Comparison of the design of composite prestressed concrete hollow-core floor units between Eurocode 2 and ACI 318

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- The second in a two-part series, this paper compares *Eurocode 2: Design of Concrete Structures, Part 1-1: General Rules and Rules for Buildings (with National Application parameters)* and the American Concrete Institute's *Building Code Requirements for Structural Concrete (ACI 318-08) and Commentary (ACI 318R-08)* methodologies for composite hollow-core floor slabs with a structural topping application.
- The first paper focused on the noncomposite design of the hollow-core unit. This second paper reviewed the hollow-core unit designed as a composite slab with a deep structural topping.
- The methodology comparison included two-stage composite service and ultimate moments of resistance, ultimate composite shear capacities, camber and deflections, and the interface shear between the hollow-core unit and cast-in-place concrete topping.

This is the second of two papers[1] presenting procedures, equations, and design examples to compare *Eurocode 2: Design of Concrete Structures, Part 1-1: General Rules and Rules for Buildings (with National Application parameters)* (EC2-1-1)[2] and the American Concrete Institute's *Building Code Requirements for Structural Concrete (ACI 318-08) and Commentary (ACI 318R-08)*[3] methodologies for composite hollow-core floor slabs with a structural topping application. The first paper dealt with the noncomposite design of the hollow-core unit alone.

In the examples in this paper, the hollow-core unit is designed as a composite slab with a 75 mm (3 in.) deep structural topping. The unit is assumed to be 1200 mm (48 in.) wide × 200 mm (8 in.) deep, and pretensioned using four 9.3 mm (0.37 in.) diameter strands and six 12.5 mm (0.49 in.) diameter strands with a characteristic strength f_{pk} of 1770 N/mm^2 (256.7 ksi). The slab is simply supported over an effective span of 8.0 m (26 ft) to carry imposed dead uniformly distributed loading of 1.5 kN/m^2 (0.23 psi), the self-weight of the topping (allowing for 15 mm [0.6 in.] upward camber) of 1.98 kN/m^2 (0.3 psi) and live load (including 1 kN/m^2 [0.145 psi] for partitions) of 8.5 kN/m^2 (1.275 psi).

The hollow-core unit shown in cross section in **Fig. 1** is analyzed and designed in order to make a comparison between the procedures according to EC2-1-1[2] and the ACI 318.[3] The purpose of this paper is to compare the design methodologies as well as present standard calculations and worked

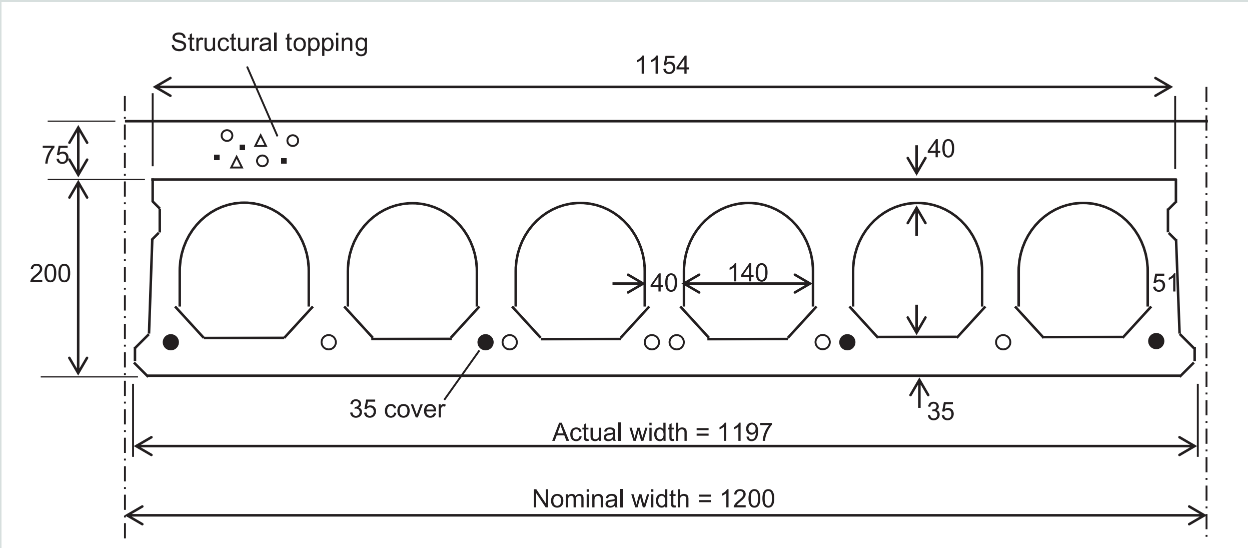

Figure 1 Cross-sectional hollow-core floor unit examined in this paper. Strands are represented by solid dots (four no. 9.3 mm diameter) and open dots (six no. 12.5 mm diameter). Note: All measurements shown in millimeters. 1 mm = 0.0394 in.

examples to serve as a future design reference.

In the design examples—following both EC2 and ACI 318 methodologies—the service moment of resistance is calculated in a two-stage analysis before and after hardening of the topping, the ultimate moment of resistance, and the ultimate composite shear capacities. The author then calculated the amount of precamber and short-term deflection at installation using the section properties of the precast concrete unit alone and then the final long-term deflection, and active deflection due to imposed loads using the composite section of the completed slab. Last, the interface shear between the topping and hollow-core unit is designed. Cover to pretensioning tendons for durability and fire resistance and material properties are the same as for the noncomposite design.1 Both EC2 and ACI 318 variables appear throughout the paper. For clarification between the two codes, see the notation at the end of the paper.

Composite construction of precast, prestressed concrete hollow-core floor slab

The structural capacity of precast concrete hollow-core units may be increased by adding a layer of structural reinforced concrete to the top of the unit. The preparation of a composite hollow-core slab with steel mesh and the completed topping is shown in **Fig. 2.** Providing that the topping concrete is fully anchored and bonded to the precast concrete unit, both precast and cast-in-place concrete may be designed as monolithic. The section properties of the precast concrete unit plus the topping are used to determine the structural performance of the composite floor. A composite floor may be made using any type of precast concrete unit, but clearly there is more to be gained from using voided prestressed units, such as hollow-core unit or double tee, which are lightweight and therefore cheaper to transport and erect than solid reinforced concrete units.

The minimum depth of the topping should not be less than 40 mm (1.6 in.) (a lower limit of 50 mm [2 in.] is more practical). There is no limit to the maximum depth, though 75 to 100 mm (3 to 4 in.) is a practical limit. When calculating the average depth of the topping, an allowance for the upward camber of the hollow-core unit should be made: a depth equal to the span of the slab divided by 300 will suffice. The 28-day compressive cylinder strength of cast-in-place concrete f_{ck} is usually 25 to 30 N/mm^2 (3625 to 4350 psi), but there is no reason why a higher strength cannot be used except that the increased strength of the composite floor resulting from the higher grade will not justify the additional costs of materials and quality control. The topping must be reinforced, but as explained later, there need only be tie steel at the interface between the precast concrete and cast-in-place topping if the design dictates. Steel mesh reinforcement Types A142 to A252 (142 to 252 mm^2/m [0.067 to 0.119 in.^{2}/ft] area of bars) are the preferred choice.

The main benefit from composite action is increased bending resistance and flexural stiffness; shear and bearing resistance are barely increased. There are a number of other reasons why a structural topping may be specified, such as to achieve the following:

- improve vibration and thermal and acoustic performance of the floor
- provide floor diaphragm action by using the topping as a thin horizontal plate[4]
- provide horizontal stability ties (using the mesh) across floors[4]

Figure 2 Hollow-core floor slabs (left) prepared with mesh for casting of the structural topping (right). The hollow-core unit should be dampened, without ponding, with water prior to casting the topping. The steel mesh should be lapped with reinforcing bars that are continuous through the construction joint.

- provide a continuous and monolithic floor (for example, where brittle finishes are applied)

Composite floor design is carried out in two stages: before (stage 1) and after (stage 2) the cast-in-place topping becomes structural. The hollow-core unit supports its own weight plus the self-weight of the wet cast-in-place concrete (plus a temporary construction traffic allowance of about 1.5 kN/m^2 [0.23 psi]). The composite floor, which is the hollow-core unit plus the hardened cast-in-place topping, carries imposed loads. In the final analysis, the stresses and forces resulting from the two cases (minus the construction traffic allowance) are additive.

The flexural capacity of the composite slab can be increased in certain situations by propping the hollow-core unit prior to casting the topping. Props must be rigid and well founded and may be placed at midspan or twin propped at $\frac{1}{3}$ span positions, often considered to be good practice in case one of the props is not fully rigid. The effect of twin propping on the ratio of design to structural capacity is small, about 2% greater than a single prop.

In Europe, the product standard *Precast Concrete Products—Hollow Core Slabs* (EN 1168)[5] provides additional normative rules and information for EC2-1-1[2] and *Eurocode 2: Design of Concrete Structures, Part 1-2: Structural Fire Design (with National Application Parameters)* (EC2-1-2).[6] In addition to the items listed in the previous paper,[1] in relation to composite hollow-core slabs, EN 1168 provides the following:

- normative rules for punching shear, torsion, localized point or line loads, splitting stresses due to prestress in narrow webs, and acoustic or thermal properties, fire resistance, and reference to EC2-1-1 for surface characteristics, such as roughness of the top surface

- information on the shear capacity of hollow-core slabs, stating that the stress in the webs due to stage 1 loads (self-weight plus topping) should be subtracted from the principal tension in the composite section, and fire resistance in shear

- lateral load distribution in composite slabs

- floor diaphragm action with a topping of at least a 40 mm (1.6 in.) depth

There are other rules, such as the following:

- the depth of the topping should not be greater than 0.4 times the depth of the hollow-core unit

- the minimum depth at the highest point at midspan should not be less than 50 mm (2 in.)

- due to precamber of the hollow-core unit, the depth of the topping at the support is greater than at the highest point at midspan by a depth equal to the span of the slab divided by 300 to 400, depending on the amount of prestress and curing conditions

The compressive cylinder strength f_{ckT} for the cast-in-place topping used in this paper is 25 N/mm^2 (3625 psi), equivalent to a cube strength greater than 30 N/mm^2 (4350 psi), which is the most common strength. In practice, strengths f_{ckT} of up to 35 N/mm^2 (5075 psi) (equivalent to cube strength greater than 45 N/mm^2 [6525 psi]) are possible.

Design procedures and equations per EC2-1-1

Definitions according to EC2-1-1

The following refers to the cross-section in **Fig. 3**:

- The depth of topping at the highest point at midspan, called the crown, is t.

- The average depth of topping t' allowing for net camber at installation δ_{ins} is $t + \delta_{ins}/2$.

- The density of concrete with a light mesh, such as A252 (4 kg/m^2) is taken as 2400 + 4/0.075 = 2450 kg/m^3 = 24 kN/m^3 (153 l_b/ft^3).

- The self-weight of the topping per unit width w_T is b $24t' \times 10^{-3}$.

- Young's modulus of cast-in-place topping E_{cmT} is $22(f_{ckT} + 8)^{0.3}$.

In the following procedures and throughout this paper, code references are given on the left, and the text/calculations/formulae are to the right.

Elastic section properties of composite compound area for service stress

The effective width of topping at service b_{ef} is b_{EcmT}/E_{cm}, where b is the nominal width of the hollow-core unit and E_{cm} is Young's modulus of concrete.

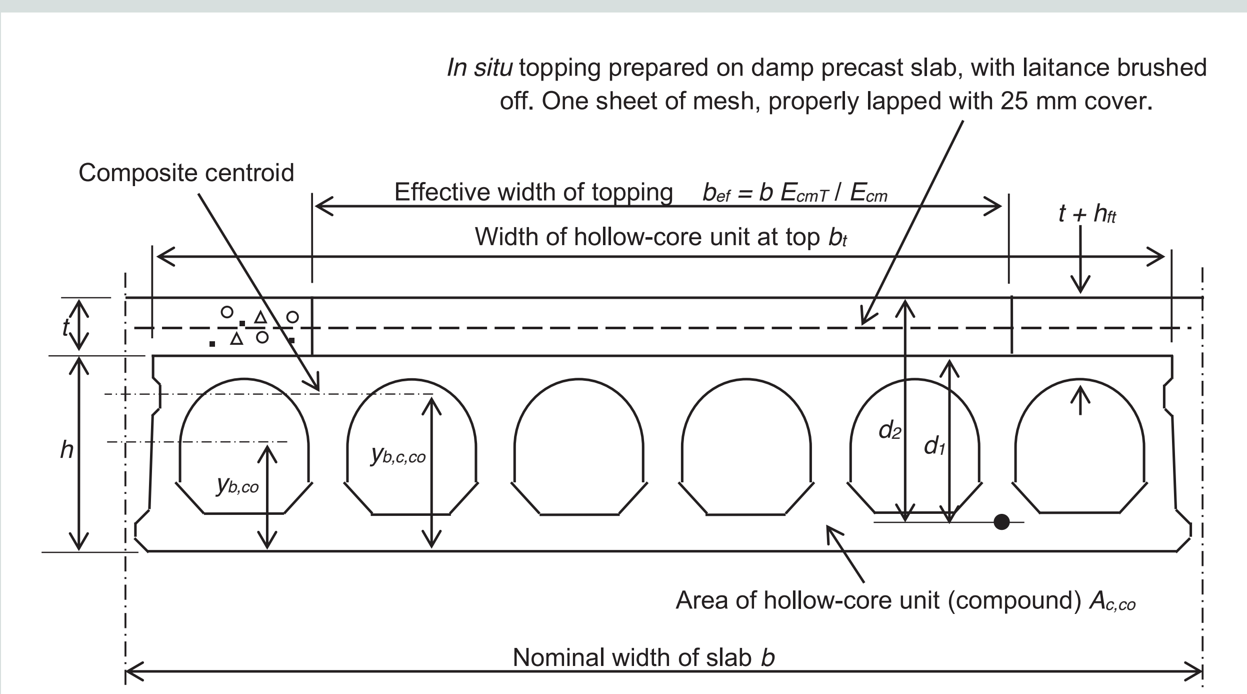

Figure 3 Geometry of the composite slab: cast-in-place concrete topping prepared on dampened precast concrete slab, with laitance brushed off, and one sheet of mesh properly lapped with 25 mm cover. The compound area and centroid heights include the transformed area of strands. Note: b_{ef} = effective width of cast-in-place concrete topping at service; d_1 = effective depth of tendons in tension zone at stage 1; d_2 = effective depth of tendons in tension zone at stage 2; E_{cm} = Young's modulus of concrete; E_{cmT} = Young's modulus of cast-in-place topping; h = depth of hollow-core unit; h_{ft} = depth of top flange; t = depth of topping; $y_{b,co}$ = height to centroid of compound section; and $y_{b,c,co}$ = height to centroid of composite compound section. 1 mm = 0.0394 in.

The composite compound area of concrete $A_{c,c,co}$ is $A_{c,co} + b_{ef}t$, where $A_{c,co}$ is the compound area of the hollow-core unit.

The height to composite compound centroid $y_{b,c,co}$ is $[A_{c,co}y_{b,co} + b_{ef}t(h + 0.5t)]/A_{c,c,co}$, where $y_{b,co}$ is the height to the compound centroid of the hollow-core unit (including the transformed area of strands) and h is the depth of the hollow-core unit.

The composite compound second moment of area $I_{c,c,co}$ is $I_{c,co} + A_{c,co}(y_{b,c,co} - y_{b,co})^2 + b_{ef}t^3/12 + b_{ef}t(h + 0.5t - y_{b,c,co})^2$, where $I_{c,co}$ is the second moment of area of the compound section.

Then the section modulus of the composite compound section at the bottom $Z_{b,c,co}$ is $I_{c,c,co}/y_{b,c,co}$, and the section modulus of the composite compound section at the top $Z_{t,c,co}$ is $I_{c,c,co}/(h - y_{b,c,co})$.

Analysis of service and ultimate moments of resistance and ultimate shear capacities

Serviceability limit state of bending per EC2-1-1

This is a two-stage analysis: before (stage 1) and after (stage 2) the topping has hardened.

Stage 1 maximum service moment M_{s1} is $(w_1 + w_T)\, L^2/8$, where w_1 is stage 1 service load due to the self-weight of the precast concrete unit plus infill and w_T is the self-weight of wet cast-in-place topping and L is the effective span of the composite hollow-core slab.

Stage 1 stress at the bottom f_{b1} is $M_{s1}/Z_{b,co}$ and at the top f_{t1} is $M_{s1}/Z_{t,co}$, where $Z_{b,co}$ is the section moment of the basic compound section at the bottom and $Z_{t,co}$ is the section moment of the basic compound section at the top.

The service stress available to resist stage 2 loading due to all imposed dead and live loads at the bottom $f_{b2} \le \sigma_b - f_{b1} - f_{ctm}$ (**Fig. 4**), where σ_b is the maximum surface stresses in service at the bottom and f_{ctm} is the permissible tension for exposure category XC1 (for dry or permanently wet concrete conditions). For exposure categories greater than XC1, that is XC2 to XS3, EC2-1-1 Table 7.1 states that the permissible tension should be zero. The terminology used is "decompression" meaning that the concrete should not be decompressed.

The service stress available to resist stage 2 loading due to all imposed dead and live loads at the top is $f_{t2} \le 0.45\, f_{ck} - \sigma_t - f_{t1}$, where σ_t is the maximum surface stresses in service at the top.

The service moment of resistance $M_{sr,c}$ is the lesser of $M_{s1} + f_{b2}Z_{b,c,co}$ or $M_{s1} + f_{t2}Z_{t,c,co}$.

These equations are not code dependent except for the limit of $0.45f_{ck}$.

Note that the service moment of resistance $M_{sr,c}$ decreases as the stage 1 moment increases and will, therefore, vary along the span. If stage 1 moment is large, then the available bending stress at the bottom due to stage 2 moment f_{b2} may be negative and is not allowed.

Ultimate limit state of bending per EC2-1-1

The ultimate moment of resistance for the composite slab $M_{Rd,c}$ is calculated for the net area of tendons in the tension zone A_{p2} as the remainder after subtracting the area required to cater for stage 1 loading due to the self-weight of the unit and the topping (including for the effects of propping) A_{p1} from the area of tendons A_p.

The following equations are derived or taken from EC2-1-1.[2]

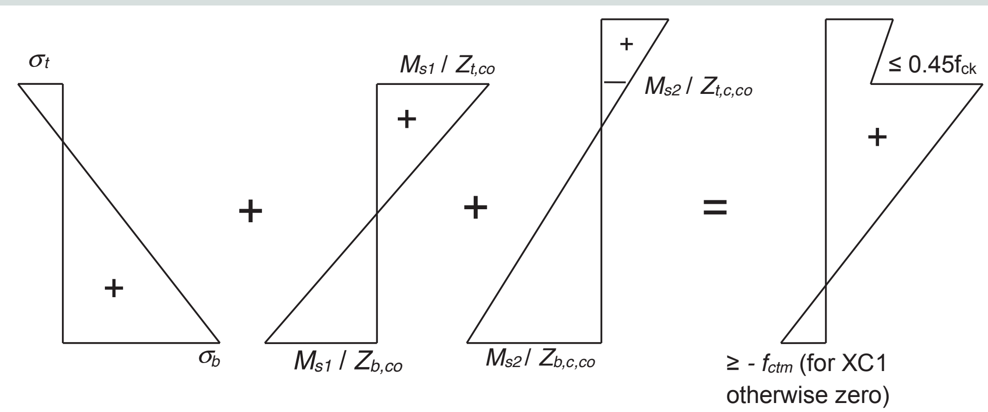

Figure 4 Prestress, service stresses and limits according to *Eurocode 2: Design of Concrete Structures, Part 1-1: General Rules and Rules for Buildings (with National Application parameters)*. Note: f_{ck} = 28-day characteristic cylinder strength; f_{ctm} = mean tensile strength of concrete; M_{s1} = service moment at stage 1; M_{s2} = service moment at stage 2; XC1 = exposure class for dry or permanently wet concrete; $Z_{b,co}$ = section modulus of basic compound section at bottom; $Z_{b,c,co}$ = section modulus of composite compound section at bottom; $Z_{t,co}$ = section modulus of basic compound section at top; $Z_{t,c,co}$ = section modulus of composite compound section at top; and σ_b = maximum surface stresses in service at bottom.

For stage 1, let $M_{Ed,1}$ = the ultimate moment due to self-weight of the slab plus topping (and propping), derived from clause 3.1.7:

3.1.7 $K_1 = M_{Ed,1}/f_{ck}(t_i)b_t d_1^2$

where

K_1 = bending moment factor at stage 1

$f_{ck}(t_i)$ = strength of precast concrete at installation at t_i days

t_i = installation age

b_t = actual width at top of hollow-core unit

d_1 = effective depth of tendons in tension zone at stage 1

Eq. (3.2) $f_{cm}(t_i) = \exp\left\{s\left[1-\sqrt{28/t_i}\right]\right\} f_{cm}$

where

$f_{cm}(t_i)$ = mean compressive strength at t_i days

The mean compressive strength at 28 days f_{cm} is f_{ck} + 8 N/mm^2.

The cement factor s is 0.2 (Class R), 0.25 (Class N), or 0.38 (Class S).

Lever arm at stage one z_1 is

$$z_1 = \left[0.5+\sqrt{\left(0.25-K_{1w}/1.134\right)}\right]d_1.$$

The neutral axis depth x_1 is $(d_1 - z_1)/0.4$.

If $0.8x_1 \le h_{ft}$, then $A_{p1} = M_{Ed,1}/z_1 f_{p1}$, where h_{ft} is the depth of the top flange and f_{p1} is stress in tendons at a limiting strain ε_s of 0.02.

Fig. 3.10 Where f_{p1} is stress in tendons at a limiting strain ε_s of 0.02.

3.1.7 If $0.8x_1 > h_{ft}$, the compression force is the sum of the force in the top flange above the cores and webs of width b_w.

Restart the analysis using the bending moment factor based on the width of the webs at stage 1

$K_{1w} = M_{Ed,1}/f_{ck}(t_i)b_w d_1^2$

$z_1 = \left[0.5+\sqrt{\left(0.25-K_{1w}/1.134-\beta\right)}\right]d_1$

where

β = the factor for the design of T-shaped sections = $(b_t - b_w)h_{ft}(d_1 - 0.5h_{ft})/(2b_w d_1^2)$

then $A_{p1} = M_{Ed,1}/z_1 f_{p1}$,

and the area available for stage 2 A_{p2} is $A_p - A_{p1}$.

The derivation for the lever arm at stage one z_1 and the factor for the design of T-shaped sections β is given in the appendix.

The ultimate moment of resistance for the composite slab at stage 2 based on an area of tendons A_{p2} is $M_{Rd,c2}$. It is calculated per the noncomposite procedures but the area available for stage 2 A_{p2} is used instead of the area of tendons A_p, the effective width of cast-in-place topping at ultimate b_{eff} is bf_{ckT}/f_{ck} instead of b_t, and the effective depth of tendons in the tension zone at stage 2 d_2 is used instead of the effective depth of tendons in the tension zone d.

$M_{Rd,c} = M_{Rd,c2} + M_{Ed,1}$

Where $M_{Ed,1} = 0$ (for example, at supports) $A_{p2} = A_p$ and let $M_{Rd,c2}$ = ultimate moment of resistance of the composite section at stage 2.

The variation of the ultimate moment of resistance of the composite section $M_{Rd,c}$ along the span is assumed to be a parabolic curve to reflect the variation in the ultimate design bending moment at stage 1 $M_{Ed,1}$. At a distance x from the support for an effective span L,

$$M_{Rd,c}(x) = M_{Rd,c} - [x/0.5L(2 - x/0.5L)(M_{Rd,c} - M'_{Rd,c})]$$

Ultimate limit state of shear per EC2-1-1

Section uncracked in flexure

Annex F in EN 1168[5] requires that a shear stress due to stage 1 loads $v_{Ed,1}$ of $V_{Ed,1}S_c/b_w I_c$ be subtracted from the principal tensile resistance f_{ctd}, where $V_{Ed,1}$ is the ultimate dead load acting on the unit, including the self-weight of wet topping; S_c is the first moment of area of the hollow-core unit; and I_c is the second moment of area of the basic section. The residual shear stress is multiplied by the shape factor $I_{c,c}b_w/S_{c,c}$ (where $I_{c,c}$ is the second moment of area of the composite section and $S_{c,c}$ is first moment of area for the composite section above the centroidal axis of the composite section [using ultimate effective width b_{eff}]) for the composite slab as follows. The calculation for $V_{Rd,c,c}$ is based on clause F2.2 of EN 1168[5] but has been adapted by Elliott[4] as follows.

$$V_{Rd,c,c} = \frac{I_{c,c}b_w}{S_{c,c}}\left[\sqrt{f_{ctd}^2 + \alpha_l \sigma_{cp} f_{ctd}} - \frac{V_{Ed,1}S_c}{b_w I_c}\right] + V_{Ed,1}$$

where

$V_{Rd,c,c}$ = flexurally uncracked shear capacity of composite section

α_1 = distance to shear plane ratio = l_x/l_{pt2}

l_x = distance to critical section = $l_b + y_b$

l_b = bearing length

y_b = height to centroid of precast unit, not to the composite section*

l_{pt2} = design transmission length

Note that the shear capacity of the composite section is calculated at the position of the centroid of the precast unit, where shear failures occur in composite hollow core slabs.

$$S_{c,c} = b_{eff}t(h + 0.5t - y_{b,c}) + b(h - y_{b,c})^2/2$$

where

$y_{b,c}$ = height to the centroid of the composite section

If the centroidal axis is below the top flange, in other words, if $y_{b,c} < h - h_{ft}$,

$$S_{c,c} = b_{efft}(h + 0.5t - y_{b,c}) + b(h - y_{b,c})^2/2 - (b - b_w)(h - h_{ft} - y_{b,c})^2/2$$

Section cracked in flexure

F4[5] This uses the same procedure as the noncomposite units except that a composite effective depth d_2 of $d + t$ replaces d. The following equations are according to EC2-1-1.

6.2.2 $$k = 1 + \sqrt{(200/d_2)} \leq 2.0$$

where

k = shear strength depth factor

Eq. (6.3) $$v_{min} = 0.035k^{3/2}f_{ck}^{1/2}$$

where

v_{min} = minimum concrete shear strength

The steel area ratio ϱ_1 of $A_p/b_wd_2 \leq 0.02$ extends beyond the shear plane.

Eq. (6.2a) $$V_{Rd,cr,c} = [C_{Rd,c}k(100\varrho_1 f_{ck})^{1/3} + 0.15\sigma_{cp}]b_wd_2$$

where

$V_{Rd,cr,c}$ = flexurally cracked shear capacity of composite section

$C_{Rd,c}$ = concrete shear strength factor

σ_{cp} = axial prestress after losses

Eq. (6.2b)

$$\text{Minimum } V_{Rd,cr,c} = [0.70 + 0.15\sigma_{cp}]b_wd_2$$

6.2.2.(2) Use $V_{Rd,cr,c}$ where service moment $M_s = M_{s1} + M_{s2}$ > composite cracking moment $M_{cr,c}$ for 2 stage loading as follows, where M_{s2} is service moment at stage 2.

The section is cracked at ultimate when $\sigma_b - M_{s1}/Z_{b,co} - M_{s2}/Z_{b,c,co} \geq f_{ctd}$, creates a paradox as the service moment is checked against an ultimate stress.

$$M_{s2} = (\sigma_b - M_{s1}/Z_{b,co} + f_{ctd})Z_{b,c,co}$$

$$M_{cr,c} = M_{s1} + (\sigma_b - M_{s1}/Z_{b,co} + f_{ctd})Z_{b,c,co} = (\sigma_b + f_{ctd})Z_{b,c,co} - M_{s1}\,[(Z_{b,c,co}/Z_{b,co}) - 1]$$

where

$Z_{b,co}$ = section modulus at bottom of basic compound section

In other words, the cracking moment of resistance of the composite section $M_{cr,c}$ varies along the span according to stage 1 service moment.

Where service moment at stage 1 M_{s1} is 0, the cracking moment of resistance of the composite section $M_{cr,c}$ is $(\sigma_b + f_{ctd})Z_{b,c,co}$.

Camber at installation of topping and deflections per EC2-1-1

At transfer At transfer, the upward camber of the hollow-core unit due to prestress δ_1 and deflection due to self-weight δ_2 are as in the previous paper.[1]

At installation The deflections at installation δ_{ins} are $\delta_3 + \delta_4$, as in the previous paper,[1] where δ_3 is upward camber due to prestress at installation and δ_4 is deflection due to self-weight of the unit and infill at installation.

Net camber at installation δ_{ins} is the camber on which the mean depth of the topping is based.

After installation, due to the self-weight of the topping w_T, uniformly distributed loading per unit,

$$\delta_5 = +5w_TL^4/384E_{cm}I_{c,co}$$

where $I_{c,co}$ is for the hollow-core unit alone and δ_5 is deflection due to self-weight of the topping at installation.

Deflection after casting of topping is $\delta_3 + \delta_4 + \delta_5$.

Long term The imposed dead load is taken as being applied immediately after installation at 28 days.

The effective creep coefficient after installation as in the previous paper[1] ψ_{28} is 0.8 × 2.5 × (1.0 − 0.4) = 1.2. This is in agreement with full scale experimental tests on composite hollow core slabs,[4] that changes in deflection due to imposed dead load are closer to the effect of $1 + \psi_{28}$ than $1 + 0.8\psi_\infty$.

The creep coefficient for cast-in-place concrete topping $0.8\psi_\infty$ is 2.0, where ψ_∞ is long-term creep coefficient for deflections. This controls the long-term deflection for imposed dead and live loads that comes on after the topping has hardened. Young's modulus of concrete E_{cm} is used, not Young's modulus of cast-in-place topping $E_{cmT,}$ because the second moment of area of composite compound section $I_{c,c,co}$ is based on the transformed section.

Long-term camber is due to camber at installation plus changes in pretensioning force.

$$\delta_7 = \delta_3 - [\psi_{28}F_{pmi} - (F_{pmi} - F_{po})]z_{cp}L^2/8E_{cm}I_{c,c,co}$$

where

δ_7 = long-term deflection due to imposed dead and live loads

F_{pmi} = pretensioning force at installation

F_{po} = final prestress force

z_{cp} = eccentricity of pretensioning force

Long-term deflection due to self-weight of unit and infill w_1 and topping w_T is due to previous static deflection $\delta_4 + \delta_5$ plus further creep movement due to w_1 and w_T using residual creep factor ($0.8\psi_\infty - \psi_1$, where ψ_1 is effective creep coefficient for deflection at installation) multiplied by the ratio of the respective compound moments of area $I_{c,co}/I_{cc,co}$ before and after hardening of the topping.

$$\delta_8 = +\delta_4 + \delta_5 + [5(w_1 + w_T)(0.8\psi_\infty - \psi_1)(I_{c,co}/I_{c,c,co})L^4/384E_{cm}I_{c,c,co}]$$

where

δ_8 = total long-term deflection

Deflection is due to imposed dead load w_2 and quasi-permanent live load $\psi_2 w_3$, where ψ_2 is quasi-permanent live load factor and w_3 is imposed live load.

$$\delta_9 = +5[(1 + \psi_{28})w_2 + (1 + 0.8\psi_\infty)\psi_2 w_3]L^4/384E_{cm}I_{c,c,co}$$

of which live load active deflection is

$$\delta_{10} = +5(1 + 0.8\psi_\infty)\psi_2 w_3 L^4/384E_{cm}I_{c,c,co}$$

where

δ_9 = long-term deflection due to dead and live loads after installation

δ_{10} = long-term active deflection due to live loads only

Overall long-term active deflection δ_{11} is due to live load plus creep deflections due to self-weight and topping using the effective creep coefficient after installation ψ_{28}, dead loads using $0.8\psi_\infty$, and changes in camber.

$$\delta_{11} = \delta_{10} + 5[\psi_{28}(w_1 + w_T) + 0.8\psi_\infty w_2]L^4/384E_{cm}I_{c,c,co} + (\delta_7 - \delta_3)$$

where

δ_{11} = long-term active deflection due to changes in camber and live load, plus creep deflections due to self-weight, topping, and imposed dead loads

Summary deflections and limits are same as in the previous paper.[1]

Effect of propping

Service stress during propping

The calculation is as above, except that referring to **Fig. 5**, stage 1 service moment is as follows.

For single prop at midspan, service moment at stage 1 M_{s1} is $w_1L^2/8 - 5w_TL/32$.

For two props at 1/3 points, service moment at stage 1 M_{s1} is $w_1L^2/8 - 11\ w_TL/90$.

The prop is assumed to be founded on a rigid base or ground. If the prop is founded on a flexible floor slab one story below the prop force should be multiplied by a reduction factor $1 - I_{c,co}/I_{c,c,co}$, where $I_{c,c,co}$ is the composite value for the completed composite slab beneath.

When props are removed, stage 2 moment is $+5w_TL/32$ or $+11w_TL/90$ for one or two props, respectively, plus all imposed dead and live loads.

Effect of propping on deflections per EC2-1-1

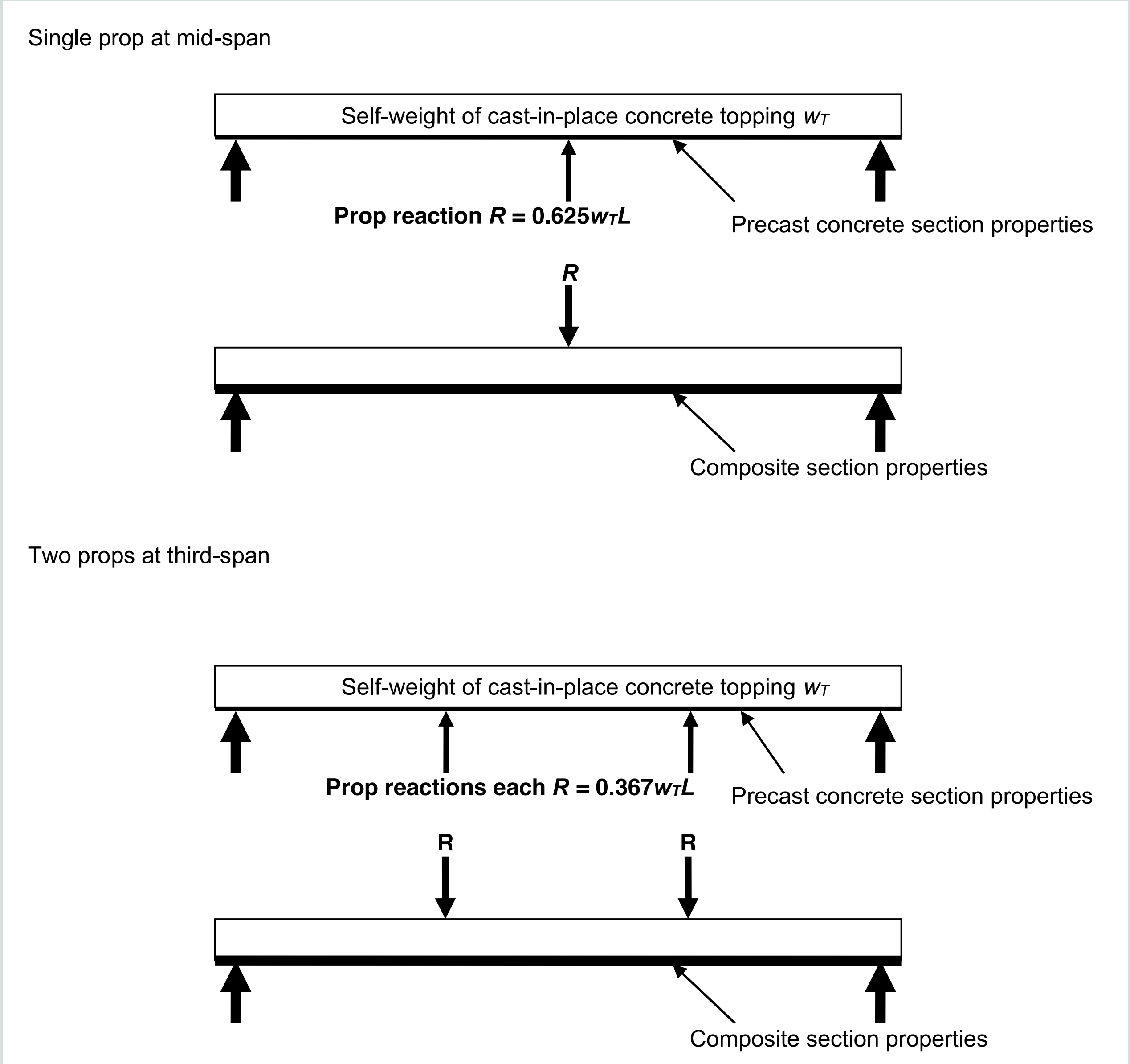

Figure 5 Positions of single and twin props. Note: L = effective span of composite hollow-core slab; R = prop reaction; and w_T = self-weight of cast-in-place concrete topping.

The equivalent upward deflection due to single propping of wet topping of average self-weight w_T at midspan is as follows:

$$\delta_6 = -0.625w_TL^4/48E_{cm}I_{c,co}$$

where

δ_6 = long-term deflection due to self-weight of unit and infill

The equivalent upward deflection due to twin propping of wet topping at ⅓-span is as follows:

$$\delta_6 = -w_TL^4/76.84E_{cm}I_{c,co}$$

The resulting deflection at installation if propped is $\delta_3 + \delta_4 + \delta_5 + \delta_6$.

There is no effect of creep due to the removal of the prop δ_{12} (that is, the instantaneous downward deflection) acting on the composite section.

$$\delta_{12} = +0.625w_TL^4/48E_{cm}I_{c,c,co}$$

There is no effect of creep due to the removal of twin props acting on the composite section.

$\delta_{12} = +w_T L^4/76.84 E_{cm} I_{c,c,co}$

The same equations apply for ACI 318, except that the notation E_{cm} is replaced with E_c.

Interface shear at precast–to–cast-in-place concrete topping

Interface shear stress and reinforcement per EC2-1-1

Fig. 6 describes the interface shear mechanism and terminology used in EC2-1-1 clause 6.2.5. It is based on the complimentary shear force due to stage 2 vertical shear force $V_{Ed,2}$.

6.2.5.(1) $v_{Edi} = \dfrac{F_{c,t}}{F_c}\dfrac{V_{Ed,2}}{b_i z_2} \le v_{Rdi}$

$$v_{Rdi} = cf_{ctd} + \mu\sigma_n + \frac{A_c f_{yd}}{A_j}$$

where

v_{Edi} = ultimate design interface shear stress

$F_{c,t}$ = ultimate compression in topping at stage 2

F_c = total ultimate compression in topping and hollow-core unit at stage 2

b_i = width of interface = width of precast unit

z_2 = lever arm at stage 2

v_{Rdi} = ultimate interface shear strength

c = cohesion factor for interface shear strength

μ = coefficient of friction for interface shear strength

σ_n = normal (vertical) stress acting on interface

A_s = area of interface shear reinforcement

f_{yd} = design strength of interface shear bars

A_j = area of interface joint

3.1.6.2.(P) Design cast-in-place concrete tensile strength f_{ctd} is $0.3 f_{ckT}^{2/3} \times 0.7/1.5$.

6.2.5.(2) The cohesion factor for interface shear strength c is 0.2 and the coefficient of friction for interface shear strength μ is 0.6 (smooth after casting).

$$\therefore A_s = \left(\frac{F_{c,t} V_{Ed,2}}{F_c b_i z_2} - cf_{ctd}\right)\frac{b_i}{f_{yd}}$$

where

V_{Ed} = ultimate design shear force

Fig 6.10. Distribution of the area of interface shear reinforcement A_s should respond to the distribution of the complimentary shear force due to stage 2 vertical shear force $V_{Ed,2}$.

The only guidance in EC2-1-1 section 9.2.2 on the minimum area of interface shear links is for where the transverse bars form part of the shear cage, which is not the case in hollow-core units. Therefore the "industry-adopted" arrangement of minimum loops with two legs is H12 at 1200 mm (48 in.) spacing, equivalent to 188 mm^2/m (0.089 in. 2/ft) or $A_{s,min}$ = 0.015% of the contact area.

Flexural compression due to stage 2 loading

The section is flexurally uncracked at the support where the interface shear is a maximum. Therefore, only stage 2 loads after hardening of the topping contribute to v_{Edi}.

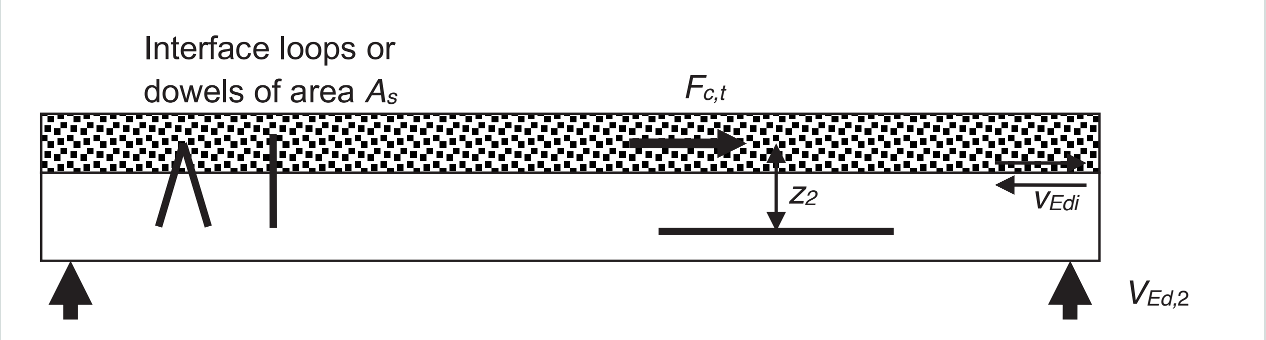

Figure 6 Definitions used in the calculation for interface shear per *Eurocode 2: Design of Concrete Structures, Part 1-1: General Rules and Rules for Buildings (with National Application parameters)*. Note: A_s = area of interface shear reinforcement; $F_{c,t}$ = ultimate compression in topping at stage 2; v_{Edi} = ultimate design interface shear stress; $V_{Ed,2}$ = ultimate design shear force at stage 2; and z_2 = lever arm at stage 2.

The ultimate force in the topping F_c is as follows:

$F_c = 0.567 f_{ckT} b_i 0.8 x_2$

where

$z_2 = \left[0.5 + \sqrt{\left(0.25 - K_2/1.134\right)}\right] d_2$

x_2 = depth to neutral axis at ultimate at stage 2 $= (d_2 - z_2)/0.4 \le 0.6 d_2$

K_2 = bending moment factor at stage 2 (for interface shear) $= M_{Ed,2}/f_{ckT} b_t d_2^2$

$M_{Ed,2}$ = maximum ultimate moment due to stage 2 loads based on *Eurocode 0: Basis of Design (with National Application Parameters)* (EC0),[7] the greater of Eq. (6.10[a]) or (6.10[b])

If $0.8x_2 > t$, $F_{c,t} = 0.567 f_{ckT} b_i t$

If $0.8x_2 \le t$, $F_{c,t} = 0.567 f_{ckT} b_i 0.8 x_2$

Worked example for composite 1200 mm wide × 200 mm deep hollow-core unit plus 75 mm topping per EC2-1-1 and EN 1168

Referring to the cross section of the composite hollow-core slab in Fig. 1, calculate the service $M_{sr,c}$ and ultimate $M_{Rd,c}$ moments of resistance and ultimate shear capacities $V_{Rd,c,c}$ and $V_{Rd,cr,c}$ and determine the position where the section is flexurally cracked. Calculate the camber at installation of topping and after casting the topping, short-term deflection at installation, final long-term deflection, and active deflection due to imposed loads. All prestressing and geometric and material data are given in the previous paper.[1]

Exposure is XC1, the effective span is 8.0 m (26 ft), floor finishes are 1.0 kN/m^2 (0.145 psi), services and ceiling are 0.5 kN/m^2 (0.0725 psi), imposed live load (including 1 kN/m^2 for partitions) is 8.5 kN/m^2 (1.275 psi), and bearing length is 100 mm (4 in.). For office loading the quasi-permanent live load factor ψ_2 is 0.3.

Composite section properties of compound section with transformed area of tendons per EC2-1-1

Young's modulus of cast-in-place concrete topping $E_{cmT} = 22 \times (25 + 8)^{0.3} = 31.48$ kN/mm^2 (4566 ksi)

Modular ratio m for cast-in-place to precast concrete = 31.48/35.22 = 0.894

Effective width of cast-in-place concrete topping at service $b_{ef} = 1200 \times 0.894 = 1072$ mm (42 in.)

Height to centroid of topping $y_{bT} = 200 + 0.5 \times 75 = 237.5$ mm (9.4 in.)

Area of compound section $A_{c,co} = 155{,}475 + (1072 \times 75) = 235{,}906$ mm^2 (366 in.^{2})

Height to centroid of composite compound section $y_{bc,co}$ = [155,475 × 99.0] + [1072 × 75 × 237.5]/235,906 = 145.4 mm (5.7 in.)

Second moment of area of composite compound section $I_{c,c,co} = 1782.2 \times 10^6$ mm^4 (4282 in.^{4})

Section modulus of composite compound section at bottom $Z_{b,c,co} = 1782.2 \times 10^6/145.4 = 12260 \times 10^6$ mm^3 (749,151 in.^{3})

Section modulus of composite compound section at top $Z_{t,c,co} = 1782.2 \times 10^6/(200 - 145.4) = 32620 \times 10^6$ mm^3 (1.99×10^6 in.^{3})

Accounting for the losses of prestress in both the hollow-core slab prior to installation and the composite slab in the long term, the final prestress in the bottom and top of the hollow-core unit and the section modulus are as follows:

- Maximum surface stresses in service at bottom σ_b are +11.52 N/mm^2 (1670 psi), and maximum surface stresses in service at top σ_t are -1.44 N/mm^2 (209 psi).
- The section modulus of the basic compound section at the bottom $Z_{b,co}$ is 7.252×10^6 mm^3 (442.5 in.^{3}), and the section modulus of the basic compound section at the top $Z_{t,co}$ is 6.926×10^6 mm^3 (422.7 in.^{3})

Note that the maximum surface stresses in service at bottom σ_b are slightly greater than the prestress in the composite unit (11.37 N/mm^2 [1649 psi] in the previous paper[1]) due to reduced creep and shrinkage losses because of greater bending moments due to greater self-weight and a greater perimeter.

Calculation for service moment of resistance of composite section $M_{sr,c}$ per EC2-1-1

For stage 1, allow 15 mm (0.6 in.) camber at installation before casting the topping (checked later in the calculation for camber at installations and deflections).

Self-weight of precast unit including infill from the previous paper[1] is 3.26 kN/m^2 (0.47 psi)

Then self-weight of slab per unit width w_1 = 1.2 x 3.26 = 3.91 kN/m (0.27 kip/ft)

Equivalent depth of topping $t' = 75 + 0.5 \times 15 = 82.5$ mm (3.25 in.)

Self-weight of topping per square meter = $24 \times 82.5/10^3$ = 1.98 kN/m^2 (0.29 psi)

Self-weight of topping per unit width = 1.2 × 1.98 = 2.38 kN/m (0.16 kip/ft)

Service load due to self-weight of precast concrete unit plus wet cast-in-place topping $w_1 + w_T$ = 3.91 + 2.38 = 6.29 kN/m (0.43 kip/ft)

Service moment at stage 1 M_{s1} = 6.29 × 8.000²/8 = 50.3 kN-m (37.1 kip-ft)

Stage 1 stress at bottom f_{b1} = 50.3/7.252 = -6.93 N/mm² (1005 psi) tension

Stage 1 stress at top f_{t1} = 50.3/6.926 = +7.26 N/mm² (1053 psi)

Stage 2 imposed loads after topping $w_2 + w_3$ = 1.2 × (1.5 + 1.0 + 7.5) = 12.00 kN/m² (1.75 psi)

Service moment at stage 2 M_{s2} = 12.00 × 8.000²/8 = 96.0 kN-m (70.8 kip-ft)

Stage 2 stress at bottom f_{b2} = 96.0/12.260 = -7.83 N/mm² (1136 psi) tension

Stage 2 stress at top f_{t2} = 96.0/32.620 = +2.94 N/mm² (426 psi)

Final stresses

Net tension in bottom f_b = 11.52 − 6.93 − 7.83 = -3.24 N/mm² (470 psi) > -3.51 N/mm² (509 psi) **OK**

Net compression in top f_t = -1.44 +7.26 +2.94 = +8.76 N/mm² (1271 psi) < 0.45 × 40 = +18.0 N/mm² (2611 psi) **OK**

Service moment of resistance of composite section $M_{sr,c}$ = 50.3 + (11.52 − 6.93 + 3.51) × 12.260 = 149.6 kN-m > 146.3 kN-m (107.9 kip-ft) where total M_s = 1.2 × (3.26 + 1.98 + 1.0 + 0.5 + 8.5) × 8,000²/8 = 146.3 kN-m (107.9 kip-ft)

Calculation for ultimate moment of resistance of composite section $M_{Rd,c}$ per EC2-1-1

Consider stage 1 moment acting on the precast concrete hollow-core unit alone at installation at an age t_i of 28 days.

Ultimate design bending moment at stage one $M_{Ed,1}$ = 1.25 × 50.3 = 62.77 kN-m (46.3 kip-ft)

Value of 28-day characteristic cylinder strength at installation age $f_{ck}(t_i)$ for precast concrete = 40 N/mm² (5800 psi)

Actual width at top of hollow-core unit b_t = 1154 mm (45.4 in.)

Effective depth of tendons in tension zone at stage 1 d_1 = 159.2 mm (6.3 in.)

Bending moment factor at stage 1 K_1 = 62.77 × 10⁶/(40 × 1154 × 159.2²) = 0.054

Lever arm at stage 1 $z_1 = \min[0.95;\ 0.5+\sqrt{\left(0.25-0.054/1.134\right)} \times 159.2 = 151.2\text{mm}\ \left(5.95\text{ in.}\right)$

Design stress of tendons at stage 1 f_{p1} is not known until $M_{Rd,c2}$ is found. Therefore, take the limiting value at 0.02 strain = 1516 N/mm² (219,877 psi).

Area of tendons required in stage 1 A_{p1} = 62.77 × 10⁶/(151.2 × 1516) = 274 mm² (0.42 in.²)

Remainder of $A_p - A_{p1}$ (available for stage 2) A_{p2} = 766 − 274 = 492 mm² (0.76 in.²)

Total strain ε_p = 0.001353 + 0.820/X Eq. (1)

where X is the depth to the neutral axis

For equilibrium X/f_p = 492/13,600 Eq. (2)

where f_p is the design ultimate stress in tendons

Eq. (3) is given in the previous paper.[1]

Combining Eq. (1) through (3) gives X = 53.7 mm (2.1 in.) < 115 mm (4.5 in.) (75 mm [2.9 in.] topping + 40 mm [1.6 in.] top flange)

Depth to the centroid of the concrete area of composite section $d_{n,c}$ = 0.4 × 53.7 = 21.5 mm (0.8 in.)

Effective depth of tendons in tension zone at stage 2 d_2 = 275 − 40.8 = 234.2 mm (9.2 in.)

Lever arm at stage 2 z_2 = 234.2 − 21.5 = 212.7 mm (8.4 in.)

From Eq. (1), the total ultimate strain in the tendons ε_p = 0.016629

From Eq. (3), the design ultimate stress in tendons f_p = 1482 N/mm² (214,946 psi)

Ultimate moment of resistance of composite section $M_{Rd,c} = f_pA_{p2}z_2 + M_{Ed,1}$ = (1482 × 492 × 212.7)/10⁶ + 62.77 = 218.0 kN-m (160.8 kip-ft) > 203.2 kN-m (149.9 kip-ft)

Ultimate uniformly distributed loading (for EC07 Eq. 6.10[b]) ultimate design load w_{Ed} = 1.25 × [3.91 + 1.2 × (1.98 + 1.5)] + 1.5 × 1.2 × 8.5 = 25.39 kN/m (1.7 kip/ft)

Ultimate design bending moment M_{Ed} = 25.39 × 8.0²/8 = 203.2 kN-m (149.9 kip-ft)

At the support where the ultimate design bending moment at stage 1 $M_{Ed,1}$ is 0, it is found that the design ultimate stress in tendons f_p is 1430 N/mm² (207,404 psi), the depth to the centroid of the concrete area of the composite section $d_{n,c}$ is 32.2 mm (1.3 in.), the lever arm at stage 2 z_2 is 234.2 − 32.2 = 202.0 mm (7.95 in.). Then $M'_{Rd,c}$ = 1430 × 766 × 202.0/10^6 = 221.3 kN-m (163.2 kip-ft).

The distribution between midspan and the support is taken as parabolic.

The results for the service $M_{sr,c}$ and ultimate design bending moments $M_{Ed,1}$ plus $M_{Ed,2}$ and moments of resistance $M_{Rd,c}$ and are shown in **Fig. 7**.

Calculation for flexurally uncracked shear capacity of composite section $V_{Rd,c,c}$ per EC2-1-1

From the previous paper,[1] bearing length l_b is 100 mm (4 in.), the first moment of area of hollow-core unit S_c is 4.80 × 10^6 mm^3 (293 in.^{3}), the second moment of area of basic section (hollow-core unit alone) I_c is 697 × 10^6 mm^4 (1675 in.^{4}), design tensile strength of concrete at transfer $f_{ctd}(t)$ is 1.23 N/mm^2 (178 psi), the ultimate bond strength of precast concrete at transfer $f_{bd}(t)$ is 3.93 N/mm^2 (570 psi), design transmission length l_{pt2} is 761 mm (30 in.), axial prestress after losses σ_{cp} is 4.29 N/mm^2 (622 psi), and design tensile strength of concrete f_{ctd} is 1.64 N/mm^2 (238 psi).

Critical distance from end to center of webs l_x = 100 + 99.0 = 199.0 mm (7.8 in.)

Distance to shear plane ratio $\alpha_1 = l_x/l_{pt2}$ = 199.0/761 = 0.262

First moment of area above composite centroid including topping $S_{c,c}$ = 8,894,326 mm^3 (543 in.^{3})

Stage 1 ultimate shear force at shear plane $V_{Ed,1}$ = 30.248 kN (6.8 kip)

Shear stress due to stage 1 = 30248 × 4.80 × 10^6/(303 × 697.0 × 10^6) = 0.69 N/mm^2 (100 psi)

Available shear resistance in stage 2 $V_{Rd,c,c2} = \left(1743.6\times10^6\right.$

$\left.\times303/8.894\times10^6\right)\times\left[\sqrt{\left(1.64^2+0.262\times4.29\times1.64\right)}-0.69\right]$

$=85.4\text{ kN }\left(19.2\text{ kip}\right)$

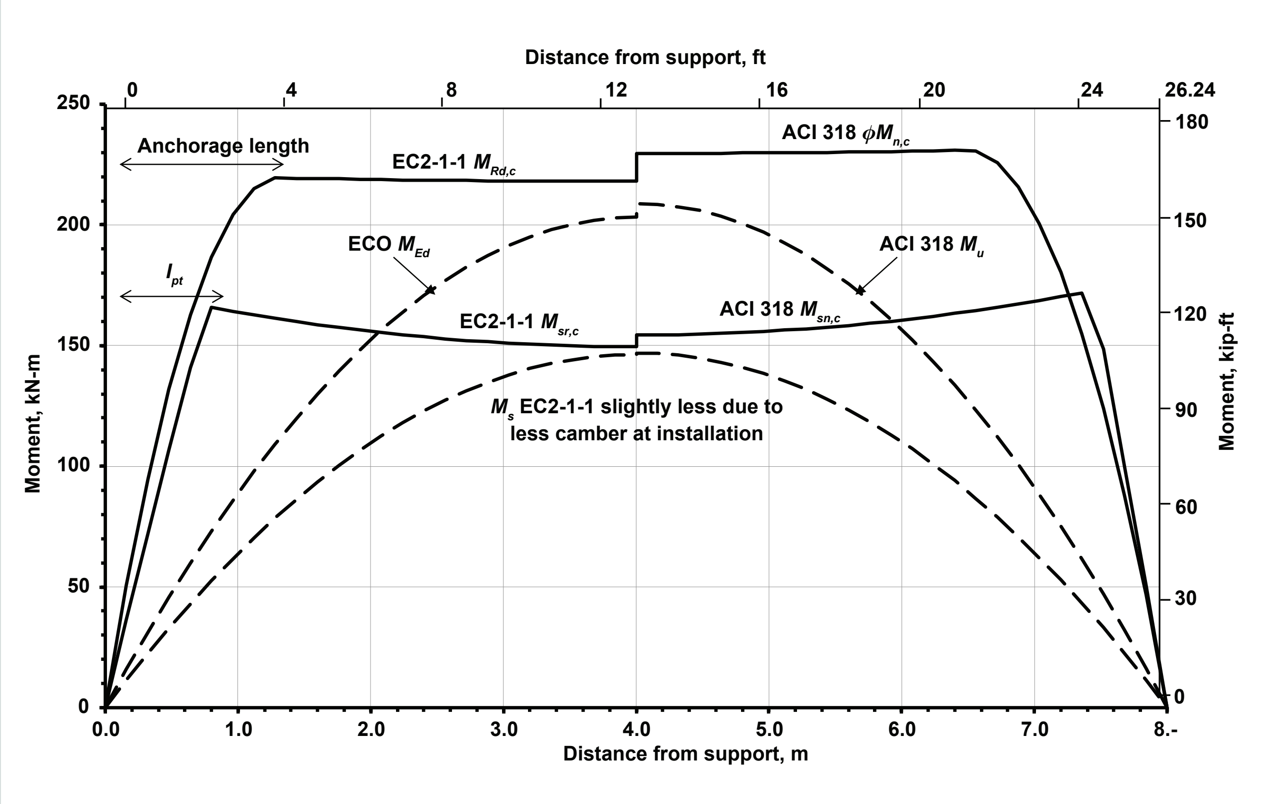

Figure 7 Distribution of service and ultimate design bending moments and composite moments of resistance to *Eurocode 2: Design of Concrete Structures, Part 1-1: General Rules and Rules for Buildings (with National Application parameters)* (EC2-1-1) (left side) and American Concrete Institute's *Building Code Requirements for Structural Concrete* (ACI 318) (right side) in the example. Note: ECO M_{Ed} = ultimate moment based on ECO[7]; l_{pt} = EC2 basic transmission length = l_{pt2}/1.2 M_{Ed} = EC2 ultimate design bending moment; $M_{Rd,c}$ = EC2 ultimate moment of resistance of composite section; M_s = EC2 service moment; $M_{sn,c}$ = ACI 318 service moments of resistance of composite section; $M_{sr,c}$ = EC2 service moments of resistance of composite section; M_u = ACI 318 ultimate design bending moment; and $\phi M_{n,c}$ = ACI 318 ultimate moments of resistance of composite section.

Uncracked shear resistance $V_{Rd,c,c}$ = 85.44 + 30.25 = 115.7 kN (26 kip)

Ultimate shear resistance is equal to the flexurally uncracked shear capacity of the composite section $V_{Rd,c,c}$ when the service moment $M_s > M_{cr,c}$ = (10.76 + 0.7 × 3.51/1.5) × 12.26 × 10^6 − [(12.26 × 10^6/7.252 × 10^6) − 1] × M_{s1} kN-m. To find the point where $M_s < M_{cr,c}$, it is best to calculate M_s and $M_{cr,c}$ at several points along the span at intervals of say the span of the beam divided by 50.

Calculation for flexurally cracked shear capacity of composite section $V_{Rd,cr,c}$ per EC2-1-1

Shear strength depth factor $k = 1 + \sqrt{(200/234.2)} = 1.92$

Minimum concrete shear strength $v_{\min} = 0.035 \times 1.92^{1.5} \times \sqrt{40}$

$= 0.59\ \text{N/mm}^2\ (85.6\ \text{psi})$

Steel area ratio ϱ_1 = 766/(303 × 234.2) = 0.0108

Flexurally cracked shear capacity of composite section $V_{Rd,cr,c}$ = [0.12 × 1.92 × (100 × 0.0108 × $40^{1/3}$ + 0.15 × 4.29)] × 303 × 234.2 = 103.2 kN (23.2 kip)

Minimum flexurally cracked shear capacity of composite section $V_{Rd,cr,c}$ = (0.59 + 0.15 × 4.29) × 303 × 234.2 = 87.6 kN (19.7 kip)

Final flexurally cracked shear capacity of composite section $V_{Rd,cr,c}$ = 103.2 kN (23.2 kip)

Ultimate shear force per EC0[7]

Ultimate design load w_{Ed} (from "Calculation for ultimate moment of resistance of composite section $M_{Rd,c}$") = 25.41 kN/m (1.7 kip/ft)

Shear span $L_v = L - l_b - 2y_b$ = 8000 − 100 − 2 × 99 = 7702 mm (303 in.), where y_b is the height to the centroid of the basic section

Ultimate design shear force V_{Ed} = 25.39 × 7.702/2 = 97.79 kN (21.985 kip) < 103.2 kN (23.2 kip) **OK**

The distribution of the ultimate design shear force V_{Ed}, the flexurally uncracked shear capacity of the composite section

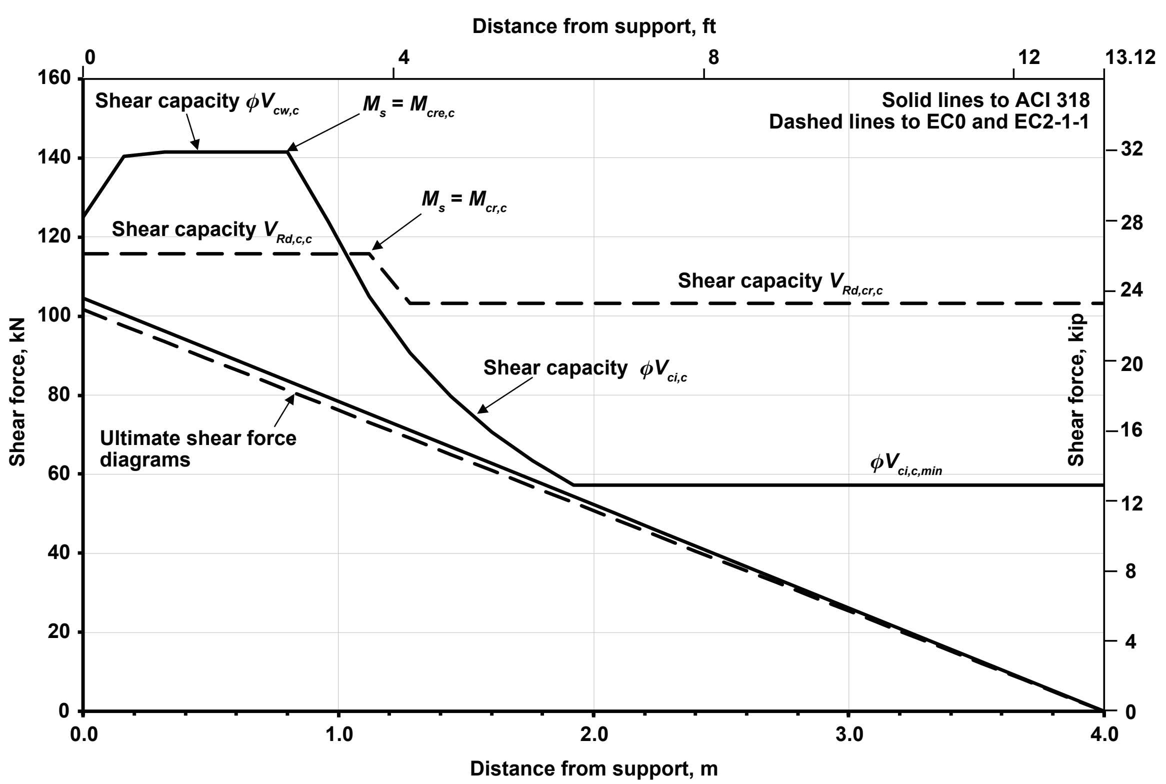

Figure 8 Distribution of shear force and shear capacity to American Concrete Institute's *Building Code Requirements for Structural Concrete* (ACI 318) in the example. Note: $M_{cr,c}$ = ACI 318 cracking moment of resistance of composite section; $M_{cre,c}$ = ACI 318 moment required to cause flexural cracking in composite section; M_s = EC2-1-1 service moment; $V_{Rd,c,c}$ = EC2 flexurally uncracked shear capacity; $V_{Rd,cr,c}$ = EC2 flexurally cracked shear capacity; $\phi V_{ci,c}$ = ACI 318 flexurally cracked shear capacity of composite section; $\phi V_{ci,min}$ = ACI 318 minimum value of flexurally cracked shear capacity ϕV_{ci}; and $\phi V_{cw,c}$ = ACI 318 flexurally uncracked shear capacity of composite section.

$V_{Rd,c,c}$, and the flexurally cracked shear capacity of the composite section $V_{Rd,cr,c}$ along the span is shown in **Fig. 8**.

Calculation for camber at installations and deflections per EC2-1-1

From the previous paper,[1] deflection at installation $\delta_{ins} = \delta_3 + \delta_4$ = -30.6 +14.9 = 15.7 mm (0.62 in.) (therefore 15 mm [0.6 in.] allowed to determine average depth of topping is **OK**).

Deflection due to self-weight of topping $\delta_5 = 5 \times 1.2 \times 1.98 \times 8000^4/(384 \times 35{,}220 \times 708.5 \times 10^6)$ = 5.5 mm (0.22 in.)

Net deflection after casting topping = -10.2 mm (0.4 in.)

Long-term deflections

From the previous paper,[1] $X\psi_\infty$ is 0.8 × 2.5 = 2.00, effective creep coefficient for deflection at installation ψ_1 is 0.72, effective creep coefficient after installation ψ_{28} is 2.00 × (1 − 0.4) = 1.20, pretensioning force at installation F_{pmi} is 857,588 N (192.8 kip), and $F_{pmi} - F_{po}$ = 132,734 N (29.8 kip).

Long-term camber $\delta_7 = -30.6 - [(1.20 \times 857{,}588) - 132{,}734] \times 58.2 \times 8000^2/(8 \times 35{,}220 \times 1782 \times 10^6)$ = -37.2 mm (1.5 in.)

Creep factor for creep of self-weight and topping acting on composite section = $(2.00 - 0.72) \times 708.5 \times 10^6/1782 \times 10^6$ = 0.513

Due to creep of self-weight of slab and static plus creep of topping $\delta_8 = 5 \times 1.2 \times [0.513 \times 3.26 + (1+ 0.513) \times 1.98] \times 8000^4/(384 \times 35{,}220 \times 708.5 \times 10^6)$ = 12.0 mm (0.5 in.)

Due to uniformly distributed dead loading = $5 \times 1.2 \times (1 + 1.20) \times 1.50 \times 8000^4/(384 \times 35{,}220 \times 1782 \times 10^6)$ = 3.4 mm (0.13 in.)

Due to quasi-permanent uniformly distributed live loading = $5 \times 1.2 \times (1 + 2.00) \times (0.3 \times 8.50) \times 8000^4/(384 \times 35{,}220 \times 1782 \times 10^6)$ = 7.8 mm (0.3 in.)

Net maximum deflection due to all loads 14.9 + 12.0 + 3.4 + 7.8 − 37.2 = +0.9 mm (0.035 in.) < 8000/250 = 32 mm (1.3 in.)

Active deflection due to quasi-permanent uniformly distributed live loading = $5 \times 1.2 \times (1 + 1.20) \times (0.3 \times 8.50) \times 8000^4/(384 \times 35{,}220 \times 1782 \times 10^6)$ = 5.7 mm (0.224 in.) < 8000/350 = 22.9 mm (0.9 in.)

Active deflection due to creep of dead and imposed loads and change in camber after installation = $\{[5 \times 1.2 \times 0.513 \times (3.26 + 1.98) \times 8000^4/(384 \times 35220 \times 708.5 \times 10^6)] + [5 \times 1.20 \times 1.2 \times 1.50 \times 8000^4/(384 \times 35{,}220 \times 1782 \times 10^6)]\} + 5.7 - (37.2 - 30.6)$ = +7.8 mm (0.3 in.) < 8000/350 = 22.9 mm (0.9 in.) for floors with no brittle finishes

Interface shear per EC2-1-1

Ultimate design bending moment at stage 1 $M_{Ed,1}$ = 62.8 kN-m (46.3 kip-ft)

Ultimate design bending moment at stage 2 M_{Ed2} = 140.4 kN-m (103.6 kip-ft)

Bending moment factor at stage 2 $K_2 = 140.4 \times 10^6/(25 \times 1200 \times 234.2^2)$ = 0.085

Lever arm at stage 2 $z_2 = \min\left[0.95;\ 0.5+\sqrt{\left(0.25-0.085/1.134\right)}\right]\times 234.2 = 215.0\text{ mm }\left(8.5\text{ in.}\right)$

Depth to neutral axis at ultimate at stage 2 x_2 = 48.0 mm (1.9 in.), thus $0.8x_2$ < topping depth of 75 mm (3 in.), then F_{cT}/F_c = 1.00

Ultimate design shear force at stage 1 V_{Ed1} = 31.4 kN (7.1 kip)

Ultimate design shear force at stage 2 loads V_{Ed2} = 70.2 kN (15.8 kip)

Ultimate design interface shear stress $v_{Edi} = 1.00 \times 70.2 \times 10^3/1200 \times 215.0$ = 0.27 N/mm² (39.2 psi)

Ultimate interface shear strength $v_{Rdi} = 0.2 \times 0.3 \times 0.7 \times 25^{2/3}/1.5$ = 0.24 N/mm² (34.8 psi) but ≤ 0.5 × 25/1.5 = 8.33 N/mm² (1208.2 psi)

Interface shear loops $A_s/s = 1000 \times (0.27 - 0.24) \times 1154/(0.87 \times 500)$ = 90 mm²/m (0.04 in.²/ft) run

Minimum area $A_{s,min}$ = 0.015% = 0.00015 × 1200 × 1000 = 180 mm²/m (0.085 in.²/ft)

Provide H12 U-loops at s = 1200 mm (48 in.) spacing (A_s/s = 226/1.2 = 188 mm²/m [0.089 in.²/ft])

Design example following ACI 318 methodology

A design example for a 1200 mm (48 in.) wide × 200 mm (8 in.) deep hollow-core unit is designed with a 75 mm (3 in.) deep structural topping following ACI 318 methodology. The example is for a simply supported span of 8.0 m (26 ft) to carry imposed dead uniformly distributed loading of 1.5 kN/m² (0.23 psi), the self-weight of the topping (allowing for a 15 mm [0.6 in.] upward camber) of 1.98 kN/m² (0.3 psi), and live load (including 1 kN/m² [0.15 psi] for partitions) of 8.5 kN/m² (1.275 psi). The example is arranged as follows. Design procedures and equations are presented followed by worked examples of the calculation for the moments of resistance, shear capacity and deflections of the composite slab, and the interface shear between the topping and hollow-core unit.

In the following procedures and throughout this paper, code references are given on the left and the text, calculation, and formulae are to the right.

Design procedures and equations per ACI 318

Definitions according to ACI 318

Referring to the cross-section in Fig. 3, the differences relative to the EC2-1-1 are Young's modulus of cast-in-place concrete topping E_{cT} is $4700\sqrt{f'_{cT}}$, where f'_{cT} is the 28-day characteristic cylinder strength and the effective width of cast-in-place concrete topping at service b_{ef} is bE_{cT}/E_c, where b is the nominal width of the hollow-core unit (1200 mm [48 in.] in this paper) and E_c is Young's modulus of concrete.

Serviceability limit state of bending per ACI 318

Referring to Fig. 4, the variations relative to Eurocode include the mean tensile strength of concrete f_{ctm} is replaced by f_t and $0.45f_{ck}$ by $0.6f'_c$ for 28-day characteristic cylinder strength.

The service stresses available to resist stage 2 loading due to all imposed dead and live loads at bottom $f_{b2} \leq f_{pbe} - f_{b1} - f_t$, where f_{pbe} is maximum surface stress due to prestress in service at bottom, f_{b1} is bending stress at bottom due to stage 1 moment, and mean tensile strength of concrete f_t is $0.63\sqrt{f'_{cT}}$ for Class U and $1.00\sqrt{f'_c}$ for Class T.

The service stresses available to resist stage 2 loading due to all imposed dead and live loads at top $f_{t2} \leq 0.6f'_c - f_{pte} - f_{t1}$, where f_{pte} is maximum surface stress due to prestress in service at top and f_{t1} is bending stress at top due to stage 1 moment.

The service moment of resistance of the composite section $M_{sn,c}$ is the lesser of $M_{s1} + f_{b2}Z_{b,c,co}$ (usually critical) or $M_{s1} + f_{t2}Z_{t,c,co}$, where M_{s1} is the service moment at stage 1, $Z_{b,c,co}$ is the section modulus of the composite compound section at the bottom, and $Z_{t,c,co}$ is the section modulus of the composite compound section at the top.

Ultimate limit state of bending per ACI 318

The ultimate moment of resistance of the composite section $\phi M_{n,c}$ is calculated for the net area of tendons A_{ps2} as $A_{ps} - A_{ps1}$, where A_{ps} is the area of tendons and A_{ps1} is the area of tendons required in stage 1.

For stage 1, $M_{u,1}$ is the ultimate moment due to self-weight plus topping (and propping).

$$K_1 = (M_{u,1}/0.9)/\, b_t d_{p1}^2$$

where

K_1 = bending moment factor at stage 1

$f'_c(t_i)$ = strength of precast concrete at installation at installation age t_i

b_t = actual width at top of hollow-core unit

d_{p1} = effective depth of tendons in tension zone at stage 1

Assume ¼ power function

$$f'_c(t_i) = \sqrt[4]{(t_i/28)}\,f'_c$$

$$z_1 = \left[0.5 + \sqrt{(0.25 - K_1/1.7)}\right] d_{p1}$$

where

z_1 = lever arm at stage 1

$$c_1 = (d_{p1} - z_1)/0.5\beta_1$$

where

c_1 = neutral axis depth

β_1 = stress block factor calculated for precast concrete strength

If $\beta_1 c_1 \leq$ the depth of the top flange h_{ft}, then the area of tendons required in stage 1 A_{ps1} is $M_{u,1}/z_1 f_{ps1}$.

Because the design stress of tendons at stage 1 f_{ps1} is not known because it is dependent on the area of tendons required in stage 1 A_{ps1}, the stress is taken as the design stress of tendons at stage 1 $f_{ps1} = f_{pu}/1.1$ without loss of accuracy, where f_{pu} is the characteristic strength of tendons.

If $\beta_1 c_1 >$ the depth of the top flange h_{ft}, compression is in webs of width b_w.

Restart the analysis using the bending moment factor at stage 1 $K_{1w} = M_{u,1}/\, f'_c(t_i) b_w d_{p1}^2$.

$$z_1 = \left[0.5 + \sqrt{(0.25 - K_{1w}/1.7 - \zeta)}\right] d_{p1}$$

where

ζ = factor for the design of T-shaped sections $= (b_t - b_w)h_{ft}(d_{p1} - 0.5h_{ft})/(2b_w d_{p1}^2)$

The derivation for the lever arm at stage 1 z_1 is given in the appendix. So, too, is the derivation of the factor for the design of T-shaped sections ζ, which follows the same procedure as for the EC2-1-1 term β.

Then the net area of tendons A_{ps2} is $A_{ps1} - A_{ps1}$.

The ultimate moment of resistance of the composite section

$\phi M_{n,c2}$ is calculated per the noncomposite procedures using the net area of tendons A_{ps2} instead of the area of tendons A_{ps}, the effective width of cast-in-place concrete topping at ultimate b_{eff} is bf'_{cT} / f'_c instead of the actual width at the top of hollow-core unit b_t, and the effective depth of tendons in the tension zone at stage 2 d_{p2} instead of the effective depth d_p.

The total ultimate moment of resistance of the composite section $\phi M_{n,c}$ is $\phi M_{n,c2} + M_{u,1}$.

Where the ultimate design bending moment at stage 1 $M_{u,1}$ is 0 (for example, at the supports), the net area of the tendons A_{ps2} is the area of tendons A_{ps} and the ultimate moment of resistance of the composite section at stage 2 is $\phi M'_{n,c}$.

The variation of the ultimate moment of resistance of the composite section $\phi M_{n,c}$ along the span is assumed as a parabolic curve to reflect the variation in the ultimate design bending moment at stage 1 $M_{u,1}$. At a distance x from the support for an effective span L,

$$\phi M_{n,c}(x) = \phi M_{n,c} - \left[x / 0.5L\left(2 - x / 0.5L\right)\left(\phi M_{n,c} - \phi M'_{n,c}\right)\right]$$

18.8.2 Ultimate moment of resistance of the composite section $\phi M_{n,c} > 1.2M_{cr,c}$

9.5.2.3 where

$M_{cr,c}$ = cracking moment of resistance, allowing for reduced bottom stress due to stage 1 loads = $M_{s1} + (f_{pbe} + f_r - M_{s1}/Z_b)Z_{b,c,co}$

f_r = modulus of rupture

Z_b = section modulus at bottom of basic section (hollow-core unit only)

Therefore, the cracking moment of resistance of the composite section $M_{cr,c}$ reduces along the span according to service moment at stage 1 M_{s1}. A parabolic distribution is taken.

Ultimate shear force capacity criterion per ACI 318

The same equations as in the previous paper[1] apply except that the effective depth d_p is replaced by the effective depth of tendons in tension zone at stage 2 d_{p2} and shear web failure V_{ci} is replaced with ultimate shear capacity in the flexurally cracked section $V_{ci,c}$ as follows. There is no change to concrete strength because failure takes place in the webs of the hollow-core and not in the topping.

Eq. (11-10)

$$V_{ci,c} = 0.05\sqrt{f'_c}b_w d_{p2} + V_d + M_{cre,c}V_i / M_{max}$$

$$V_{ci,c} \geq 0.17\sqrt{f'_c}b_w d_{p2}$$

where

$V_{ci,c}$ = ultimate shear capacity in flexurally cracked section

V_d = shear force at distance along the span of slab from the center of support *x* for service (unfactored) dead load in stages 1 and 2

$M_{cre,c}$ = moment required to cause flexural cracking in composite section

V_i = ultimate shear force at distance along the span of slab from the center of support *x* due to externally imposed dead and live loads

M_{max} = ultimate shear and moment at *x* due to externally imposed dead and live loads only

Eq. (11-11)

$M_{cre,c}$ = $Z_{b,c,co}(0.5 + f_{pbe} - f_{d1} - f_{d2})$

where

$M_{cre,c}$ = moment required to cause flexural cracking in composite section

f_{d1} = tension stress due to service dead load in stage 1 = $M_{d1}/Z_{b,co}$

f_{d2} = tension stress due to service dead load in stage 2 = $M_{d2}/Z_{b,c,co}$

Table 1. Creep coefficients according to *PCI Design Handbook* Table 5.8.2

	Hollow-core unit only	Composite slab
Self-weight after installation	1.7	1.4
Camber after installation	1.45	1.2
Imposed dead and live after installation	2.0	2.0
Topping self-weight after installation	n/a	1.3

Note: n/a = not applicable.

M_{d1} = moment due to dead load at stage 1

$Z_{b,co}$ = section modulus of basic compound section at bottom

M_{d2} = moment due to dead load at stage 2

Camber at installation of topping and deflections per ACI 318

Creep coefficients after installation ψ are given in **Table 1**.

At installation Deflection at installation δ_{ins} is $\delta_3 + \delta_4$ as the previous paper,[1] where δ_3 is upward camber due to prestress at installation and δ_4 is deflection due to self-weight of unit and infill at installation. Deflection at installation δ_{ins} is the camber on which the mean depth of the topping is based.

After installation, due to the self-weight of the topping w_T, uniformly distributed loading per unit,

$$\delta_5 = +5w_T L^4/384E_c I_{c,co}$$

where

δ_5 = long-term deflection due to self-weight of the topping at installation

The second moment of area of compound section $I_{c,co}$ is for the hollow-core unit alone.

Deflection after casting of topping is $\delta_3 + \delta_4 + \delta_5$.

Long term Imposed dead load is taken as being applied immediately after installation at 28 days.

The creep coefficient for imposed loading onto the cast-in-place concrete topping $\psi\infty$ is 2.0. This controls the long-term deflection for imposed dead and live loads that come on after the topping has hardened. Young's Modulus of concrete E_c is used, not Young's modulus of cast-in-place concrete topping E_{cT}, because the second moment of area of composite compound section $I_{c,c,co}$ is based on the transformed section.

Long-term camber is due to camber at installation, where the pretensioning force is P_{ins}, plus changes in pretensioning force using an effective creep coefficient after installation ψ_{28} of 1.2. (Note that the notation for prestressing force is given in ACI 318 as *P*.)

$$\delta_7 = \delta_3 - [\psi_{28} P_{ins} - (P_{ins} - P)]eL^2/8E_c I_{c,c,co}$$

where

δ_7 = long-term upward camber due to prestress

e = eccentricity of prestressing force

The long-term deflection is due to previous deflection $\delta_4 + \delta_5$ plus further visco elastic movement due to the self-weight of the precast unit plus infill w_1, using an effective creep coefficient after installation ψ_1 of 1.7, plus further visco elastic movement of the topping w_T, using a long-term creep coefficient ψ_T of 1.3.

$$\delta_8 = +\delta_4 + \delta_5 + [5(\psi_1 w_1 + \psi_T w_T)L^4]/384E_c I_{c,c,co}$$

where

δ_8 = long-term deflection due to self-weight of unit, infill, and topping

Deflection due to imposed dead w_2 and quasi-permanent live load $\psi_2 w_3$ is

$$\delta_9 = +5[(1 + \psi_\infty)(w_2 + \psi_2 w_3)]L^4/384E_c I_{c,c,co}$$

where

w_2 = floor dead load per unit width

ψ_2 = quasi-permanent live load factor

w_3 = floor live load per unit width

Of which live load active deflection only is

$$\delta_{10} = +5(1 + \psi_\infty)\psi_2 w_3 L^4/384E_c I_{c,c,co}$$

Overall long-term active deflection δ_{11} due to live load plus creep deflections due to self-weight ψ_1, topping ψ_T, dead loads using long-term creep coefficient for deflections ψ_∞, and changes in camber is

$$\delta_{11} = \delta_{10} + 5[(\psi_1 w_1 + \psi_T w_T) + \psi_\infty w_2] L^4/384E_c I_{c,c,co} + (\delta_7 - \delta_3)$$

Summary deflections and limits are the same as in Table 9 of the previous paper.[1]

Interface shear at precast–to–cast-in-place concrete topping

Interface shear stress and reinforcement per ACI 318

Figure 9 describes the interface shear mechanism and terminology used in ACI 318. It is based on the horizontal force in the topping due to stage 2 bending moments.

General definitions The contact width of interface b_i from EC2 is the nominal width of hollow-core unit b, which is 1154 mm (45.4 in.) in this paper.

The contact length of interface l_v is min{l_{v1}, l_{v2}} from support to zero shear (Fig. 9), where l_{v1} is the distance to zero shear force from one support (in this case left side in Fig. 9) and l_{v2}

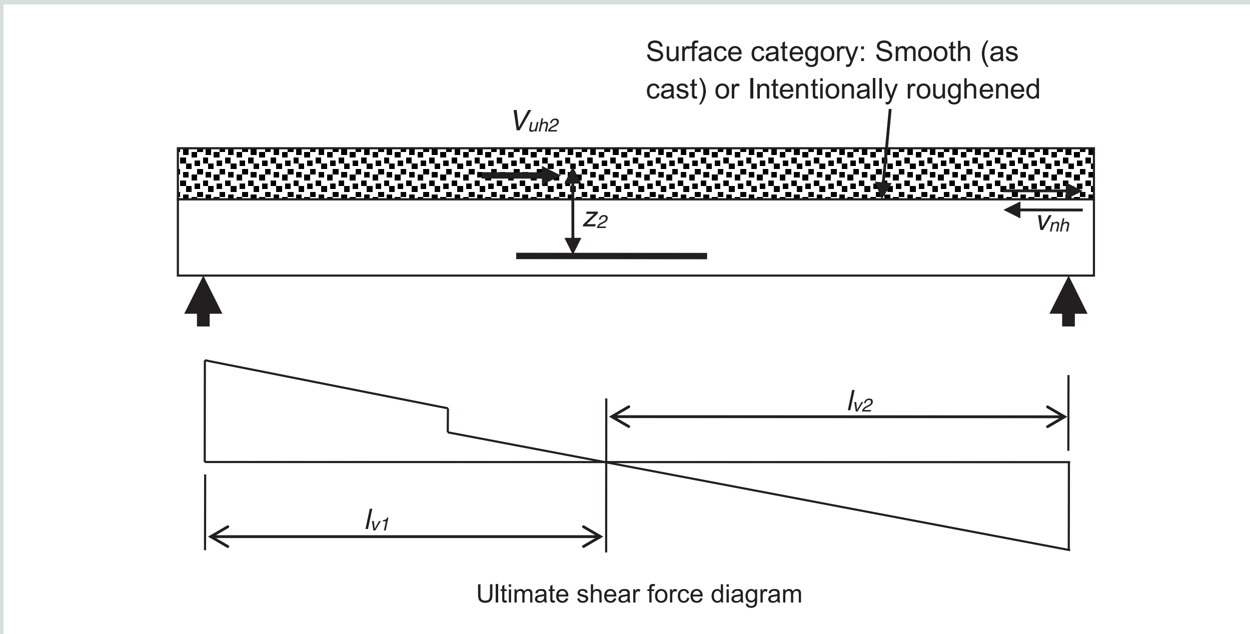

Figure 9 Definitions used in the calculation for interface shear in American Concrete Institute's *Building Code Requirements for Structural Concrete* (ACI 318). Note: l_{v1} = distance to zero shear force from left end support; l_{v2} = distance to zero shear force from right end support; v_{nh} = shear stress; V_{uh2} = horizontal shear force in topping at stage 2; and z_2 = lever arm at stage 2.

is likewise from the other support.

17.5.4 The term $b_v d$ in clause 17.5.3 may be replaced by the interface shear contact area $A_c = l_v b_i$.

Horizontal shear force in topping V_{uh2} is equal to the compressive force in the topping due to stage 2 loading, then:

V_{uh2} = $\max\{0.85 f'_c b_i t; (M_u - M_{u1})/z_2\}$ acting over area $A_c = l_v b_i$

where

t = depth of topping

M_u = ultimate design bending moment

z_2 = lever arm at stage 2

17.5.3.2 If the surface is smooth with minimum interface links, horizontal shear force in topping at stage 2 $V_{uh2} \le \phi V_{nh}$ is $\phi v_{nh} b_i l_v$.

where

ϕV_{nh} = ultimate interface shear capacity

ϕV_{nh} = ultimate interface shear strength

17.5.3.1 If the surface is intentionally roughened without links alone, horizontal shear force in topping at stage 2 $V_{uh2} \le$ ultimate interface shear capacity ϕV_{nh} is $\phi v_{nh} b_i l_v$.

9.3.2.1 Ultimate partial safety factor $\phi = 0.75$

17.5.3.1 Shear stress v_{nh} = 0.56 N/mm² (81 psi)

11.6.9 Intentionally roughened means to an amplitude of 6 mm (0.2 in.).

Smooth surface

11.6.5 Smooth horizontal interface shear strength $V_{nh} \le \min\left\{0.2 f'_c l_v b_i; 5.5 l_v b_i\right\}$

11.4.6.3 Even if horizontal shear force in topping at stage 2 V_{uh2} < ultimate interface shear capacity ϕV_{nh}, provide

$$A_{vf,min}/s = \max\left\{0.062\sqrt{f'_{cT}}\, b_i/f_y;\ 0.35 b_i/f_y\right\},$$

where $A_{vf,min}$ is the minimum value of the area of interface shear reinforcement A_{vf}, s is spacing of interface shear reinforcing bars, b_i is the width of the interface and f_y is the characteristic strength of interface shear bars.

Note that this gives quite a high area of links (840 mm²/m [0.4 in.²/ft]) compared with the adopted minimum of 188 mm²/m (0.1 in.²/ft) given in the "Interface shear stress and reinforcement per EC2-1-1" section.

17.6.1 Interface shear links in the form of dowels or U loops are anchored to the center of the longitudinal cast-in-place concrete joints between hollow-core units. The loops should stand in a vertical position across the interface and have an anchorage of at least 10 times the diameter into the joint. Otherwise, have a bend at the bottom. Loop diameter is best suited to 10 or 12 mm (0.4 to 0.5 in.).

17.5.3.4 But if horizontal shear force in topping at stage 2 $V_{uh2} > \min\{\phi V_{nh}; \phi 3.5 b_i l_v\}$

11.6.4.1 Provide $A_{vf}/s = V_{uh2}/\mu f_y l_v$, where A_{vf} is the area of interface shear reinforcement.

11.6.4.3 where the coefficient of friction at the precast–to–cast-in-place concrete interface μ is 0.6 for a smooth surface after casting

11.6.6 f_y = characteristic yield strength of high tensile reinforcing bar, taken as 420 N/mm² (60,916 psi)

17.6.1 Spacing of interface shear reinforcing bars $s \leq \min\{\text{spacing } 4 \times \text{depth of composite slab; } 600 \text{ mm } (23.6 \text{ in.})\}$

Intentionally roughened surface

11.6.5 Roughened $V_{nh} \leq \min\left\{0.2 f'_{cT} l_v b_i;\ \left(3.3 + 0.08 f'_{cT}\right) l_v b_i;\ 11 l_v b_i\right\}$

17.5.3.4 If horizontal shear force in topping at stage 2 $V_{uh2} > \min\{\phi V_{nh}; \phi 3.5 b_i l_v\}$

11.6.4.1 Provide $A_{vf}/s = V_{u2}/\phi\mu f_y$ mm²/m run, where V_{u2} is ultimate vertical design shear force due to stage 2 loading and $\mu = 1.0$

17.5.3.3 But $V_{nh} = V_{uh2}/\phi \leq 1.8 b_i l_v + 0.6 f_y l_v A_{vf}/s$

Then $A_{vf}/s = (V_{uh2}/\phi - 1.8 b_i l_v)/0.6 f_y l_v$ but not less than $V_{u2}/\phi\mu f_y$

If horizontal shear force in topping at stage 2 $V_{uh2} \leq \min\{\phi V_{nh}; \phi 3.5 b_i l_v\}$, no designed interface links are required, but it is not mentioned whether minimum links are required.

Worked example for composite 1200 mm wide × 200 mm deep hollow-core unit plus 75 mm topping per ACI 318

Composite section properties of compound section with transformed area of tendons per ACI 318

The 28-day characteristic cylinder strength of cast-in-place concrete topping f'_{cT} = 25 N/mm² (3625 psi)

Young's modulus of cast-in-place concrete topping $E_{cT} = 4700 \times \sqrt{25}$ = 23.50 kN/mm² (3408 ksi)

Modular ratio m of cast-in-place to precast concrete = 23,500/29,725 = 0.791

Effective width of cast-in-place concrete topping at service b_{ef} = 1200 × 0.791 = 949 mm (37.4 in.)

Height to centroid of topping y_{bT} = 200 + 0.5 × 75 = 237.5 mm (9.4 in.)

Area of compound section $A_{c,co}$ = 156,388 + (949 × 75) = 227,539 mm² (353 in.²)

Height to centroid of composite compound section $y_{b,c,co}$ = (156,388 × 97.4 + 949 × 75.0 × 237.5)/227,539 = 141.2 mm (5.6 in.)

Second moment of area of composite compound section $I_{cc,co} = 1705.0 \times 10^6$ mm⁴ (4096 in.⁴)

Section modulus of composite compound section at bottom $Z_{b,c,co} = 1705.0 \times 10^6/141.2 = 12.08 \times 10^6$ mm³ (737 in.³)

Section modulus of composite compound section at top $Z_{t,c,co} = 1705.0 \times 10^6/58.8 = 28.99 \times 10^6$ mm³ (1769 in.³)

The final prestress in the bottom and top of the hollow-core unit and the section modulus are as follows:

- The maximum surface stress due to prestress in service at the bottom f_{pbe} is +11.56 N/mm² (1677 psi), and the maximum surface stress due to prestress in service at the top f_{pte} is -1.43 N/mm² (207 psi).
- The section modulus of the basic compound section at the bottom $Z_{b,co}$ is 7.307×10^6 mm³ (446 in.³), and the section modulus of the basic compound section at the top $Z_{t,co}$ is 6.932×10^6 mm³ (423 in.³).

Calculation for service moment of resistance of composite section $M_{sn,c}$ per ACI 318

Self weight of precast unit including infill from the previous paper[1] is 3.26 kN/m² (0.47 psi)

Then self-weight of slab per unit width w_1 = 1.2 x 3.26 = 3.91 kN/m (0.27 kip/ft)

For stage 1, allow 20 mm (0.8 in.) camber at installation before casting topping (and check later).

Equivalent depth of topping t' = 75 + 0.5 × 20 = 85 mm (3.3 in.)

Self-weight of topping per square meter = $24 \times 85/1000$ = 2.04 kN/m^2 (0.3 psi)

Self-weight of topping per unit width $w_T = 1.2 \times 2.04 = 2.45$ kN/m (0.17 kip/ft)

Service load due to self-weight of precast concrete unit plus wet cast-in-place concrete topping $w_1 + w_T = 3.91 + 2.45 = 6.36$ kN/m (0.44 kip/ft) per 1.2 m (4.0 ft) wide unit

Service moment at stage 1 $M_{s1} = 6.36 \times 8.000^2/8 = 50.89$ kN-m (37.54 kip-ft)

Bending stress at bottom due to stage 1 moment $f_{b1} = 50.89 \times 10^6/7.307 \times 10^6 = -6.97$ N/mm^2 (1011 psi) tension

Bending stress at top due to stage 1 moment $f_{t1} = 50.89 \times 10^6/6.932 \times 10^6 = +7.34$ N/mm^2 (1065 psi)

Maximum service moment due to imposed loads after topping $M_{s2} = 96.0$ kN-m (70.8 kip-ft)

Available bending stress at bottom due to stage 2 moment $f_{b2} = 96.0 \times 10^6/12.08 \times 10^6 = -7.95$ N/mm^2 (1153 psi) tension

Available bending stress at top due to stage 2 moment $f_{t2} = 96.0 \times 10^6/28.99 \times 10^6 = +3.31$ N/mm^2 (480 psi)

Final stresses

Net tension in bottom = $11.56 - 6.97 - 7.95 = -3.36 > -3.98$ N/mm^2 (577 psi) for service Class U

Net compression in top = $-1.44 + 7.34 + 3.31 = +9.21 < 0.6 \times 40 = +24.00$ N/mm^2 (3626 psi)

Service moment of resistance of composite section $M_{sn,c} = 50.89 + (11.56 - 6.97 + 3.98) \times 12.08 = 154.4 > 146.9$ kN-m (108.4 kip-ft)

Calculation for ultimate moment of resistance of the composite section $\phi M_{n,c}$ per ACI 318

Consider stage 1 moment acting on precast concrete alone at installation age ti of 28 days.

Ultimate design bending moment at stage 1 $M_{u,1} = 61.1$ kN-m (45.1 kip-ft)

Precast concrete 28-day characteristic cylinder strength f_c' at installation = 40.0 N/mm^2 (5800 psi)

Actual width of precast unit $b_1 = 1154$ mm (45.4 in.)

Effective depth of tendons in tension zone at stage 1 $d_{p1} = 159.2$ mm (6.3 in.)

Bending moment factor at stage 1 $K_1 = (61.1 \times 10^6/0.9)/(40 \times 1154 \times 159.22) = 0.058$

Lever arm at stage 1 $z_1 = \left[0.5 + \sqrt{(0.25 - 0.058/1.7)}\right]$

$\times 159.2 = 153.6$ mm (6.05 in.)

Depth to neutral axis at ultimate at stage 1 $c_1 = (159.2 - 153.6)/(0.5 \times 0.85) = 13.2$ mm (0.52 in.) $< h_{ft} = 40$ mm (1.6 in.)

Ultimate stress is not known until the ultimate moments of resistance of composite section at stage 2 $\phi M_{n,c2}$ are found. Therefore, take as design ultimate stress in tendons $f_{ps} = f_{pu}/1.1 = 1609$ N/mm^2 (233,366 psi).

Area of tendons required in stage 1 $A_{ps1} = 61.1 \times 10^6/(153.6 \times 1609) = 247$ mm^2 (0.38 in.^{2})

Net area of tendons $A_{ps2} = 766 - 247 = 519$ mm^2 (0.80 in.^{2})

$A_{ps2}/bd_{p2} = 519/(1200 \times 234.2) = 0.0018$

Design ultimate stress in tendons $f_{ps} = 1770 \times [1 - (0.40/0.850) \times 0.0018 \times (1770/25)] = 1661$ N/mm^2 (240,908 psi)

Depth to neutral axis at ultimate at stage 2 $c_2 = 519 \times 1661/(0.85 \times 25 \times 1200 \times 0.850) = 39.8$ mm (1.6 in.)

Depth of compressive stress block at ultimate $a = 0.850 \times 39.8 = 33.8$ mm (1.3 in.) < 115 mm (4.5 in.) (75 mm [2.9 in.] topping + 40 mm [1.6 in.] top flange)

Depth to centroid of concrete area of composite section $d_{n,c} = 0.5 \times 33.8 = 16.9$ mm (0.7 in.)

Lever arm at stage 2 $z_2 = 234.2 - 16.9 = 217.3$ mm (8.6 in.)

Ultimate moment of resistance of the composite section $\phi M_{n,c} = 61.1 + 0.9 \times 1661 \times 519 \times 217.3/10^6 = 229.6$ kN-m (169.4 kip-ft) > 208.9 kN-m (154.1 kip-ft)

Ultimate design load $w_u = 1.2 \times (3.91 + 1.2 \times 2.04 + 1.2 \times 1.5) + 1.6 \times 1.2 \times 8.5 = 26.11$ kN/m (1.8 kip/ft)

Ultimate design bending moment $M_u = 26.11 \times 8.0^2/8 = 208.9$ kN-m (154.1 kip-ft)

At the support where M_{u1} is 0, the ultimate moment of resistance of the composite section $\phi M_{n,c}$ is $0.9 \times 1609 \times 766 \times 210.0 = 233.0$ kN-m (171.9 kip-ft). The distribution between is taken as parabolic.

The results for the service and ultimate design bending moments and moments of resistance $M_{sn,c}$ and ultimate moment of resistance of the composite section $\phi M_{n,c}$ are shown in Fig. 7.

Calculation for cracking moment of resistance $M_{cr,c}$ per ACI 318

Concrete modulus of rupture $f_r = 0.63 \times = 3.98$ N/mm² (577.3 psi)

At the support where conditions are more onerous, the cracking moment of resistance of the composite section $M_{cr,c} = 11.59 \times 10^6 \times (10.84 + 3.98)/10^6 = 172.0$ kN-m (126.9 kip-ft)

$\phi M_{n,c}/M_{cr,c} = 233.0/172.0 = 1.35 > 1.2$ **Pass**

Calculation for flexurally uncracked shear capacity $\Phi V_{cw,c}$ per ACI 318

From the previous paper,[1] the bearing length l_b is 100 mm (4 in.), the design transmission length l_t is 625 mm (24.6 in.), axial prestress after losses f_{cp} is 4.81 N/mm² (697.6 psi), and the mean tensile strength of concrete f_t is 1.64 N/mm² (237.9 psi).

Critical distance from end to center of webs $l_x = 100 + 100.0 = 200$ mm (8 in.)

Distance to shear plane ratio $\alpha = l_x/l_t = 200/561 = 0.357$

Ultimate partial safety factor $\alpha = 0.75$

Ultimate shear strength in flexurally uncracked section $v_{cw} = 0.29 \times = 1.83$ N/mm² (265.4 psi)

Flexurally uncracked shear capacity of composite section $\phi V_{cw,c}$ at shear plane $= 0.75 \times [1.83 + 0.357 \times 0.3 \times 4.81) \times 303 \times 234]/10^3 = 125.0$ kN (28.1 kip)

Flexurally uncracked shear capacity of composite section $\phi V_{cw,c}$ at end of the design transmission length $l_t = 0.75 \times [1.83 + 0.3 \times 4.81) \times 303 \times 234]/10^3 = 174.4$ kN (39.2 kip)

The increase in $\phi V_{cw,c}$ is linear between the position at the shear plane and the end of the transmission length.

Calculation for flexurally cracked shear capacity of composite section $\Phi V_{ci,c}$ per ACI 318

The flexurally cracked shear capacity of the composite section $\phi V_{ci,c}$ varies according to factored imposed ultimate shear and moment at the distance along the span of the slab from the center of the support x due to externally imposed dead and live loads only M_{max} and ultimate shear force at the distance along the span of the slab from the center of the support x due to externally imposed dead and live loads V_i, unfactored dead load V_d, and moment causing cracking $M_{cre,c}$ along the span.

Flexurally cracked shear capacity of composite section $\phi V_{ci,c} = 0.75\,(0.05\sqrt{f'_c}\,b_w d_{p2} + V_d + M_{cre,c} V_i/M_{max})$

where $M_{cre,c} = Z_{b,co}\left(0.5\sqrt{f'_c} + f_{pe} - f_{d2}\right)$

If the minimum value of flexurally cracked shear capacity

$\phi V_{ci,c,min} \geq 0.75 \times 0.17\sqrt{f'_c}\,b_w d_{p2}$ and

$\phi V_{ci,c,max} \geq 0.75 \times 0.42\sqrt{f'_c}\,b_w d_{p2}$

Then minimum value of the flexurally cracked shear capacity $\phi V_{ci,c,min} = 57.2$ kN (12.9 kip) and the maximum value of the flexurally cracked shear capacity $\phi V_{ci,c,max} = 141.36$ kN (31.8 kip)

The distribution of the flexurally cracked shear capacity of the composite section $\phi V_{ci,c}$ along the span is shown in Fig. 8.

Calculation for camber and deflections per ACI 318

From the previous paper,[1] deflection at installation is -41.2 + 20.8 = -20.4 mm (0.80 in.) (20 mm [0.79 in.] allowed to determine if average depth of topping is OK).

Note that in the previous paper[1] the creep coefficient for the deflection due to the self-weight of the precast unit at installation was 0.85. This coefficient can then be deducted from the long-term deflections (in the following section) for dead loads applied to the composite slab.

Deflection due to self-weight of topping $\delta_5 = 5 \times 1.2 \times 2.04 \times 8000^4/(384 \times 29{,}725 \times 711.4 \times 10^6) = 6.2$ mm (0.24 in.)

Net deflection after casting topping = -14.2 mm (0.56 in.)

Long-term deflections

Creep coefficients for deflections due to:

camber $\psi_{28} = 1.20$

self-weight of precast concrete $\psi_{28} = 1.40$

self-weight of topping $\psi_{28} = 1.30$

imposed live load $\psi_{\infty} = 2.00$

imposed dead load after topping hardened $\psi'_{\infty} = 2.00 - 0.85 = 1.15$

Long-term upward camber due to prestress $\delta_7 = -41.2 - [(1.20 \times 846{,}988) - (846{,}988 - 730{,}628)] \times 58.2 \times 8000^2/(8 \times 29725 \times 1705.0 \times 10^6) = -49.4$ mm (1.95 in.)

Static and initial creep deflection due to self-weight of precast concrete $\delta_4 = 20.8$ mm (0.82 in.)

Static deflection of topping $\delta_5 = 6.2$ mm (0.24 in.)

Static plus creep deflection due to self-weight of precast concrete and topping $\delta_8 = 20.8 + 6.2 + [5 \times 1.2 \times (1.40 \times 3.256 + 1.30 \times 2.04) \times 8000^4/(384 \times 29725 \times 1705.0 \times 10^6)] = 36.1$ mm (1.42 in.)

Static plus creep deflection for dead and live loads $\delta_9 = 5 \times 1.2 \times [(1 + 1.15) \times 1.50 + (1 + 2.0) \times 0.3 \times 8.50] \times 8000^4/(384 \times 29725 \times 1705.0 \times 10^6) = 4.1 + 9.7^* = 13.8$ mm (0.54 in.)

Long-term midspan deflection due to all uniformly distributed loading = 20.8 + 6.2 + 9.1 + 13.8 − 49.4 = +0.5 < 8000/240 = 33.3 mm (1.31 in.)

*Active deflection after installation due to live load $\delta_{10} = 9.7$ mm (0.38 in.)

Active deflection due to all imposed loads and changes in camber $\delta_{11} = 9.1 + 13.8 - (49.4 - 41.2) = 14.7 < 8000/360 = 22.2$ mm (0.87 in.) for floors with no brittle finishes

Interface shear per ACI 318

Roughened surface texture with the coefficient of friction at the precast–to–cast-in-place concrete interface $\mu = 1$

Calculate the horizontal force in the topping due to Stage 2 loads

Ultimate design bending moment $M_u = 208.9$ kN-m (154.1 kip-ft)

Ultimate design bending moment at stage 1 $M_{u1} = 61.1$ kN-m (45.1 kip-ft)

Lever arm at stage 2 $z_2 = 217.3$ mm (8.6 in.)

Horizontal shear force in topping at stage 2 $V_{uh2} = \min\{0.85 \times 25 \times 1200 \times 75 = 1839.2$ kN [413.5 kip]; $(208.9 - 61.1)/217.3 = 680.4$ kN [152.97 kip]$\}$

Width of interface $b_i = 1154$ mm (45.4 in.)

Shear span distance $l_v = 4000$ mm (157.5 in.)

Ultimate partial safety factor $\phi = 0.75$

Shear stress $v_{nh} = 0.56$ N/mm^2 (81.22 psi)

Ultimate interface shear capacity $\phi V_{nh} = 0.75 \times 0.56 \times 1154 \times 4000/10^3 = 1938.7$ kN (435.86 kip) > 680.4 kN (152.97 kip)

Use a roughened surface on the top of the precast unit with no minimum links.

Conclusion

A typical 1200 mm (48 in.) wide × 200 mm (8 in.) deep prestressed concrete hollow-core floor unit is analyzed and designed as a composite slab with a 75 mm (3 in.) deep structural topping in order to make a comparison between Eurocode 2 and ACI 318. The slab is simply supported over an effective span of 8.0 m (26 ft) to carry uniformly distributed dead and live loads of 3.5 and 8.5 kN/m^2 (0.525 and 1.275 psi), respectively.

The comparison includes two-stage composite service and ultimate moments of resistance, ultimate composite shear capacities, camber and deflections, and the interface shear between the hollow-core unit and cast-in-place concrete topping. The worked examples for Eurocode 2 and ACI 318 gave the following conclusions regarding structural capacities (Table 1).

The ratio of the service moment/resistance $M_s/M_{sr,c}$ for EC2 and $M_s/M_{sn,c}$ for ACI 318 is 5% greater than for the ratio of the ultimate design bending moment to the ultimate moment of resistance of the composite section $M_{Ed}/M_{Rd,c}$ for EC2 and $M_u/\phi M_{n,c}$ for ACI 318. This means that the composite slab is just critical at the serviceability limit state. Compared with the hollow-core unit alone, composite action increases the moments of resistance by 39% to 42% according to EC2, and 35% to 44% according to ACI 318, the latter value showing larger increases at ultimate.

Regarding shear capacity $V_{Rd,c,c}$, EC2 is only limiting when the span–to–total depth ratio (hollow-core plus topping) is less than about 26, but for ACI 318 the flexurally cracked shear capacity of the composite section $\phi V_{ci,c}$ is limiting when the span–to–total depth ratio is 30, showing that the flexurally cracked shear capacity is often the governing criteria. Compared with the hollow-core unit alone, composite action increases the flexurally uncracked shear capacity by 24% according to EC2, the reduced increase being in part due to the two-stage analysis. The increase is 46% according to ACI 318, exactly the increase in the effective depth of the strands.

Overall, for an effective span of 8.0 m (26 ft), the area of strand required to resist the maximum possible uniformly distributed loading is 746 mm^2 (1.16 in.^{2}) in EC2 and 716 mm^2 (1.11 in.^{2}) in ACI 318, about 4% less, reflecting the greater resistance for ACI 318.

The effect of propping increased $M_{sr,c}$ for EC2 by 11.0% and $M_{sn,c}$ for ACI 318 by 10.4%. This would enable the imposed live load to be increased from 8.5 to 9.7 kN/m^2 (1.2 to 1.4 psi), or otherwise enable the span to be increased from 8.0 to 8.4 m (26 to 27.6 ft).

For the interface shear between the hollow-core unit and cast-in-place concrete topping, there are no interface links to ACI 318 because hollow-core units have a roughened surface. A small number of interface links (U loops cast into the longitudinal joints), which have a factory produced surface, are required by EC2.

References

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4. Elliott, K. S. 2017. *Precast Concrete Structures*. 2nd ed. Boca Raton, FL: CRC Press/Taylor Francis.

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Notation

EC2-1-1 and EN 1168

$A_{c,c,co}$ = area of composite compound section

$A_{c,co}$ = area of compound section

A_j = area of interface joint

A_p = area of tendons

A_{p1} = area of tendons required in stage 1

A_{p2} = remainder of area of tendons A_p minus area of tendons required in stage 1 A_{p1} (available for stage 2)

A_s = area of interface shear reinforcement

$A_{s,min}$ = minimum value of area of interface shear reinforcement

b = nominal width of hollow-core unit

b_{ef} = effective width of cast-in-place concrete topping at service

b_{eff} = effective width of cast-in-place concrete topping at ultimate

b_i = width of interface = b_t

b_t = actual width at top of hollow-core unit

b_w = total width of webs

c = cohesion factor for interface shear strength

$C_{Rd,c}$ = concrete shear strength factor

d = effective depth of tendons in tension zone

d_1 = effective depth of tendons in tension zone at stage 1

d_2 = effective depth of tendons in tension zone at stage 2

$d_{n,c}$ = depth to centroid of concrete area of composite section

E_{cm} = Young's modulus of concrete

E_{cmT} = Young's modulus of cast-in-place concrete topping

$f_{bd}(t)$ = ultimate bond strength of precast concrete at transfer

f_b = final stress at bottom due to prestress and stage 1 and 2 moments

f_{b1} = bending stress at bottom due to stage 1 moment

f_{b2} = available bending stress at bottom due to stage 2 moment

f_{ck} = 28-day characteristic cylinder strength

$f_{ck}(t_i)$ = value of 28-day characteristic cylinder strength f_{ck} at time t_i

f_{ckT} = 28-day characteristic cylinder strength of cast-in-place concrete topping

f_{cm} = mean compressive strength at 28 days

$f_{cm}(t_i)$ = mean compressive strength at t_i days

f_{ctd} = design tensile strength of concrete

$f_{ctd}(t)$ = design tensile strength of concrete at transfer

f_{ctm} = mean tensile strength of concrete

f_{ctmT} = mean tensile strength of cast-in-place concrete topping

f_p = design ultimate stress in tendons

f_{p1} = design stress of tendons at stage 1

f_{pk} = characteristic yield strength of tendons

f_t = final stress at top due to prestress and stage 1 and 2 moments

f_{t1} = bending stress at top due to stage 1 moment

f_{t2} = available bending stress at top due to stage 2 moment

f_{yd} = design strength of interface shear bars

f_{yk} = characteristic strength of interface shear bars

F_c = total ultimate compression in topping and hollow-core unit at stage 2

$F_{c,t}$ = ultimate compression in topping at stage 2

F_{pmi} = pretensioning force at installation

F_{po} = final prestress force

h = depth of hollow-core unit

h_{ft} = depth of top flange

I_c = second moment of area of basic section (hollow-core unit alone)

$I_{c,c}$ = second moment of area of composite section

$I_{c,c,co}$ = second moment of area of composite compound section

$I_{c,co}$ = second moment of area of basic compound section

k = shear strength depth factor

K_{1w} = bending moment factor based on width of web at stage 1

K_1 = bending moment factor at stage 1

K_2 = bending moment factor at stage 2 (for interface shear)

l_b = bearing length

l_{pt} = basic transmission length

l_{pt2} = design transmission length

l_x = distance to critical section (shear plane)

L = effective span of composite hollow-core slab

L_v = shear span of composite hollow-core slab

m = modular ratio

$M_{cr,c}$ = cracking moment of resistance of composite section

M_{Ed} = ultimate design bending moment

$M_{Ed,1}$ = ultimate design bending moment at stage 1

$M_{Ed,2}$ = ultimate design bending moment at stage 2

M_{Rd} = total ultimate moment of resistance $M_{Rd,f} + M_{Rd,w}$

$M_{Rd,c}$ = ultimate moment of resistance of composite section

$M'_{Rd,c}$ = ultimate moment of resistance of composite section $M_{Rd,c}$ at supports

$M_{Rd,c2}$ = ultimate moment of resistance of composite section at stage 2

$M_{Rd,f}$ = ultimate moment of resistance due to top flange

$M_{Rd,w}$ = ultimate moment of resistance due to webs

M_s = service moment = $M_{s1} + M_{s2}$

$M_{sr,c}$ = service moment of resistance of composite section

M_{s1} = service moment at stage 1

M_{s2} = service moment at stage 2

R = prop reaction

s = cement factor

s = spacing of interface shear reinforcement bars

S_c = first moment of area of hollow-core unit

$S_{c,c}$ = first moment of area of hollow-core unit of composite section

t = depth of topping

t' = mean depth of topping (allowing for camber)

t_i = installation age

v_{Edi} = ultimate design interface shear stress

$v_{Ed,1}$ = ultimate shear stress at stage 1

v_{min} = minimum concrete shear strength

v_{Rdi} = ultimate interface shear strength

V_{Ed} = ultimate design shear force

$V_{Ed,1}$ = ultimate design shear force at stage 1

$V_{Ed,2}$ = ultimate design shear force at stage 2

$V_{Rd,c,c}$ = flexurally uncracked shear capacity of composite section

$V_{Rd,c,c2}$ = flexurally uncracked shear capacity of composite section at stage 2

$V_{Rd,cr,c}$ = flexurally cracked shear capacity of composite section

w_{Ed} = ultimate design load

w_T = self-weight of topping per unit width

w_1 = self-weight of slab per unit width (precast unit plus infill)

w_2 = imposed dead load per unit width (service)

w_3 = imposed live load per unit width (service)

x = distance along span from the center of support

x_1 = depth to neutral axis at ultimate at stage 1

x_2 = depth to neutral axis at ultimate at stage 2

X = depth to neutral axis in composite section

y_b = height to centroid of basic section

$y_{b,c}$ = height to centroid of composite section

$y_{b,c,co}$ = height to centroid of composite compound section

$y_{b,co}$ = height to centroid of basic compound section

y_{bT} = height to centroid of topping

z_{cp} = eccentricity of pretensioning force

z_1 = lever arm at stage 1

z_2 = lever arm at stage 2

Z_b = section modulus at bottom of basic section (hollow-core unit only)

$Z_{b,c,co}$ = section modulus of composite compound section at bottom

$Z_{b,co}$ = section modulus of basic compound section at bottom

$Z_{t,c,co}$ = section modulus of composite compound section at top

$Z_{t,co}$ = section modulus of basic compound section at top

α_1 = distance to shear plane ratio = l_x/l_{pt2}

β = factor for the design of T-shaped sections

δ_{ins} = net camber at installation = $\delta_3 + \delta_4$

δ_1 = upward camber due to prestress at transfer

δ_2 = deflection due to self-weight of unit at transfer

δ_3 = upward camber due to prestress at installation

δ_4 = deflection due to self-weight of unit and infill at installation

δ_5 = deflection due to self-weight of the topping at installation

δ_6 = equivalent upward deflection due to propping

δ_7 = long-term upward camber due to prestress

δ_8 = long-term deflection due to self-weight of unit, infill, and topping

δ_9 = long-term deflection due to imposed dead and live loads

δ_{10} = long-term active deflection due to live loads only

δ_{11} = long-term active deflection due to changes in camber and live load, plus creep deflections due to self-weight, topping, and imposed dead loads

δ_{12} = downward deflection due to removal of props

ε_p = total ultimate strain in tendons

ε_s = limiting strain in tendons

μ = coefficient of friction for interface shear strength

ϱ_1 = steel area ratio

σ_b = maximum surface stresses in service at bottom

σ_{cp} = axial prestress after losses

σ_n = normal (vertical) stress acting on interface

σ_t = maximum surface stresses in service at top

χ = long-term concrete aging coefficient

ψ_1 = effective creep coefficient for deflection at installation

ψ_2 = quasi-permanent live load factor

ψ_{28} = effective creep coefficient after installation

ψ_{∞} = long-term creep coefficient for deflections

ACI 318

a = depth of compressive stress block at ultimate

A_c = interface shear contact area

$A_{c,co}$ = area of compound section

A_{ps} = area of tendons

A_{ps1} = area of tendons required in stage 1

A_{ps2} = remainder area of tendons A_{ps} minus area of tendons required in stage 1 A_{ps1} (available for stage 2)

A_{vf} = area of interface shear reinforcement

$A_{vf,min}$ = minimum value of area of interface shear reinforcement A_{vf}

b = nominal width of hollow-core unit

b_{ef} = effective width of cast-in-place concrete topping at service

b_{eff} = effective width of cast-in-place concrete topping at ultimate

b_t = actual width at top of hollow-core unit

b_v = width of interface (only referred to in clause 17.5.3)

b_w = total width of webs

b_1 = width of interface = b_t

c_1 = depth to neutral axis at ultimate at stage 1

c_2 = depth to neutral axis at ultimate at stage 2

d = effective depth to tendons (only referred to in clause 17.5.3)

$d_{n,c}$ = depth to centroid of concrete area of composite section

d_p = effective depth of tendons in previous paper1

d_{p1} = effective depth of tendons in tension zone at stage 1

d_{p2} = effective depth of tendons in tension zone at stage 2

e = eccentricity of pretensioning force

E_c = Young's modulus of concrete

E_{cT} = Young's modulus of cast-in-place concrete topping

f_{b1} = bending stress at bottom due to stage 1 moment

f_{b2} = available bending stress at bottom due to stage 2 moment

f'_c = 28-day characteristic cylinder strength

$f'_c(t_i)$ = strength of precast concrete at installation at t_i days

f_{cp} = axial prestress after losses

f'_{cT} = 28-day characteristic cylinder strength of cast-in-place concrete topping

f_{d1} = tension stress due to service dead load at stage 1

f_{d2} = tension stress due to service dead load at stage 2

f_{pbe} = maximum surface stress due to prestress in service at bottom

f_{ps} = design ultimate stress in tendons

f_{ps1} = design stress of tendons at stage 1

f_{pte} = maximum surface stress due to prestress in service at top

f_{pu} = characteristic yield strength of tendons (known as specified strength in ACI 318)

f_r = modulus of rupture

f_t = mean tensile strength of concrete

f_{t1} = bending stress at top due to stage 1 moment

f_{t2} = available bending stress at top due to stage 2 moment

f_y = characteristic strength of interface shear bars

h_{ft} = depth of top flange

$I_{c,c,co}$ = second moment of area of composite compound section

$I_{c,co}$ = second moment of area of compound section

K_1 = bending moment factor at stage 1

K_{1w} = bending moment factor based on width of web at stage 1

l_b = bearing length

l_t = design transmission length

l_v = distance to point of zero shear from ends = $\min\{l_{v1}, l_{v2}\}$

l_{v1} = distance to zero shear force from left end support

l_{v2} = distance to zero shear force from right end support

l_x = distance to critical section

L = effective span of composite hollow-core slab

m = modular ratio

$M_{cr,c}$ = cracking moment of resistance of composite section

$M_{cre,c}$ = moment required to cause flexural cracking in composite section

M_{d1} = moment due to dead load at stage 1

M_{d2} = moment due to dead load at stage 2

M_{max} = ultimate moment at distance along span of slab from the center of support x due to externally imposed dead and live loads

M_s = service moment = $M_{s1} + M_{s2}$

$M_{sn,c}$ = service moment of resistance of composite section

M_{s1} = service moment at stage 1

M_{s2} = service moment at stage 2

M_u = ultimate design bending moment

$M_{u,1}$ = ultimate design bending moment at stage 1

P = final prestressing force

P_{ins} = pretensioning force at installation

s = spacing of interface shear reinforcing bars

t = depth of topping

t' = mean depth of topping (allowing for camber)

t_i = installation age

v_{cw} = ultimate concrete shear strength in flexurally uncracked section

v_{nh} = shear stress

$V_{c,c}$ = the least of ultimate shear capacity in flexurally uncracked section $V_{cw,c}$ or ultimate shear capacity in flexurally cracked section $V_{ci,c}$

V_{ci} = shear web capacity in previous paper[1]

$V_{ci,c}$ = ultimate shear capacity in flexurally cracked section

$V_{cw,c}$ = ultimate shear capacity in flexurally uncracked section

V_d = shear force at distance along span of slab from the center of support x for service (unfactored) dead load

V_i = ultimate shear force at distance along span of slab from the center of support x due to externally imposed dead and live loads

V_{nh} = horizontal interface shear strength

V_u = ultimate design shear force

V_{u2} = ultimate vertical design shear force due to stage 2 loading

V_{uh2} = horizontal shear force in topping at stage 2

w_T = self-weight of topping per unit width

w_u = ultimate design load per unit width

w_1 = self-weight of slab per unit width (precast unit plus infill)

w_2 = floor dead load per unit width

w_3 = floor live load per unit width

x = distance along span of slab from the center of support

$y_{b,c,co}$ = height to centroid of composite compound section

y_{bT} = height to centroid of topping

z_1 = lever arm at stage 1

z_2 = lever arm at stage 2

Z_b = section modulus at bottom of basic section (hollow-core unit only)

$Z_{b,c,co}$ = section modulus of composite compound section at bottom

$Z_{b,co}$ = section modulus of basic compound section at bottom

$Z_{t,c,co}$ = section modulus of composite compound section at top

$Z_{t,co}$ = section modulus of basic compound section at top

α = distance to shear plane ratio = l_x/l_{pt2}

β_1 = rectangular stress block factor

δ_{ins} = net camber at installation = $\delta_3 + \delta_4$

δ_3 = upward camber due to prestress at installation

δ_4 = deflection due to self-weight of the hollow-core unit and infill at installation

δ_5 = deflection due to self-weight of the topping at installation

δ_7 = long-term upward camber due to prestress

δ_8 = long-term deflection due to self-weight of unit, infill, and topping

δ_9 = long-term deflection due to imposed dead and live loads

δ_{10} = long-term active deflection due to live loads only

δ_{11} = long-term active deflection due to changes in camber and live load, plus creep deflections due to self-weight, topping, and imposed dead loads

ζ = factor for the design of T-shaped sections

μ = coefficient of friction at precast–to–cast-in-place concrete interface

ϕ = ultimate partial safety factor

$\phi M_{n,c}$ = ultimate moment of resistance of composite section

$\phi M_{n,c}(x)$ = ultimate moment of resistance of composite section at distance x from support

$\phi M'_{n,c}$ = ultimate moment of resistance of composite section $\phi M_{n,c}$ at supports

$\phi M_{n,c2}$ = ultimate moments of resistance of composite section at stage 2

ϕv_{nh} = ultimate interface shear strength

$\phi V_{ci,c}$ = flexurally cracked shear capacity of composite section

$\phi V_{ci,c,max}$ = maximum value of flexurally cracked shear capacity $\phi V_{ci,c}$

$\phi V_{ci,c,min}$ = minimum value of flexurally cracked shear capacity $\phi V_{cc,i}$

$\phi V_{cw,c}$ = flexurally uncracked shear capacity of composite section

ϕV_{nh} = ultimate interface shear capacity

ψ = creep coefficient

ψ_T = long-term creep coefficient for deflections due to cast-in-place concrete topping

ψ_1 = effective creep coefficient for deflection at installation

ψ_2 = quasi-permanent live load factor

ψ_{28} = effective creep coefficient after installation

ψ_∞ = long-term creep coefficient for deflections due to live loads

ψ'_∞ = long-term creep coefficient for deflections due to dead loads

Appendix: Derivation of solution for the lever arm and area of reinforcement in a flanged cross-section

This appendix contains the derivation of a solution for the lever arm and area of reinforcement in a flanged cross-section (**Fig. A.1**).

The compressive force in flanges $F_{c,f}$ is calculated as follows:

$F_{c,f}$ $= 0.567 f_{ck}(b - b_w)h_f$

where

f_{ck} = 28-day characteristic cylinder strength

b = nominal width of hollow-core unit

b_w = total width of webs

h_f = depth of top flange

The ultimate moment of resistance due to the force $F_{c,f}$ in the top flange $M_{Rd,f}$ is calculated as follows:

$M_{Rd,f}$ $= 0.567 f_{ck}(b - b_w)h_f(d - h_f/2)$

or $= 1.134 f_{ck}(b - b_w)h_f(d - h_f/2)/2$

where

d = effective depth of tendons in tension zone

The compressive force in the webs $F_{c,w}$ is calculated as follows:

$F_{c,w}$ $= 0.567 f_{ck} b_w 0.8x$

where

x = depth to the neutral axis

The ultimate moment of resistance due to the force $F_{c,f}$ in the webs $M_{Rd,w}$ is calculated as follows:

$M_{Rd,w}$ $= 0.567 f_{ck} b_w 0.8xz = 1.134 f_{ck} b_w (d - z)z$

or $= 1.134 f_{ck} b_w dz - 1.134 f_{ck} b_w z_2$

where

z = lever arm

Adding $M_{Rd,f} + M_{Rd,w} = M_{Rd}$ and dividing by $f_{ck} b_w d_2$ gives $M_{Rd}/f_{ck} b_w d^2$ equal to $1.134(b - b_w)h_f(d - h_f/2)/(2b_w\, d_2) + 1.134(z/d) - 1.134(z/d)^2$.

Let the factor for the design of T-shaped sections β equal $(b - b_w)h_f(d - 0.5\, h_f)/(2b_w d_2)$.

Let $K_w = M_{Ed}/f_{ck} b_w d_2$ because in the limit, ultimate design bending moment M_{Ed} is equal to M_{Rd}.

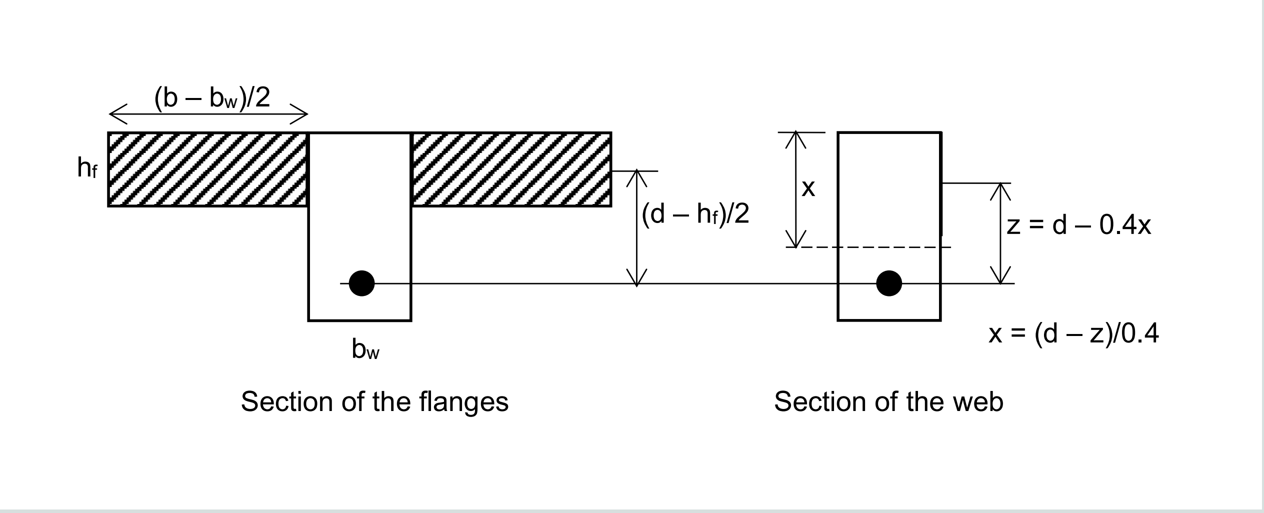

Figure A.1

Then $K_w = 1.134\beta + 1.134(z/d) - 1.134(z/d)^2$.

Giving a quadratic $1.134(z/d)^2 - 1.134(z/d) + (Kw - 1.134\beta) = 0$

$(z/d)^2 - (z/d) + (K_w/1.134 - \beta) = 0$

$$z/d = \left[1 \pm \sqrt{1^2 - 4\left(\frac{K_w}{1.134} - \beta\right)}\right]/2$$

or rewritten,

$$z = \left[0.5 + \sqrt{0.25 - \left(\frac{K_w}{1.134} - \beta\right)}\right]d \le 0.95d$$

Then $A_s = [M_{Rd,f}/(0.87f_{yk}\{d - h_f\}/2)] + [(M_{Ed} - M_{Rd,f})/(0.87f_{yk}z)]$

where

A_s = area of bars

f_{yk} = characteristic strength of main bars

Notation

A_s = area of bars

b = nominal width of hollow-core unit

b_w = total width of webs

d = effective depth of tendons in tension zone

f_{ck} = 28-day characteristic cylinder strength

f_{yk} = characteristic strength of main bars

$F_{c,f}$ = compressive force in top flange

$F_{c,w}$ = compressive force in web

h_f = depth of top flange

K_w = bending moment factor based on width of web

M_{Ed} = ultimate design bending moment

M_{Rd} = total ultimate moment of resistance $M_{Rd,f} + M_{Rd,w}$

$M_{Rd,f}$ = ultimate moment of resistance due to top flange

$M_{Rd,w}$ = ultimate moment of resistance due to webs

x = depth to the neutral axis

z = lever arm

β = factor for the design of T-shaped sections

About the author

Kim S. Elliott, PhD, is a consultant to the precast concrete industry in the United Kingdom. He was senior lecturer at Nottingham University in the United Kingdom from 1987 to 2010 and was formerly at Trent Concrete Structures Ltd. UK. He has also been a consultant to three organizations in Malaysia. He is a member of *fib* Commission 6 on Prefabrication, where he has made contributions to six manuals and technical bulletins and is the author of *Multi-Storey Precast Concrete Frame Structures* and *Precast Concrete Structures*. He was chairman of the European research project COST C1 on semirigid connections in precast concrete structures from 1992 to 1999. He has lectured on precast concrete structures 45 times in 16 countries, including Malaysia, Singapore, Australia, Korea, Brazil, Bahrain, South Africa, Barbados, Austria, Spain, and Scandinavia, and at 30 universities in the UK.

Abstract

A typical 1200 mm (48 in.) wide × 200 mm (8 in.) deep prestressed concrete hollow-core floor unit is analyzed and designed as a composite slab with a 75 mm (3 in.) deep structural topping in order to make a comparison between *Eurocode 2: Design of Concrete Structures* (EC2-1-1) and the American Concrete Institute's *Building Code Requirements for Structural Concrete (ACI 318-08) and Commentary (ACI 318R-08)*. This is the second of two papers. The first was published in *PCI Journal* in the March–April 2021 issue and designed a noncomposite hollow-core unit. The present comparison includes two-stage composite service and ultimate moments of resistance, ultimate composite shear capacities, camber and deflections, and interface shear between the hollow-core unit and cast-in-place concrete topping. Precast concrete cylinder strength was 40 N/mm^2 (5800 psi) and the cast-in-place concrete topping was 25 N/mm^2 (3625 psi). The hollow-core unit was pretensioned using 10 number seven-wire helical strands. Worked examples are presented in parallel formation for EC2 and ACI 318.

The service and ultimate moments of resistance were respectively 3.2% and 5.1% greater when calculated based on ACI 318. The uncracked shear capacity was7.4% greater for the ACI 318 calculation, but the flexurally cracked capacity was 80% lower. This can often be the limiting criteria for ACI 318, but rarely for EC2. The effect of propping increased the service moments of resistance by about 11% in both codes.

The ratio of the service moment to resistance $M_s/M_{sr,c}$ for EC2 and $M_s/M_{sn,c}$ for ACI was 5% greater than for the ratio of the ultimate design bending moment to the ultimate moment of resistance of the composite section $M_{Ed}/M_{Rd,c}$ for EC2 and $M_u/\phi M_{n,c}$ for ACI 318. This means that the composite slab is just critical at the serviceability limit state. Regarding shear capacity $V_{Rd,c,c}$, EC2 was only limiting when the span to total depth ratio for the hollow-core plus topping was less than about 26, but for ACI 318, the flexurally cracked shear capacity of the composite section $\phi V_{ci,c}$ was limiting when the span to total depth ratio was 30, showing that shear cracking in flexure is often the governing criteria.

For the interface shear between the hollow-core unit and cast-in-place concrete topping, there are no interface links to ACI 318 because hollow-core units have a roughened surface. A small number of interface links, which have a factory-produced surface, are required by EC2.

Keywords

ACI 318, composite floor slab, deflection, Eurocode 2, floor unit, hollow-core, pretensioning, propped slab, service stress, shear capacity, structural topping, ultimate strength.

Review policy

This paper was reviewed in accordance with the Precast/Prestressed Concrete Institute's peer-review process. The Precast/Prestressed Concrete Institute is not responsible for statements made by authors of papers in *PCI Journal*. No payment is offered.

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