Analysis of bonded link slabs in precast, prestressed concrete girder bridges

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- When using link slabs in bridge construction, current practice in the United States and abroad is to debond the precast concrete girders from the link slab to minimize stress development at the connecting zone.
- This paper proposes bonding the link slabs to the adjacent precast, prestressed concrete girders rather than debonding.
- The paper investigates the structural behavior of the bonded link slab and identifies a step-by-step procedure for the design of bonded link slabs for bridge structures in the medium-span-length range.

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Precast, prestressed concrete girders offer a cost-effective solution for the construction of bridge structures in the medium-span-length range (between 12 and 42 m [40 and 140 ft]). Generally, the precast concrete girders are erected as simply supported (**Fig. 1**) and the reinforced concrete deck slab is placed afterward (either cast in place or by using precast concrete slab units). In multiple-span construction (Fig. 1), the deck slab is made continuous over the intermediate supports to minimize the use of expansion joints. The portion of the deck slab connecting two adjacent simple-span girders is referred to as the link slab (Fig. 1). The link slab can be cast with the deck slab or separately after all dead loads are placed.[1]

The general practice in the United States and abroad consists of providing solid reinforced concrete end diaphragms and debonding the precast concrete girders from the link slab over the diaphragms to minimize stress development and cracking at the connection zone.[2] A survey of bridges in the United States indicated that full-depth cracks can still develop in the link slab, and therefore some practices are maintaining the bond between the link slab and girder and eliminating end diaphragms for simplification.[2] Although extensive analytical and experimental studies have been performed for debonded link slabs,[3] information related to bonded link slabs is scarce.

This paper investigates the structural behavior of link slabs bonded to adjacent precast, prestressed concrete girders of various lengths and types. A proposed design methodology that allows calculating shear and bending moments in the

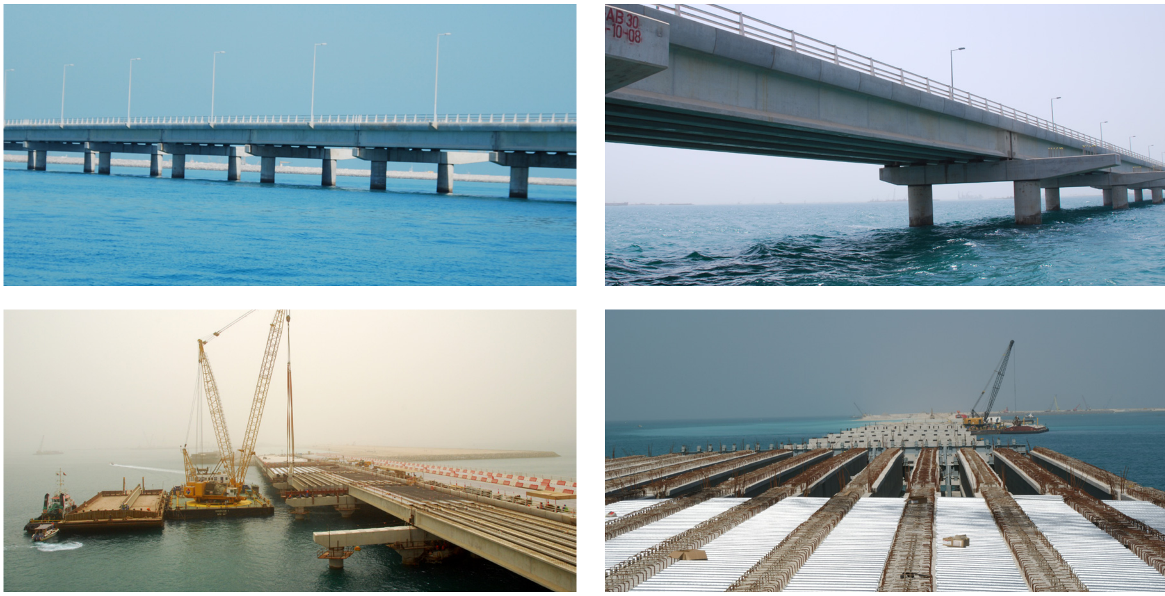

Figure 1. The bottom left photo shows a picture of the Khalifa Port bridge structure girder placement in Abu Dhabi, UAE. The bottom right photo shows the placement of deck form. The top right photo shows the 1.64 km (5412 ft) long multispan trestle bridge structure after completion. The top left photo shows continuity of the deck slab and link slab over the intermediate support. Photos courtesy of Archirodon Construction, UAE.

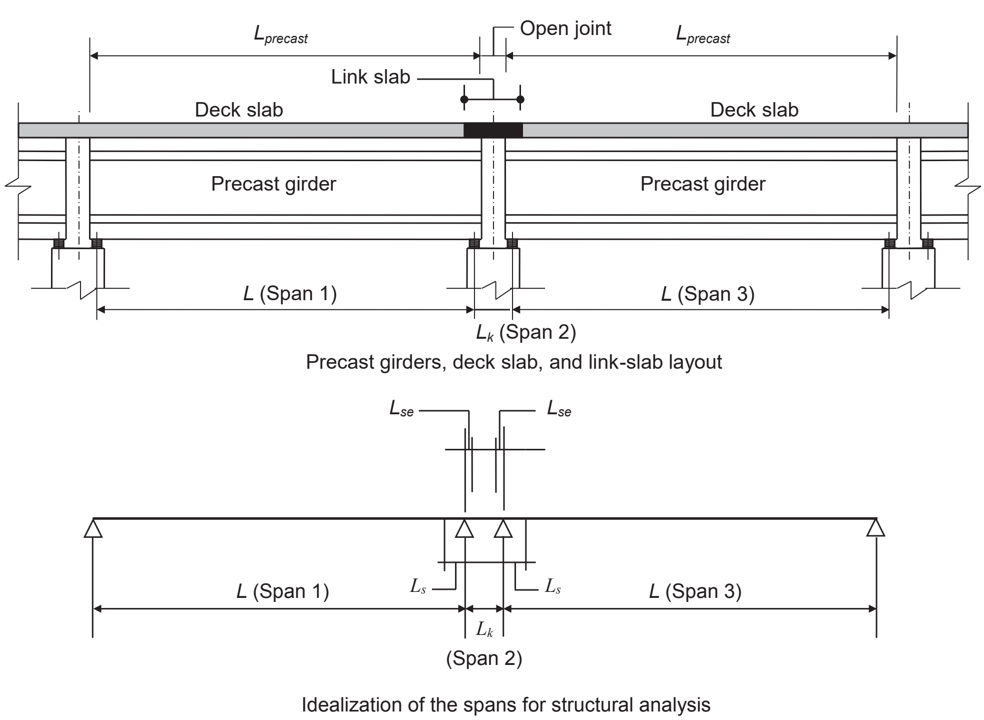

Figure 2. Layout and span arrangement for two symmetrical spans connected by a link slab for which the analysis was performed. Note: L = design length; L_k = span length between intermediate supports; $L_{precast}$ = precast girder length; L_s = distance from centerline of support to edge of link slab in the main span; L_{se} = distance from centerline of support to edge of girder; ($L_k - 2L_{se}$) = open joint dimension.

link slab based on its stiffness and the structural response of the adjacent girders is developed. Recommendations for the analysis and design of bonded link slabs in precast, prestressed concrete girder applications in the medium-span range are made and illustrated by a numerical example.

Background and approach

Previous research focused on experimental and theoretical evaluation of link slabs that are debonded from the girders' ends. Some researchers considered a simple design method assuming that the link slab is in bending,[2] while others conducted detailed investigations based on the offset between the centroid of the composite section (girder and slab) and link slab and the axial stiffness of the link slab rather than its flexural stiffness.[4] A comprehensive analytical and experimental study by Caner and Zia[2] confirmed that for structures subjected to vertical loads, the link slab is in bending and behaves like a beam rather than a tension member.

This paper quantifies the forces that develop in bonded link slabs due to superimposed dead load and live load based on beam member shear and flexural behavior. A two-symmetrical-span case (**Fig. 2**) connected by a link slab for which closed-form solutions can be easily derived is considered. Both uniform and concentrated loads that are representative of bridge dead load and American Association of State Highway and Transportation Officials' AASHTO HL-93 live load (truck and lane) are used in the analysis.[5]

The outline of the paper is as follows:

- Derive closed-form solutions for shear and bending moments in the link slab and precast concrete girders.
- Identify the main parameters that influence the link slab shear and flexural stiffness (length, thickness, concrete cylinder strength, and reinforcement size), and present the closed-form solutions as a function of these parameters.
- Conduct a parametric study to show the implications of varying these parameters on the structural response of the link slab in the medium-span-length range (12 m [40 ft] $\le L \le$ 42 m [140 ft]).
- Develop plots that illustrate the variation of bending moments in the link slab for standard AASHTO girder applications (Type II to VI precast, prestressed concrete girders).
- Identify controlling parameters that contribute to inducing bending moments in the link slab without considerably increasing shear forces.
- Present a step-by-step procedure for the design of bonded link slabs and provide recommendations for optimizing design. Illustrate applicability by a numerical example.

It shall be noted that although the two-symmetrical-span case connected by the link slab may not always be representative of actual span arrangements, it provides conservative indication of the magnitude of forces that develop in the link slab if more than two spans or asymmetrical spans are used.

Structural response

The structural response of a link slab connecting two adjacent symmetrical spans is carefully examined (Fig. 2). The precast concrete girders are standard AASHTO girders of precast length $L_{precast}$ and design span between supports L (Fig. 2). The precast concrete girders are assumed to be simply supported, and continuity is achieved after casting the link slab. The length of the inner span between bearings is designated as L_k. The distance from the centerline of the intermediate support to the edge of the precast concrete girders is defined as L_{se}, and the effective link slab length between the girder ends is $L_k - 2L_{se}$ (Fig. 2). The link slab extends over the precast concrete girders in the first and last span distances L_s measured from centerline of support (Fig. 2).

Precast concrete girders in the medium-span-length range are usually supported by composite elastomeric bearing pads (Fig. 2). In the structural analysis, these were initially idealized as pin supports to calculate the flexural moments, and the bearing pad stiffness is accounted for later, during the shear force analysis.

Stiffness

The structural response of the three-span unit shown in Fig. 2 depends on the flexural stiffness of the beam elements (EI/L) in spans 1, 2, and 3 (that is, the beam modulus of elasticity E multiplied by its moment of inertia I and divided by its length L). Spans 1 and 3 represent the girder and deck slab, while span 2 represents the connecting link slab.

The flexural stiffness in spans 1 and 3 (EI/L) is based on the composite action of the precast concrete girder and deck slab. The intermediate (short) span stiffness (span 2) comprises two parts: the link slab of effective length $L_k - 2L_{se}$ between the edges of the girders and the composite action of the girder and link slab over distance L_{se} and distance L_s (Fig. 2).

Design loads

The primary loads supported by the structure after continuity is achieved are the composite dead load and live load. Note that lateral loads (relating mainly to temperature, creep, and shrinkage) were not considered in this paper.

Composite dead load includes traffic barriers and parapets, wearing surface, sidewalks, fixtures, and utilities. These are assumed to be uniformly distributed throughout the spans (**Fig. 3**). Composite dead loads are subdivided into two categories: component and attachment (labeled DC) and wearing surface and utility (labeled DW).[5]

Live load consists of a combination of AASHTO HL-93 truck or tandem load (whichever is larger) and lane load.[5] Truck load magnitude is 325 kN (72 kip) multiplied by 1.33 for impact distributed over three axles: 35 kN (8 kip) front, 145 kN (32 kip) middle, and 145 kN rear axles. Spacing is 4.3 m (14 ft) between the front and middle axles and varies from 4.3 to 9 m (29 ft) between the middle and rear axles. Note that tandem load did not govern for span length larger than 12 m (40 ft). Lane load magnitude is 9.3 kN/m (0.64 kip/ft) and can be uniformly applied to the three spans or to one span only.

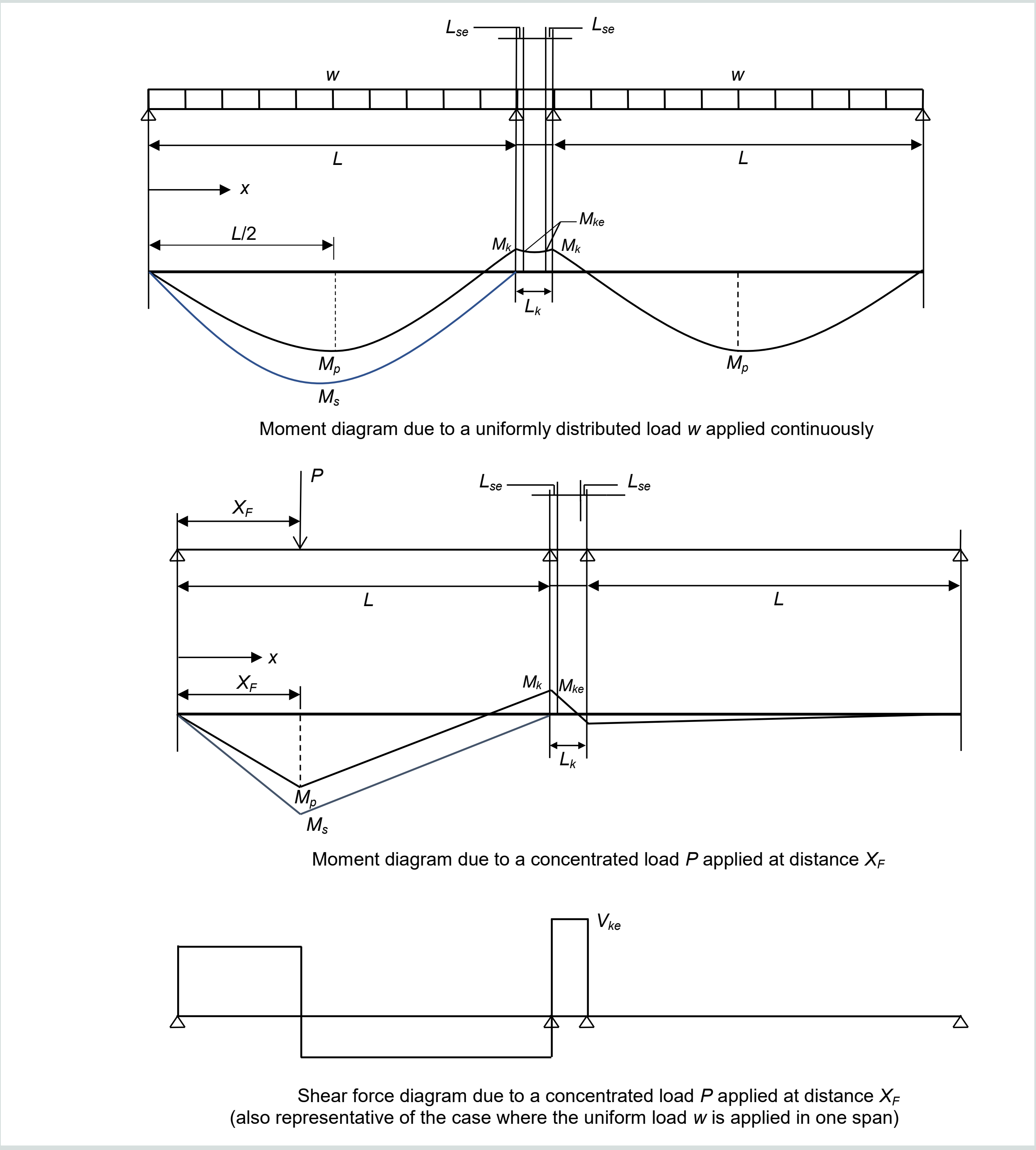

Figure 3. Load configurations for which the structural analysis was performed as well as the moment and shear diagrams that develop due to continuity in the link slab. Note: L = design length; L_k = span length between intermediate supports; L_{se} = distance from centerline of support to edge of girder; M_k = moment at centerline of support; M_{ke} = moment at edge of link slab ($x = L + L_{se}$); M_p = girder reduced moment due to continuity; M_s = girder moment based on simply supported beam analysis; P = concentrated load; V_{ke} = maximum shear force in link slab; w = uniform load; x = offset; X_F = offset of concentrated load from left end support.

Analysis was conducted for a uniformly distributed load w (Fig. 3), representative of the superimposed dead loads and lane live load if continuously applied, and a single concentrated load P, representative of the truck axles' load, applied at distance X_F in one span (Fig. 3, middle). It was shown in this study that the effect of a lane load partially applied over one span on redistribution of moment and shear forces is similar to the case where a concentrated load is applied at midspan distance $X_F = L/2$ (Fig. 3). Consequently, analysis was not conducted for the case of a uniform load applied over one span.

Analytical solution

The analytical solution determined the flexural (bending) moments that developed in spans 1, 2, and 3 due to dead loads and live loads (Fig. 3). These were calculated by integration of deformations using the differential equation

$$M = EI\frac{d^2u}{dx^2}$$

where

M = bending moment

E = Young's modulus

I = moment of inertia

u = displacement

x = horizontal distance from left end support.

The loading cases considered (uniformly distributed load w and concentrated load P applied at X_F [Fig. 3]) relate to the first derivative of the shear force V.

$$dV/dx = -w$$

where

w = uniformly distributed load (constant throughout the beam for a uniform load and zero for a concentrated load)

The shear force V is the first derivative of the bending moment M.

$$V = dM/dx$$

The analytical solution consisted of integrating the relevant beam-bending differential equations and developing polynomials of known degree and coefficients based on boundary and continuity conditions. For this purpose, the continuous beam was divided into segments and displacements, and shear and bending moments were determined over each segment by solving the differential equations at critical points. Critical points consisted of the segments' end points, locations of point loads and supports, beginnings and ends of uniformly distributed loads, and beginnings and ends of sudden changes in flexural stiffness EI/L. These functions thus provided a piecewise definition of the corresponding fields over the continuous beam.[6] Wolfram Mathematica V.11.0, a symbolic mathematical computation program, was used to calculate practical solutions to those equations where independent fields for displacement, shear force, and internal moment can be assumed in each segment.

Parameters

To simplify derivation and presentation of the closed-form solutions, parameters based on girder and link slab geometric and material properties were introduced as follows:

- It was based on span length in Fig. 2: $\alpha_s = L_s/L$, $\alpha_{se} = L_{se}/L$, $\alpha_k = L_k/L$ where α_s is the ratio of the link slab length in the main span to the girder design length, α_{se} is the ratio of the link slab length in the continuity zone to the girder design length, and α_k is the ratio of the link slab length between the intermediate supports to the girder design length.

- For concentrated load P applied at distance X_f (Fig. 3), parameter $x_f = X_f/L$ is introduced as the ratio of the point load offset from the left support to the design length.

- It was also based on the beam member properties in spans 1, 2, and 3 in Fig. 2: $ei_s = EI_s/EI$ (ratio of the flexural rigidity of the girder and link slab to the flexural rigidity of the girder and deck slab) and $ei_k = EI_k/EI$ (ratio of the flexural rigidity of the link slab to the flexural rigidity of the girder and deck slab). E is the concrete modulus of elasticity (in MPa) = $4800\sqrt{f_c}$, where f'_c is the compressive 28-day cylinder strength (in MPa) (labeled f'_{cg} for the girder, f'_{cs} for the deck slab, and f_{ck} for the link slab). I is the moment of inertia of the girder and deck slab, I_s is the moment of inertia of the girder and link slab, and I_k is the moment of inertia of the link slab. Note that I and I_s were based on the gross concrete section because the girder tension stress zone is precompressed due to prestress, while I_k is for the cracked section of the link slab calculated based on the composite action between the concrete and steel reinforcing bar as shown in the following section.[2]

Link slab cracked section properties

The moment of inertia of the cracked section of the link slab[2] I_k is given by Eq. (1).

$$I_k = \frac{by^3}{3} + nA_s\left(d - y\right)^2 \tag{1}$$

where

b = effective link slab width (the girder spacing S for the range of parameters considered in this paper)

y = depth of the compression zone for service load design ([Fig. 4] determined from Eq. [2])[2]

n = ratio of the modulus of elasticity of reinforcing steel (200,000 MPa [29,000 ksi]) to the modulus of elasticity of the link slab (4800, $\sqrt{f'_{ck}}$ in MPa)

A_s = area of reinforcing steel in link slab

$$y = -n\rho d + d\sqrt{\left((n\rho)^2 + 2\rho n\right)} \qquad (2)$$

where

ρ = steel reinforcement ratio = A_s/bd

Tension steel should ensure that the ratio of the offset of the extreme compression strain from the neutral axis to the depth of the reinforcement c/d does not exceed 0.6, where c is the depth of equivalent rectangular stress block a divided by the factor that relates this stress block to depth of neutral axis $c = a/\beta_1$.[5] In this paper, ρ ranges from 37.5% to 75% of the ratio of reinforcement for balanced strain conditions ρ_b calculated from Eq. (3):

$$\rho_b = \left(\frac{0.85\beta_1 f'_{ck}}{f_y}\right)\left(\frac{580}{580+f_y}\right) \qquad (3)$$

where

f_y = yield stress of reinforcing steel = 415 MPa (60,000 psi)

β_1 = ratio of compression stress block to actual compression zone = 0.84 for 28-day concrete cylinder strength of link slab f_{ck} = 30 MPa (4350 psi)

β_1 = 0.76 for 28-day concrete cylinder strength of link slab f_{ck} = 40 MPa (5800 psi)

It is shown that the ratio of cracked section moment of inertia I_k to gross moment of inertia I_g depends on the link slab thickness t_k and steel ratio ρ only, irrespective of the concrete cylinder strength f_{ck} and effective slab width b (where b = S as shown in Fig. 5). Numerical values of the ratio of I_k (using Eq. [1]) to I_g (equal to $St_k^3/12$) are shown in **Table 1** for the different ratio of tension steel ρ considered. Note that the depth of reinforcement d was based on a clear cover of 50 mm (2 in.) and a reinforcing steel bar size of T20 or 20 mm (0.8 in.) (Fig. 4). For a 200 mm (8 in.) thick link slab, d is calculated as follows:

$d = 0.7t_k = 0.7 \times 200 = 140$ mm (5.5 in.)

Table 1. Ratio of the cracked moment of inertia I_k to the gross moment of inertia I_g

Steel reinforcement ratio ρ	I_k/I_g for t_k = 200 mm
$0.375\rho_b$	0.21
$0.5\rho_b$	0.26
$0.75\rho_b$	0.34

Note: t_k = thickness of link slab; ρ = steel reinforcement ratio; ρ_b = balance steel reinforcement ratio. 1 mm = 0.0394 in.

Closed-form solutions

Closed-form solutions were derived for the positive moment at the midspan of the girder M_p and the negative moment in the link slab M_{ke} at the edge of the girder. Moment diagrams for the uniform load w and concentrated load P are shown in Fig. 3. Equations for shear forces in the link slab due to the concentrated load P applied at X_F were also derived, as their magnitude in the link slab region is considerably large when asymmetrical load is applied (Fig. 3).

Moments M_p and M_{ke} are presented as ratios to the positive moment from the simple-span case M_s (also shown in Fig. 3) to quantify the effect of the link slab stiffness on the structural response of the girder. The shear force (Fig.

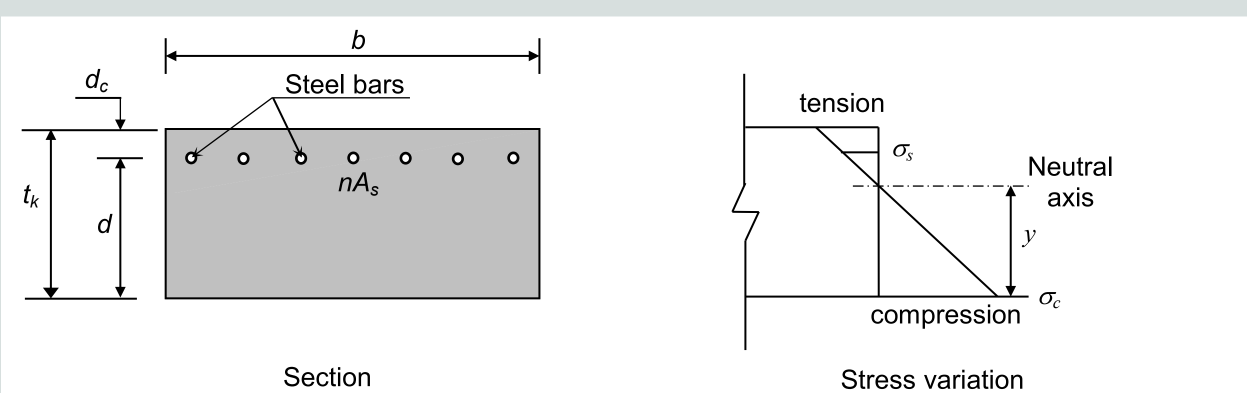

Figure 4. Parameters for link slab service load analysis based on cracked moment of inertia I_k (tension steel distribution of link slab). Note: A_s = area of steel; b = effective link slab width = girder spacing S; d = distance from extreme compression fiber to center of flexural reinforcement; d_c = distance from extreme tension fiber to center of flexural reinforcement; n = modular ratio; t_k = link slab thickness; y = depth of compression zone for service load design; σ_c = stress in concrete; σ_s = stress in reinforcement.

3) at the inner face of the link slab V_{ke} is shown as a ratio to the applied concentrated load P. Closed-form solutions are presented as a function of parameters α_s, α_{se}, α_k, x_f, ei_s, and ei_k defined earlier.

For the uniform load case, the ratio of the negative moment in the link slab M_{ke} to the positive moment from the simple span case M_s is given by Eq. (4) as follows:

$$\frac{M_{ke}}{M_s}=\frac{\left\{2\left(ei_k\left(\begin{array}{l}3\alpha_s^4-8\alpha_s^3+6\alpha_s^2-4\alpha_{se}^3+6\alpha_k\alpha_{se}^2\\-ei_s\left(\alpha_s-1\right)^3\left(3\alpha_s+1\right)\end{array}\right)+ei_s\left(\alpha_k^3-6\alpha_{se}^2\alpha_k+4\alpha_{se}^3\right)\right)\right\}}{\left\{\begin{array}{l}\left(2ei_k\left(ei_s\left(\alpha_s-1\right)^3-\alpha_s^3+3\alpha_s^2-3\alpha_s-3\alpha_{se}\right)\right.\\\left.-3ei_s\left(\alpha_k-2\alpha_{se}\right)\right)\end{array}\right\}} \tag{4}$$

where

$M_s = wL^2/8$

For the concentrated load case, the ratio of the negative moment in the link slab M_{ke} to the positive moment from the simple span case M_s is given by Eq. 5 as

$$\frac{M_{ke}}{M_s}=\frac{-\left\{\begin{array}{l}4ei_k\left[\begin{array}{l}2ei_k\left(\begin{array}{l}ei_s\alpha_k\left(\alpha_k-\alpha_{se}\right)\left(\alpha_s-1\right)^3-2\alpha_{se}^3\\-\alpha_k^2\left(\alpha_s^3-3\alpha_s^2+3\alpha_s+3\alpha_{se}\right)\\+\alpha_k\alpha_{se}\left(\alpha_s^3-3\alpha_s^2+3\alpha_s+6\alpha_{se}\right)\end{array}\right)\\-ei_s\left(\alpha_k-2\alpha_{se}\right)^2\left(2\alpha_k-\alpha_{se}\right)\end{array}\right]x_f\\\left[\left(2\alpha_s-3\right)\alpha_s^2+ei_s\left(-2\alpha_s^3+3\alpha_s^2+x_f^2-1\right)\right]\end{array}\right\}}{\left\{\begin{array}{l}3ei_s^2\left(\alpha_k-2\alpha_{se}\right)^4\\+4ei_kei_s\left[\begin{array}{l}-2ei_s\left(\alpha_k^2-\alpha_{se}\alpha_k+\alpha_{se}^2\right)\left(\alpha_s-1\right)^3\\+2\alpha_k^2\left(\alpha_s^3-3\alpha_s^2+3\alpha_s+3\alpha_{se}\right)\\+2\alpha_{se}^2\left(\alpha_s^3-3\alpha_s^2+3\alpha_s+6\alpha_{se}\right)\\-\alpha_{se}\alpha_k\left(2\alpha_s^3-6\alpha_s^2+6\alpha_s+15\alpha_{se}\right)\end{array}\right]\left(\alpha_k-2\alpha_{se}\right)\\+4ei_k^2\left[\begin{array}{l}ei_s^2\alpha_k^2\left(\alpha_s-1\right)^6\\-2ei_s\left(\begin{array}{l}2\alpha_{se}^3-3\alpha_k\alpha_{se}^2\\+\alpha_k^2\left(\alpha_s^3-3\alpha_s^2+3\alpha_s+3\alpha_{se}\right)\end{array}\right)\left(\alpha_s-1\right)^3\\+\left(\alpha_s^3-3\alpha_s^2+3\alpha_s+3\alpha_{se}\right)\\\times\left(4\alpha_{se}^3-6\alpha_k\alpha_{se}^2+\alpha_k^2\left(\alpha_s^3-3\alpha_s^2+3\alpha_s+3\alpha_{se}\right)\right)\end{array}\right]\end{array}\right\}} \tag{5}$$

where

$M_s = PX_F(1 - X_F/L)$

For the concentrated load case, the ratio of the maximum shear force in the link slab V_{ke} to the concentrated load P is given by Eq. (6) as

$$\frac{V_{ke}}{P}=\frac{\left\{ei_kx_fa_k\left[\begin{array}{l}\alpha_s^2\left(2\alpha_s-3\right)\\+ei_s\left(-2\alpha_s^3+3\alpha_s^2+x_f^2-1\right)\end{array}\right]\right\}}{\left\{\begin{array}{l}-ei_s\left(\alpha_k-2\alpha_{se}\right)^3\\+2ei_k\left[\begin{array}{l}ei_s\alpha_k^2\left(\alpha_s-1\right)^3+6\alpha_k\alpha_{se}^2\\-4\alpha_{se}^3-\alpha_k^2\left(3\alpha_s-3\alpha_s^2+\alpha_s^3+3\alpha_{se}\right)\end{array}\right]\end{array}\right\}} \tag{6}$$

Parametric study

Closed-form solutions allow calculating the maximum moment and shear force in the link slab. To make the equations practice oriented, ranges of parameters were identified for geometric and material properties of the girder, deck slab, and link slab in the medium-span range (12 m [40 ft] ≤ L ≤ 42 m [140 ft]) (Fig. 3).

Girder type and strength

Standard AASHTO girders[7,8] were used: Type II (12 m [40 ft] ≤ L ≤ 18 m [60 ft]), Type III (18 m ≤ L ≤ 24 m [80 ft]), Type IV (24 m ≤ L ≤ 30 m [100 ft]), Type V (30 m ≤ L ≤ 36 m [120 ft]), and Type VI (36 m ≤ L ≤ 42 m [140 ft]).

The concrete cylinder compressive strength for the girder f'_{cg} was initially set at 40 MPa (5800 psi). Higher strengths were then used (f'_{cg} = 50, 60, and 70 MPa [7250, 8700, and 10,150 psi]) to identify the impact of the girder strength on the link slab moments and shear forces.

Deck slab dimensions and strength

The deck slab thickness t_s is typically 200 mm (8 in.) for highway bridges for a maximum girder spacing S of 3 m (10 ft). The concrete cylinder strength of the slab f'_{cs} was set at 30 MPa (4350 psi). A concrete strength of 40 MPa (5800 psi) was also considered because some practices prefer to use it for the girder and deck slab.[9]

The flexural stiffness of the composite section (precast concrete girder and slab) was based on an effective slab width b equal to the girder spacing S multiplied by the modular ratio $n=\sqrt{f'_{cs}}/\sqrt{f'_{cg}}$ (governing case for the span-length range and girder spacing considered) (**Fig. 5**). In this paper, S varies from 1.75 to 3 m (5.75 to 10 ft) depending on the AASHTO girder type, length, loading, and service load stress limits.[7]

Link slab dimensions and strength

The key variable in the analysis was the link slab flexural stiffness equal to its modulus of elasticity E multiplied by its cracked moment of inertia I_k and divided by its effective length $L_k - 2L_{se}$. E is calculated as a function of the concrete cylinder strength f'_{ck} and I_k is determined from Eq. (1) as a

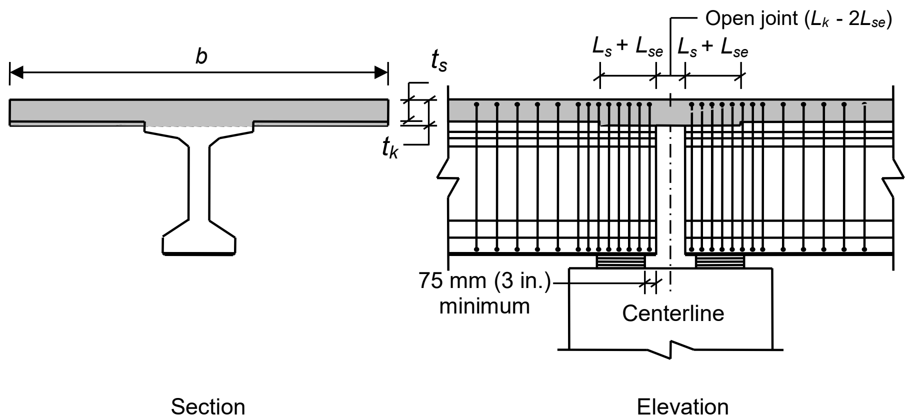

Figure 5. Elastomeric bearing pad and girder end details near link slab. The figure shows that the link slab can have a larger thickness than the deck slab, but this case was not considered in this paper. Note: b = effective link slab width = girder spacing S; L_k = span length between intermediate supports; L_s = distance from centerline of intermediate support to edge of link slab in the main span; L_{se} = distance from centerline of intermediate support to edge of girder; t_s = deck slab thickness; t_k = link slab thickness.

function of the reinforcing steel ratio ρ, modular ratio n, and distance from extreme compression fiber to center of reinforcing steel d (taken as 0.7 times the link slab thickness t_k as shown in the derivations of the link slab cracked section properties). The main goal of the parametric study was to investigate the impact of these parameters on the redistribution of moments.

- Link slab thickness t_k: The link slab thickness t_k is typically set at 200 mm (8 in.), similar to the deck slab thickness t_s (Fig. 5). Larger thickness t_k tends to increase its stiffness and, thereby, cause more redistribution of forces. It drastically increases the shear forces for the asymmetrical concentrated load case P (Fig. 3). Consequently, in this study t_k is set equal to 200 mm for the link slab and for the deck slab portion over length L_s (Fig. 5).

- Link slab effective length $L_k - 2L_{se}$: The link slab effective length (Fig. 5) relates to the open joint between girders. In practice, it varies from 200 to 400 mm (8 to 16 in.). The distance from the centerline of support to the girder edge L_{se} also varies from 200 to 400 mm, depending on the bearing width (Fig. 5).[10] As a result, the distance L_k between supports ranges from 600 to 1200 mm (24 to 48 in.). In this study, two values for L_k were considered: L_k = 600 mm (open joint of 200 mm) and L_k = 1200 mm (open joint of 400 mm) to examine its impact on induced link slab moments and shear forces for the asymmetrical loads.

- Link slab concrete cylinder strength f'_{ck} : f'_{ck} was set at 30 and 40 MPa (4350 and 5800 psi) as for the deck slab cylinder strength f'_{cs}.

- Link slab tension reinforcement A_s: The stiffness of the link slab is determined based on the cracked section using Eq. (1). Different ratios of tension reinforcement $\rho = A_s/bd$ were considered ($\rho = 0.375\rho_b$, $0.5\rho_b$, and $0.75\rho_b$, where ρ_b is determined from Eq. (3) as the ratio of reinforcement for balanced strain conditions) to examine its effects on induced moments and shear forces.

Plots

Closed-form solutions (Eq. [4] and [5]) were used to construct plots for moments for the range of parameters specified. Plots were generated as a function of the girder type (Type II to VI girders) for the cases of uniform load w applied over three spans and concentrated load P applied at X_F (Fig. 3). In the plots, X_F was set at $L/2$ because it produced the largest link slab moments.

Note that AASHTO requires placing double trucks per lane spaced 15 m (50 ft) apart[5] for negative moment calculations but did not govern for link slabs of short lengths. Therefore, only one truck load per lane was considered. It is shown later that if the truck load is placed at the middle of one span (span 1 or 3) (Fig. 3), induced bending moments and shear forces are maximized in the link slab.

Plots are provided for the link slab thickness t_k = 200 mm (8 in.); concrete cylinder strength f'_{ck} = 30 MPa (4350 psi); $\rho = 0.375\rho_b$, $0.5\rho_b$, and $0.75\rho_b$; and open joint dimension $L_k - 2L_{se}$ = 200 and 400 mm (16 in.). These are presented in **Fig. 6** for $L_k - 2L_{se}$ = 200 mm and **Fig. 7** for $L_k - 2L_{se}$ = 400 mm for the concrete girder cylinder strength f'_{cg} = 40 MPa (5800 psi). Each figure contains two plots: one for uniform load w

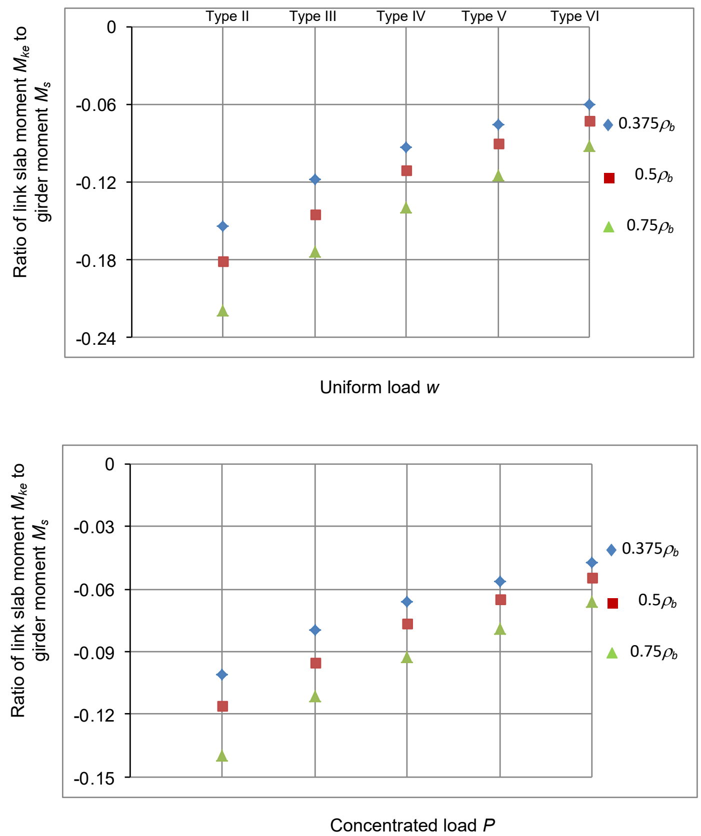

Figure 6. Charts from which the ratio of link slab moment M_{ke} to simply supported girder moment M_s can be calculated. The abscissa is the girder type, while the ordinate is the ratio. These are provided for a girder cylinder strength f'_{cg} = 40 MPa (5800 psi), link slab cylinder strength f'_{ck} = 30 MPa (4350 psi), and open joint dimension $L_k - 2L_{se}$ = 200 mm (8 in.). Girder moments were calculated by analyzing the beam as simply supported. Note: L_k = span length between intermediate supports; L_{se} = distance from centerline of intermediate support to edge of girder; ρ_b = balance steel reinforcement ratio.

(Fig. 3) and the other for concentrated load P applied at X_F = 0.5L (Fig. 3) for the ratio of tension reinforcement provided. The ordinates of the plots represent the ratio of the maximum negative moment at the edge of the link slab M_{ke} to midspan positive moment from the simple-span case M_s (Fig. 3).

The effects of increasing the girder cylinder strength f'_{cg} to 50, 60, and 70 MPa (7250, 8700, and 10,150 psi) were considered. Based on the higher cylinder strength, the girder flexural rigidity will increase and consequently $ei_k = EI_k/EI$ (ratio of the flexural rigidity of the link slab to the flexural rigidity of the girder and deck slab) will decrease. It is shown that M_{ke}/M_s (calculated using Eq. [4] and Eq. [5] as a function of parameter ei_k) reduces if f'_{cg} increases and that the percentage reductions are almost equal for each girder cylinder strength f'_{cg} irrespective of the girder type and length. Ratios of Mk_e/M_s for f'_{cg} = 50, 60, and 70 MPa (7250, 8700, 10,150 psi) to M_{ke}/M_s for f'_{cg} = 40 MPa (5800 psi) are presented in **Table**

Table 2. Ratios of link slab moment to girder moment M_{ke}/M_s for increased concrete strength relative to M_{ke}/M_s for link slab cylinder strength f'_{ck} = 30 MPa and girder cylinder strength f'_{cg} = 40 MPa

Concrete cylinder strength f'_c, MPa	Ratio for uniform load w	Ratio for concentrated load P
f'_{ck} = 40	1.1	1.075
f'_{cg} = 50	0.94	0.94
f'_{cg} = 60	0.88	0.9
f'_{cg} = 70	0.83	0.86

Note: M_{ke} = service load moment at edge of link slab; M_s = precast concrete girder moment based on simply supported beam analysis. 1 MPa = 0.145 ksi.

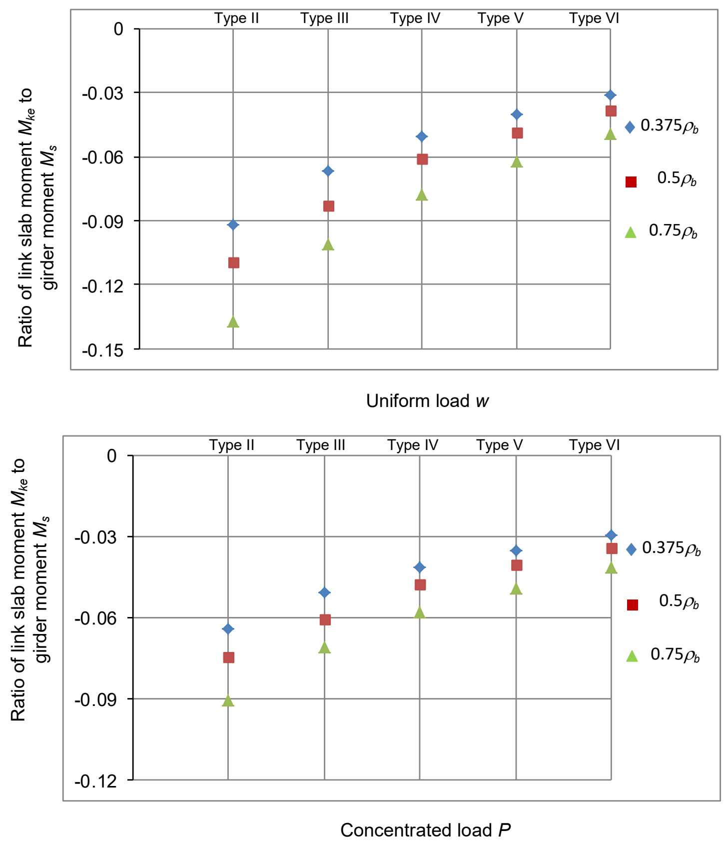

Figure 7. Charts from which the ratio of link slab moment M_{ke} to simply supported girder moment M_s can be calculated. The abscissa is the girder type, while the ordinate is the ratio. These are provided for a girder cylinder strength f'_{cg} = 40 MPa (5800 psi), link slab cylinder strength f'_{ck} = 30 MPa (4350 psi), and open joint dimension $L_k - 2L_{se}$ = 400 mm (16 in.). Girder moments were calculated by analyzing the beam as simply supported. Note: L_k = span length between intermediate supports; L_{se} = distance from centerline of intermediate support to edge of girder; ρ_b = balance steel reinforcement ratio.

2. Consequently, M_{ke}/M_s for specific girder cylinder strength f'_{cg} can be calculated by multiplying values depicted from the plots in Fig. 6 and 7 for f'_{cg} = 40 MPa (5800 psi) by the ratios in Table 2.

Similarly, the effects of increasing the link slab cylinder strength f'_{ck} from 30 to 40 MPa (4350 to 5800 psi) were considered. Ratios of M_{ke}/M_s for f'_{ck} = 40 MPa (5800 psi) relative to M_{ke}/M_s for f'_{ck} = 30 MPa (4350 psi) and f'_{cg} = 40 MPa (5800 psi) are also shown in Table 2.

Interpretation of the plots

From the plots, the effects of varying the link slab stiffness parameters on redistribution of moments in the link slab M_{ke} are presented as follows:

- From Fig. 6, the ratio M_{ke}/M_s varies from 0.22 (uniform load w) and 0.14 (concentrated load P) for the Type II girder to 0.09 (uniform load w) and 0.07 (concentrated load P) for the Type VI girder. These ratios correspond to f'_{cg} = 40 MPa (5800 psi), f'_{ck} = 30 MPa (4350 psi), $\rho = 0.75\rho_b$, and $L_k - 2L_{se}$ = 200 mm (8 in.). These ratios decrease for $\rho = 0.5\rho_b$ and $\rho = 0.375\rho_b$.
- The ratio M_{ke}/M_s reduces to 0.13 (load w) and 0.09 (load P) for the Type II girder and to 0.05 (load w) and 0.04 (load P) for the Type VI girder if the open joint dimension ($L_k - 2L_{se}$) increases to 400 mm (16 in.) (Fig. 7).
- The ratio M_{ke}/M_s increases to 0.24 (0.22 × 1.1) and 0.15 (0.14 × 1.075) for the Type II girder and 0.1 (0.09 × 1.1) and 0.075 (0.07 × 1.075) for the Type VI girder if f'_{ck} is increased to 40 MPa (5800 psi) (ratios obtained from Table 2).
- The ratio M_{ke}/M_s increases by about 20% if the steel ratio ρ is increased from $0.375\rho_b$ to $0.5\rho_b$ and by about 50% if the steel ratio ρ is increased to $0.75\rho_b$ (Fig. 6 and 7).
- The ratio M_{ke}/M_s decreases by about 6% if the cylinder strength of the girder f'_{cg} is increased from 40 to 50 MPa (5800 to 7250 psi) for the steel ratios considered (Table 2). For f'_{cg} = 60 MPa (8700 psi), M_{ke}/M_s decreases by about 10% and at 70 MPa (10,150 psi) it reduces by about 17%.

Note that positive moments in the girder (M_p in Fig. 3) also reduce by about 10% for the Type II girder and less than 5% for the Type VI girder. Because the effect is minimal, it is not accounted for in this study.

Shear force analysis and effects of elastomeric bearing pad stiffness

Shear forces in the link slab are of negligible magnitude when the loads are symmetrically applied; however, for the asymmetrical concentrated load P and uniform load w applied to one span (Fig. 3), they are considerably large. For the range of parameters considered, shear due to point load P applied at $X_F = L/2$ (Fig. 3) varies from $2P$ (for the Type II girder) to $5P$ (for the Type VI girder) for pin supports (Fig. 3) (calculated using Eq. [6]).

In practice, precast concrete girders in the medium-span-length range are supported by composite elastomeric bearing pads that have a vertical stiffness K_v, which if accounted for, the shear force in the link slab reduces considerably, making it possible for the bonded link slab to provide adequate resistance for the induced shear forces.

The vertical stiffness of the bearing pad K_v is determined by Eq. (7).

$$K_v = \frac{E_c A}{h} \tag{7}$$

where

E_c = modulus of elasticity of elastomeric bearing pad determined from Eq. (8) for SI units[5]

A = area of elastomeric bearing pad (equal to its length L' multiplied by its width W)

h = total thickness of elastomer in the bearing pad

$$E_c = 4.8GS_f^2 \tag{8}$$

where

G = shear modulus of elastomeric bearing pad (equal to 0.9 MPa [130 psi] for a nominal hardness of 50 on the Shore A scale[5]

S_f = the shape factor given by Eq. (9)[5]

$$S_f = \frac{L' \times W}{2h_{ri}\left(L' \times W\right)} \tag{9}$$

where

h_{ri} = thickness of the elastomer internal layer (equal to 12.7 mm (0.5 in.) in this paper)

Standard expansion bearing pad size for Type II to VI girders in the medium-span range are shown in **Table 3**.[10] Three internal layers of elastomer of thickness h_{ri} = 12.7 mm (0.5 in.) each and two cover layers (6.35 mm [0.25 in.] thick each) are used, which results in a total elastomer thickness $h = 3 \times 12.7 + 2 \times 6.35$ = 50.8 mm (2 in.). The vertical stiffness K_v is consequently calculated for each girder type using Eq. (7) through (9), and it varies from 575,000 kN/m (3300 kip/in.) for Type II to 1,215,000 kN/m (6935 kip/in.) for Type VI girders (Table 3).

The maximum shear force in the link slab develops due to truck load (multiplied by 1.33 for impact) and lane load both applied to one span (Fig. 3). Values are obtained from computer analysis using elastic supports with a vertical stiffness K_v (instead of pin supports) for the different girder types, lengths, and spacing shown in Table 3. As shear force increases with span length, the upper bound of girder length L and corresponding girder spacing S are considered for each

Table 3. Effect of elastomeric bearing pad stiffness on link slab shear force for open joint dimension $L_k - 2L_{se}$ = 400 mm

Girder	Span length L, m	Bearing pad area $L' \times$ bearing pad width W, mm	Shape factor S_f	Maximum steel ratio ρ	Vertical stiffness E_cA/h, kN/m	Girder spacing S, m	Shear distribution factor DF	Link slab shear capacity ΦV_c, kN	Factored shear V_u, kN
Type II	18	300 × 350	7.5	$0.75\rho_b$	575,000	1.75	0.66	200	110
Type III	24	350 × 450	7.9	$0.75\rho_b$	725,000	1.9	0.7	220	160
Type IV	30	300 × 550	7.8	$0.75\rho_b$	735,000	2.1	0.74	240	145
Type V	36	350 × 600	8.8	$0.55\rho_b$	1,215,000	2.15	0.76	245	230
Type VI	42	350 × 600	8.8	$0.4\rho_b$	1,215,000	1.85	0.68	215	205

Note: A = area of elastomeric bearing pad (length L' × width W); E_c = modulus of elasticity of elastomeric bearing pad; h = total thickness of elastomer; L_k = span length between intermediate supports; L_{se} = distance from centerline of intermediate support to edge of girder; V_c = nominal shear capacity of link slab; ρ_b = balanced steel reinforcement ratio; Φ = resistance factor (load factored design). 1 mm = 0.0394 in.; 1 m = 3.281 ft; 1 kN = 0.225 kip.

girder type. Girder spacing S is determined for normal design conditions (HL-93 live load and service load tension stress $\sigma_{eT} = 0.5\sqrt{f'_{cg}}$ in MPa [$6\sqrt{f'_{cg}}$ in psi]).[7] Calculated values are presented in Table 3.

Based on the girder spacing S, shear distribution factor DF is consequently calculated using Eq. (10) for SI units.[5]

$$DF = 0.2 + \frac{S}{3600} - \left(\frac{S}{10{,}700}\right)^2 \tag{10}$$

Note that the shear distribution factor is determined for the interior beam only, which is considered to be the governing case for the range of parameters specified.

Factored shear forces[5] V_u are presented in Table 3 for an open joint dimension $L_k - 2L_{se}$ = 400 mm (16 in.). Contrary to bending moments, shear forces reduce for $L_k - 2L_{se}$ = 200 mm (8 in.) if the vertical stiffness of the bearing pad is accounted for, and therefore maximum values that correspond to $L_k - 2L_{se}$ = 400 mm are used in the calculations.

The factored shear force for each girder type is verified against the link slab shear capacity ΦV_c determined from Eq. (11), for SI units, based on a simplified procedure.[5]

$$\Phi V_c = \Phi\left(0.083\beta_e\sqrt{f'_{ck}}b_w d\right) \tag{11}$$

where

Φ = resistance factor (equal to 0.9 for shear)[5]

β_e = shear factor (equal to 2 for the simplified procedure)

b_w = link slab width = girder spacing S

Values for shear capacity are calculated for f'_{ck} = 30 MPa (4350 psi), $d = 0.7t_k = 0.7 \times 200$ = 140 mm (5.5 in.) (noted earlier in link slab cracked section properties) and presented in Table 3 for Type II to VI girders for the upper limit of the span length and corresponding girder spacing. Note that increasing f'_{ck} from 30 to 40 MPa (4350 to 5800 psi) will increase the shear force V_u and the shear capacity ΦV_c as well. Therefore, its effect is not incorporated in the shear analysis.

Interpretation of results

Table 3 shows that the maximum factored shear force that develops in the link slab V_u due to truck and lane load applied to one span only is smaller than the link slab shear capacity ΦV_c. V_u determined using the bearing pad vertical stiffness K_v is also a function of the flexural rigidity of the link slab EI_k calculated based on the cracked section properties and steel reinforcement ratio ρ. While values of V_u in Table 3 are based on the ratio of tension reinforcement taken at its maximum value ($\rho = 0.75\rho_b$) for Type II to IV girders, it is recommended to limit the steel ratio ρ to $0.5\rho_b$ to $0.55\rho_b$ for Type V and $0.375\rho_b$ to $0.4\rho_b$ for Type VI girders so that link slab shear capacity ΦV_c is always larger than factored shear force V_u (Table 3). Alternatively, since shear force V_u reduces for $L_k - 2L_{se}$ = 200 mm (8 in.) when the bearing pad stiffness is accounted for, $\rho = 0.75\rho_b$ can be used for Type V and VI girders in this case.

It shall be noted that equations for maximum shear force V_u considering the vertical bearing pad stiffness were derived and these equations are currently being validated with test data and will be presented in a future publication.

Design procedure

The design of the bonded link slab can be optimized by selecting appropriate parameters so that it can be adequately reinforced without exhausting it in shear. A step-by-step procedure for designing bonded link slabs is presented:

1. Based on the span length, girder type, and specified strengths f'_{cg} for the girder and f'_{ck} for the link slab, use the appropriate plot (Fig. 6 for $L_k - 2L_{se}$ = 200 mm [8 in.] and Fig. 7 for $L_k - 2L_{se}$ = 400 mm [16 in.]) to determine the ratio of the negative moment induced in the link slab to the simple-span positive moment M_{ke}/M_s for the concentrated load P and uniform load w. Initially, ρ can be set equal to $0.375\rho_b$ to $0.4\rho_b$.

2. Calculate the link slab negative moment M_{ke} due to live load (truck and lane load) using the ratio for concentrated load P and due to superimposed dead load using the ratio for uniform load w. Calculate the service and factored load moments M_{ke} and M_u.

3. Based on the steel ratio, check that the ultimate moment capacity of the link slab is larger than the factored load moment ($\Phi M_n > M_u$). If not, increase the steel ratio or change the construction sequence to exclude superimposed dead loads by achieving continuity in the deck slab after they are placed.[1]

   Check reinforcement limits to confirm that $\frac{c}{d} = \frac{\frac{a}{\beta_1}}{d} < 0.6$.

4. Check the service limit state based on the service load moment M_{ke}. Calculate the depth of the compression zone y using Eq. (2) and the tension force in the reinforcement $T_s = \frac{M_{ke}}{\left(d - \frac{y}{3}\right)}$. Calculate the corresponding stress in the reinforcing steel $f_s = T_s/A_s$ and compare with the limiting value of $0.6f_y$.

5. Check the spacing of reinforcement s in the tension side of the slab for crack control (AASHTO LRFD section 5.7.3.4) for SI units.[5]

   $$S \le \frac{122{,}580\gamma_e}{\beta_s f_s} - 2d_c$$

   where

γ_e = factor based on exposure of steel = 0.75 for class 2 exposure (top reinforcement)

β_s = ratio of flexural strain at the extreme tension face to the strain at the centroid of the reinforcement layer nearest the tension face $= 1 + \dfrac{d_c}{0.7\left(t_k - d_c\right)}$

d_c = distance from extreme tension fiber to center of flexural reinforcement (in mm) (Fig. 4)[5]

Check that the crack width is smaller than 0.33 mm (0.013 in.) for class 2 exposure ($\gamma_e = 0.75$) using the Gergely-Lutz expression[2] $\omega = 0.011\beta f_s(d_c A_e)^{1/3}$, where ω is the surface crack in units of 0.001 mm (0.00004 in.), β is the ratio of distances to the neutral axis from the extreme tension fiber and from the centroid of reinforcement $= \dfrac{t_k - y}{d - y}$, and A_e is the effective area per reinforcing bar (equal to $2Sd_c$/number of reinforcing bars) in mm^2.

Numerical example

A bridge structure consists of AASHTO Type VI precast concrete girders with a length equal to 39.6 m (130 ft) and design span length L = 38.8 m (128 ft). The overall width is 28.9 m (95 ft), accommodating three 3.65 m (12 ft) lanes, a 2 m (6.6 ft) walkway, and a 0.5 m (1.6 ft) edge barrier in each direction separated by a 2 m median. The open joint dimension $L_k - 2L_{se}$ is 400 mm (16 in.).

Superimposed dead load includes a 100 mm (4 in.) thick asphalt layer (2.3 kN/m^2 [48 lb/ft^2]), 150 mm (6 in.) thick concrete walkway (3.75 kN/m^2 [78 lb/ft^2]), a median barrier (10 kN/m [680 lb/ft]), and two edge barriers (7.5 kN/m [510 lb/ft] each). Construction is based on an unshored system, so the deck slab is not part of the composite dead load.

Simple-span girder analysis

For a service load tension stress $\sigma_{eT} = 0.5\sqrt{f'_{cg}}$ in MPa ($6\sqrt{f'_{cg}}$ in psi) and f'_{cg} = 40 MPa (5800 psi), 13 Type VI girders spaced at 2.3 m (7.5 ft) prestressed with forty-two 15.2 mm (0.6 in.) diameter strands each were used. The deck slab and link slab concrete cylinder strength is 30 MPa (4350 psi).

Composite dead load and live load moments based on simple beam analysis are as follows.

Composite dead load (DC and DW)

DC

Barriers: (2 × 7.5 kN/m [edge] + 10 kN/m [median])/13 girders = 1.9 kN/m (130 lb/ft)

Walkways: (3.75 kN/m^2 × 2 m) × 2 sides/13 girders = 1.2 kN/m (80 lb/ft)

The slab is considered rigid enough to distribute the barrier and walkway load equally to the girders.[5]

Moment for component and attachments dead load M_{DC} = (1.9 + 1.2) × (38.8)^2/8 = 579 kN-m (427 kip-ft)

DW

Asphalt: (2.3 m × 2.3 kN/m^2) = 5.3 kN/m (360 lb/ft)

Moment for wearing surface dead load M_{DW} = (5.3)(38.8)^2/8 = 552 kN-m (407 kip-ft)

DC + DW

Service load: M_{cdl} = 579 + 552 = 1131 kN-m (834 kip-ft)

Factored load: M_{cdl} = (1.25 × 579) + (1.5 × 552) = 1552 kN-m (1145 kip-ft)

Live load

For a girder spacing of 2.3 m (7.5 ft), the live load distribution factor for an interior girder is 0.6. M_{LL+I} (truck load only) = 2200 kN-m (1620 kip-ft) and M_{LL} (lane load only) = 1050 kN-m (775 kip-ft).

Total live load moment M_{LL+I} = 3250 kN-m (2400 kip-ft)

Link slab design using a girder cylinder strength of 40 MPa

Step 1

Select the steel ratio ρ and determine the link slab moment factors from the plots.

For a Type VI girder, select $\rho = 0.375\rho_b$ (Table 1). Design parameters are f'_{cg} = 40 MPa (5800 psi), f'_{ck} = 30 MPa (4350 psi), $L_k - 2L_{se}$ = 400 mm (16 in.), and M_{ke}/M_s = 0.031 for the uniform load w (Fig. 7) and 0.03 for the point load P (Fig. 7). Note that the uniform load comprises the composite dead load and the lane live load.

Step 2

Calculate the maximum service and factored negative moment in the link slab.

The link slab negative moment M_{ke} = 0.031 × (1131 + 1050) + 0.03 × (2200) = 134 kN-m (99 kip-ft) (service) and M_u = 1.75 × (0.03 × 2200 + 0.031 × 1050) + 0.031 × 1552 = 221 kN-m (163 kip-ft) (factored).

Step 3

Check whether the steel ratio selected is adequate based on factored loads.

Using Eq. (3),

$$\rho_b = \left(\frac{0.85\beta_1 f'_{ck}}{f_y}\right)\left(\frac{580}{580+f_y}\right) = \left(\frac{0.85\times0.84\times30}{415}\right)\left(\frac{580}{580+415}\right)$$

$= 0.03$

$\rho = 0.375\rho_b = 0.375 \times 0.03 = 0.0112$

$A_s = \rho bd = 0.0112 \times 2.3 \times 0.14 = 0.003614$ m^2 $= 3614$ mm^2 (5.6 in.^{2})

For 20 mm (0.8 in.) diameter steel (T20), the number of bars = 3614/314 = 12. Therefore $A_s = 12 \times 314 = 3770$ mm^2 (5.8 in.^{2}). Spacing $s = 2300/(12 - 1) = 209$ mm (8.2 in.). Therefore, use 200 mm (8 in.) spacing.

Checking for equilibrium:

$0.85 f'_{ck}\, ab = A_s f_y$

Solving for a:

$a = (3770 \times 10^{-6} \times 415)/(0.85 \times 30 \times 2.3) = 0.0267$ m $= 26.7$ mm (1.05 in.)

$\Phi M_n = A_s f_y (d - a/2) = (0.9 \times 3770 \times 10^{-6} \times 415 \times 10^3)/(0.14 - 0.0267/2) = 178$ kN-m (131 kip-ft)

$M_u = 221$ kN-m (163 kip-ft)

Therefore $\Phi M_n < M_u$, which is not adequate.

As discussed before, induced moments in the link slab can be reduced by changing the construction sequence[1] to exclude superimposed dead loads or by increasing the girder cylinder strength from 40 to 70 MPa (5800 to 10,150 psi).

Note that the steel ratio ρ for the Type VI girder is limited to $0.4\rho_b$ to avoid overstressing the link slab in shear. Both alternatives are presented in the following sections.

Alternative 1: Repeat steps 2 and 3 without considering composite dead load

The link slab negative moment is recalculated to exclude composite dead load.

$M_{ke} = 0.03 \times 2200 + 0.031 \times 1050 = 99$ kN-m (73 kip-ft) (service)

$M_u = 1.75 \times 99 = 173$ kN-m (127 kip-ft) (factored)

$M_u < \Phi M_n = 178$ kN-m (131 kip-ft) from the previous section

Step 4

Check link slab for service load.

$E_{ck} = 4800\sqrt{30} = 26{,}290$ MPa (3800 ksi)

$n = E_s/E_{ck} = 200{,}000/26{,}290 = 7.6$

From Eq. (2),

$$y = -n\rho d + d\sqrt{\left((n\rho)^2 + 2\rho n\right)} = -7.6\times0.0112\times0.14 + 0.14\sqrt{\left(7.6\times0.0112\right)^2 + 2\left(7.6\times0.0112\right)}$$

$= 0.047$ m $= 47$ mm (1.85 in.)

Reinforcing bar tension $T_s = \dfrac{M_{ke}}{\left(d-\dfrac{y}{3}\right)} = \dfrac{99}{\left(0.14-\dfrac{0.047}{3}\right)}$

$= 793$ kN (178 kip)

Reinforcing bar stress $f_s = T_s/A_s = 793 / (3770 \times 10^{-6}) \times 10^{-3} = 208$ MPa (30 ksi)

$f_s < 0.6f_y = 0.6 \times 415 = 249$ MPa (36 ksi)

Step 5

Check spacing of reinforcing bar s in tension side and surface crack.

$$s = \frac{122{,}580\gamma_e}{\beta_s f_s} - 2d_c$$

where

Concrete cover from tension fiber to center of reinforcing bar $d_c = 50 + 20/2 = 60$ mm (2.4 in.)

$$\beta_s = 1 + \frac{d_c}{0.7\left(t_k - d_c\right)} = 1 + \frac{60}{0.7\left(200-60\right)} = 1.61$$

$\gamma_e = 0.75$ for class 2 exposure (top steel)

Reinforcing bar tension stress $f_s = 208$ MPa (30 ksi)

Therefore, $s \le \dfrac{122{,}580\times0.75}{1.61\times208} - 2\times60 = 155$ mm (6 in.).

For 12 T20 (0.8 in.) at 200 mm (8 in.), the steel bar spacing limitation is not satisfied and therefore 19 T16 (0.6 in.) at 125 mm (5 in.) are used ($A_s = 3820$ mm^2 > 3614 mm^2 from step 3). Note that $d = 140$ mm (5.5 in.) is still conservatively used, though the bar size is reduced from T20 to T16.

The surface crack in units of 0.001 mm (0.00004 in.) is

$\omega = 0.011\beta f_s (d_c A)^{1/3}$, where $\beta = \dfrac{t_k - y}{d - y} = \dfrac{\left(200-47\right)}{\left(140-47\right)} = 1.33$,

$A = 2Sd_c$/number of bars $= 2 \times 2300 \times 60/19 = 14{,}526$ mm^2 (22.5 in.^{2}), and $f_s = 208$ MPa (3020 psi). Therefore $\omega = [0.011 \times$

$1.33 \times 208 \times (60 \times 14{,}526)^{1/3}] \times 0.001 = 0.29$ mm (0.011 in.) < 0.33 mm (0.013 in.) for class 2 exposure.

Alternative 2: Increase girder cylinder strength to 70 MPa

If the girder cylinder strength is increased from 40 to 70 MPa (5800 to 10,150 psi), the design of the link slab (steps 1 through 5) is repeated as shown. Although the higher concrete strength allows reducing the number of girders, 13 Type VI girders spaced at 2.3 m [7.5 ft] are maintained in this example.

Step 1

Select the steel ratio ρ and determine the link slab moment factors from the plots.

$\rho = 0.375\rho_b$ (Table 1)

For f'_{cg} = 70 MPa (10,150 psi), f'_{ck} = 30 MPa (4350 psi), $L_k - 2L_{se}$ = 400 mm (16 in.), $M_{ke}/M_s = 0.031 \times 0.83 = 0.0257$ for the uniform load w and $0.03 \times 0.86 = 0.0258$ for the point load P (Fig. 7). Note that the ratios for f'_{cg} = 70 MPa are obtained from Table 2.

Step 2

Calculate the maximum service and factored negative moment in the link slab.

The link slab negative moment $M_{ke} = 0.0257 \times (1131 + 1050) + 0.0258 \times (2220) = 113$ kN-m (83 kip-ft) (service) and $M_u = 1.75 \times (0.0258 \times 2200 + 0.0257 \times 1050) + 0.0257 \times 1552 = 187$ kN-m (138 kip-ft) (factored).

Step 3

Check whether the steel ratio selected is adequate based on factored loads.

Try 19 T16 (0.6 in.) at 125 mm (5 in.) (from the previous case), for which A_s = 3820 mm² (5.9 in.²) and d = 142 mm (5.6 in.).

$0.85 f'_{ck}\, ab = A_s f_y$

Solving for a,

$a = 3820 \times 10^{-6} \times 415/(0.85 \times 30 \times 2.3)$
$= 0.027$ m = 27 mm (1.06 in.)

$\Phi M_n = \Phi A_s f_y (d - a/2) = 0.9 \times 3820 \times 10^{-6} \times 415 \times 103/(0.142 - 0.027/2) = 184$ kN-m (135 kip-ft)

$\Phi M_n < M_u$ = 187 kN-m (138 kip-ft). As it is slightly smaller, increase the number of bars to 20 T16 spaced at 120 mm (4.8 in.)

For A_s = 4020 mm² (6.2 in.²), ΦM_n = 192 kN-m (142 kip-ft) > M_u = 187 kN-m (138 kip-ft). Check the steel ratio $\rho = 4020/(2300 \times 142) = 0.0123$, which is approximately equal to $0.4\rho_b$ (limiting value for Type VI girder from Table 3).

Step 4

Check link slab for service load.

$E_{ck} = 4800\sqrt{30} = 26{,}290$ MPa (3800 psi)

$n = (E_s/E_{ck}) = (200{,}000/26{,}290) = 7.6$

From Eq. (2), $y = -n\rho d + d\sqrt{\left(\left(n\rho\right)^2 + 2\rho n\right)} = -7.6 \times 0.0123 \times 0.142 + 0.142\sqrt{\left(7.6 \times 0.0123\right)^2 + 2\left(7.6 \times 0.0123\right)}$

$= 0.05$ m = 50 mm (2 in.)

Reinforcing bar tension $T_s = \dfrac{M_{ke}}{\left(d - \dfrac{y}{3}\right)} = \dfrac{113}{\left(0.142 - \dfrac{0.05}{3}\right)}$

= 900 kN (202 kip)

Reinforcing bar stress $f_s = T_s/A_s = 900/(4020 \times 10^{-6}) \times 10^{-3}$ = 224 MPa (33 ksi)

$f_s < 0.6 f_y = 0.6 \times 415 = 249$ MPa (36 ksi)

Step 5

Check the spacing of reinforcing bar s in the tension side and surface crack.

$$s \le \frac{122{,}580\gamma_e}{\beta_s f_s} - 2d_c$$

where

$d_c = 50 + 20/2 = 60$ mm (2.4 in.)

$$\beta_s = 1 + \frac{d_c}{0.7\left(t_k - d_c\right)} = 1 + \frac{60}{0.7\left(200 - 60\right)} = 1.61$$

$\gamma_e = 0.75$ for class 2 exposure (top steel)

f_s = 224 MPa (33 ksi)

Therefore, $s \le \dfrac{122{,}580 \times 0.75}{1.61 \times 224} - 2 \times 60 = 135$ mm (5.3 in.) > 120 mm (4.8 in.).

The surface crack in units of 0.001 mm (0.00004 in.) is

$\omega = 0.011\beta f_s (d_c A)^{1/3}$, where $\beta = \dfrac{t_k - y}{d - y} = \dfrac{\left(200 - 50\right)}{\left(142 - 50\right)} = 1.33$,
$A = 2Sd_c$/number of bars $= 2 \times 2300 \times 60/20 = 13{,}800$ mm² (21.4 in.²), and f_s = 224 MPa (33 ksi). Therefore $\omega = (0.011 \times$

$1.33 \times 224 \times (60 \times 13{,}800)^{1/3}) \times 0.001 = 0.31$ mm (0.012 in.) < 0.33 mm (0.013 in.) for class 2 exposure.

Discussion

The numerical application illustrates applicability of the proposed procedure and charts for the design of link slabs. For the Type VI girders used, it is advised to limit the steel ratio to $0.375\rho_b$ to $0.4\rho_b$. Two alternatives were presented:

- Use 19 T16 (0.6 in.) bars spaced at 125 mm (5 in.) for a link slab cylinder strength of 30 MPa (4350 psi) and girder cylinder strength of 40 MPa (5800 psi).

- Use 20 T16 (0.6 in.) bars spaced at 120 mm (4.8 in.) and increase the girder cylinder strength to 70 MPa (10,150 psi).

In alternative 1, the construction sequence is adjusted to exclude composite dead load, while in alternative 2 composite dead loads were maintained because the higher girder cylinder strength reduces the link slab moments. The steel ratios in options 1 and 2 are $0.375\rho_b$ and $0.4\rho_b$, respectively, both within limits.

Conclusion

This paper provides a detailed analysis of bonded link slabs in precast, prestressed concrete girder applications in the medium-span-length range (12 m [40 ft] $\le L \le$ 42 m [140 ft]). AASHTO Type II to VI girders were used[8] with concrete cylinder strengths varying from 40 to 70 MPa (5800 to 10,150 psi). Closed-form solutions representative of the link slab's negative moments due to composite dead load and live load were derived. These are presented as a function of the link slab parameters (thickness t_k, concrete cylinder strength f'_{ck}, steel ratio ρ, and open joint dimension $L_k - 2L_{se}$). A parametric study was then conducted and plots were consequently developed for a practical range of parameters: t_k = 200 mm (8 in.), $0.375\rho_b \le \rho \le 0.75\rho_b$, f'_{ck} = 30 and 40 MPa (4350 and 5800 psi), and $L_k - 2L_{se}$ = 200 and 400 mm (16 in.).

From the analysis, the following was concluded:

- The ratio of the link slab moment to the precast concrete girder simple span moment varies from 0.22 for a uniform load w and 0.14 for a point load P for Type II girder to 0.09 (uniform load w) and 0.07 (point load P) for Type VI girder. These correspond to a girder cylinder strength f'_{cg} = 40 MPa (5800 psi), link slab cylinder strength f'_{ck} = 30 MPa (4350 psi), steel ratio $\rho = 0.75\rho_b$, and open joint $L_k - 2L_{se}$ = 200 mm (8 in.).

- These ratios will reduce if the open joint dimension is increased ($L_k - 2L_{se}$ = 400 mm [16 in.]), if the steel ratio is reduced ($\rho = 0.5\rho_b$ and $\rho = 0.375\rho_b$), and if the girder cylinder strength is increased (f'_{cg} = 50, 60, and 70 MPa [7250, 8700, and 10,150 psi]).

- While a reinforcement steel ratio $\rho = 0.75\rho_b$ can be used for Type II to IV girders, it is recommended to limit the steel ratio to $0.55\rho_b$ for Type V girders and $0.4\rho_b$ for Type VI girders to avoid overstressing the link slab in shear.

A step-by-step procedure is presented that allows selecting the appropriate design parameters for the link slab and its application is illustrated by a numerical example. Reinforcement provided in bonded link slabs satisfied service and ultimate load requirements as well as the crack-control criteria. Shear forces were within the shear capacity of the concrete slab as the vertical stiffness of the composite elastomeric bearing pads was accounted for (Table 3).

This paper paves the way for more rigorous numerical and experimental investigations of the structural behavior and deformed shapes exhibited by link slabs, which are ongoing.

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Notation

a = depth of equivalent rectangular compression stress block (load factored design)

A = area of elastomeric bearing pad

A_e = effective area per reinforcing bar

A_s = area of reinforcing steel in link slab

b = effective link slab width

b_w = effective web width in calculation of shear capacity

c = distance from the extreme compression fiber to the neutral axis

d = distance from extreme compression fiber to center of flexural reinforcement

d_c = distance from extreme tension fiber to center of flexural reinforcement

ei_k = ratio of flexural rigidity of link slab to precast concrete girder and deck slab

ei_s = ratio of flexural rigidity of girder and link slab to girder and deck slab

E = modulus of elasticity

E_c = modulus of elasticity of elastomeric bearing pad

f'_c = 28-day compressive cylinder strength

f'_{ck} = 28-day cylinder strength of link slab

f'_{cs} = 28-day cylinder strength of deck slab

f_s = tensile stress in steel reinforcement

f_y = yield stress of reinforcing steel

G = shear modulus of elastomeric bearing pad

h = total thickness of elastomer in the bearing pad

h_{ri} = thickness of the elastomer internal layer

I = moment of inertia of precast concrete girder and deck slab

I_g = gross moment of inertia of link slab

I_k = cracked moment of inertia of link slab

I_s = gross moment of inertia of precast concrete girder and link slab

K_v = vertical stiffness of the elastomeric bearing pad

L = design span length

L' = length of elastomeric bearing pad

L_k = span length between intermediate supports

$L_{precast}$ = precast concrete girder length

L_s = distance from centerline of intermediate support to edge of link slab in the main span

L_{se} = distance from centerline of intermediate support to edge of girder

M = bending moment

M_{cdl} = total composite dead load moment

M_{DC} = composite dead load moment for component and attachment

M_{DW} = composite dead load moment for wearing surface and utility

M_k = service load moment at centerline of intermediate support

M_{ke} = service load moment at edge of link slab

M_{LL} = live load moment due to lane load

M_{LL+I} = live load moment plus impact

M_n = nominal moment capacity of link slab

M_p = precast concrete girder reduced moment due to continuity

M_s = precast concrete girder moment based on simply supported beam analysis

M_u = factored load moment

n = ratio of the modulus of elasticity of reinforcing steel

NA = neutral axis

P = concentrated load

s = spacing of reinforcing steel in deck slab

S = spacing of precast concrete girders

S_f = the shape factor

t_k = thickness of link slab

t_s = thickness of deck slab

T_s = tension force in reinforcing steel

u = displacement

V = shear force

V_c = nominal shear capacity of link slab

V_{ke} = maximum shear force in link slab due to point load P

V_u = factored load shear force

w = uniformly distributed load

W = width of elastomeric bearing pad

x = horizontal distance from left end support

x_F = ratio of the offset of the concentrated load from left support to the girder design length

X_F = offset of concentrated load from left end support

y = depth of compression zone for service load design

α_k = ratio of link slab length between supports to the girder design length

α_s = ratio of link slab length in the main span to the girder design length

α_{se} = ratio of link slab length in the continuity zone to the girder design length

β = ratio of distances to neutral axis from extreme tension fiber and from centroid of steel

β_e = shear factor

β_s = ratio of flexural strain at the extreme tension face to the strain at the centroid of the reinforcement layer nearest the tension face

β_1 = ratio of the compression stress block to the actual compression zone

γ_e = factor based on exposure of steel

ρ = steel reinforcement ratio

ρ_b = balance steel reinforcement ratio

σ_c = stress in concrete

σ_{eT} = allowable tension stress in precast, prestressed concrete girder

σ_s = stress in reinforcement

Φ = resistance factor (load factored design)

ω = surface crack width

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Abstract

Link slabs provide continuity in the deck slab over the intermediate supports of multiple-span bridges where precast, prestressed concrete girders are used. Current practice recommends debonding the link slab from the girders' ends to reduce its stiffness. Steel reinforcement is consequently provided to satisfy the dead- and live-load criteria. A survey of bridges in the United States indicated that full-depth cracks still develop in the link slab and some practices are placing the link slab without debonding. Although design procedures for debonded link slabs are currently available, data related to bonded link slabs is scarce. This paper investigates the structural behavior of bonded link slabs due to dead and live loads. It examines the contribution of the stiffness of the link slab on redistribution of shear and moments. Closed-form solutions were derived for a three-span symmetrical case and a parametric study conducted based on standard American Association of State Highway and Transportation girders commonly used in the medium-span-length range. It was shown that the link slab parameters, if optimized, can strengthen the connection in the continuity zone to resist the induced loads without affecting its resisting capacity.

Keywords

AASHTO girder, bond, girder, link slab, load, parametric study, stiffness.

Review policy

This paper was reviewed in accordance with the Precast/Prestressed Concrete Institute's peer-review process.

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