Simplified design for positive restraint continuity moment in bridge girders

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- Many U.S. states design precast, prestressed concrete continuous bridges as simple spans for both dead and live loads without considering any moments developed by the span connections.
- The effect of thermal expansion and contraction is hardly considered in the analysis, and there is no consensus on the best method to calculate restraint moments that develop in the continuity diaphragm or how to detail positive moment connections.
- The objective of this paper is to provide a simplified spreadsheet calculation of the positive restraint moments and the number of strands to be extended into the cast-in-place concrete diaphragm in order to control cracking.

Bridges composed of simple-span precast concrete beams that are made continuous by the deck reinforcement are also known as continuous-for-live-load bridges. Design of these bridges for continuity over piers is explained in the American Association of State Highway and Transportation Officials' *AASHTO LRFD Bridge Design Specifications*, article 5.14.1.4.[1]

In these bridges, the beams carry their own dead load and the slab dead load as simple spans, but all subsequent loads are carried as continuous spans. Deck reinforcement provides the negative moment resistance. In these designs, beams camber upward due to beam creep. If net camber due to simple-span loads (girder weight, prestressing, and deck weight) is upward, a positive moment develops due to concrete creep and the connection cracks. For this reason, the AASHTO LRFD specifications require a positive moment connection at the joint.

Literature survey

When National Cooperative Highway Research Program (NCHRP) report 519[2] was published, article 5.14.1.4 contained a provision stating that a connection could be considered continuous if the net stress at the bottom of the diaphragm from superimposed permanent loads, settlement, creep, shrinkage, temperature gradient, and 50% of the live load was compressive. Because this provision was already in the AASHTO LRFD specifications and was consistent with the experimental results showing that the connection could tolerate some cracking, the provision was retained as article 5.14.1.4.5.

iven that the calculation of diaphragm stresses is complex, there was a desire for a simple rule. At the time, the Tennessee Department of Transportation had a provision requiring the beams be aged 90 days before continuity was established. By that time, it is estimated that 60% of the creep has already taken effect. This was adopted as article 5.14.1.4.4, which requires the engineer to provide a positive moment connection with a strength of $1.2M_{cr}$, where M_{cr} is the cracking moment, and to specify in the contract documents that the beams are to be at least 90 days old when continuity is established.

Chebole[3] conducted a parametric study to assess the effect of girder age on restraint moment in continuous bridges. The study was based on the method and the computer program developed in NCHRP report 519,[2] which are the basis for the current AASHTO LRFD specifications for restraint moment reinforcement. The required amount of restraint moment reinforcement is excessive due to using a strength limit state.

The hand-calculation method presented herein is a simplification of the initial strain method developed in the 1970s by Tadros et al.[4] and used in commercial time-step software. The solution aims to control cracking at the joint. It relates the required reinforcement to a serviceability limit state. Using the proposed approach frees the designer from having to wait 90 days before the girder is allowed to be made continuous over the piers.

Current provisions

Article 5.14.1.4 of the AASHTO LFRD specifications summarizes the provisions and commentary of AASHTO, providing four design options:

- Provide a positive moment connection with a strength of $1.2M_{cr}$, and require the beams to be at least 90 days old at the time continuity is established.
- Provide a positive moment connection with a strength of $1.2M_{cr}$, and use the provisions of article 5.4.2.3 with time development factor $k_{td} = 0.7$ to establish the minimum age at which continuity can be established.
- Use the provisions of article 5.14.1.4.5, and consider the bridge continuous if the net stress at the bottom of the diaphragm from superimposed permanent loads, settlement, creep, shrinkage, temperature gradient, and 50% of live load is compressive.
- Calculate the actual restraint moments, and determine the degree of continuity from the analyses (article 5.14.1.4.2).

This paper offers a simplified procedure for application of the fourth option.

Calculation of restraint moments

The initial strain theory that can be used to perform time-dependent analysis of a composite prestressed concrete bridge member of any cross section is described herein. This method is based on traditional composite section analysis, using pseudoelastic properties to reflect creep characteristics. The so-called initial strains are introduced in the analysis to account for the effects of member restraint due to concrete creep. In the analysis, the effect of restraint caused by noncreeping steel is ignored. It will be shown that no creep effects take place due to any loads introduced after continuity is made. Superimposed dead loads, live loads, temperature, deck shrinkage, and the like, can be assumed to be elastic (instantaneous) with negligible creeping effect.

Stress-strain-time relationship

The stress-strain-time relationship for the concrete is used to predict the total strain ε at a future time t that results from a stress increment applied at time t_o. The total concrete strain at any time t can be separated into three components:

- ε_o = immediate concrete strain due to the applied stress f
- ε_{cr} = time-dependent creep concrete strain
- ε_{sh} = free shrinkage concrete strain (this term will be ignored in future discussion because it does not contribute significantly to the restraint moments)

Constant stress

The total concrete strain $\varepsilon = \varepsilon_o + \varepsilon_{cr}$ is usually expressed as Eq. (1).

$$\varepsilon = \frac{f(t_o)}{E_c(t_o)}\left[1+\psi(t,t_o)\right] \qquad (1)$$

where

$f(t_o)$ = applied stress at time t_o

$E_c(t_o)$ = modulus of elasticity of concrete at time t_o

$\psi(t, t_o)$ = creep coefficient during a time interval from t_o to t for stress applied at time t_o and kept constant

Equation (1) applies as long as stress f is a constant, sustained stress. **Figure 1** shows the gradual development of creep strain associated with constant stress over time. Both the modulus of elasticity E and the creep coefficient ψ are functions of time. In addition, because concrete is an aging material, ψ depends on the loading age t_o as well.

Variable stress

Where the applied stress f is variable, Eq. (1) cannot be directly used. **Figure 2** depicts the development of creep strain

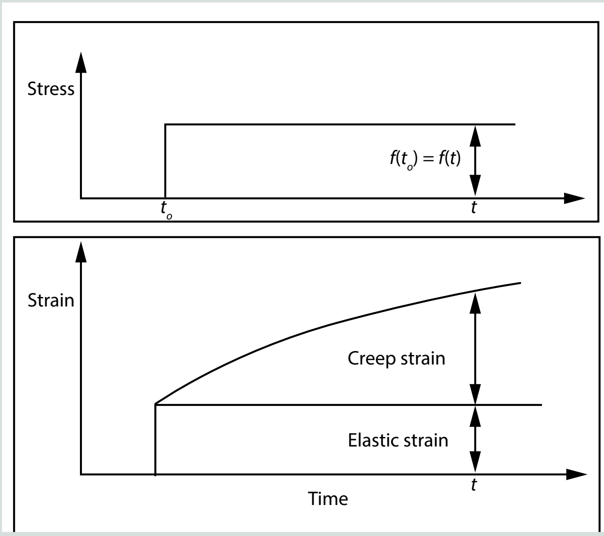

Figure 1. Concrete strain over time under constant stress, shrinkage excluded. Note: $f(t)$ = applied stress at time t; $f(t_o)$ = applied stress at time t_o; t = time (infinity); t_o = beginning of the interval under loading being considered.

under the effect of a gradually introduced stress. Using the principle of superposition, the effects of a series of applied stress increments can be determined individually using Eq. (1) and then combined to give the total time-dependent concrete strain. This approach is often called the time-step method and is suitable for numerical computer modeling. Another approach is called the age-adjusted effective modulus method, in which an aging factor is applied to the creep coefficient to account for the effect of the stress being gradually introduced to an aging concrete with gradually increasing modulus of elasticity and decreasing creep effect.[5] The aging coefficient at a certain time $\chi(t, t_o)$ is a function of the age of the concrete at the time of initial load introduction and a number of other factors. Equation (2) calculates the total strain.

$$\varepsilon = \frac{f(t_o)}{E_c(t_o)}\left[1+\chi(t,t_o)\psi(t,t_o)\right] \tag{2}$$

A pseudoelastic analysis can be performed by assuming a reduced modulus of elasticity that accounts for creep effects. Equation (3) gives the age-adjusted effective modulus of elasticity of concrete for constant sustained stress $E^*_{ctc}(t,t_o)$.

$$E^*_{ctc}(t,t_o) = \frac{E_c(t_o)}{1+\psi(t,t_o)} \tag{3}$$

Equation (1) can be rewritten as Eq. (4) to take advantage of the effective modulus concept.

$$\varepsilon = \frac{f(t)}{E^*_c(t,t_o)} \tag{4}$$

For gradually developing stress, Eq. (5) calculates the age-adjusted effective modulus.

$$E^*_{ctv}(t,t_o) = \frac{E_c(t_o)}{1+\chi(t,t_o)\psi(t,t_o)} \tag{5}$$

The age-adjusted effective modulus of elasticity is referred to in this paper as defined by Eq. (5). Equation (3) is considered to represent a special condition: instantaneously applied load with the aging coefficient χ equal to 1. Further simplification is to assume a constant aging coefficient χ of 0.7, which has been shown to be reasonable by Dilger[6] and by Tadros et al.[7] for the type of application (prestressed concrete beams with prestress and self-weight introduced at age of concrete of one to three days).

Calculation of restraint moment in precast concrete beams based on initial strain theory

Creep analysis for positive restraint moments over piers has been studied in detail by Mattock and Kaar,[8] Freyermuth,[9] Tadros et al.,[4] Dilger,[6] Tadros et al.,[7] Oesterle et al.,[10] and Miller et al.[2] Creep of the beam under the net effects of prestressing, self-weight, deck weight, and superimposed dead loads will tend to produce additional upward camber with time. Shrinkage of the deck concrete will tend to produce downward deflection of the composite system with time. In addition, loss of prestress due to creep, shrinkage, and relaxation will result in downward deflection. Depending on the properties of the concrete materials and the age at which the beams are erected and subsequently made continuous, either positive or negative moments may occur over continuous supports.

In situations where beams are made continuous at a relatively young age, it is more likely that positive moments will develop with time at the supports. These positive restraint moments are the result of the tendency of the beams to continue to

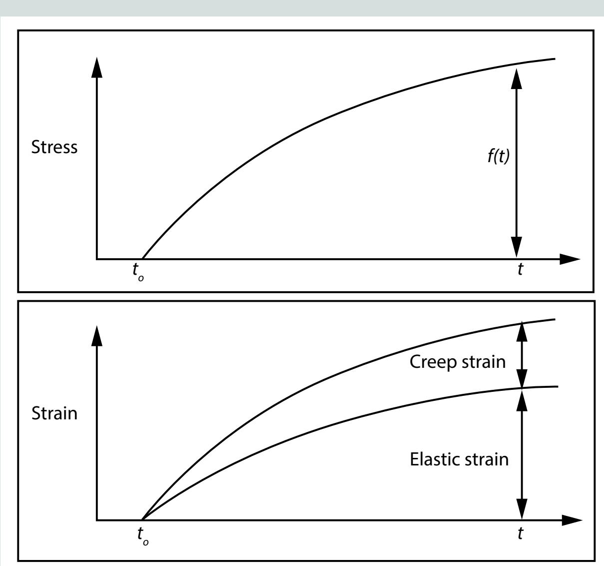

Figure 2. Concrete strain over time under variable stress, shrinkage excluded. Note: $f(t)$ = applied stress at time t; t = time (infinity); t_o = beginning of the interval under loading being considered.

camber upward as a result of ongoing creep strains associated with the prestressing forces. Shrinkage of the deck concrete, loss of prestressing, and creep strains due to self-weight, deck weight, and superimposed dead loads all have a tendency to reduce this positive moment.

When beams are mature when they are made continuous, negative moments at the supports can result. This is especially true when the beams have short spans and large depths, thus requiring relatively low prestress levels. In these conditions, the time-dependent creep strains associated with prestressing have diminished to the point that the creep effects that produce downward deflection (beam weight and deck weight) are larger. This will induce negative restraint moments at the supports.

The effects of positive moments and associated diaphragm cracking on bridge performance continue to be hotly debated. An argument can be made that continuity for live loads becomes unreliable after a small crack has opened near the bottom of the diaphragm.[10] A finite end rotation is required to close this crack, forcing the beam to carry live loads as a simple-span member. Theoretically, this simple-span action results in live-load moments that are significantly higher than those predicted by calculations that assume full continuity.

Countering this argument, however, is the successful experience of the many agencies that routinely design precast, prestressed concrete bridges under the assumption of full continuity for live loads. Cracking at the bottom fibers of the midspan region of these bridges due to smaller live-load mo-

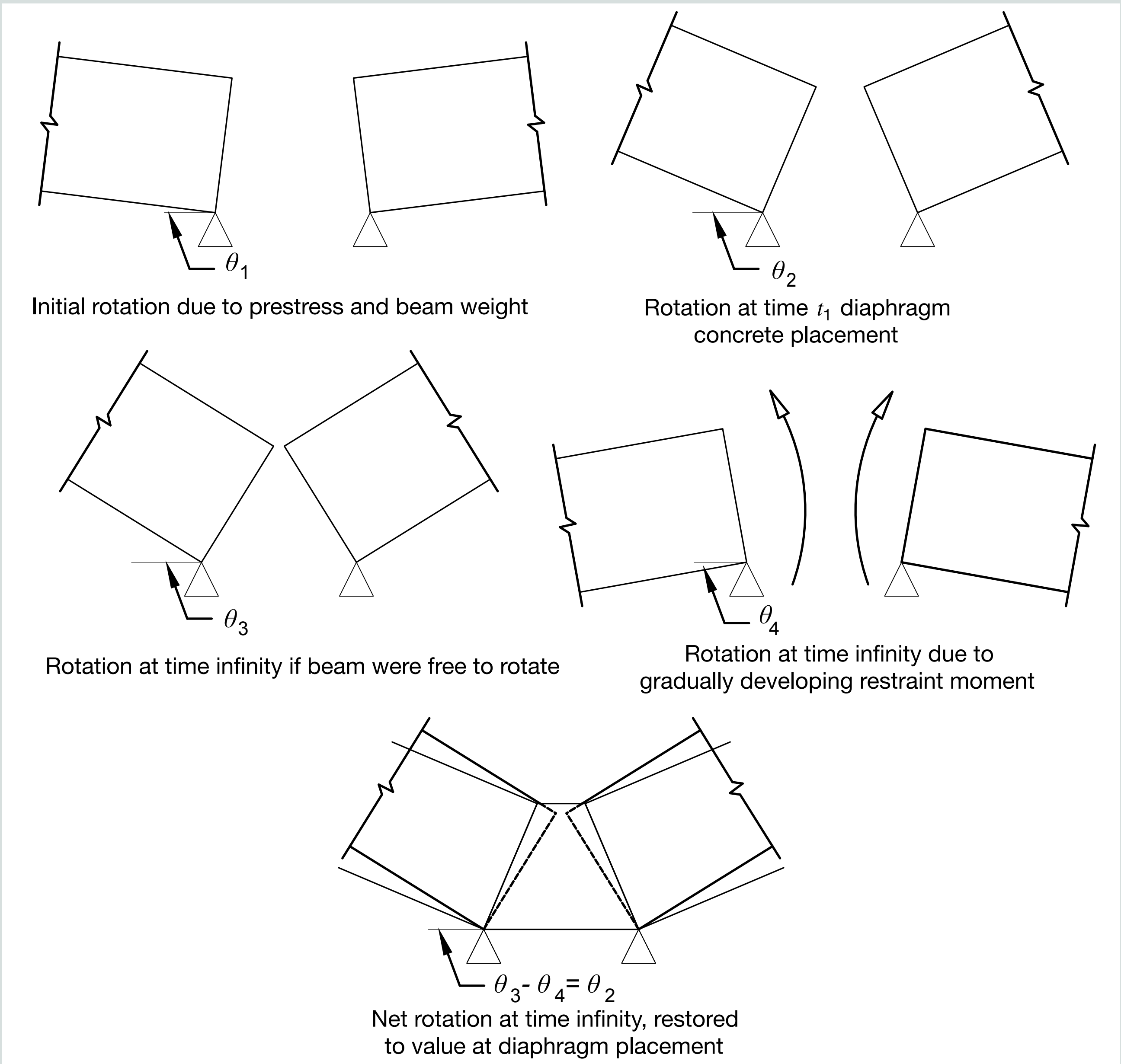

Figure 3. Progression of beam end rotation at pier support.

ments in a continuous-span scenario has not been reported. In addition, only service load behavior is significantly affected. Under ultimate loads, end rotations of the beams will be large enough to close any crack that may have opened at the piers, restoring full continuity. It should be agreeable, therefore, that ultimate capacity is unaffected by existence of controlled bottom cracks in the beams at the pier areas.

The detailed pseudoelastic analysis is generally done using full, uncracked concrete section properties. It does not account for loss of stiffness due to cracking, which can create considerable relief of the calculated restraint moment. In some countries, a reduction factor of 0.9 is applied to the uncracked section restraint moment. Even with this reduced moment, the analysis given herein is considered conservative.

Understanding which loads create creep restraint

As a general statement, loads (including the effect of prestressing) applied before continuity is made create creep restraint moments in the continuous beam. Alternatively, loads introduced after the beam becomes continuous only create elastic (no-creep) restraint moments.[11–14]

The analysis is made for a continuous beam with rigid vertical supports while allowing the beam to rotate and to translate horizontally over the supports. Only one support is fixed against horizontal translation to maintain stability. With these assumptions, uniform shrinkage and temperature change only induce axial deformation and not curvature or flexure.

The analysis, using pseudoelastic properties, allows for the superposition of various effects. As shown later, the analysis is given separately for each of the following effects and then superimposed to create the net effect. These effects are: prestressing, beam weight, deck weight, superimposed dead loads (barriers and overlays), deck shrinkage, and temperature gradient.

The analysis is illustrated with a two-span example. Assume the rotation at the center support of the left beam is θ_1 (**Fig. 3**) at the initial time when the prestressing is released. That rotation increases to θ_2 as concrete creeps at the time the beams are ready to be connected with cast-in-place concrete diaphragms. If the beams were still free to rotate, the end rotation would increase to θ_3. However, the continuity restraint moment would cause the rotation to remain unchanged by imposing an offsetting rotation θ_4 such that $\theta_3 - \theta_4 = \theta_2$.

The value of θ_1 represents the elastic deformation at the beam end θ_{el}. Creep causes the rotation to grow by an increment $\theta_3 - \theta_2 = \theta_{el}\psi_1$, where ψ_1 is the creep coefficient between t_o and time infinity. That deformation cannot happen if the two beam ends are joined with a rigid connection. Thus a restraint moment develops gradually and causes the ends to have an equal and opposite rotation to that caused by the applied loads. That rotation θ_4 is calculated as $\theta_{r,el}(1 + \chi\psi_2)$, where $\theta_{r,el}$ is the elastic restraint rotation, χ is the aging coefficient, and ψ_2 is the creep coefficient for concrete loaded from time of deck placement to time infinity. By setting these two rotations equal to each other, the resulting restraint moment is equal to the moment that would result elastically if it were multiplied by the factor $\psi_1/(1 + \chi\psi_2)$.

Alternatively, a load that is introduced just after continuity is made would have a free rotation of $\theta_{el}(1 + \psi_2)$ and a restraining rotation of $\theta_{r,el}(1 + \chi\psi_2)$. For this case, the restraint moment is equal to the elastic moment multiplied by $(1 + \psi_2)/(1 + \chi\psi_2)$, which would be equal to the elastic moment if χ is approximated as 1.0. If the loading after continuity is gradually introduced at the same rate as creep development, the restraint moment would be exactly equal to the elastic moment. Therefore, it is reasonable to assume that there is no creep restraint moment due to loads that are introduced after continuity is made. In a typical design, the restraint moment would consist of creep moment due to prestressing, beam weight, deck weight, and elastic moment due to superimposed dead load, live load, and daily temperature gradients.

Steps of analysis

Specifically, the following procedure is used for each load:

1. Calculate time-dependent material properties.

   $\psi(t, t_o)$ = creep coefficient at time t for concrete loaded at time t_o, specifically prestressing and beam weight

   $\psi(t, t_1)$ = creep coefficient at time t for concrete loaded at time t_1, specifically deck weight and the restraint moment

   $\psi(t_1, t_o)$ = creep coefficient at time t_1, for concrete loaded at time t_o

   Time t is generally assumed equal to 75 years or 27,000 days. Several authors have assumed 2000 days and 20,000 days to represent the time at which creep growth becomes nearly zero.

   The age-adjusted effective modulus of elasticity of concrete subjected to gradual loading at time t_1 with creep developing in the period $(t - t_1)$ $E^*_{ctv}(t,t_1)$ is calculated by Eq. (6).

   $$E^*_{ctv}(t,t_1) = \frac{E_c(t_1)}{1+0.7\psi(t,t_1)} \tag{6}$$

   The age-adjusted effective modulus of elasticity of concrete subjected to constant stress introduced at time t_o with creep developing in the period $(t - t_1)$ is calculated by Eq. (7).

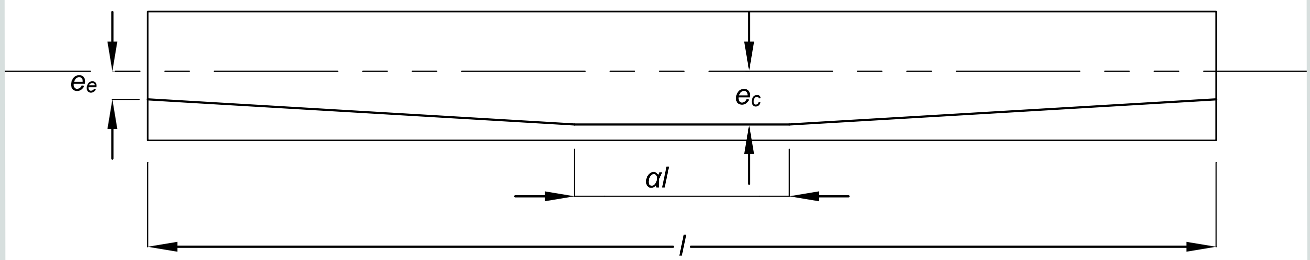

Figure 4. Pretensioned strand profile. Note: e_c = eccentricity of the strand at midspan; e_e = eccentricity of the strand at beam ends; l = overall beam span; α = coefficient of linear thermal expansion.

$$E^*_{ctv}(t,t_o) = \frac{E_c(t_o)}{\psi(t,t_o) - \psi(t_1,t_o)} \tag{7}$$

2. Perform elastic analysis, assuming the load is introduced to a continuous member. Determine the fictitious elastic restraint moments at the supports M_{el}.

3. Determine the time-dependent multiplier corresponding to the load δ_c using Eq. (8).

$$\frac{E^*_{ctv}(t,t_1)}{E^*_{ctc}(t,t_o)} \tag{8}$$

4. Determine the cracking moment $M_{cr}(t)$ at time t using Eq. (9).

$$M_{cr}(t) = \delta_c M_{el} \tag{9}$$

5. Calculate the net total moments by adding the creep restraint moments due to all loads applied before continuity is effected to the elastic continuity moments after continuity becomes effective. There needs to be two values for design, the maximum and minimum values. For example, the maximum positive moment should not include the negative moment due to live load. Although the future wearing surface load is considered to be dead load, its negative moment effect should not be included because its application may happen many years after the bridge is constructed.

Calculation of continuity moments due to pretensioning

Pretensioning is conventionally done on simple-span beams. As such, there are no continuity forces as long as the beam remains a simple span for its life. However, because it is made continuous with the adjacent spans along the bridge, creep effects generate continuity moments. As discussed in previous sections, the creep effects are determined as factors of the elastic moments that would happen if the beam were made continuous from the start. Thus, we need to determine these fictitious elastic continuity moments. The process is similar to the calculation of the secondary moments due to post-tensioning.[15–19]

Consider the pretensioning strand profile in **Fig. 4**. This profile is a general condition of other simpler profiles, such as single-point depression and straight profiles.

The steps of analysis are as follows:

1. Determine the equivalent loads due to prestressing.

2. Run continuous-span analysis for the loads in step 1.

3. The statically indeterminate moments at the joints (not counting the moment due to prestressing eccentricity Pe_e, where P is the effective prestressing force and e_e is the eccentricity of the strand at beam ends) are the elastic moment needed in the creep analysis.

Equation (10) gives the elastic continuity moment due to prestress $M_{p,el}$ for the special case of a two-span bridge with two equal spans prestressed with strands that have symmetrical profiles (Fig. 4).

$$M_{p,el} = \frac{3P}{4}\left[2e_e + (1+\alpha)(e_c - e_e)\right] \tag{10}$$

where

α = coefficient of linear thermal expansion

e_c = eccentricity of the strand at midspan

e_e = eccentricity of the strand at the ends

Effects of deck shrinkage and temperature gradient

Both deck shrinkage and temperature gradient occur after the bridge beams have been made continuous over the pier supports. As indicated earlier, effects that occur after continuity should have negligible creep redistribution. Their effects can thus be most closely modeled as elastic effects. Because shrinkage and temperature gradient are volume-change effects, the initial strain theory can be employed for their analysis, except that only the elastic modulus without adjustment is used.

For simplified analysis, shrinkage of the deck may be taken in isolation without consideration of shrinkage of the beam because the latter is already accounted for implicitly when estimating the prestress losses. If a computer program is used, as in the analysis of segmental bridges, then a more precise differential shrinkage could be used. If the approach proposed here is adopted, then analysis for deck shrinkage is greatly simplified. Because analysis for temperature gradient is significantly more complex, it can be adapted to deck shrinkage. Roberts-Wollman et al. provides more discussion of these effects.[20]

The AASHTO LRFD specifications, article 3.12.3, outlines the temperature gradient that should be used to calculate thermal effects. **Figure 5** shows the temperature gradient used for this type of bridge and in the design example section in this paper.

The coefficient of linear thermal expansion α is taken as 6×10^{-6} in./in./°F (1×10^{-5} mm/mm/°C) for normalweight concrete. Analysis for temperature effects using the initial strain theory is described in the next section. It is valid for all span and support conditions. Another option is offered in the commentary in the AASHTO LRFD specifications, which gives a closed-form solution. However, the solution is limited to a two-span beam with equal spans.

This treatment of temperature effects is highly simplified and should be considered as such in design. It assumes that the temperature rises instantaneously. It does not distinguish between daily and seasonal temperature changes. It does not explicitly account for concrete's ability to absorb and store heat. Thus, it should not be treated as more than an approximate quantification of a much more complex effect.

Analysis steps

1. Calculate the free strain due to the temperature rise (Fig. 5) $\alpha \Delta T$ at various fibers along the cross-section depth. This calculation assumes that the fibers are free to deform without any restraint.

2. Divide the section into layers of similar geometric and material properties. Apply restraining forces in these layers to cancel the free strains in step 1. Each force is equal to the strain times the modulus of elasticity times the area. The force is a stress resultant and is located at the geometric center of the stress diagram. Determine the total moment of the restraining forces about the centroid of the composite section.

3. Restore equilibrium by applying equal and opposite forces and moments on the composite section, assuming the beam to be free of external supports. Place the supports and perform a structural analysis of the continuous beam for the equilibrium forces. Determine the internal stresses due to the forces resulting from the statically indeterminate beam analysis.

4. The summation of the deformations and stresses in steps 1, 2, and 3 produces the net effects of the temperature gradient.

Design of bottom continuity reinforcement

The preceding part of this paper discusses methods of calculating the time-dependent positive restraint moment in continuous precast, prestressed concrete bridge beams at the joints over the intermediate supports (piers). In all previous discussion, the analysis is completed using pseudoelastic analysis with uncracked section properties. If the positive restraint moment due to the most critical loading combination is less than the moment needed to create flexural cracking at the bottom of the beam at the face of the cast-in-place concrete diaphragm, then no reinforcement would be necessary. However, it is common to expect higher moments than the cracking moment and, thus, flexural cracking. In this case, two issues emerge: how to use the uncracked member analysis to extrapolate for estimation of the applied moment while the member is cracked and how to use the applied moment to calculate the required steel reinforcement and to detail it for acceptable serviceability performance. A related issue is how to design the continuous span beam for negative moments due to full live-load effects, considering that it may be cracked over some lengths in the pier regions.

The issues given here have not been fully resolved through research. This explains the wide variation of practices, which rely on experience and engineering judgment. Until research produces better results, the following recommendations are made:

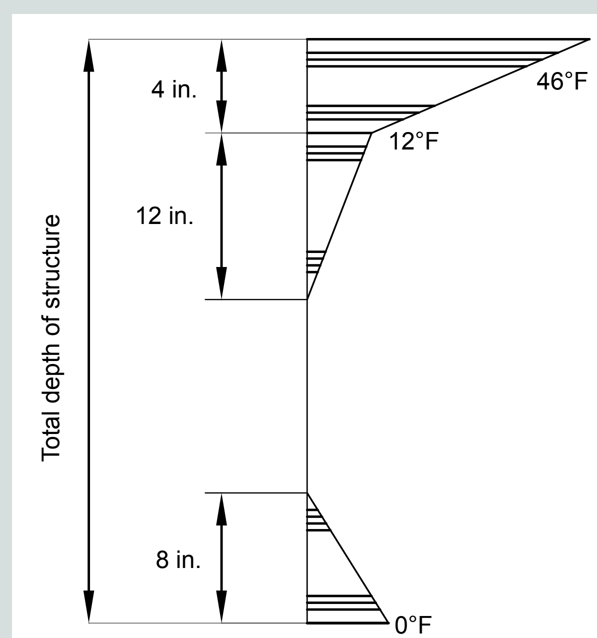

Figure 5. Design temperature gradient throughout the cross section according to the seventh edition of the *AASHTO LRFD Bridge Design Specifications*. Note: 1 in. = 25.4 mm; °C = (°F – 32)/1.8

- The restraint moment is calculated using uncracked section analysis. The moment may be reduced by as much as 10% to account for reduction due to cracking in the bottom fibers near the pier supports. Although the 10% reduction is quite conservative, it is somewhat consistent with the redistribution allowed for negative moment design.

- Reinforcement is designed using cracked-section capacity analysis. A simplified calculation of the steel stress may be used assuming the lever arm between the compression and tension forces is equal to 0.9*d*, where *d* is the effective depth from the compression face to the center of the tensile reinforcement. Axial force from deck shrinkage and temperature can be accounted for by assuming half of the axial force carried by the bottom reinforcement.

- Tensile steel stress is limited to the value obtained from crack control formulas, such as the one recommended by Frosch.[21] A maximum limit on steel stress of 36 ksi (250 MPa) would correspond to a maximum crack width of 0.01 in. (0.25 mm). This limit would also allow justification for assuming full continuity for negative moment analysis, according to Miller.[2]

Design example

The following example is a typical overpass bridge over Interstate 80 near Omaha, Neb. The geometric properties, material properties, and environmental conditions are similar to what is generally assumed in that area of the United States. The bridge is designed for the HL-93 live loading specified by the AASHTO LRFD specifications. **Figure 6** shows the cross section of the composite NU900 girder with the cast-in-place concrete deck as used in the example.

Bridge data

Bridge width = 50 ft (15 m)

Bridge length l = 180 ft (55 m) (two 90 ft [27 m] spans)

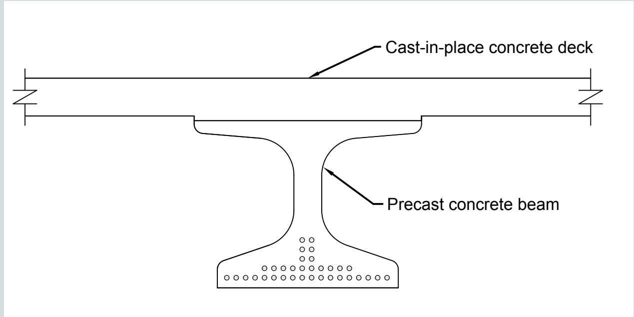

Figure 6. Composite cross section of NU900 beam used in example.

Beam depth for NU900 = 35.4 in. (0.900 m)

Beam spacing = 10 ft (3 m)

Beam concrete strength = 5.5 ksi (38 MPa) cylinder strength at release

Beam strength = 8 ksi (55 MPa) cylinder strength at 28 days

Beam prestressing = thirty-four 0.6 in. (15 mm) low-relaxation strands (18 + 10 + 2 + 2 + 2 at a 2 × 2 in. [50 × 50 mm] spacing), with 2 in. bottom cover to strand centerline and draped profile (**Fig. 7**)

Deck thickness = 8 in. (200 mm) cast-in-place concrete, plus constant 1 in. (25 mm) thick haunch (assumed in design)

Deck strength = 4 ksi (28 MPa) cylinder strength at 28 days

Barrier = 20 lb/ft^2 (1 kN/m^2)

Future wearing surface = 25 lb/ft^2 (1.1 kN/m^2) (not used)

Design live load = HL-93 (not used)

Relative humidity = 70%

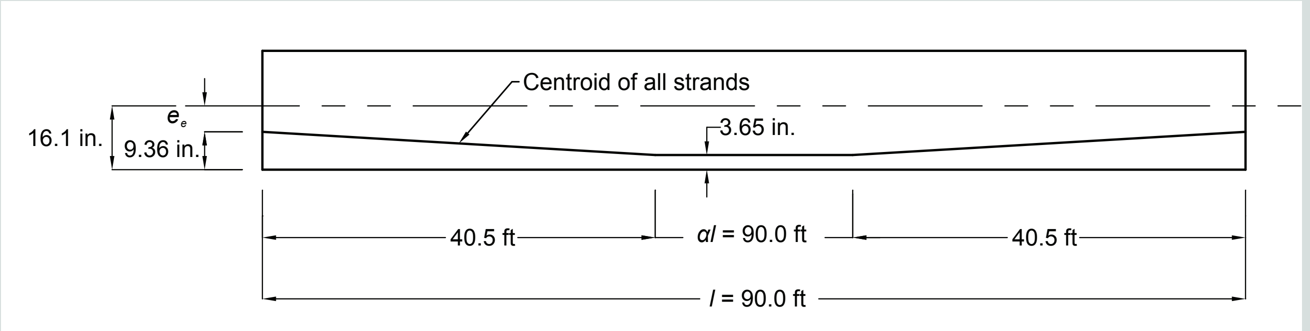

Figure 7. Beam elevation with strand profile. Note: e_e = eccentricity of the strand at beam ends; l = beam span; α = coefficient defining drape point location relative to span length.

Time prestressing strand released t_i = 1 day

Time diaphragm and deck placed t_d = 28 days

Time of end of beam life t = 20,000 days

Precast concrete beam moment of inertia I = 110,444 in.^{4} (0.0459686 m^4)

Precast concrete beam area A_b = 649 in.^{2} (0.419 m^2)

Beam depth h = 35.43 in. (0.900 m)

Beam centroid to bottom fiber y_b = 16.10 in. (0.409 m)

Composite section centroid to bottom fibers y_{bc} = 28.48 in. (0.723 m)

Beam weight w = 0.676 kip/ft (9.86 kN/m)

Modulus of elasticity of beam concrete E_{ci} = 4406 ksi (30,390 MPa) at initial time

Modulus of elasticity of beam concrete E_c = 5314 ksi (36,650 MPa) at 28 days

Modulus of elasticity of deck concrete E_{cd} = 3644 ksi (25,130 MPa) at 28 days

These moduli of elasticity were calculated using the AASHTO prediction equation.

$$E_c = 33{,}000 w^{1.5}\sqrt{f_c'}$$

where

w = 0.145 kip/ft^3 (22.8 kN/m^3) for $f_c' \le$ 5 ksi (34 MPa)
= 0.14 + 0.001 $f_c' \le$ 0.155 kip/ft (2.26 kN/m) for f_c' > 5 ksi (34 MPa)

f_c' = compressive strength of concrete

Beam shrinkage strain between initial time of loading and final time ε_{bif} = 0.000393

Beam shrinkage strain between prestress transfer and deck placement ε_{bid} = 0.000161

Beam shrinkage strain between time of deck placement and final time ε_{bdf} = 0.000232

Shrinkage strain of deck concrete between placement and final time ε_{ddf} = 0.000274

Beam creep coefficient between initial time of loading and final time ψ_{bif} = 1.526

Beam creep coefficient at beam between prestress transfer and deck placement ψ_{bid} = 0.626

Beam creep coefficient between time of deck placement and final time ψ_{bdf} = 1.030

Deck creep coefficient between deck placement and final time ψ_{ddf} = 2.126

Creep coefficients are defined as ratios of creep strain to initial strain for a constant sustained stress.

Restraint moment due to prestressing, beam weight, and deck weight

Only loads introduced before continuity can cause time-dependent restraint moment due to creep. Typically, these are the pretensionsing forces, member self-weight, and deck weight. Each loading case is considered separately, though prestressing and self-weight cannot actually be separated. The total effect is obtained by simple superposition. Elastic analysis is performed first, assuming that the load was introduced to a noncreeping continuous member. The fictitious elastic restraint moments M_{el} are then determined at the supports. The sign convention is that a positive moment creates tension in the bottom fiber.

The elastic moment due to the effect of self-weight of the beam M_o is calculated as

$$M_o = -\frac{wl^2}{8} = -\frac{(0.676)(90)^2}{8} = \text{-684.5 kip-ft (-928.2 kN-m)}$$

where

The elastic moment due to the effect of deck weight M_d (load applied before continuity is made) is calculated as

$$M_d = -\frac{w_d l^2}{8} = -\frac{(1.02)(90)^2}{8} = \text{-1028.1 kip-ft (-1394.2 kN-m)}$$

where

w_d = weight of the deck, including haunch

The following equation calculates the elastic moment due to prestressing release M_p, assuming the beam was continuous and composite with the deck.

$$M_p = \frac{3P}{4}\left[2e_e + (1+\alpha)(e_c - e_e)\right]$$

$$= \frac{3(1269.9)}{4}\frac{\left[2(19.12)+(1+0.1)(24.83-19.12)\right]}{12}$$

= 3533.6 kip-ft (4790.7 kN-m)

where

e_e = 28.48 – 9.36 = 19.12 in. (485.6 mm)

e_c = 28.48 – 3.65 = 24.83 in. (630.6 mm)

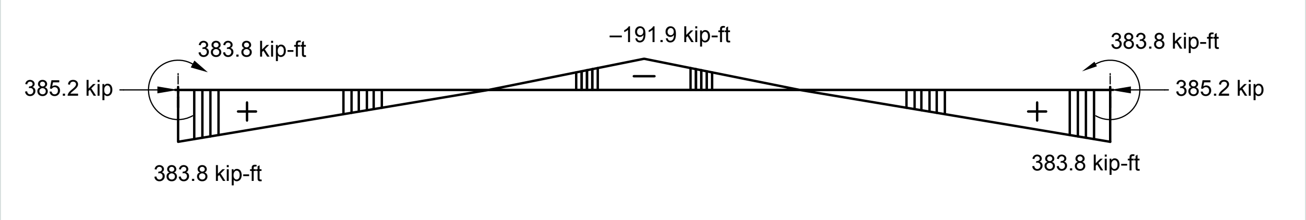

Figure 8. Restraint moment due to deck shrinkage. Note: 1 kip = 4.448 kN; 1 kip-ft = 1.356 kN-m.

P = 85% of the initial prestressing force = (0.85)(34)(0.217)(202.5) = 1269.9 kip (5648.5 kN)

Moment at beam ends due to prestressing (simple span) = -1269.9(16.1 – 9.36)/12 = -713.3 kip-ft (-967.3 kN-m)

The elastic moment due to the effect of self-weight of the barrier $M_{Barrier}$ is calculated as

$$M_{Barrier} = -\frac{wl^2}{8} = -\frac{(0.20)(90)^2}{8} = \text{-202.5 kip-ft (-274.5 kN-m)}$$

where

w = (0.020)(10) = 0.20 kip/ft (2.9 kN/m)

The age-adjusted effective modulus for concrete subjected to gradually introduced restraint moment from time of deck placement to time infinity $E^*_{ctv}(t,t_d)$ is calculated as

$$E^*_{ctv}(t,t_d) = \frac{E_c(t_d)}{1+0.7\psi(t,t_d)} = \frac{5314}{1+0.7(1.030)}$$

$$= 3088 \text{ ksi (21.29 GPa)}$$

where

$\psi(t, t_d)$ = creep coefficient for load applied at time t_d and sustained to time t

The age-adjusted effective modulus of elasticity of concrete subjected to a constant stress introduced at t_i with creep determined to the period $(t – t_d)$, $E^*_{ctc}(t,t_i)$ is calculated as

$$E^*_{ctc}(t,t_i) = \frac{E_c(t_i)}{\psi_{bif} - \psi_{bid}} = \frac{4406}{1.526-0.626} = 4896 \text{ ksi (33.76 GPa)}$$

where

$E_c(t_i)$ = modulus of elasticity of concrete at time t_i

Determine the time-dependent multiplier corresponding to prestressing and beam self-weight δ_1.

$$\delta_1 = \frac{E^*_{ctv}(t,t_d)}{E^*_{ctc}(t,t_o)} = \frac{3088}{4896} = 0.631$$

The age-adjusted effective modulus of elasticity for beam concrete due deck weight E_{cd} is calculated as

$$E_{cd} = \frac{E_c(t_d)}{1+1.0(1.030)} = \frac{5314}{1+1.0(1.030)} = 2618 \text{ ksi (18.05 GPa)}$$

where

$E_c(t_d)$ = modulus of elasticity of concrete at time of deck placement t_d

Determine the time-dependent multiplier due to deck weight δ_2.

$$\delta_2 = \frac{E_{cd}}{E^*_{ctc}(t,t_o)} = \frac{2618}{4896} = 0.535$$

Table 1. Calculation of restraint and equilibrium due to temperature gradient

Area number	Width, in.	Depth, in.	Centroidal distance, in.	Temperature gradient	Restraining stress, ksi	Restraining force, kip	Restraint moment, kip-ft
1	82.286	4	13.96	29.00	0.925	304.36	4247.8
2	82.286	4	9.96	6.00	0.191	62.97	627.0
3	33.209	1	7.46	6.00	0.191	6.35	47.4
4	48.430	2.5625	5.68	6.00	0.191	23.74	134.7
5	27.070	1.77	3.51	6.00	0.191	9.17	32.2
6	11.240	2.6875	1.28	6.00	0.191	5.78	7.4
					Total	412.37	424.70
						Total restraint moment	212.4

Note: 1 in. = 25.4 mm; 1 kip = 4.448 kN; 1 kip-ft = 1.356 kN-m; 1 ksi = 6.895 MPa.

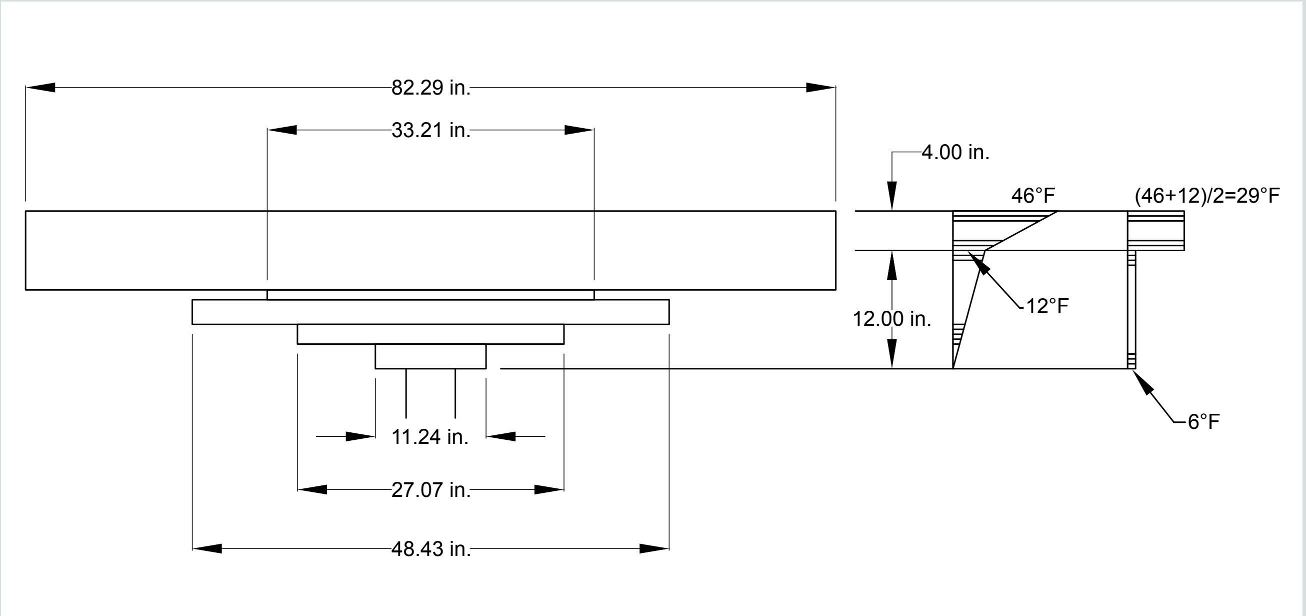

Figure 9. Simplified model of temperature gradient. Note: 1 in. = 25.4 mm; °C = (°F – 32)/1.8

Determine the restraint moment M_r:

The restraint moment due beam weight M_{r1} is calculated as $M_{r1} = \delta_1 M_o$ = (0.631)(-684.5) = -431.9 kip-ft (-585.5 kN-m)

The restraint moment due to prestressing M_{r2} is calculated as $M_{r2} = \delta_1 M_p$ + initial elastic moment = (0.631)(3533.6) – 713.3 = 1516.1 kip-ft (2055.4 kN-m)

The moment due to prestressing is positive, and the moment due to self-weight of the beam is negative.

The restraint moment due to deck weight M_{r3} is calculated as $M_{r3} = \delta_2 M_d$ = (0.535)(-1028.1) = -550.0 kip-ft (-745.7 kN-m)

Restraint moments due to deck shrinkage

Modular ratio = 3644/5314 = 0.686

Beam spacing = 10 ft (3 m)

Span = 90 ft (27 m)

Deck thickness = 8 in. (200 mm)

Area of the deck A_d = 960 in.^{2} (620,000 mm^2)

Moment of inertia of composite section = 308,248 in.^{4} (1.28303 × 10^{11} mm^4)

Centroid of the section from the top = 15.956 in. (405.28 mm)

Total depth of the cross section = 44.43 in. (1129 mm)

Deck shrinkage strain ε_s = 0.000274

Compressive force due to deck shrinkage

$$= \left(\varepsilon_s A_d E_{cd}\right)\left(\frac{1}{1+0.7\psi_{ddf}}\right)$$

$$= (0.000274)(960)(3644)\left[\frac{1}{1+0.7(2.126)}\right] = 385.2 \text{ kip}$$
(1714 kN)

Moment due to shrinkage

$$M_{sh} = \frac{(385.2)\left(15.956-\frac{8}{2}\right)}{12} = 383.8 \text{ kip-ft (520.5 kN-m)}$$

This moment is applied as external end moment on the beams. Using continuous beam analysis software, the restraint moment caused at the interior support can be found. **Figure 8** shows the restraint moment due to deck shrinkage.

Total restraint moment = -191.9 kip-ft (-260.2 kN-m)

Restraint moment due to thermal gradient

For this example, the cross-section geometry is simplified into layers of rectangular shapes. Further, the temperature gradient is simplified into a series of constant rises in temperature (**Fig. 9**).

Table 1 gives the calculations of the initial strains and restraining forces. As for the shrinkage forces (Fig. 8), the pier restraint moment is half of the applied end moment. The exception here is that the temperature restraint moment is

Figure 10. Strand bending at beam end. Courtesy of Coreslab Structures, Omaha, Neb.

Figure 11. Bridge under construction near Des Moines, Iowa. Note the bent strands.

Figure 12. Fairview Road overpass over Interstate 80 in Omaha, Neb.

positive (requiring bottom steel), while the shrinkage restraint moment is negative. In other examples where the beam has more than two spans, the moments at the piers would be calculated with a continuous beam with two equal end moments applied at the abutments.

Net restraint moment due to total effects

Restraint moment due to beam weight = -431.9 kip-ft (-585.5 kN-m)

Restraint moment due to prestressing = 1516.1 kip-ft (2055.4 kN-m)

Restraint moment due to deck weight = -550.0 kip-ft (-745.7 kN-m)

Restraint moment due to temperature = 212.4 kip-ft (288.0 kN-m)

Restraint moment due to deck shrinkage = -191.9 kip-ft (-260.2 kN-m)

Elastic moment due to barrier weight = -202.5 kip-ft (-274.5 kN-m)

Total net moment M_{res} = 351.6 kip-ft (476.8 kN-m)

Net restraint axial force due to total effects

Restraint axial force due to beam weight = 0.0 kip (0.0 kN)

Restraint axial force due to prestressing = 0.0 kip (0.0 kN)

Restraint axial force due to deck weight = 0.0 kip (0.0 kN)

Restraint axial force due to temperature = -412.37 kip (-1834.2 kN) (tension)

Restraint axial force due to deck shrinkage = +385.2 kip (+1714 kN) (compression)

Elastic axial force due to barrier weight = 0.0 kip (0.0 kN)

Total net axial force F_{res} = -27.17 kip (-120.9 kN)

Moments or axial force at the piers due to future wearing surface and live loads is not included in this example. These loads create negative moments causing compression at the bottom of the connection and are not permanent.

Restraint moment reinforcement extended from beam end into diaphragm

The stress limit is assumed to be 36 ksi (250 MPa).

The minimum required area of steel $A_{s,required}$ is calculated as

$$A_{s,required} = 0.9\left(\frac{\frac{M_{res}}{jd}+\frac{F_{res}}{2}}{36}\right) = 0.9\left(\frac{\frac{(351.6)(12)}{39.99}-\frac{(-27.17)}{2}}{36}\right)$$

$$= 2.977 \text{ in.}^2 \ (1921 \text{ mm}^2)$$

where

jd = lever arm between tension and compression, assumed to be $0.9h$ = 39.99 in. (1016 mm)

Use fourteen 0.6 in. (15 mm) strands bent into the diaphragm.

Commentary on example results

The number of strands required to be extended for this bridge example is greater than the empirical number of eight strands generally used in the state of Nebraska. The empirical approach in Nebraska has worked well for more than 30 years with no visible signs of bottom-fiber cracking. This would seem to indicate that the proposed analysis is perhaps too conservative. For example, the restraint moment is calculated using uncracked section analysis. Reducing that moment by 10% to 20% could be justified to allow for the fact that cracking relieves the positive moment zone from the moment for which reinforcement is being provided. The reinforcement is only for crack control and should never be thought of as a strength design requirement.

The use of strands in the joint to control cracking is superior to the use of additional L-shaped bars, as some owners seem to specify. Strands are already in the beam and are continuous for the full length. Thus, they do not cause stress concentration as additional short bars placed only at the ends could create. Also, additional bars are inserted between strand positions in the already occupied beam bottom flange. They create additional detailing complications and result in additional costs.

Bending strands into the cast-in-place concrete diaphragms is a common practice in many states. It is done with a simple tool (**Fig. 10**).

In Nebraska, the strands are bent at 6 in. (150 mm) from the end face of the beam. The strands are extended at least 18 in. (460 mm) for a total strand extension beyond the face of the beam of 24 in. (610 mm). The strands are embedded in a 24 in. wide cast-in-place concrete diaphragm. The beams are embedded 8 in. (200 mm) into the diaphragms with an 8 in. gap where the continuity steel exists. The state of Iowa use similar details (**Fig. 11**). The result is an aesthetically pleasing exterior face, as shown for a bridge on Interstate 80 in Nebraska (**Fig. 12**).

Conclusion

A simplified step-by-step method of for calculating the reinforcement needed to control cracking due to creep restraint moment in precast, prestressed concrete beams is presented. It can be used by designers using a simple Excel workbook. It is valid for cases involving any number of spans of any length. The analysis shows that only a few strands need to be extended beyond the end of the beam and embedded in the cast-in-place concrete diaphragms. Such process is common at virtually no additional cost. It is superior to adding reinforcing bars at the ends, which tend to congest the end zone of the beam and create unnecessary stress concentrations. With the proposed analysis, it is no longer needed to place excessive amounts of steel corresponding to flexural strength being equal to $1.2M_{cr}$ or to wait a period of 90 days between girder casting and diaphragm concrete placement.

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Notation

A_b = area of cross section of the precast concrete beam

A_d = area of the deck

$A_{s,required}$ = minimum required area of steel

d = effective depth from the compression face to the center of the tensile reinforcement

e_c = eccentricity of the strand at midspan

e_e = eccentricity of the strand at beam ends

E = modulus of elasticity

E_c = modulus of elasticity of the beam

E_{cd} = modulus of elasticity of the deck

E_{ci} = modulus of elasticity of the beam

$E_c(t_o)$ = modulus of elasticity of concrete at time t_o the beginning of the interval being considered

$E_c(t_d)$ = modulus of elasticity of beam at time t_d

$E_c(t_i)$ = modulus of elasticity of beam at time t_i

$E_c^*(t, t_o)$ = age-adjusted effective modulus of elasticity for stress applied at t_o and sustained to t

$E_{ctc}^*(t, t_o)$ = age-adjusted effective modulus of elasticity of concrete for constant sustained stress

$E_{ctc}^*(t, t_i)$ = age-adjusted effective modulus of elasticity of concrete subjected to a constant stress introduced at t_i with creep determined to the period $(t - t_d)$

$E_{ctv}^*(t, t_o)$ = age-adjusted effective modulus of elasticity of concrete for gradually developing stress

$E_{ctv}^*(t, t_1)$ = age-adjusted effective modulus of elasticity of concrete subjected to gradual loading at time t_1 with creep developing in the period $(t - t_1)$

$E_{ctv}^*(t, t_d)$ = age-adjusted effective modulus of elasticity of concrete subjected to gradual loading introduced at time t_d with creep developing in the period $(t - t_d)$, where t is generally taken in design as time infinity

f = applied stress

f_c' = compressive strength of concrete

$f(t_o)$ = applied stress at time t_o

F_{res} = total net axial force

h = depth of precast concrete beam

I = precast concrete beam moment of inertia

jd = lever arm between tension and compression, assumed to be $0.9h$

k_{td} = time development factor

l = span length

$M_{Barrier}$ = elastic moment due to effect of self-weight of the barrier

M_{cr} = cracking moment

$M_{cr}(t)$ = cracking moment at time t

M_d = elastic moment due to the effect of weight of the deck

M_{el} = elastic restraint moments at the supports

M_o = elastic moment due to self-weight of the beam

M_p = elastic moment due to prestress release

$M_{p,el}$ = elastic continuity moment due to prestress for the special case of a two-span bridge with two equal spans prestressed with strands that have symmetrical profiles

M_r = total restraint moment

M_{r1} = restraint moment due to beam weight

M_{r2} = restraint moment due to prestressing

M_{r3} = restraint moment due to deck weight

M_{res} = total net moment

M_{sh} = moment due to shrinkage

P = effective prestressing force

s = spacing of reinforcement

t = time in days, generally in design assumed as final time, which is 20,000 days

t_o = beginning of the interval under loading being considered

t_d = time of diaphragm and deck placement

t_i = time of prestress release

w = beam weight

w_d = deck weight, including haunch

y_b = distance from the centroid to the bottom fiber of the precast concrete beam

y_{bc} = composite section centroid to bottom fibers

α = coefficient of linear thermal expansion

$\alpha\Delta T$ = free strain due to the temperature change

γ_e = exposure factor

δ_1 = time-dependent multiplier corresponding to prestressing and beam self-weight

δ_2 = time-dependent multiplier corresponding to deck weight

δ_c = time-dependent multiplier corresponding to the load being considered

ε = total strain

ε_o = immediate concrete strain due to applied stress f

ε_{bdf} = beam shrinkage strain between time of deck placement and final time

ε_{bid} = beam shrinkage strain between transfer and deck placement

ε_{bif} = beam shrinkage strain between initial time of loading and final time

ε_{cr} = time-dependent concrete creep strain

ε_{ddf} = shrinkage strain of deck concrete between placement and final time

ε_s = free shrinkage strain of deck

ε_{sh} = free shrinkage strain of concrete

θ_1 = rotation at the center support of the left beam

θ_2 = rotation as concrete creeps at the time the beams are ready to be connected with cast-in-place concrete diaphragms

θ_3 = end rotation

θ_4 = offsetting rotation caused by the continuity restraint momen, causing the rotation to remain unchanged by imposing an offsetting rotation such that $\theta_3 - \theta_4 = \theta_2$

θ_{el} = elastic rotation at beam end = θ_1

$\theta_{r,el}$ = elastic restraint rotation

χ = aging coefficient

$\chi(t, t_o)$ = aging coefficient at a certain time

ψ = creep coefficient

ψ_1 = creep coefficient between t_1 and time infinity

ψ_2 = creep coefficient for concrete loaded from time t_1 to time infinity

ψ_{bdf} = creep coefficient of beam between time of deck placement and final time

ψ_{bid} = creep coefficient at beam between prestress transfer and deck placement

ψ_{bif} = creep coefficient at beam between initial time of loading and final time

ψ_{ddf} = creep coefficient of deck concrete between deck placement and final time

$\psi(t, t_o)$ = creep coefficient during a time interval from t_o to t for stress applied at time t_o and kept constant

$\psi(t, t_1)$ = creep coefficient at time t for concrete loaded at time t_1, specifically deck weight and the restraint moment

$\psi(t, t_d)$ = creep coefficient for load applied at time t_d and sustained to time t

$\psi(t_1, t_o)$ = creep coefficient at time t_1, for concrete loaded at time t_o

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Abstract

Precast, prestressed concrete continuous bridges have been constructed in many countries around the world. Although these bridges have been in service for many years, there has been limited verification of the ability of the connection to provide the predicted continuity. Subsequently, many U.S. states design the girders as simple spans for both dead and live loads without considering any moments developed by the diaphragm connection. The effect of thermal expansion and contraction is hardly considered in the analysis, though it is found to have significant effects on continuity. Apart from this, there is no consensus on the best method to calculate restraint moments that develop in the continuity diaphragm or how to detail positive moment connections.

The objective of this paper is to provide a simplified spreadsheet analysis for the restraint moments and the required crack control reinforcement. Detailed numerical analysis was performed using a two-span NU-girder bridge. It is recommended that the strands already in the beam be used to meet the crack control requirements with no additional mild reinforcement.

Keywords

Aging effect, bridge, continuity diaphragm, restraint moment, thermal contraction, thermal expansion.

Review policy

This paper was reviewed in accordance with the Precast/Prestressed Concrete Institute's peer-review process.

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