Hanger reinforcement for corbels

Gary Klein, Harry Gleich, Ralf Leistikow, and Greg Lucier

- This paper explores available design methods for hanger reinforcement in corbels supported by spandrel beams or wall panels.
- Hanger reinforcement design equations that account for eccentricity of the applied load and the effect of shear and torsion carried by the supporting member web below the applied load are proposed.
- Failure mechanisms from previous research are evaluated with the proposed equations, and additional research to refine corbel design methods is anticipated.

PCI Journal (ISSN 0887-9672) V. 64, No. 6, November–December 2019.

PCI Journal is published bimonthly by the Precast/Prestressed Concrete Institute, 200 W. Adams St., Suite 2100, Chicago, IL 60606.

Many precast concrete producers are using isolated brackets and corbels to support double-tee floor members in buildings and parking structures instead of continuous ledges. Corbels are frequently located along the bottom of spandrel beams (**Fig. 1**) or just above openings in wall panels. Hanger reinforcement is needed to transfer the reaction from the corbel to the upper region of the supporting member. Loads from double-tee members are increasing as wider double tees are used and heavier loads, such as soil weight from green roofs, are supported on these isolated brackets and corbels. As such, attention to hanger reinforcement details for corbels is more important than ever.

This paper summarizes the development of industry design methods for proportioning hanger reinforcement and proposes revised equations for the design of hanger reinforcement for spandrel beams with corbels, as well as for corbels located above openings in wall panels.

Historically, hanger reinforcement has been proportioned such that its design strength is equal to the load on the corbel. However, the force resisted by the hanger reinforcement is amplified by the eccentricity between the load and the hanger reinforcement—based on summing moments about the outside face of the member. Conversely, hanger reinforcement demand is reduced by the combined effects of shear and torsion in the portion of the supporting member below the top of the corbel. For brackets or corbels located near the bottom of a supporting member, the load amplification due to the eccentricity is much greater than the

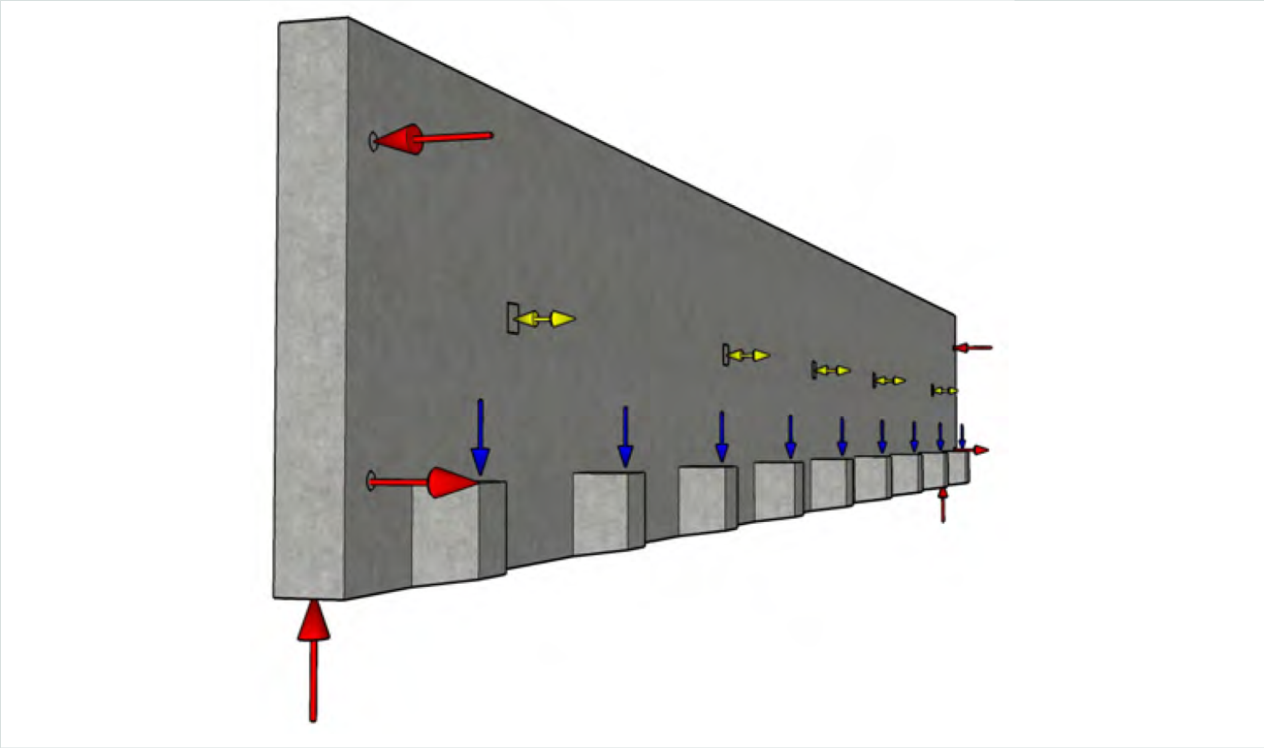

Figure 1. Spandrel beam with corbels.

Figure 2. Distress in wall panel corbel with inadequate hanger reinforcement. Note: The steel angle was installed as a retrofit.

reduction in demand due to shear and torsion in the supporting member web below the top of the corbel. **Figure 2** shows distress at a wall panel corbel caused by eccentricity that was not considered in the design.

Review of hanger reinforcement design recommendations

The structural capacity of ledges and corbels in precast and prestressed concrete construction is largely dependent on the configuration and size of the hanger reinforcement. Designers have used various approaches to determine the spacing and size of the hanger reinforcement. The American Concrete Institute's (ACI's) *Building Code Requirements for Structural Concrete (ACI 318-14) and Commentary (ACI 318R-14)*[1] and *Building Code Requirements for Structural Concrete (ACI 318-19) and Commentary (ACI 318R-19)*[2] do not provide requirements or guidance for hanger reinforcement in corbels. Instead, the publications frequently used by designers are the Portland Cement Association's (PCA's) *Notes on ACI 318-11 Building Code Requirements for Structural Concrete*[3] and the *PCI Design Handbook: Precast and Prestressed Concrete*.[4]

A brief review of the hanger reinforcement design approaches provided in these two publications follows.

PCA notes on ACI 318

PCA introduced a design procedure for the detailing of a continuous concrete beam ledge in its *Notes on ACI 318-83 Building Code Requirements for Structural Concrete*[5] in chapter 16, "Brackets, Corbels and Beam Ledges." The PCA approach calculates the minimum required area of reinforcement as follows:

$$A_{sh} = \frac{V_u}{\phi f_y}$$

where

A_{sh} = area of hanger reinforcement

V_u = factored vertical force acting on ledge, corbel, or bracket

ϕ = strength reduction factor for shear

f_y = specified yield strength of nonprestressed reinforcement

In PCA's *Notes on ACI 318-95 Building Code Requirements for Structural Concrete*[6] in chapter 17, "Brackets, Corbels and Beam Ledges," the equation was modified to include a hanger reinforcement spacing factor as follows:

$$V_u \le \phi \frac{A_v f_y}{s} S \qquad \text{(PCA 17-4)}$$

where

A_v = area of hanger reinforcement as defined in the various editions of the PCA Notes

s = spacing of hanger reinforcement

S = distance between ledge loads

The modified equation remained unchanged in chapter 15 of PCA's *Notes on ACI 318-11 Building Code Requirements for Structural Concrete*.[3] It should be noted that there

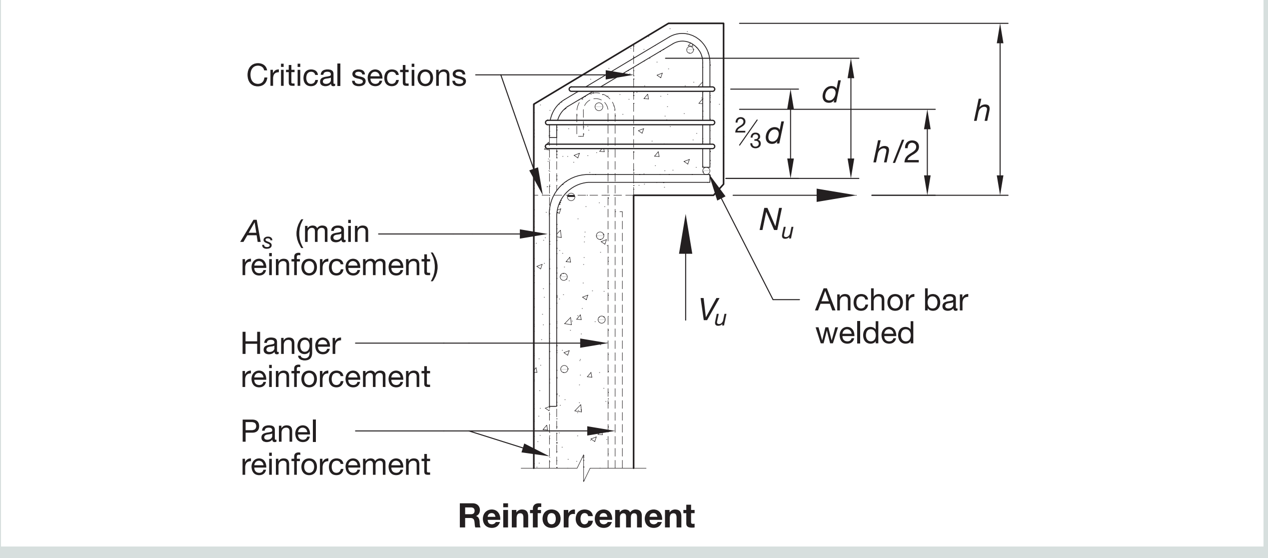

Figure 3. Illustration of reinforcement for wall corbels. Source: Reproduced from PCI (2017), Fig. 5.7.2. Note: A_s = area of nonprestressed main reinforcement; d = distance from extreme compression fiber to centroid of main reinforcement; h = overall height of member or element; N_u = factored horizontal force occurring simultaneously with V_u; V_u = factored vertical force acting on ledge, corbel, or bracket.

is no plan to update the PCA notes for ACI 318-14 and ACI 318-19.

The equations provided in the PCA notes for the calculation of the required hanger reinforcement neglect the influence of the eccentricity between the reaction on the corbel and the location of the hanger reinforcement. The required hanger reinforcement calculated using these equations is inaccurate and unconservative. Additional design procedures are provided in the PCA notes, but these procedures focus on shear failure modes and do not account for the forces due to the eccentricity between the corbel load and the location of the hanger reinforcement.

PCI Design Handbook, eighth edition

Section 5.7, "Concrete Corbels," of the eighth edition of the *PCI Design Handbook* addresses general corbel design, including hanger reinforcement. Two design approaches are recommended for the design of concrete corbels: the cantilever beam method and the strut-and-tie method. The cantilever beam design method is based on chapter 16 of ACI 318-19, while the strut-and-tie method follows chapter 23 of ACI 318-19. For the cantilever beam design method, *PCI Design Handbook* Fig. 5.7.2 for wall corbels is reproduced in **Fig. 3** for reference. The design equations provided in section 5.7.1 of the *PCI Design Handbook* for the cantilever beam design method are limited to the calculation of the necessary flexural and shear friction reinforcement and do not cover hanger reinforcement.

Designs using the strut-and-tie method account for eccentricity. However, unless a more complex three-dimensional model is considered, the method may be conservative because it would not account for a reduction in demand due to the combined effects of shear and torsion in the portion of the supporting member web or wall below the corbel. "Below" assumes downward load on the corbel (Fig. 1 and 2). Figure 3 shows an upward load on a corbel.

Section 5.6.4 of the *PCI Design Handbook* provides an equation to calculate the hanger reinforcement needed to attach a ledge to a spandrel beam web. The design approach is based on research[7] on the design of spandrel beams with continuous ledges. The design model and notation are illustrated in **Fig. 4**. The hanger reinforcement area A_{sh} is calculated using the load amplification factor m as follows:

$$A_{sh} = \frac{V_u}{\phi f_y}(m) \quad \text{(PCI 5-83)}$$

where

$$m = \frac{\left[(d_s + a) - \left(3 - \frac{2h_\ell}{h}\right)\left(\frac{h_\ell}{h}\right)^2\left(\frac{b_\ell}{2}\right) - e\gamma_t \frac{(x_\ell^2 y_\ell)}{(x_\ell^2 y_\ell + x_w^2 y_w)}\right]}{d_s} = 0.6$$

(PCI 5-84)

where

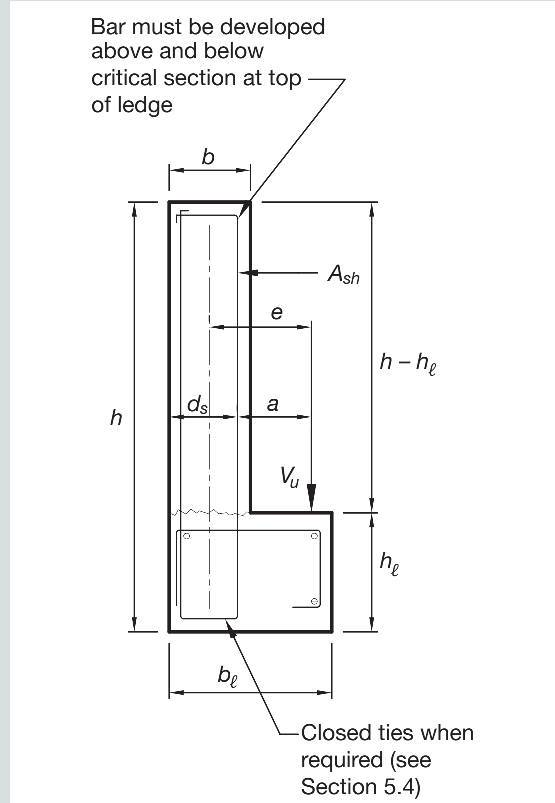

Figure 4. Design model and notation for calculation of hanger reinforcement in beams with ledges. Source: Reproduced from PCI (2017), Fig. 5.6.3. Note: a = horizontal distance between the applied load and centroid of the hanger reinforcement; A_{sh} = area of hanger reinforcement; b = width of wall or spandrel beam web; b_l = width across the bottom of a ledger beam including the web and ledge projection; d_s = distance from outside face of L beam to centroid of hanger reinforcement; e = eccentricity, horizontal distance between applied load and centerline of wall or spandrel beam web; h = overall height of member or element; h_l = overall height of bracket or corbel; V_u = factored vertical force acting on ledge, corbel, or bracket.

d_s = distance from outside face of L beam to centroid of hanger reinforcement

a = horizontal distance between the applied load and centroid of the hanger reinforcement

h_ℓ = overall height of bracket or corbel

h = overall height of member or element

b_ℓ = width across the bottom of a ledger beam including the web and ledge projection

e = eccentricity, horizontal distance between applied load and centerline of wall or spandrel beam web

γ_t = 0 when closed ties are not used in the ledge and is 1.0 when closed ties are used in the ledge

x_ℓ = h_ℓ when $b_\ell > h_\ell$ and b_ℓ otherwise

x_w = b when $(h - h_\ell) > b$ and $h - h_\ell$ otherwise

y_ℓ = b_ℓ when $b_\ell > h_\ell$ and h_ℓ otherwise

y_w = $h - h_\ell$ when $(h - h_\ell) > b$ and b otherwise

The load amplification factor m is dependent on the eccentricity of the applied load relative to the location of the hanger reinforcement. In addition, m considers the shear and torsional resistance of the continuous ledge.

Proposed design approach for corbel hanger reinforcement

The critical consideration for the design of the hanger reinforcement is the transfer of the factored eccentric load V_u onto the corbel and into the upper portion of the supporting member (assuming a vertical downward-acting load). The design model and notation are illustrated in **Fig. 5**. Summing the moments about the outside face of the member (point x in Fig. 5) accounts for the eccentricity between the corbel reaction and hanger reinforcement. The design model also accounts for the reduction in demand due to the combined effects of shear and torsion in the portion of the supporting member web or wall below the corbel. Accordingly, the following equation for nominal shear strength V_n provided by the hanger reinforcement for spandrel and wall corbels is proposed.

$$V_n = \left[\frac{\left(A_{sh} f_y d_s\right)}{\left(d_s + a\right) - \left(\frac{V_b}{V_u}\right)\left(\frac{b}{2}\right) - \left(\frac{T_b}{V_u}\right)}\right] \quad (1)$$

where

$$\frac{V_b}{V_u} = \left(3 - \frac{2h_b}{h}\right)\left(\frac{h_b}{h}\right)^2 \quad \text{and} \quad \frac{T_b}{V_u} = e\frac{h_b}{h}$$

V_b = vertical force in a ledger beam web or wall below the applied load on the bracket or corbel

b = width of wall or spandrel beam web

T_b = torsional moment in a ledger beam web or wall below the applied load on the bracket or corbel

h_b = vertical distance between the top of the bracket or corbel and the bottom of the ledger beam web or wall to which it is attached

As would be expected, the required hanger reinforcement decreases as the distance between the top of the bracket or corbel and the bottom of the member increases. The subscript ℓ used in the equations from section 5.6.4 of the *PCI Design Handbook* for a continuous ledge-to-web attachment has been replaced by b for bracket or corbel in the proposed equations. The proposed equations will require a trial and error approach to solve for the shear strength. Similar to the approach used for beam ledges, Eq. (1) can be simplified by using a modification factor m_b to account for eccentricity of the load as well as shear and torsion in the portion of the member below the load. Including the strength reduction factor ϕ, A_{sh} is given by Eq. (2):

$$A_{sh} = \frac{V_u}{\phi f_y}\left(m_b\right) \quad (2)$$

where

$$m = \frac{\left[\left(d_s + a\right) - \left(3 - \frac{2h_b}{h}\right)\left(\frac{h_b}{h}\right)^2\left(\frac{b}{2}\right) - e\left(\frac{h_b}{h}\right)\right]}{d_s}$$

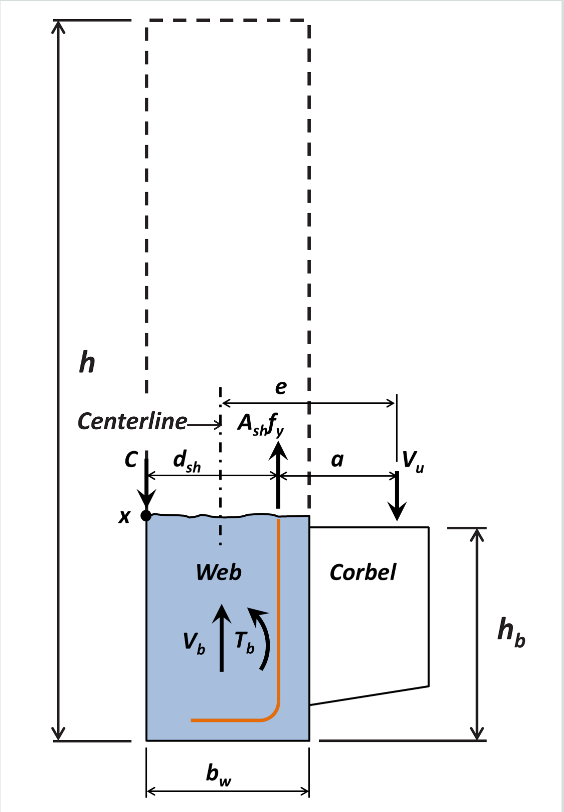

Figure 5. Design model and notation for corbel hanger reinforcement. Note: a = horizontal distance between the applied load and centroid of the hanger reinforcement; A_{sh} = area of hanger reinforcement; b_w = width of the supporting member web; C = compression block resultant; d_{sh} = distance from extreme compression fiber to centroid of hanger reinforcement; f_y = specified yield strength of nonprestressed reinforcement; h = overall height of member or element; h_b = vertical distance between the top of the bracket or corbel and the bottom of the ledger beam web or wall to which it is attached; V_b = vertical force in a ledger beam web or wall below the applied load on the bracket or corbel; V_u = factored vertical force acting on ledge, x = point used to sum the moments about the outside face of the member.

The second and third term in the numerator of the equation are for the shear and torsion, respectively, in the wall or spandrel beam web located below the top of the corbel. If desired, the shear V_b and torsion T_b in the ledger beam web or wall below the applied load on the bracket or corbel can be conservatively neglected for the corbel design, and these terms can be excluded from Eq. (1). An alternative and simplified design equation can be derived to calculate the required hanger reinforcement as follows:

$$A_{sh} = \frac{V_u}{\phi f_y}\left(\frac{d_s + a}{d_s}\right)$$

Detailing considerations

A corbel failure that occurred in specimen SP19 from a PCI research project on slender spandrel beams[8] is shown in **Fig. 6**. Hanger reinforcement for SP19 was provided by three no. 5 (16M) reinforcing bars, which were designed considering eccentricity. The hanger reinforcement did not yield; rather, the corbel failed when the corbel flexural reinforcement pulled out at the top of the corbel and the bottom of the corbel broke through the web of the spandrel beam. Based on Eq. (1), the shear strength provided by the hanger reinforcement was 37.7 kip (168 kN), somewhat greater than the failure load of 35.9 kip (160 kN). As such, the hanger reinforcement would not have been expected to yield.

The top pullout failure of the flexural reinforcement is similar to breakout failure of embedded anchors, which is addressed in section 17.6.2 of ACI 318-19. The failure of specimen SP19 occurred at a load of 35.9 kip (160 kN), well above the factored design load; however, evaluation in accordance with section 17.6.2 indicates that the breakout failure occurred prematurely. The estimated horizontal force in the top reinforcement was about half that predicted by the ACI 318-19 equations for tension breakout. It appears that the premature failure was due to tension that developed perpendicular to the direction of breakout: horizontal tension from global flexure and vertical tension from the corbel reaction. In addition, horizontal cracking due to vertical tension in the hanger reinforcement appears to have cut off the top portion of the breakout cone. Referring to Fig. 6, it is also apparent that due to the through-thickness breakout failure the outer two hanger reinforcing bars, which are located at the outside edges of the corbel, were not as effective as the hanger reinforcement located at the center of the corbel. Ideally, hanger reinforcement should be located within or adjacent to the corbel horizontal reinforcement. In all cases, hanger reinforcement should be developed across potential inclined cracks, such as the crack shown in Fig. 2.

The authors are not aware of any through-thickness breakout failures in service. Nevertheless, through-thickness breakout should be considered where corbels are attached to relatively thin walls and spandrel beam webs, especially thin wythes of sandwich panels. In accounting for through-thickness breakout, the strength-reducing effect of tension perpendicular to the breakout surface should be considered. Methods have been developed for considering punching-shear strength reduction due to global shear and tension in beam ledges.[9]

Figure 6. Corbel failure of specimen SP19.

Conclusion

The effect of eccentricity on corbel hanger reinforcement design is a critical but often overlooked consideration. A design approach was developed for corbel hanger reinforcement that considers both eccentricity and reduction in demand due to the combined effects of shear and torsion in the portion of the supporting member web or wall below the corbel.

A corbel failure observed in previous research[7] indicates that tension in the hanger reinforcement and global flexural tension reduce the breakout strength of corbel flexural reinforcement. This effect requires further research. In the meantime, ACI 318-19 breakout strength equations should be used conservatively.

References

1. ACI (American Concrete Institute) Committee 318. 2014. *Building Code Requirements for Structural Concrete (ACI 318-14) and Commentary (ACI 318R-14)*. Farmington Hills, MI: ACI.

2. ACI Committee 318. 2019. *Building Code Requirements for Structural Concrete (ACI 318-19) and Commentary (ACI 318R-19)*. Farmington Hills, MI: ACI.

3. Kamara, M. E., and L. C. Novak. 2013. *Notes on ACI 318-11 Building Code Requirements for Structural Concrete*. Skokie, IL: PCA (Portland Cement Association).

4. PCI Industry Handbook Committee. 2017. *PCI Design Handbook: Precast and Prestressed Concrete*. MNL-120. 8th ed. Chicago, IL: PCI.

5. PCA. 1984. *Notes on ACI 318-83 Building Code Requirements for Reinforced Concrete*. Skokie, IL: PCA.

6. PCA. 1996. *Notes on ACI 318-95 Building Code Requirements for Structural Concrete*. Skokie, IL: PCA.

7. Klein, Gary J. 1986. "Design of Spandrel Beams." *PCI Journal* 31 (5): 76–124.

8. Lucier, G., C. Walter, S. Rizkalla, P. Zia, and G. Klein. 2011. "Development of a Rational Design Methodology for Precast Concrete Slender Spandrel Beams: Part 1, Experimental Results." *PCI Journal* 56 (2): 88–112.

9. Nafadi, Mohamed K., Gregory W. Lucier, Sami H. Rizkalla, Paul Z. Zia, and Gary J. Klein. 2018. "Ledge Behavior and Strength of Long-Span L-shaped Beams." *PCI Journal* 63 (2): 50–66.

Notation

a = horizontal distance between the applied load and centroid of the hanger reinforcement

A_s = area of nonprestressed main reinforcement

A_{sh} = area of hanger reinforcement

A_v = area of hanger reinforcement as defined in the various editions of the PCA Notes

b = width of wall or spandrel beam web

b_ℓ = width across the bottom of a ledger beam including the web and ledge projection

b_w = width of the supporting member web

C = compression block resultant

d = distance from extreme compression fiber to centroid of longitudinal reinforcement

d_s = distance from outside face of L beam to centroid of hanger reinforcement

d_{sh} = distance from extreme compression fiber to centroid of hanger reinforcement

e = eccentricity, horizontal distance between applied load and centerline of wall or spandrel beam web

f_y = specified yield strength of nonprestressed reinforcement

h = overall height of member or element

h_b = vertical distance between the top of the bracket or corbel and the bottom of the ledger beam web or wall to which it is attached

h_ℓ = overall height of bracket or corbel

m = modification factor for hanger reinforcement design defined in *PCI Design Handbook* section 5.6.4

m_b = modification factor for design of hanger reinforcement for brackets and corbels that accounts for eccentricity of the applied load as well as shear and torsion below the applied load

N_u = factored horizontal force occurring simultaneously with V_u

s = spacing of hanger reinforcement

S = distance between ledge loads

T_b = torsional moment in a ledger beam web or wall below the applied load on the bracket or corbel

V_b = vertical force in a ledger beam web or wall below the applied load on the bracket or corbel

V_n = nominal shear strength of hanger reinforcement

V_u = factored vertical force acting on ledge, corbel, or bracket

x = point used to sum the moments about the outside face of the member (see Fig. 5)

x_ℓ = h_ℓ when $b_\ell > h_\ell$ and b_ℓ otherwise

x_w = b when $(h - h_\ell) > b$ and $h - h_\ell$ otherwise

y_ℓ = b_ℓ when $b_\ell > h_\ell$ and h_ℓ otherwise

y_w = $h - h_\ell$ when $(h - h_\ell) > b$ and b otherwise

γ_t = 0 when closed ties are not used in the ledge and is 1.0 when closed ties are used in the ledge

ϕ = strength reduction factor for shear

About the authors

Gary Klein is a PCI member and executive vice president and senior principal at Wiss, Janney, Elstner Associates in Northbrook, Ill. A licensed structural engineer specializing in failure investigation, he is a member of ACI Committees 318, Structural Concrete Building Code, and 445, Shear and Torsion. He has led or contributed to several PCI-sponsored investigations, including research on ledge-to-web attachment, torsion of slender spandrel beams, volume change of precast concrete structures, dapped double tees, and punching shear of beam ledges.

Harry Gleich, PE, FACI, FPCI, is vice president of engineering for Metromont Corp. in Greenville, S.C., serves on many PCI committees and is chair of the Technical Activities Council, and is the immediate past chair of the PCI Research and Development Council and past chair of the PCI Precast Insulated Wall Panels Committee. At ACI, he also serves on numerous committees and is the former chair of ACI Committee 533, Precast Panels, and former chair of ACI Committee 550, Precast Concrete Structures.

Ralf Leistikow is a principal at Wiss, Janney, Elstner Associates in Raleigh, N.C. He is a licensed professional engineer specializing in structural investigation, with particular focus on precast concrete structures. He is active on ACI Committee 224, Cracking, and currently serves as the committee's secretary. In addition, he is active on Post-Tensioning Institute Committees DC-20, Building Design, and DC-80, Repair, Rehabilitation and Strengthening.

Gregory Lucier, PhD, is a PCI member and manager of the Constructed Facilities Laboratory at North Carolina State University in Raleigh. He has led or contributed to several PCI-sponsored investigations, including research on torsion of slender spandrel beams, dapped double tees, and punching shear of beam ledges.

Abstract

Brackets and corbels are increasingly used to transfer loads to beam and wall elements. Where corbels are located along the bottom of spandrel beams or just above wall openings, hanger reinforcement is needed to transfer the applied load to the upper region of the member. This paper reviews past practices and proposes revised equations for the design of corbel hanger reinforcement. The recommended procedure accounts for the eccentricity of the applied load as well as shear and torsion carried by the member below the applied load. Unlike past practices, accounting for eccentricity of the applied load will avoid hanger reinforcement deficiencies that could require retrofit or cause structural failure.

Keywords

Bracket, corbel, eccentricity, hanger reinforcement, spandrel beam, wall panel.

Review policy

This paper was reviewed in accordance with the Precast/Prestressed Concrete Institute's peer-review process.

Reader comments

Please address any reader comments to *PCI Journal* editor-in-chief Emily Lorenz at elorenz@pci.org or Precast/Prestressed Concrete Institute, c/o *PCI Journal*, 200 W. Adams St., Suite 2100, Chicago, IL 60606.