reviewer recommends that in comparing two different members for ductility (such as with steel or FRP reinforcement) it is not enough to compare their ductility index, but there is also need, for completeness, to compare their total energy to failure. It would be interesting to have such numbers added to Table 2.

Table 2. Summary of the test results of the full-scale beams

Beam identifier	Concrete strength, ksi		Cracking load, kip	Ultimate		Failure mode	Predicted load capacity, kip		Deformability index		Total energy to failure, kip-in.
	Girder	Deck		Load, kip	Deflection, in.		ACI 440.4	AASHTO LRFD specifications	Abdelrahman et al.	Naaman and Jeong	
CPouSM	10.4	9.7	61	135	9.9	Concrete crushing	144.7	104.2	14.4	1.21	999.3
CPouSF	10.9	8.2	65	122	7.4				10.8	1.33	697.6
CPouDF	10.9	9.8	72	143	8.9				12.4	1.26	978.0
CPoDM#01	10.9	11.5	81.4	175	5.2	Cable rupture	151.6	n/a	6.4	1.23	677.7
CPoDM#02			62.8	174	6.7				9.5	1.26	859.6

Note: CPouDF = CFRP post-tensioned beam with unbonded draped cables subjected to the flexural fatigue loading condition; CPouSF = CFRP post-tensioned beam with unbonded straight cables subjected to the flexural fatigue loading condition; CPouSM = CFRP post-tensioned beam with unbonded straight cables subjected to the monotonic flexure loading condition; CPoDM#01 = CFRP post-tensioned beam with bonded draped cables subjected to the monotonic flexure loading condition (first specimen of this type); CPoDM#02 = CFRP post-tensioned beam with bonded draped cables subjected to the monotonic flexure loading condition (second specimen of this type); n/a = not applicable. 1 in. = 25.4 mm; 1 kip = 4.448 kN; 1 ksi = 6.895 MPa.

New updated equations for f_{pf} to replace Eq. (3) to (5)

Equation (3) was first recommended in a study published in 1991.[3]

$$f_{pf} = f_{pe} + \Delta f_{pf} = f_{pe} + \Omega_u E_{pf} \varepsilon_{cc} \left(\frac{d_p - c}{c} \right) \tag{3}$$

where

f_{pe} = effective stress in prestressing steel at the section under consideration after all losses
Ω_u = strain reduction factor
E_{pf} = prestressing CFRP tendon elastic modulus
ε_{cc} = concrete compressive strain
d_p = depth of the tendon from the extreme compression fiber
c = distance from the extreme compression fiber to the neutral axis

All variables and parameters in Eq. (3) are fundamentally derived from satisfying equilibrium, strain compatibility, and stress-strain relation of the reinforcement, except for the strain reduction coefficient Ω_u, which was obtained from comparing analytical predictions with experimental results. Note that the main purpose of using Ω_u is to augment the prediction from a strain compatibility to a deflection compatibility analysis.

In the original study,[3] 143 beams with steel tendons were analyzed. Since then, a new two-part extensive study[4,5] was carried out with an augmented set of data comprising 227 beams with steel tendons and 8 beams with FRP tendons. The data are considered a representative sample, having a broad range of L/d_p , where L is the length of the tendon between the anchorages, between 6.0 and 55, with both internal and external tendons, and using rectangular (81%) as well as T sections (19%).

In order to strengthen the statistical results and related conclusions, the new study[4,5] evaluated not only the mean and coefficient of correlation but also three additional finer measures, namely the sum of least squares (LS), the percentage of predicted data that is less than the experimental one, and the normalized LS ratio (the value of LS for a given equation divided by the value of LS for the smallest LS recorded in the study).

The statistical analysis identified the best overall equation to predict stress at the ultimate state f_{ps}, not only among six commonly available code equations but also among 25 other equations.[4,5] Moreover, the data were used to fine-tune the recommended strain reduction coefficient at ultimate Ω_u for code implementation. The related papers contain a large number of figures comparing experimental data versus analytical predictions for a total of 33 prediction equations of f_{ps}. Special remarks are provided for each code equation, including those from AASHTO Eq. (7) and ACI.

$$f_{pf} = f_{pe} + \left(\frac{900}{n}\right)\left(\frac{d_p - c}{c}\right) \le f_{pu} \tag{7}$$

where

n = modular ratio = E_s/E_{pf}

f_{pu} = ultimate tensile strength of the CFRP tendon

E_s = prestressing steel elastic modulus

Best overall results were obtained with Eq. (9) to (11) (using the same notation as in Manaa[1]), which are recommended for code implementation and to replace Eq. (3) to (5) in the paper:

$$f_{pf} = f_{pe} + \Delta f_{pf} = f_{pe} + \Omega_u E_{pf} \varepsilon_{cc}\left(\frac{d_p - c}{c}\right)\frac{L_1}{L_2} \le 0.82 f_{pu} \tag{9}$$

where

L_1 = span length of simply supported beam or beam analyzed for the loading considered

L_2 = total length of prestressed tendon between anchorages

For single point loading,

$$\Omega_u = \frac{e_m}{d_p}\left(0.02 + \frac{3.6}{L/d_p}\right) \tag{10}$$

where

e_m = eccentricity of the prestressed tendons at midspan or the critical section analyzed

For uniform or third point loading,

$$\Omega_u = \frac{e_m}{d_p}\left(0.029 + \frac{6.48}{L/d_p}\right) \tag{11}$$

Other notation is as in the paper. Note that Eq. (9) is fundamentally similar to Eq. (3); for a simply supported beam, the ratio L1/L2 = 1.

Because the beams tested in the paper under discussion are simply supported, using Eq. (9) instead of Eq. (3) would lead to only a slight difference in the numerical value of Ω_u, thus f_{pf}. However, given the scale used in Fig. 14 comparing predicted moment versus moment capacity, it is likely that the location of the data shown will not change enough to be visible to the naked eye. Keep in mind, as shown in the Alqam papers,[4,5] that the most appropriate way to evaluate a prediction equation is to compare the predicted values of Δf_{pf} from Eq. (3) or (9) with the experimental results.

The reader interested in this topic is strongly advised to review the material in the Alqam papers[4,5] where extensive additional information and a large number of graphs can be found. Note finally that in these papers there is a typo in Eq. (11) where the number 0.029 appears as 0.29.

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Notation

c = distance from the extreme compression fiber to the neutral axis
d_p = depth of the tendon from the extreme compression fiber
e_m = eccentricity of the prestressed tendons at midspan or the critical section analyzed
E_{ela} = elastic energy, which is part of the total energy
E_{pf} = prestressing CFRP tendon elastic modulus
E_s = prestressing steel elastic modulus
E_{tot} = total energy under the load-deflection curve
f_{pe} = effective stress in prestressing steel at the section under consideration after all losses
f_{pf} = total force in the unbonded tendon
f_{ps} = stress at the ultimate state
f_{pu} = ultimate tensile strength of the CFRP tendon
L = the length of the tendon between the anchorages
L_1 = span length of simply supported beam or beam analyzed for the loading considered
L_2 = total length of prestressed tendon between anchorages
n = modular ratio = E_s/E_{pf}
Δ_u = deflection at ultimate
Δ_y = deflection at first yield tension steel
ε_{cc} = concrete compressive strain
μ_{en} = energy ductility index
ϕ_u = curvature at ultimate
ϕ_y = curvature at first yield of tension reinforcement
Ω_u = strain reduction factor

Authors’ response

The authors thank Antoine Naaman for his valuable discussion of the paper.[1] In his discussion, Namaan correctly points out a clerical error in Eq. (1) of the original manuscript. This equation was correctly typed, and all the results are based on the correct equation shown in Naaman’s discussion. The typo occurred during the typesetting process, and the authors did not catch this mistake. However, since all the calculations in the original paper are based on the correct equation, the results and the ensuing discussions do not require any changes, including those shown in Table 2.

The authors agree with Naaman that a comparison of the total energy to failure for the test beams is important. We have added this information to Table 2. The results follow a similar trend to deformability indices because the total energy to failure is largely affected by the deflection of the beams at failure.

Naaman correctly points out in his discussion that the Alqam papers[2,3] were not available to the authors at the time of completing the research project and writing the original paper. Naaman provides a valuable discussion of how Eq. (3) in the original paper has been further refined with additional test data. As mentioned in the discussion, the change in the value of the strain reduction coefficient Ω_u will be negligible for the simply supported beams tested in our paper, hence the results and the ensuing discussions are still valid and do not require any changes. The reader is referred to Alqam[2,3] for further details on the methodology developed by Naaman and his colleagues to predict stress at ultimate in prestressed unbonded tendons.

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Notation

Ω_u = strain reduction factor

COMMENTS?