APPENDIX

Dapped ends of prestressed concrete thin-stemmed members: Part 2, design

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Design example: Vertical Z scheme, two strands in nib

This section presents a design example for a 60 ft (18 m) long dapped-end 12 ft (3.7 m) wide double-tee beam with the vertical Z scheme. Two pretensioning strands pass through the nib.

Design loads

Table A1 gives the design loads used for the double-tee beam. The service and the factored load combinations considered in the design are based on the load combinations given in sections 2.3 and 2.4 in ASCE 7-10.[1] **Table A2** gives the geometry and details of the double tee.

Table A1. Design loads used for the beam

Load type	Service load $D + L$	Factored load $1.2D + 1.6L$
Dead load D, lb/ft²	78	94
Live load L, lb/ft²	60	96
Combination, lb/ft²	138	190
Vertical dap reaction, kip	25	34

Note: 1 kip = 4.448 kN; 1 lb/ft² = 47.88 Pa.

Table A2. Geometry and details of the double-tee beam

Characteristic	Value
Beam length, ft	60
Overall width, ft	12
Overall depth, in.	30
Flange thickness, in.	4
Cross-sectional area, in.²	859
Top stem width, in.	6.25
Bottom stem width, in.	4.5
Dap depth, in.	14
Dap length, in.	8
Concrete compressive strength f_c', psi	7000 (typical)
Prestressing strand (½ in. special strand) ultimate strength, ksi	270
ASTM A615 reinforcing bar yield strength, ksi	60
Stem shear reinforcement (custom A185 WWR) minimum yield strength, ksi	80

Note: WWR = welded-wire reinforcement. 1 in. = 25.4 mm; 1 ft = 0.305 m; 1 psi = 6.895 kPa; 1 ksi = 6.895 MPa.

Given information

Factored vertical reaction at end of beam V_u = 34 kip (150 kN)

Factored horizontal dap reaction N_u = (0.2)(34) = 6.8 kip (30 kN)

Shear strength reduction factor ϕ for dapped-end design = 0.75

Distance measured from the vertical reaction to center of hanger reinforcement a = 7.5 in. (190 mm)

Clear distance between the face of the dap and the hanger reinforcement at the bottom of the section ℓ_c = 1.25 in. (190 mm)

Distance from extreme compression fiber to nib flexural reinforcement d_n = 15.25 in. (387 mm)

Distance from extreme compression fiber to centroid of prestressing reinforcement (not less than 80% of the total height of the section, 0.8h) d_p = 24 in. (610 mm)

Width of web taken at midheight of the full-depth section b_w = 5.51 in. (140 mm)

Width of web taken at midheight of the nib b_w = 5.98 in. (152 mm)

Height of the nib h_n = 16 in. (406 mm)

Yield strength f_y of deformed reinforcement bars = 60 ksi (414 MPa)

Yield strength f_y of welded-wire reinforcement = 80 ksi (552 MPa)

Specified clear cover of embedded reinforcement from bottom c_{cb} = 1.25 in. (32 mm)

Specified clear cover of embedded reinforcement from side face c_{cs} = 1.75 in. (44 mm)

Length of bearing pad ℓ_{pad} = 4 in. (100 mm)

Width of bearing pad b_{pad} = 4 in. (125 mm)

Design procedure

Step 1: Verify sectional strength in B region (flexure, shear)

Beyond a distance of 2h from the face of the dap, the beam region of dapped double tees was designed for flexure and shear following the section design requirements of ACI 318-14.[2] **Figure A1** shows the beam cross section and the strand pattern. The flexural capacity of the section was designed to carry an applied factored design load of 190 lb/ft^2 (9.10 kPa). The nominal flexural capacity was calculated using sectional analysis assuming a rectangular concrete stress distribution to satisfy both equilibrium conditions and strain compatibility. **Figure A2** shows the reinforcement in the dapped end, which will be proportioned in subsequent steps.

Step 2: Verify shear strength in full section of D region

Full section nominal shear strength V_n (two strands in nib):

$$V_n = V_c + V_s$$

where

V_c = nominal shear strength provided by concrete

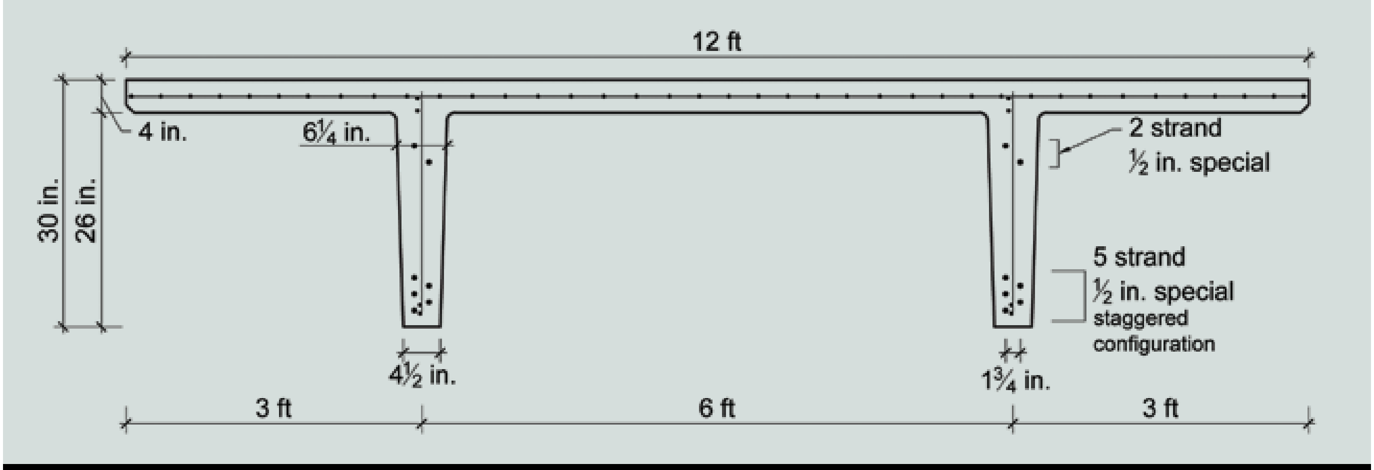

Figure A1. Double-tee cross section. Note: 1 in. = 25.4 mm; 1 ft = 0.305 m.

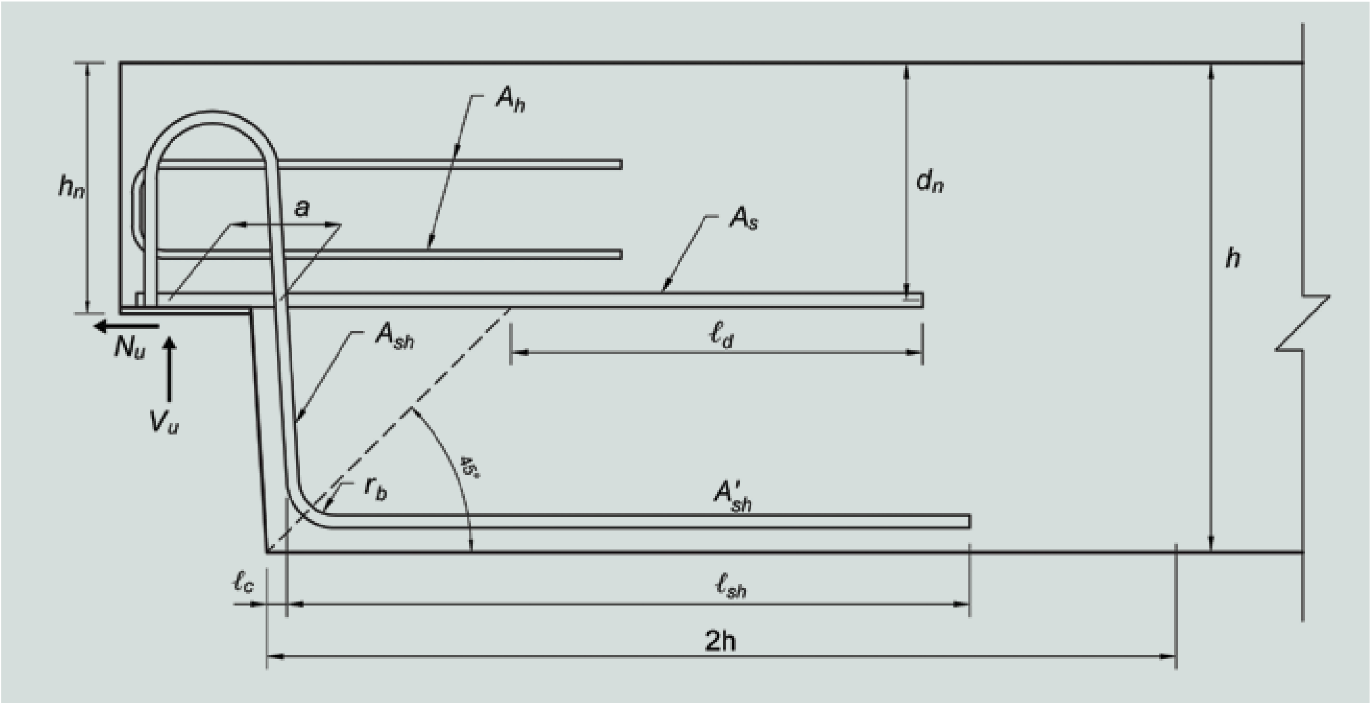

Figure A2. Dapped end with vertical Z reinforcement. Note: a = shear span measured from the vertical reaction to center of hanger reinforcement; A_h = area of shear-friction reinforcement across vertical crack at dapped ends and corbels; A_s = area of nib flexural reinforcement; A_{sh} = area of hanger reinforcement for dapped end; A'_{sh} = area of horizontal extension of hanger reinforcement; d_n = distance from extreme compression fiber to nib flexural reinforcement; h = member height; h_n = height of the nib; ℓ_c = clear distance between the face of the dap and the hanger reinforcement at the bottom of the section; ℓ_d = development length of reinforcement; ℓ_{sh} = length of hanger reinforcement bar tail; N_u = factored horizontal or axial force; r_b = bend radius of hanger reinforcement measured to the inside of the bar; V_u = factored vertical reaction at end of beam.

V_s = nominal shear strength provided by steel reinforcement

$V_c = 3.0\sqrt{f'_c}b_w d_p$ for two strands in the nib

where

f'_c = specified compressive strength of concrete

$$V_c = 3.0\sqrt{7000}(5.51)(24)$$

$$= 33{,}200 \text{ lb} = 33.2 \text{ kip (148 kN)}$$

$$V_s \geq \frac{V_u}{\phi} - V_c$$

$$\geq \frac{34}{0.75} - 33.2$$

$$\geq 12.1 \text{ kip (53.8 kN)}$$

$$\frac{A_v}{s} \geq \frac{V_s}{f_y d}$$

where

A_v = area of diagonal tension reinforcement in full section of beam

s = spacing between reinforcement bars

d = effective depth from the centroid of reinforcement to the extreme fiber of the compression zone

$$\frac{A_v}{s} \geq \frac{12.1}{(80)(24)}$$

$$\geq 0.0063 \text{ in.}^2\text{/in. } (0.16 \text{ mm}^2\text{/mm})$$

Use one layer of W4 (5.7 mm) wires at 6 in. (150 mm), which provides a reinforcement area of 0.0067 in.^{2}/in. (0.16 mm^2/mm).

Check minimum shear reinforcement.

$$\text{Minimum } \frac{A_v}{s} = 0.75\frac{\sqrt{f'_c}b_w}{f_y} = 0.75\frac{\sqrt{7000}(5.51)}{80{,}000}$$

$= 0.0043$ in.²/in. (0.11 mm²/mm)

$\geq \frac{50b_w}{f_y} = \frac{(50)(5.51)}{80,000}$

$= 0.0034$ in.²/in. (0.086 mm²/mm)

where

Minimum $\frac{A_v}{s} = 0.0043$, which is less than that provided.

Thus $\frac{A_v}{s} \longrightarrow$ OK

Check maximum nominal shear strength provided by steel reinforcement $V_{s,max}$.

$$V_{s,max} = 2.0\sqrt{f_c'}\,b_w d_p$$

$$= 2.0\sqrt{7000}(5.51)(24)$$

$$= 22{,}100 \text{ lb} = 22.1 \text{ kip } (98.3 \text{ kN}) > 12.1 \text{ kip} \longrightarrow \text{OK}$$

Step 3: Proportion hanger reinforcement

Determine area of hanger reinforcement A_{sh}.

$$A_{sh} = \frac{V_u}{\phi f_y} = \frac{34}{(0.75)(60)}$$

$$= 0.76 \text{ in.}^2 \ (490 \text{ mm}^2)$$

Use one no. 8 (25M) bar (area equals 0.79 in.² [510 mm2]). The one no. 8 hanger bar should be inserted between the two columns of staggered strands.

Step 4: Check bend region of hanger reinforcement

Calculate bend radius of hanger reinforcement measured to the inside of the bar r_b.

$$r_b \geq \frac{2A_{sh}f_y}{b_b f_c'}$$

where

b_b = width of web at the bend region because the dap reinforcement is placed in the middle of the stem

$$r_b = \frac{2(0.79)(60{,}000)}{(4.5)(7000)}$$

$$\geq 3.01 \text{ in. } (76.5 \text{ mm})$$

Increase r_b by a ratio of $2d_b/c_c$, where d_b is the nominal diameter of reinforcement, and c_c is the specified clear cover of embedded reinforcement.

$$2d_b/c_c = (2)(1.0)/1.75 = 1.14$$

$$r_b \geq (3.01)(1.14) = 3.44 \text{ in. } (87.4 \text{ mm})$$

Make the radius of bend 3.5 in. (89 mm), which is greater than half the standard diameter of bend for this size bar, which is 3 in. (75 mm).

Step 5: Determine length of hanger reinforcement horizontal extension ℓ_{sh}

Check confinement–to–bar diameter ratio c_b/d_b, which should be at least 1.5 and preferably 2.5 or more. Concrete confinement c_b is controlled by bottom cover in this case.

$$c_b = c_{cb} + d_b/2$$

$$= 1.25 + 1.0/2$$

$$= 1.75 \text{ in. } (44.4 \text{ mm})$$

$$\frac{c_b}{d_b} = \frac{1.75}{1.0}\ 1.75 > 1.5 \longrightarrow \text{OK}$$

The hanger bar tails should be extended into the span from the face of the dap at least 1.5 times the strand transfer length ℓ_t, and the horizontal extension from the bend should be at least 2.0 times the bar development length ℓ_d.

Check development length per ACI 318-14 (Eq. 25.4.2.3.a).

$$\ell_d = \left\{\left(\frac{3}{40}\right)\left(\frac{f_y}{\sqrt{f_c'}}\right)\left[\frac{\psi_e\psi_t\psi_s}{\left(\frac{c_b+K_{tr}}{d_b}\right)}\right]\right\}d_b \quad \text{(ACI 25.4.2.3.a)}$$

where

ψ_e = factor used to modify development length based on reinforcement coating

ψ_t = factor used to modify development length for locating reinforcing in concrete placement

ψ_s = factor used to modify development length based on reinforcement size

K_{tr} = transverse reinforcement index (0 in this case)

$$\ell_d = \left\{\left(\frac{3}{40}\right)\left(\frac{60{,}000}{\sqrt{7000}}\right)\left[\frac{(1)(1)(1)}{\left(\frac{1.75+0}{1.0}\right)}\right]\right\}1.0$$

$$= 30.7 \text{ in. } (780 \text{ mm})$$

Strand transfer length $\ell_t = 50d_s$

where

d_s = strand diameter

ℓ_t = (50)(0.522) = 26.1 in. (663 mm)

ℓ_{sh} = maximum $2\ell_d$ or $(1.5\ell_t - \ell_c)$

= maximum (2)(30.7) or [(1.5)(26.1) – 1.25)]

= maximum 61.4 or 37.9

= 61.4 in. (1560 mm)

Therefore, use hanger reinforcement tail length ℓ_{sh} of 62 in. (1575 mm).

Step 6: Proportion reinforcement for nib flexure and axial tension

Calculate area of nib flexural reinforcement A_s.

$$A_s = \frac{1}{\phi f_y}\left[V_u\left(\frac{a}{d_n}\right)+N_u\left(\frac{h_n}{d_n}\right)\right]$$

$$= \frac{1}{(0.75)(60)}\left[34\left(\frac{7.5}{15.25}\right)+6.8\left(\frac{16.0}{15.25}\right)\right]$$

= 0.53 in.^{2} (340 mm^2)

Use two no. 5 (16M) bars (area = 0.62 in.^{2} [400 mm^2]).

This reinforcement should be welded to the bearing plate and extend at least a distance ℓ_d beyond the potential 45-degree crack intersecting the bottom corner of the full section.

Step 7: Design for nib shear

Nib shear strength (non–C bar detail):

$$\phi V_n = \phi 6.0\sqrt{f_c'}\,b_w d_n$$

$$= (0.75)(6.0)\sqrt{7000}(5.98)(15.25)$$

= 34,300 lb = 34.3 kip (153 kN) > V_u ⟶ OK

Nib shear friction:

A_h = $0.5(A_s - A_n)$

where

A_h = area of shear-friction reinforcement across vertical crack at dapped ends and corbels

A_n = area of nib reinforcement resisting tensile force

$$A_n = \frac{N_u}{\phi f_y}\left(\frac{h_n}{d_n}\right) = \frac{6.8}{(0.75)(60)}\left(\frac{16}{15.25}\right)$$

= 0.16 in.^{2} (100 mm^2)

A_h = 0.5(0.53 – 0.16) = 0.19 in.^{2} (120 mm^2)

Use one no. 4 (13M) hairpin (U-shaped) bar (area = 0.40 in.^{2} [258 mm^2]).

Step 8: Design for direct bearing

The design checks for nib flexure and shear (steps 6 and 7) ensure adequate flexure, shear, and shear-friction resistance. Check for maximum direct bearing in accordance with Eq. (9).

$\phi V_n \le \phi 1.1 \ell_{pad} b_{pad} f_c'$

$34 \le (0.65)(1.1)(4.0)(4.0)(7.0)$

34 kip ≤ 80.1 kip (356 kN) ⟶ OK

Also check bearing pad stress per *PCI Design Handbook: Precast and Prestressed Concrete*.[3]

References

1. ASCE (American Society of Civil Engineers) Committee on *Minimum Design Loads for Buildings and Other Structures*. 2010. *Minimum Design Loads for Buildings and Other Structures.* ASCE/SEI 7-10.

2. ACI (American Concrete Institute). 2014. *Building Code Requirements for Structural Concrete (ACI 318-14) and Commentary (ACI 318R-14)*. Farmington Hills, MI: ACI.

3. PCI Industry Handbook Committee. 2010. *PCI Design Handbook: Precast and Prestressed Concrete*. MNL-120. 7th ed. Chicago, IL: PCI.

Notation

a = shear span measured from the vertical reaction to center of hanger reinforcement

A_h = area of shear-friction reinforcement across vertical

A_s = area of nib flexural reinforcement

A_{sh} = area of hanger reinforcement for dapped end

$A_{sh}^{'}$ = area of horizontal extension of hanger reinforcement

A_v = area of diagonal tension reinforcement in section under consideration

b_b = width of the web at the bend region

b_{pad} = width of bearing pad, but not greater than the stem width

b_w = width of web taken at mid-height of the portion of the tapered stem under consideration (full section or nib)

c_b = concrete confinement = lesser of: distance from the center of bar or wire to the nearest concrete surface or one half the center-to-center spacing of bars or wires being developed

c_c = specified clear cover of embedded reinforcement

c_{cb} = specified clear cover of embedded reinforcement from bottom

c_{cs} = specified clear cover of embedded reinforcement from side face

d = effective depth from the centroid of reinforcement to the extreme fiber of the compression zone

d_b = nominal diameter of reinforcement

d_n = distance from extreme compression fiber to nib flexural reinforcement

d_p = distance from extreme compression fiber to centroid of prestressing reinforcement but not less than 0.8h

d_s = strand diameter

D = dead load

$f_c^{'}$ = specified compressive strength of concrete

f_y = specified yield strength of reinforcement

h = member height

h_n = height of the nib

K_{tr} = transverse reinforcement index

ℓ_c = clear distance between the face of the dap and the hanger reinforcement at the bottom of the section

ℓ_d = development length of reinforcement

ℓ_{pad} = length of bearing pad measured parallel to stem

ℓ_{sh} = length of hanger reinforcement horizontal extension

ℓ_t = strand transfer length

L = live load

N_u = factored horizontal or axial force

r_b = bend radius of hanger reinforcement measured to the inside of the bar

s = spacing between reinforcement bars

V_c = nominal shear strength provided by concrete

V_n = nominal shear strength

V_s = nominal shear strength provided by steel reinforcement

$V_{s,max}$ = maximum nominal shear strength provided by steel reinforcement

V_u = factored vertical reaction at end of beam

ϕ = shear-strength reduction factor

ψ_e = factor used to modify development length based on reinforcement coating

ψ_s = factor used to modify development length based on reinforcement size

ψ_t = factor used to modify development length for locating of reinforcing in concrete placement