Multistep elastic analysis of the nonlinear out-of-plane load-deflection behavior of precast concrete insulated sandwich panels

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- This paper proposes a multistep linear elastic analysis (MLEA) method to estimate the nonlinear out-of-plane load-deflection behavior of precast concrete insulated sandwich panels.

- The MLEA method was used to predict the load-deflection behavior of insulated precast concrete panels with different geometries and interlayer mechanical connectors.

- Load-deflection relationships predicted using the proposed analysis method were compared to the authors' results from out-of-plane flexural testing of panel specimens and to experimental results reported by other investigators.

PCI Journal (ISSN 0887-9672) V. 65, No. 1, January–February 2020.

PCI Journal is published bimonthly by the Precast/Prestressed Concrete Institute, 200 W. Adams St., Suite 2100, Chicago, IL 60606.

Precast concrete insulated sandwich panels (ISPs) are a cladding system used to protect buildings from moisture ingress and heat loss. These panels are typically made of rigid insulation between two concrete layers. Mechanical connectors connect the concrete layers to each other. Like all cladding systems, these panels are subjected to out-of-plane wind and seismic loads. The out-of-plane loads induce out-of-plane shear forces and bending moments in the precast concrete ISP, thereby producing interlayer shear forces that are primarily transferred between the concrete layers by the mechanical connectors.

If shear is not transferred between the concrete layers, the panel exhibits noncomposite flexural behavior; that is, plane cross sections remain plane across the depth of each concrete layer but not across the entire depth of the panel. If the mechanical connectors have sufficient shear strength and stiffness to transfer the interlayer shear forces, the panel exhibits fully composite flexural behavior; that is, plane cross sections remain plane across the full depth of the panel. Any behavior between these two extreme cases is referred to as partially composite flexural behavior. Therefore, the flexural strength and stiffness of the precast concrete ISP depends on the shear strength and stiffness of the mechanical connectors between the layers.

Common types of interlayer mechanical connectors include trusses, grid connectors, M ties, pins, and clips.[1,2] Some connector systems, such as trusses and grids, are continuous along the height of the precast concrete ISP, while others, such as pins and M ties, are discrete. The top and bottom ends of precast concrete ISPs are sometimes enclosed by reinforced concrete members called end beams. End beams contribute to the interlayer shear transfer and improve the out-of-plane flexural strength and stiffness of the precast concrete ISP.

Many researchers have compared the experimentally measured out-of-plane behavior of precast concrete ISPs constructed with different interlayer mechanical connectors with the theoretical noncomposite and fully composite behaviors.[3–5] Previous researchers have also derived analytical methods to approximate the degree of composite action and estimate the deflection and strength of precast concrete ISPs.[5–7] These analytical methods, however, did not account for the effects of parameters such as the length of the panels and the thickness of the concrete layers on the degree of composite action; these parameters are known to affect the out-of-plane behaviour of precast concrete ISPs.[8,9]

Previous researchers have used nonlinear finite element analysis (FEA) to predict the out-of-plane behavior of precast concrete ISPs with the concrete layers modeled as either solid elements[10,11] or shell elements.[12] Shell elements have been shown to give better estimates of the panel's behavior.[12] Previous FEAs modeled the reinforcing bars as truss elements with nonlinear properties and the interlayer connectors as shear springs.[11] The effect of insulation was considered by using solid elements in contact with the concrete elements.[11]

In this paper, a multistep linear elastic analysis (MLEA) method is proposed to predict the out-of-plane behavior of precast concrete ISPs with different geometric properties and types of interlayer mechanical connectors. This method is faster than nonlinear FEA and can give conservative estimates of the structural design parameters of precast concrete ISPs, such as out-of-plane deflection and strength. In the MLEA method, the concrete layers and interlayer connectors are modeled as shell elements and shear springs, respectively. The insulation layer, and thus the interlayer shear transfer through the insulation, is conservatively neglected in this method. The test results of an experimental study on the out-of-plane behavior of four large-scale precast concrete ISP specimens without end beams and two precast concrete ISP specimens with end beams[13] were used to verify the adequacy of the proposed MLEA method.

In addition, the MLEA method was used to estimate the out-of-plane elastic deflection, cracking, and ultimate strength of eight precast concrete ISPs tested by Salmon et al.,[14] Naito et al.,[7] and Mouser.[3] These previously tested panels varied in length, width, thickness of the concrete and insulation layers, and type of interlayer mechanical connectors.

MLEA method

The materially-nonlinear-only formulation described by Bathe[15] was used to develop an MLEA method to predict the nonlinear load-deflection behavior of precast concrete ISPs. In the proposed MLEA method, the nonlinear load-deflection behavior was approximated by multiple linear segments. At every load step, the linear segments were constructed by performing a linear elastic analysis of the precast concrete ISP. These analysis steps continued until the entire nonlinear load-deflection behavior was constructed.

To perform the MLEA, the flexural and axial stiffness of the concrete layers and shear stiffness of the interlayer mechanical connectors create the stiffness matrix of the panel $[K]_i$ and are calculated at the beginning of each step *i,* taking into account any prior concrete cracking or yielding of reinforcement and shear yielding of the interlayer connectors. The incremental load vector $\Delta[F]_i$ is then applied and the incremental deformation vector $\Delta[D]_i$ is calculated by performing a linear elastic analysis of the panel as follows.

$$\Delta\left[D\right]_i = \left[K\right]_i^{-1} \Delta\left[F\right]_i$$

The load and deformation vectors after *N* steps are found using Eq. (1) and Eq. (2), respectively.

$$\left[F\right]_N = \sum_{i=1}^{N} \Delta\left[F\right]_i \quad (1)$$

$$\left[D\right]_N = \sum_{i=1}^{N} \Delta\left[D\right]_i \quad (2)$$

where

$[F]_N$ = force vector at end of *N*th step

$[D]_N$ = deflection vector at end of *N*th step

Numerical modeling of the concrete layers

In the proposed MLEA method, the concrete layers were modeled as shell elements, which have been shown to be accurate in predicting the out-of-plane behavior of precast concrete ISPs.[12] The shell elements were assumed to be at the midthickness of each concrete layer. The end beams were modeled as shell elements with a thickness equal to the width of the end beams.

When a precast concrete ISP is subjected to out-of-plane flexure, one of the concrete layers of the precast concrete ISP is under the combined effect of bending moment and compression and the other is under the combined effect of bending moment and tension. Therefore, the out-of-plane moment-curvature behavior of the concrete layers at different levels of axial force and the axial force-strain behavior of the concrete layers at different levels of out-of-plane bending moment were considered in the materially nonlinear analysis of the precast concrete ISPs.

To assign the proper axial and out-of-plane flexural stiffness to the concrete layers in the precast concrete ISP numerical models, the axial force–strain P–ε and out-of-plane bending moment–curvature $M–\varphi$ relationships for both concrete layers were constructed using the Hognestad parabola.[16]

The $P–\varepsilon$ and $M–\varphi$ relationships for the concrete layers should account for the tension stiffening of the reinforced concrete layers; this was accomplished by assuming the tensile strength of concrete was reduced to half the cracking strength f_t, as proposed by Collins and Mitchell.[17]

Figure 1 shows an idealized $M–\varphi$ relationship for a concrete layer with tension stiffening. The out-of-plane bending moment increases with curvature until first cracking of concrete occurs (Fig. 1). The slope of this portion of the diagram is the initial flexural stiffness of the concrete layer $K_{f,i}$, which is equal to E_cI_0, where E_c is the concrete modulus of elasticity and I_0 is the uncracked moment of inertia of the concrete layer in the direction of the out-of-plane bending moment. After concrete cracking, the bending moment keeps increasing with curvature, but at a lower rate than before cracking, until the reinforcing bars yield. The slope of this portion of the diagram is the cracked flexural stiffness of the concrete layer $K_{f,cr}$, which is equal to E_cI_{cr}, where I_{cr} is the cracked moment of inertia of the concrete layer.

To simplify calculations for the structural design of precast concrete ISPs, the cracked flexural and axial stiffness of the concrete layers can be approximated as E_cI_{cr} and E_cA_{cr}, respectively, where I_{cr} and A_{cr} are the transformed moment of inertia and area of the cracked cross section of the concrete layer, respectively.

Numerical modeling of interlayer mechanical connectors

In the proposed MLEA method, discrete interlayer mechanical connectors were modeled as shear springs and the shear stiffness was derived from the published shear test results. Continuous mechanical connectors were modeled as shell elements with the shear stiffness of the shell elements $\frac{Gtl}{h_c}$ taken equal to the shear stiffness of the continuous connectors derived from direct shear tests.

$$\frac{Gtl}{h_c} = k_s l \qquad (3)$$

where

G = shear modulus of elasticity of the shell element

t = thickness of continuous connector

l = length of the connector along the panel

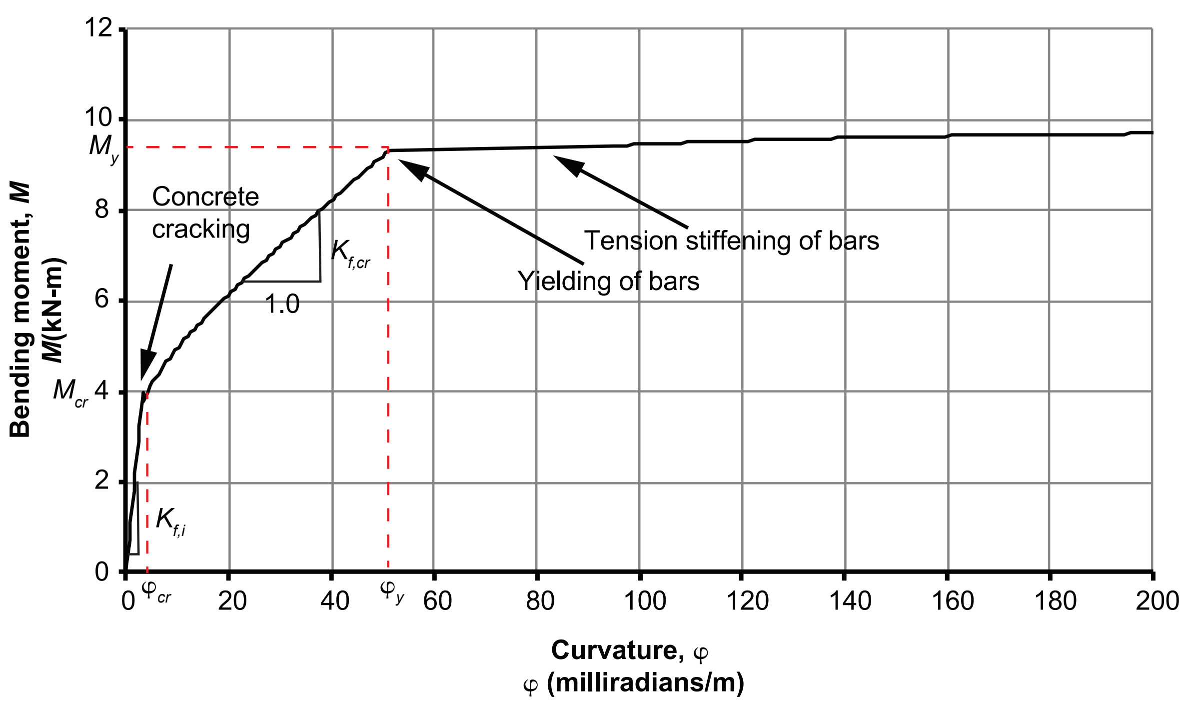


Figure 1. Idealized $M–\phi$ diagram for a concrete layer with tension stiffening of reinforcing bars. Note: $K_{f,cr}$ = out-of-plane flexural stiffness of concrete layer after cracking; $K_{f,i}$ = initial out-of-plane flexural stiffness of concrete layer; M_{cr} = out-of-plane bending moment of concrete layer at concrete cracking; M_y = out-of-plane bending moment of concrete layer at yielding of reinforcement; ϕ_{cr} = out-of-plane curvature of concrete layer at concrete cracking; ϕ_y = out-of-plane curvature of concrete layer at yielding of reinforcement. 1 m = 3.281 ft; 1 kN = 0.225 kip.

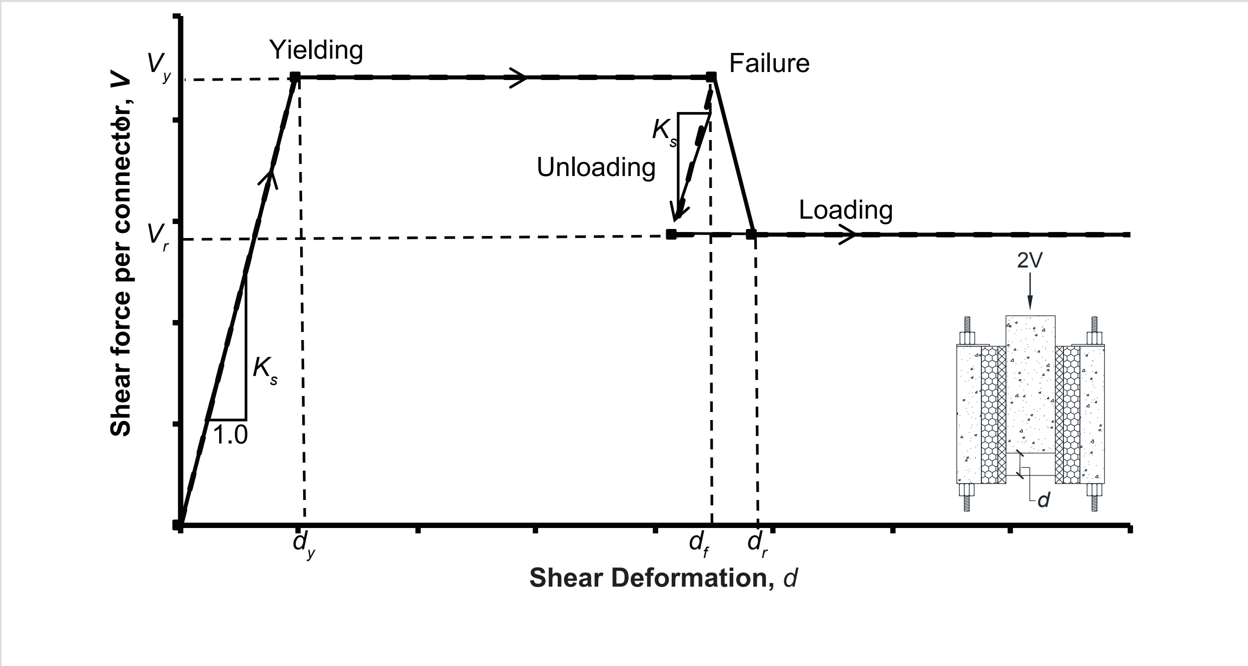


Figure 2. Loading path and idealized shear behavior of interlayer connectors. Note: d = shear deformation of interlayer mechanical connectors; d_f = shear deformation of interlayer mechanical connectors at failure strength; d_r = shear deformation of interlayer mechanical connectors at residual strength; d_y = yield shear deformation of interlayer mechanical connectors; K_s = shear stiffness of discrete connector; V_r = shear force per connector at residual strength; V_y = shear force per connector at yield strength.

h_c = distance between the centers of the two concrete layers

k_s = shear stiffness of the continuous connector per unit length of the connector

Equation (3) is rearranged to give the shear modulus of elasticity of the shell element G as follows:

$$G = \frac{k_s h_c}{t}$$

The modulus of elasticity of the shell element in the direction along the panel length should be assigned an infinitesimal value so it does not contribute to the flexural strength of the modeled precast concrete ISP. The modulus of elasticity of the shell elements along the thickness of the panel should be representative of the stiffness of the interlayer connectors and insulation along the thickness of the panel. In the proposed analysis, the stiffness of the insulation along the thickness of the panel was conservatively ignored.

Figure 2 shows the idealized shear behavior of the discrete connectors and the modeled loading path used in the numerical analyses of precast concrete ISPs using the MLEA model. The idealized shear behavior shows that the shear force increases at the rate of the shear stiffness K_s up to the yield point, after which the shear force remains constant but the shear deformation increases up to the failure point (buckling of the connector's web). After failure, the shear force decreases to a residual shear force, after which the shear force remains constant with increased shear deformation.

In the MLEA of precast concrete ISPs with discrete connectors, the initial stiffness of the connectors was assigned to the shear springs at the onset of loading. When any of the shear springs yielded, the shear stiffness of that shear spring was set to zero. Loading continued until a connector reached its failure point. Then the load was decreased to unload the failed connector until the connector's shear force reached the residual shear force. For steel shear connectors, the shear stiffness of the connector during unloading can be assumed to be the same as its initial shear stiffness. When the shear force of the connector reached the residual shear force, the shear stiffness of the connector was reduced to zero and loading of the panel was resumed.

MLEA procedure

The proposed MLEA method is described in this section for the 1118 mm (44 in.) wide by 3556 mm (140 in.) long precast concrete ISPs tested by Goudarzi et al.[18] These panels had six rows of Z-shaped steel plate connectors along each panel's length, with each row containing two connectors. **Figure 3** shows the panel layout with three rows in each half panel. Row 1 was the closest to the panel's midspan, row 3 was the closest to the panel's end, and row 2 was between rows 1 and 3. For the proposed MLEA method, an arbitrary incremental out-of-plane load ΔF_i was applied at every step i, then the resulting incremental out-of-plane midspan deformation of the panel Δs_i, shear deformation of the nth row of connectors $\Delta d_{n,i}$ and curvature of the concrete layers $\Delta\varphi_i$ during step i were computed. **Figure 4** shows schematic representations of the results of the MLEA at every step. The out-of-plane load on the precast concrete ISP, shear force of the shear connectors at every row, and bending moment of the top and bottom concrete layers were plotted against the out-of-plane midspan deflection of the panel (Fig. 4).

In step 1, a linear elastic FEA was conducted for the precast concrete ISP under self-weight. At the end of this step, linear elastic FEA was used to calculate the out-of-plane midspan deformation of the panel s_1; out-of-plane load of the panel F_1; shear deformation of the Z-shaped steel plate connector at the n^{th} row $d_{n,1}$; curvature at the midspan of the top and bottom concrete layers $\varphi_{top,1}$ and $\varphi_{bot,1}$, respectively; and axial strains of the top and bottom concrete layers $\varepsilon_{top,1}$ and $\varepsilon_{bot,1}$, respectively.

In step 2, the same parameters were calculated under an arbitrary incremental out-of-plane four-point loading ΔF; this can be a unit out-of-plane load. The incremental out-of-plane midspan deformation of the panel Δs_2; incremental shear deformation of the Z-shaped steel plate connector at the n^{th} row $\Delta d_{n,2}$; incremental curvature at the midspan of the top and bottom concrete layers $\Delta\varphi_{top,2}$ and $\Delta\varphi_{bot,2}$, respectively; and incremental axial strains of the top and bottom concrete layers $\Delta\varepsilon_{top,2}$ and $\Delta\varepsilon_{bot,2}$, respectively, were calculated using linear elastic FEA. Then the midspan curvature of the top and bottom concrete layers at the end of step 2 $\varphi_{top,2}$ and $\varphi_{bot,2}$, respectively; axial strains of top and bottom concrete layers at the end of step 2 $\varepsilon_{top,2}$ and $\varepsilon_{bot,2}$, respectively; and shear deformation of the Z-shaped steel plate connector at the nth row at the end of step 2 $d_{n,2}$ were calculated as follows:

$$\varphi_{top,2} = \varphi_{top,1} + \Delta\varphi_{top,2}$$

$$\varphi_{bot,2} = \varphi_{bot,1} + \Delta\varphi_{bot,2}$$

$$\varepsilon_{top,2} = \varepsilon_{top,1} + \Delta\varepsilon_{top,2}$$

$$\varepsilon_{bot,2} = \varepsilon_{bot,1} + \Delta\varepsilon_{bot,2}$$

$$d_{n,2} = d_{n,1} + \Delta d_{n,2}$$

The curvature and axial strains of the concrete layers and the shear deformation of the connectors were compared with the material behavior of the concrete and connectors to determine what type of stiffness reduction occurred in the panel. The comparison at the end of the second step showed the following:

$$\frac{\varphi_{bot,2}}{\varphi_{bot,cr}} > \frac{\varphi_{top,2}}{\varphi_{top,cr}} > \frac{d_{n,2}}{d_y}$$

where

$\varphi_{bot,cr}$ = curvature of the bottom concrete layer at cracking

$\varphi_{top,cr}$ = curvature of the top concrete layer at cracking

d_y = shear deformation of the Z-shaped steel plate connector at yielding

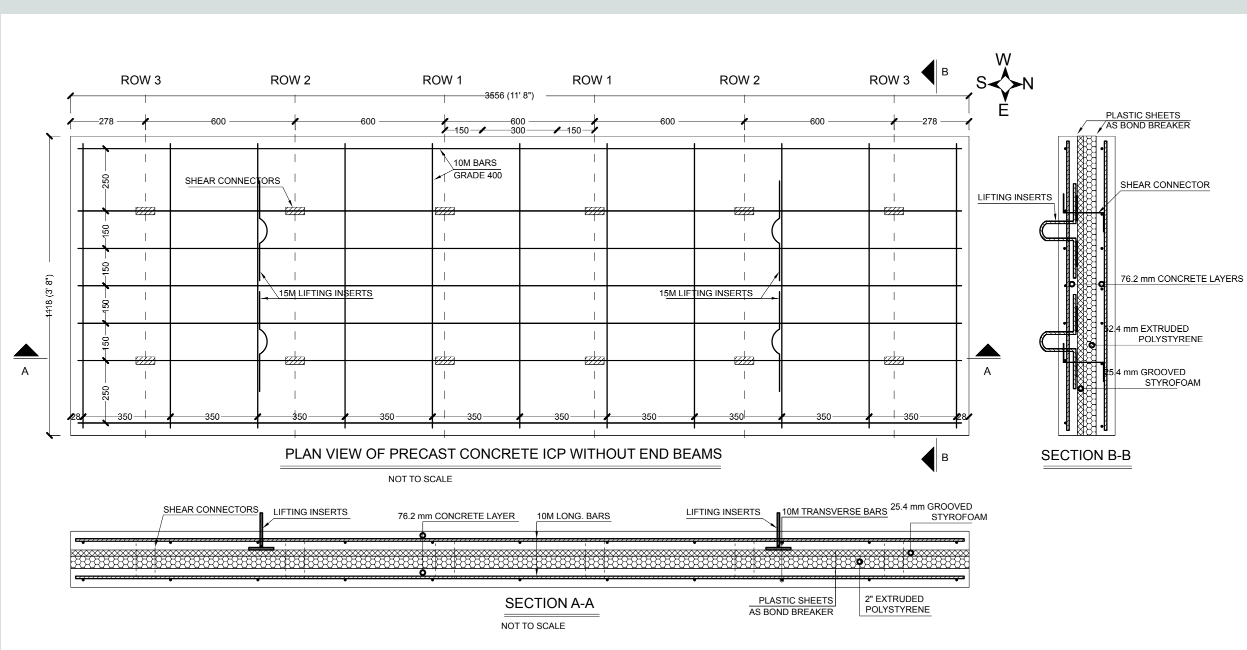


Figure 3. Details of the tested precast concrete insulated sandwich panel specimens. Note: Dimensions are in millimeters. 1 mm = 0.0394 in.; 10M = no. 3; 15M = no. 5; Grade 400 = 58 ksi.

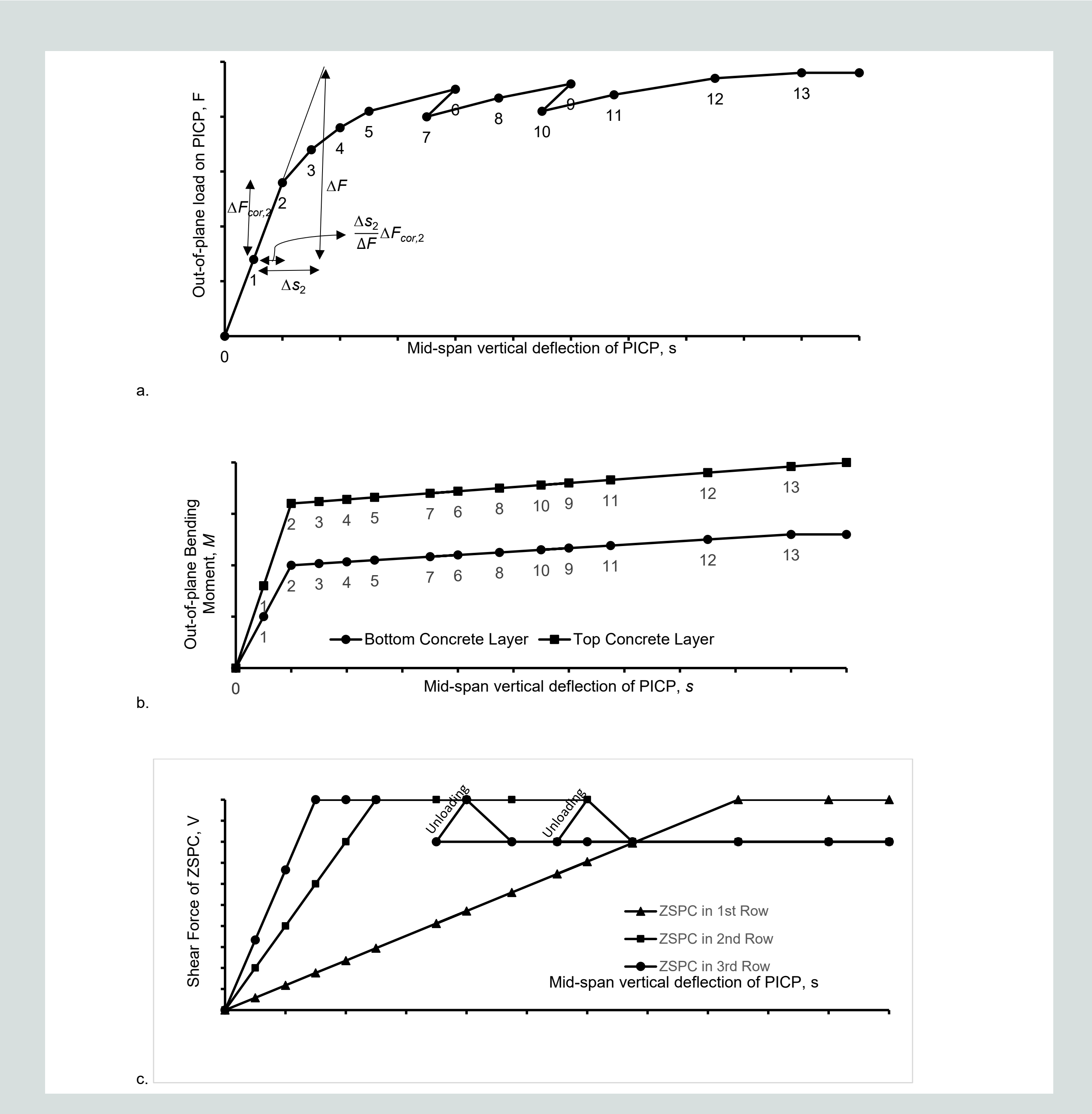


Figure 4. Schematic representation of MLEA results of precast concrete ISPs. Note: ISP = insulated sandwich panel; MLEA = multistep linear elastic analysis; PICP = precast insulated concrete panel; ZSPC = Z-shaped steel plate connector; ΔF = incremental out-of-plane load; $\Delta F_{cor,2}$ = corrected value of incremental out-of-plane load at step 2; Δ_{s2} = incremental out-of-plane midspan deformation at step 2.

This means that with increased loading, midspan cracking of the bottom concrete layer will occur first. A corrected value of incremental out-of-plane load $\Delta F_{cor,2}$ is determined such that $F_1 + \Delta F_{cor,2}$ corresponds to the concrete cracking of the bottom concrete layer. Cracking of the bottom concrete layer occurs when φ_{bot} reaches the curvature of the concrete layer corresponding to the cracking moment $\varphi_{bot,cr}$. The corrected value of the incremental out-of-plane loading $\Delta F_{cor,2}$ that causes cracking of the bottom concrete layer was calculated by solving Eq. (4).

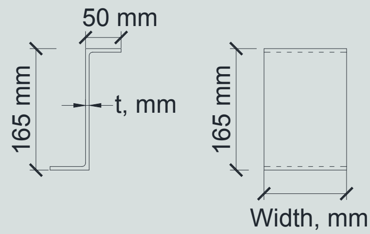
$$\varphi_{bot,2,cor} = \varphi_{bot,cr} = \varphi_{bot,1} + \frac{\Delta\varphi_{bot,2}}{\Delta F}\Delta F_{cor,2} \tag{4}$$

where

$\varphi_{bot,2,cor}$ = corrected curvature of the bottom concrete layer at step 2.

Next, the corrected values for the out-of-plane load corre-

Table 1. Details of the precast concrete insulated sandwich panel specimens tested

Panel designation	Shear connector configuration	Width, mm	Thickness, gauge	End condition
P3-16		76.2	16	Without end beams
P4-16		102	16	
P6-16		152	16	
P4-10		102	10	
PB3-16		76.2	16	With end beams
PB6-16		152	16	

Source: Data from Goudarzi et al., "Flexural Behaviour of Highly Composite Non-Loadbearing Precast Concrete Sandwich Panels" (2016).
Note: 1 mm = 0.0394 in.

sponding to the cracking of the bottom concrete layer $F_{cor,2}$, the deflection of the panel's midspan s_2, the deformation of the connectors $d_{n,2,cor}$, and the curvature and axial strains of the concrete layers ($\varphi_{bot,2,cor}$, $\varepsilon_{top,2,cor}$, and $\varepsilon_{bot,2,cor}$, respectively) at step 2 were calculated using the following equations.

$$F_{cor,2} = F_1 + \Delta F_{cor,2}$$

$$s_2 = s_1 + \frac{\Delta s_2}{\Delta F} \Delta F_{cor,2}$$

$$d_{n,2,cor} = d_{n,1} + \frac{\Delta d_{n,2}}{\Delta F} \Delta F_{cor,2}$$

$$\varepsilon_{top,2,cor} = \varepsilon_{top,1} + \frac{\Delta \varepsilon_{top,2}}{\Delta F} \Delta F_{cor,2}$$

$$\varepsilon_{top,2,cor} = \varepsilon_{top,1} + \frac{\Delta \varepsilon_{top,2}}{\Delta F} \Delta F_{cor,2}$$

$$\varepsilon_{bot,2,cor} = \varepsilon_{bot,1} + \frac{\Delta \varepsilon_{bot,2}}{\Delta F} \Delta F_{cor,2}$$

These corrected values correspond to the cracking of the bottom concrete layer and will be used in step 3. At the beginning of step 3, the flexural and axial stiffness of the bottom concrete layer were reduced to the cracked stiffness values. A linear elastic analysis was performed for the panel under ΔF, and the incremental deformations of the panel, shear connectors, and concrete layers were calculated, similar to step 2.

Additional analysis steps were continued until the out-of-plane load-deformation behavior of the panel was constructed up to the failure of the concrete layers, defined as crushing of the compressive concrete layer. As the steps progressed, when a concrete layer cracked or the reinforcement yielded, the flexural and axial stiffness of the concrete layer was adjusted to the corresponding stiffness values derived from the M–φ and P–ε relationships. Similarly, when a shear connector yielded or failed, the shear stiffness of that connector was adjusted according to the shear force–deformation behavior of the connector. Also, at every step, the shear deformation of connectors, the curvature and axial strains of the concrete layers were corrected, similar to step 2. The proposed MLEA method is applicable to any precast concrete ISP with generic geometric properties constructed using any type of interlayer mechanical connectors.

Validation of the proposed MLEA method

Description of the analytical model of the tested precast concrete ISPs

In this section, modeling of the precast concrete ISP specimens tested in a previous study is explained.[13] **Table 1** gives the dimensions of the Z-shaped steel plate connector and the designation of the tested panels. The tested precast concrete ISPs were modeled with SAP 2000 as an assembly of two concrete layers and interlayer mechanical connectors (**Fig. 5**). Loading and support beams were modeled as beam elements with W200×46 (W8×31) steel sections.

The boundary conditions of the support beams allowed horizontal displacement along the panel length (x axis) and rotation about the longitudinal axis of the beam (y axis) to simulate the pin-roller supports used in the experimental tests. Horizontal displacement along the panel length was constrained at midlength of the concrete layers to satisfy static equilibrium along the x axis in the numerical models.

The concrete layers were modeled as 76.2 mm (3.0 in.) thick shell elements. The end beams of the PB-series panels were modeled as 150 mm (6.0 in.) thick shell elements, equal to the end-beam widths of the specimens. The measured tensile behavior of the longitudinal reinforcing bars and the 28-day E_c and f_c of the concrete cylinders (**Table 2**) were used to construct the axial force–strain P–ε and out-of-plane bending moment–curvature M–φ relationships for the concrete layers.

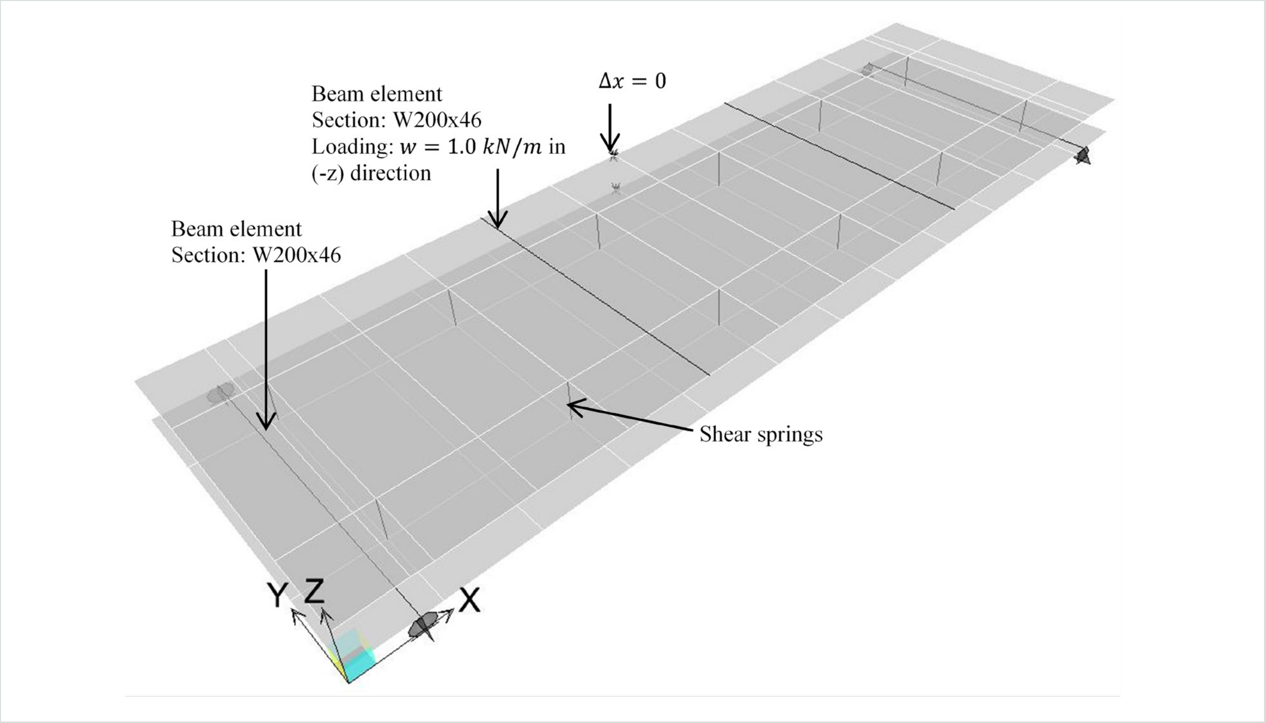


Figure 5. Three-dimensional view of the numerical models for the precast concrete insulated sandwich panels tested. Source: Reproduced with permission from Goudarzi, "Effect of Z-shaped Steel Plate Connectors on Out-of-Plane Flexural Behaviour of Precast Insulated Concrete Panels" (2016, Fig. 4–8).

The Z-shaped steel plate connectors were modeled as shear springs, and the results of push-off shear tests of Z-shaped steel plate connectors found in Goudarzi et al.[19] were used to construct the simplified shear behavior of the connectors (**Fig. 6**). These idealized shear-displacement behaviors were used in the numerical models of the tested precast concrete ISPs.

The self-weight of the panels was applied as body forces on the concrete layers and the out-of-plane, four-point loading was simulated by applying uniform line load w on the top loading beams.

Predicted out-of-plane load-deflection behavior of the tested precast concrete ISPs

Figure 7 shows the numerical and experimental out-of-plane total load F compared with the midspan deflection s for the tested P-series panels. The results of the MLEA show that the failure of the P-series panels predicted using the numerical model starts with cracking of the bottom concrete layer followed by cracking of the top concrete layer. Yielding of the connectors starts from the Z-shaped steel plate connec-

Table 2. Concrete properties of the precast concrete insulated sandwich panel test specimens at 28 days

Panel designation	Bottom concrete layer*			Top concrete layer*		
	f_c, MPa	E_c, GPa	v	f_c, MPa	E_c, GPa	v
P3-16	27.93	21.74	0.156	37.19	24.72	0.158
P4-16	29.30	21.26	0.163	40.28	26.23	0.153
P6-16	28.52	21.57	0.150	36.65	26.00	0.167
P4-10	27.11	22.82	0.177	40.21	25.77	0.165
PB3-16	27.26	24.18	0.154	36.45	22.08	0.150
PB6-16	30.89	24.71	0.164	40.28	26.85	0.152

Note: E_c = 28-day concrete modulus of elasticity; f_c = 28-day compressive strength of concrete; v = Poisson's ratio. 1 MPa = 0.145 ksi; 1 GPa = 145 ksi.
*Concrete properties were measured by testing 100 × 200 mm (4.0 × 8.0 in.) concrete cylinders.

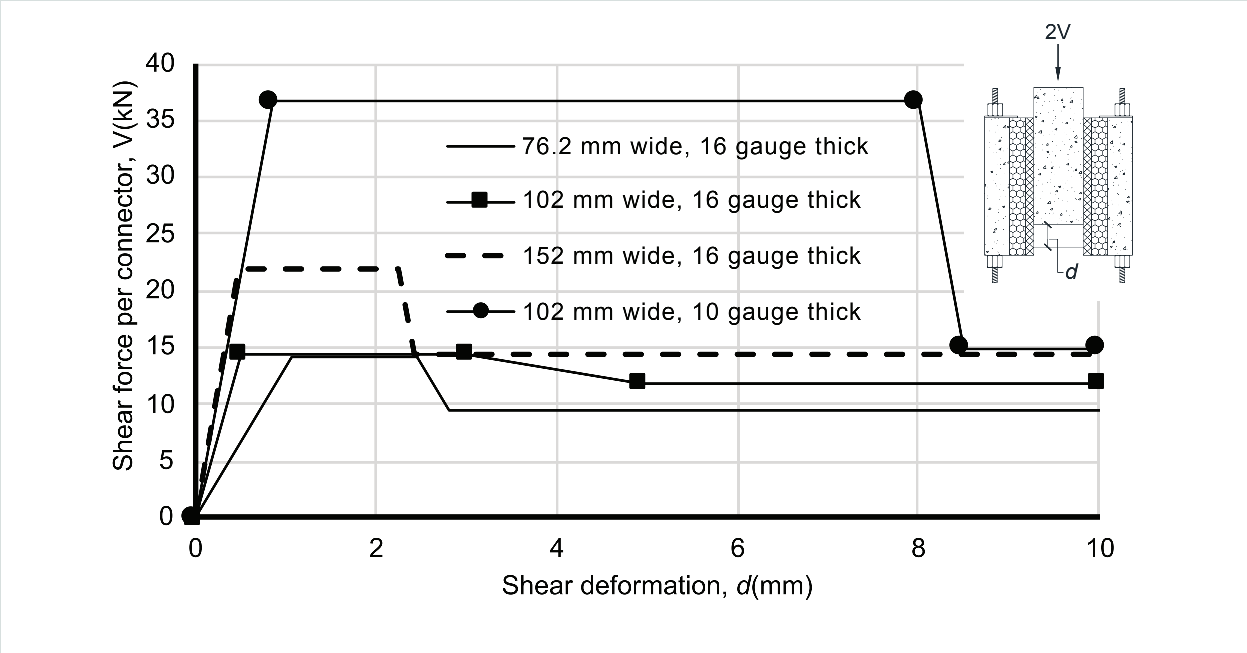


Figure 6. Idealized shear behavior of Z-shaped steel plate connectors. Note: d = shear deformation of interlayer mechanical connectors; V = shear force per connector. 1 mm = 0.0394 in.; 1 kN = 0.225 kip. Source: Reproduced with permission from Goudarzi et al., "Characterization of Shear Behavior of Z-shaped Steel Plate Connectors Used in Insulated Concrete Panels" (2016).

tors closest to the ends of the panel; that is, the yielding of Z-shaped steel plate connectors occurs in row 3, then row 2, then row 1 of the connectors.[18]

Figure 8 compares the numerical results with the experimental out-of-plane load-deflection behavior of PB-series panels. The MLEA predicted that cracking of the PB-series panels would take place in the following order: cracking of the ends of the bottom layer, cracking of the midspan of the bottom layer, cracking of the ends of the top layer, and then cracking of the midspan of the top concrete layer.[18]

After the cracking stages of the PB-series panels, the Z-shaped steel plate connectors in row 2 of the connectors reached their yield strength. Then, the ends and midspan of the bottom concrete layer reached their flexural yield strength M_y, which is the flexural strength at yielding of tensile reinforcement in the concrete layer. After reinforcement yielding in the bottom concrete layer, the slope of the F–s diagram decreased as the flexural stiffness of the bottom concrete layer approached zero.

Figure 8 shows that in the PB-series panels, the Z-shaped steel plate connectors in connector row 2 yielded first, and then the Z-shaped steel plate connectors in rows 2 and 3 yielded at a midspan deflection of about 75 mm (2.9 in.). This suggests that the interlayer shear force near the panel ends is primarily resisted by the end beams. This might be due to the large shear stiffness of the end beams relative to the shear stiffness of the connectors.

MLEA model adequacy

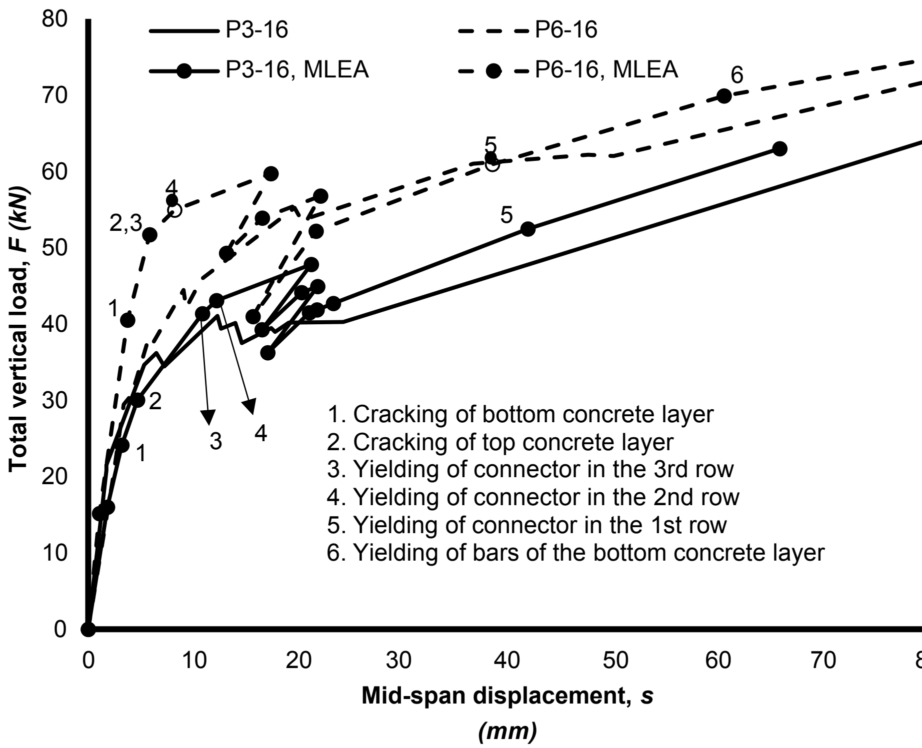
Table 3 gives the root mean square of the predicted out-of-plane load F_{rms} estimated by the MLEA models. F_{rms} was calculated using the following equation:

$$F_{rms} = \sqrt{\frac{\int_{s=0}^{s_f}\left(F_{\exp} - F_{MLE}\right)^2 \cdot ds}{s_f}}$$

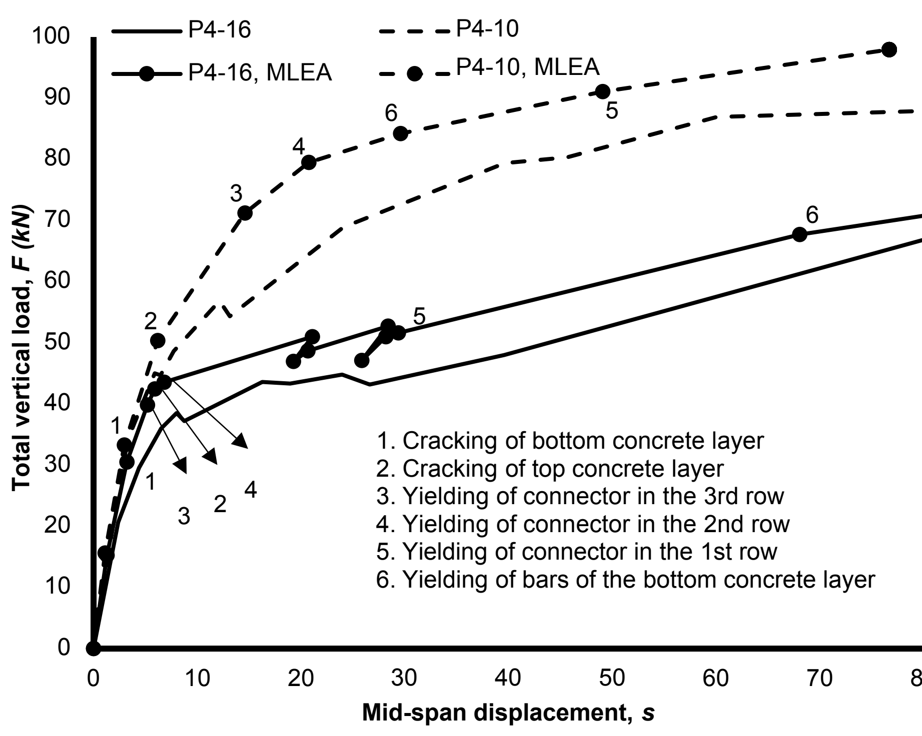
Table 3. Root mean square of the parameters predicted by multistep linear elastic analysis method

Panel	s_f, mm	E_{abs}, kJ	F_{rms}, kN	*COV*, %
P3-16	65.8	2.896	4.04	9.2
P4-16	99.4	5.251	6.13	11.6
P6-16	91.4	5.428	5.15	8.7
P4-10	76.7	3.920	7.30	14.3
PB3-16	76.1	5.986	7.07	9.0
PB6-16	73.7	5.718	13.49	17.4
Average				11.7

Note: *COV* = coefficient of variation; E_{abs} = absorbed energy; F_{rms} = root mean square of the predicted out-of-plane strength; s_f = final out-of-plane deflection. 1 mm = 0.0394 in.; 1 kN = 0.225 kip; 1 kJ = 1 kN-m = 0.738 kip-ft.



Specimens P3-16 and P6-16



Specimens P4-10 and P4-16

Figure 7. Predicted and measured out-of-plane load compared with the midspan deflection response. Note: MLEA = multistep linear elastic analysis. 1 mm = 0.0394 in.; 1 kN = 0.225 kip.

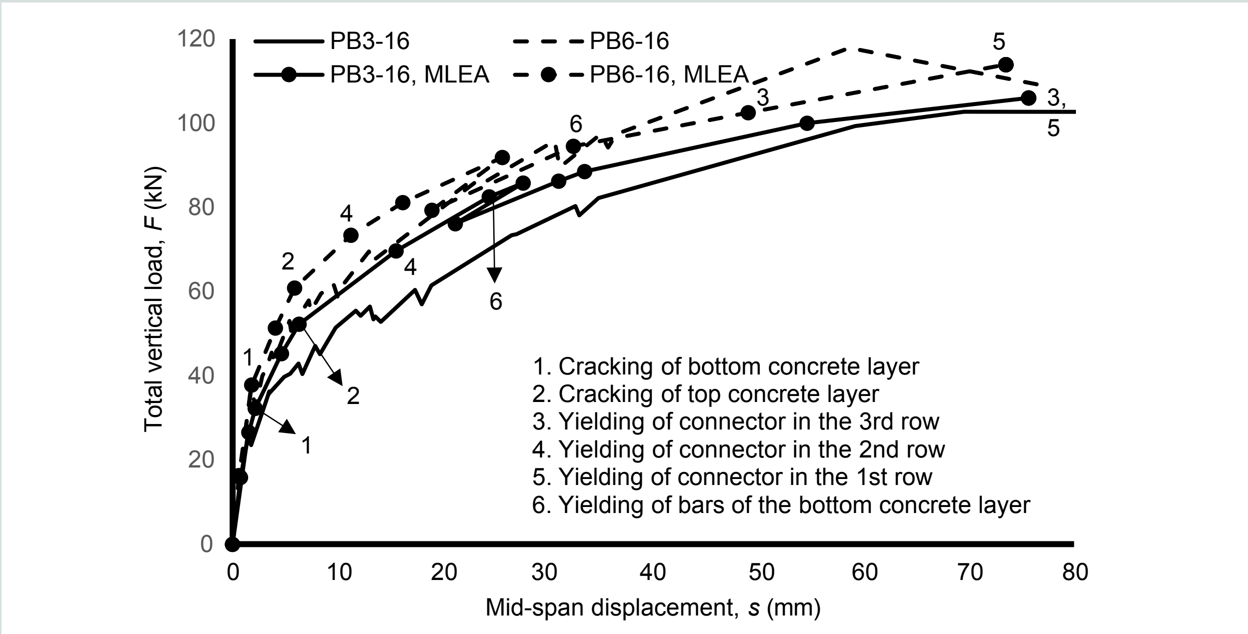


Figure 8. Numerical analysis and experimental out-of-plane force compared with midspan deflection for specimens PB3-16 and PB6-16. Note: MLEA = multistep linear elastic analysis. 1 mm = 0.0394 in.; 1 kN = 0.225 kip.

where

F_{exp} = total out-of-plane loads determined from the experiments

F_{MLE} = total out-of-plane loads determined from the MLEA models

s = out-of-plane midspan deflection

s_f = final out-of-plane midspan deflection for which the experimental and numerical results are available

Table 3 also gives the absorbed energy of the flexural tests E_{abs}, which is defined as the area under the out-of-plane load-deflection graphs. The average experimental out-of-plane load carried by each panel throughout loading is equal to $\frac{E_{abs}}{s_f}$. The coefficient of variation $COV\%$ is then defined as the root mean square of the estimated out-of-plane load F_{rms} divided by the average experimental out-of-plane load, as given in the following equation:

$$COV\% = \frac{F_{rms} s_f}{E_{abs}}\%$$

The coefficient of variation is a measure of the accuracy of the MLEA method. Table 3 shows that the coefficient of variation $COV\%$ for the MLEA models varied from 9.0% to 17.4%, with an average value of 11.7%. This suggests that the MLEA method overestimated the out-of-plane stiffness of the tested panels, which is not uncommon for numerical models because the models cannot account for all degradation mechanisms.

Applicability of the MLEA method to precast concrete ISPs tested by others

The proposed MLEA method was used to estimate the out-of-plane deflection, cracking, and ultimate strength of eight prestressed precast concrete ISPs with different discrete and continuous interlayer connector systems tested by other researchers.[3,7,14] The tested panels varied in width, length, type of connector system, and thickness of concrete layers and insulation. A summarized description of these specimens is given in **Table 4**. Two of the tested panels were 9.14 m (30.0 ft) long and six were 3.05 m (10.0 ft) long. Five of the tested precast concrete ISPs had continuous interlayer connectors including fiber-reinforced polymer and steel truss connectors, carbon-fiber-reinforced polymer grid, and expanded metal connectors. Three of the tested precast concrete ISPs had discrete interlayer connectors including composite pins, C clips, and M ties. The concrete layers of the panels varied from 63.5 to 82.5 mm (2.5 to 3.25 in.) thick and the insulation thickness varied from 50.8 to 76.2 mm (2.0 to 3.0 in.).

Table 5 provides a comparison between the out-of-plane midspan deflection of the tested precast concrete ISPs s_e and the analytical midspan deflection s_{MLE} determined by the proposed MLEA method. The uniform load q in Table 5 corresponds to the end of the linear elastic region of the experimental out-of-plane load-deflection graphs. Table 5 shows that $\frac{s_e}{s_{MLE}}$ ratios varied from 0.53 to 1.24 and that $s_{MLE} - s_e$ for the panels varied from -2.1 to 9.6 mm (-0.0827 to 0.38 in.). It should be

Table 4. Summary of the dimensional and physical properties of the precast concrete insulated sandwich panels tested in other sources

Panel number	Source	Type of interlayer connector	*b*, m	*L*, m	t_c, mm	t_{ins}, mm	f_c, MPa	Reinforcement type
1	Salmon et al. (1997)	FRP truss	2.44	9.14	63.5	76.2	34.4	Five 10 mm diameter strands and 152 × 152 MW26 × MW26 (WWR)
2		Steel truss						
3	Naito et al. (2012)	CFRP grid	0.813	3.05	76.2	50.8	48.2	Two 10 mm diameter strands
4		Composite pin						
5		C clip						
6	Mouser (2003)	Expanded metal	2.39	3.05	76.2	76.2	54.8	Five 10 mm diameter strands
7		Expanded metal			82.5	50.8	51.1	
8		M tie			82.5	50.8	40.1	Six 10 mm diameter strands

Note: 152 × 152 MW26 × MW26 = 6 × 6 W4.0 × W4.0. *b* = panel width; CFRP = carbon-fiber-reinforced polymer; f_c = 28-day compressive strength of concrete; FRP = fiber-reinforced polymer; *L* = panel length; t_c = thickness of concrete layer; t_{ins} = thickness of insulation; WWR = welded wire reinforcement. 1 mm = 0.0394 in.; 1 m = 3.281 ft; 1 MPa = 0.145 ksi.

Table 5. Midspan deflection of tested precast concrete insulated sandwich panels estimated by the proposed multistep linear elastic analysis method

Panel number	Source	Type of interlayer connector	*q*, kPa	s_e, mm	s_{MLE}, mm	$\frac{s_e}{s_{MLE}}$	($s_e - s_{MLE}$), mm
1	Salmon et al. (1997)	FRP truss	3.0	17.4	27.0	0.64	9.6
2		Steel truss	1.5	16.2	14.1	1.15	-2.1
3	Naito et al. (2012)	CFRP grid	18	3.0	3.4	0.88	0.4
4		Composite pin	15	3.1	5.8	0.53	2.7
5		C clip	16	5.2	7.5	0.69	2.3
6	Mouser (2003)	Expanded metal	10	2.6	4.1	0.64	1.5
7		Expanded metal	10	3.6	2.9	1.24	-0.7
8		M tie	10	3.4	4.1	0.83	0.7
Average						0.82	
Coefficient of variation, %						31.0	

Note: CFRP = carbon-fiber-reinforced polymer; FRP = fiber-reinforced polymer; *q* = out-of-plane uniform load; s_e = experimental midspan out-of-plane deflection; s_{MLE} = midspan out-of-plane deflection determined from multistep linear elastic analysis model. 1 mm = 0.0394 in.; 1 kPa = 0.145 psi.

noted that for panels 2 and 7, $\frac{s_e}{s_{MLE}}$ is larger than 1, which means that the proposed MLEA method underestimates the experimental midspan deflection for these two panels. However, this underestimation is only 2.1 and 0.7 mm (0.083 and 0.028 in.) for panels 2 and 7, respectively.

Table 6 provides a comparison between the measured out-of-plane cracking and ultimate flexural strength of the experimentally tested precast concrete ISPs $q_{cr,e}$ and $q_{u,e}$, respectively, and the cracking and ultimate strength values predicted by the proposed MLEA method $q_{cr,MLE}$ and $q_{u,MLE}$, respectively. The experimental cracking strength was taken as the out-of-plane strength, after which the load-deflection relationship deviates from linear proportionality. The estimated cracking strength is associated with the cracking of the compressive concrete layer.

Table 6. Out-of-plane cracking and ultimate strength of tested precast concrete insulated sandwich panels estimated by the proposed multistep linear elastic analysis method

Panel number	Source	Type of interlayer connector	Cracking strength			Ultimate strength		
			$q_{cr,e}$, kPa	$q_{cr,MLE}$, kPa	$\frac{q_{cr,e}}{q_{cr,MLE}}$	$q_{u,e}$, kPa	$q_{u,MLE}$, kPa	$\frac{q_{u,e}}{q_{u,MLE}}$
1	Salmon et al. (1997)	FRP truss	5.74	4.10	1.40	8.14	5.41	1.50
2		Steel truss	3.83	3.32	1.15	5.67	4.11	1.38
3	Naito et al. (2012)	CFRP grid	23.4	23.3	1.01	32.7	24.0	1.36
4		Composite pin	15.2	13.9	1.09	31.7	32.33	0.98
5		C clip	14.47	10.7	1.33	28.9	22.8	1.27
6	Mouser (2003)	Expanded metal	20.5	19.94	1.03	25.0	19.9	1.25
7		Expanded metal	17.0	15.3	1.11	23.0	19.3	1.19
8		M tie	13.0	10.07	1.29	21.0	20.4	1.03
Average					1.14			1.24
Coefficient of variation, %					11.0			18.0

Note: CFRP = carbon-fiber-reinforced polymer; FRP = fiber-reinforced polymer; $q_{cr,e}$ = experimental out-of-plane cracking strength; $q_{cr,MLE}$ = out-of-plane cracking strength determined from multistep linear elastic analysis model; $q_{u,e}$ = experimental out-of-plane ultimate flexural strength; $q_{u,MLE}$ = out-of-plane ultimate flexural strength determined from multistep linear elastic analysis model. 1 kPa = 0.145 psi.

Table 6 shows that $\frac{q_{cr,e}}{q_{cr,MLE}}$ varies from 1.01 to 1.40, with an average value of 1.14 and coefficient of variation of 10.76%, and $\frac{q_{u,e}}{q_{u,MLE}}$ varies from 0.98 to 1.50, with an average value of 1.24 and coefficient of variation of 17.59%. This means that the proposed MLEA method provides conservative estimates of the cracking and ultimate strength of the panels with 14% and 24% error, respectively.

Conclusion

This study proposed an MLEA method to predict the out-of-plane load-deflection behavior of precast concrete ISPs with any type of interlayer connector system. The method is also applicable to precast concrete ISPs with end beams. In the MLEA, the concrete layers and continuous interlayer mechanical connectors are modeled as shell elements and the discrete interlayer mechanical connectors are modeled as shear springs. The nonlinear out-of-plane load-deflection behavior of the precast concrete ISP is broken into multiple connected linear segments. Each linear segment of the behavior is approximated by conducting a linear elastic analysis of the panel using adjusted flexural and axial stiffness for the concrete layers and shear stiffness for the interlayer connectors.

To validate the adequacy of the proposed MLEA method, out-of-plane four-point bending tests were conducted on six panels with Z-shaped steel plate connectors, two of which were enclosed by end beams. The MLEA model predicted the behavior of the experimentally tested panels with about a 12% margin of error.

Predictions using the proposed method were also compared to the experimental results of eight precast concrete ISPs tested under out-of-plane loading by other researchers. These eight panels varied in length, width, thickness of concrete and insulation layers, and interlayer mechanical connector systems. The proposed MLEA method gave conservative estimates for the out-of-plane flexural elastic deflection, cracking, and ultimate strength of the eight precast concrete ISPs with a 31%, 11%, and 18% coefficient of variation, respectively. Part of this deviation is due to the shear transfer through the insulation-to-concrete interface neglected in the MLEA method and the uncertainty in the material properties. The proposed method can be used to analyze and design precast concrete ISPs with different geometric properties and types of interlayer connectors. The proposed method is faster and easier to use than performing nonlinear FEA.

Acknowledgments

Nabi Goudarzi was funded by the Natural Sciences and Engineering Research Council of Canada through an industrial sponsorship from Read Jones Christoffersen Consulting Engineers. The experimental investigations by the authors cited in this paper were conducted at the I. F. Morrison Structures Laboratory at the University of Alberta in Edmonton, AB, Canada. The authors are grateful to FERO Corp. for donating the metal interlayer connectors used to construct the insulated precast concrete panels.

References

1. Einea, A. A., D. C. Salmon, M. K. Tadros, T. Culp, and G. J. Fogarasi. 1991. "State-of-the-Art of Precast Concrete Sandwich Panels." *PCI Journal* 36 (6): 78–98.

2. PCI Committee on Precast Sandwich Wall Panels. 2011. "State of the Art of Precast/Prestressed Sandwich Wall Panels." *PCI Journal* 56 (2): 131–176.

3. Mouser, L. A. D. 2003. "Partially Composite Concrete Sandwich Panels." MS thesis, University of Alberta, Edmonton, AB, Canada.

4. Pessiki, S., and A. Mlynarczyk. 2003. "Experimental Evaluation of the Composite Behavior of Precast Concrete Sandwich Panels." *PCI Journal* 48 (2): 54–71.

5. Salmon, D. C., and A. Einea. 1995. "Partially Composite Sandwich Panel Deflections." *Journal of Structural Engineering* 121 (4): 778–783.

6. Hassan, T., and S. Rizkalla. 2010. "Analysis and Design Guidelines of Precast, Prestressed Concrete, Composite Load-Bearing Sandwich Wall Panels Reinforced with CFRP Grid." *PCI Journal* 55 (2): 147–162.

7. Naito, C., J. Hoemann, M. Beacraft, and B. Bewick. 2012. "Performance and Characterization of Shear Ties for Use in Insulated Precast Concrete Sandwich Panels." *Journal of Structural Engineering* 138 (1): 52–61.

8. Carbonari, G., S. H. P. Cavalaro, M. M. Cansario, and A. Aguado. 2012. "Flexural Behaviour of Light-Weight Sandwich Panels Composed by Concrete and EPS." *Construction and Building Materials* 35: 792–799.

9. Pfeifer, D. W., and J. A. Hansen. 1964. *Precast Concrete Wall Panels: Flexural Stiffness of Sandwich Panels.* Special publication SP-11. Farmington Hills, MI: American Concrete Institute.

10. Lee, B. J. 2003. "Development of a Precast Prestressed Concrete Three-Wythe Sandwich Wall Panel." PhD diss., Lehigh University, Bethlehem, PA.

11. Newberry, C., J. Davidson, J. Hoemann, and B. Bewick. 2010. "Simulation of Prestressed Concrete Sandwich Panels Subjected to Blast Loads." Report AFRL-RX-TY-TP-2010-0014. Tyndall Air Force Base, FL: Air Force Research Laboratory.

12. Benayoune, A., A. A. Abdul Samad, D. N. Trikha, A. A. Abang Ali, and S. H. M. Ellinna. 2008. "Flexural Behaviour of Pre-cast Concrete Sandwich Composite Panel—Experimental and Theoretical Investigations." *Construction and Building Materials* 22 (4): 580–592.

13. Goudarzi, N., Y. Korany, S. Adeeb, and R. Cheng. "Flexural Behaviour of Highly Composite Non-Loadbearing Precast Concrete Sandwich Panels." Paper STR-909 presented at Canadian Society for Civil Engineering Annual Conference, Fifth International Structural Specialty Conference, London, ON, Canada, June 1–4, 2016.

14. Salmon, D. C., A. Al-Einea, M. K. Tadros, and T. D. Culp. 1997. "Full Scale Testing of Precast Concrete Sandwich Panels." *ACI Structural Journal* 94 (4): 354–362.

15. Bathe, K. J. 1996. *Finite Element Procedures*. Englewood Cliffs, NJ: Prentice Hall.

16. Hognestad, E. 1951. "A Study on Combined Bending and Axial Load in Reinforced Concrete Members." Engineering Experiment Station Bulletin 399. Urbana, IL: University of Illinois.

17. Collins, M. P., and D. Mitchell. 1997. *Prestressed Concrete Structures.* Toronto, ON, Canada: Response Publications.

18. Goudarzi, N. 2016. "Effect of Z-Shaped Steel Plate Connectors on Out-of-Plane Flexural Behaviour of Precast Insulated Concrete Panels." PhD thesis, University of Alberta, Edmonton, AB, Canada.

19. Goudarzi, N., Y. Korany, S. Adeeb, and R. Cheng. 2016. "Characterization of Shear Behavior of Z-Shaped Steel Plate Connectors Used in Insulated Concrete Panels." *PCI Journal* 61 (2): 23–37.

Notation

A_{cr} = cracked equivalent cross-sectional area of concrete layer

b = panel width

COV = coefficient of variation

d = shear deformation of interlayer mechanical connectors

d_f = shear deformation of interlayer mechanical connectors at failure strength

$d_{n,i}$ = shear deformation of interlayer mechanical connectors in the *n*th connector row at end of step *i*

$d_{n,i,cor}$ = corrected value of out-of-plane deformation of interlayer mechanical connectors in the *n*th connector row at end of step *i*

d_r = shear deformation of interlayer mechanical connectors at residual strength

d_y = yield shear deformation of interlayer mechanical connectors

$[D]$ = deformation matrix of the panel

E_{abs} = absorbed energy

E_c = modulus of elasticity of concrete

f_c = 28-day compressive strength of concrete

f_t = cracking strength of concrete

F = total out-of-plane load

$[F]$ = force matrix of the panel

F_{exp} = experimental out-of-plane strength

F_i = out-of-plane load at step i

$F_{cor,i}$ = corrected value of out-of-plane load at step i

F_{MLE} = out-of-plane strength estimated by multistep linear elastic analysis method

F_{rms} = root mean square of the predicted out-of-plane strength

G = shear modulus of elasticity of the shell element

h_c = distance between the centers of the concrete layers

i = analysis step number

I_{cr} = out-of-plane cracked equivalent moment of inertia of concrete

I_0 = out-of-plane uncracked moment of inertia of concrete layer

k_s = shear stiffness of continuous connector per unit length of the connector

K_f = out-of-plane flexural stiffness of concrete layer

$K_{f,cr}$ = out-of-plane flexural stiffness of concrete layer after cracking

$K_{f,i}$ = initial out-of-plane flexural stiffness of concrete layer

$[K]_i$ = stiffness matrix of the panel

K_s = shear stiffness of discrete connector

l = length of connector along the panel

L = panel length

M = out-of-plane bending moment of concrete layer

M_{cr} = out-of-plane bending moment of concrete layer at concrete cracking

M_y = out-of-plane bending moment of concrete layer at reinforcing bar yielding

n = connector row number

N = total number of analysis steps

P = axial force of the concrete layer

q = out-of-plane uniform surface load

$q_{cr,e}$ = experimental out-of-plane cracking strength

$q_{cr,MLE}$ = out-of-plane cracking strength determined from multistep linear elastic analysis model

$q_{u,e}$ = experimental out-of-plane ultimate flexural strength

$q_{u,MLE}$ = out-of-plane ultimate flexural strength determined from multistep linear elastic analysis model

s = midspan out-of-plane deflection

s_e = experimental midspan out-of-plane deflection

s_f = final out-of-plane deflection

s_i = midspan out-of-plane deflection at step i

s_{MLE} = midspan out-of-plane deflection determined from multistep linear elastic analysis model

t = thickness of continuous connector

t_c = thickness of concrete layer

t_{ins} = thickness of insulation

V = shear force per connector

V_r = shear force in connector at residual strength

V_y = shear force in connector at yield strength

w = uniform line load

$\Delta d_{n,i}$ = incremental shear deformation of the nth connector row at step i

$\Delta[D]_i$ = incremental deflection vector of the panel at step i

$\Delta F_{cor,i}$ = corrected value of incremental out-of-plane load at step i

ΔF_i = incremental out-of-plane load at step i

$\Delta[F]_i$ = incremental force vector of the panel at step i

Δs_i = incremental out-of-plane midspan deformation at step i

Δx = displacement in the x direction

$\Delta\varepsilon_{bot,i}$ = incremental axial strain of bottom concrete layer at step i

$\Delta\varepsilon_{top,i}$ = incremental axial strain of top concrete layer at step i

$\Delta\varphi_{bot,i}$ = incremental out-of-plane curvature of bottom concrete layer at step i

$\Delta\varphi_i$ = incremental curvature of concrete layers at step i

$\Delta\varphi_{top,i}$ = incremental out-of-plane curvature of top concrete layer at step i

ε = axial strain of concrete layer

$\varepsilon_{bot,i}$ = axial strain of bottom concrete layer at step i

$\varepsilon_{bot,i,cor}$ = corrected axial strain of bottom concrete layer at step i

$\varepsilon_{top,i}$ = axial strain of top concrete layer at step i

$\varepsilon_{top,i,cor}$ = corrected axial strain of top concrete layer at step i

φ = out-of-plane curvature of concrete layer

$\varphi_{bot,cr}$ = out-of-plane curvature of bottom concrete layer at concrete cracking

$\varphi_{bot,i}$ = out-of-plane curvature of bottom concrete layer at step i

$\varphi_{bot,i,cor}$ = corrected value of out-of-plane curvature of bottom concrete layer at step i

φ_{cr} = out-of-plane curvature at concrete cracking

$\varphi_{top,cr}$ = out-of-plane curvature of top concrete layer at concrete cracking

$\varphi_{top,i}$ = out-of-plane curvature of top concrete layer at step i

$\varphi_{top,i,cor}$ = corrected value of out-of-plane curvature of top concrete layer at step i

φ_y = out-of-plane curvature at yielding of the reinforcement

ν = Poisson's ratio

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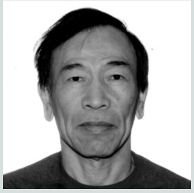

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Abstract

This paper presents a multistep linear elastic analysis (MLEA) method to estimate the nonlinear out-of-plane load-deflection behavior of precast concrete insulated sandwich panels (ISPs), idealized as multiple segments of linear behavior. The proposed method was verified using the results of out-of-plane flexural tests conducted on six precast concrete ISPs, two of which were enclosed by end beams. The MLEA method predicted the experimental load-deflection behavior of the tested precast concrete ISPs with 12% deviation. The results of the MLEA method were also compared with the experimental results of eight precast concrete ISP specimens tested by other researchers; these precast concrete ISP specimens varied in geometry and type of interlayer mechanical connectors. The MLEA method estimated the elastic out-of-plane deflection, cracking, and ultimate strength of the eight precast concrete ISP specimens within 31%, 14%, and 24% margin of error, respectively. This suggests that the MLEA method can be used to estimate the out-of-plane behavior of precast concrete ISPs with any geometric properties and type of interlayer mechanical connectors.

Keywords

ISP, MLEA, multistep linear elastic analysis, out-of-plane behavior, out-of-plane deflection, out-of-plane strength, precast concrete insulated sandwich panel.

Review policy

This paper was reviewed in accordance with the Precast/Prestressed Concrete Institute's peer-review process.

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