Dapped ends of prestressed concrete thin-stemmed members: Part 2, design

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- This paper describes the design of dapped ends of prestressed concrete thin-stemmed members based on an experimental program conducted to identify the most effective reinforcement schemes and develop design guidelines for dapped ends.
- These design guidelines were developed from experimental research findings presented in "Dapped Ends of Prestressed Concrete Thin-Stemmed Members: Part 1, Experimental Testing and Behavior," which also appears in the March–April 2017 *PCI Journal*.
- Several modified design practices for dapped double tees are recommended, and recommendations for control of cracking in the end region are discussed.

Behavior of the dapped ends of prestressed concrete thin-stemmed members is significantly different from the behavior of the end regions of conventionally supported beams. The dap increases tension in the end region, especially at the reentrant corner, which requires special reinforcing details (**Fig. 1**). The procedure provided in the *PCI Design Handbook: Precast and Prestressed Concrete*[1] for proportioning dapped-end reinforcement is based on shear friction theory and the free body diagrams of forces acting across potential failure planes. The procedure was based on the research reported by Mattock and Chan[2] and conservatively treats the dapped-end details as a reinforced concrete inverted corbel. The literature also includes several examples that employ strut-and-tie models for the design of dapped-end members.[3]

Field performance of dapped ends suggests a need for improved dapped-end reinforcing details, which are not yet standardized within the precast concrete industry. As such, this research is intended to develop rational methodologies for proportioning key reinforcement in dapped-end double tees and to develop standard details that have been rigorously reviewed by industry experts and have proven to be effective by extensive analyses and tests. The research findings are reported in two parts: part 1 (a companion paper[4]) describes the experimental program under which promising reinforcement schemes and key parameters were tested, and part 2 (this paper) describes the development of design guidelines for dapped thin-stemmed members. The PCI research report[5] provides a complete description of the

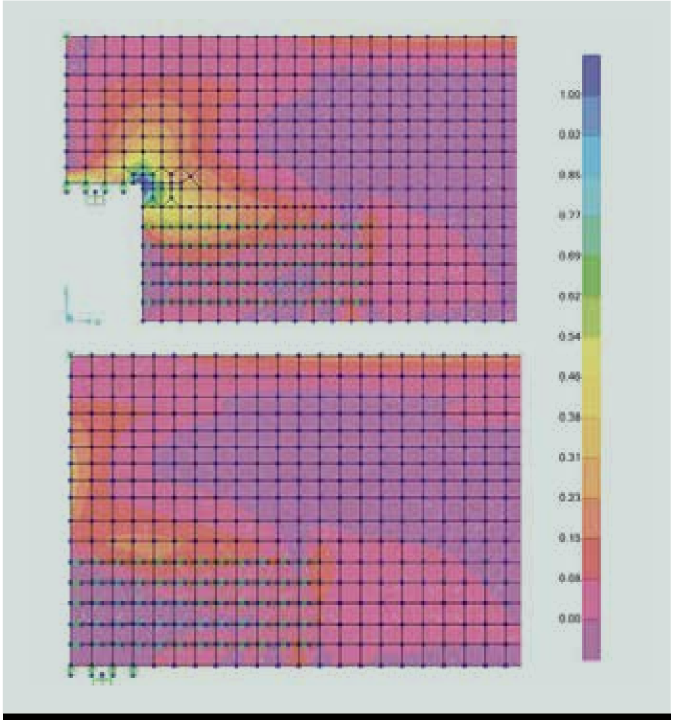

Figure 1. Comparison of principal tension stresses in a dapped-end double tee (top) versus a conventionally supported tee (bottom) subjected to pretensioning and dead loads. Note: All measurements are in kip per square inch. 1 ksi = 6.895 MPa.

research study and findings including a literature review, summary of industry experience, an extensive analytical study, an experimental study of the lap splice between the hanger reinforcement tails and pretensioning strand, full-scale testing of 24 dapped ends, and the development of design recommendations and examples. **Table 1** provides a summary of full-scale testing results and corresponding analytical predictions.

Dapped-end load path

A dapped beam relies on a reduced section to support the member. The notch itself is known as the dap, and the reduced concrete section remaining above the dap is referred to as the nib. The strength of a dapped end is dependent on the load path that transfers the vertical shear and moment from the section inside the end region to the bearing supporting the shallow nib. **Figure 2** illustrates critical elements in a typical dapped-end load path and defines the dapped-end reinforcing necessary to maintain this load path, as well as related notation.

Each element of the dapped-end load path is essential in ensuring the integrity of the end region, and the behavior of one or more of these elements may influence the performance of another critical element because dapped-end behavior is discontinuous, highly nonlinear, and highly coupled. The following sections outline the authors' findings regarding the design of each element and notable behaviors influencing the overall strength of the dapped end. The section numbers correspond to the load path elements identified in Figure 2.

1. Beam region flexure and shear

Full sectional flexural and shear capacities are well defined by the American Concrete Institute's (ACI's) *Building Code Requirements for Structural Concrete (ACI 318-14) and Commentary (ACI 318R-14)*[6] and *PCI Design Handbook* provisions for the continuous beam region (B region) of a prestressed concrete member. Sectional strength was not investigated in this research study, but the test results provided guidance for locating the transition between the B region and the discontinuous region at the end of the beam near the dap face (D region).

As load flows from the B region into the D region of a dapped-end beam, the stresses and strains become increasingly nonlinear through the depth of the section. The effective prestressing force in the concrete decreases to zero approaching the dap face. As a result, current B region design practices are unconservative in this D region, particularly with respect to shear in the full section within the end region. Further than about 1.5 to 2.0 times the member height h from the face of the dap, sectional design procedures apply. The exact distance beyond which the current section design procedures apply is not clear because no beam specimens failed in diagonal tension cracking outside of the 1.5h distance (the centers of the cracks were all within 1.5h of the dap face). However, the top ends of the diagonal tension cracks that precipitated failure extended more than 1.5h but less than 2h from the dap face. This suggests that reinforcing required in the full section at the dap face should be extended to a distance of 2h to intercept the full length of the typical diagonal tension cracks that form. The interesting behavioral trends in the D region revealed by this research effort are the focus of the subsequent sections.

2. End region shear strength

Concrete contribution to shear strength According to ACI 318-14 and the *PCI Design Handbook*, Eq. (1) can determine the nominal shear strength V_n for a prestressed concrete member.

$$V_n = V_c + V_s \quad (1)$$

where

V_c = nominal shear strength provided by concrete

V_s = nominal shear strength provided by steel reinforcement

Table 1. Summary of full-scale experimental results and comparison with finite element model predictions

Specimen	Dap reinforcing scheme	Concrete strength, psi	Dap reaction at failure, kip		Experimental/analysis	Reentrant corner crack width at service load, in.	
			Measured	Analysis		Measured	Analysis
1A	Vertical L	6970	42.8	42.1	1.02	0.015	0.019
1B	Vertical L	6970	52.7	51.0	1.03	0.015	0.017
2A	Vertical Z	8450	51.2	47.9	1.07	0.005	0.010
2B	Vertical Z	8450	59.3	55.7	1.06	0.015	0.022
3A	Inclined L	7400	50.2	50.9	0.98	0.005	0.009
3B	Inclined L	7400	53.8	60.0	0.89	0.010	0.011
4A	Custom WWR	8450	40.0	36.2	1.10	0.020	0.024
4B	Vertical C	8450	45.9	42.1	1.09	0.015	0.022
5A	Vertical Z	8340	55.3	51.0	1.09	0.005	0.009
5B	Vertical Z	8340	67.4	63.0	1.07	0.005	0.007
6A	Vertical Z	12,767	59.6	62.4	0.95	0.015	0.020
6B	Vertical L	12,767	59.2	59.5	0.99	0.010	0.017
7A	Custom WWR	7650	43.4	50.9	0.85	0.020	0.026
7B	Vertical C	7650	52.7	55.2	0.95	0.015	0.020
8A	Vertical L	8650	44.3	44.9	0.98	0.015	0.023
8B	None	8650	44.6	50.8	0.87	0.005	0.001
9A	Vertical L	8100	51.0	45.0	1.13	0.015	0.021
9B	Vertical L	8100	38.6	42.1	0.92	0.015	0.020
10A	CZ	8340	49.1	45.0	1.09	0.010	0.024

Note: WWR = welded-wire reinforcement. 1 in. = 25.4 mm; 1 kip = 4.448 kN; 1 psi = 6.895 kPa.

The shear strength provided by the concrete V_c is taken as the lesser value of the web shear cracking strength V_{cw} and the flexural shear cracking strength V_{ci} (Eq. [2] and [3], respectively).

$$V_{cw} = \left(3.5\lambda\sqrt{f_c'} + 0.3\ f_{pc}\right)b_w d_p + V_p \tag{2}$$

where

λ = modification factor of lightweight concrete

f_c' = specified compressive strength of concrete

f_{pc} = stress in concrete (after allowance for all prestress losses) at centroid of section

b_w = width of web taken at midheight of the full section $h/2$

d_p = distance from extreme compression fiber to centroid of prestressing reinforcement, but not less than $0.8h$

V_p = vertical component of effective prestress force at section

$$V_{ci} = 0.6\lambda\sqrt{f_c'}b_w d_p + V_d + \frac{V_i M_{cre}}{M_{max}} \tag{3}$$

where

V_d = shear force at section due to unfactored dead load

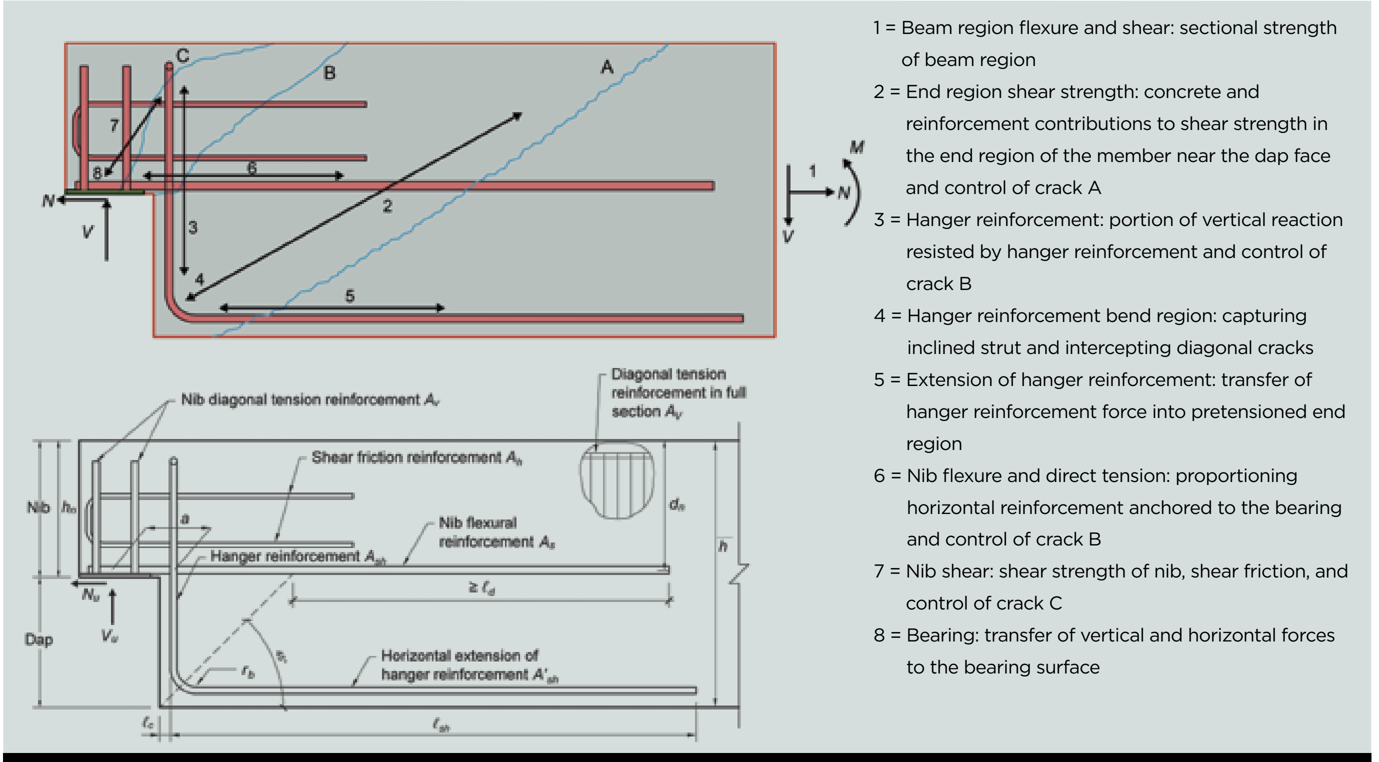

Figure 2. Critical elements in the dapped-end load path (numbers 1 through 8) and potential crack locations (letters A through C). Note: a = shear span measured from the vertical reaction to center of hanger reinforcement; d_n = distance from extreme compression fiber to nib flexural reinforcement; h = member height; h_n = height of the nib; l_c = clear distance between the face of the dap and the hanger reinforcement at the bottom of the section; l_d = development length of reinforcement; l_{sh} = length of hanger reinforcement bar tail; M = moment; N = horizontal reaction; N_u = factored horizontal or axial force; r_b = bend radius of hanger reinforcement measured to the inside of the bar; V_u = factored vertical reaction at end of beam.

V_i = factored shear force at section due to externally applied loads

M_{cre} = moment causing flexural cracking due to externally applied loads

M_{max} = maximum factored moment at section due to externally applied loads

ACI 318-14 and *PCI Design Handbook* design provisions are unconservative in their predictions of shear strength in the full section within the D region of dapped-end members. In numerous test specimens, diagonal tension failure occurred in the full section following yielding of the shear reinforcement. While other failure mechanisms compounded or were concurrently precipitated by the diagonal tension failures, the testing program results as a whole consistently demonstrated that ACI 318-14 and *PCI Design Handbook* sectional-strength equations (Eq. [2] and [3]) for predicting the concrete contribution to the shear strength V_c overestimate this component of the section's resistance in the D region.

Figure 3 plots the measured concrete contribution to shear strength versus the calculated steel contribution to shear strength, in terms of roots of concrete strength, for the test specimens of two independent experimental programs.[4,7]

The results indicate that the concrete contribution to shear strength in the two testing programs was between $2.0\sqrt{f'_c}b_w d_p$ and $3.0\sqrt{f'_c}b_w d_p$ for most of the tested specimens.

Figure 3 also shows the total measured shear strength versus the steel contribution to shear strength for all tested specimens. The figure indicates that the maximum developed shear strength within the D region of the beam was between $4.0\sqrt{f'_c}b_w d_p$ and $5.0\sqrt{f'_c}b_w d_p$ for most of the tested specimens.

Figure 4 presents the concrete contribution to shear strength versus the number of strands in the nib for the test specimens of the two experimental programs.[4,7] The figure indicates that the concrete contribution to shear strength was higher in specimens with strands in the nib. The concrete contribution to shear strength was on the order of $2.0\sqrt{f'_c}b_w d_p$ for specimens without strands in the nib and above $3.0\sqrt{f'_c}b_w d_p$ for specimens with strands in the nib.

Figure 4 shows the maximum measured shear strength versus the number of strands in the nib. The results indicate that the maximum developed shear strength within the D region of the beam was on the order of $4.0\sqrt{f'_c}b_w d_p$ for specimens without strands in the nib and close to or greater than $5.0\sqrt{f'_c}b_w d_p$ for specimens with strands in the nib.

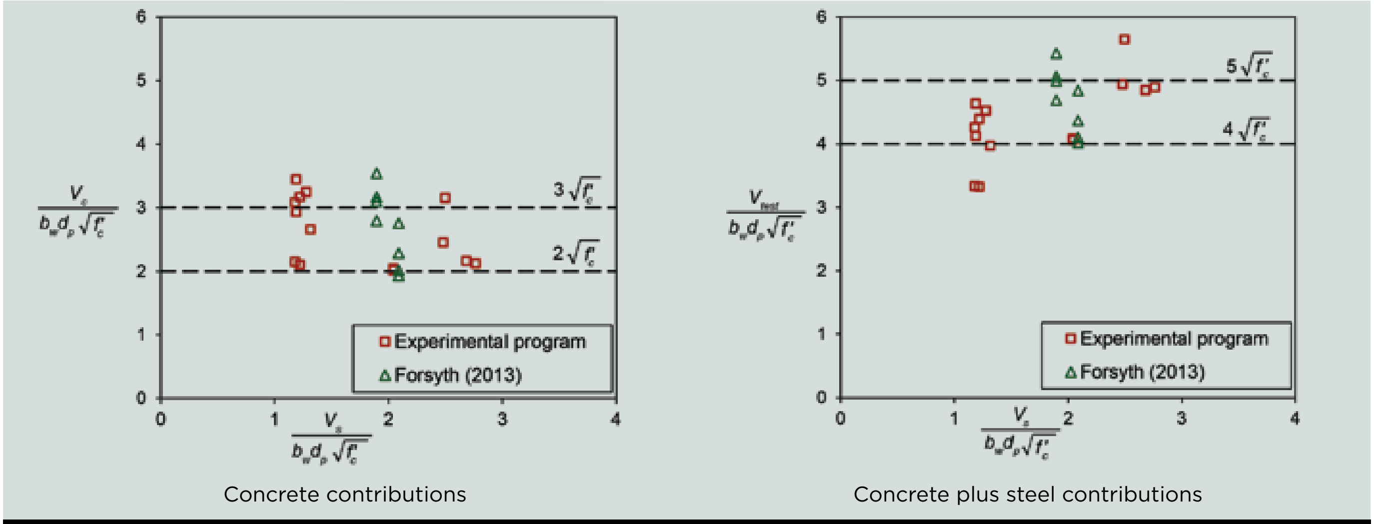

Figure 3. Contributions to shear strength from test results.
Sources: Experimental program results from Botros et al. (2017); prior research results from Forsyth (2013).
Note: b_w = width of web taken at midheight of the full section; d_p = effective depth of prestressing reinforcement; f_c' = specified compressive strength of concrete; V_c = nominal shear strength provided by concrete; V_s = steel contribution to shear strength; V_{test} = measured shear force at failure.

The test results vary from about 50% to 80% of the design values predicted by ACI 318-14 and *PCI Design Handbook* equations for shear strength (which predict minimum concrete contributions to shear strength of $3.5\sqrt{f_c'}b_w d_p$).

Given these results, current unconservative design practices should be modified to increase the overall reliability of dapped-end shear designs to match those provided by code provisions for the design of other structural members. The following modifications are recommended:

- The concrete contribution to shear strength should be taken as $2\sqrt{f_c'}$ within the D region of the beam (2*h* from the dap face), where there are no prestressing strands passing through the nib.

- Where there are at least two strands passing through the nib (that is, two bonded, fully pretensioned strands per stem, prequalified for bond and ½ in. (13 mm) or greater in diameter, located between the bearing plate and the bottom of the flange), the value of the concrete contribution can be increased to $3\sqrt{f_c'}$ within the D region of the beam (2*h* from the dap face).

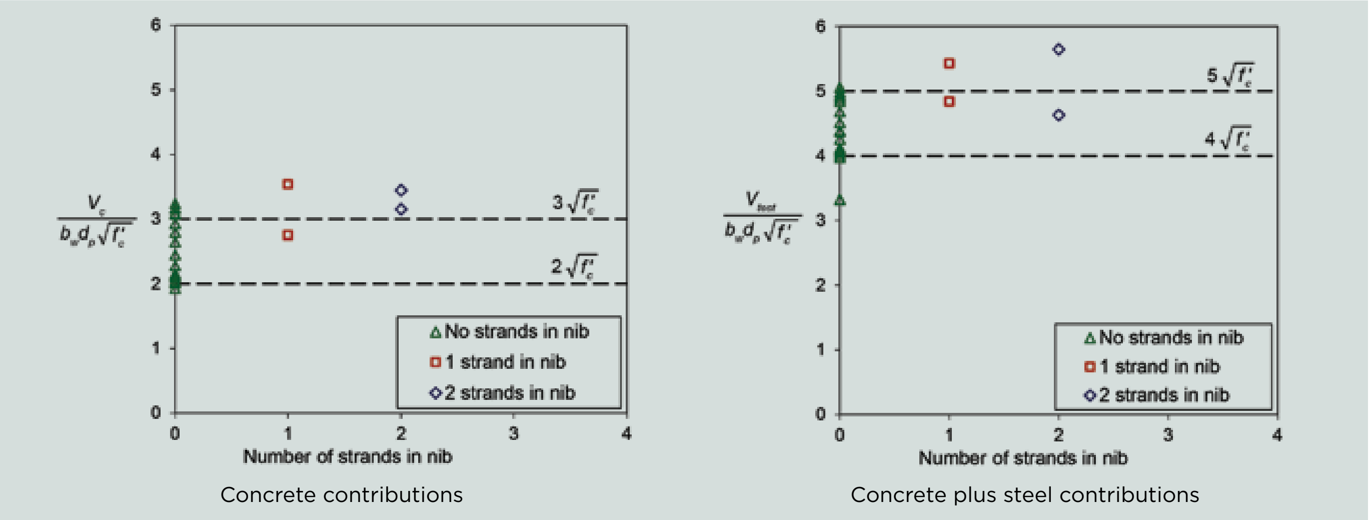

Figure 4. Comparison of contributions to shear strength versus number of strands in nib.
Sources: Experimental program results from Botros et al. (2017); prior research results from Forsyth (2013).
Note: b_w = width of web taken at midheight of the full section; d_p = effective depth of prestressing reinforcement; f_c' = specified compressive strength of concrete; V_c = nominal shear strength provided by concrete; V_{test} = measured shear force at failure.

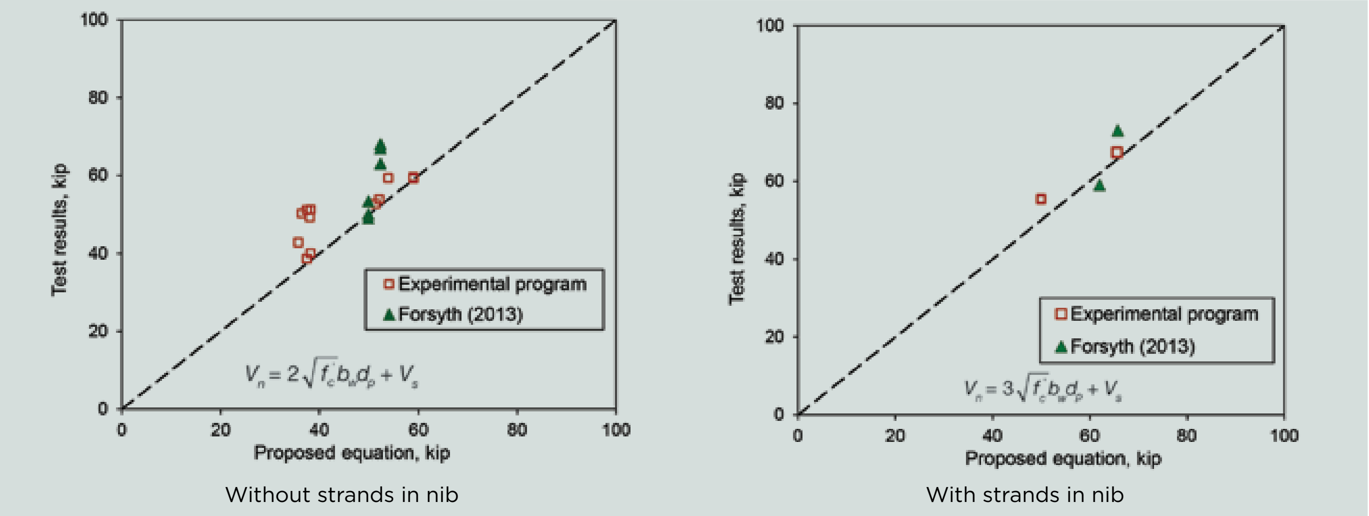

Figure 5. Proposed equations versus experimental results for beams.
Sources: Experimental program results from Botros et al. (2017); prior research results from Forsyth (2013).
Note: b_w = width of web taken at midheight of the full section; d_p = effective depth of prestressing reinforcement; f_c' = specified compressive strength of concrete; V_n = nominal shear strength; V_s = steel contribution to shear strength. 1 kip = 4.448 kN.

- The design concrete strength $\sqrt{f_c'}$ should be limited to 100 psi (700 kPa) (in accordance with ACI 318-14).

- The reinforcing steel contribution to shear strength V_s should be calculated in accordance with ACI 318-14 and *PCI Design Handbook* provisions but limited to $2\sqrt{f_c'}$. This limit on the reinforcing steel contribution limits the total shear strength of the end region to $4\sqrt{f_c'}$ where there are no pretensioned strands through the nib and $5\sqrt{f_c'}$ where at least two pretensioned strands extend through the nib of each stem, which is consistent with the lower-bound test results.

Figure 5 compares the proposed nominal shear strength V_n based on this approach with the measured values for test specimens for the two experimental programs.[4,7] The comparison indicates that the proposed equations conservatively predict the measured values for the two experimental programs.

Design of shear reinforcement The testing program results underscore the importance of shear reinforcement in developing the strength of a dapped-end beam. Control of crack A in Fig. 2, development of the hanger reinforcement, post-cracking strength gain, and overall ductility of the specimen were heavily influenced by the amount of shear reinforcement present. All test specimens had stem welded wire reinforcement (WWR) in excess of ACI 318-14 minimum requirements, yet brittle diagonal tension failures still occurred in the full section of some specimens without premature interference from other secondary failure mechanisms. In more heavily reinforced stems, additional load carrying capacity after diagonal tension cracking afforded considerable ductility until another failure mechanism controlled.

At a minimum, shear reinforcement should be designed in accordance with ACI 318-14 and *PCI Design Handbook* procedures for calculating V_s and the previous recommendations for calculating V_c to provide a nominal design shear strength greater than the factored shear acting at the end of the beam. As required by ACI 318-14, minimum shear reinforcement must be provided when the factored shear force exceeds one half of ϕV_c, where ϕ is the strength reduction factor (0.75 for shear). Because the pretensioning strands are poorly developed across the critical cracks, the ACI 318-14 minimum shear reinforcement provisions for non-prestressed concrete members apply.

To enhance ductility in the end region, consideration should be given to providing more than the minimum required shear reinforcement (up to a contribution equivalent of $2\sqrt{f_c'}$ especially in regions of high seismicity. As demonstrated by the experimental program, targeting V_s to $2.0\sqrt{f_c'}b_w d_p$ adds ductility and assures that the end region does not fail in shear immediately after formation of diagonal cracks.

3. Hanger reinforcement

Together with the horizontal reinforcement extending from the bearing, hanger reinforcement controls the opening of the diagonal crack extending upward from the reentrant corner (crack B in Fig. 2). *PCI Design Handbook* procedure requires that the area of the hanger reinforcement A_{sh} in a dapped end be proportioned with sufficient capacity to carry the entire factored vertical reaction V_u. Reinforcing schemes with inclined hanger bars may tend to induce higher forces due to the inclination. According to the *PCI Design Handbook*, Eq. (5-60) can determine the area of the required hanger reinforcement A_{sh}.

Table 2. Measured hanger reinforcement force versus dap vertical reaction

Specimen	Dap reinforcing scheme	Dap vertical reaction at failure, kip	Measured hanger force, kip	Hanger force/ vertical reaction, %	Hanger yield force, kip	Hanger force/ yield force, %
1A	Vertical L	42.8	28	65	40	69
1B	Vertical L	52.7	55	105	55	100
2A	Vertical Z	51.2	37	71	37	100
2B	Vertical Z	59.3	49	83	49	100
3A	Vertical L	50.2	40	80	40	100
3B	Vertical L	53.8	55	103	55	100
4B	Vertical C	45.9	33	72	37	89
5A	Vertical Z	55.3	34	61	37	91
5B	Vertical Z	67.4	49	73	49	100
6A	Vertical Z	59.6	52	87	56	92
6B	Vertical L	59.2	55	93	55	100
7B	Vertical C	52.7	40	75	40	100

Note: 1 kip = 4.448 kN.

$$A_{sh} = \frac{V_u}{\phi f_y} \quad (5\text{-}60)$$

where

f_y = specified yield strength of reinforcement

Using electric resistance strain gauges installed on the hanger reinforcement of the test specimens, the force in the vertical leg of the hanger bars was measured. **Table 2** highlights the variability of the measured force carried by the hanger reinforcement at failure relative to the vertical dap reaction. The measured forces were less than the vertical reaction; however, in two cases the measured force in the hanger exceeded the vertical reaction. In some specimens, strain gauges were not located in the immediate vicinity of the reentrant corner crack. Accordingly, the measured force could be lower than the actual force developed in the hanger reinforcement. The measured forces in the hanger reinforcement varied from 65% to 105% of the *PCI Design Handbook* design equations; therefore, it is recommended that no changes be introduced to the current equation used for proportioning the hanger reinforcement.

4. Hanger reinforcement bend region

In a dapped-end member, the bend region of the hanger reinforcement must allow for the flow of tension through the bend and capture the inclined strut extending along the potential diagonal shear crack toward the bottom corner of the full section (**Fig. 6**). In such a curved bar node, the compressive stress can be calculated based on the bend radius and the inclination and width of the strut. Concrete crushing in the hanger reinforcement bend region was observed in specimens 1A, 6B, and 9A. The failures of these specimens illustrate the propensity of thin-stemmed dapped ends to develop bend-region distress due to bar placement errors and the small bends conventionally used in fabrication of the hanger reinforcement. The ACI 318-14 standard minimum bends were used for all specimens in this test program.

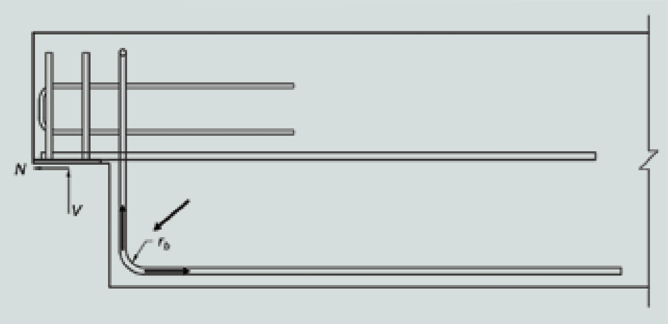

Figure 6. Hanger reinforcement bend radius. N = horizontal reaction; r_b = the bend radius of hanger reinforcement measured to the inside of the bar; V = vertical reaction.

The compressive distress associated with the failures of these specimens in the test program highlights the need for control of the detailing and placement of the hanger

A'_{sh}

reinforcement in the bend region. Accordingly, it is recommended that, if necessary, the bend radius of the hanger reinforcement be increased above the ACI 318-14 standard minimum such that the compressive stress in the bend region is less than the ACI 318-14 limit for nodes consisting of one strut and two ties.

Based on the work done by Klein,[8] the bend radius of the hanger reinforcement should satisfy Eq. (4) to ensure that nodal zone compressive stress does not exceed the ACI 318-14 limit for nodes consisting of one strut and two ties.

$$r_b = \frac{2A_{sh}f_y}{b_b f'_c} \tag{4}$$

where

r_b = bend radius of hanger reinforcement measured to the inside of the bar

b_b = width of web at the bend region

If the specified side clear cover of the hanger bar is less than twice the bar diameter, the required bend radius given by Eq. (4) should be multiplied by the ratio $2d_b/c_c$ (where d_b is the nominal diameter of reinforcement and c_c is the specified clear cover of embedded reinforcement).

5. Splice length of hanger reinforcement

At the bend region, the force from the vertical leg of the hanger steel is transmitted into the tail, which must be transferred into the precompressed concrete stem. This region is vulnerable to splitting. Before loading, the hanger reinforcement tail is in compression. Dilation of the pretensioning strand after release generates splitting forces (the Hoyer effect). As the hanger reinforcement tail is stressed, the tensile force decompresses the concrete stem without significantly increasing the stress in the pretensioning strand. After the tensile force overcomes the precompression, transverse cracks develop. These cracks disrupt the bond between the pretensioning strand and surrounding concrete, resulting in strand slip and loss of pretensioning force. As tension in the deformed reinforcement increases to failure, transverse cracks develop further into the section, causing strand slip and increasing bond stress, which leads to splitting, complete loss of bond, and failure. As such, the splice between the hanger reinforcement tail and the pretensioning strand is unlike a conventional splice, where tensile forces are transferred between the spliced bars as tensile cracks develop.

Test results indicated that longer splices helped to avoid premature splitting, loss of strand bond, and flexure-shear failure. The hanger reinforcement tail of specimen 9B, with a splice length of 15 in. (381 mm), which is equal to 0.95 times the development length ℓ_d of the bar, terminated within the transfer zone of the strands. This short splice length provided insufficient development, and accordingly, web splitting followed by a flexure-shear failure occurred. Specimens 1A and 9A, with longer splices of $2\ell_d$ and $3.75\ell_d$, respectively, avoided web splitting and flexure-shear failure and experienced diagonal tension failure with concrete crushing in the hanger reinforcement bend region. Accordingly, a longer splice is essential to fully mobilize the strut-and-tie action and full-section shear capacity.

The development length ℓ_d for the horizontal extension of the hanger reinforcement bars was calculated in accordance with ACI 318-14 provisions and compared with the actual provided splice length of the beam specimens (**Table 3**). Column 6 in Table 3 gives the ratio of the concrete confinement c_b (typically the distance from the edge of the concrete to the center of the reinforcing bar) to bar diameter d_b. The c_b/d_b values for the test specimens ranged from 1.2 to 2.14. The specimens in the experimental program that failed in flexure shear due to splitting had c_b/d_b values ranging from 1.33 to 1.70. The experimental results indicated that increased concrete confinement of the hanger reinforcement tail c_b/d_b and longer hanger reinforcement tails (area of horizontal extension of hanger reinforcement A'_{sh}) help avoid premature splitting, loss of strand bond, and flexure-shear failure. Eccentric placement of the hanger reinforcement resulting in shallow side and bottom concrete covers for the bars could increase the potential for splitting. Therefore, the size and location of the hanger reinforcement tail bars should be configured such that c_b/d_b is preferably not less than 2.5 and never less than 1.5. Consideration should be given to bending the hanger reinforcement tail slightly upward from horizontal to help maintain cover to the bottom of the member.

Figure 7 shows the ratio of the provided splice length of the hanger bars to the development length of the bars and the ratio of the measured force in the hanger bars to the yield strength of the bar. The figure indicates that in most cases, to develop the yield strength of the bars, the splice length of the hanger bars should be greater than twice the development length specified by ACI 318-14.

Similarly, Fig. 7 shows the ratio of the splice length of the hanger bars to the transfer length of the prestressing strand based on a transfer length equal to 50 times the strand diameter, versus the ratio of the force in the hanger bars to the yield strength of the bar. The figure indicates that in most cases, to develop the yield strength of the bars, the splice length should be greater than 1.5 times the transfer length of the prestressing strand. Accordingly, it is recommended that the hanger reinforcement be extended

Table 3. Comparison of hanger reinforcement splice length to bar development length

Specimen	Concrete Strength f'_c, psi	Hanger reinforcement		Concrete confinement c_b, in.	Bar diameter d_b, in.	Concrete confinement to bar diameter ratio c_b/d_b	Splice length, in.	Transfer length of strand ℓ_t, in.	Development length of hanger bars ℓ_d, in.	Splice length-to-transfer length ratio	Splice length-to-development length ratio
		Bars	Area, in.²								
1A	6970	Two no. 5	0.62	1.06	0.63	1.70	36.0	28.1	18	1.28	2.00
1B	6970	Four no. 4	0.80	0.75	0.50	1.50	36.0	28.1	17	1.28	2.12
2A	8450	One no. 7	0.60	1.88	0.88	2.14	36.0	26.0	22	1.38	1.64
2B	8450	One no. 8	0.79	2.00	1.00	2.00	36.0	26.0	28	1.38	1.29
3A	7400	Two no. 5	0.62	1.06	0.63	1.70	25.3	28.1	17	0.90	1.49
3B	7400	Four no. 4	0.80	0.75	0.50	1.50	25.7	28.1	17	0.92	1.51
4B	8450	One no. 7	0.60	1.88	0.88	2.14	36.0	26.0	22	1.38	1.64
5A	8340	One no. 7	0.60	1.88	0.88	2.14	36.0	26.0	23	1.38	1.57
5B	8340	One no. 8	0.79	2.00	1.00	2.00	36.0	26.0	28	1.38	1.29
6A	12,767	Two no. 6	0.88	1.00	0.75	1.33	36.0	28.1	21	1.28	1.71
6B	12,767	Four no. 4	0.80	0.75	0.50	1.50	36.0	28.1	13	1.28	2.77
7B	7650	Two no. 5	0.62	0.75	0.63	1.20	36.0	28.1	24	1.28	1.50
9A	8100	Two no. 5	0.62	1.06	0.63	1.70	60.0	28.1	16	2.14	3.75
9B	8100	Two no. 5	0.62	1.06	0.63	1.70	15.0	28.1	16	0.53	0.94
10B	8340	One no. 7	0.60	1.88	0.88	2.14	36.0	26.0	23	1.38	1.57

Note: no. 4 = 13M; no. 5 = 16M; no. 6 = 19M; no. 7 = 22M; no. 8 = 25M; 1 in. = 25.4 mm; 1 psi = 6.895 kPa.

horizontally a distance equal to the greater of 1.5 times the strand transfer length measured from the face of the dap and 2.0 times the bar development length measured from the front end of the hanger reinforcement bar.

6. Nib flexure and direct tension

Together with the hanger reinforcement, horizontal reinforcement extending from the bearing controls the opening of the diagonal crack extending upward from the reentrant corner (crack B in Fig. 2). Based on the design procedure in the *PCI Design Handbook* (Eq. [5-56]), Eq. (5) gives the force in the nib due to flexure and axial tension F_{A_S}.

$$F_{A_S} = V_u\left(\frac{a}{d_n}\right) + N_u\left(\frac{h_n}{d_n}\right) \tag{5}$$

where

a = shear span measured from the vertical reaction to center of hanger reinforcement

N_u = factored horizontal or axial force

d_n = distance from extreme compression fiber to nib flexural reinforcement

h_n = height of the nib

Table 4 summarizes the forces calculated in accordance with Eq. (5) and provides measured forces from the strain data. Specimens 4B and 7B failed due to propagation of a crack that initiated from the reentrant corner and propagated over the top bend of the C-shaped reinforcement (**Fig. 8**). The nibs of the specimens failed in shear (diagonal tension), though the cracks that caused failure crossed the flexural reinforcement near the reentrant corner and were wide at failure, indicating yielding of the flexural reinforcement at the cracks. For specimens 4B and 7B, the calculated forces in the flexural reinforcement were 88% and 110% of the actual yield strength, respectively.

Specimens 8A and 10B also exhibited nib shear failures. The wide cracks that precipitated failure were located

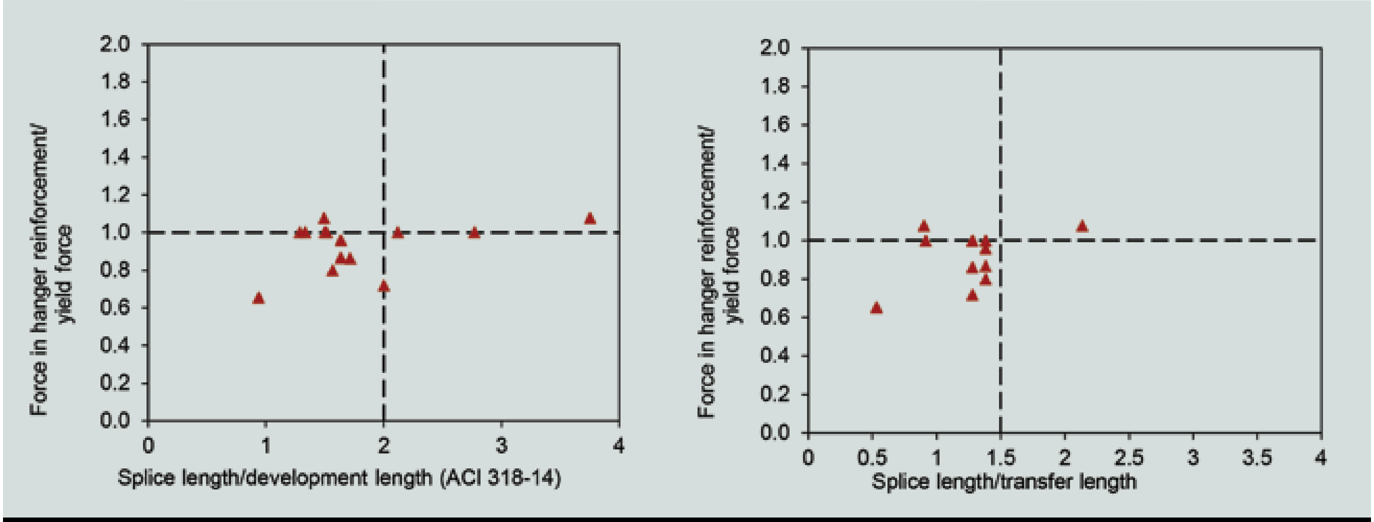

Figure 7. Ratios of splice length to development length or transfer length versus the ratio of hanger reinforcement force to yield force.

within the nib and not at the reentrant corner. The calculated forces in the flexural reinforcement in specimens 8A and 10B were 111% and 61% of the actual yield strength, respectively.

A comparison between calculated and measured forces in the nib flexural reinforcement can be made for specimens 1A, 3A, and 4A. The calculated and measured forces compare well for specimen 1A, which is a vertical L scheme. The calculated force is much higher than the measured force for specimen 3A, which is an inclined L scheme, and much less for specimen 4A, which is a custom WWR scheme. Although these data are limited, it appears that Eq. 5) is accurate for typical vertical hanger reinforcement schemes but overpredicts the force in the reinforcement for the inclined schemes. This is logical because some of the force that might otherwise be resisted by the flexural reinforcement may be resisted in the inclined hanger reinforcement. The data also suggests that forces are higher than predicted in the custom WWR scheme, potentially contributing to its poorer performance compared with the other schemes.

Measured forces in the flexural reinforcement in the full section, away from the reentrant corner, were less than or equal to the yield strength of the reinforcing, though not much less in most cases. Specimen 8B did not have dap reinforcement, and the diagonal crack that precipitated failure would only have been controlled by the stem WWR and the flexural reinforcement. In the case of specimen 5A, which had strands in the nib, the applied forces from external loading in the reinforcement did not relieve the prestress prior to failure. These results suggest that the flexural reinforcement contributes to resisting diagonal cracking in the full section, though this reinforcement does not appear to influence the strength of the full section. Full section strength appears to be limited more by other factors (prestressing the nib, amount of stem WWR, and concrete strength).

Overall, the data from the analytical study and single-tee test results do not indicate a need for changes to the design of the flexural reinforcement. Thus, *PCI Design Handbook* Eq. (5-56) (reproduced as Eq. [6]) gives the required reinforcement.

$$A_s = A_f + A_n = \frac{1}{\phi f_y}\left[V_u\left(\frac{a}{d_n}\right) + N_u\left(\frac{h_n}{d_n}\right)\right] \tag{6}$$

where

A_s = area of nib flexural reinforcement

A_f = area of reinforcement resisting factored moment in the extended end of dap

A_n = area of nib reinforcement resisting tensile force

ϕ = shear-strength reduction factor

This reinforcing element is important for resisting nib failures and, to a lesser degree, failures in the full section. All specimens except for specimen 10B (the pocket nib detail) achieved their target strength, indicating that the current design equations for the flexural reinforcement result in satisfactory designs. *PCI Design Handbook* Eq. (5-57), which calculates the area of nib flexural reinforcement A_s based on shear friction, need not be checked for thin-stemmed members.

7. Nib shear

Two nib shear failure modes should be considered: a diagonal tension failure across the crack that extends from the inside of the bearing and over the top of the hanger reinforcement (crack C in Fig. 2) and shear friction across

Table 4. Forces in nib flexure and axial tension reinforcement								
Specimen*	Dap reinforcing scheme	Calculated forces		Measured force at reentrant corner		Measured force in full section		Notes
		Force,† kip	Yield,‡ %	Force,§ kip	Yield,‡ %	Force,‖ kip	Yield,‡ %	
1A	Vertical L	33.2	89	37.2	100	37.2	100	
1B	Vertical L	40.9	110	n.d.	n.d.	n.d.	n.d.	
2A	Vertical Z	39.7	99	n.d.	n.d.	40.3	100	
2B	Vertical Z	46.0	114	n.d.	n.d.	n.d.	n.d.	
3A	Inclined L	38.9	105	27.8	75	n.d.	n.d.	
3B	Inclined L	41.7	112	n.d.	n.d.	n.d.	n.d.	
4A	Custom WWR	31.0	70	48.5	109	3.6	8	
4B#	Vertical C	35.6	88	n.d.	n.d.	40.3	100	
5A	Vertical Z	42.9	106	n.d.	n.d.	−1.8	−4	Strands in nib
5B	Vertical Z	52.3	130	n.d.	n.d.	28.8	71	Strands in nib
6A	Vertical Z	46.2	124	n.d.	n.d.	31.3	84	
6B	Vertical L	45.9	123	n.d.	n.d.	37.2	100	
7A	Custom WWR	33.6	76	n.d.	n.d.	33.2	75	
7B#	Vertical C	40.9	110	n.d.	n.d.	31.3	84	
8A#	Vertical L	41.2	111	n.d.	n.d.	n.d.	n.d.	12 in. nib
8B	None	24.6	89	n.d.	n.d.	27.6	100	24 in. nib
9A	Vertical L	39.5	106	n.d.	n.d.	n.d.	n.d.	
9B	Vertical L	29.9	80	n.d.	n.d.	n.d.	n.d.	
10A	CZ	38.1	94	n.d.	n.d.	40.3	100	
10B#	Vertical Z	24.4	61	n.d.	n.d.	30.6	76	8 in. nib

Note: n.d. = no data; WWR = welded-wire reinforcement. 1 in. = 25.4 mm; 1 kip = 4.448 kN.

*Specimens 1 to 10 included the horizontal reaction force N_u in the test setup.

†Calculated using Eq. (5).

‡Based on measured yield strength of reinforcement.

§Calculated based on readings from strain gauges at reentrant corner, the measured strength of the reinforcement, and assuming an elastic–perfectly plastic material response.

||Calculated based on readings from strain gauges located 19 in. from the reentrant corner, the measured strength of the reinforcement, and assuming an elastic–perfectly plastic material response.

#Specimens that failed in the nib region.

a potential vertical crack extending upward from the reentrant corner.

Table 5 gives the calculated strength of the nib for the tested beams and the measured failure loads. Column 4 in Table 5 lists the direct shear stress acting in the nib expressed in terms of square roots of concrete strength. The nib stress was determined by dividing the failure load by the effective area of the nib. The effective area of the nib was determined by multiplying the average b_w and d_n. The effective depth of the nib d_n was measured from the centroid of nib flexural reinforcement to the extreme fiber

Specimen 4B

Specimen 7B

Figure 8. Diagonal tension failures within the nib for vertical C scheme.

of the compression zone (top of the nib or double-tee flange).

The experimental results[4] indicated that the specimens, with the exception of specimens 4B, 7B, 8A, and 10B, were controlled by full-depth section failures away from the nib region. Specimens 4B, 7B, 8A, and 10B failed due to diagonal tension cracking within the nib. This observation indicates that neither of the two failure modes described controlled failure for most of the test specimens and that the dapped-end reinforcement within the nib region is adequate to transfer the forces from the nib to the full-depth section.

Shear stresses within the nib varied from $5.0\sqrt{f_c'}$ to $8.6\sqrt{f_c'}$ for all specimens, and from $5.8\sqrt{f_c'}$ to $8.0\sqrt{f_c'}$ for specimens 4B, 7B, 8A, and 10B, which failed due to diagonal tension cracking within the nib (Table 5). Specimen 4B, reinforced with the vertical C scheme, failed in the nib at a lower value of applied stress $5.8\sqrt{f_c'}$ than other moderate-target-load specimens 1A and 2A, which realized stresses in the nib on the order of $6.0\sqrt{f_c'}$ to $6.5\sqrt{f_c'}$ before failure in the full-depth section. Specimen 4B failed due to diagonal tension cracking within the nib that extended over the upper bend leg of the C-shaped hanger bar (Fig. 8). Specimen 7B, also with the C-shaped hanger bar, failed due to a similar crack that appeared to avoid crossing the hanger reinforcement by extending over the top bend of the C-shaped bar (Fig. 8). This specimen realized a nib shear stress of $7.0\sqrt{f_c'}$, which is lower than similar high-target-load specimens 1B and 2B, which achieved nib shear stresses between $7.4\sqrt{f_c'}$ and $7.5\sqrt{f_c'}$ prior to failure in the full-depth section. These results indicate that the vertical C scheme is prone to fail due to diagonal tension cracking within the nib. The vertical C-scheme failures are characterized as failure of the hanger reinforcement to sufficiently capture the inclined strut that attempts to form from the nib bearing to the top of the hanger reinforcement.

These results indicate that permitting a maximum average shear stress of $5.0\sqrt{f_c'}$ over the nib area is appropriate for nominal strength computations of dapped ends reinforced with C-shaped hanger reinforcement details. The improved performance of the other reinforcing schemes relative to the C-shaped hanger with comparable nib geometry affords a higher permitted average nib shear stress, up to $6.0\sqrt{f_c'}$ over the nib area, for design calculations. Nib shear reinforcement was not effective because the vertical bars were not developed above the crack. As such, *PCI Design Handbook* Eq. (5-61) and (5-62) do not apply. Instead, Eq. (7) and (8) determine the design shear strength of the nib region.

Vertical C detail

$$\phi V_n = \phi 5.0\sqrt{f_c'}b_w d_n \tag{7}$$

All other details

$$\phi V_n = \phi 6.0\sqrt{f_c'}b_w d_n \tag{8}$$

The inclined hanger bars used in the inclined L detail serve as reinforcement across the potential shear friction failure plane extending upward from the reentrant corner. Except for the inclined L detail, shear friction reinforcement should be provided in accordance with *PCI Design Handbook* Eq. (5-59).

$$A_h = 0.5(A_s - A_n) \tag{5-59}$$

Table 5. Nib strength compared with test results

Specimen	Concrete compressive strength f_c', psi	Failure load V_{test}, kip	Shear stress in nib $\sqrt{f_c'}$	Failure mode
1A	6970	42.8	6.0	Failure in full-depth section
1B	6970	52.7	7.4	
2A	8450	51.2	6.5	
2B	8450	59.3	7.5	
4B	8450	45.9	5.8	Diagonal tension crack in the nib
5A	8340	55.3	7.0	Failure in full-depth section
5B	8340	67.4	8.6	
6A	12,767	59.6	6.2	
6B	12,767	59.2	6.1	
7B	7650	52.7	7.0	Diagonal tension crack in the nib
8A	8650	44.3	6.9	Diagonal tension crack in the nib
9A	8100	51.0	6.6	Failure in full-depth section
9B	8100	38.6	5.0	
10A	8340	49.1	6.2	
10B	8340	31.5	8.0	Diagonal tension crack in the nib

Note: 1 kip = 4.448 kN; 1 psi = 6.895 kPa.

The shear friction reinforcement should be detailed to be developed on both sides of the potential vertical crack emanating from the reentrant corner. It should also extend as far as practical toward the end of the member to be developed to resist cracks that could occur in the nib. A horizontally oriented no. 4 (13M) U bar was used in this research and is considered a practical minimum and therefore recommended for thin-stemmed members.

Except for the vertical Z detail, a deformed bar anchor that is 0.5 in. (13 mm) in diameter or larger and welded to the bearing plate is recommended. If only one bar is used, it should be positioned within the first third of the length of the bearing plate, away from the end of the beam, to restrain potential cracking that could occur near the end of the beam from nonuniform bearing.

8. Bearing

The *PCI Design Handbook* includes recommendations proportioning reinforcement for direct bearing. The *PCI Design Handbook* also indicates that confinement reinforcement in all directions may be necessary where the bearing stress exceeds $1.1f_c'$. Eq. (9) calculates the design bearing strength.

$$\phi V_n \le \phi 1.1 \ell_{pad} b_{pad} f_c' \tag{9}$$

where

b_{pad} = width of bearing pad, but not greater than the stem width

ℓ_{pad} = length of bearing pad (dimension parallel to span)

It is recommended that the bearing pad be at least 4 in. (100 mm) long.

Serviceability and constructibility recommendations

Cracking is likely to occur near the reentrant corner of the dapped end below service load levels due to the corner stress concentration and the fact that in most cases the end region of the beam is essentially nonprestressed. Results of the experimental program[4] indicated variability in the effectiveness of the dapped-end reinforcing schemes in controlling cracks. The test results showed that prestressing the nib region was effective in restraining reentrant corner cracking and significantly reduced cracking at service loads. The results also indicated that the vertical Z and inclined L schemes performed well in terms of strength and crack control at service load compared with other reinforcement schemes.

Recommended design procedure

The recommended design procedure is based on the analysis and testing performed in this research program as well as on previous research results, all of which pertained to thin-stemmed dapped-end members. The recommendations are generally applicable to dapped-end members that have the following characteristics:

- member height between 24 and 36 in. (610 and 910 mm)
- dapped-end reinforcement that is no. 3 (10M) to no. 8 (25M) in size
- normalweight concrete
- stem widths between approximately 4 and 8 in. (100 and 200 mm)
- stems with one column of strand or two staggered columns of strand
- nib height greater than or equal to 50% of the total height of the beam
- nib length up to 8 in.

With these limitations in mind, the following paragraphs summarize the recommended design procedures arising from this research. Where appropriate, the recommendations refer to design procedures and equations in the *PCI Design Handbook*. A sample design is provided in the appendix to this paper, which is posted online at www.pci.org.

The allowable shear stress in the nib (step 7) may control the dapped-end strength where the dap is near 50% of the overall depth of the member. In such cases, the designer is advised to check diagonal tension in the nib (step 7) early in the design process.

Step 1: Verify sectional strength in B region (flexure, shear)

Beyond a distance 2*h* from the face of the dap, the beam region of dapped double tees should be designed for flexure and shear following the sectional design requirements of ACI 318-14.

Step 2: Verify shear strength in full section of D region

Within a distance of 2*h* from the face of the dap, the concrete contribution to shear strength may be taken as $2.0\sqrt{f_c'}b_w d_p$ for typical members and $3.0\sqrt{f_c'}b_w d_p$ for members with at least two fully pretensioned strands extending through the nib of each stem (two bonded pretensioned strands per stem, prequalified for bond and ½ in. [13 mm] or greater in diameter, located between the bearing plate and the bottom of the flange). The reinforcing steel contribution to shear strength in the end region should be limited to $2.0\sqrt{f_c'}b_w d_p$. The minimum shear reinforcement requirements for nonprestressed sections in ACI 318-14 apply, as does the ACI 318-14 limit to the concrete strength of 10,000 psi (69 MPa) for shear design.

Step 3: Proportion hanger reinforcement

The force in the hanger reinforcement may be taken as equal to the vertical reaction. Accordingly, *PCI Design Handbook* Eq. (5-60) is recommended for proportioning hanger reinforcement.

$$A_{sh} = \frac{V_u}{\phi f_y} \tag{5-60}$$

The experimental program verified that this equation may be safely applied to the inclined L scheme, though the vertical component of the yield strength of the inclined hanger bars is somewhat less than that of comparable vertical hanger bars. Where WWR is used, it is necessary to design A_{sh} to be the same as A_{sh}'.

The hanger reinforcement should be placed as close as practical to the dap face at the reentrant corner and should be extended as high as practical in the section and at least to the bottom of the flange.

Step 4: Check bend region of hanger reinforcement

Except for the custom WWR schemes, the dapped reinforcing schemes considered in this research include hanger reinforcement that extends horizontally toward the midspan. The minimum bend radius of the hanger reinforcement at the bottom of the full section should satisfy Eq. (4), such that

$$r_b \geq \frac{2A_{sh}f_y}{b_b f_c'} \tag{4}$$

For symmetrically placed hanger reinforcement bars, b_b is the width of the stem. For a single asymmetrically placed hanger bar, b_b is twice the least specified distance from the center of the hanger bar to the side face c_b.

Where a side clear cover of less than $2d_b$ is specified, r_b required by the above equation should be increased in proportion to $2d_b$ divided by the specified side clear cover c_c.

Where the bend regions of more than one hanger reinforcement bar are in the same plane, A_{sh} must be taken as the total area of hanger reinforcement and r_b must be taken as bend radius of the inside layer.

The previous equation for minimum radius is most likely to govern over the standard bend radius where two no. 4 or 5 (13M or 15M) hanger bars are used in the lower corner of the full section or where more than one layer of hanger reinforcement is used.

Step 5: Determine length of hanger reinforcement tail

The hanger bar tails should be extended into the span from the face of the dap at least 1.5 times the strand transfer length, and the horizontal extension from the bend should be at least 2.0 times the bar development length. The size and location of the tail bars should be configured such that c_b/d_b is preferably not less than 2.5 and never less than 1.5.

Step 6: Proportion reinforcement for nib flexure and axial tension

The horizontal reinforcement at the bottom of the nib should be proportioned according to *PCI Design Handbook* Eq. (5-56) (reproduced as Eq. [6]).

$$A_s = A_f + A_n = \frac{1}{\phi f_y}\left[V_u\left(\frac{a}{d_n}\right) + N_u\left(\frac{h_n}{d_n}\right)\right] \qquad (6)$$

This reinforcement should be welded to the bearing plate and extend at least a distance ℓ_d beyond the potential 45-degree crack intersecting the bottom corner of the full section.

Step 7: Design for nib shear

Eq. (7) and (8) give the design shear strength of the nib region.

Vertical C detail

$$\phi V_n = \phi 5.0\sqrt{f_c'}b_w d_n \qquad (7)$$

All other details

$$\phi V_n = \phi 6.0\sqrt{f_c'}b_w d_n \qquad (8)$$

Except for the inclined L detail, shear friction reinforcement should also be provided in accordance with *PCI Design Handbook* Eq. (5-59).

$$A_h = 0.5(A_s - A_n) \qquad (5\text{-}59)$$

The shear friction reinforcement should be detailed to be developed on both sides of the potential vertical crack emanating from the reentrant corner. It should also extend as far as practical toward the end of the member to be developed to resist cracks that could occur in the nib.

Except for the vertical Z detail, specify a deformed bar anchor that is 0.5 in. (13 mm) in diameter or larger and welded to the bearing plate. If only one bar is used, it should be positioned within the first third of the length of the bearing plate away from the end of the beam to restrain potential cracking that could occur near the end of the beam from nonuniform bearing.

Step 8: Design for direct bearing

To maintain stresses in the nib region below the limit in the *PCI Design Handbook*, Eq. (9) gives the allowable bearing stress. Of course, the stress acting on the bearing pad must also be checked in accordance with *PCI Design Handbook* procedures. The allowable stress depends on the pad material. In addition, the bearing pad length ℓ_{pad} should be at least 4 in. (100 mm).

Conclusion

The following findings were developed from extensive three-dimensional nonlinear finite element analysis and full-sized specimen tests, which are presented in the companion paper.[4]

- The end region of dapped beams, within two times the member height, is a discontinuity region. Within this region, sectional design procedures for shear strength do not apply and are unconservative. Alternative design equations for the concrete and steel contributions to shear strength of the end-region full section are provided.

- Design and detailing recommendations are provided for the bend radius of the hanger reinforcement at the bottom corner of the full section to preclude compression failure of the strut extending into this corner.

- Design recommendations for the minimum length of the horizontal extension of hanger reinforcement into the full section have been developed based on bar development length and strand transfer length. Detailing recommendations intended to preclude premature splitting are also provided.

- Shallow nibs are vulnerable to diagonal tension failures across cracks extending from the reentrant corner and over the top of the hanger reinforcement, especially where C-shaped hanger reinforcing bars are used. Equations to determine the design shear strength of the nib are provided.

- Procedures in the *PCI Design Handbook* apply to other aspects of dapped-end design, including the equations for proportioning hanger reinforcement, horizontal reinforcement extending from the bearing, shear friction reinforcement, and the bearing area.

- Cracks extending diagonally upward from the reentrant corner at service load are likely, especially if pretensioning strands do not extend through the nib. Recommendations for controlling these cracks are provided.

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Notation

a = shear span measured from the vertical reaction to center of hanger reinforcement

A_f = area of reinforcement resisting factored moment in the extended end of dap

A_h = area of shear-friction reinforcement across vertical crack at dapped ends and corbels

A_n = area of nib reinforcement resisting tensile force

A_s = area of nib flexural reinforcement

A_{sh} = area of hanger reinforcement for dapped end

A'_{sh} = area of horizontal extension of hanger reinforcement

A_v = area of diagonal tension reinforcement in section under consideration (full section or nib)

b_b = width of web at the bend region

b_{pad} = width of bearing pad, but not greater than the stem width

b_w = width of web taken at midheight of the portion of the tapered stem under consideration (full section or nib)

c_b = concrete confinement = lesser of distance from the center of bar or wire to the nearest concrete surface or one half the center-to-center spacing of bars or wires being developed

c_c = specified clear cover of embedded reinforcement

d_b = nominal diameter of reinforcement

d_n = distance from extreme compression fiber to nib flexural reinforcement

d_p = distance from extreme compression fiber to centroid of prestressing reinforcement, but not less than 0.8h

f_c' = specified compressive strength of concrete

f_{pc} = stress in concrete (after allowance for all prestress losses) at centroid of section

f_y = specified yield strength of reinforcement

F_{A_s} = force in the nib flexure and axial tension reinforcement

h = member height

h_n = height of the nib

ℓ_c = clear distance between the face of the dap and the hanger reinforcement at the bottom of the section

ℓ_d = development length of reinforcement

ℓ_{pad} = length of bearing pad

ℓ_{sh} = length of hanger reinforcement bar tail (splice length)

ℓ_t = strand transfer length

M = moment

M_{cre} = moment causing flexural cracking due to externally applied loads

M_{max} = maximum factored moment at section due to externally applied loads

N = horizontal reaction

N_u = factored horizontal or axial force

r_b = bend radius of hanger reinforcement measured to the inside of the bar

V = vertical reaction

V_c = nominal shear strength provided by concrete

V_{ci} = flexural shear cracking strength

V_{cw} = web shear cracking strength

V_d = shear force at section due to unfactored dead load

V_i = factored shear force at section due to externally applied loads

V_n = nominal shear strength

V_p = vertical component of effective prestress force at section

V_s = nominal shear strength provided by steel reinforcement

$V_{s,max}$ = maximum nominal shear strength provided by steel reinforcement

V_{test} = measured shear force at failure

V_u = factored vertical reaction at end of beam

λ = modification factor of lightweight concrete

ϕ = shear-strength reduction factor

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Abstract

This paper describes the design of dapped ends of prestressed concrete thin-stemmed members based on an experimental program conducted to identify the most effective reinforcement schemes and develop design guidelines for dapped ends. The testing was part of a research program that included 20 full-scale tests and extensive finite element modeling. The experimental program, under which promising reinforcement schemes and key parameters were tested, is described in a companion paper. This paper describes the development of design guidelines for dapped thin-stemmed members based on analytical studies and an experimental program.

Several modified design practices for dapped double tees are recommended. Recommendations for control of cracking in the end region are also discussed.

Keywords

Bearing, cracking, dapped end, double tees, hanger reinforcement, nib, notched tees, thin-stemmed members, shear, shear friction.

Review policy

This paper was reviewed in accordance with the Precast/Prestressed Concrete Institute's peer-review process.

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