

Development of design guidelines for ledges of L-shaped beams

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- This paper presents the development of a design procedure for the eighth edition of the PCI Design Handbook to evaluate the punching shear strength of ledges of L-shaped beams.
- Based on the failure surfaces observed throughout a comprehensive experimental program, an idealized failure surface was determined.
- A procedure to evaluate the punching shear strength of the ledge was developed to provide an improved margin of safety for ledge capacity under a wide range of loading conditions.

This paper is part of a series of three that reports research on the behavior and punching shear strength of ledges of L-shaped beams. The research program included experimental tests of 21 short beams of 15.5 ft (4.72 m) span, 8 long beams of 45.5 ft (13.9 m) span, and 1 long beam of 36.5 ft (11.1 m) span. All short beams were reinforced with mild steel only, while all long beams were prestressed, except for one 45.5 ft span beam. All beams were subjected to multiple tests at different locations along the ledge, resulting in 106 total tests in the program. The results of these tests were presented in the first two papers in this series.^{1,2} In addition to the test program, the research also included the development of a three-dimensional nonlinear finite element model (FEM), validated by the experimental data and other data reported in the literature.³⁻⁹ The FEM was used initially to examine the possible effects of various design parameters, thus providing a basis for design of the experimental program. The FEM was also used to generate additional data supplementing the experimental data.¹⁰

Research findings revealed that several parameters significant to ledge punching shear strength are not considered by the design procedure presented in the seventh edition of the *PCI Design Handbook: Precast and Prestressed Concrete*¹¹ for the ledges of L-shaped beams, referred to as the PCI procedure in this paper. These parameters include the global flexural and shear stresses, prestressing, and load eccentricity. While high levels of global stress and increased load eccentricity cause reductions in ledge capacity, the use of prestressing enhances the load-carrying capacity of the

ledge. The PCI procedure assumes 45-degree failure planes; however, observed failure planes were generally inclined at shallower angles, resulting in a relatively larger failure surface. Furthermore, the PCI procedure specifies a load spacing of $b_t + h_l$ or greater to prevent the overlapping of failure cones from adjacent loads (where b_t is the width of the double-tee stem or the width of the bearing pad, whichever is less, and h_l is the height of beam ledge). Nevertheless, test results indicate that failure cones overlap when adjacent loads are spaced at a distance much larger than this specified value.²

This paper proposes a design procedure that takes into consideration the effects of the previously mentioned parameters along with the parameters traditionally considered for ledge resistance. Recommendations are presented for certain reinforcement details found to improve the behavior and to enhance the capacity of the ledge without changing ledge geometry. This proposed procedure has been incorporated in the eighth edition of the *PCI Design Handbook*.¹²

Observed failure surface

Figure 1 compares the symmetric failure surface observed from a typical test with the surface assumed by the PCI procedure. If a load is applied sufficiently close to the end of the ledge, then there exists a potential for asymmetric failure (**Fig. 1**). If two concentrated loads are placed relatively close to one another, their failure planes may overlap. In such a case, a combined failure plane will develop and the failure surface will generally follow the same configuration as the isolated failure surface, whether symmetric or asymmetric (**Fig. 2**). In all cases, test results demonstrated that the slopes of the faces of the observed failure surfaces are affected by

five main parameters: global flexure and shear stresses, prestressing, load eccentricity, concentrated ledge reinforcement, and ledge height.^{1,2}

Proposed idealized failure surface

To determine the punching shear strength of a ledge, the PCI procedure uses an idealized rectangular failure surface, based on assumed 45-degree failure planes developed from both sides of the bearing area (**Fig. 3**). The width of the idealized rectangular failure surface is assumed to be the ledge projection l_p regardless of the load eccentricity from the inner web face. The depth of failure surface is assumed to be equal to the full ledge height h_l .

Based on the actual failure surfaces measured from the 106 experimental tests, the length of the idealized rectangular failure surface is determined by a similar approach. **Figure 4** shows the ratios of the average extension of the idealized rectangular failure surface from each side of the bearing area to the ledge height. Statistical analysis of these plotted data indicates that the average extension of the idealized failure surface on each side of the bearing area is $1.1h_l$, with a coefficient of variation of 19%. Therefore, for simplicity, the average is taken as $1.0h_l$.

Given these results, it is proposed to modify the PCI procedure to consider the extension of the idealized rectangular failure surface from each side of the bearing area as $1.0h_l$ instead of $0.5h_l$, reflecting the relatively larger failure surface observed in the tests. No change is proposed to either the width or the height of the idealized failure surfaces. Accordingly, idealized design surfaces for various loading cases can be easily derived.

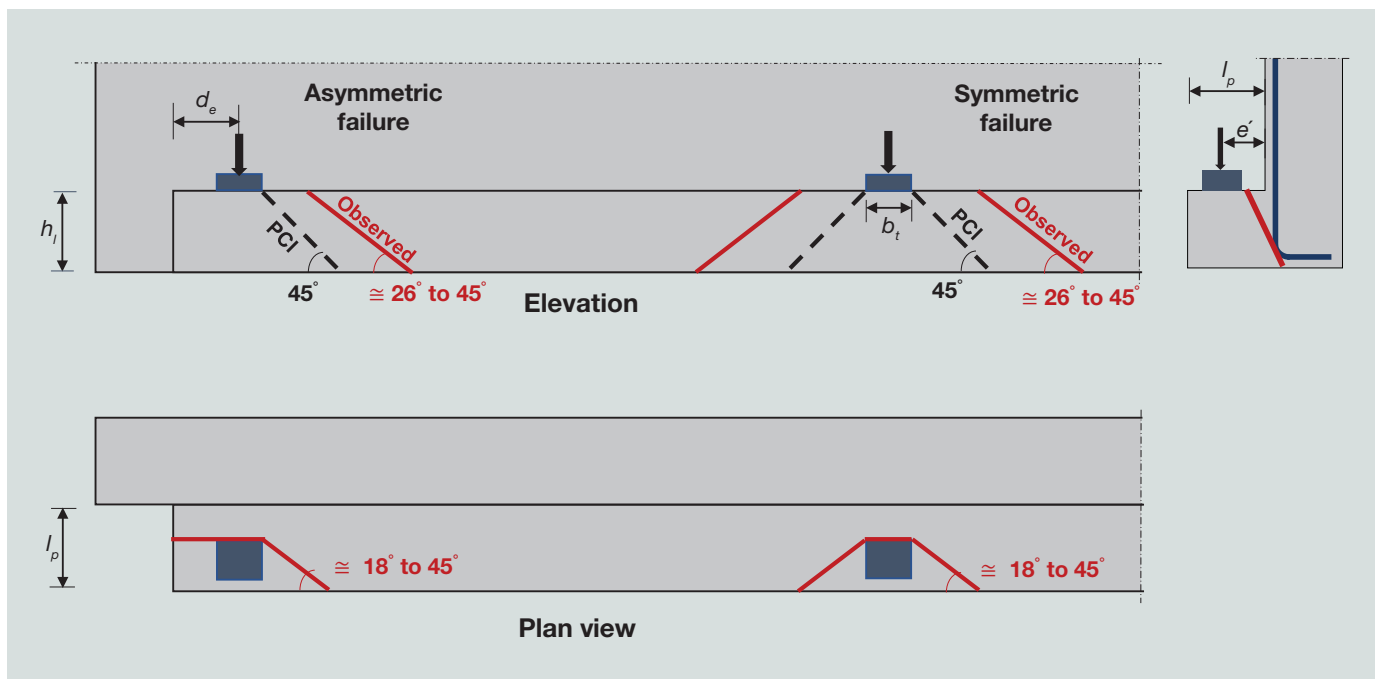


Figure 1. Isolated symmetric and asymmetric failures. Note: b_t = width of the double-tee stem or the width of the bearing pad, whichever is less; d_e = distance from the center of an applied concentrated load to end of the ledge; e' = eccentricity of the factored ledge vertical load to the inner web face; h_l = height of beam ledge; l_p = projection of the ledge.

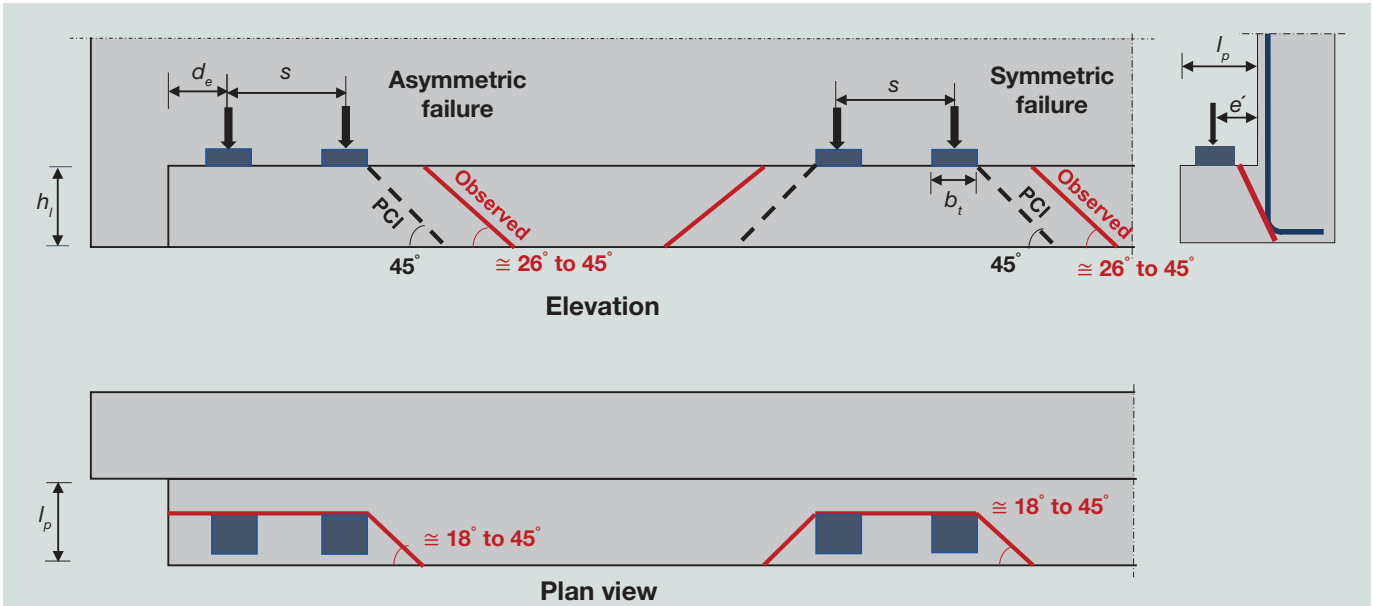


Figure 2. Combined symmetric and asymmetric failures. Note: b_t = width of the double-tee stem or the width of the bearing pad, whichever is less; d_e = distance from the center of an applied concentrated load to end of the ledge; e' = eccentricity of the factored ledge vertical load to the inner web face; h_l = height of beam ledge; l_p = projection of the ledge; s = spacing between applied concentrated loads.

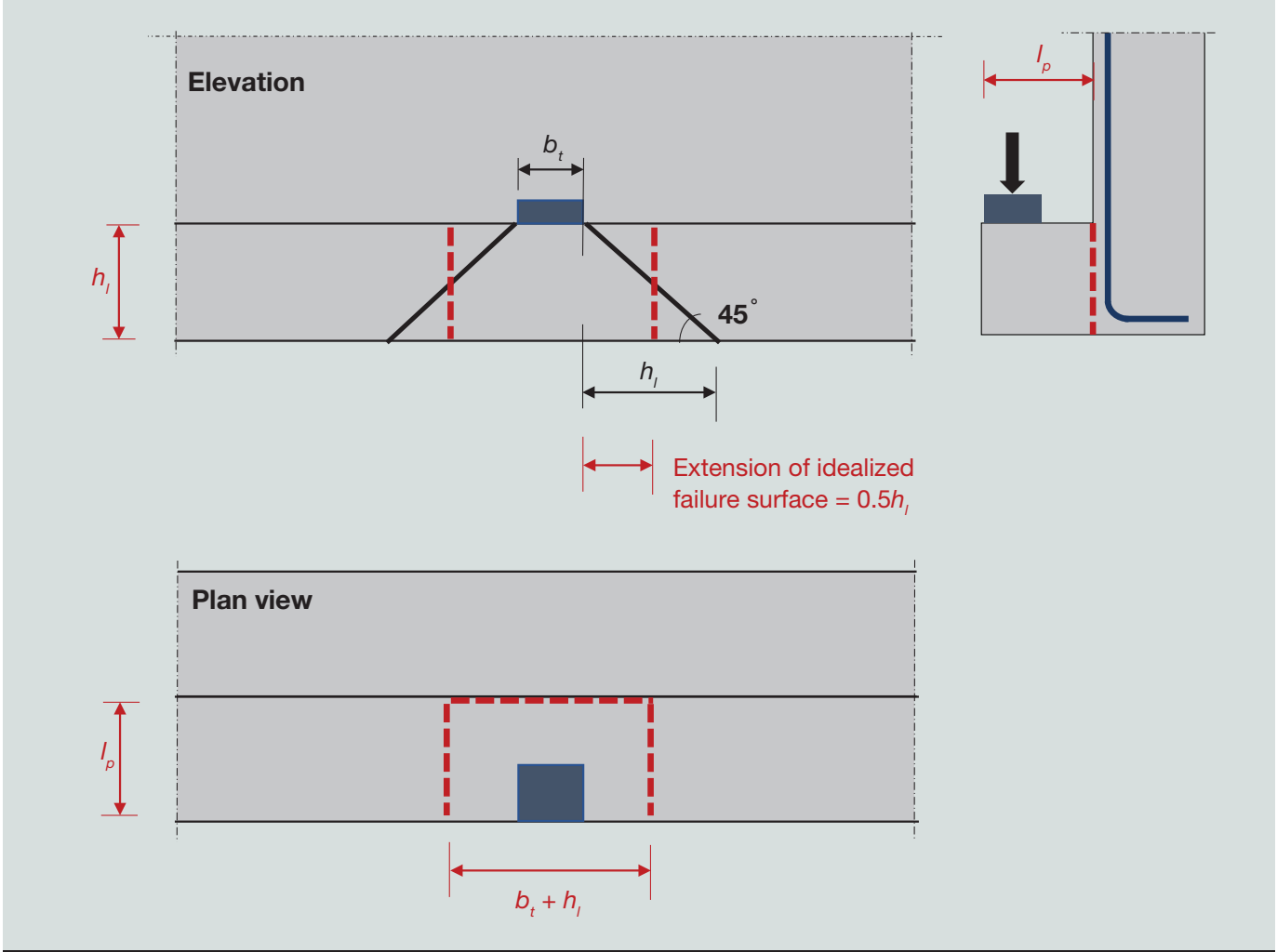


Figure 3. Idealized failure surface by PCI procedure (isolated symmetric failure). Note: b_t = width of the double-tee stem or the width of the bearing pad, whichever is less; h_l = height of beam ledge; l_p = projection of the ledge.

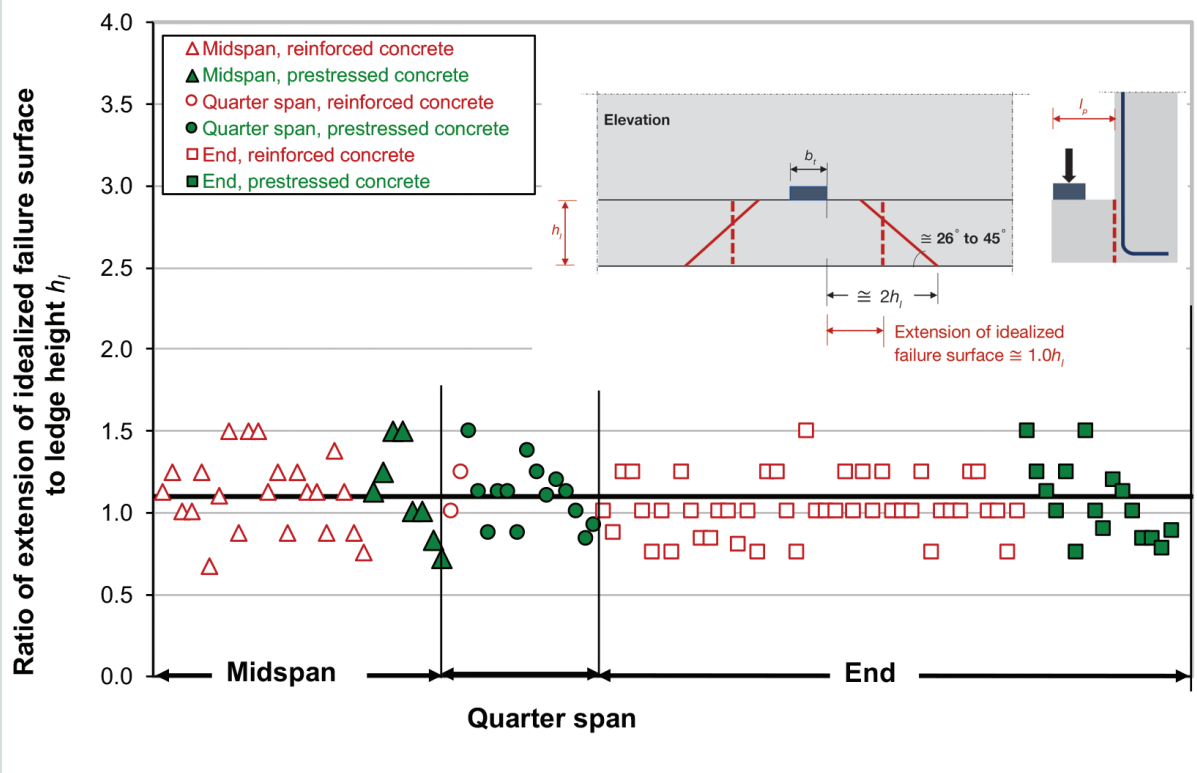


Figure 4. Ratios of the extension of idealized failure surface to ledge height for 106 observed failure surfaces. Note: b_t = width of the double-tee stem or the width of the bearing pad, whichever is less; h_l = height of beam ledge; l_p = projection of the ledge.

Nominal shear stress

The shear stress on the ledge due to applied eccentric concentrated loads is composed of two components:

- direct shear stress due to the vertical applied load, which is assumed uniformly distributed along the perimeter of the idealized failure surface
- torsional shear stress assumed linearly distributed due to the eccentricity of the applied load with respect to the centroid of the critical section of the idealized failure surface (Fig. 5) for symmetric and asymmetric failures

For symmetric failures, the distribution of shear stress on the back face of the failure surface due to the vertical load is typically uniform. However, the presence of the torsional moment around the y axis at the centroid of the critical section M_{ny} induces a nonuniform distribution of shear stress on the side faces of the failure surface with a minimum value at the back plane and a maximum value at the edge of the ledge (Fig. 6). However, some of the torsional moment is counteracted by cantilever ledge flexure in the same way that the unbalanced moment at a slab-column connection is resisted by a combination of flexure and eccentric shear.¹³ For asymmetric failures, the distribution of shear stress on both the back face and the side face of the failure surface is nonuniform due to the presence of the torsional moments in two directions M_{ny} and the

torsional moment around the x axis at centroid of critical section M_{nx} (Fig. 6). Again, some of the torsional moment in both directions is counteracted by ledge flexure. To simplify these complex stress distributions, the shear stress is assumed to be uniform whether for the symmetric or asymmetric failures. The value of this uniform shear stress is expressed as $\beta\sqrt{f'_c}$, where β is a shear strength coefficient dependent on the level of global stress and f'_c is the compressive strength of concrete.

Effect of global stresses in reinforced concrete beams

Effect of global flexural stress in reinforced concrete beams Results of the analytical and experimental studies indicate that increasing the level of the global flexural stress reduces the nominal punching shear strength of the ledge. To account for this effect, the level of global flexural stress at a given location along the span can be represented by the ratio of the applied moment M to the nominal moment capacity M_n of the beam at the location of interest (M/M_n). Using the idealized failure surface, the applied ledge load, and the concrete strength, the shear strength coefficient β was determined as follows.

Selected results of the finite element analysis (FEA)¹⁰ for two locations—midspan and quarter span—were used to determine the effect of global flexural stress on the shear strength coefficient β . For each location, different load cases were

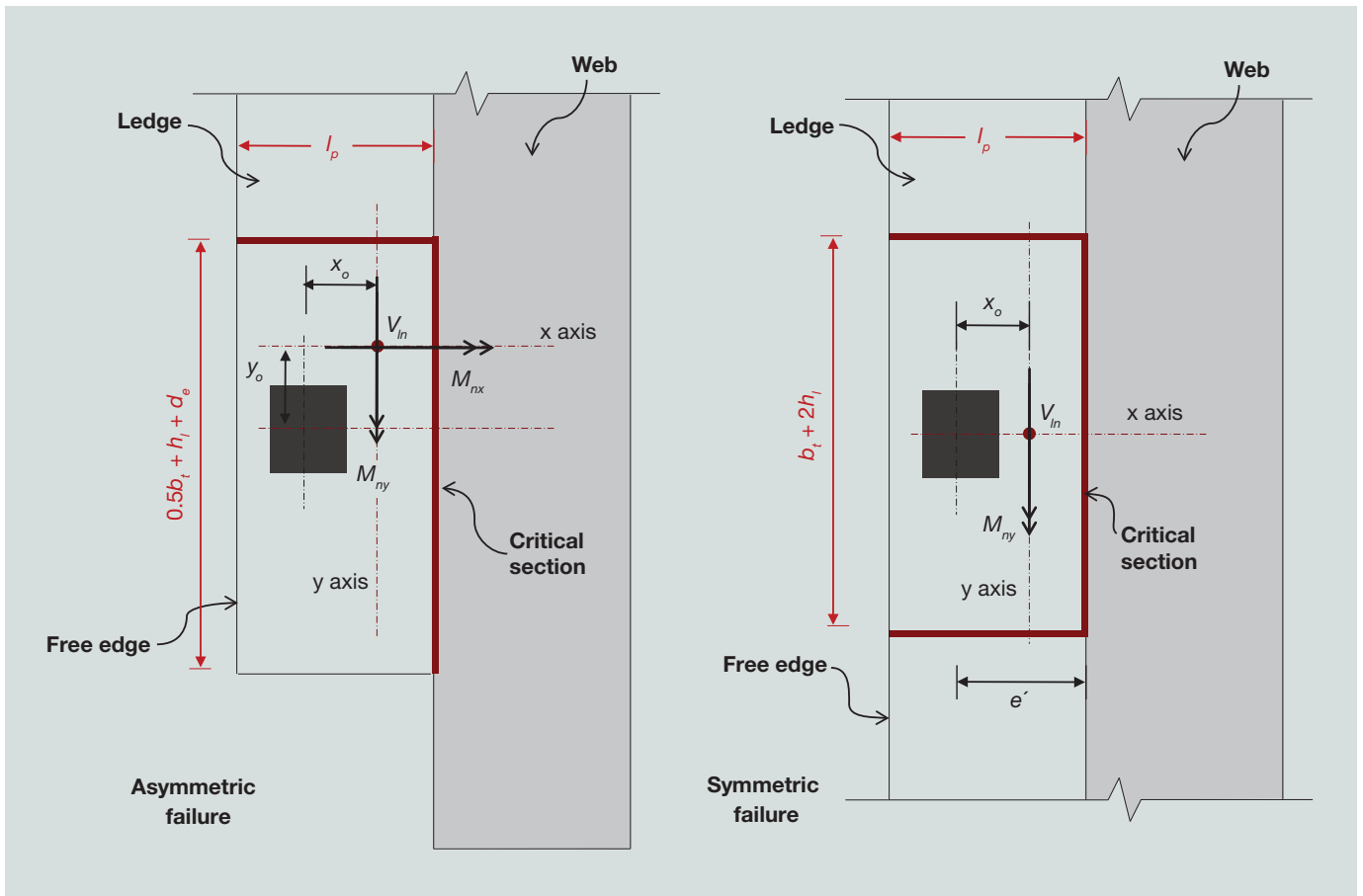


Figure 5. Applied torsional moments in asymmetric and symmetric failures. Note: b_t = width of the double-tee stem or the width of the bearing pad, whichever is less; d_e = distance from the center of an applied concentrated load to end of the ledge; e' = eccentricity of the factored ledge vertical load to the inner web face; h_l = height of beam ledge; l_p = projection of the ledge; M_{nx} = torsional moment around x axis at centroid of critical section (parallel to ledge projection); M_{ny} = torsional moment around y axis at centroid of critical section (perpendicular to ledge projection); V_{ln} = nominal ledge capacity; x_o = distance from ledge load to centroid of critical section in a parallel direction to ledge projection; y_o = distance from ledge load to centroid of critical section in a perpendicular direction to ledge projection.

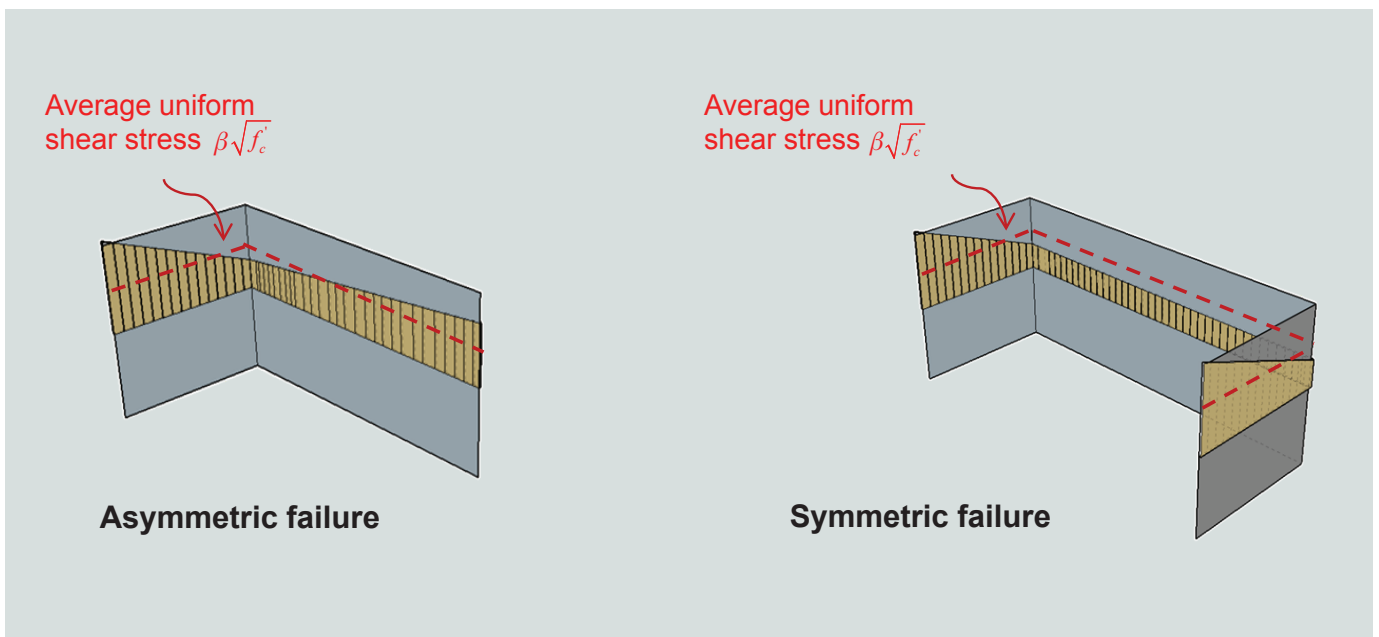


Figure 6. Shear stress distribution for asymmetric and symmetric failures. Note: f'_c = specified compressive strength of concrete; β = shear strength coefficient dependent on the level of global stress.

studied by holding loads at auxiliary locations constant at a specified level while increasing the load at a selected location to failure. By varying the magnitude of the auxiliary loads, different levels of global flexural stress were achieved. Using the failure load predicted by the FEA for each case and the ratio of applied moment to nominal moment capacity M/M_n , the shear strength coefficient β was determined for each case (Fig. 7). Such correlation indicates that increasing the ratio M/M_n reduces the shear strength coefficient β from 2.0 to 1.0. Most of the reduction of shear strength occurred when the ratio M/M_n ranged from 0.2 to 0.6, while it became insignificant at ratios higher than 0.6. It should be noted that the data used in this analysis represent industry-typical cases with uniformly distributed ledge reinforcement and maximum practical load eccentricity (with the ledge load placed at $3/4$ of the ledge projection l_p from the inner web face, in accordance with the seventh edition of the *PCI Design Handbook*).

Effect of global shear stress in reinforced concrete beams

A similar analysis was performed to correlate the level of global shear stress to the uniform shear stress on the idealized failure surface of the ledge. In this case, the level of global shear stress is represented by the ratio V/V_n , where V is the applied shear and V_n is the nominal shear capacity of the beam at a given location, as determined by the procedure developed by Lucier et al.⁶⁻⁸

Different loading cases were analyzed by FEA for two selected locations at the end region and the quarter span to simulate different levels of global shear stress. Using the failure load predicted by the FEA for each loading case, both the ratio of applied shear to nominal shear capacity V/V_n and the shear strength coefficient β were determined for each loading case (Fig. 8). The correlation clearly indicates that increasing the ratio V/V_n reduces the shear strength coefficient β from 2.0 to 1.0, similar to the reduction associated with increasing global flexural stress. Similarly, most of the reduction occurs when the ratio V/V_n ranges from 0.2 to 0.6.

Proposed relationship for the effects of global stress

Results of the FEA indicate that the relationships between the ratios M/M_n and V/V_n and the shear strength coefficient β are almost identical. Therefore, one bilinear relationship can be used to estimate the shear strength coefficient β at a given location, based on the larger of the two ratios M/M_n and V/V_n .

Figure 9 shows the experimental results plotted against the proposed relationship using the larger of M/M_n and V/V_n and the corresponding shear strength coefficient β . Similar trends were obtained from the FEA parametric study that was performed to study the effects of various parameters on ledge capacity at various locations. These results are presented elsewhere.¹⁰ The proposed relationship is based on an optimized correlation between the shear strength coefficients determined by FEA and the experimental program and the predictions.

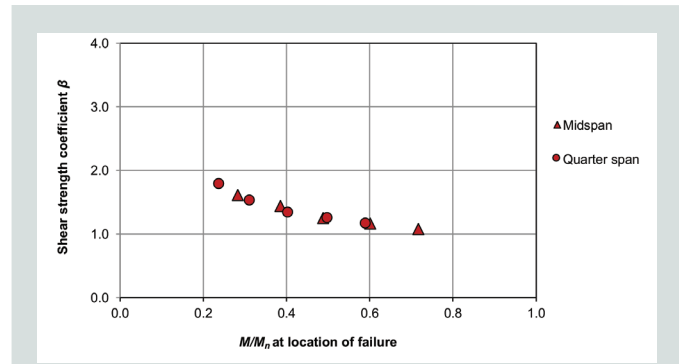


Figure 7. Effect of global flexural stress in reinforced concrete beams (finite element analysis). Note: M = applied moment; M_n = nominal moment capacity of the beam at the given location determined in accordance with section 5.2 of the seventh edition of the *PCI Design Handbook*; β = shear strength coefficient dependent on the level of global stress.

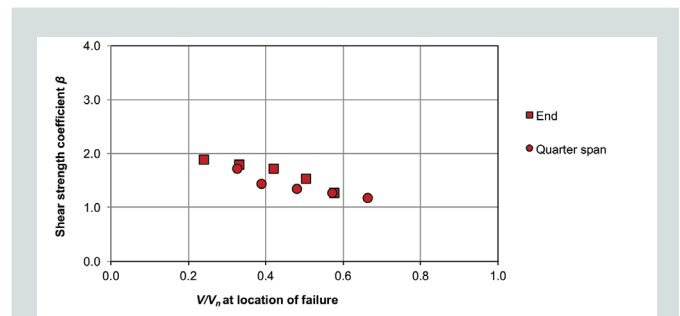


Figure 8. Effect of global shear stress in reinforced concrete beams (finite element analysis). Note: V = applied shear; V_n = nominal shear strength of the beam at the given location; β = shear strength coefficient dependent on the level of global stress.

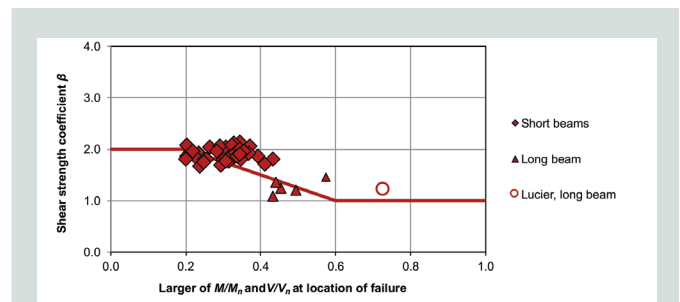


Figure 9. Experimental results overlaid on the proposed relationship for the effect of global stresses in reinforced concrete beams. Note: M = applied moment; M_n = nominal moment capacity of the beam at the given location determined in accordance with section 5.2 of the seventh edition of the *PCI Design Handbook*; V = applied shear; V_n = nominal shear strength of the beam at the given location; β = shear strength coefficient dependent on the level of global stress.

Effect of prestressing

Research findings clearly indicate that the use of prestressing generally enhances the punching shear strength of a ledge.² The influence of prestressing is dependent on the level of prestressing in the beam.

Using the same approach for reinforced concrete beams, the results of FEA¹⁰ for the ledge capacities of prestressed concrete beams were analyzed to determine the relationship between the larger of the ratios M/M_n and V/V_n and the corresponding shear strength coefficient β (Fig. 10). The data used to establish the relationship were based on beams with the same prestressing level and the same concrete strength. The figure clearly indicates that prestressing increases the shear strength coefficient for different levels of global stress at all locations along the beam.

To account for the effect of prestressing, it is proposed to modify the shear strength coefficient β by a factor accounting for the level of prestressing γ . The coefficient γ was derived based on the increase of the principal tensile strength of concrete in the prestressed section, subjected to the combined effects of shear and torsion.¹⁴⁻¹⁶ For reinforced concrete beams, γ equals 1.0, and for prestressed concrete beams, the following equation can be used:

$$\gamma = \sqrt{1 + 10 \frac{f_{pc}}{f'_c}}$$

where

f_{pc} = average prestress after losses

f'_c = specified compressive strength of concrete

For the data presented at the top of Fig. 10, γ was 1.47, based on the average prestress after losses f_{pc} of 690 psi (4.8 MPa) and a concrete compressive strength f'_c of 6000 psi (41 MPa). The comparison between the modified shear strength coefficients $\gamma\beta$ and those predicted by FEA indicates the validity of the proposed approach to predict the punching shear strength of a ledge. The same conclusion is drawn when this proposed approach is compared with the available experimental results for beams having an average γ equal to 1.32, corresponding to an average f_{pc} of 690 psi (4.8 MPa) and measured f'_c ranging from 8670 to 10,190 psi (59.8 to 70.26 MPa) (Fig. 10). The proposed approach for considering the effect of prestressing can be used to conservatively predict the modified shear strength coefficient $\gamma\beta$.

Proposed procedure to evaluate the punching shear strength of ledges

Based on the analyses listed previously, a step-by-step procedure for evaluating ledge punching shear capacity is proposed:

1. For a given location of concentrated ledge load along the

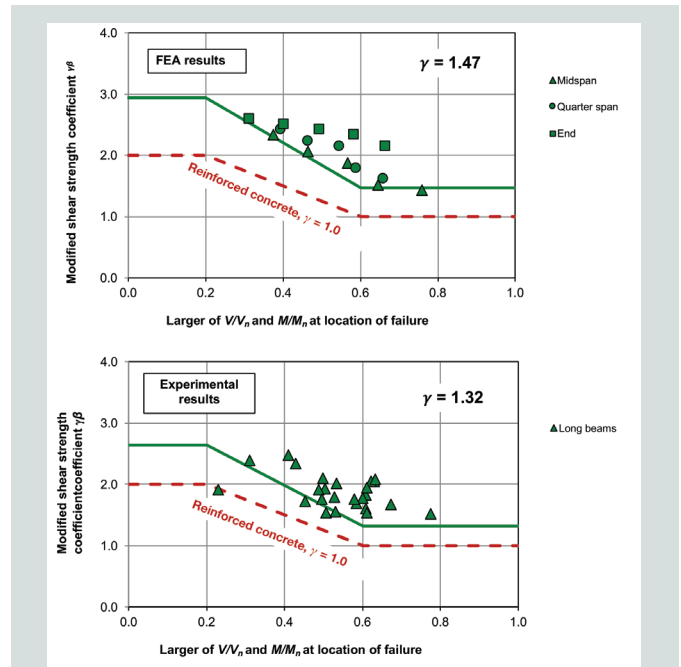


Figure 10. Finite element and experimental results overlaid on the proposed idealization for the effect of prestressing. Note: FEA = finite element analysis; M = applied moment capacity; M_n = nominal moment capacity of the beam at the given location determined in accordance with section 5.2 of the seventh edition of the *PCI Design Handbook*; V = applied shear; V_n = nominal shear strength of the beam at the given location; β = shear strength coefficient dependent on the level of global stress; γ = factor accounting for the level of prestressing.

span of an L-shaped beam, the ratios M_u/M_n and V_u/V_n are determined, where M_u is the factored moment in the beam at the given location and V_u is the factored shear in the beam at the same location.

$$V_n = V_c + V_s$$

where

V_c = nominal shear strength provided by concrete, determined in accordance with section 5.3 of the seventh edition of the *PCI Design Handbook*

V_s = nominal shear strength provided by shear reinforcement, determined in accordance with the slender spandrel procedure developed by Lucier et al.⁶

$$= 2 \left(\frac{A_{so}}{s} \right) f_y d$$

A_{so}/s = vertical shear reinforcement on the outer web face (that is, non-ledge web face)

f_y = specified yield strength of shear reinforcement

d = distance from the extreme compression fiber to the centroid of longitudinal tension reinforcement (per ACI 318-14¹³), but not less than $0.8h$ for prestressed components

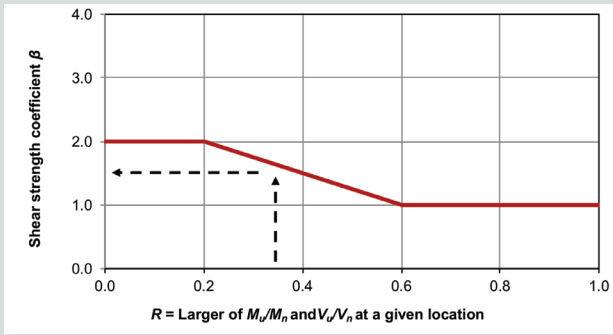


Figure 11. Shear strength coefficient β for ledges of L-shaped beams. Note: M_n = nominal moment capacity of the beam at the given location determined in accordance with section 5.2 of the seventh edition of the *PCI Design Handbook*; M_u = factored moment in the beam at the given location; V_n = nominal shear strength of the beam at the given location; V_u = factored shear in the beam at the given location.

h = height of beam

- Let R be the larger of the two ratios M_u/M_n and V_u/V_n . Determine the shear strength coefficient of the ledge β based on the following conditions (**Fig. 11**).

For $R \leq 0.2$, $\beta = 2$

For $0.2 < R < 0.6$, $\beta = 1 + 2.5(0.6 - R)$

For $R \geq 0.6$, $\beta = 1$

For typical designs, the ratio R will likely exceed 0.6 in regions of maximum shear or flexure. Thus, it is usually reasonable and always conservative to take β as 1.0. Generally, ledge punching shear strength near the support or the midspan will control the design of the ledge.

- For interior concentrated loads, where the distance from the center of an applied concentrated load to the end of the ledge $d_e \geq 0.5b_t + h_t + l_p$, a symmetric failure would control the design punching shear strength of the ledge ϕV_{ln} (where ϕ is the strength-reduction factor and V_{ln} is the nominal ledge capacity). The design strength should be taken as the lesser of the values given by Eq. (1) and (2). Typically, Eq. (1) controls the strength for single interior ledge loads, while Eq. (2) controls the strength for closely spaced interior ledge loads.

$$\phi V_{ln} = \phi \lambda \gamma \beta \sqrt{f'_c} h_t (b_t + 2h_t + 2l_p) \quad (1)$$

$$\phi V_{ln} = \phi 0.5 \lambda \gamma \beta \sqrt{f'_c} h_t (b_t + 2h_t + s + 2l_p) \quad (2)$$

where

λ = modification factor for density of concrete

s = spacing between applied concentrated loads

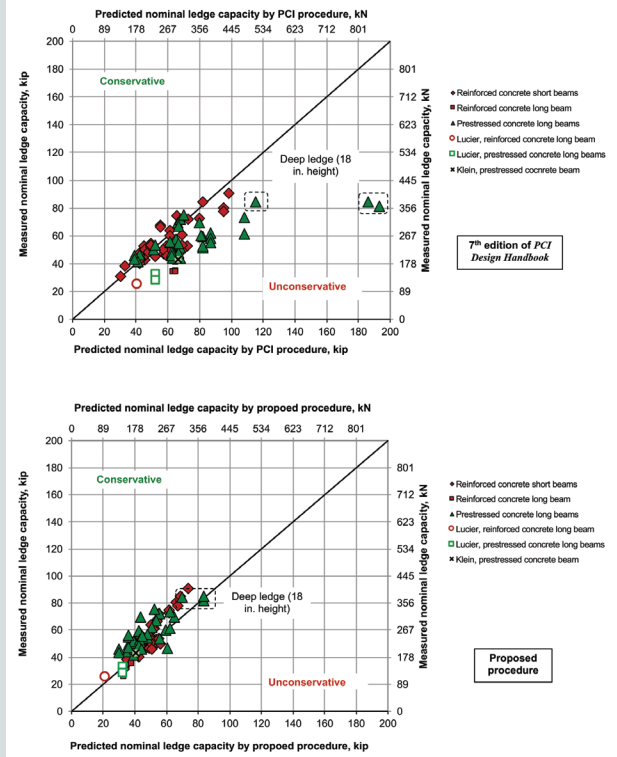


Figure 12. Comparison of measured nominal ledge capacities and the values predicted by the equations included in the seventh edition of the *PCI Design Handbook: Precast and Prestressed Concrete* and by the proposed procedure. Note: 1 in. = 25.4 mm.

For end concentrated loads, where $d_e < 0.5b_t + h_t + l_p$, an asymmetric failure would control the design punching shear strength of the ledge ϕV_{ln} . The design strength should be taken as the lesser of the values given by Eq. (3) and (4). Typically, Eq. (3) controls the strength for single ledge loads close to the end of the ledge, while Eq. (4) controls the strength for closely spaced ledge loads close to the end of the ledge.

$$\phi V_{ln} = \phi \lambda \gamma \beta \sqrt{f'_c} h_t (0.5b_t + h_t + d_e + l_p) \quad (3)$$

$$\phi V_{ln} = \phi 0.5 \lambda \gamma \beta \sqrt{f'_c} h_t (0.5b_t + h_t + d_e + s + l_p) \quad (4)$$

The minimum load spacing s along the ledge should be used to determine the design punching shear strength. For nonprestressed sections, γ is 1.0. If d_e is less than the transfer length of the strands, the average prestress f_{pc} should be reduced accordingly.

This research did not consider the case where a ledge supports a continuous load (such as cored slabs or box beams placed side by side) or a continuous series of closely spaced concentrated loads (such as channel beams).

These conditions are addressed by Eq. (5-48) of the seventh edition of the *PCI Design Handbook*.

$$\phi V_{ln} = 24\phi h_l \lambda \sqrt{f'_c} \quad (5-48)$$

ϕV_{ln} = design shear strength, lb/ft

PCI Eq. (5-48) assumes a shear strength of $2\sqrt{f'_c}$ applied over the height of the ledge. The observed behavior of ledges under concentrated loads in this research suggests that PCI Eq. (5-48) may overestimate ledge strength; however, uniformly loaded ledges were not expressly studied in the research. Accordingly, it is recommended that the shear strength term be reduced to $\sqrt{f'_c}$ and Eq. (5) be conservatively used in lieu of PCI Eq. (5-48).

$$\phi V_{ln} = 12\phi h_l \lambda \sqrt{f'_c} \quad (5)$$

Reliability of the proposed procedure

Figure 12 compares the experimentally measured ledge capacities to predictions calculated per the design equations included in the seventh edition of the *PCI Design Handbook* and also per the proposed procedure. The ledge capacity predicted by PCI equations for each case is determined using the specified equation for the selected failure location (that is, Eq. [5-44] for interior locations and Eq. [5-45] for end locations). The experimental data represent typical cases where ledge reinforcement is uniformly distributed and maximum eccentricity is induced by placing the ledge load at $\frac{3}{4}$ of the ledge projection l_p from the inner web face. The comparisons demonstrate that the proposed procedure can be conservatively applied to predict the punching shear strength of the ledges of L-shaped beams. The average ratio between the measured and predicted values using the proposed design procedure is 1.15 ± 0.03 , based on a 95% confidence level, with much of the variation on the conservative side. The study was performed on L-shaped beams with the following characteristics:

- varying levels of global flexure and shear stresses
- reinforced and prestressed concrete beams
- ledge heights from 8 to 18 in. (200 to 460 mm)
- ledge projections from 6 to 10 in. (150 to 250 mm)
- concrete strengths from 5000 to 15,000 psi (34 to 100 MPa)
- bearing widths from 4 to 12 in. (100 to 300 mm)
- distances of end load to end of ledge ranging from 4 to 36 in. (100 to 910 mm)
- normalweight concrete

The research findings were mainly developed for ledges of L-shaped beams that support thin-stemmed precast concrete members, such as double tees, and further research is needed

to verify the research findings for other cases, such as the much larger ledges of bridge piers or for cases where a ledge supports continuous loads. A size effect would need to be considered if the proposed procedure is applied to larger ledge beams, especially deep ledges supporting bridge girders. Although limited test results demonstrate the applicability of the proposed procedure for concrete strengths up to 15,000 psi (100 MPa), it is recommended to conservatively limit f'_c to 10,000 psi (69 MPa), in accordance with ACI 318-14.¹³ While it is true that the experimental program was conducted using normalweight concrete only, the proposed design equations are equally applicable to lightweight concrete if the traditional factor λ is included; however, additional research on ledges cast with lightweight concrete is recommended.

Detailing recommendations

The ledge shear resistance mechanism is primarily governed by failure in the concrete. However, test results^{1,2} indicate that detailing of the ledge reinforcement can influence ledge behavior.

- Test results indicate that turning the bottom leg of the hanger reinforcement into the ledge may improve the ledge shear failure mechanism by intercepting the diagonal compressive strut that extends downward from the ledge load, thus reducing the brittleness of the failure. It is recommended to provide a bar or strand at the inside corner of hanger bars that are turned outward towards the ledge.
- Limited test results indicate that the nominal shear capacity of the ledge can be increased by 25% or more by concentrating all required ledge transverse and hanger reinforcement over a distance of $b_l + 2h_l$ at the location of the applied concentrated load while turning the bottom leg of the hanger reinforcement into the ledge. The amount of reinforcement concentrated near the load should be determined using the equations in sections 5.5.2 and 5.5.4 of the seventh edition of the *PCI Design Handbook*, with additional minimum reinforcement placed elsewhere between the ledge loads. For practical reasons, the ledge height is often kept constant throughout a project, so this approach may be particularly useful in cases where supporting an unusually heavy load is required but increasing the ledge height is undesirable. The strut-and-tie method of ACI 318-14¹³ provides an alternative means for proportioning concentrated hanger and ledge reinforcement.
- Limited test results indicate that welded-wire reinforcement performs as well as conventional ledge reinforcement and it reduces the time required for fabrication and placement of ledge reinforcement.

Conclusion

Based on extensive FEMs and a comprehensive experimental program that included 106 tests performed on 21 short-span

and 9 long-span L-shaped beams,^{1,2} the following conclusions are drawn for evaluating the punching shear strength of ledges of L-shaped beams.

- The procedure provided by the seventh edition of the *PCI Design Handbook* can significantly overestimate the punching shear strength of the ledge, especially for interior locations.
- The level of global stress is the most influential parameter affecting ledge capacity. High levels of global flexural or shear stress in the region of a concentrated ledge load significantly reduce the punching shear strength of the ledge.
- The use of prestressing enhances the punching shear strength of the ledge. The enhanced capacity is dependent on the level of prestressing in the beam.
- Increasing the eccentricity of the applied load from the inner web face reduces the punching shear strength of the ledge. Therefore, it is conservative to determine the punching shear strength of the ledge by assuming that the ledge load is applied at $\frac{3}{4}$ of the ledge projection l_p from the inner web face, in accordance with PCI guidelines.
- For typical ledges, the crack angles of the observed failure surfaces were generally shallower than the 45-degree angles assumed by the PCI procedure.
- A simple and practical procedure is introduced for calculating the punching shear strength of the ledge by assuming a uniform nominal shear stress acting on an idealized failure surface.
- Special reinforcement details are recommended to improve and enhance the behavior and capacity of the ledge without changing its geometry.

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Notation

A_{so}/s	= vertical shear reinforcement on the outer web face (that is, non-ledge web face)	M_{ny}	= torsional moment around y axis at centroid of critical section (perpendicular to ledge projection)
b_t	= width of the double-tee stem or the width of the bearing pad, whichever is less	M_u	= factored moment in the beam at a given location
d	= distance from the extreme compression fiber to the centroid of longitudinal tension reinforcement (per ACI 318-14 ¹³), but not less than $0.8h$ for prestressed components (d_p is used for prestressed components when a distinction from d for nonprestressed reinforcement is relevant)	R	= larger of the two ratios M_u/M_n and V_u/V_n
d_e	= distance from the center of an applied concentrated load to the end of the ledge	s	= spacing between applied concentrated loads
e'	= eccentricity of the factored ledge vertical load to the inner web face	V	= applied shear
f'_c	= specified compressive strength of concrete	V_c	= nominal shear strength provided by concrete, determined in accordance with section 5.3 of the seventh edition of the <i>PCI Design Handbook</i>
f_{pc}	= average prestress after losses at a given location	V_n	= nominal ledge capacity (V_n in section 5.5 of the seventh edition of the <i>PCI Design Handbook</i>)
f_y	= specified yield strength of shear reinforcement	V_n	= nominal shear strength of the beam at a given location
h	= height of beam	V_s	= nominal shear strength provided by shear reinforcement, determined in accordance with the slender spandrel procedure developed by Lucier et al. ⁶
h_l	= height of beam ledge	V_u	= factored shear in the beam at a given location
l_p	= projection of the ledge ($[b_l - b]$ in section 5.5 of the seventh edition of the <i>PCI Design Handbook</i>)	x_0	= distance from ledge load to centroid of critical section in a parallel direction to ledge projection
M	= applied moment	y_0	= distance from ledge load to centroid of critical section in a perpendicular direction to ledge projection
M_n	= nominal moment capacity of the beam at a given location determined in accordance with section 5.2 of the seventh edition of the <i>PCI Design Handbook</i>	β	= shear strength coefficient dependent on the level of global stress
M_{nx}	= torsional moment around x axis at centroid of critical section (parallel to ledge projection)	γ	= factor accounting for the level of prestressing
		λ	= modification factor for density of concrete (section 5.3.3 in the seventh edition of the <i>PCI Design Handbook</i>)
		ϕ	= strength-reduction factor

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Abstract

The design procedure presented in the seventh edition of the *PCI Design Handbook: Precast and Prestressed Concrete* for ledges of L-shaped beams has been called into question by many engineers and researchers. Research findings from previous experimental studies have indicated that the PCI ledge design equations can overestimate ledge punching shear strength. This paper presents the development of the design procedure for the eighth edition of the *PCI Design Handbook* to evaluate the punching shear strength of ledges of L-shaped beams. Based on the failure surfaces observed throughout a comprehensive experimental program, an idealized failure surface was determined. The results of extensive finite element analyses and a large experimental program were used to evaluate the effects of global stress on ledge capacity, and a procedure to evaluate the punching shear strength of the ledge was developed. The proposed procedure is presented in this paper and is intended to provide an improved margin of safety for ledge capacity under a wide range of loading conditions. Consideration was given to ensure simplicity and practicality of the proposed design procedure.

Keywords

Global stress, ledge, ledge capacity, L-shaped beam, punching shear, spandrel.

Review policy

This paper was reviewed in accordance with the Precast/Prestressed Concrete Institute's peer-review process.

Reader comments

Please address any reader comments to *PCI Journal* editor-in-chief Emily Lorenz at elorenz@pci.org or Precast/Prestressed Concrete Institute, c/o *PCI Journal*, 200 W. Adams St., Suite 2100, Chicago, IL 60606.

APPENDIX

Design example

Problem

Investigate the shear strength of the ledge of the following L-shaped beam and determine the required ledge reinforcement (Fig. A1).

Data

Prestressing

Fourteen ½ in. (13 mm) special, 270 ksi (1860 MPa) low-relaxation strands

Area of prestressing strands $A_{ps} = 2.338 \text{ in.}^2$ (1510 mm²)

Pertinent results

Nominal moment capacity $M_n = 1405 \text{ kip-ft}$ (1906 kN-m) at first stem, near support

$= 2138 \text{ kip-ft}$ (2900 kN-m) at fifth stem, midspan

Nominal shear strength provided by concrete $V_c = 215 \text{ kip}$ (956 kN, at first stem, near support

$= 66 \text{ kip}$ (290 kN) at fifth stem, midspan

Vertical shear reinforcement on the outer web face $A_{sv}/s = 0.06 \text{ in.}^2/\text{ft}$ (127 mm²/m), continuous

Additional information

Height of beam $h = 60 \text{ in.}$ (1520 mm)

Distance from extreme compression fiber to centroid of prestressing reinforcement (not less than 80% of the total height of the section, $0.8h$) $d_p = 48 \text{ in.}$ (1220 mm)

Factored ledge vertical load $V_{lu} = 24.7 \text{ kip/stem}$ (110 kN/stem)

Height of beam ledge $h_l = 8 \text{ in.}$ (200 mm)

Projection of the ledge $l_p = 8 \text{ in.}$ (200 mm)

Width of the double-tee stem or the width of the bearing pad $b_l = 4 \text{ in.}$ (100 mm)

Spacing between applied concentrated loads $s = 60 \text{ in.}$ (1520 mm)

Specified concrete compressive strength $f'_c = 10,000 \text{ psi}$ (69 MPa)

Distance from extreme compression fiber to the centroid of hanger reinforcement in L beam $d_s = 6.75 \text{ in.}$ (170 mm)

Depth of centroid of reinforcement in L beam ledges $d_l = 6 \text{ in.}$ (150 mm)

Modification factor for density of concrete $\lambda = 1.0$

Distance between torsional equilibrium reactions $h_s = 48 \text{ in.}$

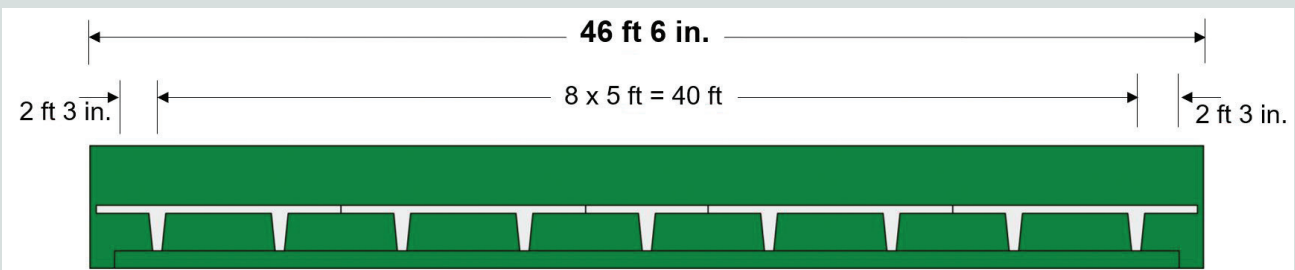
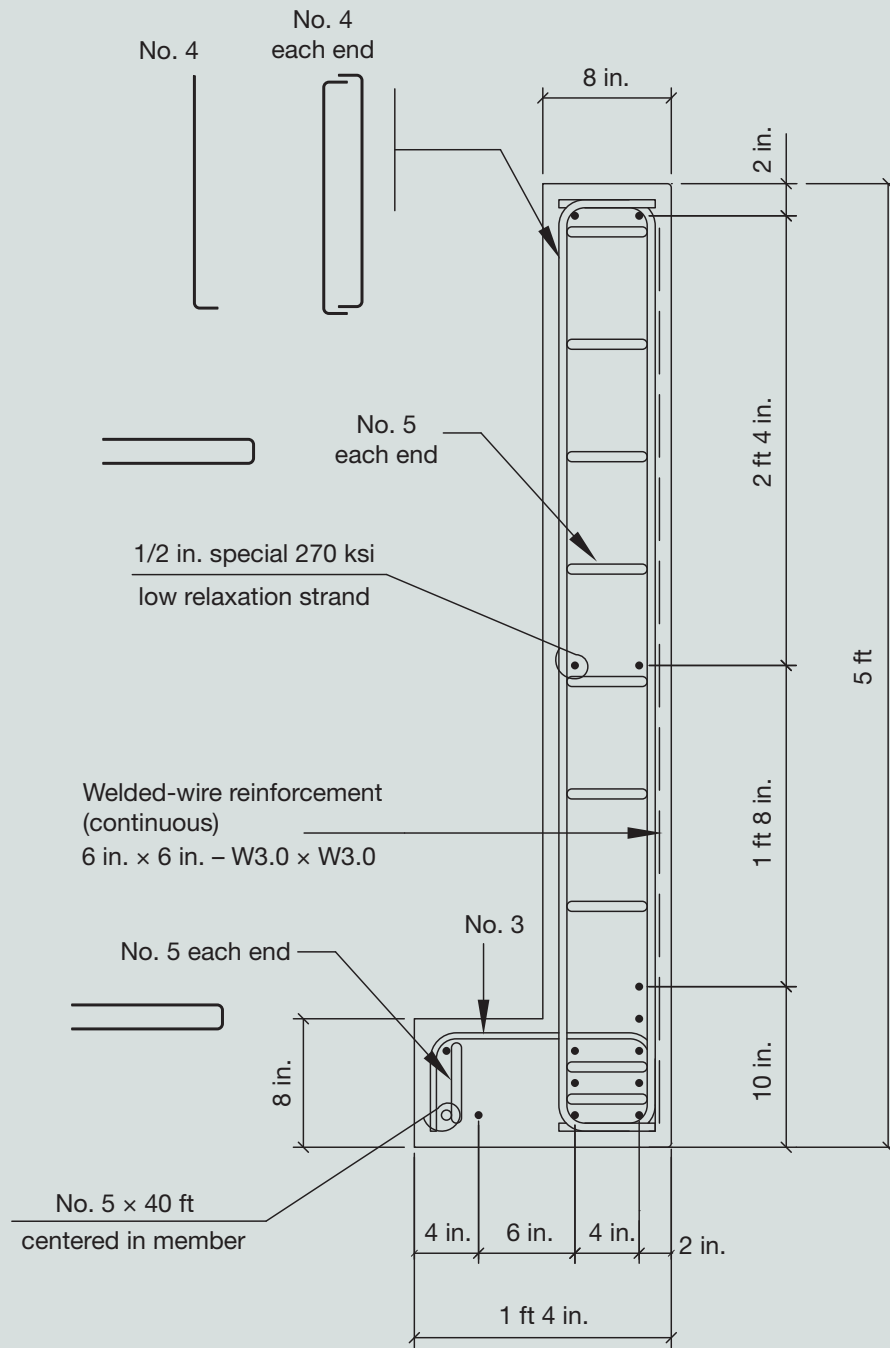


Figure A1. Elevation view of double-tees bearing on L-shaped beam and cross-section view of the example prestressed concrete L-shaped beam. Note: no. 3 = 10M; no. 4 = 13M; no. 5 = 16M; 1 in. = 25.4 mm; 1 ft = 0.305 m.

Solution

Check the shear strength of the ledge

At the stem location of maximum shear (first stem)

Step 1

Calculate the ratios V_u/V_n and M_u/M_n .

Factored shear in the beam $V_u = 123$ kip (547 kN)

Factored moment in the beam $M_u = 340$ kip-ft (460 kN-m)

$$V_s = 2(A_{so}/s)f_y(d/12) = 2(0.06)(60)(48/12) = 28.8 \text{ kip (128 kN)}$$

where

- V_s = nominal shear strength provided by shear reinforcement
- f_y = specified yield strength of shear reinforcement
- d = effective depth from the centroid of reinforcement to the extreme fiber of the compression zone

$$V_n = V_c + V_s = 215 + 28.8 = 243.8 \text{ kip (1084 kN)}$$

where

- V_n = nominal shear strength of the beam
- V_c = nominal shear strength provided by concrete
- $V_u/V_n = 123/243.8 = 0.50$ (larger)
- $M_u/M_n = 340/1405 = 0.24$

Step 2

Determine the coefficient of shear strength of the ledge β .

$$R = \text{larger of the two ratios } M_u/M_n \text{ and } V_u/V_n = 0.50$$

for $0.2 < R < 0.6$

$$\beta = 1 + 2.5(0.6 - R) = 1 + 2.5(0.6 - 0.5) = 1.25$$

Step 3

Determine the design shear strength of the ledge.

From Design Aid 15.3.4 of the seventh edition of the *PCI Design Handbook*, strand develops 170 ksi at 29.6 in. (750 mm) $< 27 + 12 = 39$ in. (990 mm). Thus, prestress force is fully transferred.

$$f_{pc} = \frac{P_{pd}}{A} = \frac{(2.338)(170)}{544} = 0.731 \text{ ksi (5.04 MPa)}$$

where

f_{pc} = average prestress after losses at a given location

P_{pd} = prestress force limited by strand development

A = cross-sectional area

$$\gamma = \sqrt{1 + 10 \frac{f_{pc}}{f'_c}} = \sqrt{1 + 10 \left(\frac{0.731}{10} \right)} = 1.316$$

where

γ = factor accounting for the level of prestressing

$$d_e = 27 \text{ in. (690 mm)} \geq 0.5b_t + h_t + l_p = 18 \text{ in. (460 mm)}$$

where

d_e = distance from the center of an applied concentrated load to the end of the ledge

Use the lesser of equations (1) and (2).

$$\begin{aligned} \phi V_m &= \phi \lambda \gamma \beta \sqrt{f'_c} h_t (b_t + 2h_t + 2l_p) \quad (1) \\ &= (0.75)(1.0)(1.316)(1.25)\sqrt{10,000}(8)[4 + 2(8) + 2(8)]/1000 \\ &= 35.5 \text{ kip (158 kN)} \rightarrow \text{lesser} \end{aligned}$$

where

- ϕ = strength-reduction factor
- V_m = nominal ledge capacity
- λ = modification factor for density of concrete

$$\begin{aligned} \phi V_m &= \phi 0.5 \lambda \gamma \beta \sqrt{f'_c} h_t (b_t + 2h_t + s + 2l_p) \quad (2) \\ &= (0.75)(0.5)(1.0)(1.316)(1.25)\sqrt{10,000}(8) \\ &\quad \times [4 + 2(8) + 60 + 2(8)]/1000 \\ &= 47.4 \text{ kip (211 kN)} \end{aligned}$$

At the stem location of maximum moment (fifth stem)

Step 1

Calculate the ratios V_u/V_n and M_u/M_n .

$$V_u = 12 \text{ kip (53 kN)}$$

$$M_u = 1446 \text{ kip-ft (1961 kN-m)}$$

$$V_s = 28.8 \text{ kip (128 kN)}$$

$$V_n = V_c + V_s = 66 + 28.8 = 94.8 \text{ kip (422 kN)}$$

$$V_u/V_n = 12/94.8 = 0.13$$

$$M_u/M_n = 1446/2138 = 0.68 \text{ (larger)}$$

Step 2

Determine the coefficient of shear strength of the ledge β .

$$R = M_u/M_n = 0.68$$

for $R \geq 0.6$, $\beta = 1.0$

Step 3

Determine the design shear strength of the ledge.

$$d_e = 267 \text{ in. (6780 mm)} \geq 0.5b_l + h_l + l_p = 18 \text{ in. (460 mm)}$$

Use the lesser of equations (1) and (2)

Applying Eq. (1)

$$= (0.75)(1.0)(1.316)(1.0)\sqrt{10,000}(8)\left[4 + 2(8) + 2(8)\right]/1000$$

$$= 28.4 \text{ kip (126 kN)} \rightarrow \text{lesser}$$

Applying Eq. (2)

$$= (0.75)(0.5)(1.0)(1.316)(1.0)\sqrt{10,000}(8)$$

$$\times [4 + 2(8) + 60 + 2(8)]/1000$$

$$= 37.9 \text{ kip (169 kN)}$$

therefore, $\phi V_n = 28.4 \text{ kip (126 kN)} > 24.7 \text{ kip/stem (110 kN/stem)} \rightarrow \text{OK}$

Determine the required transverse reinforcement

Using factored ledge friction load $N_{lu} = 0.15V_{lu} = 3.71 \text{ kip (16.5 kN)}$

Assuming the maximum load eccentricity of $0.75l_p$ from the inner web face

Applied load eccentricity measured to the centroid of the hanger reinforcement $a = 0.75l_p + \text{cover}$

$$= (0.75)(8) + 1.25 = 7.25 \text{ in. (184 mm)}$$

$$d = 6.75 \text{ in. (171 mm)}$$

$$6h_l = 48 > s/2 = 30 \text{ in. (760 mm)}$$

Distribute reinforcement over a distance of $s/2$ on each side of the load:

$$A_s = \left(\frac{1}{\phi f_y}\right) \left[V_{lu} \left(\frac{a}{d}\right) + N_{lu} \left(\frac{h_l}{d}\right) \right]$$

$$= \left(\frac{1}{(0.75)(60)}\right) \left[24.7 \left(\frac{7.25}{6.75}\right) + 3.71 \left(\frac{8}{6.75}\right) \right]$$

$$= 0.69 \text{ in.}^2 \text{ (445 mm}^2\text{)}$$

where

A_s = area of transverse flexural reinforcement

$$\frac{A_s}{s} = \frac{(0.69)(12)}{60} = 0.138 \text{ in.}^2/\text{ft (292 mm}^2/\text{m)}$$

Maximum bar spacing = 8 in. (200 mm)

Use no. 3 at 8 in. (10M at 200 mm), $A_s = 0.165 \text{ in.}^2/\text{ft (349 mm}^2/\text{m)}$ at all locations

Check longitudinal bending of the ledge

$$A_l = 200l_p \frac{d_l}{f_y} = 200(8) \left(\frac{6}{60,000}\right) = 0.16 \text{ in.}^2 \text{ (100 mm}^2\text{)}$$

where

A_l = area of longitudinal reinforcement to resist longitudinal bending in ledges

Use one 1/2 in. (13 mm) special strand at top of the ledge, providing 0.167 in.^2 (108 mm²), and use one no. 5 (16M) bar at the bottom of the ledge, providing 0.31 in.^2 (200 mm²).

Determine the required hanger reinforcement A_{sh}

$$m = \frac{\left[(d_s + a) - \left(3 - 2\frac{h_l}{h} \right) \left(\frac{h_l}{h} \right)^2 \left(\frac{b_l}{2} \right) - e\gamma_t \frac{(x^2 y)_l}{\sum x^2 y} \right]}{d_s}$$

$$= \frac{\left\{ (6.75 + 7.25) - \left[3 - 2\left(\frac{8}{60}\right) \right] \left(\frac{8}{60}\right)^2 \left(\frac{16}{2}\right) - 0 \right\}}{6.75}$$

$$= 2.02 \geq 0.6 \text{ (OK)}$$

where

γ_t = factor used in designing hanger reinforcement as defined in section 5.5.4 in the seventh edition of the *PCI Design Handbook* = 0 (open reinforcement)

m = modification factor for hanger reinforcement design defined in section 5.5.4 in the seventh edition of the *PCI Design Handbook*

b_l = width of web and one ledge

e = eccentricity of design load measured from centroid of section

x = shorter side of the component rectangles forming the ledge and the web

y = longer side of the component rectangles forming the ledge and the web

$$A_{sh} = \left(\frac{V_u}{\phi f_y} \right) m = \left[\frac{24.7}{(0.75)(60)} \right] = 1.11 \text{ in.}^2 \text{ (716 mm}^2\text{)}$$

$$\frac{A_{sh}}{s} = \frac{(1.11)(12)}{60} = 0.22 \text{ in.}^2/\text{ft (470 mm}^2/\text{m)}$$

Use no. 4 at 8 in. (13M at 200 mm), $A_{sh} = 0.30 \text{ in.}^2/\text{ft}$ (635 mm²/m)

Determine reinforcement for out-of-plane bending near beam end

$$A_{wl} = A_{wv} = \frac{V_u e}{2\phi f_y d_w}$$

$$= \frac{(24.7)(9/2)(10)}{2(0.75)(60)(6.5)}$$

$$= 1.9 \text{ in.}^2 \text{ (1226 mm}^2\text{)}$$

(to be distributed over a height and length of $h_s = 48 \text{ in.}$ [1200 mm])

where

A_{wl} = area of steel in longitudinal direction at beam end for torsional equilibrium

A_{wv} = area of steel in vertical direction at beam end for torsional equilibrium

d_w = depth of A_{wl} and A_{wv} reinforcement from outside face of L beam

Notation

a = applied load eccentricity measured to the centroid of the hanger reinforcement

A = cross-sectional area

A_l = area of longitudinal reinforcement to resist longitudinal bending in ledges

A_{ps} = area of prestressing strand

A_s = area of transverse flexural reinforcement

A_{sh} = area of hanger reinforcement

A_{so}/s = vertical shear reinforcement on the outer web face (that is, non-ledge web face)

A_{wl} = area of steel in longitudinal direction at beam end for torsional equilibrium

A_{wv} = area of steel in vertical direction at beam end for torsional equilibrium

b = width of web

b_l = width of web and one ledge

b_t = width of the double-tee stem or the width of the bearing pad, whichever is less

d = distance from the extreme compression fiber to the centroid of longitudinal tension reinforcement (per ACI 318-14¹³), but not less than $0.8h$ for prestressed components (d_p is used for prestressed components when a distinction from d for nonprestressed reinforcement is relevant)

d_e = distance from the center of an applied concentrated load to the end of the ledge

d_l = depth of centroid of reinforcement in L beam ledges

d_p = distance from extreme compression fiber to centroid of prestressing reinforcement (not less than $0.8h$)

d_s = distance from extreme compression fiber to the centroid of hanger reinforcement in L beam

d_w = depth of A_{wl} and A_{wv} reinforcement from outside face of L beam

e = eccentricity of design load measured from centroid of section

f_c = specified compressive strength of concrete

f_{pc} = average prestress after losses at a given location

f_y = specified yield strength of shear reinforcement

h = height of beam

h_l = height of beam ledge

h_s = distance between torsional equilibrium reactions

- l_p = projection of the ledge
- m = modification factor for hanger reinforcement design defined in section 5.5.4 in the seventh edition of the *PCI Design Handbook*
- M_n = nominal moment capacity of the beam at the given location determined in accordance with section 5.2 of the seventh edition of the *PCI Design Handbook*
- M_u = factored moment in the beam at the given location
- N_{lu} = factored ledge friction load (N_u in section 5.5 of the seventh edition of the *PCI Design Handbook*)
- P_{pd} = prestress force limited by strand development
- R = larger of the two ratios M_u/M_n and V_u/V_n
- s = spacing between applied concentrated loads
- V_c = nominal shear strength provided by concrete, determined in accordance with section 5.3 of the seventh edition of the *PCI Design Handbook*
- V_{ln} = nominal ledge capacity (V_n in section 5.5 of the seventh edition of the *PCI Design Handbook*)
- V_{lu} = factored ledge vertical load (V_u in section 5.5 of the seventh edition of the *PCI Design Handbook*)
- V_n = nominal shear strength of the beam at the given location
- V_s = nominal shear strength provided by shear reinforcement, determined in accordance with the slender spandrel procedure developed by Lucier et al.⁶
- V_u = factored shear in the beam at the given location
- x = shorter side of the component rectangles forming the ledge and the web
- y = longer side of the component rectangles forming the ledge and the web
- β = shear strength coefficient dependent on the level of global stress
- γ = factor accounting for the level of prestressing
- γ_t = factor used in designing hanger reinforcement as defined in section 5.5.4 in the seventh edition of the *PCI Design Handbook*
- λ = modification factor for density of concrete (section 5.3.3 in the seventh edition of the *PCI Design Handbook*)
- ϕ = strength-reduction factor