Comparison of the design of prestressed concrete hollow-core floor units with Eurocode 2 and ACI 318

Kim S. Elliott

- The paper presents procedures, equations, and design examples to compare *Eurocode 2: Design of Concrete Structures—Part 1-1: General Rules and Rules for Buildings (with National Application Parameters)* and the American Concrete Institute's *Building Code Requirements for Structural Concrete (ACI 318-08) and Commentary (ACI 318R-08)* methodologies for hollow-core unit applications.

- In the examples, the prestressed concrete hollow-core unit is assumed to be 1200 mm (48 in.) wide × 200 mm (8 in.) deep with four 9.3 mm (0.37 in.) diameter strands and six 12.5 mm (0.49 in.) diameter strands for a simply supported span of 8.0 m (26 ft) to carry imposed uniformly distributed dead loads of 2.0 kN/m² (0.3 psi) and live loads of 5.0 kN/m² (0.75 psi).

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The 1200 mm (48 in.) wide × 200 mm (8 in.) deep prestressed concrete hollow-core unit shown in cross section in **Fig. 1** is analyzed and designed to make a comparison between the procedures according to *Eurocode 2: Design of Concrete Structures—Part 1-1: General Rules and Rules for Buildings (with National Application Parameters)* (EC2)[1] and the American Concrete Institute's *Building Code Requirements for Structural Concrete (ACI 318-08) and Commentary (ACI 318R-08)*.[2] The purpose of this paper is to compare the design methodologies as well as present standard calculations and worked examples to serve as design references.

In the design examples—following both EC2 and ACI 318 methodologies—the author calculated the service and ultimate moments of resistance and ultimate shear capacities and then determined the position where the section was flexurally cracked. The author then calculated the amount of precamber, short-term deflection at installation, final long-term deflection, and active deflection due to imposed loads. Both EC2 and ACI 318 variables appear throughout this paper. For clarification, see the notation section at the end of the paper.

Hollow-core unit manufacturing and design

Precast concrete hollow-core units are manufactured by extrusion or slip forming concrete through a machine that creates the cores in a continuous process along a steel bed that is typically 100 m (330 ft) long (**Fig. 2**). Approximately

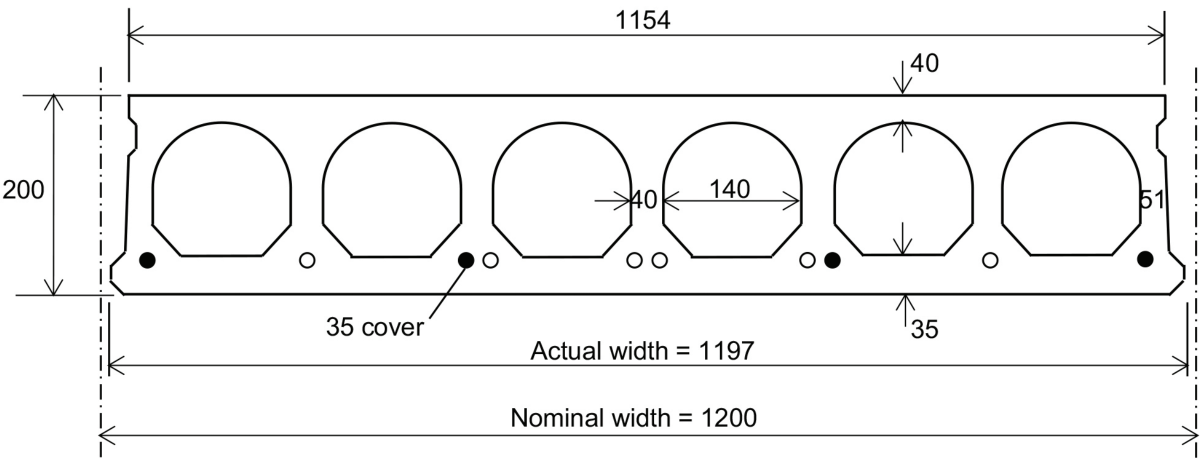

Figure 1. Cross-sectional hollow-core unit examined in this paper. Strands are represented by solid dots (four 9.3 mm diameter) and open dots (six 12.5 mm diameter). Note: All dimensions are in millimeters. 1 mm = 0.0394 in.

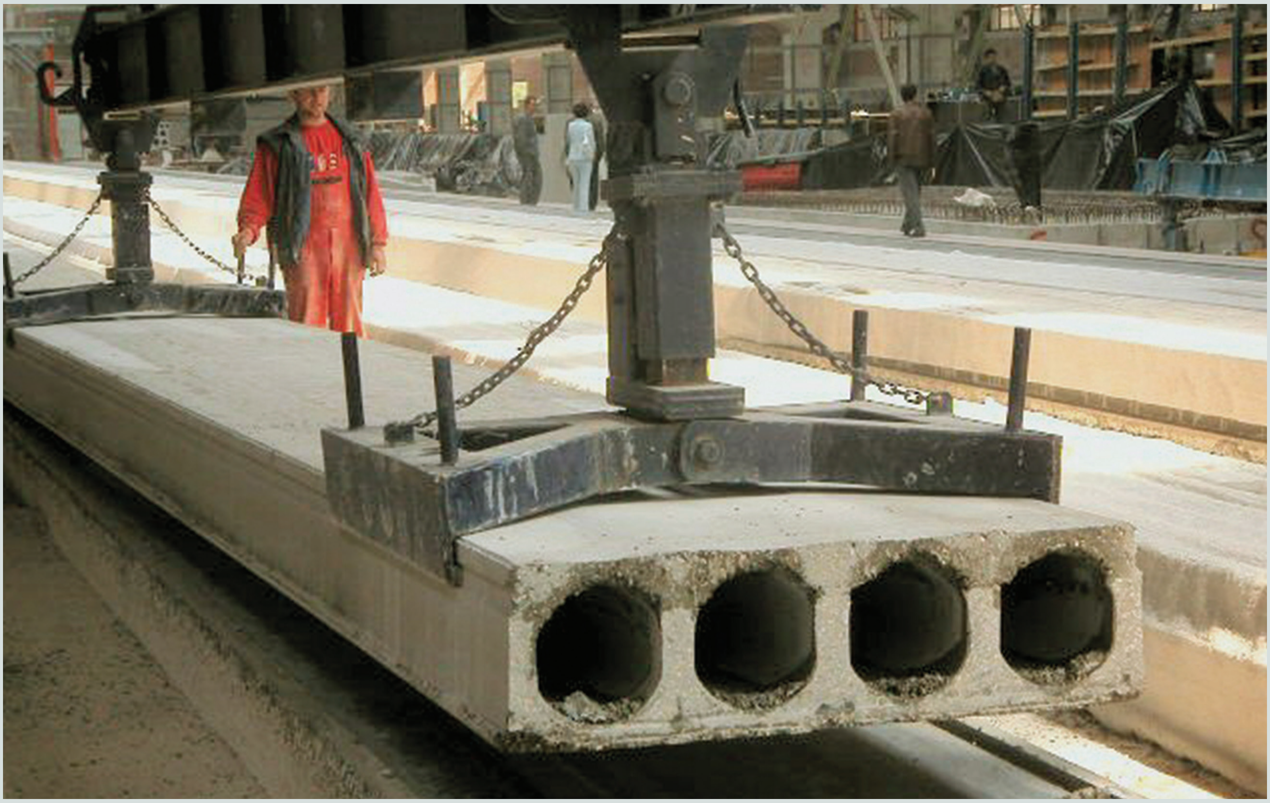

Figure 2. Hollow-core manufacturing technique.

18 hours after extrusion, the pretensioning strands are detensioned and the units are sawed to length.

Key issues in the manufacturing procedure that may affect the design of hollow-core units are as follows.

- The tendons (strands in this case) are pretensioned by stretching to 70% of their ultimate strength f_{pk} prior to casting and are sawn without anchorage hooks or bends at the flat ends of the units.[1]

- Shear and flexural strength of the hollow-core unit relies on the rapid development of bond around the strands.

- The units are one-way spanning and rely on shear keys in the longitudinal joints to form a floor plate.

- The ultimate shear capacity relies on the tensile capacity of flexurally uncracked concrete because no shear (or torsion) links are possible.

- Transverse reinforcing bars, top reinforcing bars, and projecting reinforcing bars (to connect with adjacent units) are not possible.

As a result of these issues, the design of hollow-core units does not fully comply with the usual rules given in codes and therefore requires type approval from the appropriate regulatory agency. In Europe, this is provided according to the British Standards Institution's (BSI's) *Precast Concrete Products—Hollow Core Slabs* (EN 1168).[3] EN 1168 provides the following:

- normative rules on matters such as geometry (web and flange thickness, joint shape), tolerances, splitting stresses, shear capacity and torsion, punching shear, point and edge loads, and testing schemes

- information on floor diaphragm action, transverse load distribution, three-line supports, unintended restraints (for example, due to walls), and further procedures and data for shear capacity in fire

There is no similar complement to ACI 318, though the *PCI Design Handbook: Precast and Prestressed Concrete*[4] supplies important information related to hollow-core units.

Hollow-core unit properties and design assumptions

For the design examples, the hollow-core unit is assumed to be 1200 mm (48 in.) wide × 200 mm (8 in.) deep with four 9.3 mm (0.37 in.) diameter strands and six 12.5 mm (0.49 in.) diameter strands. The hollow-core unit examples assume a simply supported span of 8.0 m (26 ft) carrying a superimposed uniformly distributed dead load of 2.0 kN/m^2 (0.3 psi) and a live load of 5.0 kN/m^2 (0.75 psi).

Using notation from EC2, concrete compressive strength is assumed for two situations: design strength at prestress transfer $f_{ck}(t)$ (typically 16 to 20 hours after casting) and 28-day characteristic cylinder strength f_{ck} for long-term service and ultimate design. The expressions for short-term strength given in EC2-1-1 clause 3.1.2(6) are not used. Although f_{ck} based on cylinder strength is used in design, it is still common to test cubes and convert that strength to an equivalent cylinder strength. In this paper, the concrete compressive strength at prestress transfer $f_{ck}(t)$ will be assumed to be 28 N/mm^2 (4060 psi) (cube > 35 N/mm^2 [5076 psi]) and the 28-day characteristic strength is 40 N/mm^2 (5800 psi) (cube > 50 N/mm^2 [7350 psi]). It is common for the actual 28-day cube strength to be in the order of 60 to 70 N/mm^2 (8700 to 10,150 psi).

Seven-wire helical strands of 9.3 and 12.5 mm (0.37 and 0.49 in.) diameter of ultimate strength f_{pk} of 1770 N/mm^2 (257 ksi) are used in these examples. Super strands of 9.6 and 12.9 mm (0.38 and 0.51 in.) diameter with an ultimate strength f_{pk} of 1860 N/mm^2 (270 ksi) are used in some countries. Plain or indented 5 and 7 mm (0.20 and 0.28 in.) diameter wire, of ultimate strength f_{pk} of 1860 and 1760 N/mm^2 (255 ksi), respectively, is also popular in shallow hollow-core units (up to 250 mm [9.8 in.] deep).

The initial prestress is typically $0.70f_{pk}$ (1239 N/mm^2 [180 ksi]), but approximately $0.65f_{pk}$ is sometimes used to control splitting cracks or excessive camber. Greater values up to $0.75f_{pk}$ may be used with care.

The initial prestressing force is 70% of the strength of standard seven-wire helical strand, which is 1770 N/mm^2 (257 ksi). The manufacturer's data gives relaxation Class 2 as detensioned at 20 hours after curing at a mean temperature of 50ºC (122ºF). Additional assumptions include the following:

- environmental condition = XC1 is the classification in EC2 for indoor exposure with low ambient humidity

- effective span = 8.0 m (26 ft)

- superimposed dead load due to floor finishes = 1.5 kN/m^2 (0.22 psi)

- superimposed dead load due to services and ceiling load = 0.5 kN/m^2 (0.07 psi)

- superimposed live load (including partitions) = 5.0 kN/m^2 (0.73 psi)

- bearing length = 100 mm (4 in.)

Geometric and material data given by the manufacturer are as follows:

- area = 152 × 10^3 mm^2 (235 in.^{2})

- second moment of area $I_c = 697 \times 10^6$ mm^4 (1675 in.^{4})
- height to centroid $y_b = 99$ mm (3.9 in.)
- cylinder strength at 28 days $f_{ck} = 40$ N/mm^2 (5800 psi)
- transfer cylinder strength $f_{ck}(t) = 28$ N/mm^2 (4060 psi)
- ultimate strength of tendons $f_{pk} = 1770$ N/mm^2 (257 ksi)
- area of tendons $A_p = 4 \times 52 + 6 \times 93 = 766$ mm^2 (1.2 in.^{2})
- self-weight of hollow-core unit = 3.724 kN/m (0.25 kip/ft)
- aggregate = gravel
- cement = CEM I grade 52.5R
- area of infilled joints = 7500 mm^2 (12 in.^{2})

Derived properties are as follows:

- section modulus at bottom $Z_b = 697 \times 10^6/99 = 7.040 \times 10^6$ mm^3 (430 in.^{3})
- section modulus at top $Z_t = 697 \times 10^6/101 = 6.901 \times 10^6$ mm^3 (421 in.^{3})
- mean height to tendons $y_s = (4 \times 52 \times 39.65 + 6 \times 93 \times 41.25)/766 = 40.8$ mm (1.6 in.)
- eccentricity of pretensioning force $z_{cp} = 99.0 - 40.8 = 58.2$ mm (2.3 in.)
- section modulus at level of tendons $Z_{cp} = 697 \times 10^6/58.2 = 11.979 \times 10^6$ mm^3 (731 in.^{3})

Design example following EC2 methodology

A design example for a 1200 mm (48 in.) wide × 200 mm (8 in.) deep hollow-core unit is presented following EC2 methodology, according to EC2-1-1 and *Eurocode: Basis of Design (with National Application Parameters)*[5] (EC0). The example is arranged as follows: design procedures and equations are presented followed by worked examples of the calculations for moment of resistance, shear capacity, deflection, and fire resistance.

In the following procedures and throughout this paper, code references are given on the left and the text, calculations, and formulas are to the right.

Design procedures and equations per EC2

Load combinations per EC2 This section sets out the various service and ultimate load combinations for permanent (dead) and variable (live) loads and the quasi-permanent live load for deflections. A typical value for the quasi-permanent factor ψ_2 is 0.3 for office structures.

Note that the Eurocodes refer to loads as "actions." The service load is taken as the characteristic combination, as shown in **Table 1**. The ultimate load is obtained from EC0, Exp. 6.10(a) or (b). The quasi-permanent load factor is from EC0 Table A.1.1, as given in Table 1.

Minimum and nominal cover per EC2 and BS 8500-1
This section presents the background information required to determine the cover for strands based on environmental conditions XC1 (for indoor exposure with low ambient humidity) and XC3 (for outdoor exposure with medium to high humidity and no chlorides) according to EC2-1-1 and BSI's *Concrete—Complementary British Standard to BS EN 206, Part 1: Method of Specifying and Guidance for the Specifier* (BS 8500-1).[6]

In the United Kingdom, the EC2 National Annex Tables NA.2 and NA.3 are replaced by BS 8500-1:2015 Table A.1 for the environmental classification and Table A.4 for the cover (50-year service life). Tables 4.3N and 4.5N in EC2-1-1 are not used.

BS 8500-1 Table A.1 — for XC1 Grade C20/25, minimum and nominal cover to

Table 1. Ultimate load combinations according to Eurocode 0 and ACI 318

	Eurocode 0	ACI 318
Service load	$G_k + Q_k$	$D + L$
Ultimate load Use the greater of	$1.35G_k + 1.05(Q_{k,1} + \Sigma Q_{k,l}\ ..)$	$1.4D$
	$1.25G_k + 1.5Q_{k,1} + 1.05\Sigma Q_{k,l}\ ..$	$1.2D + 1.6L$
	$1.25G_k + 1.5Q_{k,i} + 1.05\Sigma Q_{k,i}\ ..$	$1.2D + 1.6(L_r$ or $S)$
Quasi-permanent load	$G_k + \psi_2 Q_k$	$D + \psi_2 L$ (or L_r)

Note: D = dead load (sustained); G_k = dead load (sustained); L = live load (imposed); L_r = live roof load; Q_k = live load (imposed); $Q_{k,1}$ = one dominant variable action, such as live uniformly distributed loads; $Q_{k,i}$ = secondary point or linear variable action from another source; S = snow load; ψ_2 = quasi-permanent live-load factor.

	tendons C_{min} = 15 mm (0.6 in.)
	for XC3 Grade C40/50 using CEM I (ordinary or rapid-hardening cement), C_{min} = 20 mm (0.8 in.)
EC2-1-1 4.4.1.1.(1)	nominal cover $C_{nom} = C_{min} + \Delta C_{dev}$
EC2-1-1 4.4.1.2(2)P	$C_{min} \geq C_{min,dur} + \Delta C_{dur} \geq C_{min,b}$
EC2-1-1 4.4.1.2.(3) Table 4.2	minimum cover for bond $C_{min,b} \geq 1.5 \times$ strand diameter = 1.5 × 12.5 = 19 mm (0.75 in.)
EC2-1-1 4.4.1.2.(6)	safety distance for cover for durability $\Delta C_{dur} = 0$
EC2-1-1 4.4.1.3.(1)P	generally allowance for deviation of cover ΔC_{dev} = 10 mm (0.4 in.)
EC2-1-1 4.4.1.3.(3)	but hollow-core unit cover is regulated by steel guides, known as soldiers, which hold the tendons at the correct level. Therefore ΔC_{dev} may be reduced to 5 mm (0.2 in.). $\therefore C_{nom} = C_{min}$ + 5 mm.

The final cover for XC1 $C_{nom} \geq$ 20 mm (0.8 in.), and for XC3 $C_{nom} \geq$ 25 mm (1 in.). A cover of 20 mm (0.8 in.) may be used for smaller tendons, such as 5 mm (0.2 in.) diameter wires, but for 9.3 and 12.5 mm (0.37 and 0.49 in.) diameter strands the concrete around the strands is subject to radial tension, leading to $C_{nom} \approx 2½ \times$ diameter in order to avoid longitudinal splitting. In this paper a C_{nom} of 35 mm (1.4 in.) is used.

Axis distance and effective thickness for fire resistance per EC2-1-2 and EN 1168 This section determines the axis distance to the centroid of the strands resisting tension for 60-minute fire resistance according to *Eurocode 2: Design of Concrete Structures, Part 1-2: Structural Fire Design (with National Application Parameters)*[7] and EN 1168.

The data presented in EN 1168 Annex G Table G.1 are for siliceous aggregates. If calcareous aggregates, such as limestone, are used, the required effective thickness t_e and average axis distance to centroid of tendons in tension zone a are increased by 10%.

The actual depth t_e is based on a ratio ζ of solid material (including infilled joints) to the whole of 0.4. When $\zeta \geq 0.85$, the unit may be considered solid.

The required effective thickness t_e and axis distance a based on 60-, 90-, and 120-minute fire resistance are given in **Table 2**.

For prestressed concrete using strand or wire, axis distance a (provided in Table 2) must be decreased by 15 mm (0.6 in.) according to EC2-1-2 clause 5.2.(5) unless a check on the service fire–to–ultimate load ratio $E_{d,fi}/E_d$ is carried out according to clause 5.2.(6–8), Fig. 5.1, and clause 2.4.2.

Eq. (5.3)	additional axis distance for tendons $\Delta a = 0.1(500 - \theta_{cr})$ mm, where θ_{cr} *is* critical temperature, obtained from EC2-1-2 Fig. 5.1 for tendons curve 3 as follows: If $k_p(\theta_{cr})$ = 0.55 to 1.0, $\theta_{cr} = 655.5 - 555.5k_p(\theta_{cr})$ If $k_p(\theta_{cr})$ = 0.1 to 0.55, $\theta_{cr} = 594.4 - 444.4k_p(\theta_{cr})$ If $k_p(\theta_{cr}) < 0.1$, $\theta_{cr} = 1200 - 6500k_p(\theta_{cr})$
Eq. (5.2)	strength reduction coefficient for tendons in fire $k_p(\theta_{cr}) = \sigma_{p,fi}/p_{yk}(20°C)$, where $\sigma_{p,fi}$ is strength of tendons in fire = $(E_{d,fi}/E_d)(p_{yk}(20°C)/1.15)(A_{p,required}/A_{p,provided})$, $p_{yk}(20°C)$ is strength of tendons at room temperature, $A_{p,required}$ is area of tendons required by design, and $A_{p,provided}$ is area of tendons provided
Eq. (2.4)	fire load ratio (may also be taken as $M_{Ed,fi}/M_{Ed}$, according to EC2-1-2 clause 2.4.2[4]) $\eta_{fi} = E_{d,fi}/E_d$, where the smaller of Eq. (EC2-1-2 2.5a) $\eta_{fi} = G_k + \psi_{fi}Q_k/(1.35G_k + 1.05Q_k)$ or Eq. (EC2-1-2 2.5b) $\eta_{fi} = G_k + \psi_{fi}Q_k/(1.25G_k + 1.5Q_k)$; G_k is dead load (sustained); ψ_{fi} is quasi-permanent combination factor $\psi_{2,1}$ from EC0 Table A1.1, according to EC1-1-2 clause 4.3; Q_k is live load (imposed); $M_{Ed,fi}$ is ultimate design moment in fire; and M_{Ed} is ultimate design bending moment

Material data per EC2-1-1 This section lists the material properties for concrete at 28 days and at transfer and for the pretensioning strands.

Table 2. Effective thickness and axis distance for fire resistance according to EN 1168 and Eurocode 2 Part 1-2

Fire resistance, minutes	Effective thickness according to EN 1168 Annex G, mm	Axis distance according to EC2-1-2, mm
60	130	20
90	160	30
120	200	40

Note: 1 mm = 0.0394 in.

Concrete

Type of cement: strength class CEM 52.5R Class R

Type of aggregate: gravel

28-day characteristic cylinder strength $f_{ck} = 40\ \text{N/mm}^2$ (5800 psi)

28-day characteristic cube strength $f_{ck,cube} = 50\ \text{N/mm}^2$ (7350 psi)

Table 3.1	mean compressive strength at 28 days $f_{cm} = f_{ck} + 8 = 48\ \text{N/mm}^2$ (6960 psi)
3.1.6.(1)P	concrete strength coefficient $\alpha_{cc} = 0.85$ flexure, otherwise 1.0, concrete strength coefficient in tension $\alpha_{ct} = 1.0$ and partial safety factor $\gamma_c = 1.5$
Eq. (3.15)	design strength $f_{cd} = 0.85 \times 40/\gamma_c = 22.67\ \text{N/mm}^2$ (3288 psi) for flexure, otherwise 26.67 N/mm² (3868 psi)
Table 3.1	mean tensile strength $f_{ctm} = 0.3 \times 40^{2/3} = 3.51\ \text{N/mm}^2$ (509 psi)
Table 3.1	5% fractile strength $f_{ct,0.05} = 0.7 \times 3.51 = 2.46\ \text{N/mm}^2$ (357 psi)
Eq. (3.16)	design tensile strength $f_{ctd} = 2.46/\gamma_c = 1.64\ \text{N/mm}^2$ (238 psi)
Table 3.1	Young's modulus $E_{cm} = 22\ (48/10)^{0.3} = 35.22\ \text{kN/mm}^2$ (5108 ksi)

Transfer cylinder strength $f_{ck}(t) = 28\ \text{N/mm}^2$ (4060 psi)

Transfer cube strength > 35 N/mm² (5076 psi)

Table 3.1	mean strength at transfer $f_{cm}(t) = f_{ck}(t) + 8 = 36\ \text{N/mm}^2$ (5220 ksi)
Table 3.1	design strength at transfer $f_{cd}(t) = 0.85 \times 28/\gamma_c = 15.87\ \text{N/mm}^2$ (2302 psi)
Eq. (3.1)	mean tensile strength $f_{ctm}(t) = \beta_{cc}(t)^{\alpha} f_{ctm}$ where $\alpha = 1$ for 1 day
Eq. (3.4)	strength ratio $\beta_{cc}(t) = f_{cm}(t)/f_{cm} = 36/48 = 0.75$. $\therefore f_{ctm}(t) = 0.75 \times 3.51 = 2.63\ \text{N/mm}^2$ (381 psi)
Eq. (3.1.6.2.[P])	design tensile strength $f_{ctd}(t) = 0.7 \times 2.63/\gamma_c = 1.22\ \text{N/mm}^2$ (177 psi)
Table 3.1	Young's modulus $E_{cm}(t) = 22(36/10)^{0.3} = 32.31\ \text{kN/mm}^2$ (4686 ksi)

Steel tendons

Diameter: 9.3 and 12.5 mm (0.37 and 0.49 in.) (EN 1168 permits < 16 mm [0.63 in.] maximum)

Ultimate strength $f_{pk} = 1770\ \text{N/mm}^2$ (257 ksi)

5% fractile tensile strength of concrete $f_{p,0.1k} = 0.9 f_{pk} = 1593\ \text{N/mm}^2$ (231 ksi)

2.4.2.2.(1)	favorable partial safety factor at ultimate $\gamma_{p,fav} = 0.9 \times$ prestress at ultimate and 1.0 at service
5.10.9	factors for the direct measurement of prestress $r_{sup} = r_{inf} = 1.0$
2.4.2.4	partial safety factor $\gamma_s = 1.15$
3.3.6.(7)	design stress at ultimate $f_{pd} = 1593/1.15 = 1385\ \text{N/mm}^2$ (201 ksi)
3.3.6.(3)	Young's modulus $E_p = 195{,}000\ \text{N/mm}^2$ (28,281 ksi)
3.3.6.(7) Fig. 3.10	ultimate strain limit $\varepsilon_{uk} = \varepsilon_{ud}/0.9 = 0.02/0.9 = 0.0222$, where ε_u is the limiting strain
3.3.2.(4)	relaxation class: 2
3.3.2.(6)	relaxation loss at 1000 hours $\rho_{1000} = 2.5\%$
5.10.2.1.(P)	degree of pretensioning $\eta = 70\% < 80\%$ limit
8.10.2.2.(2)	detensioning rate: gradual initial pretensioning stress $\sigma_{pi} = \eta f_{pk}$ initial pretensioning force $F_{pi} = A_p \sigma_{pi}$ pretensioning force F_{pmo} and stress after initial losses $\sigma_{pmo} \leq 0.75 f_{pk}$

Note that final prestressing force F is used in this paper to distinguish it from P in ACI calculations.

Analysis of prestress losses and service and ultimate moments of resistance per EC2-1-1 This section presents standard calculation procedures leading to the service M_{sr} and ultimate M_{Rd} moments of resistance according to EC2-1-1.

Service stress due to bending at transfer

5.10.3	losses at transfer
3.3.2.(7)	relaxation of tendon at t hours
Eq. (3.29)	ratio of initial prestress is μ initial prestress $\sigma_{pi} = \eta f_{pk}$
Eq. (3.29)	prestress loss due to relaxation in tendon

$$\Delta\sigma_{pr} = \sigma_{pi} \times 0.66 \times 2.5 \times e^{(9.1 \times \mu)}$$

$$\left(\frac{t}{1000}\right)^{0.75(1-\mu)}$$

prestress at release $\sigma_r = \sigma_{pi} - \Delta\sigma_{pr}$
prestressing force at release $F_r = \sigma_r A_p$

5.10.5.1 instantaneous deformation due to elastic shortening determined after relaxation loss concrete stress at level of tendons (ignoring self-weight) $\sigma_{pb} = F_r/A_c + F_r z_{cp}/Z_z$, where A_c is area of tendons and Z_z is the section modulus at the level of the tendons

Eq. (5.44) elastic shortening loss of prestress $\Delta\sigma_{el} = \sigma_{pb}E_p/E_{cm}(t)$, where $E_{cm}(t)$ is Young's modulus at transfer

5.10.3.(2) prestress after initial losses $\sigma_{pmo} = \sigma_{pi} - \Delta\sigma_{p,r} - \Delta\sigma_{el}$ should be $\leq k_p f_{pk} = 0.75 f_{pk}$
prestressing force at transfer
$F_{pmo} = \sigma_{pmo} A_p$
$R_{tr} = \sigma_{pmo}/\sigma_{pi}$

Maximum surface stress at transfer

Maximum surface stress at transfer at bottom

$$\sigma_b(t) = \frac{F_{pmo}}{A_c} + \frac{F_{pmo} z_{cp}}{Z_b}$$

5.10.2.2.(5) limit for $\sigma_b(t) \leq 0.6 \times f_{ck}(t)$

Maximum surface stress at transfer at top

$$\sigma_t(t) = \frac{F_{pmo}}{A_c} - \frac{F_{pmo} z_{cp}}{Z_t}$$

5.10.2.2.(5) limit for $\sigma_t(t) \geq -f_{ctm}(t)$

Serviceability limit state of bending

Long-term losses are first calculated up to installation time t_i using relative humidity RH of 70% with all faces of the hollow-core unit exposed, and then to 500,000 hours using relative humidity in service RH_s of 50% with only the bottom exposed (top and sides protected).

5.10.6.(1a) Loss due to creep to installation is covered in annex B.1. Although the strength of concrete at 1 day will be the transfer strength $f_{ck}(t)$, after a few days it will reach the 28-day strength f_{ck}, so the mean strength f_{cm} is taken for the strength factors in this calculation.

During this period, notional depth $h_o = 2A_c/(b_t + b_b + 2h)$ is for all faces exposed, ignoring the cores, where b_t is the actual breadth at the top and b_b is the actual breadth at the bottom of the hollow-core unit and h is the depth of the hollow-core unit.

Eq. (B.1) creep coefficient $\phi_{(ti,to)} = \phi_{RH}\beta(f_{cm})\beta(t_o)\beta_c(t,t_o)$ for installation in days t_i, transfer age in days t_o, and a relative humidity RH of 70%, where ϕ_{RH} is the relative humidity factor, $\beta(t_o)$ is age at release loading factor, and $\beta_c(t,t_o)$ is detensioning age factor to service

Eq. (B.3b/B.8c) relative humidity factor

$$\phi_{RH} = \left(1 + \frac{1 - RH/100}{0.1x^3\sqrt{h_O}}\left(\frac{35}{f_{cm}}\right)^{0.7}\right)\left(\frac{35}{f_{cm}}\right)^{0.2}$$

Eq. (B.4) strength factor $\beta(f_{cm}) = \dfrac{16.8}{\sqrt{f_{cm}}}$

Eq. (B.5) age at release loading factor

$$\beta(t_o) = \frac{1}{0.1 + t_0^{0.2}}$$

Eq. (B.9) here calculations may use equivalent age at transfer

$$t_0 = t_T\left(\frac{9}{2 + t_T^{1.2}} + 1\right)^{\alpha} \geq 0.5$$

days, where t_{0T} is equivalent age at curing after transfer and α is 1, 0, and -1 for Classes R, N, and S cement, respectively.

Eq. (B.10) temperature adjusted age $t_{oT} = t_o e^{-[4000/(273 + T) - 13.65]}$, where T is mean temperature in degrees Celsius during curing time in days, taken as 50°C (122°F)

Eq. (B.7) detensioning age factor to service

$$\beta_c(t,t_o) = \left(\frac{t_i - t_o}{\beta_H + t_i - t_o}\right)^{0.3}$$

Eq. (B.8b) relative humidity RH factor in days

$$\beta_H = 1.5\left[1 + (0.012 \times RH)^{18}\right]h_o + 250\left(\frac{35}{f_{cm}}\right)^{0.5}$$

stress at level of tendons after

initial losses $\sigma_{cpo} = \dfrac{F_{pmo}}{A_c} + \dfrac{F_{pmo} Z_{cp}}{Z_z}$

Eq. (5.46) prestress loss due to creep at installation

$$\sigma_{p,c,i} = \frac{\frac{E_p}{E_{cm}} \times \phi_{(ti,to)}\sigma_{cpo}}{1 + \left[\frac{E_p}{E_{cm}} \times \frac{A_p}{A_c}\left(1 + \frac{A_c \times z_{cp}^2}{I_c}\right)\left(1 + 0.8\phi_{(ti,to)}\right)\right]}$$

5.10.6.(1a) loss due to creep from transfer age t_o to relaxation time of tendon, in other words, time in service t = 500,000 hours (20,833 days); service RH_s = 50% notional depth for bottom only exposed $h_o = 2A_c/b_b$

Eq. (B.2) creep coefficient $\phi_{(t,to)} = \phi_{RH}\beta(f_{cm})\beta(t_o)\beta_c(t,t_o)$

Eq. (B.3b/B.8c) relative humidity factor

$$\phi_{RH} = \left(1 + \frac{1 - RH_S/100}{0.1\sqrt[3]{h_o}} \times \left[\frac{35}{f_{cm}}\right]^{0.7}\right)\left(\frac{35}{f_{cm}}\right)^{0.2}$$

Eq. (B.4 and B.5) strength factor $\beta(f_{cm})$ and age at release loading factor $\beta(t_o)$ as above

Eq. (B.7)

$$\beta_c(t,t_o) = \left(\frac{20833 - t_o}{\beta_H + 20833 - t_o}\right)^{0.3}$$

Eq. (B.8b) relative humidity RH factor

$$\beta_H \text{ in days} = 1.5\left[1 + \left(0.012 \times RH_s\right)^{18}\right]h_o + 250\left(\frac{35}{f_{cm}}\right)^{0.5}$$

stress at level of tendons after losses at installation (ignoring self-weight)

$$\sigma_{pmi} = \frac{F_{pmi}}{A_c} + \frac{F_{pmi}z_{cp}}{Z_z}$$

Eq. (5.46) prestress loss due to creep at service

$$\Delta\sigma_{p,c} = \frac{\frac{E_p}{E_{cm}} \times \phi_{(t,ti)}\sigma_{pmi}}{1 + \left[\frac{E_p}{E_{cm}} \times \frac{A_p}{A_c}\left(1 + \frac{A_c \times z_{cp}^2}{I_c}\right)\left(1 + 0.8\phi_{(t,ti)}\right)\right]}$$

where $\phi_{(t,ti)}$ is the internal creep coefficient

5.10.6.(1a) loss due to shrinkage from transfer age to relaxation time of tendon t_s, assuming time in service t of 500,000 hours (see annex B.2)
relative humidity RH during the period of shrinkage RH_s is taken as 50% notional depth is for bottom only exposed $h_o = 2A_c/b_b$

Eq. (B.12) relative humidity RH factor

$$\beta_{RH} = 1.55\left[1 - \left(\frac{RH_s}{100}\right)^3\right]$$

type of cement: Class R (α_{ds1} = 6 and α_{ds2} = 0.11, where α_{ds1} and α_{ds2} are cement factors)

Eq. (B.11) basic drying shrinkage strain

$$\varepsilon_{cd,o} = 0.85 \times \left(220 + 110 \times 6\right)^{-\left(0.11 \times f_{cm}/10\right)}\beta_{RH}$$

Table 3.3 size coefficient
$k_n = 1.0 - 0.0015\,(h_o - 100)$ for $100 \le h_o <$ 200 mm ($4 \le h_o < 8$ in.)
$= 0.85 - 0.001\,(h_o - 200)$ for $200 \le h_o <$ 300 mm ($8 \le h_o < 12$ in.)
$= 0.75 - 0.0005\,(h_o - 300)$ for $300 \le h <$ 500 mm ($12 \le h_o < 20$ in.)

Eq. (3.10) age factor

$$\beta_{ds}(t,t_s) = \frac{\left(20833 - t_s\right)}{\left(20833 - t_s\right) + 0.04\sqrt{h_o^3}}$$

Eq. (3.9) drying shrinkage strain
$\varepsilon_{cd} = \beta_{ds}(t,t_s)k_n\varepsilon_{cd,o}$

10.3.1.2 autogenous shrinkage strain is taken as zero

Eq. (3.8) total shrinkage strain $\varepsilon_{cs} = \varepsilon_{cd}$
creep coefficient $\phi_{(t,to)} = \phi_{RH}\beta(f_{cm})\beta(t_o)\beta_H$
using values from earlier

Eq. (5.46) prestress loss due to shrinkage

$$\Delta\sigma_{p,s} = \frac{E_p\varepsilon_{cs}}{1 + \left[\frac{E_p}{E_{cm}} \times \frac{A_p}{A_c}\left(1 + \frac{A_c \times z_{cp}^2}{I_c}\right)\left(1 + 0.8\phi_{(t,to)}\right)\right]}$$

5.10.6.(1b) loss due to tendon relaxation

Eq. (3.29) ratio of initial prestress $\mu = \sigma_{pmo}/f_{pk}$

Eq. (3.29) prestress due to tendon relaxation

$$\Delta\sigma_{pr} = \sigma_{pmo} \times 0.66 \times \rho_{1000} \times e^{(9.1^{\mu})}\left(\frac{500000}{1000}\right)^{0.75(1-\mu)}$$

Eq. (5.46) prestress loss due to relaxation in tendon

$$\Delta\sigma_{p,r} = \frac{0.8\sigma_{pr}}{1 + \left[\frac{E_p}{E_{cm}} \times \frac{A_p}{A_c}\left(1 + \frac{A_c \times z_{cp}^2}{I_c}\right)\left(1 + 0.8\phi_{(ti,to)}\right)\right]}$$

prestress after final losses since installation
$\sigma_{po} = \sigma_{pmi} - \Delta\sigma_{p,ci} - \Delta\sigma_{p,c} - \Delta\sigma_{p,s} - \Delta\sigma_{p,r}$
final prestressing force $F_{po} = \sigma_{po}A_p$
therefore, total loss $R = 1 - \sigma_{po}/\sigma_{pi}$
maximum surface stresses in service at bottom

$$\sigma_b = \frac{F_{po}}{A_c} + \frac{F_{po}z_{cp}}{Z_b}$$

5.10.2.2.(5) limit $\sigma_b \le 0.45f_{ck}$ for exposure > XC1, otherwise no limit is given, though $\sigma_b \le 0.45f_{ck}$ overcomes the possibility of nonlinear

	creep.
	maximum surface stresses in service at top $$\sigma_t = \frac{F_{po}}{A_c} - \frac{F_{po} z_{cp}}{Z_t}$$
7.3.2(4) and (2)	limit $\sigma_t \geq -\sigma_{ct,p} = -f_{ctm}$, where $\sigma_{ct,p}$ is mean value of the tensile strength of the concrete
	With reference to **Fig. 3**, the service moment of resistance is the lesser of the following:
	based on bottom surface $M_{sr} = (\sigma_b + f_{ctm})\, Z_{b,co}$ (usually critical for hollow-core units), where $Z_{b,co}$ is the compound section modulus at bottom using the transformed area of tendons based on a modular ratio without creep effects $m = E_p/E_{cm}$
	based on top surface $M_{sr} = (\sigma_t + 0.45 f_{ck}) Z_{t,co}$ where $Z_{t,co}$ is the compound section modulus at bottom using the transformed area of tendons based on a modular ratio without creep effects $m = E_p/E_{cm}$
	Compound section applies only to bending stresses, not to prestress.
	area of compound section $A_{c,co} = A_c + (m-1)A_p$
	height to centroid of compound section $y_{b,co} = (A_{cyb} + (m-1)A_{pys})/A_{c,co}$, where y_b is height to centroid of basic section

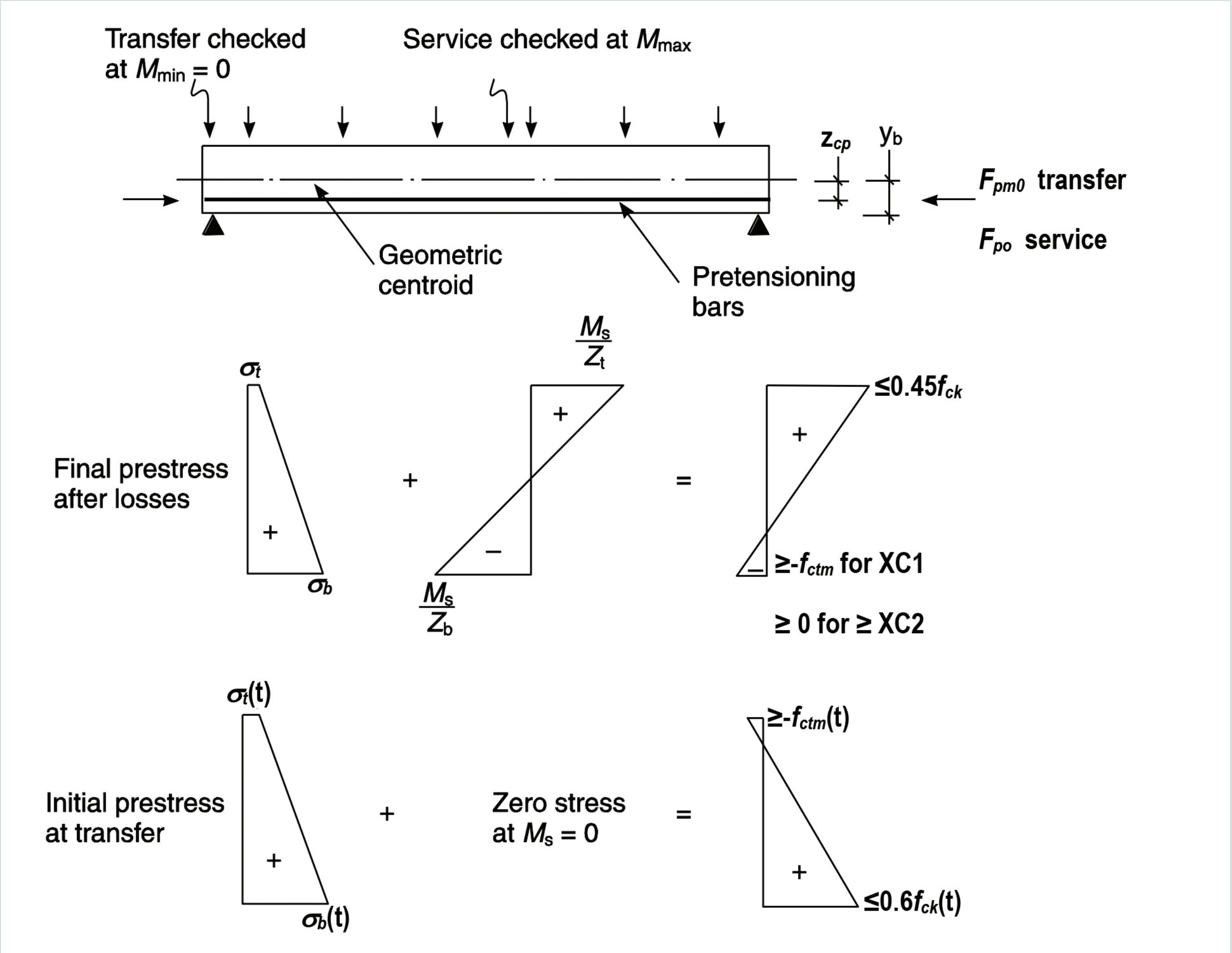

Figure 3. Principles of serviceability stress limitations for prestressed concrete elements according to Eurocode 2 Part 1-1. Note: f_{ck} = 28-day characteristic cylinder strength; $f_{ck}(t)$ = transfer cylinder strength of concrete; f_{ctm} = mean tensile strength of concrete; $f_{ctm}(t)$ = mean tensile strength of concrete at transfer; F_{pm0} = prestressing force at transfer; F_{po} = final prestressing force in service; M_{max} = maximum service moment in the slab; M_{min} = minimum service moment in the slab; M_s = service moment; y_b = height to centroid of basic section; z_{cp} = eccentricity of pretensioning force; Z_b = basic section modulus at bottom; Z_t = basic section modulus at top; σ_b = maximum surface stresses in service at bottom; $\sigma_b(t)$ = maximum surface stress at transfer at bottom; σ_t = maximum surface stresses in service at top; $\sigma_t(t)$ = maximum surface stress at transfer at top.

	second moment of area of compound section $I_{c,co} = I_c + A_c(y_{b,co} - y_b)^2 + \Sigma(m-1)A_p(y_s - y_{b,co})^2$ per layer of tendons
	$Z_{b,co} = I_{c,co}/y_{b,co}$
	$Z_{t,co} = I_{c,co}/(h - y_{b,co})$
Table 7.1N.	For exposure greater than XC1, the section is also checked for zero tension stress f_{ctm} under quasi-permanent load combination $(G_k + \psi_2 Q_k)$ for XC2 to XC4 and frequent load $(G_k + \psi_1 Q_k)$ for XD and XS, where ψ_1 is effective creep coefficient for deflection at installation. Then based on bottom surface service moment of resistance $M_{sr} = \sigma_b Z_{b,co}$
7.3.2.(4)P	Where exposure > XC1, the section is also checked for the characteristic load combination $(G_k + Q_k)$ for the value of the tensile stress $\leq \sigma_{ct,p} = f_{ctm}$ and the most critical used in design.

Ultimate limit state of bending per EC2 This section presents calculation procedures for ultimate moment of resistance M_{Rd} according to EC2-1-1.

Fig. 3.4 and Table 3.1	ultimate strain in concrete $\varepsilon_{cu3} = 0.0035$ for $f_{ck} \leq 50$ N/mm²
3.3.6.(7) Fig. 3.10	limit of proportionality of tendons $\varepsilon_{LOP} = 0.9f_{pk}/(\gamma_s E_p)$ at which stress $= 0.9f_{pk}/\gamma_s$
	where $\gamma_s = 1.15$
3.3.6.(7) Fig. 3.10	ultimate strain in tendons $\varepsilon_{uk} = 0.0222$ at which design stress $f_{pd} = f_{pk}/\gamma_s$
5.10.9	prestrain due to prestress after losses $\varepsilon_{po} = r_{sup}\sigma_{po}/E_p$, but $r_{sup} = 1.0$. This varies along the span due to changes in creep losses, so the value at the support is used.

effective depth of tendons in tension zone $d = h - y_{sT}$, where y_{sT} is mean height to tendons in tension zone

first assuming that $0.8X < h_{ft}$, where X is depth to neutral axis and h_{ft} is depth of top flange

$$\varepsilon_p = \varepsilon_{po} + \varepsilon_{cu3}\left(\frac{d}{X} - 1\right) \quad (1)$$

where

ε_p = total ultimate strain in tendons

If $\varepsilon_{LOP} < \varepsilon_p < \varepsilon_{uk}$, from EC2-1-1 Fig. 3.10 (reproduced in **Fig. 4**) inclined branch

$$f_p = 0.9 f_{pd} + 0.1 f_{pd}\left(\frac{\varepsilon_p - \varepsilon_{LOP}}{\varepsilon_{uk} - \varepsilon_{LOP}}\right) \quad (2)$$

where

f_p = design ultimate stress in tendons

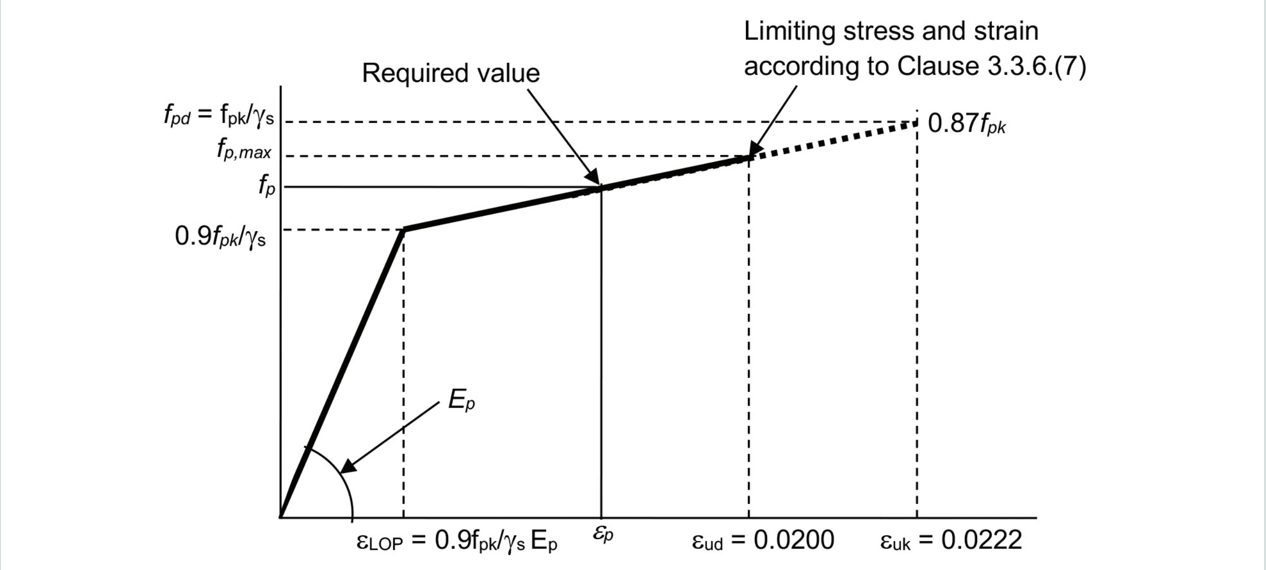

Figure 4. Idealized stress versus strain curve for tendons according to Eurocode 2 Part 1-1 Fig. 3.10. Note: E_p = Young's modulus for tendons; f_p = design ultimate stress in tendons; f_{pd} = maximum stress at ultimate of tendons; f_{pk} = ultimate strength of tendons; γ_s = partial factor of safety for steel tendons; ε_{LOP} = limit of proportionality of tendons; ε_p = design ultimate strain in tendons; ε_{ud} = limiting strain in tendons; ε_{uk} = ultimate strain in tendons.

but where the strain is limited $\varepsilon_p \leq \varepsilon_{ud} = 0.02$

With reference to **Fig. 5**, the force equilibrium is

$$f_p A_{pT} = 0.8 f_{cd} b_X \quad (3)$$

where

A_{pT} = area of tendons in tension zone, in this case equal to A_p

f_{cd} = design strength of concrete = $0.85 f_{ck}/\gamma_m$

b = breadth of unit plus infill = 1200 mm (47 in.)

Combining Eq. (1) through (3) yields

$$0.8 f_{cd} b(\varepsilon_{uk} - \varepsilon_{LOP})X^2 - [0.9(\varepsilon_{uk} - \varepsilon_{LOP}) + 0.1(\varepsilon_{po} - \varepsilon_{cu3} - \varepsilon_{LOP})]A_{pT} f_{pd} X - 0.1\varepsilon_{cu3} d A_{pT} f_{pd} = 0 \quad (4)$$

Solving yields depth to neutral axis X. depth to centroid of concrete area $d_n = 0.5 \times 0.8X$

If the compression zone lies beneath the level of the top flange (that is, $0.8X > h_{ft}$), the compression force is the sum of the force in the top flange above the cores and webs between the cores. Thus Eq. (3) is amended to $f_p A_{pT} = b_c h_{ft} f_{cd} + 0.8 f_{cd}(b - b_c)X$, where b_c is the total breadth of cores, such that Eq. (4) is amended to Eq. (5).

$$0.8 f_{cd}(b - b_c)(\varepsilon_{uk} - \varepsilon_{LOP})X^2 - \{[0.9(\varepsilon_{uk} - \varepsilon_{LOP}) + 0.1(\varepsilon_{po} - \varepsilon_{cu3} - \varepsilon_{LOP})]A_{pT} f_{pd} - f_{cd} b_c h_{ft}(\varepsilon_{uk} - \varepsilon_{LOP})\}X - 0.1\varepsilon_{cu3} d A_{pT} f_{pd} = 0 \quad (5)$$

Solving yields $X > h_{ft}/0.8$, from which d_n is obtained.

lever arm $z = d - d_n$

From Eq. (1),

$$\varepsilon_p = \varepsilon_{po} + 0.0035 - \left(\frac{d}{X} - 1\right) \leq \varepsilon_{ud} = 0.02.$$

Design ultimate stress f_p is found from Eq. (2), but its maxiumum allowed value $f_{p,max} \leq 0.9 f_{pd} + 0.1 f_{pd} \left(\dfrac{\varepsilon_{ud} - \varepsilon_{LOP}}{\varepsilon_{uk} - \varepsilon_{LOP}}\right)$

Therefore ultimate moment of resistance $M_{Rd} = f_p A_{pT} z$.

Ultimate limit state of shear per EC2-1-1

Section uncracked in flexure $V_{Rd,c}$, clause 6.2.2.(2)

8.10.2.2 design tensile strength at transfer $f_{ctd}(t) = 0.7 f_{ctm}(t)/1.5$

Eq. (8.15) bond stress $f_{pbt} = 3.2 \times 1.0 f_{ctd}(t)$

8.10.2.2 transmission length coefficient $\alpha_1 = 1.0$ for gradual release

transmission length coefficient $\alpha_2 = 0.19$ for seven-wire strand, 0.25 for wire

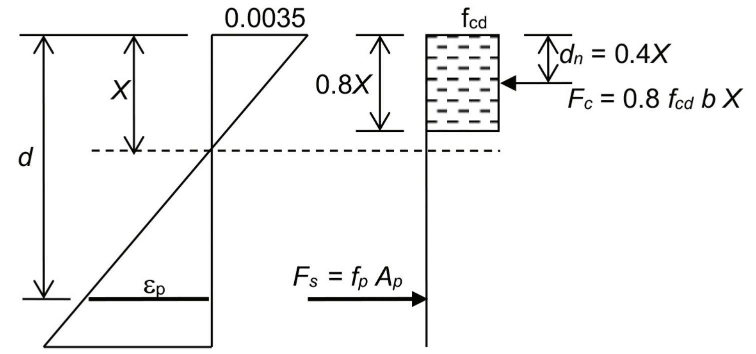

Figure 5. Force equilibrium according to Eurocode 2 Part 1-1, clause 3.1.7(3). Note: A_p = area of tendons; b_t = breadth of hollow-core unit plus infill; d = effective depth of tendons in tension zone; d_n = depth to centroid of concrete area; f_{cd} = design strength of concrete; f_p = design ultimate stress in tendons; F_c = ultimate force in concrete; F_s = ultimate force in tendons; X = depth to neutral axis at ultimate; ε_p = design ultimate strain in tendons.

Eq. (8.16) basic transmission length $l_{pt} = 1.0 \times 0.19 \sigma_{pm0}/f_{pbt}\phi$, where diameter of strands $\phi = 9.3$ or 12.5 mm (0.37 or 0.49 in.)

Eq. (8.18) design transmission length $l_{pt2} = 1.2l_{pt}$

distance to critical section $l_x = l_b + y_b$, where l_b is bearing length

6.2.2.(2) $\alpha_1 = l_b + y_b/l_{pt2}$

2.4.2.2.(1) $\gamma_{p,fav} = 0.9 \times$ prestress at ultimate

6.2.2.(2) stress at centroidal axis $\sigma_{cp} = 0.9F_{po}/A_c$

3.1.6.2.(P) design tensile strength $f_{ctd} = 0.3f_{ck}^{2/3}0.7/1.5$

Eq. (6.4) $V_{Rd,c} = I_c b_w / S_c \times \sqrt{f_{ctd}^{\;2} + \alpha_1 \sigma_{cp} f_{ctd}}$,

where b_w is total breadth of webs and S_c is first moment of area of hollow-core unit

Section cracked in flexure $V_{Rd,cr}$, clause 6.2.2.(1)

6.2.2.(1) shear strength depth factor

$$k = 1 + \sqrt{\frac{200}{d}} \leq 2.0$$

Eq. (6.3) minimum concrete shear strength $v_{min} = 0.035k^{3/2}f_{ck}^{\;1/2}$

steel ratio extends beyond section $\rho_1 = A_p/(b_w d) \leq 0.02$

concrete shear strength factor $C_{Rd,c} = 0.18/1.5 = 0.12$

Eq. (6.2a) $V_{Rd,cr} = [C_{Rd,c}k(100\rho_1 f_{ck})^{1/3} + 0.15\sigma_{cp}]b_w d$

Eq. (6.2b) minimum $V_{Rd,cr} = [v_{min} + 0.15\sigma_{cp}]b_w d$

6.2.2.(2) Use $V_{Rd,cr}$ where service moment $M_s >$ cracking moment $= (\sigma_b + f_{ctk,0.05}/\gamma_c)Z_{b,co}$

Anchorage length for ultimate limit state, clause 8.10.2.3

Eq. (8.21) ultimate anchorage/development length $l_{bpd} = l_{pt2} + \alpha_2\phi(\sigma_{pd} - \sigma_{po})/f_{bpd}$

where

$\alpha_2 = 0.25$ for wire, or 0.19 for seven-wire strand

σ_{pd} = ultimate stress f_p

σ_{po} = prestress after all losses

f_{bpd} = ultimate bond strength

Eq. (8.20) $f_{bpd} = \eta_{p2}\eta_1 f_{ctd}$

8.10.2.3(1) anchorage length factors for tendons $\eta_{p2} = 1.4$ for wire or 1.2 for seven-wire strand

8.10.2.2 concrete bond factor $\eta_1 = 1$ for good bond (as for dry-cast hollow-core manufacture)

Precamber and deflections per EC2 This section presents a standard procedure for the determination of precamber and short-term and long-term deflections according to EC2-1-1.

Deflections are determined for the immediate elastic deformation plus the viscoelastic effects of creep, which is reflected in the changing values of pretension force and Young's modulus with time. The creep coefficients ψ are obtained from the Association of Manufacturers of Prestressed Hollow Core Floors' *Hollow Core Floor Design and Applications*[8] manual used in current *fib* (International Federation for Structural Concrete) publications[9] and by the British Precast Concrete Federation.[10,11] Because the stress conditions in the top and bottom of the unit are different, the values of ψ determined for the effect of dead and imposed loads are *not the same* as the internal creep coefficients $\phi_{(t,ti)}$ used for calculating losses in prestress.

Long-term creep coefficient $\psi_\infty = 2.5$

Creep coefficients of development at specific times are as follows:

- transfer = 0.1
- 15 days = 0.3
- 28 days = 0.4
- 2 months = 0.5
- 3 months = 0.6
- 6 months = 0.7
- 12 months = 0.8
- ∞ = 1.0

The long-term concrete aging coefficient to allow for stress increments after initial loads X is 0.8.

At transfer

Upward camber due to negative moment due to prestress $\delta_1 = -F_{pmo}z_{cp}L^2/8E_{cm}(t)I_{c,co}$, where L is effective span of hollow-core unit. Strictly, L is the actual length of the unit, but because effective length is used in later equations, it is also used here.

Deflection due to self-weight w_o is

$\delta_2 = 5w_o L^4/384E_{cm}(t)I_{c,co}$

Resultant deflection at transfer is $\delta_1 + \delta_2$, where $I_{c,co}$ is the compound value.

At installation

The effective creep coefficient, which also takes into account the mean change in Young's modulus until installation, is

$\psi_1 = \{E_{cm}(t)/0.5 \times [E_{cm} + E_{cm}(t)]\} \times 2.5 \times (0.4 - 0.1)$

Camber at installation is due to the camber at transfer plus further viscoelastic movement $\psi_1\delta_1$ minus a reduction due to the reduction in pretensioning force from transfer F_{pmo} to installation F_{pmi} and is

$\delta_3 = -(1 + \psi_1)F_{pmo}z_{cp}L^2/8E_{cm}(t)I_{c,co} + (F_{pmo} - F_{pmi})z_{cp}L^2/8E_{cm}I_{c,co}$

Deflection at installation due to self-weight w_o is due to further viscoelastic movement of the self-weight at transfer, plus the static deflection $\delta_4 = + \delta_2(1 + \psi_1)$.

Resultant deflection of precast concrete only at installation is $\delta_3 + \delta_4$.

Long term

Imposed loads are applied after 28 days. Effective creep coefficient after installation ψ_{28} takes into account the increase in Young's modulus after 28 days.

$\psi_{28} = 0.8 \times 2.5 \times (1.0 - 0.4) = 1.2$

Long-term camber is due to camber at installation plus further viscoelastic movement due to the pretensioning force at installation F_{pmi} minus a reduction due to the reduction in pretensioning force from installation F_{pmi} to long-term F_{po}.

$\delta_5 = \delta_3 - [\psi_{28}F_{pmi} - (F_{pmi} - F_{po})]z_{cp}L^2/8E_{cm}I_{c,co}$

Long-term deflection for self-weight and infill w_1 is due to the deflection at installation, plus further viscoelastic movement of w_1.

$\delta_6 = + \delta_4 + 5w_1\psi_{28}L^4/384E_{cm}I_{c,co}$

Deflection due to uniformly distributed load (UDL) w_2 added after installation plus quasi-permanent live load $\psi_2 w_3$ is

$\delta_7 = + 5\ [(1 + \psi_{28})w_2 + (1 + 0.8\psi_\infty)\psi_2 w_3]L^4/384E_{cm}I_{c,co}$

Total deflection is

$\delta_8 = \delta_5 + \delta_6 + \delta_7$

Overall long-term active deflection due to creep-induced self-weight and dead loads plus static and creep-induced live load after installation using post-installation creep factor ψ_{28} and changes in camber after installation is

$\delta_9 = 5[\psi_{28}(w_1 + w_2) + (1 + \psi_{28})\ \psi_2 w_3]L^4/384E_{cm}I_{c,co} + (\delta_5 - \delta_3)$

Long-term active deflection due to static and creep-induced live load only is

$\delta_{10} = + 5\ (1 + \psi_{28})\ \psi_2 w_3 L^4/384E_{cm}I_{c,co}$

Summary deflections

Final long-term deflection $\delta_8 \leq$ span/250.

Long-term active deflections δ_9 or $\delta_{10} \leq$ span/500, or span/350 for nonbrittle partitions, finishes, and so forth.

Worked example for 1200 mm wide × 200 mm deep hollow-core unit per EC2

This section presents a worked example for a 1200 mm (48 in.) wide × 200 mm (8 in.) deep hollow-core unit according to EC2-1-1. This is for a simply supported span of 8.0 m (26 ft) to carry imposed dead UDL of 2.0 kN/m^2 (0.3 psi) and live load of 5.0 kN/m^2 (0.75 psi).

Section properties of compound section with transformed area of tendons per EC2-1-1

Young's modulus for tendons E_p = 195 kN/mm^2 (28,281 ksi)

Modular ratio $m - 1$ = (195,000/35,220) – 1 = 4.54

Area of compound section $A_{c,co}$ = 152,000 + 4.37 × 766 = 155,475 mm^2 (240 in.^{2})

Height to centroid of compound section $y_{b,co}$ = 97.7 mm (3.85 in.)

Second moment of area of compound section $I_{c,co}$ = 708.5 × 10^6 mm^4 (1702 in.^{4})

Compound section modulus at bottom $Z_{b,co}$ = 7.252 × 10^6 mm^3 (443 in.^{3})

Compound section modulus at top $Z_{t,co}$ = 6.926 × 10^6 mm^3 (423 in.^{3})

Compound section modulus at level of tendons $Z_{cp,co}$ = 708.5 × 10^6/58.2 = 12.173 × 10^6 mm^3 (743 in.^{3})

These values are used only for calculating stresses due to loading (not for prestress) and for service moment of resistance M_{sR} and deflections.

Calculations for prestress per EC2-1-1

Initial prestress σ_{pi} = 0.7 × 1770 = 1239.0 N/mm^2 (180 ksi)

Initial pretensioning force F_{pi} = 1239.0 × 766 × 10^{-3} = 949,074 kN (213,370 kip)

Prestress loss due to initial relaxation in tendon $\Delta\sigma_{pr}$ = 1239.0 × 0.66 × 2.5 × $e^{(9.1 \times 0.7)}$ × (20/1000)$^{[(0.75 \times (1 - 0.7)]}$ × 10^{-5} = 4.95 N/mm^2

Pretensioning force after initial losses F_{pmo} = (1239.0 – 4.95) × 766 = 945,281 N

Axial prestress after losses σ_{cp} = (945,281/152,000) + (945,281 × 58.2/11.979 × 10^6) = 10.81 N/mm^2

Axial prestress after losses at midspan, bending moment due to self-weight M_{s0} = 3.724 × 8.0^2/8 = 29.79 kN-m

Stress at level of tendons after initial losses $\sigma_{cp0} = M_{s0}Z_{cp}$ = -29.79/12.173 = -2.45 N/mm^2

Then stress at level of tendons after initial losses at midspan σ_{cp} = 10.81 – 2.45 = 8.36 N/mm^2

Elastic shortening loss of prestress $\Delta\sigma_{el}$ = 195,000 × 10.81/32,308 = 65.25 N/mm^2

Elastic shortening loss of prestress at midspan = 50.48 N/mm^2

Prestress after initial losses σ_{pm0} = 1168.8 N/mm^2

Prestress after initial losses at midspan σ_{pm0} = 1183.6 N/mm^2 (172 ksi) < 0.75 × 1770 = 1328 N/mm^2 (193 ksi) **OK**

Pretensioning force after initial losses F_{pm0} = 1168.8 × 766 = 895,302 N (201 kip)

Check transfer stresses at support:

Maximum surface stress at transfer at bottom $\sigma_b(t)$ = (895,302/152,000) + (895,302 × 58.2/7.040 × 10^6) = 13.29 N/mm^2 (1927 psi) < 0.6 × 28 = 16.8 N/mm^2 (2437 psi) **OK**

Maximum surface stress at transfer at top $\sigma_t(t)$ = (895,302/152,000) – (895,302 × 58.2/6.901 × 10^6) = -1.66 N/mm^2 (-240 psi) > -2.63 N/mm^2 (-381 psi) **OK**

Note that the self-weight of the unit at the end of the transfer length may be considered if $\sigma_b(t)$ or $\sigma_t(t)$ transfer stresses are not within the limits.

Long-time losses to life using a relative humidity *RH* of 50% with bottom only exposed

Maturity of concrete for mean temperature during 20 hours curing = 50ºC (122ºF)

Temperature adjusted age t_{0T} = (20/24) $e^{-[4000/(273 + 50) - 13.65]}$ = 2.96 days

Factor for cement Class R = 1

Equivalent age after curing at transfer t_o = 2.96 × [9/(2 + 2.96$^{1.2}$) + 1]^1 = 7.65 days

Then age at release loading factor $\beta(t_o)$ = 1/(0.1+ 7.65$^{0.2}$) = 0.624

Notional depth h_o = 2 × area/bottom = 2 × 152,000/1200 = 254.0 mm (10 in.)

Transmission length coefficients α_1 = 0.80; α_2 = 0.94; α_3 = 0.85

Relative humidity factor ϕ_{RH} = [1+ (1 – (50/100) × 0.80/(0.1 × 254.0$^{1/3}$)] × 0.94 = 1.533

Strength factor $\beta(f_{cm})$ = 16.8/$\sqrt{48}$ = 2.425

Relative humidity *RH* factor β_H = {1.5 × [1+ (0.012 × 50)^{18}] × 254.0} + (250 × 0.85) = 594 days

Detensioning age factor to installation $\beta_c(t_i,t_o)$ = [(20,833 – 1)/(594 + 20,833 – 1]$^{0.3}$ = 0.992

Creep coefficient in service $\phi(t,t_o) = \phi_{RH}\beta(f_{cm})\beta(t_o)\beta_c(t,t_o)$ = 1.533 × 2.425 × 0.624 × 0.992 = 2.301

Axial prestress after loss σ_{cp} = (895,302/152,000) + (895,302 × 58.2^2/697 × 10^6) = 10.24 N/mm^2

Axial prestress after loss at midspan = 10.37 N/mm^2

Axial prestress after loss at midspan after subtracting self-weight (3.72 + 7500 × 24.5 × 10^{-6} = 3.91 kN/m) and UDL finishes (1.5 × 1.2 = 1.8 kN/m)

At midspan, bending moment due to self-weight and finishes M_{s0} = 5.71 × 8.0^2/8 = 45.68 kN-m

Due to M_{s0}, σ_{cp0} = -45.68 × 12.17 = -3.75 N/mm^2

Then σ_{cp} = 10.37 – 3.75 = 6.62 N/mm^2

Denominator in the EC2 Exp. 5.46 = 1.138

Prestress loss due to creep at service $\Delta\sigma_{p,c}$ = 195,000 × 2.301 × 10.24/(35,220 × 1.138) = 114.63 N/mm^2 (16.6 ksi)

Prestress loss due to creep at service at midspan = 74.10 N/mm^2 (10.7 ksi)

Concrete shrinkage

Size coefficient k_n = 0.80

β_{RH} = 1.55 × [1 – (50/100)^3] = 1.356

Age factor for shrinkage $\beta_{ds}(t,t_s)$ = (20,833 – 1)/[(20,833 – 1) + 0.04 × 254.0$^{1.5}$] = 0.992

Total shrinkage strain $\varepsilon_{cs} = \beta_{ds}(t,t_s)k_n\varepsilon_{cd,o}$ = 0.992 × 0.80 × 0.85 × (220 + 110 × 6) × $e^{-0.11 \times 48/10}$ × 1.356 × 10^{-6} = 0.000473

Prestress loss due to shrinkage $\Delta\sigma_{p,s}$ = 0.000473 × 195,000/1.138 = 81.0 N/mm^2 (11.7 ksi)

At midspan, prestress loss also equals 81.0 N/mm^2

Tendon relaxation

Ratio of initial prestress μ = 1168.8/1770 = 0.660

Ratio of initial prestress at midspan = 1183.6/1770 = 0.669

Prestress due to tendon relaxation σ_{pr} = 1168.8 × 0.66 × 2.5 × $e^{(0.660 \times 9.1)}$ × [(500,000/1000)$^{0.75 \times (1 - 0.660)}$] × 10^{-5} = 38.24 N/mm^2 (5.5 ksi)

Prestress due to tendon relaxation at midspan σ_{pr} = 40.18 N/mm^2 (5.8 ksi)

Prestress loss due to final relaxation in tendon $\Delta\sigma_{p,r}$ = 0.8 × 38.24/1.138 = 26.89 N/mm^2 (3.9 ksi)

Prestress loss due to final relaxation in tendon at midspan $\Delta\sigma_{p,r}$ = 28.25 N/mm^2 (4.1 ksi) (slightly greater due to smaller losses)

Final prestress

Prestress after final losses σ_{po} = 1168.8 – 114.63 – 81.00 – 26.89 = 946.3 N/mm^2 (137 ksi)

Prestress at midspan σ_{po} = 1002.2 N/mm^2 (145 ksi)

Final prestressing force F_{po} = 946.3 × 766 = 724,854 N (163 kip)

Force at midspan = 766,168 N (172 kip)

Prestress ratio in service (working) R_{wk} = 724,854/949,074 = 0.764

At midspan ratio R_{wk} = 0.807

Final prestress at midspan

Maximum surface stresses in service at bottom σ_b = (766,168/152,000) + (766,168 × 58.2/7.040 × 10^6) = 11.37 N/mm^2 (1.6 ksi) < 0.45 × 40 = 18.0 N/mm^2 (2.6 ksi) **OK**

Maximum surface stresses in service at top σ_t = (766,168/152,000) – (766,168 × 58.2/6.900 × 10^6) = -1.42 N/mm^2 (-0.2 ksi) > -3.51 N/mm^2 (-0.5 ksi) **OK**

Service moment of resistance M_{sR} at midspan is the lesser of the following:

At the bottom $M_{sR,b}$ = (11.37 + 3.51) × 7.252 = 107.9 kN-m (79.6 kip-ft)

At the top $M_{sR,t}$ = (18.00 + 1.42) × 6.926 = 134.5 kN-m (99.2 kip-ft) > 107.9 kN-m

Therefore, M_{sR} at midspan = 107.9 kN-m.

Calculation for ultimate moment of resistance M_{Rd} per EC2-1-1

Concrete design strength $f_{cd} = 0.85f_{ck}/1.5$ = 0.567 × 40 = 22.67 N/mm^2 (3.3 ksi)

Area of tendons in tension zone A_{pT} = 766 mm^2 (1.2 in.^{2})

Mean height to tendons in tension zone $y_{sT} = y_s$ = 40.8 mm (1.6 in.) (all strands are in tension)

Effective depth of tendons in tension zone d = 200 – 40.8 = 159.2 mm (6.3 in.)

Prestrain due to prestress after losses ε_{po} = 946.3/195,000 = 0.004853

Ultimate strain tendons using Eq. (1) ε_p = 0.004853 + 0.0035 (159.2/X – 1)

Force on the concrete F_c = 0.8 × 22.67 × 1200X = 21,760X; force in the steel $F_s = f_p$ × 766

Then using Eq. (2) for equilibrium: X/f_p = 766/21,760 = 0.0352

Limit of proportionality of tendons ε_{LOP} = 0.9 × (1770/1.15)/195,000 = 0.007104

If ε_p > 0.007104, then using Eq. (3), f_p = 1385 + [154 × (0.0222 – ε_p)/(0.0222 – 0.007104)].

Combining Eq. (1) through (3) gives 328X^2 – 15,340X – 65,686 = 0.

Then X = 50.6 mm (2 in.), but because 0.8X = 40.5 mm (1.6 in.) > h_{ft} = 40 mm (1.6 in.), the compression block is just below the top of the circular cores. Further analysis using the breadth of the concrete beneath the top flange of 900 mm (35in.) finds X = 50.8 mm (2 in.).

Depth to centroid of concrete area d_n = 0.4 × 50.8 = 20.3 mm (0.8 in.)

Lever arm z = 159.2 – 20.3 = 138.9 mm (5.5 in.)

From Eq. (1): ε_p = 0.012313

From Eq. (3): f_p = 1438 N/mm^2 < limit at ε_s = 0.02 = 1516 N/mm^2 (220 ksi)

M_{Rd} = 1438 × 766 × 138.9 × 10^{-6} = 153.1 kN-m (113 kip-ft)

Anchorage bond length l_{bpd} to full M_{Rd} = 761 + 0.19 × 11.2 × (1438 – 946)/1.97 = 1295 mm (51 in.), where l_{pt2} = 761 mm (30 in.), α_2 = 0.19, average diameter of strands = 11.2 mm (0.4 in.), η = 1.2 for strands, and f_{bpd} = 1.2 × 0.7 × 3.51/1.5 = 1.97 N/mm^2 (0.3 ksi)

The results for the service and ultimate design bending moments and moments of resistance M_{sr} and M_{Rd} are shown in **Fig. 9**.

The service UDL w_s = 3.91 + 1.2 × (2.0 + 5.0) = 12.32 kN/m (0.8 kip/ft)

Service moment M_s = 12.31 × 8.0^2/8 = 98.5 kN-m (72.7 kip-ft)

The ultimate UDL w_{Ed} (for EC0, Exp. 6.10[b]) = 1.25 × (3.91 + 1.2 × 2.0) + 1.5 × 1.2 × 5.0 = 16.88 kN/m (1.2 kip/ft)

Ultimate design bending moment M_{Ed} = 16.88 × 8.0^2/8 = 135.0 kN-m (99.6 kip-ft)

Calculation for flexurally uncracked shear capacity $V_{Rd,c}$ per EC2-1-1

Nominal bearing length l_b = 100 mm (4 in.)

$f_{ctd}(t)$ = 0.7 × 2.63/1.5 = 1.23 N/mm^2 (0.18 ksi); f_{bpt} = 3.2 × 1.23 = 3.93 N/mm^2 (0.57 ksi)

Average diameter of strands = 11.2 mm (0.4 in.)

At the support σ_{pm0} = 1168.8 N/mm^2 (169.5 ksi)

l_{pt2} = 1.2 × 0.19 × 1168.8 × 11.2/3.93 = 761 mm (30 in.)

Prestress at neutral axis. $\gamma_{p,fav}\sigma_{cp}$ = 0.9 × 946.3 × 10^3 × 766/152,000 = 4.29 N/mm^2 (0.62 ksi)

f_{ctd} = 0.7 × 3.51/1.5 = 1.64 N/mm^2 (0.24 ksi)

$l_x = l_b + y_b$ = 100 + 99.0 = 199.0 mm (7.8 in.)

Distance to shear plane ratio α_l = 199.0/761 = 0.262

I_c = 697.0 × 10^6 mm^4 (1675 in.^{4}), b_w = 303 mm (11.9 in.), S_c = 4.80 × 10^6 mm^3 (293 in.^{3})

$V_{Rd,c}$ = (697.0 × 10^6 × 303/4.80 × 10^6) × [$\sqrt{1.64^2 + 0.262 \times 4.29 \times 1.64}$] × 10^{-3} = 93.5 kN (21 kip)

Ultimate design shear force V_{Ed}

w_{Ed} (from above) = 16.88 kN/m (1.16 kip/ft)

Shear span $l - l_b - 2_{yb}$ = 8000 – 100 – 2 × 99 = 7702 mm (303 in.)

V_{Ed} = 16.88 × 7.702/2 = 65.0 kN (14.6 kip) < 93.5 kN (21 kip) **OK**

Calculation for flexurally cracked shear capacity $V_{Rd,cr}$ per EC2-1-1

k (≤ 2.0) = 1 + $\sqrt{\frac{200}{159.2}}$ = 2.12 use 2.0

ρ_1 (≤ 0.02) = 766/(303 × 159.2) = 0.0159

Maximum $V_{Rd,cr}$ = [(0.18/1.5) × 2.0 × (100 × 0.0159 × 40)$^{1/3}$ + 0.15 × 4.29)] × 303 × 159.2 × 10^{-3} = 77.2 kN (17.4 kip)

v_{min} = 0.035 × 2.0$^{3/2}$ $\sqrt{40}$ = 0.63 N/mm^2 (0.09 ksi)

Minimum $V_{Rd,cr}$ = (0.63 + 0.15 × 4.29) × 303 × 159.2 × 10^{-3} = 61.3 kN (13.8 kip)

Critical $V_{Rd,cr}$ = 77.2 kN

To check the cracking moment of resistance, the compound section modulus and prestress at the bottom $Z_{b,co}$ = 7.252 × 10^6 mm^3 (442 in.^{3}), σ_b = 10.76 N/mm^2 (1.6 ksi) at the support

Use $V_{Rd,cr}$ where $M_s > M_c$ = 7.252 × 10^6 × (10.76 + 1.64) × 10^{-6} = 89.9 kN-m (66.3 kip-ft). This occurs at 2.72 m (8.9 ft) from the center of the supports. V_{Ed} at this point = 21.6 < 77.2 kN **OK**

Calculation for camber and deflections Considering the prestress at the support (not midspan), the initial loss at transfer is 5.7% and final losses are 23.6%. Also (but not included above) the losses at installation at 28 days after transfer are 9.6%.

Then the prestressing forces at different stages F_i = 949.1 kN (213.4 kip), F_{pm0} = 895.3 kN (201.3 kip), F_{pi} = 857.6 kN (192.8 kip), and F_{po} = 724.9 kN (163 kip).

L = 8000 mm (315 in.), $E_{cm}(t)$ = 32,308 N/mm^2 (4686 ksi), E_{cm} = 35,220 N/mm^2 (5108 ksi), z_{cp} = 58.2 mm (2.3 in.), $I_{c,co}$ = 708.5 × 10^6 mm^4 (1702 in.^{4})

Deflection at transfer

δ_1 = -895,302 × 58.2 × 8000^2/(8 × 32,308 × 708.5 × 10^6) = -18.2 mm (-0.71 in.)

δ_2 = 5 × 3.724 × 8000^4/(384 × 32,308 × 708.5 × 10^6) = +8.7 mm (+0.34 in.)

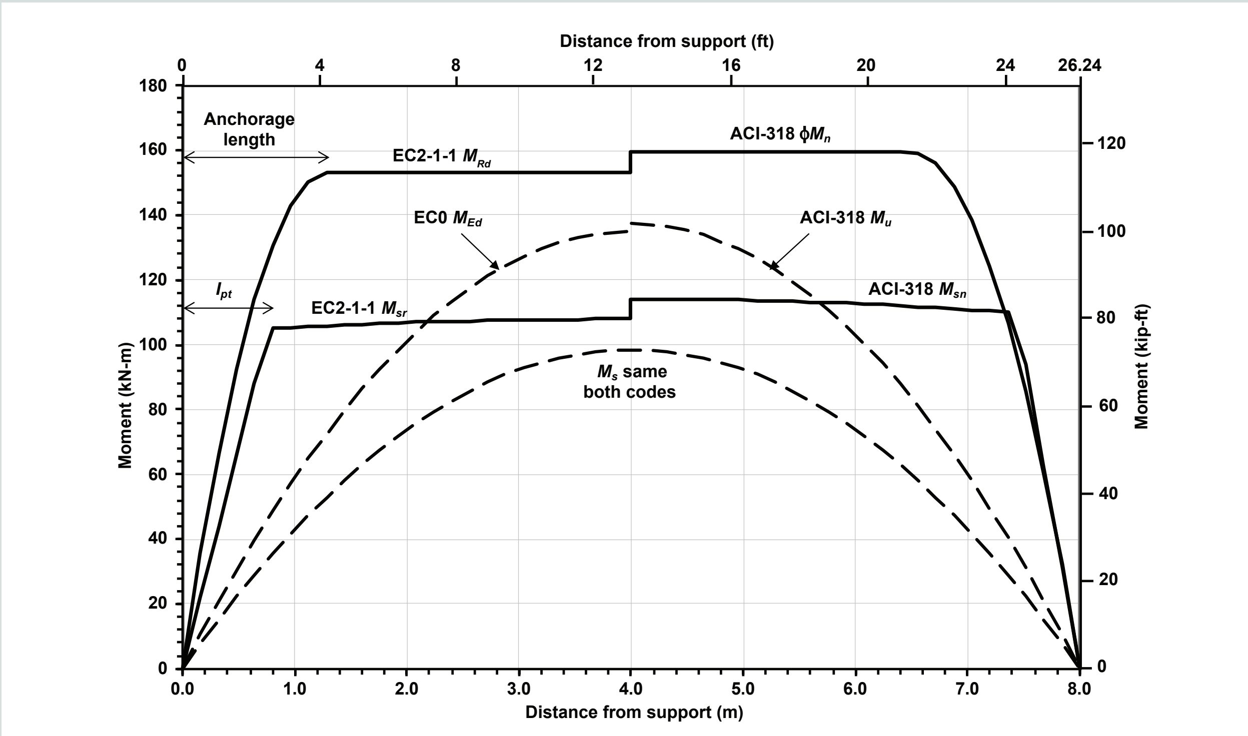

Figure 9. Distribution of service and ultimate design bending moments and moments of resistance from the design example according to Eurocode 2 and ACI 318. Note: l_{pt} = design transmission length; M_{Ed} = ultimate design bending moment; M_{Rd} = ultimate moments of resistance; M_s = service moment; M_{sn} = service moment of resistance; M_{sr} = service moment of resistance; M_u = ultimate design bending moment; ϕM_n = ultimate moments of resistance.

Net deflection at transfer = -9.5 mm (-0.37 in.)

Deflection at installation

ψ_∞ = 2.5, where one-day transfer ψ = 0.1 and 28-day transfer ψ = 0.4

ψ_1 = {[32,308/[0.5 × (32,308 + 35,220)]} × 2.5 × (0.4 – 0.1) = 0.72

$F_{pm0} - F_{pmi}$ = 37,713 N (8.5 kip)

δ_3 = 1.72 × -18.2 + [37,713 × 58.2 × 8000^2/(8 × 35,220 × 708.5 × 10^6)] = -30.6 mm (-1.2 in.)

δ_4 = 1.72 × 8.7 = +14.9 mm (+0.59 in.)

Net deflection at installation = -15.7 mm (-0.62 in.)

Long-term deflections

$X\psi_\infty$ = 0.8 × 2.5 = 2.00; ψ_{28} = 2.00 × (1 – 0.4) = 1.20

$F_{pmi} - F_{po}$ = 132,734 N (29.8 kip)

δ_5 = -30.6 – [(857,588 × 1.20) – 132,734] × 58.2 × 8000^2/(8 × 35,220 × 708.5 × 10^6) = -47.3 mm (-1.86 in.)

Self-weight of slab w_1 = 3.91 kN/m (0.27 kip/ft)

Floor dead load per unit width w_2 = 2.00 × 1.2 = 2.40 kN/m (0.16 kip/ft)

Offices ψ_2 = 0.3, then the quasi-permanent live load per unit width $\psi_2 w_3$ = 0.3 × 5.00 × 1.2 = 1.80 kN/m (0.12 kip/ft)
$\delta_6 + \delta_7$ = 14.9 + [1.20 × 3.91 + (1 + 1.20) × 2.40 + (1 + 2.00) × 1.80] × 5 × 8000^4/(384 × 35,220 × 708.5 × 10^6) = +47.8 mm (+1.88 in.)

Final deflection δ_8 = -47.3 + 47.8 = +0.5 < 8000/250 = 32 mm (1.26 in.)

Active deflections for floors with no brittle finishes
δ_9 = [1.20 × (3.91 + 2.4) + 2.2 × 1.8] × 5 × 8000^4/(384 × 35,220 × 708.5 × 10^6) – (47.3 – 30.6) = 7.9 mm (0.31 in.) < 22.9 mm (0.9 in.)

δ_{10} = (1 + 1.20) × 1.80 × [5 × 8000^4/(384 × 35,220 × 708.5 × 10^6)] = +8.5 mm (0.33 in.) < 8000/350 = 22.9 mm (0.9 in.)

Calculation for reduced axis distance for fire resistance per EC2-1-2 clause 2.4.2 and 5.2

$M_{Ed,fi} = M_{s,dead} + \psi_2 M_{s,live}$ = 50.5 + 0.3 × 48.0 = 64.9 kN-m (47.9 kip-ft), where $M_{s,dead}$ is the midspan moment due to self-weight of the slab plus dead loads and $M_{s,liv}$ is the midspan moment due to self-weight of the slab plus live loads

$M_{Ed,fi}/M_{Ed} = 64.9/135.0 = 0.48$

A_p required for M_{Ed} of 135.0 kN-m (99.6 kip-ft). flexural stiffness $K = M_{Ed}/f_{ck}bd^2 = 0.111$; z = 142 mm (5.6 in.) and the depth to the neutral axis x = 44 mm (1.7 in.)

The strain in the tendons $\varepsilon_p = 0.014069$; f_p = 1456 N/mm^2 (211.2 ksi); $A_{p,required} = M_{Ed}/zf_p$ = 654 mm^2 (1 in.^{2})

Then $A_{p,required}/A_{p,provided} = 654/766 = 0.85$

From $M_{Ed,fi}/M_{Ed}$ and $A_{p,required}/A_{p,provided}$, $k_p(\theta_{cr}) = 0.48 \times 0.85/1.15 = 0.357$

Figure 5.1 for prestressing steel curve 3, θ_{cr} = 436ºC (122ºF)

Eq. (EC2-1-2 5.3) $\Delta a = 0.1\ (500 - 436) = 6.4$ mm (0.25 in.)

Reduced axis distance a to strands = $y_s - \Delta a = 40.8 - 6.4 =$ 34.4 mm (1.35 in.) > 30 mm (1.2 in.) for 90 minutes (from Table 2). Then fire resistance R = 90 minutes. Also *REI* for t_e = 200 mm (8 in.) for 120 minutes.

Critical fire resistance rating *REI* = 90 minutes > 60 minutes required.

Design example following ACI 318 methodology

A design example for a 1200 mm (48 in.) wide × 200 mm (8 in.) deep hollow-core unit is presented following ACI 318 methodology. The hollow-core unit example is for a simply supported span of 8.0 m (26 ft) to carry imposed dead UDL of 2.0 kN/m^2 (0.3 psi) and live load of 5.0 kN/m^2 (0.75 psi). The example is arranged as follows. Design procedures and equations are presented followed by worked examples of the calculations for moment of resistance, shear capacity, deflection, and fire resistance.

In the following procedures and throughout this paper, code references are given on the left and the text/calculations/formulae are to the right.

Design procedures and equations per ACI 318

Load combinations per ACI 318 This section sets out the various service and ultimate load combinations for permanent (dead) and variable (live) loads and the quasi-permanent live load for deflections. A typical value for the quasi-permanent factor is a ψ_2 of 0.3 for office structures.

The service load is taken as the characteristic combination, as shown in Table 1. The ultimate load is obtained from ACI 318 section 9.2.1.

Minimum and nominal cover per ACI 318 Refer to ACI 318 Table 4.2.1 sections 7.7.3 and 7.7.6, as summarized in **Table 3**.

Axis distance and effective thickness for fire resistance per ACI 318 and the *PCI Design Handbook* Referring to *PCI Design Handbook* section 10.5.1, the effective thickness (or depth) h_{ef} is $(A_{c1}/bh)h$, where A_{c1} is the net cross-sectional area (after removing voids) including the infill in joints, b is the nominal breadth (1200 mm [48 in.]), and h is depth (200 mm [8 in.]).

The data from Fig. 10.5.1 and Table 6.3.1 of the *PCI Manual for the Design of Hollow-Core Slabs*[12] are based on service moment/nominal ultimate moment of resistance M_s/M_n, as given in **Table 4**, where M_s is service moment and M_n is ultimate moments of resistance without partial safety factor ϕ.

Material data per ACI 318

Concrete

Type of aggregate: gravel

28-day characteristic cylinder strength: f'_c = 40 N/mm^2 (5800 psi)

10.2.7.1 design strength $f_{cd} = 0.85 \times 40 = 34.0$ N/mm^2 (4930 psi)

18.4.2(b) limiting service strength $f_{cs} = 0.6 \times 40 =$ 24.0 N/mm^2 (3480 psi) due to total load

18.4.2(a) limiting service strength $f_{cs} = 0.45 \times 40 =$ 18.0 N/mm^2 (2610 psi) due to dead (sustained) load

18.3.3(a) tensile strength Class U $f_t = 0.63\sqrt{40} =$ -3.98 N/mm^2 (-577 psi)

18.3.3(b) tensile strength Class T $f_t = 1.00\sqrt{40} =$ -6.32 N/mm^2 (-916 psi)

8.5.1 Young's modulus $E_c = 4700\sqrt{f'_c}$ = 29.72 kN/mm^2 (4310 ksi)

Transfer cylinder strength of concrete f'_{ci} = 28 N/mm^2 (4060 psi) (cylinder strength)

18.4.1(c) tensile strength at ends at transfer $f_{ti} = 0.5\sqrt{30} =$ 2.74 N/mm^2 (397 psi)

18.4.1(c) tensile strength elsewhere at transfer $f_{ti} = 0.25\sqrt{30}$ = 1.27 N/mm^2 (184 psi)

18.4.1(b) limiting service compressive strength at ends f_{csi} =

$0.7 \times 30 = 21.0$ N/mm² (3046 psi)

18.4.1(a) limiting service compressive strength elsewhere $f_{csi} = 0.6 \times 30 = 18.0$ N/mm² (2610 psi)

8.5.1 Young's modulus at transfer $E_{ci} = 4700\sqrt{f'_{ci}} = 24.87$ kN/mm² (3607 ksi)

Steel tendons

Characteristic strength $f_{pu} = 1770$ N/mm² (257 ksi)

$f_{py}/f_{pu} = 0.851$ to 0.855 as reported by the manufacturer, where f_{pu} is the ultimate strength of the tendon

8.5.3 Young's modulus $E_p = 200{,}000$ N/mm² (29,007 ksi)

18.5.1 degree of pretensioning $\eta = 70\% < 80\% f_{pu}$ limit initial pretensioning stress $f_i = \eta f_{pu}$ initial pretensioning force $P_i = f_i A_{ps}$, where A_{ps} is area of tendons

Analysis of prestress losses and service and ultimate moments of resistance per ACI 318 This section presents standard calculation procedures leading to the service M_{sn} and ultimate ϕM_n moments of resistance according to ACI 318.

Service stress due to bending at transfer

The following guidance is from the *PCI Manual for the Design of Hollow-Core Slabs*.[12]

2.2.3 initial stress $f_{si} = \eta f_{pu}$
There is no loss given for initial relaxation before transfer. Instantaneous deformation due to elastic shortening:concrete stress at level of tendons (ignoring self-weight) $f_{cir} = K_{cir}(P_i/A_c + P_{ie}/Z_z)$, where A_c is net cross-sectional area of hollow-core unit, e is eccentricity of pretensioning force, and Z_z is section modulus at level of tendons

2.2.3(1) elastic shortening loss $ES = K_{es} f_{cir} E_p/E_{ci}$, where elastic shortening loss factors $K_{cir} = 0.9$ and $K_{es} = 1.0$

18.5.1 prestress after initial losses $f_o = f_{si} - ES$ should be $\leq 0.74 f_{pu}$

prestressing force at transfer $P_o = f_o A_{ps}$

Maximum surface stresses at transfer

Maximum surface stress at transfer at bottom $f_{pb} = \dfrac{P_o}{A_c} + \dfrac{P_o e}{Z_b}$,

Table 3. Requirements for durability according to ACI 318 Table 4.2.1 clauses 7.7.3 and 7.7.6

Category	Exposure	Severity	$f_{c,min}$, N/mm²	Cover C_{nom}, mm
C0	Dry or protected from moisture	n/a	17	20
C1	Exposed to moisture, no chlorides	Moderate	17	30

Note: $f_{c,min}$ = minimum compressive strength of concrete required for durability. n/a = not applicable. 1 mm = 0.0394 in.; 1 N/mm² = 0.145 ksi.

Table 4. Effective thickness and axis distance for fire resistance according to ACI 318

Fire endurance, hours	Effective thickness h_{ef}, mm		Mean axis distance to bottom tendons ≥ durability cover + radius, mm		
	Other aggregate	Limestone aggregate	Moment ratio M_s/M_n	Other aggregates	Limestone aggregate
1	89	81	0.5	32	27
			0.4	27	24
			0.3	24	21
2	127	119	0.5	49	46
			0.4	44	40
			0.3	40	33
3	157	147	0.5	64	59
			0.4	56	51
			0.3	49	43

Note: M_n = ultimate moment of resistance without partial safety factor ϕ; M_s = service moment. 1 mm = 0.0394 in.

where Z_b is the basic section modulus at the bottom of the slab

18.4.1 limit for $f_{pb} \leq 0.7f_{ci}$ at the ends, elsewhere $\leq 0.6f_{ci}$ not critical

Maximum surface stress at transfer at top $f_{pt} = \frac{P_o}{A_c} - \frac{P_o e}{Z_t}$,

where Z_t is the basic section modulus at the top of the slab

18.4.1 limit for $f_{pt} \geq -f_{ti} = -0.5\sqrt{f'_{ci}}$ at ends, elsewhere

$\geq -0.25\sqrt{f'_{ci}}$ not critical

Serviceability limit state of bending

Long-term losses are first calculated up to installation time t_i using creep loss factor at installation K_{cr1} of 0.8, and then to long term using a relative humidity RH_s of 50% with the bottom only exposed (top and sides protected) and a creep loss factor in service K_{cr2} of 2.0.

PCI 2.2.3(2) concrete stress at level of tendons after transfer $f_{co} = K_{cr1}(P_o/A_c + P_o e/Z_z)$

PCI 2.2.3(2) creep loss $CR1 = K_{cr1}(E_p/E_c)f_{co}$ prestress after creep losses at installation $f_{ins} = f_o - CR1$

$\therefore$ prestressing force at installation $P_{ins} = f_{ins}A_{ps}$ loss due to creep at long-term service RH_s of 50% and concrete stress at level of tendons after losses at installation (ignoring self-weight)

$$f_{ins} = \frac{P_{ins}}{A_c} + \frac{P_{ins}e}{Z_z},$$

PCI 2.2.3(2) creep loss $CR2 = K_{cr2}(E_p/E_c)f_{ins}$

PCI 2.2.3(3) For loss due to shrinkage from transfer to long term, relative humidity RH during the period of shrinkage is taken as 50% and notional depth for bottom only exposed $V/S = 2A_c/b_b$, where b_b is actual breadth at bottom of hollow-core unit.

shrinkage loss $SH = 8.2 \times 10^{-6}K_{sh}E_p[1 - (0.06/25.4)V/S](100 - RH)$, where shrinkage loss factor $K_{sh} = 1.0$

PCI 2.2.3(4) loss due to tendon relaxation $RE = [K_{re} - J(SH + CR + ES)]C$, where CR is creep

Values for tendon relaxation loss factor K_{re} and shrinkage loss factor J as a function of relaxation type and ultimate strength f_{pu} are given in **Table 5** and for shrinkage loss factor C in **Table 6**.

shrinkage loss factor C for stress-relieved strands for $f_{si}/f_{pu} > 0.7$, $C = 9.0f_{si}/f_{pu} - 5.3$, otherwise $5.1f_{si}/f_{pu} - 2.57$

shrinkage loss factor C for low-relaxation strand for $f_{si}/f_{pu} > 0.69$, $C = 5.0f_{si}/f_{pu} - 2.75$, otherwise $4.11f_{si}/f_{pu} - 2.14$

prestress after final losses since installation $f_{se} = f_{ins} - CR2 - SH - RE$

final prestressing force $P = f_{se}A_{ps}$

18.4.2(a) maximum surface stresses in service at

bottom $f_{pbe} = \frac{P}{A_c} + \frac{Pe}{Z_b}$

limit for $f_{pbe} \leq 0.45$

18.3.3(a) maximum surface stress in service at top

$f_{pte} = \frac{P}{A_c} - \frac{Pe}{Z_t}$, where Z_t is the basic section modulus at the top

limit for $f_{pte} \geq -0.63\sqrt{f'_c}$

Service moment of resistance M_{sn} is the lesser of the following:

Based on bottom surface

18.3.3(a) for Class U $M_{sn} = (f_{pbe} + 0.63\sqrt{f'_c})Z_{b,co}$, where $Z_{b,co}$ is the compound section modulus using the transformed area of tendons based on a modular ratio without creep effects $m = E_p/E_c$

18.3.3(b) for Class T $M_{sn} = (f_{pbe} + 1.00\sqrt{f'_c})Z_{b,co}$

Based on top surface

18.4.2(b) for total load $M_{sn} = (f_{pte} + 0.6)Z_{t,co}$ where is the compound section modulus using the transformed area of tendons based on a modular ratio without creep effects $m = E_p/E_c$

18.4.2(a) for dead load only $M_{sn} = (f_{pte} + 0.45)Z_{t,co}$

The compound section applies only to bending stresses, not to prestress.

Ultimate limit state of bending per ACI 318 This section presents calculation procedures for ultimate moment of resistance M_n according to ACI 318.

18.9.2 minimum area of tendons $A_{psT,min} = 0.004A_{ct}$, where A_{ct} is area between tension face and center of gravity of unit, approximately

$0.5A_c$

effective depth of tendons in tension zone $d_p = h - y_{sT}$, where h is depth of hollow-core unit and y_{sT} is mean height to tendons in tension zone

10.2.7.1. depth of rectangular stress block $a = \beta_1 c$, where β_1 is rectangular stress block factor and c is depth to neutral axis

first assume that $a < h_{ft}$, where h_{ft} is depth of top flange

18.7.1 with reference to **Fig. 6**, for $f_{se} \geq 0.5 f_{pu}$

or as the tensions reinforcement index ω = 0 without static reinforcement

18.7.2 design ultimate stress in tendons $f_{ps} = f_{pu}[1 - (\gamma_p/\beta_1)(A_{psT}/bd_p)(f_{pu}/f'_c)]$, where γ_p is tendon stress factor and A_{psT} is area of tendons in tension zone

Values for γ_p and β_1 are given in **Table 7**.

force equilibrium is $f_{ps}A_{psT} = 0.85 f'_c b \beta_1 c$

depth of compressive stress block at ultimate $a = f_{ps}A_{psT}/0.85 f'_c b$

depth to centroid of concrete area $d_n = 0.5a$

9.3.2.1 ultimate partial safety factor $\phi = 0.9$

ultimate moment of resistance $\phi M_n = 0.9 f_{ps} A_{psT} d_n$

18.8.2 $\phi M_n \geq M_{cr}$ except when both $\phi M_n \geq 2M_u$ and $\phi V_n \geq 2V_u$, where M_u is ultimate design bending moment, V_n is shear capacity check, and V_u is ultimate design shear force

9.5.2.3 cracking moment of resistance $M_{cr} = (f_{pbe} + f_r)Z_b$, where modulus of rupture

$$f_r = 0.63\sqrt{f'_c}$$

If the compression zone lies beneath the level of the top flange (that is, $a > h_{ft}$), the compression force is the sum of the force in the top flange above the cores and webs between the cores. Then $f_{ps}A_{psT} = 0.85 f_c(b - b_w) h_{ft} + 0.85 f'_c b_w a$, where b_w is total breadth of webs

Solving yields $a > h_{ft}$

Table 5. PCI MNL-126 Table 2.2.3.1 values for K_{re} and J as a function of f_{pu}

	Strength of tendons f_{pu}, N/mm²	Tendon relaxation loss factor K_{re}, N/mm²	Shrinkage loss factor J
Stress relieved	1860	137.9	0.15
	1725	127.6	0.14
	1655	121.4	0.13
Low relaxation	1860	34.5	0.040
	1725	31.9	0.037
	1655	30.3	0.035

Note: 1 N/mm² = 0.145 ksi.

Table 6. PCI MNL-126 Table 2.2.3.2 values of shrinkage loss factor C

Initial prestress f_{si}/strength of tendons f_{pu}	C for stress-relieved strands	C for low-relaxation strands
0.80	n/a	1.28
0.79	n/a	1.22
0.78	n/a	1.16
0.77	n/a	1.11
0.76	n/a	1.05
0.75	1.45	1.00
0.74	1.36	0.95
0.73	1.27	0.90
0.72	1.18	0.85
0.71	1.09	0.80
0.70	1.00	0.75
0.69	0.94	0.70
0.68	0.89	0.66
0.67	0.83	0.61
0.66	0.78	0.57
0.65	0.73	0.53
0.64	0.68	0.49
0.63	0.63	0.45
0.62	0.58	0.41
0.61	0.53	0.37
0.60	0.49	0.33

Note: n/a = not applicable.

Depth to centroid of concrete area $d_n = 0.5 \times 0.8X$

Lever arm = $d_p - d_n$

Therefore ultimate moment of resistance $\phi M_n = 0.9 f_{ps} A_{psT}(d_p - d_n)$

18.8.2 ultimate moment of resistance $\phi M_n > 1.2 M_{cr}$, where $M_{cr} = (f_{pbe} + f_r) Z_b$

Ultimate limit state of shear per ACI 318 This section presents calculation procedures for ultimate uncracked ϕV_{cw} and cracked ϕV_{ci} shear capacities according to ACI 318.

Ultimate shear force versus capacity criterion

11.3.1 effective depth $d_p \geq 0.8h$

For $f_{se} \geq 0.4 f_{py}$, where f_{py} is the yield stress (if $f_{py}/f_{pu} = 0.9$, then for $f_{se} \geq 0.36 f_{pu}$). If $\eta = 0.7$, then this implies 51% losses, meaning that the following equations may be used.

9.3.2.1 ultimate partial safety factor $\phi = 0.75$

11.1.1 ultimate design shear force $V_u \leq \phi V_n$ where $V_n = V_c + V_s$, where V_s is not applicable to hollow-core units and V_c is the lesser of V_{cw} for shear-web failure and V_{ci} for shear tension in the flexurally cracked part of the span

11.3.3 V_c shall be permitted to be the lesser of V_{cw} and V_{ci}

Section uncracked in flexure ϕV_{cw}, section 11.3.3.2

11.3.5 tendons bonded to end

11.3.3.2 transmission length $l_t = 50 \times d_b$, where d_b is diameter of strand

distance to critical section l_x = bearing length $l_b + 0.5h$

stress at centroidal axis after losses $f_{pc} = P/A_c$

11.3.4 Let distance to shear plane ratio $\alpha = \min\{1,$

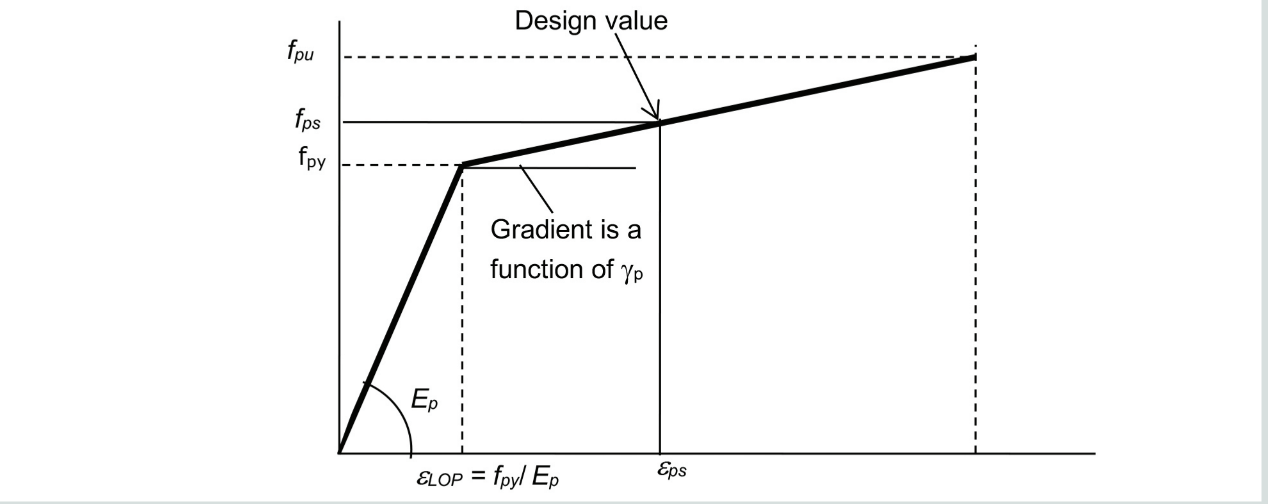

Figure 6. Idealized stress versus strain curve for tendons according to ACI 318. Note: E_p = Young's modulus for tendons; f_{ps} = design ultimate stress in tendons; f_{pu} = ultimate strength of tendons; f_{py} = 0.1% proof stress; yield stress; γ_p = tendon stress factor; ε_{LOP} = limit of proportionality of tendons; ε_{ps} = .

Table 7. Values for γ_p and β_1 as a function of f_{py}/f_{pu}

Yield stress f_{py}/ultimate strength of tendons f_{pu}	Clause 18.7.2 tendon stress factor γ_p	Concrete strength , N/mm²	Clause 10.2.7.3 rectangular stress block factor β_1
>0.8	0.55	17 to 28	0.85
>0.85	0.40	28 to 56	1.05 – f_c/140
>0.9	0.28	>56	0.65

Note: f_c = . 1 N/mm² = 0.145 ksi.

	$l_x/l_t\}$ representing a linear increase in f_{cp} for distance from center of support $x < l_t$ $V_{cw} = 0.29\sqrt{f_c b_w d_p} + \alpha 0.3 f_{cp} b_w d_p$
11.3.4	When distance from center of support $x \le l_b/2 + h/2$
Eq. (ACI 318 11-9)	$V_{cw} \le 0.42\sqrt{f'_c b_w d_p}$

Section cracked in flexure ϕV_{ci}, section 11.3.3.1

Shear capacity varies along the span at x according to

Eq. (ACI 318 11-10)	$V_{ci} = 0.05\sqrt{f_c b_w d_p} + V_d + M_{cre} V_i/M_i$, where V_d is shear force at x for service (unfactored) dead load, V_i is ultimate shear at x due to externally imposed dead and live loads only, and M_i is ultimate moment at x due to externally imposed dead and live loads only

$$V_{ci} \ge 0.17\sqrt{f_c b_w d_p}$$

Eq. (ACI 318 11-11)	moment required to cause flexural cracking $M_{cre} = Z_{b,co}\,(0.5\sqrt{f_c} + f_{pbe} - f_d)$, where tension stress due to service dead load $f_d = M_d/Z_{b,co}$ and M_d is moment due to dead load

Development length for ultimate limit state, section 12.9.1

ultimate anchorage/development length $L_d = [f_{se}/21 + (f_{ps} - f_{se})/7]d_b$

Note that L_d is a very large value, almost three times the transmission length (and approximately 75% greater than in British and European codes of practice).

Precamber and deflections per ACI 318 Deflections are determined for the immediate elastic deflection plus the viscoelastic effects of creep, which is also reflected in the changing values of pretension force and Young's modulus with time. The creep coefficients (referred to as long-term multipliers in *PCI Design Handbook*[4] Table 5.8.2) ψ multiplier is -1. Creep coefficients ψ are given in **Table 8**.

9.5.4.1	For Class U flexural members, deflection calculations are based on gross compound section $I_{c,co}$.
9.5.4.2	For Class T flexural members, deflection calculations are based on a cracked transformed section I_e. $I_e = (M_{cr}/M_a)^3 I_{c,co} + [1 - (M_{cr}/M_a)^3]I_{cr}$, where M_a is service moment at the stage where deflection is computed and I_{cr} is second moment of area for flexurally cracked section

The exponent (3) is the tension stiffening coefficient and shape factor for hollow-cores.

At transfer

Upward camber $\delta_1 = -P_o eL^2/8E_{ci}I_{c,co}$, where L is the effective span

Deflection due to self-weight $\delta_2 = +\,5w_o L^4/384E_{ci}I_{c,co}$, where $I_{c,co}$ is the compound value

Resultant deflection at transfer is $\delta_1 + \delta_2$.

At installation (taken as 28 days)

Camber at installation is due to the camber at transfer plus further viscoelastic movement $\psi_1\delta_1 = 0.8\delta_1$ minus a reduction due to the reduction in pretensioning force from transfer P_o to installation P_{ins} and is $\delta_3 = [-(1 + \psi_1)P_o + (P_o - P_{ins})]eL^2/8E_{ci}I_{c,co}$

Deflection at installation due to self-weight w_o is due to further viscoelastic movement $\psi_1\delta_2 = 0.85\delta_2$ of the self-weight at transfer, plus the static deflection $\delta_4 = +\,\delta_2(1 + \psi_1)$.

Resultant deflection of precast concrete only at installation is $\delta_3 + \delta_4$.

Long term

Imposed loads are applied after 28 days. Long-term camber is due to camber at installation plus further viscoelastic movement ψ_{28} of 1.45 due to the pretensioning force at installation P_{ins} less a reduction due to the reduction in pretensioning force from installation P_{ins} to long-term P.

$$\delta_5 = \delta_3 - \psi_{28}P_{ins}eL^2/8E_cI_{c,co} + (P_{ins} - P)eL^2/8E_cI_{c,co}$$

Long-term deflection due to self-weight and infill w_1 is due to the deflection at installation plus further viscoelastic movement using a ψ_{28} of 1.7.

$$\delta_6 = +\,\delta_4 + 5w_1\psi_{28}L^4/384E_cI_{c,co}$$

Deflection due to dead UDL w_2 added after installation plus further viscoelastic movement using long-term creep coefficient for deflection ψ_∞ is 2.0 of w_2 plus quasi-permanent live load $\psi_2 w_3$ using a long-term creep coefficient ψ_∞ of 2.0.

$$\delta_7 = +\,5\,[(1 + \psi_\infty)w_2 + (1 + \psi_\infty)\psi_2 w_3]L^4/384E_{cm}I_{c,co}$$

Total deflection is

$$\delta_8 = \delta_5 + \delta_6 + \delta_7$$

Overall long-term active deflection due to creep-induced self-weight using an effective creep coefficient after installation ψ_{28} of 1.7, creep-induced dead loads using a long-term creep coefficient ψ_∞ of 2.0 plus static and creep-induced live load using a long-term creep coefficient ψ_∞ of 2.0, and changes in camber after installation.

$$\delta_9 = 5[\psi_{28}w_1 + \psi_\infty w_2 + (1 + \psi_\infty)\psi_2 w_3]L^4/384E_cI_{c,co} + (\delta_5 - \delta_3)$$

Long-term active deflection due to static and creep-induced live load only is

$$\delta_{10} = + 5(1 + \psi_\infty)\psi_2 w_3 L^4/384E_cI_{c,co}$$

The limiting deflections are from ACI 318 clause 9.5.2.6 as summarized in **Table 9**.

Flexurally cracked section I_{cr}

Determined according to the geometry shown in cross section in **Fig. 8**.

modular ratio for deflection at installation $m_1 = (1 + \psi_1)E_p/E_c$, where $\psi_1 = 0.85$

modular ratio for long term deflection $m_\infty = (1 + \psi_\infty)E_p/E_c$ where $\psi_\infty = 2.0$

Assume first that $x_c > h_{ft}$

depth to centroidal axis

$$x_c = \frac{(b-b_w)h_{ft}^2/2 + mA_sd_p + (m-1)A_s'd'}{(b-b_w)h_{ft} + b_wx_c + mA_s + (m-1)A_s'}$$, where is the area of the top steel and is the depth of the top steel.

Solve for x_c.

Therefore $I_{cr} = bx_c^3/3 - (b - b_w)(x_c - h_{ft})^3/3 + A_{sm}(d_p - x_c)^2 + (m - 1)(x_c - d')^2$

If $x_c \le h_f$

$$x_c = \frac{bx_c^2/2 + mA_sd_p + (m-1)A_s'd'}{bx_c + mA_s + (m-1)A_s'}$$

Therefore $I_{cr} = bx_c^3/3 + A_{sm}(d_p - x_c)^2 + (m - 1)(x_c - d')^2$

Worked example for 1200 mm wide × 200 mm deep hollow-core unit per *PCI Design Handbook*

This section presents a worked example for a 1200 mm (48 in.) wide × 200 mm (8 in.) deep hollow-core unit according to ACI 318. This is for a simply supported span of 8.0 m (26 ft) to carry a UDL of 2.0 kN/m^2 (0.3 psi) and live load of 5.0 kN/m^2 (0.75 psi).

Section properties of compound section with transformed area of tendons per ACI 318

Young's modulus for tendons E_p = 200 kN/mm^2 (29,000 ksi)

Modular ratio $m - 1$ = (200,000/29,725) – 1 = 5.73

Area of compound section $A_{c,co}$ = 152,000 + 5.73 × 766 = 156,338 mm^2 (242 in.^{2})

Height to centroid of compound section $y_{b,co}$ = 97.4 mm (3.83 in.)

Second moment of area of compound section $I_{c,co}$ = 711.4 × 10^6 mm^4 (1709 in.^{4})

Compound section modulus at bottom $Z_{b,co}$ = 7.307 × 10^6 mm^3 (446 in.^{3})

Compound section modulus at top $Z_{t,co}$ = 6.932 × 10^6 mm^3 (423 in.^{3})

Compound section modulus at level of tendons $Z_{z,co}$ = 711.4 × 10^6/58.2 = 12.23 × 10^6 mm^3 (746 in.^{3})

Table 8. Creep coefficients according to *PCI Design Manual* Table 5.8.2

	Creep coefficient for hollow-core unit only	Creep coefficient for composite slab
Self-weight at installation	0.85	0.85
Camber at installation	0.8	0.8
Self-weight after installation	1.7	1.4
Camber after installation	1.45	1.2
Imposed dead and live load after installation	2.0	2.0
Topping self-weight after installation	n/a	1.3

Note: n/a = not applicable.

Calculations for prestress per ACI 318

Hollow-core unit designed uncracked with service Class U

As calculated for EC2, initial prestress f_{si} = 1239.0 N/mm² (180 ksi) and initial pretensioning force P_{pi} = 949,074 kN (213,370 kip)

Elastic shortening loss factor K_{cir} = 0.9

Concrete stress at level of tendons f_{cir} = 0.9 × [(949,074/152,000) + (949,074 × 58.2/11.979 × 10⁶)] = 9.77 N/mm²

Concrete stress at level of tendons at midspan = 7.33 N/mm²

Elastic shortening loss of prestress ES = 200,000 × 9.77/24,870 = 78.55 N/mm²

Elastic shortening loss of prestress at midspan after subtracting self-weight = 58.96 N/mm²

Prestress at transfer $f_o = f_{si} - ES$ = 1160 N/mm² (168 ksi) < 0.75 × 1770 = 1328 N/mm² (193 ksi) **OK**

Prestressing force at transfer P_o = 1160 × 766 = 888,901 N (200 kip)

Check maximum surface stresses at transfer:

Maximum surface stress at transfer at bottom f_{pb} = (888,901/152,000) + (888,901 × 58.2/7.04 × 10⁶) = 13.19 N/mm² (1913 psi)< 0.7 × 28 = 19.6 N/mm² (2843 psi) **OK**

Maximum surface stress at transfer at top f_{pt} = (888,901/152,000) – (888,901 × 58.2/6.90 × 10⁶) = -1.65 N/mm² (-239 psi) > -2.65 N/mm² (384 psi) **OK**

Short-term losses from transfer to installation

Creep coefficient for loading at 1 day to installation at 28 days = 0.8

Stress at level of tendons after initial losses f_{co} = (888,901/152,000) + (888,901 × 58.2²/697,000,000) = 10.17 N/mm²

Stress at level of tendons after initial losses at midspan after

Table 9. Summary of limiting deflections

Type of member	Deflection considered	Limit
Flat roof not supporting finishes likely to be damaged by deflection	Immediate live load δ_{10}	l/180
Floor not supporting finishes likely to be damaged by deflection	Immediate live load δ_{10}	l/360
Floor or flat roof supporting finishes likely to be damaged by deflection	After finishes δ_9	l/480*
Floor or flat roof supporting finishes not likely to be damaged by deflection	After finishes δ_9	l/240†
Floor or flat roof	Total minus camber δ_8	l/240

Note: *l* = effective span; δ_8 = long-term total deflection and active deflections; δ_9 = long-term total deflection and active deflections; δ_{10} = long-term total deflection and active deflections.

* May be exceeded if measures are taken to prevent damage.

† May be exceeded if camber is provided so that total deflection minus camber is less than *l*/240.

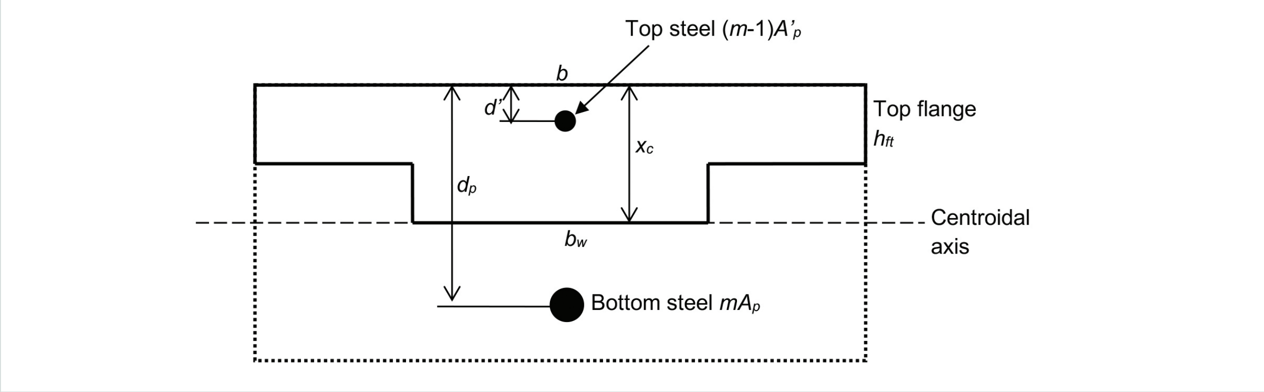

Figure 8. ACI 318 section definitions in a flexurally cracked section. Note: A_p = area of tendons; b =; b_w = total breadth of webs; d' =; d_p = effective depth in tendons in tension zone; h_{ft} = depth of top flange; I_{cr} = second moment of area for flexurally cracked section; m =; x_c = depth to centroid of axis (in calculation for I_{cr}).

subtracting self-weight = 8.51 N/mm^2

Prestress loss due to creep at installation $CR1$ = 200,000 × 0.800 × 10.17/29,725 = 54.7 N/mm^2

Prestress loss due to creep at installation at midspan = 45.80 N/mm^2

Prestress at installation f_{ins} = 1160.4 – 54.7 = 1105.7 N/mm^2 (160 ksi)

Prestress at installation at midspan = 1134.2 N/mm^2 (165 ksi)

Pretensioning force at installation P_{ins} = 1105.7 × 766 = 846,988 N (190 kip)

Pretensioning force at installation at midspan = 868,823 N (195 kip)

Long-term losses to life using relative humidity *RH* of 50% with bottom only exposed

Creep loss factor in service K_{cr2} = 2.0

Prestress at installation f_{ins} = (846,988/152,000) + (846,988 × 58.2^2/697.0 × 10^6) = 9.69 N/mm^2 (1.4 ksi)

Prestress at installation at midspan after subtracting self-weight and dead UDL = 5.81 N/mm^2 (0.8 ksi)

Prestress loss due to creep at service $CR2$ = 200,000 × 2.000 × 9.69/29,725 = 130.3 N/mm^2 (18.9 ksi)

Prestress loss due to creep at service at midspan = 78.17 N/mm^2 (11.3 ksi)

Concrete shrinkage

Notional depth V/S = 2 × area/bottom = 2 × 152,000/1200 = 111 mm (4.4 in.)

Prestress loss due to shrinkage SH = 8.2 × 10^{-6} × 1.0 × 200,000 × [1 –{(0.06/25.4) × 111}] × (100 – 50) = 60.6 N/mm^2 (8.8 ksi)

At midspan, prestress loss also equals 60.6 N/mm^2

Note that 0.06 factor is in inches.

Shrinkage loss factor K_{sh} = 1

Tendon relaxation

Relaxation factor K_{re} = 30.34 N/mm^2

Shrinkage loss factor J = 0.035 and stress relieved factor C = 0.75

Prestress loss due to final relaxation in tendon RE = {30.34 – [0.04 × losses (60.60 + 130.34 + 78.55)]} × 0.75 = 15.68 N/mm^2 (2.3 ksi)

Prestress loss due to final relaxation in tendon at midspan = 17.57 N/mm^2 (2.5 ksi)

Final prestress

Prestress after final losses f_{se} = 1160.45 – 130.34 – 60.60 – 15.68 = 953.8 N/mm^2 (138 ksi)

Prestress after final losses at midspan = 1023.7 N/mm^2 (148 ksi)

Final prestressing force P = 953.8 × 766 = 730,628 N (164 kip)

Final prestressing force at midspan = 784,160 N (176 kip)

Ratio of the final (working) prestressing force to the initial force R_{wk} = 730,628/949,074 = 0.770

At midspan, this ratio = 0.826

Final prestress at midspan

Maximum surface stresses in service at bottom f_{pbe} = (784,160/152,000) + (784,160 × 58.2/7.04 × 10^6) = 11.64 N/mm^2 (1.7 ksi) < 0.45 × 40 = 18.00 N/mm^2 (2.6 ksi) **OK**

Maximum surface stresses in service at top f_{pte} = (784,160/152,000) – (784,160 × 58.2/6.90 × 10^6) = -1.45 N/mm^2 (-0.2 ksi) > -3.98 N/mm^2 (-0.6 ksi) **OK**

Service moment of resistance M_{sn} at midspan for service Class U is the lesser of the following:

Top stress M_{sn} = (0.6 × 40.0 + 1.45) × 6.932 × 10^6 = 176.4 kN-m (130 kip-ft)

Bottom stress M_{sn} = (11.64 + 3.98) × 7.307 × 10^6 = 114.2 kN-m (84 kip-ft) < 176.4 kN-m

Therefore critical M_{sn} at midspan = 114.2 kN-m (84.3 kip-ft).

Calculation for cracking moment of resistance M_{cr} per ACI 318

Modulus of rupture $f_r = 0.63 \times \sqrt{40}$ = 3.98 N/mm^2

Cracking moment of resistance M_{cr} = 7.04 × 10^6 × (10.82 + 3.98)/10^6 = 104.2 kN-m (76.9 kip-ft)

$\phi M_n/M_{cr}$ = 1.53 > 1.2 **OK**

Calculation for ultimate moment of resistance *ϕMn* per ACI 318

Ultimate strength of tendons f_{pu}, area of tendons A_{ps}, mean height to tendons in the tension zone ys, and effective depth d_p

are the same as in the EC2 calculation.

Because $f_{py}/f_{pu} = 0$, tendon stress factor $\gamma_p = 0.40$

Because $f'_c = 40$ N/mm² (5.8 ksi), stress block ratio $\beta_1 = 1.05 - 40/140 = 0.764$

$A_{ps}/bd_p = 766/(1200 \times 159.2) = 0.0040$

$f_{ps} = 1770 \times [1 - (0.40/0.764) \times 0.0040 \times (1770/40)] = 1606$ N/mm² (233 ksi)

$c = 766 \times 1606/(0.85 \times 40 \times 1200 \times 0.764) = 39.4$ mm (1.6 in.)

$a = 0.764 \times 39.4 = 30.1 < h_{ft} = 40$ mm (1.6 in.), then compression block is rectangular

$d_n = 15.1$ mm (0.6 in.)

lever arm $z = 159.2 - 15.1 = 144.1$ mm (5.7 in.)

$\phi = 0.9$

$\phi M_n = 0.9 \times 1606 \times 766 \times 144.1 \times 10^{-3} = 159.5$ kN-m (117.6 kip-ft)

The results for the service M_{sn} and ultimate ϕM_n design bending moments and moments of resistance are shown in Fig. 9.

The service moment is the same as for EC2-1-1; that is, $M_s = 98.5$ kN-m (72.7 kip-ft).

The ultimate UDL $w_n = 1.2 \times (3.91 + 1.2 \times 2.0) + 1.6 \times 1.2 \times 5.0 = 17.17$ kN/m (1.2 kip/ft)

$M_u = 16.88 \times 8.0^2/8 = 137.4$ kN-m (101.3 kip-ft)

Calculation for flexurally uncracked shear capacity ϕV_{cw} per ACI 318

Average diameter of tendons = 11.22 mm (0.4 in.)

$l_t = 50 \times 11.22 = 561$ mm (22 in.)

Prestress at neutral axis $f_{cp} = 951.3 \times 766/152{,}000 = 4.81$ N/mm² (0.7 ksi)

l_x = bearing length $l_b + 0.5h = 100 + 100/2 = 200$ mm (8 in.)

$l_x/l_t = 200/561 = 0.357$

$\phi = 0.75$

Design concrete shear strength $v_{cw} = 0.29 \times \min\sqrt{\{69; 40\}} = 1.83$ N/mm² (0.27 ksi)

ϕV_{cw} at shear plane $= 0.75 \times [1.83 + 0.357 \times 0.3 \times 4.81) \times 303 \times 160]/10^3 = 85.4$ kN (19.2 kip)

ϕV_{cw} at end of $l_t = 0.75 \times [1.83 + 0.3 \times 4.81) \times 303 \times 160]/10^3 = 119.1$ kN (26.8 kip)

with a linear increase in between

Calculation for flexurally cracked shear capacity ϕV_{ci} per ACI 318

ϕV_{ci} varies according to factored imposed M_i and V_i, unfactored dead V_d, and moment causing cracking M_{cre} along the span.

$$\phi V_{ci} = 0.75\,(0.05\sqrt{f'_c}\,b_w d_p + V_d + M_{cre}V_i/M_i)$$

$$M_{cre} = Z_{b,co}\,(0.5\sqrt{f'_c} + f_{pe} - f_d)$$

but $\phi V_{ci,min} \geq 0.75 \times 0.17\sqrt{f'_c}\,b_w d_p$ and $\phi V_{ci,max} \leq 0.75 \times 0.42\sqrt{f'_c}\,b_w d_p$

The distribution of ϕV_{ci} along the span is shown in **Fig. 10**.

Calculation for camber and deflections per ACI 318

Considering the prestress at the support (not midspan), the initial loss at transfer is 6.3% and final losses are 23.0%. Also (but not previously included), the losses at installation at 28 days after transfer are 10.7%.

Then $P_i = 949.1$ kN (213.4 kip), $P_o = 888.9$ kN (199.8 kip), $P_{ins} = 847.0$ kN (190.4 kip), and final $P = 730.6$ kN (164.3 kip).

$L = 8000$ mm (315 in.), = 24,870 N/mm² (3607 ksi), $E_c = 29{,}725$ N/mm² (4311 ksi), $e = 58.2$ mm (2.3 in.), $I_{c,co} = 711.44 \times 10^6$ mm⁴ (1709 in.⁴)

Deflection at transfer

Due to the prestressing force $\delta_1 = -888{,}901 \times 58.2 \times 8000^2/(8 \times 24{,}870 \times 711.44 \times 10^6)$
$= -23.4$ mm (-0.92 in.)

Due to the self-weight of the slab $\delta_2 = (5 \times 3.72 \times 8000^4)/(384 \times 24{,}870 \times 711.44 \times 10^6) = +11.2$ mm (+0.44 in.)

Net deflection at transfer = -12.2 mm (-0.48 in.)

Deflection at installation

Creep coefficients at installation for camber $\psi_1 = 0.8$ and self-weight = 0.85

$\delta_3 = -[888{,}901 \times (1+0.80) - (888{,}901 - 846{,}988)] \times 58.2 \times 8000^2/(8 \times 29{,}725 \times 711.44 \times 10^6) = -41.2$ mm (-1.62 in.)

$\delta_4 = 11.2 \times (1 + 0.85) = +20.8$ mm (+0.82 in.)

Net deflection at installation = -20.4 mm (-0.80 in.)

Long-term deflections

Creep coefficients for camber ψ_{28} = 1.45, self-weight ψ_{28} = 1.7, and imposed dead and live ψ_{∞} = 2.0

δ_5 = -41.2 -[(846,988 × 1.45) – (846,988 – 730,628)] × 58.2 × 8000²/(8 × 29,725 × 711.44 × 10⁶) = -65.6 mm (-2.58 in.)

$\delta_6 + \delta_7$ = 20.8 + [(1.7 × 3.91 + (1 + 2.0) × (1.2 × 2.00) + (1 + 2.0) × (1.2 × 0.3 × 5.00)] × 5 × 8000⁴/(384 × 29,725 × 711.44 × 10⁶) = +69.3 mm (2.73 in.)

Net maximum deflection due to all loads δ_8 = +3.7 mm (0.15 in.) < 8000/240 = 33.3 mm (1.3 in.)

Active deflections for floors with no brittle finishes

δ_9 = (1.7 × 3.91 + 2.0 × 2.4 + 3.0 × 1.8) × 5 × 8000⁴/(384 × 29,725 × 711.44 × 10⁶) – (65.6 – 41.2) = 18.1 mm (0.7 in.) < 8000/360 = 22.2 mm (0.87 in.)

δ_{10} = (3.0 × 1.8) × 5 × 8000⁴/(384 × 29,725 × 711.44 × 10⁶) = 13.6 mm (0.54 in.) < 22.2 mm (0.87 in.)

Conclusion

For UDLs, the design criterion between the service moment to the service moment of resistance (M_s/M_{sr} for EC2 and M_s/M_{sn} for ACI 318) and the ultimate design bending moment to the ultimate moment of resistance (M_{Ed}/M_{Rd} for EC2 and $M_u/\phi M_n$ for ACI 318) is well balanced for this example. Usually the service moment is critical unless the amount of prestress is small. For EC2-1-1, flexurally uncracked shear capacity $V_{Rd,c}$ is only limiting when the span-to-depth ratio in this example is less than about 35. For ACI 318, flexurally cracked shear capacity ϕV_{ci} is limiting when the span-to-depth ratio is 42, showing that shear cracking in flexure will often be the governing criterion.

The calculations for prestressing losses in EC2 are complex but thorough, whereas ACI 318 uses simple tabulated data; however, the final losses differ by only 1%. The least amount of cover for indoor exposure (protected, low humidity) is 20 mm (0.8 in.) according to both codes. The axis height to tendons for 60-minute fire resistance using gravel aggregates is 35 mm (1.4 in.) according to EC2-1-2 and 24 to 32 mm (0.9 to 1.25 in.) according to ACI 318, depending on the M_s/M_n ratio (32 mm in this example). In EC2-1-2 this may be reduced by about 5 to 10 mm (0.2 to 0.4 in.), typically using the fire/ultimate moment method.

Overall, for an effective span of 8.0 m (26 ft), the area of strand required to resist the maximum possible UDL is 670 mm² (1.04 in.²) for EC2-1-1 and 640 mm² (0.99 in.²) for ACI 318, which is almost 5% less, reflecting the greater resistance in ACI 318 of M_{sn} equal to an EC2 1.055M_{sr}.

A summary of the worked EC2 and ACI 318 structural capacity examples is presented in **Table 10**.

References

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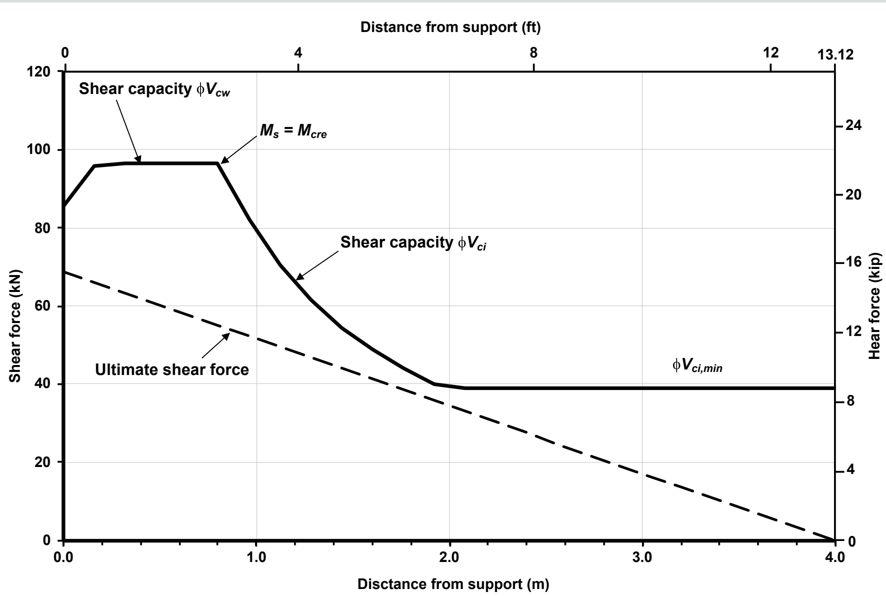

Figure 10. Distribution of shear force and shear capacity from the design example according to ACI 318. Note: M_{cre} = moment required to cause flexural cracking; M_s = service moment; M_u = ultimate design bending moment; V_{ci} = flexurally cracked shear capacity; $V_{ci,min}$ = ; V_{cw} = flexurally uncracked shear capacity.

Table 10. Summary of Eurocode 2 and ACI 318 worked examples structural capacities

	Unit	Eurocode 2		ACI 318		Difference between EC2 and ACI 318, %
Residual prestress after all losses	Ratio	0.807		0.826		-2.3
Permissible tension in soffit	N/mm2	f_{ctm}	= -3.51	f_t	= -3.98	-11.9
Service moment	kN-m	M_{sr}	= 107.9	M_{sn}	= 114.2	-5.5
Ultimate moment	kN-m	M_{Rd}	= 153.1	ϕM_n	= 159.5	-4.0
Uncracked shear	kN	$V_{Rd,c}$	= 93.5	ϕV_{cw}	= 85.4	+9.6
Cracked shear	kN	$V_{Rd,cr}$	= 77.2	ϕV_{ci}	= 39.1	+98
Flexural stiffness $M_s/E_{cm}I$	mm-1	K	= 4.01 × 10-6	S	= 4.75×10^{-6}	-15.6

Note: 1 mm = 0.0394 in.; 1 kN = 0.2248 kip; 1 kN-m = 0.7376 kip-ft; 1 N/mm² = 0.145 ksi.

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12. Buettner, Donald R., and Roger J. Becker. 1998. *PCI Manual for the Design of Hollow-Core Slabs*. MNL-126. 2nd ed. Chicago, IL: PCI.

Notation

Eurocode 2

a = axis distance to centroid of tendons in tension zone

A_c = net cross-sectional area of hollow-core unit

A_{c1} = net cross-sectional area (after removing voids) including the infill in joints

$A_{c,co}$ = area of compound section

A_p = area of tendons

$A_{p,provided}$ = area of tendons provided

$A_{p,required}$ = area of tendons required by design

A_{pT} = area of tendons in tension zone

b = nominal breadth of hollow-core unit

b_b = actual breadth at bottom of hollow-core unit

b_c = total breadth of cores

b_t = actual breadth at top of hollow-core unit

b_w = total breadth of webs

C_{min} = minimum and nominal cover to tendons

$C_{min,b}$ = minimum cover for bond

$C_{min,dur}$ = minimum cover for durability

C_{nom} = nominal cover

$C_{Rd,c}$ = concrete shear strength factor

d = effective depth of tendons in tension zone

d_n = depth to centroid of concrete area

E_{cm} = Young's modulus of concrete

$E_{cm}(t)$ = Young's modulus at transfer

E_d = ultimate design action

$E_{d,fi}$ = design action in fire

E_p = Young's modulus for tendons

f_{bpd} = ultimate bond strength

f_{cd} = design strength of concrete

$f_{cd}(t)$ = design strength at transfer

f_{ck} = 28-day characteristic cylinder strength

$f_{ck,cube}$ = 28-day characteristic cube strength

$f_{ck}(t)$ = transfer cylinder strength of concrete

f_{cm} = mean compressive strength at 28 days

$f_{c,min}$ = minimum compressive strength of concrete required for durability

$f_{cm}(t)$ = mean compressive strength at transfer

f_{ctd} = design tensile strength of concrete

$f_{ctd}(t)$ = design tensile strength of concrete at transfer

f_{ctm} = mean tensile strength of concrete

$f_{ctm}(t)$ = mean tensile strength of concrete at transfer

$f_{ct,0.05}$ = 5% fractile tensile strength of concrete

f_p = design ultimate stress in tendons

f_{pbt} = bond stress at transfer

f_{pd} = maximum stress at ultimate of tendons

f_{pk} = ultimate strength of tendons

$f_{p,max}$ = maximum prestress allowed

$f_{p,0.1k}$ = 0.1% proof stress; yield stress

F = final prestressing force

F_c = force on the concrete

F_{pi} = initial pretensioning force

F_{pmi} = pretensioning force at installation

F_{pm0} = pretensioning force after initial losses

F_{po} = final prestressing force

F_r = prestressing force at release/transfer

F_s = force in the steel

G_k = dead load (sustained)

h = depth of hollow-core unit

h_{ft} = depth of top flange

h_o = notional depth

I_c = second moment of area of basic section

$I_{c,co}$ = second moment of area of compound section

k = shear strength depth factor

k_n = size coefficient

k_p = strength reduction coefficient for tendons in fire

k_T = limiting ratio of prestress after losses to initial pre-stressing

K = flexural stiffness

l = effective span

l_b = bearing length

l_{bpd} = ultimate anchorage/development length

l_{pt} = basic transmission length

l_{pt2} = design transmission length

l_x = distance to critical section

L = effective span of hollow-core unit

m = modular ratio

M_{Ed} = ultimate design bending moment

$M_{Ed,fi}$ = ultimate design moment in fire

M_{max} = maximum service moment to be checked in the slab

M_{min} = minimum service moment to be checked in the slab

M_{Rd} = ultimate moments of resistance

M_s = service moment

M_{s0} = service moment due to self-weight and finishes

$M_{s,dead}$ = service bending moment due to self-weight and dead loads

$M_{s,live}$ = service bending moment due to live loads

M_{sr} = service moment of resistance

$p_{yk}(20°C)$ = strength of tendons at room temperature

Q_k = live load (imposed)

$Q_{k,i}$ = secondary point or linear variable action from another source

$Q_{k,1}$ = one dominant variable action, such as uniformly distributed live load

r_{inf} = factor for the direct measurement of prestress

r_{sup} = factor for the direct measurement of prestress

R = total loss ratio

R_{tr} = prestress ratio at installation

R_{wk} = prestress ratio in service (working)

REI = critical fire resistance rating

RH = relative humidity

RH_s = relative humidity in service

S_c = first moment of area of hollow-core unit

t = relaxation time of tendon; time in service

t_e = effective thickness (depth) in fire

t_i = installation age

t_o = transfer age

t_o = equivalent age after curing at transfer

t_{oT} = temperature-adjusted age

t_s = transfer age to relaxation time of tendon

T = mean temperature during curing

v_{min} = minimum concrete shear strength

V_{Ed} = ultimate design shear force

$V_{Rd,c}$ = flexurally uncracked shear capacity

$V_{Rd,cr}$ = flexurally cracked shear capacity

w_{Ed} = ultimate design load

w_o = self-weight of unit

w_1 = self-weight of slab (unit plus infill)

w_2 = floor dead load per unit width

w_3 = floor live load per unit width

X = depth to neutral axis at ultimate

y_b = height to centroid of basic section

$y_{b,co}$ = height to centroid of compound section

y_s = mean height to all tendons

y_{sT} = mean height to tendons in tension zone

z = lever arm

z_{cp} = eccentricity of pretensioning force

Z_b = basic section modulus at bottom of slab

$Z_{b,co}$ = compound section modulus at bottom

Z_{cp} = section modulus at level of tendons

$Z_{cp,co}$ = compound section modulus at level of tendons

Z_t = basic section modulus at top of slab

$Z_{t,co}$ = compound section modulus at top

$Z_{z,co}$ = compound section modulus at level of tendons

α_{cc} = concrete strength coefficient

α_{ct} = concrete strength coefficient in tension

α_{ds1} = cement factor

α_{ds2} = cement factor

α_l = distance to shear plane ratio = l_x/l_{pt2}

α_1 = transmission length coefficient

α_2 = transmission length coefficient

$\beta(f_{cm})$ = strength factor

$\beta(t_o)$ = age at release loading factor

$\beta_c(t,t_o)$ = detensioning age factor to service

$\beta_c(t_i,t_o)$ = detensioning age factor to installation

$\beta_{cc}(t)$ = strength ratio $f_{cm}(t)/f_{cm}$

$\beta_{ds}(t,t_s)$ = age factor for shrinkage

β_H = relative humidity time factor in days

β_{RH} = relative humidity factor

β_1 = rectangular stress block factor

γ_c = partial factor of safety for concrete

$\gamma_{p,fav}$ = favorable partial safety factor at ultimate (EC2-1-1 only)

γ_s = partial factor of safety for steel tendons

δ_1 = upward camber due to prestress at transfer

δ_2 = deflection due to self-weight of unit at transfer

δ_3 = upward camber due to prestress at installation

δ_4 = deflection due to self-weight of unit and infill at installation

δ_5 = long-term upward camber due to prestress

δ_6 = long-term deflection due to self-weight of unit and infill

δ_7 = long-term deflection due to imposed dead and live loads

δ_8 = total long-term deflection

δ_9 = long-term active deflection due to changes in camber and dead and live loads after installation

δ_{10} = long-term active deflection due to live loads only

Δa = additional axis distance for tendons

ΔC_{dev} = allowance for deviation of cover

ΔC_{dur} = safety distance for cover for durability

$\Delta\sigma_{el}$ = elastic shortening loss of prestress

$\Delta\sigma_{p,c}$ = prestress loss due to creep at service

$\Delta\sigma_{p,c,i}$ = prestress loss due to creep at installation

$\Delta\sigma_{p,r}$ = prestress loss due to initial relaxation in tendon (no data in ACI 318)

$\Delta\sigma_{p,r}$ = prestress loss due to final relaxation in tendon

$\Delta\sigma_{p,s}$ = prestress loss due to shrinkage

ε_{cd} = drying shrinkage strain

$\varepsilon_{cd,o}$ = basic drying shrinkage strain

ε_{cs} = total shrinkage strain

ε_{cu3} = ultimate strain in concrete

ε_{LOP} = limit of proportionality of tendons

ε_p = total ultimate strain in tendons

ε_{po} = prestrain due to prestress after losses

ε_{ud} = limiting design strain in tendons

ε_{uk} = ultimate strain in tendons

ζ = ratio of solid material to the whole

η = degree of pretensioning

η_{fi} = fire load ratio = $E_{d,fi}/E_d$

η_{p2} = anchorage length factors for tendons

η_1 = concrete bond factor

θ_{cr} = critical temperature for tendons in fire

μ = ratio of initial prestress

ρ_1 = steel area ratio

ρ_{1000} = relaxation loss in tendons at 1000 hours

σ_b = maximum surface stresses in service at bottom

$\sigma_b(t)$ = maximum surface stress at transfer at bottom

σ_{cp} = axial prestress after losses

σ_{cpo} = stress at level of tendons after initial losses

$\sigma_{ct,p}$ = mean tensile strength of concrete

σ_{pb} = concrete stress at level of tendons

$\sigma_{p,fi}$ = strength of tendons in fire

σ_{pi} = initial prestress

σ_{pmi} = prestress at installation

σ_{pmo} = prestress after initial losses

σ_{po} = prestress after final losses

σ_{pr} = prestress due to tendon relaxation

σ_r = prestress at release/transfer

σ_t = maximum surface stresses in service at top

$\sigma_t(t)$ = maximum surface stress at transfer at top

ϕ_{RH} = relative humidity factor

$\phi_{(t,ti)}$ = internal creep coefficient

$\phi(t,to)$ = creep coefficient in service

$\phi_{(ti,to)}$ = creep coefficient at installation

ϕ = diameter of strands

ψ_1 = effective creep coefficient for deflection at installation

ψ_2 = quasi-permanent live load factor

$\psi_{2,1}$ = quasi-permanent combination factor

ψ_{28} = effective creep coefficient after installation

ψ_∞ = long-term creep coefficient for deflections

ψ_{fi} = quasi-permanent combination factor for live load in fire

ACI 318 and *PCI Design Handbook*

a = depth of compressive stress block at ultimate

A_c = net cross-sectional area of hollow-core unit

A_{c1} = net cross-sectional area after removing voids including the infill in joints

$A_{c,co}$ = area of compound section

A_{ct} = area between tension face and centroid of unit

A_{ps} = area of tendons

A_{psT} = area of tendons in tension zone

$A_{psT,min}$ = minimum area of tendons

A_s' = area of top steel

b = nominal breadth

b_b = actual breadth at bottom of hollow-core unit

b_t = actual breadth at top of hollow-core unit

b_w = total breadth of webs

c = depth to neutral axis at ultimate

C = shrinkage loss factor

$C_{min,b}$ = minimum cover for durability and bond

C_{nom} = minimum and nominal cover to tendons

CR = creep

$CR1$ = prestress loss due to creep at installation

$CR2$ = prestress loss due to creep at service

d' = depth of top steel

d_b = diameter of strands

d_n = depth to centroid of concrete area

dp = effective depth of tendons in tension zone

D = dead load (sustained)

e = eccentricity of pretensioning force

E_c = Young's modulus of concrete

E_{ci} = Young's modulus of concrete at transfer

E_p = Young's modulus for tendons

ES = elastic shortening loss of prestress

f_c' = 28-day characteristic cylinder strength

f_{ci}' = transfer cylinder strength of concrete

f_{cd} = design strength of concrete

f_{cir} = concrete stress at level of tendons

$f_{c,min}$ = minimum compressive strength of concrete required for durability

f_{co} = stress at level of tendons after initial losses

f_{cs} = limiting service strength

f_{csi} = limiting service compressive strength

$f_{ct,0.05}$ = 5% fractile tensile strength of concrete

f_d = tension stress due to service dead load

f_i = initial prestress

f_{ins} = prestress at installation

f_o = prestress at release/transfer

f_{pb} = maximum surface stress at transfer at bottom

f_{pbe} = maximum surface stresses in service at bottom

f_{pc} = axial prestress after losses

f_{ps} = design ultimate stress in tendons

f_{pt} = maximum surface stress at transfer at top

f_{pte} = maximum surface stresses in service at top

f_{pu} = ultimate strength of tendons

f_{py} = 0.1% proof stress; yield stress

f_r = modulus of rupture

f_{se} = prestress after final losses

f_{si} = initial prestress

f_t = mean tensile strength of concrete

f_{ti} = mean tensile strength of concrete at transfer

F_c = compression force

F_{ps} = force in prestress

h = depth of hollow-core unit

h_{ef} = effective thickness (in fire) = $(A_{c1}/bh)h$

h_{ft} = depth of top flange

I_c = second moment of area of basic section

$I_{c,co}$ = second moment of area of compound section

I_{cr} = second moment of area for flexurally cracked section

I_e = cracked transformed section for Class T flexural members

J = shrinkage loss factor

K_{cir} = elastic shortening loss factor

K_{cr1} = creep loss factor at installation

K_{cr2} = creep loss factor in service

K_{es} = elastic shortening loss factor

K_{re} = tendon relaxation loss factor

K_{sh} = shrinkage loss factor

l_b = bearing length

l_t = design transmission length

l_x = distance to critical section

L = live load (imposed)

L = effective span of hollow-core unit

L_d = ultimate anchorage/development length

L_r = live roof load

m = modular ration without creep effects

m_∞ = modular ratio for long-term deflection

m_1 = modular ratio for deflection at installation

M_a = service moment at the design stage where deflection is computed

M_{cr} = cracking moment of resistance

M_{cre} = moment required to cause flexural cracking

M_d = moment due to dead load

M_i = ultimate bending moment at distance x along the span of the slab

M_n = ultimate moments of resistance without ϕ

M_s = service moment

M_{sn} = service moment of resistance

M_u = ultimate design bending moment

p_{yk}(20ºC) = strength of tendons at room temperature

P = final prestressing force

P_{ins} = pretensioning force at installation

P_o = prestressing force at release/transfer

P_{pi} = initial pretensioning force

r_{inf} = factors for the direct measurement of prestress

R_{wk} = ratio of the final (working) prestressing force to the initial force

RE = prestress loss due to final relaxation in tendon

S = snow load

SH = prestress loss due to shrinkage

t_i = installation age

V/S = notional depth

v_{cw} = concrete shear strength

V_c = the lesser of V_{cw} and V_{ci}

V_{ci} = shear web failure

V_{cw} = shear tension in flexurally cracked part of span

V_d = shear force at x for service (unfactored) dead load

V_i = ultimate shear at x due to externally imposed dead and live loads

V_n = shear capacity check = V_c

V_u = ultimate design shear force

w_0 = self-weight of unit

w_1 = self-weight of slab (unit plus infill)

w_2 = floor dead load per unit width

w_3 = floor live load per unit width

w_u = ultimate design load

x = distance along span of slab from center of support

x_c = depth to centroidal axis (in calculation for I_{cr})

y_b = height to centroid of basic section

$y_{b,co}$ = height to centroid of compound section

y_{sT} = mean height to tendons in tension zone

z = lever arm

Z_b = basic section modulus at bottom of slab

Z_t = basic section modulus at top of slab

Z_z = section modulus at level of tendons

α = distance to shear plane ratio = l_x/l_{pt2}

β_1 = rectangular stress block factor

γ_c = partial factor of safety for concrete

γ_p = tendon stress factor

δ_1 = upward camber due to prestress at transfer

δ_2 = deflection due to self-weight of unit at transfer

δ_3 = upward camber due to prestress at installation

δ_4 = deflection due to self-weight of unit and infill at installation

δ_5 = long-term upward camber due to prestress

δ_6 = long-term deflection due to self-weight of unit and infill

δ_7 = long-term deflection due to imposed dead and live loads

δ_8 = total long-term deflection

δ_9 = long-term active deflection due to changes in camber and dead and live loads after installation

δ_{10} = long-term active deflection due to live loads only

ε_{ps} = total shrinkage strain

ε_{sh} = total shrinkage strain

η = degree of pretensioning

η = ratio of initial prestress

ϕ = ultimate partial safety factor

ϕM_n = ultimate moments of resistance

ϕV_{ci} = flexurally cracked shear capacity

$\phi V_{ci,max}$ = maximum value of flexurally cracked shear capacity ϕV_{ci}

$\phi V_{ci,min}$ = minimum value of flexurally cracked shear capacity ϕV_{ci}

ϕV_{cw} = flexurally uncracked shear capacity

ψ_1 = effective creep coefficient for deflection at installation

ψ_2 = quasi-permanent live load factor

ψ_{28} = effective creep coefficient after installation

ψ_∞ = long-term creep coefficient for deflections

ω = tension reinforcement index

About the author

Kim S. Elliott, PhD, is a consultant to the precast concrete industry in the United Kingdom. He was senior lecturer at Nottingham University in the United Kingdom from 1987 to 2010 and was formerly at Trent Concrete Structures Ltd. UK. He has also been a consultant to three organizations in Malaysia. He is a member of *fib* Commission 6 on Prefabrication, where he has made contributions to six manuals and technical bulletins, and is the author of *Multi-Storey Precast Concrete Frame Structures* and *Precast Concrete Structures*. He was chairman of the European research project COST C1 on semi-rigid connection in precast concrete structures from 1992 to 1999. He has lectured on precast concrete structures 45 times in 16 countries, including Malaysia, Singapore, Australia, Korea, Brazil, Bahrain, South Africa, Barbados, Austria, Spain, and Scandinavia, and at 30 U.K. universities.

Abstract

A typical 1200 mm (48 in.) wide × 200 mm (8 in.) deep prestressed concrete hollow-core unit is analyzed and designed in order to make a comparison between Eurocode 2 and ACI 318-08. This includes calculations for serviceability limit state of stress and moment of resistance, ultimate moment of resistance, ultimate shear capacities, flexural stiffness (that is, for deflection), and cover to pretensioning tendons for conditions of environmental exposure and fire resistance. Concrete cylinder strength is 40 MPa (5.8 ksi), and concrete cube strength is 50 MPa (7.3 ksi). The hollow-core unit is pretensioned using seven-wire helical strands. Worked examples are presented in parallel formation according to Eurocode 2 and ACI 318.

For uniformly distributed loads, the design criterion between the service moment to service moment of resistance (M_s/M_{sr} for EC2 and M_s/M_{sn} for ACI 318) and the ultimate design bending moment to ultimate moment of resistance (M_{Ed}/M_{Rd} for EC2 and $M_u/\phi M_n$ for ACI 318) is well balanced for this example. Usually the service moment is critical unless the amount of prestress is small. For EC2-1-1, flexurally uncracked shear capacity $V_{Rd,c}$ is only limiting when the span-to-depth ratio in this example is less than about 35. For ACI 318, flexurally cracked shear capacity ϕV_{ci} is limiting when span-to-depth ratio is 42, showing that shear cracked in flexure will often be the governing criterion.

Keywords

ACI 318, deflection, durability, Eurocode 2, fire resistance, floor slab, hollow-core floor unit, pretensioning strand, service stress, shear capacity, ultimate strength.

Review policy

This paper was reviewed in accordance with the Precast/Prestressed Concrete Institute's peer-review process.

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