Dapped ends of prestressed concrete thin-stemmed members: Part 2, design

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Design example: Vertical Z scheme, two strands in nib

This section presents a design example for a 60 ft (18 m) long dapped-end 12 ft (3.7 m) wide double-tee beam with the vertical Z scheme. Two pretensioning strands pass through the nib.

Design loads

Table A1 gives the design loads used for the double-tee beam. The service and the factored load combinations considered in the design are based on the load combinations given in sections 2.3 and 2.4 in ASCE 7-10.\(^1\) Table A2 gives the geometry and details of the double tee.

<table>
<thead>
<tr>
<th>Characteristic</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam length, ft</td>
<td>60</td>
</tr>
<tr>
<td>Overall width, ft</td>
<td>12</td>
</tr>
<tr>
<td>Overall depth, in.</td>
<td>30</td>
</tr>
<tr>
<td>Flange thickness, in.</td>
<td>4</td>
</tr>
<tr>
<td>Cross-sectional area, in.(^2)</td>
<td>859</td>
</tr>
<tr>
<td>Top stem width, in.</td>
<td>6.25</td>
</tr>
<tr>
<td>Bottom stem width, in.</td>
<td>4.5</td>
</tr>
<tr>
<td>Dap depth, in.</td>
<td>14</td>
</tr>
<tr>
<td>Dap length, in.</td>
<td>8</td>
</tr>
</tbody>
</table>

| Concrete compressive strength \(f'_c\), psi | 7000 (typical) |
| Prestressing strand (½ in. special strand) ultimate strength, ksi | 270 |
| ASTM A615 reinforcing bar yield strength, ksi | 60 |
| Stem shear reinforcement (custom A185 WWR) minimum yield strength, ksi | 80 |

Note: WWR = welded-wire reinforcement. 1 in. = 25.4 mm; 1 ft = 0.305 m; 1 lb/ft\(^2\) = 47.88 Pa; 1 kip = 4.448 kN; 1 psi = 6.895 kPa; 1 ksi = 6.895 MPa.

<table>
<thead>
<tr>
<th>Load type</th>
<th>Service load (D + L)</th>
<th>Factored load (1.2D + 1.6L)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dead load (D), lb/ft(^2)</td>
<td>78</td>
<td>94</td>
</tr>
<tr>
<td>Live load (L), lb/ft(^2)</td>
<td>60</td>
<td>96</td>
</tr>
<tr>
<td>Combination, lb/ft(^2)</td>
<td>138</td>
<td>190</td>
</tr>
<tr>
<td>Vertical dap reaction, kip</td>
<td>25</td>
<td>34</td>
</tr>
</tbody>
</table>

Note: 1 kip = 4.448 kN; 1 lb/ft\(^2\) = 47.88 Pa.
Specified clear cover of embedded reinforcement from bottom \( c_{cb} = 1.25 \text{ in. (32 mm)} \)

Specified clear cover of embedded reinforcement from side face \( c_{cs} = 1.75 \text{ in. (44 mm)} \)

Length of bearing pad \( \ell_{pad} = 4 \text{ in. (100 mm)} \)

Width of bearing pad \( b_{pad} = 4 \text{ in. (125 mm)} \)

**Design procedure**

**Step 1: Verify sectional strength in B region (flexure, shear)**

Beyond a distance of \( 2h \) from the face of the dap, the beam region of dapped double tees was designed for flexure and shear following the section design requirements of ACI 318-14. \(^2\) **Figure A1** shows the beam cross section and the strand pattern. The flexural capacity of the section was designed to carry an applied factored design load of 190 lb/ft\(^2\) (9.10 kPa). The nominal flexural capacity was calculated using sectional analysis assuming a rectangular concrete stress distribution to satisfy both equilibrium conditions and strain compatibility. **Figure A2** shows the reinforcement in the dapped end, which will be proportioned in subsequent steps.

**Step 2: Verify shear strength in full section of D region**

Full section nominal shear strength \( V_n \) (two strands in nib):

\[
V_n = V_c + V_s
\]

where

\[
V_c = \text{nominal shear strength provided by concrete}
\]
where

\[ A_v = \text{area of diagonal tension reinforcement in full section of beam} \]

\[ s = \text{spacing between reinforcement bars} \]

\[ d = \text{effective depth from the centroid of reinforcement to the extreme fiber of the compression zone} \]

\[ A_s \geq \frac{12.1}{(80)(24)} \]

\[ \geq 0.0063 \text{ in.}^2/\text{in.} (0.16 \text{ mm}^2/\text{mm}) \]

Use one layer of W4 (5.7 mm) wires at 6 in. (150 mm), which provides a reinforcement area of 0.0067 in.²/in. (0.16 mm²/mm).

Check minimum shear reinforcement.

Minimum \[ \frac{A_s}{s} = 0.75 \frac{\sqrt{f_y b_v}}{f_y} = 0.75 \frac{\sqrt{7000 (5.51)}}{80,000} \]
APPENDIX

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\[
\begin{align*}
&= 0.0043 \text{in.}^2/\text{in.} (0.11 \text{mm}^2/\text{mm}) \\
&\geq \frac{50b}{f_y} = \frac{(50)(5.51)}{80,000} \\
&= 0.0034 \text{in.}^2/\text{in.} (0.086 \text{mm}^2/\text{mm})
\end{align*}
\]

where

Minimum \( \frac{A_s}{s} = 0.0043 \), which is less than that provided.

Thus \( \frac{A_s}{s} \rightarrow \text{OK} \)

Check maximum nominal shear strength provided by steel reinforcement \( V_{s,\text{max}} \).

\[
V_{s,\text{max}} = 2.0\sqrt{f_y b_d d_p} \\
= 2.0\sqrt{7000 (5.51)(24)} \\
= 22,100 \text{ lb} = 22.1 \text{ kip} (98.3 \text{ kN}) > 12.1 \text{ kip} \rightarrow \text{OK}
\]

**Step 3: Proportion hanger reinforcement**

Determine area of hanger reinforcement \( A_{sh} \).

\[
A_{sh} = \frac{V_{s,\text{max}}}{\phi f_y} = \frac{34}{(0.75)(60)}
\]

\[
= 0.76 \text{in.}^2 (490 \text{ mm}^2)
\]

Use one no. 8 (25M) bar (area equals 0.79 in.\(^2\) [510 mm\(^2\)]). The one no. 8 hanger bar should be inserted between the two columns of staggered strands.

**Step 4: Check bend region of hanger reinforcement**

Calculate bend radius of hanger reinforcement measured to the inside of the bar \( r_b \).

\[
r_b \geq \frac{2A_{sh} f_y}{b_d f_c}
\]

where

\[
b_d = \text{width of web at the bend region because the dap reinforcement is placed in the middle of the stem}
\]

\[
r_b = \frac{2(0.79)(60,000)}{(4.5)(7000)} \\
\geq 3.01 \text{ in.} (76.5 \text{ mm})
\]

Increase \( r_b \) by a ratio of \( 2d_{i}/c_{s} \), where \( d_{i} \) is the nominal diameter of reinforcement, and \( c_{s} \) is the specified clear cover of embedded reinforcement.

\[
2d_{i}/c_{s} = (2)(1.0)/1.75 = 1.14
\]

\[
r_b \geq (3.01)(1.14) = 3.44 \text{ in.} (87.4 \text{ mm})
\]

Make the radius of bend 3.5 in. (89 mm), which is greater than half the standard diameter of bend for this size bar, which is 3 in. (75 mm).

**Step 5: Determine length of hanger reinforcement horizontal extension \( \ell_{sh} \)**

Check confinement–to–bar diameter ratio \( c_{b}/d_{b} \), which should be at least 1.5 and preferably 2.5 or more. Concrete confinement \( c_{b} \) is controlled by bottom cover in this case.

\[
c_{b} = c_{b} + d_{b}/2
\]

\[
= 1.25 + 1.0/2 \\
= 1.75 \text{ in.} (44.4 \text{ mm})
\]

Increase \( c_{b} \) by a ratio of \( d_{b}/c_{c} \), where \( d_{b} \) is the nominal diameter of reinforcement, and \( c_{c} \) is the specified clear cover of embedded reinforcement.

\[
\frac{c_{b}}{d_{b}} = \frac{1.75}{1.0} \geq 1.5 \rightarrow \text{OK}
\]

The hanger bar tails should be extended into the span from the face of the dap at least 1.5 times the strand transfer length \( \ell_{t} \), and the horizontal extension from the bend should be at least 2.0 times the bar development length \( \ell_{d} \).

Check development length per ACI 318-14 (Eq. 25.4.2.3.a).

\[
\ell_{d} = \left[ \frac{3}{40} \sqrt{7000} \left( \frac{c_{b} + K_{tr}}{d_{b}} \right) \right] d_{b} \quad \text{(ACI 25.4.2.3.a)}
\]

where

\[
\psi_{e} = \text{factor used to modify development length based on reinforcement coating}
\]

\[
\psi_{t} = \text{factor used to modify development length for locating reinforcing in concrete placement}
\]

\[
\psi_{s} = \text{factor used to modify development length based on reinforcement size}
\]

\[
K_{tr} = \text{transverse reinforcement index (0 in this case)}
\]

\[
\ell_{d} = \left[ \frac{3}{40} \sqrt{7000} \left( \frac{1(1)(1)}{1.75 + 1.0} \right) \right] 1.0 \\
= 30.7 \text{ in.} (780 \text{ mm})
\]
Strand transfer length $\ell_t = 50d_s$

where

\[ d_s = \text{strand diameter} \]

\[ \ell_t = (50)(0.522) = 26.1 \text{ in. (663 mm)} \]

\[ \ell_{sh} = \text{maximum } 2\ell_d \text{ or } (1.5\ell_t - \ell_t) \]

\[ = \text{maximum } 2(30.7) \text{ or } [(1.5)(26.1) - 1.25)] \]

\[ = \text{maximum } 61.4 \text{ or } 37.9 \]

\[ = 61.4 \text{ in. (1560 mm)} \]

Therefore, use hanger reinforcement tail length $\ell_{sh}$ of 62 in. (1575 mm).

**Step 6: Proportion reinforcement for nib flexure and axial tension**

Calculate area of nib flexural reinforcement $A_s$.

\[ A_s = \frac{1}{f_y} \left[ \frac{V_a}{d_a} + N_u \left( \frac{h_u}{d_a} \right) \right] \]

\[ = \frac{1}{(0.75)(60)} \left[ 34 \left( \frac{7.5}{15.25} \right) + 6.8 \left( \frac{16.0}{15.25} \right) \right] \]

\[ = 0.53 \text{ in.}^2 (340 \text{ mm}^2) \]

Use two no. 5 (16M) bars (area = 0.62 in.\(^2\) [400 mm\(^2\)]).

This reinforcement should be welded to the bearing plate and extend at least a distance $\ell_t$ beyond the potential 45-degree crack intersecting the bottom corner of the full section.

**Step 7: Design for nib shear**

Nib shear strength (non–C bar detail):

\[ \phi V_n = \phi 6.0 \sqrt{f_y b_n d_n} \]

\[ = (0.75)(6.0) \sqrt{7000(5.98)(15.25)} \]

\[ = 34,300 \text{ lb} = 34.3 \text{ kip (153 kN)} > V_u \rightarrow \text{OK} \]

Nib shear friction:

\[ A_n = 0.5(A_s - A_n) \]

where

\[ A_n = \text{area of shear-friction reinforcement across vertical crack at dapped ends and corbels} \]

\[ A_n = \text{area of nib reinforcement resisting tensile force} \]

\[ A_n = \frac{N_u \left( \frac{h_u}{d_u} \right)}{\phi f_y} = \frac{6.8}{(0.75)(60)} \left( \frac{16}{15.25} \right) \]

\[ = 0.16 \text{ in.}^2 (100 \text{ mm}^2) \]

\[ A_n = 0.5(0.53 - 0.16) = 0.19 \text{ in.}^2 (120 \text{ mm}^2) \]

Use one no. 4 (13M) hairpin (U-shaped) bar (area = 0.40 in.\(^2\) [258 mm\(^2\)])

**Step 8: Design for direct bearing**

The design checks for nib flexure and shear (steps 6 and 7) ensure adequate flexure, shear, and shear-friction resistance. Check for maximum direct bearing in accordance with Eq. (9).

\[ \phi V_n \leq \frac{\phi 1.1 \ell_{pad} b_{pad} f_y}{60} \]

\[ 34 \leq (0.65)(1.1)(4.0)(4.0)(7.0) \]

\[ 34 \text{ kip} \leq 80.1 \text{ kip (356 kN)} \rightarrow \text{OK} \]

Also check bearing pad stress per PCI Design Handbook: Precast and Prestressed Concrete.\(^3\)

**References**


2. ACI (American Concrete Institute). 2014. Building Code Requirements for Structural Concrete (ACI 318-14) and Commentary (ACI 318R-14). Farmington Hills, MI: ACI.


**Notation**

\[ a = \text{shear span measured from the vertical reaction to center of hanger reinforcement} \]
\[ A_h = \text{area of shear-friction reinforcement across vertical} \]
\[ A_s = \text{area of nib flexural reinforcement} \]
\[ A_{sh} = \text{area of hanger reinforcement for dapped end} \]
\[ A_{sh} = \text{area of horizontal extension of hanger reinforcement} \]
\[ A_r = \text{area of diagonal tension reinforcement in section under consideration} \]
\[ b_h = \text{width of the web at the bend region} \]
\[ b_{pad} = \text{width of bearing pad, but not greater than the stem width} \]
\[ b_w = \text{width of web taken at mid-height of the portion of the tapered stem under consideration (full section or nib)} \]
\[ c_b = \text{concrete confinement = lesser of: distance from the center of bar or wire to the nearest concrete surface or one half the center-to-center spacing of bars or wires being developed} \]
\[ c_v = \text{specified clear cover of embedded reinforcement} \]
\[ c_{cb} = \text{specified clear cover of embedded reinforcement from bottom} \]
\[ c_{cs} = \text{specified clear cover of embedded reinforcement from side face} \]
\[ d = \text{effective depth from the centroid of reinforcement to the extreme fiber of the compression zone} \]
\[ d_b = \text{nominal diameter of reinforcement} \]
\[ d_a = \text{distance from extreme compression fiber to nib flexural reinforcement} \]
\[ d_p = \text{distance from extreme compression fiber to centroid of prestressing reinforcement but not less than 0.8h} \]
\[ d_s = \text{strand diameter} \]
\[ D = \text{dead load} \]
\[ f_{yc} = \text{specified compressive strength of concrete} \]
\[ f_y = \text{specified yield strength of reinforcement} \]
\[ h = \text{member height} \]
\[ h_n = \text{height of the nib} \]
\[ K_{tr} = \text{transverse reinforcement index} \]
\[ \ell_c = \text{clear distance between the face of the dap and the hanger reinforcement at the bottom of the section} \]
\[ \ell_d = \text{development length of reinforcement} \]
\[ \ell_{pad} = \text{length of bearing pad measured parallel to stem} \]
\[ \ell_{sh} = \text{length of hanger reinforcement horizontal extension} \]
\[ \ell_t = \text{strand transfer length} \]
\[ L = \text{live load} \]
\[ N_u = \text{factored horizontal or axial force} \]
\[ r_b = \text{bend radius of hanger reinforcement measured to the inside of the bar} \]
\[ s = \text{spacing between reinforcement bars} \]
\[ V_c = \text{nominal shear strength provided by concrete} \]
\[ V_n = \text{nominal shear strength} \]
\[ V_s = \text{nominal shear strength provided by steel reinforcement} \]
\[ V_{s,\text{max}} = \text{maximum nominal shear strength provided by steel reinforcement} \]
\[ V_a = \text{factored vertical reaction at end of beam} \]
\[ \phi = \text{shear-strength reduction factor} \]
\[ \psi_e = \text{factor used to modify development length based on reinforcement coating} \]
\[ \psi_s = \text{factor used to modify development length based on reinforcement size} \]
\[ \psi_t = \text{factor used to modify development length for locating of reinforcing in concrete placement} \]