Proposed lump-sum formulas for long-term prestress losses

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- The American Association of State Highway and Transportation Officials' *AASHTO LRFD Bridge Design Specifications* approximate method for estimating long-term prestress losses simplifies the estimates for creep and shrinkage losses by using constant multipliers but does not reflect the effect of beam configuration on the creep and shrinkage multipliers.
- This paper describes two parametric studies that used refined methods to calculate creep and shrinkage losses for common bridge girder cross sections to compare with the constant multipliers used in the AASHTO LRFD specifications' approximate method for estimating long-term prestress losses.
- Lump-sum equations with updated creep and shrinkage multipliers are proposed to provide an improved estimate of long-term prestress losses for common bridge girder types that are simple to use and appropriate for preliminary prestressed concrete girder design.

The approximate formula for estimating long-term prestress losses in the ninth edition of the American Association of State Highway and Transportation Officials' *AASHTO LRFD Bridge Design Specifications*[1] is the outcome of the research work presented in National Cooperative Highway Research Program (NCHRP) report 496 *Prestress Losses in Pretensioned High-Strength Concrete Bridge Girders*.[2] The approximate formula was derived by simplifying the detailed method and taking into account the variability of concrete properties and the interaction between precast concrete girders and a cast-in-place concrete deck. Two detailed parametric studies are presented in this paper based on the average conditions for the design and construction of commonly used bridge girders. The girders examined were two bulb tees (BT-54 and BT-72), three I-girders (NU1100, NU1600, and NU2000), two box beams (BI-48 and BIII-48), one inverted tee (IT600), and one slab beam (SIV-48). Three spans and, consequently, three levels of prestressing for each section were considered. The first study established the creep multiplier N_c, while the second study evaluated the shrinkage multiplier N_s. Both multipliers are used in lump-sum formulas for estimating long-term prestress losses for different bridge girders. The multipliers derived from these studies were compared with that of the AASHTO LRFD specifications approximate method, and new lump-sum formulas for long-term prestress losses are proposed.

Research significance

The goal of these two parametric studies was to increase the accuracy and use of the AASHTO LRFD specifications' approximate formula[1] in estimating the long-term prestress losses for commonly used pretensioned sections. The author believes that the values produced by the two studies can be a tremendous help to designers during the preliminary design stage.

Parametric studies

Variables used for the two parametric studies include the following:

- type of beam cross section
- span and spacing of beams
- concrete strengths at transfer and final time of load application
- levels of prestressing

Nine commonly used pretensioned concrete sections were selected for the two studies:

- bulb tee BT-54
- bulb tee BT-72
- I-girder NU1100
- I-girder NU1600
- I-girder NU2000
- box beam BI-48
- box beam BIII-48
- inverted tee IT600
- slab beam SIV-48

The design parameters investigated for each section included three levels of prestressing—low, medium, and high—for three different simple spans and three different spacings for each girder. Composite section properties were computed using a ½ in. (13 mm) reduction for deck thickness to allow for long-term wear, a 1 in. (25 mm) thick haunch, and low-relaxation strands (Grade 270 [1860 MPa]) with a modulus of elasticity E_p of 28,500 ksi (196,500 MPa) and a yield strength f_{py} of 243 ksi (1675 MPa). The stress in the prestressing strands immediately before initial transfer (jacking stress) f_{pi} was 0.75 times the ultimate strength f_{pu} of the prestressing strands. The dead load included girder weight, deck weight, diaphragm weight, haunch weight, and a 2 in. (50 mm) thick asphalt wearing surface.

Table 1 shows the main section properties of all nine girder cross sections. **Table 2** provides the girder and deck information used in the AASHTO LRFD specifications' refined estimate of time-dependent losses. **Table 3** illustrates the three spans, beam spacing, and levels of prestressing for each girder.

The AASHTO LRFD specifications' refined estimate of time-dependent losses, which is the result of the extensive research work presented in NCHRP report 496,[2] was used to compute the long-term losses using its concrete creep and shrinkage formulas.

In the refined estimate, the transformed section coefficients K_{id} and K_{df}, defined by AASHTO Eq. (5.9.3.4.2a-2) and (5.9.3.4.3a-2), respectively, are used to reflect the interaction between concrete and prestressing strands in the girder (transformed-section effects) and the softening effect of concrete creep on that transformed section (as opposed to instantaneous elastic analysis). Thus, the transformed section coefficients may be viewed as the creep-adjusted transformed-section coefficients.

Table 1. Section properties

Section property	BT-54	BT-72	NU1100	NU1600	NU2000	BIII-48	BI-48	IT600	SIV-48
Type	Bulb tee	Bulb tee	I-girder	I-girder	I-girder	Box beam	Box beam	Inverted tee	Slab beam
A_g, in.^{2}	659.0	767.0	694.6	810.8	903.8	812.5	692.5	245.7	703.0
h, in.	54	72	43.3	63	78.7	39	27	23.6	21
I_g, in.^{4}	268,077	545,894	182,279	458,482	790,592	168,367	65,941	11,938	34,517
y_b, in.	27.6	36.6	19.6	28.4	35.7	19.3	13.4	8.3	10.5
Weight, kip/ft^3	0.686	0.799	0.724	0.840	0.942	0.846	0.721	0.256	0.732

Note: A_g = area of gross section of girder; h = depth of concrete girder; I_g = moment of inertia of gross section of girder; y_b = eccentricity of bottom fiber with respect to centroid of gross girder section. 1 in. = 25.4 mm; 1 in.^{2} = 645.2 mm^2; 1 in.^{4} = 416,231 mm^4; 1 kip/ft^3 = 16,018 kg/m^3.

$$K_{id} = \frac{1}{1+\frac{E_p A_{ps}}{E_{ci} A_g}\left(1+\frac{A_g e_{pg}^2}{I_g}\right)\left(1+0.7\psi_{bid}\right)} \quad \text{(AASHTO 5.9.3.4.2a-2)}$$

where

E_p = modulus of elasticity of prestressing steel

A_{ps} = area of prestressing steel

E_{ci} = modulus of elasticity of girder concrete at transfer

A_g = area of gross section of girder

e_{pg} = eccentricity of strands with respect to centroid of gross section of girder, always taken as positive

I_g = moment of inertia of gross section of girder

ψ_{bid} = girder creep coefficient between initial (transfer) and deck placement times

$$K_{df} = \frac{1}{1+\frac{E_p A_{ps}}{E_{ci} A_c}\left(1+\frac{A_c e_{pc}^2}{I_c}\right)\left(1+0.7\psi_{bif}\right)} \quad \text{(AASHTO 5.9.3.4.3a-2)}$$

where

Table 2. Girder and deck section and materials properties

Beam sections	BT-54	BT-72	NU1100	NU1600	NU2000	BIII-48	BI-48	IT600	SIV-48
Low span, ft	100	110	70	100	110	60	50	44	28
Medium span, ft	110	120	90	110	130	70	56	56	38
High span, ft	110	130	110	130	150	100	60	70	46
V/S	2.96	2.96	2.95	2.95	2.95	3.44	3.36	2.60	3.47
H, %	70	70	70	70	70	70	70	70	70
f'_{ci}, ksi	8	8	5.5	5.5	5.5	5.5	5.5	6.5	5.5
f'_c, ksi	12	12	7.5	7.5	7.5	7	7	8	7
E_{ci}, ksi	5531	5531	4384	4384	4384	4362	4362	4790	4362
E_c, ksi	6774	6774	5120	5120	5120	4921	4921	5314	4921
t_i, days	1	1	1	1	1	1	1	1	1
ψ_{bid}	0.848	0.848	1.084	1.084	1.084	1.019	1.030	1.011	1.015
ψ_{bif}	1.123	1.123	1.556	1.556	1.556	1.463	1.478	1.406	1.457
Deck section	**BT-54**	**BT-72**	**NU1100**	**NU1600**	**NU2000**	**BIII-48**	**BI-48**	**IT600**	**SIV-48**
Width, ft	8	8	12	12	12	8	8	2	4
	8	8	8	10	10	8	8	2	4
	8	8	6	6	6	8	8	2	4
Thickness, in.	8	8	8	8	8	8	8	8	3
f_d, ksi	4.5	4.5	4.5	4.5	4.5	4.5	4.5	4.5	4.5
E_d, ksi	3845	3845	3845	3845	3845	3845	3845	3845	3845
t_d, days	90	90	90	90	90	90	90	90	90
t_f, days	20,000	20,000	20,000	20,000	20,000	20,000	20,000	20,000	20,000

Note: E_c = modulus of elasticity of girder concrete; E_{ci} = modulus of elasticity of girder concrete at transfer; E_d = modulus of elasticity of deck concrete; f'_c = specified compressive strength of girder concrete; f'_{ci} = specified compressive strength of concrete at transfer; f_d = specified compressive strength of deck concrete; H = average annual ambient relative humidity; t_d = deck placement time; t_f = age of concrete at final time of load application; t_i = age of concrete at time of initial loading at transfer; V/S = volume-to-surface ratio of girder; ψ_{bid} = girder creep coefficient between initial (transfer) and deck placement times; ψ_{bif} = girder creep coefficient due to sustained load applied at initial time (transfer) and kept constant until final time. 1 in. = 25.4 mm; 1 ft = 0.305 m; 1 ksi = 6.895 MPa.

Table 3. Level of prestressing, span, and spacing

Level of prestressing	BT-54	BT-72	NU1100	NU1600	NU2000	BIII-48	BI-48	IT600	SIV-48
Low									
Number and size of strands	Twenty-four 0.6 in. diameter	Thirty 0.6 in. diameter	Forty-two 0.5 in. diameter	Fifty-four 0.5 in. diameter	Fifty-six 0.5 in. diameter	Twenty-three 0.5 in. diameter	Twenty-three 0.5 in. diameter	Eight 0.5 in. diameter	Ten 0.5 in. diameter
A_{ps}, in.^{2}	5.208	6.510	6.426	8.262	8.568	3.519	3.519	1.224	1.530
Eccentricity, in.	24.63	33.00	16.22	24.24	31.34	17.29	11.37	6.49	8.25
Beam span, ft	100	110	70	100	110	60	50	44	28
Beam center-to-center spacing, ft	8	8	12	12	12	8	8	2	4
Medium									
Number and size of strands	Thirty-six 0.6 in. diameter	Forty 0.6 in. diameter	Forty-eight 0.5 in. diameter	Fifty-six 0.5 in. diameter	Fifty-eight 0.5 in. diameter	Thirty-five 0.5 in. diameter	Thirty-one 0.5 in. diameter	Ten 0.5 in. diameter	Fifteen 0.5 in. diameter
A_{ps}, in.^{2}	7.812	8.680	7.344	8.568	8.874	5.355	4.743	1.530	2.295
Eccentricity, in.	23.41	31.70	15.91	24.04	31.08	16.60	10.85	6.49	8.25
Beam span, ft	110	120	90	110	130	70	56	56	38
Beam center-to-center spacing, ft	8	8	8	10	10	8	8	2	4
High									
Number and size of strands	Forty-four 0.6 in. diameter	Fifty 0.6 in. diameter	Fifty-four 0.5 in. diameter	Fifty-eight 0.5 in. diameter	Fifty-eight 0.5 in. diameter	Forty-six 0.5 in. diameter	Forty-one 0.5 in. diameter	Eighteen 0.5 in. diameter	Twenty 0.5 in. diameter
A_{ps}, in.^{2}	9.548	10.850	8.262	8.874	8.874	7.038	6.273	2.754	3.06
Eccentricity, in.	21.81	29.08	15.44	23.78	31.08	16.29	10.49	5.80	8.25
Beam span, ft	110	130	110	130	150	100	60	70	46
Beam center-to-center spacing, ft	8	8	6	6	6	8	8	2	4

Note: A_{ps} = area of prestressing steel. 1 in. = 25.4 mm; 1 ft = 0.305 m. 1 in.^{2} = 645.2 mm^2.

A_c = area of composite section at service

e_{pc} = eccentricity of strands with respect to centroid of composite section at service, always taken as positive

I_c = moment of inertia of composite section at service

ψ_{bif} = girder creep coefficient due to sustained load applied at initial time (transfer) and kept constant until final time

Table 4 shows the calculated values of the transformed section coefficients K_{id} and K_{if} for each beam at each of the three levels of prestressing. These values can conveniently be used in early conceptual design stages. The upper bounds of K_{id} and K_{if} for the different girder sections are as follows:

Table 4. Transformed section coefficients K_{id} and K_{if}

	BT-54	BT-72	NU1100	NU1600	NU2000	BIII-48	BI-48	IT600	SIV-48
Creep ψ_{bid}	0.848	0.848	1.084	1.084	1.084	1.019	1.030	1.011	1.015
Creep ψ_{bif}	1.123	1.123	1.556	1.556	1.556	1.463	1.478	1.406	1.457
Low prestressing									
K_{id}	0.861	0.850	0.825	0.808	0.813	0.895	0.882	0.914	0.945
K_{df}	0.855	0.843	0.809	0.788	0.794	0.887	0.876	0.914	0.938
Medium prestressing									
K_{id}	0.814	0.817	0.808	0.802	0.809	0.854	0.854	0.895	0.92
K_{df}	0.805	0.807	0.789	0.782	0.789	0.843	0.846	0.895	0.91
High prestressing									
K_{id}	0.795	0.797	0.794	0.800	0.809	0.824	0.821	0.839	0.896
K_{df}	0.783	0.785	0.772	0.777	0.788	0.809	0.81	0.834	0.884
Average K_{id}	0.823	0.821	0.809	0.803	0.810	0.858	0.852	0.883	0.920
Average K_{df}	0.814	0.812	0.790	0.782	0.790	0.846	0.844	0.881	0.911

Note: K_{id} = transformed section coefficient that accounts for the interaction between concrete and steel between prestress transfer and deck placement times; K_{df} = transformed section coefficient that accounts for the interaction between concrete and steel between deck placement and final times; ψ_{bid} = girder creep coefficient between initial (transfer) and deck placement times; ψ_{bif} = girder creep coefficient due to sustained load applied at initial time (transfer) and kept constant until final time.

- bulb tee: $K_{id} = 0.823$ and $K_{df} = 0.814$
- I-girder: $K_{id} = 0.810$ and $K_{df} = 0.790$
- box beam: $K_{id} = 0.858$ and $K_{df} = 0.846$
- inverted tee: $K_{id} = 0.883$ and $K_{df} = 0.881$
- slab beam: $K_{id} = 0.920$ and $K_{df} = 0.911$

Figures 1 and **2** show summary charts that display the values of K_{id} and K_{df} for high, medium, and low levels of prestressing for the nine beams examined. The values of the transformed section coefficients for the same beam are consistently lower for higher levels of prestressing. Furthermore, they vary substantially based on the geometry and configuration of each beam.

Study 1: Concrete multiplier for creep loss N_c

This study evaluated the concrete creep multiplier N_c, which is taken equal to 10 in the first term of the approximate formula for estimating time-dependent losses, AASHTO Eq. (5.9.3.3-1). Relative humidity and concrete compressive strength factors used in the approximate formula are shown in AASHTO Eq. (5.9.3.3-2) and (5.9.3.3-3), respectively:

$$\Delta f_{pLT} = 10\left(\frac{f_{pi}A_{ps}}{A_g}\right)\gamma_h\gamma_{st} + 12\gamma_h\gamma_{st} + 2.4 \quad \text{(AASHTO 5.9.3.3-1)}$$

where

Δf_{pLT} = total long-term prestress loss that occurs between initial time and final condition

f_{pi} = stress in prestressing strands immediately before initial transfer

γ_h = correction factor for relative humidity of the ambient air

γ_{st} = correction factor for specified concrete strength at initial time

$$\gamma_h = 1.7 - 0.01H \quad \text{(AASHTO 5.9.3.3-2)}$$

where

H = average annual ambient relative humidity percentage

$$\gamma_{st} = \frac{5}{1 + f'_{ci}} \quad \text{(AASHTO 5.9.3.3-3)}$$

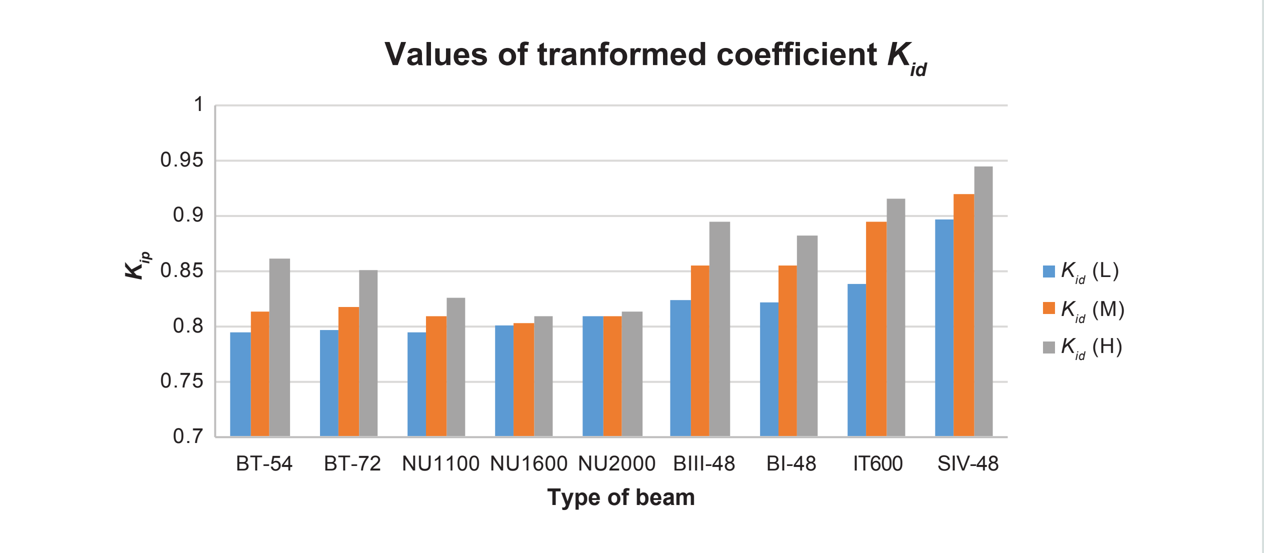

Figure 1. Values of transformed coefficient K_{id} from the time of transfer to the time of deck placement. Note: (H) = high levels of prestressing; K_{id} = transformed-section coefficient that accounts for the interaction between concrete and steel between prestress transfer and deck placement times; (L) = low levels of prestressing; (M) = medium levels of prestressing.

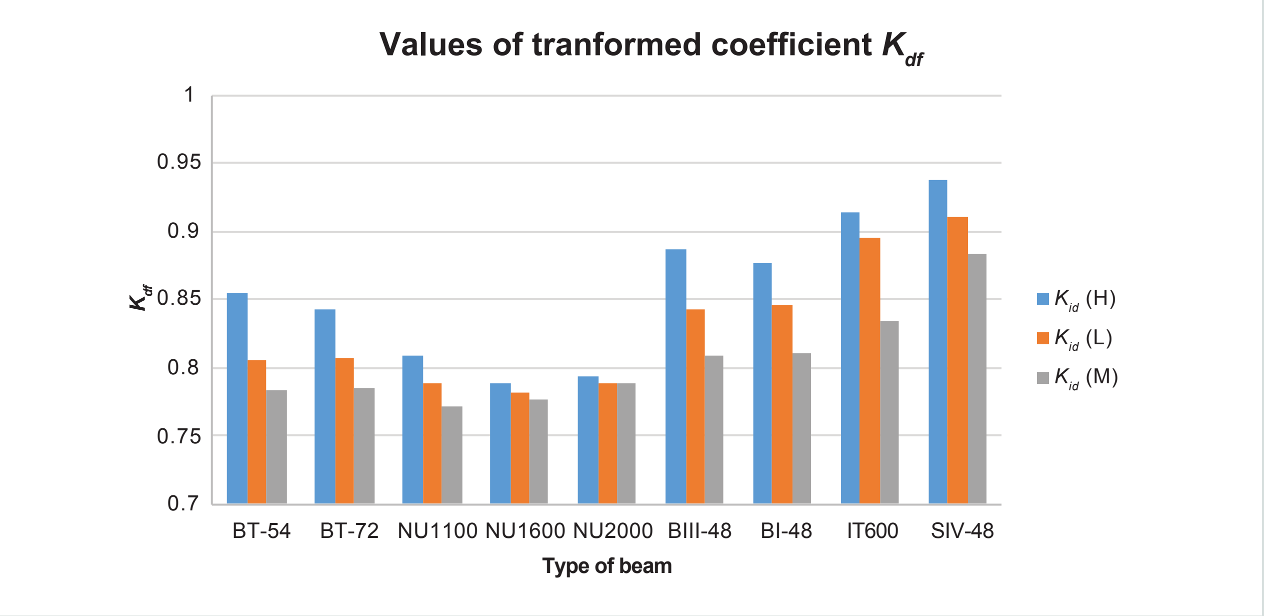

Figure 2. Values of transformed coefficient K_{df} from the time of deck placement to the final time. Note: (H) = high levels of prestressing; K_{df} = transformed-section coefficient that accounts for the interaction between concrete and steel between deck placement and final times; (L) = low levels of prestressing; (M) = medium levels of prestressing.

where

f'_{ci} = specified compressive strength of concrete at transfer

Tables 2 and 3 show the material properties and design parameters for the nine sections examined. The effects of girder type and level of prestressing on concrete creep losses were assessed.

The analysis started with computing creep losses at the three levels of prestressing for each of the nine cross sections using the AASHTO LRFD specifications' refined estimate of time-dependent losses. The value of creep loss is influenced by the cross-section configuration, magnitude and duration of stress, creep coefficient of concrete, level of prestressing, and maturity of concrete at the time of loading. Equation (1) was used for computing the total creep component of the long-term prestress losses.

$$\Delta f_{pC} = \Delta f_{pCR} + \Delta f_{pCD1} + \Delta f_{pCD2} \quad (1)$$

where

Δf_{pC} = total creep component of long-term prestress loss between initial (transfer) and final times

Δf_{pCR} = creep component of long-term prestress loss between initial (transfer) and deck placement times

Δf_{pCD1} = creep component of long-term prestress loss between deck placement and final times due to initial loads

Δf_{pCD2} = creep component of long-term prestress loss between deck placement and final times due to deck weight and superimposed dead load

Table 5 shows the range of the creep loss values based on the three levels of prestressing for the nine girder cross sections. When the AASHTO LRFD specifications approximate estimate is used, the total long-term creep prestress loss Δf_{pC} is represented by the first term of AASHTO Eq. (5.9.3.3-1) with N_c equal to 10, or as follows:

$$\Delta f_{pC} = N_c \frac{f_{pi} A_{ps}}{A_g} \gamma_h \gamma_{st} \quad (2)$$

The concrete creep multiplier can, therefore, be calculated by rearranging Eq. (2) as follows:

$$N_c = \frac{\Delta f_{pC} A_g}{f_{pi} A_{ps} \gamma_h \gamma_{st}} \quad (3)$$

For BT-54 with a low level of prestressing and section properties and area of prestressing strands shown in **Table 6**, the total long-term creep prestress loss Δf_{pC} is equal to 18.27 ksi (126.0 MPa). The value of N_c is calculated using the section

Table 5. Computed values of long-term prestress losses due to creep

Level of prestressing	BT-54	BT-72	NU1100	NU1600	NU2000	BIII-48	BI-48	IT600	SIV-48
Low									
Δf_{pCR}, ksi	10.47	11.72	17.14	17.92	17.35	9.25	10.72	7.64	5.10
Δf_{pCD1}, ksi	3.37	3.37	7.31	7.61	7.38	3.99	4.64	2.98	2.21
Δf_{pCD2}, ksi	4.44	3.77	6.20	7.22	6.70	3.52	4.07	2.47	0.80
Total creep Δf_{pC}, ksi	18.27	19.26	30.66	32.76	31.43	16.75	19.43	13.09	8.11
Medium									
Δf_{pCR}, ksi	14.36	14.37	17.40	17.80	16.44	12.86	13.24	8.48	6.94
Δf_{pCD1}, ksi	4.60	4.60	7.39	7.55	6.98	5.52	5.71	3.31	2.99
Δf_{pCD2}, ksi	5.14	4.35	6.67	7.28	7.38	4.63	4.89	3.71	1.37
Total creep Δf_{pC}, ksi	24.10	23.31	31.46	32.64	30.80	23.01	23.84	15.50	11.30
High									
Δf_{pCR}, ksi	16.29	15.87	17.14	16.56	14.85	13.06	16.35	13.26	8.61
Δf_{pCD1}, ksi	5.20	5.06	7.26	7.01	6.29	5.58	7.02	5.15	3.70
Δf_{pCD2}, ksi	5.01	4.70	7.13	6.37	6.27	7.70	5.58	5.22	1.93
Total creep Δf_{pC}, ksi	26.50	25.62	31.53	29.95	27.41	26.34	28.95	23.63	14.24

Note: Δf_{pC} = total creep component of long-term prestress loss between initial (transfer) and final times; Δf_{pCD1} = creep component of long-term prestress loss between deck placement and final times due to initial loads; Δ_{fpCD2} = creep component of long-term prestress loss between deck placement and final times due to deck weight and superimposed dead load; Δf_{pCR} = creep component of long-term prestress loss between initial (transfer) and deck placement times. 1 ksi = 6.895 MPa.

Table 6. Bulb tee BT-54 section properties at initial time

	Gross section	Transformed section
Area, in.^{2}	659	691
Area of prestressing strands, in.^{2}	5.208	5.208
Centroid of strands from the bottom fiber, in.	27.63	26.54
Moment of inertia, in.^{4}	268,077	284,820
Eccentricity of strands, in.	23.41	22.32

Note: 1 in. = 25.4 mm; 1 in.^{2} = 645.2 mm^2; 1 in.^{4} = 416,231 mm^4.

properties and the design parameters shown in Tables 2 and 3:

$$N_c = \frac{18.27(659)}{202.5(5.208)0.556} = 20.5$$

N_c is taken as 10 in the AASHTO LRFD specifications' approximate estimate equation, which is quite different from the computed value of 20.5 for a BT-54 with a low level of prestressing. **Table 7** shows all calculated values of the creep multiplier N_c for the different beams considered. The computed values of N_c are consistently higher for low levels of prestressing than for high levels of prestressing. The values of N_c range from 16.1 to 20.5 for bulb tees, with an average of 18.2; from 17.0 to 21.3, with an average of 19.4, for NU I-girders; from 19.5 to 24.8, with an average of 22.4, for box beams; from 15.6 to 19.5, with an average of 17.8, for inverted tees; and from 21.0 to 23.9, with an average of 22.4, for slab beams. Again, this is in contrast to the corresponding constant value of 10 used in the AASHTO LRFD specifica-

Table 7. Parametric study creep multiplier N_c

Level of prestressing	BT-54	BT-72	NU1100	NU1600	NU2000	BIII-48	BI-48	IT600	SIV-48
Low									
Creep loss, ksi	18.27	19.26	30.66	32.76	31.43	16.75	19.43	13.09	8.11
H	70	70	70	70	70	70	70	70	70
γ_h	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
γ_{st}	0.556	0.556	0.769	0.769	0.769	0.769	0.769	0.667	0.769
$\gamma_h\gamma_{st}$	0.556	0.556	0.769	0.769	0.769	0.769	0.769	0.667	0.769
A_{ps}/A_g	0.0079	0.0085	0.0093	0.0102	0.0095	0.0043	0.0051	0.0050	0.0022
N_c	20.5	20.2	21.3	20.6	21.3	24.8	24.5	19.5	23.9
Medium									
Creep loss, ksi	24.10	23.31	31.46	32.64	30.80	23.01	23.84	15.5	11.30
$\gamma_h\gamma_{st}$	0.556	0.556	0.769	0.769	0.769	0.769	0.769	0.667	0.769
A_{ps}/A_g	0.0119	0.0113	0.0106	0.0106	0.0098	0.0066	0.0069	0.0062	0.0033
N_c	18.1	18.3	19.1	19.8	20.1	22.4	22.3	18.4	22.2
High									
Creep loss, ksi	26.50	25.62	31.53	29.95	27.41	26.34	28.95	23.63	14.24
$\gamma_h\gamma_{st}$	0.556	0.556	0.769	0.769	0.769	0.769	0.769	0.667	0.769
A_{ps}/A_g	0.0145	0.0142	0.0119	0.0109	0.0098	0.0087	0.0091	0.0112	0.0044
N_c	16.3	16.1	17.0	17.6	17.9	19.5	20.5	15.6	21.0
Average N_c	18.2	18.2	19.1	19.3	19.8	22.3	22.5	17.8	22.4
AASHTO LRFD multiplier	10	10	10	10	10	10	10	10	10

Note: AASHTO LRFD = American Association of State Highway and Transportation Officials' *AASHTO LRFD Bridge Design Specifications*; A_g = area of gross section of girder; A_{ps} = area of prestressing steel; H = average annual ambient relative humidity; N_c = concrete creep multiplier; γ_h = correction factor for relative humidity of the ambient air; γ_{st} = correction factor for specified concrete strength at initial time. 1 ksi = 6.895 MPa.

tions' approximate estimate of time-dependent losses, which does not incorporate the effects of the cross-sectional configuration of beams and level of prestressing.

Figure 3 presents a summary chart of the values of N_c for high, medium, and low levels of prestressing of the four girder types and slab. This type of chart can be useful to designers at the early stage of design.

Study 2: Concrete multiplier for shrinkage loss N_s

Study 2 adopted the same parameters used in study 1. The main objective of study 2 was to evaluate the value of 12 that represents the concrete shrinkage multiplier N_s in the second term of AASHTO Eq. (5.9.3.3-1) for the AASHTO LRFD specifications' approximate estimate of time-dependent losses.

The change in concrete stress at the centroid of prestressing steel due to shrinkage was calculated using the AASHTO LRFD specifications' refined estimate of time-dependent losses for the three levels of prestressing of each of the nine cross sections. The prestress loss due to shrinkage is a function of the cross-section configuration, shrinkage strain of concrete, level of prestressing, ambient relative humidity, and concrete compressive strength. Equation (4) was used to compute the total shrinkage component of the long-term prestress losses:

$$\Delta f_{pS} = \Delta f_{pSR} + \Delta f_{pSD} + \Delta f_{pSS} \tag{4}$$

where

Δf_{pS} = total shrinkage component of the long-term prestress loss that occurs between initial (transfer) and final times

Δf_{pSD} = shrinkage component of the long-term prestress loss that occurs between deck placement and final time

Δf_{pSR} = shrinkage component of the long-term prestress loss that occurs between initial (transfer) and deck placement times

Δf_{pSS} = deck-slab shrinkage component of long-term prestress loss that occurs between deck placement and final time

Another departure in this study from the AASHTO LRFD specifications is in the computation of Δf_{pSS}. Differential shrinkage between the cast-in-place deck and the precast concrete girder $(\varepsilon_{ddf} - \varepsilon_{bdf})$ should be used in the calculation of Δf_{pSS} instead of the shrinkage of the deck concrete for the same time period ε_{ddf} because girder shrinkage has already been accounted for in the long-term loss. The change in concrete stress at the level of the strands' centroid was, therefore, computed using Eq. (5), which is a modified version of AASHTO Eq. (5.9.3.4.3d-2):

$$\Delta f_{cdf} = \frac{\left(\varepsilon_{ddf} - \varepsilon_{bdf}\right) A_d E_{cd}}{\left(1 + 0.7\psi_{ddf}\right)} \left(\frac{1}{A_c} - \frac{e_{pc} e_d}{I_c}\right) \tag{5}$$

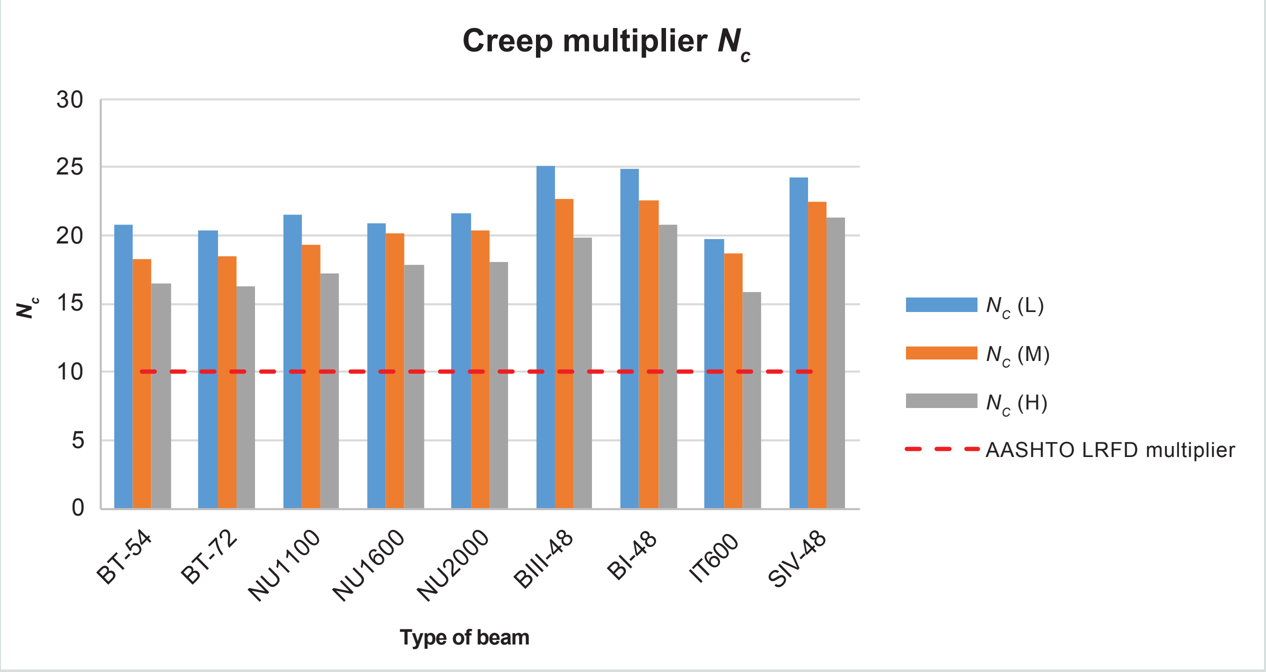

Figure 3. Values of creep multiplier N_c. Note: AASHTO LRFD = American Association of State Highway and Transportation Officials' *AASHTO LRFD Bridge Design Specifications*; (H) = high levels of prestressing; (L) = low levels of prestressing; (M) = medium levels of prestressing; N_c = creep multiplier.

where

Δf_{cdf} = change in concrete stress at the level of the strands' centroid between deck placement and final times

ε_{ddf} = shrinkage strain of deck between deck placement and the final time

ε_{bdf} = shrinkage strain of girder between deck placement and final time

A_d = area of concrete deck

E_{cd} = modulus of elasticity of deck concrete at service

ψ_{ddf} = creep coefficient of deck due to a sustained load applied at deck placement time t_d and kept constant until final time t_f

t_d = age of concrete at time of deck placement

t_f = age of concrete at final time of load application

e_d = eccentricity of deck with respect to transformed composite section at the time of application of superimposed dead load

This study used the Eq. (5) where differential shrinkage ($\varepsilon_{ddf} - \varepsilon_{bdf}$) was used instead of deck shrinkage ε_{ddf} for the period between deck placement and final time.

Table 8 shows the range of shrinkage loss values based on the three levels of prestressing for the nine girder cross sections examined in this study.

When the AASHTO LRFD specifications' approximate estimate is used, the total long-term shrinkage prestress loss Δf_{pS} is represented by the second term of AASHTO Eq. (5.9.3.3-1) with N_s equal to 12 or as follows:

$$\Delta f_{pS} = N_s \gamma_h \gamma_{st}$$

Therefore, the concrete shrinkage multiplier can be computed using Eq. (6).

$$N_s = \frac{\Delta f_{pS}}{\gamma_h \gamma_{st}} \tag{6}$$

For beam BT-54 with a low level of prestressing, the total long-term shrinkage prestress loss Δf_{pS} is equal to 8.25 ksi (56.9 MPa). The value of N_s can, therefore, be calculated using the section properties and the design parameters shown in Tables 2 and 3.

$$N_s = \frac{8.25}{0.556} = 14.8$$

Table 8. Computed values of long-term prestress losses due to shrinkage

Level of prestressing	BT-54	BT-72	NU1100	NU1600	NU2000	BIII-48	BI-48	IT600	SIV-48
Low									
Δf_{pSR}, ksi	5.36	5.29	6.57	6.43	6.47	6.70	6.67	6.79	7.05
Δf_{pSD}, ksi	1.72	1.70	2.80	2.73	2.75	2.89	2.89	2.65	3.05
Δf_{pSS}, ksi	1.16	1.10	0.91	0.87	0.92	1.30	1.31	1.03	0.84
Total shrinkage loss Δf_{pS}, ksi	8.25	8.09	10.28	10.03	10.14	10.88	10.86	10.47	10.93
Medium									
Δf_{pSR}, ksi	5.07	5.09	6.43	6.39	6.44	6.39	6.46	6.64	6.86
Δf_{pSD}, ksi	1.62	1.63	2.73	2.71	2.73	2.75	2.78	2.60	2.96
Δf_{pSS}, ksi	0.97	0.96	0.79	0.83	0.86	1.13	1.15	1.01	0.81
Total shrinkage loss Δf_{pS}, ksi	7.67	7.68	9.96	9.92	10.03	10.27	10.39	10.25	10.62
High									
Δf_{pSR}, ksi	4.95	4.97	6.32	6.37	6.44	6.17	6.21	6.23	6.68
Δf_{pSD}, ksi	1.58	1.58	2.68	2.69	2.73	2.64	2.67	2.42	2.87
Δf_{pSS}, ksi	0.79	0.75	0.67	0.68	0.73	0.98	1.02	0.77	0.79
Total shrinkage loss Δf_{pS}, ksi	7.32	7.30	9.67	9.74	9.90	9.79	9.90	9.42	10.34

Note: Δf_{pS} = total shrinkage component of the long-term prestress loss that occurs between initial (transfer) and final times; Δf_{pSD} = shrinkage component of the long-term prestress loss that occurs between deck placement and final time; Δf_{pSR} = shrinkage component of the long-term prestress loss that occurs between initial (transfer) and deck placement times; Δf_{pSS} = deck-slab shrinkage component of long-term prestress loss that occurs between deck placement and final time. 1 ksi = 6.895 MPa.

N_s is taken as 12 in the AASHTO LRFD specifications' approximate estimate equation, which is quite different from the estimated value of 14.8 when the total long-term shrinkage loss is estimated using the AASHTO LRFD specifications' refined estimate.

Table 9 illustrates the values of the shrinkage multiplier N_s. The same results are also shown in **Fig. 4**. The values of N_s are consistently higher for low levels of prestressing than for high levels of prestressing when the concrete compressive strength and the ambient relative humidity are constants. The values of N_s range from 13.1 to 14.9, with an average of 13.9, for bulb tees; from 12.6 to 13.4, with an average of 13.0, for NU I-girders; from 12.7 to 14.1, with an average of 13.5, for box beams; from 14.1 to 15.7, with an average of 15.1, for inverted tees; and from 13.4 to 14.2, with an average of 13.8, for slab beams. This is in contrast with the corresponding constant value of 12 used in the AASHTO LRFD specifications' approximate estimate of time-dependent losses, which does not consider the effects of the cross-section configuration of the beams.

Figure 4 shows a summary chart of the values of N_s for high, medium, and low levels of prestressing for the nine beam types.

Proposed lump-sum formulas

The following proposed lump-sum formulas represent the average conditions of time-dependent losses due to creep and shrinkage of concrete, and relaxation of steel prestressing strands for each of the beams examined.

Bulb tee:

$$\Delta f_{pLT} = 19.6\left(\frac{f_{pi}A_{ps}}{A_g}\right)\gamma_h\gamma_{st} + 14.4\gamma_h\gamma_{st} + 2.4$$

I-girder:

$$\Delta f_{pLT} = 20.5\left(\frac{f_{pi}A_{ps}}{A_g}\right)\gamma_h\gamma_{st} + 13.2\gamma_h\gamma_{st} + 2.4$$

Box beam:

$$\Delta f_{pLT} = 23.8\left(\frac{f_{pi}A_{ps}}{A_g}\right)\gamma_h\gamma_{st} + 13.8\gamma_h\gamma_{st} + 2.4$$

Inverted tee:

$$\Delta f_{pLT} = 18.9\left(\frac{f_{pi}A_{ps}}{A_g}\right)\gamma_h\gamma_{st} + 15.4\gamma_h\gamma_{st} + 2.4$$

Slab beam:

$$\Delta f_{pLT} = 23.4\left(\frac{f_{pi}A_{ps}}{A_g}\right)\gamma_h\gamma_{st} + 14.0\gamma_h\gamma_{st} + 2.4$$

The lump-sum formulas are useful for computing the time-dependent losses in the preliminary design but the estimated loss should be recalculated in the final design.

Conclusion

Based on the results of these parametric studies, the following conclusions are drawn:

Table 9. Parametric study shrinkage multiplier N_s

Level of prestressing	BT-54	BT-72	NU1100	NU1600	NU2000	BIII-48	BI-48	IT600	SIV-48
Low									
Shrinkage loss, ksi	8.25	8.09	10.28	10.03	10.14	10.88	10.86	10.47	10.93
$\gamma_h\gamma_{st}$	0.556	0.556	0.769	0.769	0.769	0.769	0.769	0.667	0.769
N_s	14.9	14.6	13.4	13.0	13.2	14.1	14.1	15.7	14.2
Medium									
Shrinkage loss, ksi	7.67	7.68	9.96	9.92	10.03	10.27	10.39	10.25	10.62
$\gamma_h\gamma_{st}$	0.556	0.556	0.769	0.769	0.769	0.769	0.769	0.667	0.769
N_s	13.8	13.8	12.9	12.9	13.0	13.4	13.5	15.4	13.8
High									
Shrinkage loss, ksi	7.32	7.30	9.67	9.74	9.90	9.79	9.90	9.42	10.34
$\gamma_h\gamma_{st}$	0.556	0.556	0.769	0.769	0.769	0.769	0.769	0.667	0.769
N_s	13.2	13.1	12.6	12.7	12.9	12.7	12.9	14.1	13.4
Average N_s	13.9	13.8	13.0	12.9	13.0	13.4	13.5	15.1	13.8
AASHTO LRFD multiplier	12	12	12	12	12	12	12	12	12

Note: AASHTO LRFD = American Association of State Highway and Transportation Officials' *AASHTO LRFD Bridge Design Specifications*; N_s = concrete shrinkage multiplier; γ_h = correction factor for relative humidity of the ambient air; γ_{st} = correction factor for specified concrete strength at initial time. 1 ksi = 6.895 MPa.

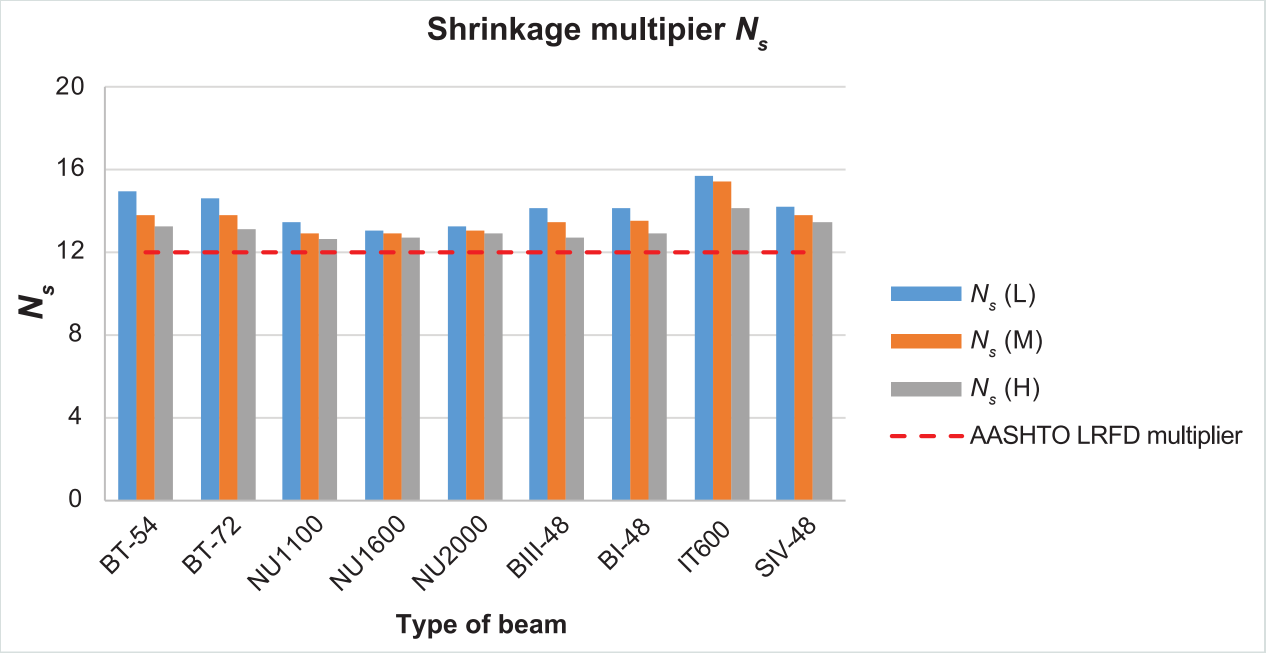

Figure 4. Values of shrinkage multiplier N_s. Note: AASHTO LRFD = American Association of State Highway and Transportation Officials' *AASHTO LRFD Bridge Design Specifications*; (H) = high levels of prestressing; (L) = low levels of prestressing; (M) = medium levels of prestressing; N_s = shrinkage multiplier.

- The values of K_{id} and K_{df} vary substantially based on the geometry and configuration of each beam.
- The values of K_{id} and K_{df} for the same beam cross section are consistently lower for higher levels of prestressing.
- The AASHTO LRFD specifications' approximate estimate of time-dependent losses does not reflect the effect of beam configuration on the creep and shrinkage multipliers.
- The computed creep multiplier N_c in the parametric study is significantly higher than that of the AASHTO LRFD specifications' approximate estimate of 10.
- The value of the creep multiplier N_c decreases with an increased level of prestressing and vice versa. This trend is observed in all nine beams analyzed in this study.
- The value of the shrinkage multiplier N_s decreases slightly with increased levels of prestressing and vice versa. This trend is observed in all the nine beams analyzed in this study.
- The computed shrinkage multiplier N_s in the parametric study is close to that of the AASHTO LRFD specifications' approximate estimate of 12.
- Five proposed lump-sum formulas have been presented for estimating long-term prestress losses that account for the effect of cross-section type for commonly used beams.

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Notation

A_c = area of composite section at service

A_d = area of concrete deck

A_g = area of gross section of girder

A_{ps} = area of prestressing steel

e_d = eccentricity of deck with respect to transformed composite section at the time of application of superimposed dead load

e_{pc} = eccentricity of strands with respect to centroid of composite section at service, always taken as positive

e_{pg} = eccentricity of strands with respect to centroid of gross section of girder, always taken as positive

E_c = 28-day modulus of elasticity of girder concrete

E_{cd} = modulus of elasticity of deck concrete at service

E_{ci} = modulus of elasticity of girder concrete at transfer

E_d = modulus of elasticity of deck concrete

E_p = modulus of elasticity of prestressing steel

f'_c = specified 28-day compressive strength of concrete

f'_{ci} = specified compressive strength of concrete at transfer

f_d = specified compressive strength of deck concrete

f_{pi} = stress in prestressing strands immediately before initial transfer

f_{pu} = ultimate stress of the prestressing strands

f_{py} = yield stress of prestressing steel

h = depth of concrete girder

H = average annual ambient relative humidity percentage

I_c = moment of inertia of composite section at service

I_g = moment of inertia of gross section of girder

K_{df} = transformed-section coefficient that accounts for the interaction between concrete and steel between deck placement and final times

K_{id} = transformed-section coefficient that accounts for the interaction between concrete and steel between prestress transfer and deck placement times

N_c = concrete creep multiplier

N_s = concrete shrinkage multiplier

t_d = age of concrete at time of deck placement

t_f = age of concrete at final time of load application

t_i = age of concrete at time of initial loading at transfer

V/S = volume-to-surface ratio of girder

y_b = eccentricity of bottom fibers with respect to centroid of gross girder section

γ_h = correction factor for relative humidity of the ambient air

γ_{st} = correction factor for specified concrete strength at initial time

Δf_{cdf} = change in concrete stress at the level of the strands' centroid between deck placement and final times

Δf_{pC} = total creep component of long-term prestress loss between initial (transfer) and final times

Δf_{pCD1} = creep component of long-term prestress loss between deck placement and final times due to initial loads

Δf_{pCD2} = creep component of long-term prestress loss between deck placement and final times due to deck weight and superimposed dead load

Δf_{pCR} = creep component of long-term prestress loss between initial (transfer) and deck placement times

Δf_{pLT} = total long-term prestress loss that occurs between initial time and final condition

Δf_{pS} = total shrinkage component of the long-term prestress loss that occurs between initial (transfer) and final times

Δf_{pSD} = shrinkage component of the long-term prestress loss that occurs between deck placement and final time

Δf_{pSR} = shrinkage component of the long-term prestress loss that occurs between initial (transfer) and deck placement times

Δf_{pSS} = deck-slab shrinkage component of long-term prestress loss that occurs between deck placement and final time

ε_{bdf} = shrinkage strain of girder between deck placement and final time

ε_{ddf} = shrinkage strain of deck between deck placement and the final time

ψ_{bid} = girder creep coefficient between initial (transfer) and deck placement times

ψ_{bif} = girder creep coefficient due to sustained load applied at initial time (transfer) and kept constant until final time

ψ_{ddf} = creep coefficient of deck due to a sustained load applied at deck placement time t_d and kept constant until final time t_f

About the author

Nabil Al-Omaishi, PhD, PEng, PE, is a professor of civil engineering at the College of New Jersey in Ewing, N.J. He received his doctorate in structural engineering from the University of Nebraska–Lincoln, his master of engineering from the University of Toronto in Canada, and his BSc in civil engineering from the University of Baghdad in Iraq. Al-Omaishi was involved in the research work of National Cooperative Highway Research Program report 496, *Prestress Losses in Pretensioned High-Strength Concrete*, which was subsequently adopted by the American Association of State Highway and Transportation Officials' *AASHTO LRFD Bridge Design Specifications*. Al-Omaishi was the first author of the research paper that won the American Society of Civil Engineers' T. Y. Lin Award in 2011. His research interests include prestress losses and long-term properties of concrete materials.

Abstract

The current American Association of State Highway and Transportation Officials' *AASHTO LRFD Bridge Design Specifications'* approximate formula for estimating long-term prestress losses is the outcome of the research work presented in National Cooperative Highway Research Program report 496. It is produced by simplifying the detailed method and taking into account the variability of concrete properties and the interaction between the precast concrete girder and cast-in-place deck. This paper presents two detailed parametric studies based on the average conditions for the design and construction of commonly used bridge girders. Three spans and, consequently, three levels of prestressing for each section have been considered. The first study establishes the creep multiplier N_c, whereas the second study evaluates the shrinkage multiplier N_s. Both multipliers are used in the lump-sum formulas for estimating long-term prestress losses for different bridge girders. The multipliers produced by these studies are compared with that of the current AASHTO LRFD specifications' approximate method, and new lump-sum formulas for long-term prestress losses are proposed.

Keywords

Creep loss, prestress loss, pretensioned girder, shrinkage loss.

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