Examination of the effective coefficient of friction for shear friction design

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- This paper presents a database of shear friction test results collected from the literature and analyzed for the approaches included in the *PCI Design Handbook: Precast and Prestressed Concrete* and ACI's *Building Code Requirements for Structural Concrete (ACI 318-14) and Commentary (ACI 318R-14)*.
- The analysis indicates that PCI Eq. (5-32b) is more accurate and has a lower standard deviation than both PCI Eq. (5-32a) and ACI 318-14 Eq. (22.9.4.2) for normalweight, sand-lightweight, and all-lightweight concrete with monolithic uncracked, monolithic precracked, and cold-joint roughened interface conditions.
- For the cold-joint smooth interface condition, the authors recommend removing the modification factor λ in the coefficient of friction μ to provide more accurate and economical designs.

The seventh edition of the *PCI Design Handbook: Precast and Prestressed Concrete*[1] includes two approaches for shear friction design. The first approach, which has existed since the second edition of the handbook,[2] uses the effective coefficient of friction μ_e to compute the required area of shear reinforcement across the shear plane A_{vf} due to the factored shear force V_u.

$$A_{vf} = \frac{V_u}{\phi f_y \mu_e} \tag{5-32b}$$

where

ϕ = strength reduction factor = 0.75 for shear

f_y = yield strength of reinforcement, which has an upper limit of 60,000 psi (420 MPa)

In the seventh edition of the *PCI Design Handbook*,[1] μ_e is calculated using Eq. (5-33).

$$\mu_e = \frac{\phi 1000 \lambda A_{cr} \mu}{V_u} \tag{5-33}$$

where

Table 1. Shear friction coefficients and maximum shear strength for different interface conditions

Case	Crack interface condition	μ	*PCI Design Handbook*		ACI 318-14
			Maximum μ_e	Maximum V_u/ϕ	Maximum V_n
1	Concrete to concrete, cast monolithically	1.4λ	3.4	$0.30\lambda f_c' A_{cr} \le 1000\lambda A_{cr}$	For normalweight concrete: $0.2f_c' A_c \le (480 + 0.08f_c')A_c \le 1600A_c$
2	Concrete to hardened concrete with roughened surface	1.0λ	2.9	$0.25\lambda f_c' A_{cr} \le 1000\lambda A_{cr}$	For all other cases: $0.2\, f_c' A_c \le 800A_c$
3	Concrete placed against hardened concrete not intentionally roughened	0.6λ	n/a	$0.20\lambda f_c' A_{cr} \le 800\lambda A_{cr}$	$0.2f_c' A_c \le 800A_c$
4	Concrete to steel	0.7λ	n/a	$0.20\lambda f_c' A_{cr} \le 800\lambda A_{cr}$	

Note: A_c = area of concrete shear interface (ACI 318-14); A_{cr} = area of concrete shear interface (*PCI Design Handbook*); f_c' = concrete compressive strength; n/a = not applicable; V_n = nominal shear strength; V_u = factored shear force; λ = modification factor reflecting the reduced mechanical properties of lightweight concrete relative to normalweight concrete of the same compressive strength; μ = coefficient of friction; μ_e = effective coefficient of friction; ϕ = strength reduction factor.

λ = modification factor reflecting the reduced mechanical properties of lightweight concrete, relative to normalweight concrete of the same compressive strength

A_{cr} = area of concrete shear interface

μ = coefficient of friction, which is intended to account for friction between the surfaces of the crack interface

The value of μ is a function of the crack interface condition and the concrete type (normalweight, sand-lightweight, or all-lightweight) (**Table 1**). The modification factor for concrete type λ is intended to account for different mechanical properties of lightweight aggregate concrete relative to normalweight concrete of the same compressive strength. The value of λ is taken as 1.0 for normalweight concrete and 0.75 for all-lightweight concrete and may be taken as 0.85 for sand-lightweight concrete.[1] Table 1 summarizes the upper limits on the effective coefficient of friction μ_e. The *PCI Design Handbook*[1] also specifies upper limits on the shear strength V_u/ϕ (Table 1), which are intended to account for the value at which the shear plane is overreinforced and the shear transfer strength increases at a reduced rate as the reinforcement ratio increases.[3]

Substituting V_u/ϕ for the nominal shear strength V_n, substituting V_n/A_{cr} for nominal shear stress v_n, and combining PCI Eq. (5-32b) and (5-33) gives Eq. (1).

$$v_n = 31.62\sqrt{\rho f_y \lambda \mu} \tag{1}$$

where

ρ = shear friction reinforcement ratio = A_{vf}/A_{cr}

PCI Eq. (5-32b) can be expressed in terms of μ_e associated with the nominal shear stress v_n in Eq. (2).

$$\mu_e = \frac{v_n}{\rho f_y} \tag{2}$$

Use of the effective coefficient of friction μ_e is based on work summarized by Shaikh,[4] who proposed revisions to the traditional shear friction design concept by Mast[5] used in the American Concrete Institute's (ACI's) *Building Code Requirements for Structural Concrete (ACI 318-14) and Commentary (ACI 318R-14)*[6] to produce more economical designs. Shaikh evaluated equations for μ_e proposed by Mattock,[7] Birkeland,[8] and Raths[9] against the experimental test data available at that time. Equations for μ_e proposed by Birkeland[8] and Raths[9] took on a parabolic form that relates v_n and ρf_y and is the form of Eq. (5-33) in the *PCI Design Handbook*.[1] Alternatively, the equation proposed by Mattock[7] was the summation of a friction term and a cohesion term and is the form of shear friction design provisions in the American Association of State Highway and Transportation Officials' *AASHTO LRFD Bridge Design Specifications*.[10]

The equation used to compute μ_e has been modified in the past several editions of the *PCI Design Handbook* due to several mathematical anomalies identified by Tanner,[11] including revisions to the load and strength reduction factors and the inclusion of the modification factor for lightweight concrete. While the method proposed by Shaikh[4] was applicable to the four crack interface conditions in Table 1, revisions to the seventh edition of the *PCI Design Handbook*[1] have excluded its use for certain crack interface conditions, namely cases 3 and 4 in Table 1 (concrete placed against hardened concrete not intentionally roughened and concrete to steel, respectively).

The second approach to determining the required area of shear friction reinforcement was introduced in the seventh edition of the *PCI Design Handbook*,[1] in which the effective coefficient of friction μ_e in Eq. (5-32b) is replaced with the coefficient of friction μ. This approach, given in Eq. (5-32a) of the *PCI Design Handbook*,[1] is applicable for all four crack interface conditions in Table 1. Values of the coefficient of friction μ are included in Table 1.

$$A_{vf} = \frac{V_u}{\phi f_y \mu} \tag{5-32a}$$

PCI Eq. (5-32a) is consistent with ACI 318-14,[6] though the limits on the shear strength are different. The ACI 318-14 approach is given in Eq. (22.9.4.2).

$$V_n = \mu A_{vf} f_y \tag{22.9.4.2}$$

PCI Eq. (5-32a) and ACI 318-14 Eq. (22.9.4.2) can be expressed in terms of nominal stress v_n in Eq. (3).

$$\mu = \frac{v_n}{\rho f_y} \tag{3}$$

where

v_n = V_n/A_{cr}

V_n = V_u/ϕ

ρ = A_{vf}/A_{cr}

Since the introduction of the effective coefficient of friction μ_e approach to the *PCI Design Handbook*,[2] several studies have been published that expand the database of test results that can be used to compare and validate the shear friction design provisions. The shear friction concept has been studied extensively by others,[12–38] especially for normalweight concrete with various reinforcement ratios, compressive strengths, and interface conditions. Recent studies have focused on the use of high-strength concretes,[3,16,21] lightweight aggregate concretes,[17,18] and nonmonolithic (cold-joint) interface conditions.[16–19] This paper presents an up-to-date database of shear friction test results collected from the literature and examines the results in terms of the effective coefficient of friction μ_e and coefficient of friction μ approaches in the *PCI Design Handbook*[1] and ACI 318-14.[6]

Database

The literature was surveyed to collect published test data on direct shear transfer of concrete to concrete (cases 1 through 3 in Table 1). Works evaluated were limited to those published in English. Various test configurations have been used to evaluate shear friction, depending on the objective of the particular research study. For the purpose of direct comparison in this paper, only classical push-off specimens[12,14–21] without an external force normal to the shear plane were included. The external force normal to the shear plane criterion excluded studies conducted by Walraven and Reinhardt,[22] Papanicolau and Triantafillou,[23] and Echegaray et al.[24] Other types of test specimens that were not included in this database were inclined push-off specimens or those that had inclined reinforcement across the interface, such as the specimens in studies conducted by Vangsirirungrang,[25] Mattock and Hawkins,[13] Dulacska,[26] Mattock,[7] and Hawkins and Kuchma.[27] Also excluded were pull-off-type specimens, such as those studied by Chatterjee[28] and Mattock and Hawkins;[13] corbel-type specimens with moment and tension across the interface, such as those studied by Mattock et al.;[29] wall footing-type specimens, such as those studied by Bass et al.[30] and Valluvan et al.;[31] and beam-slab connections, such as those studied by Saemann and Washa,[32] Ivey and Buth,[33] Loov and Patnaik,[34] and Gohnert.[35,36] Horizontal push-off specimens studied by Hanson[37] and Paulay et al.[38] were not included in this database. The database in this paper is limited to specimens subjected to monotonic loading. Specimens that were cyclically loaded or specimens with sustained loading were not included. The criteria excluded specimens in the studies by Frenay[39] and Valluvan et al.[31]

Tables 2 through **5** present the resulting database, which summarizes the reference, specimen from the original reference, compressive strength of concrete f_c', shear interface area A_{cr}, reinforcement ratio ρ, reinforcement yield strength f_y, clamping stress $\rho f_{y,limited}$, peak measured shear force V_{test}, and peak measured shear stress v_{test} for each specimen. The compressive strength of concrete f_c' is the value reported at the test date. For cold-joint specimens with different concrete strengths on each side of the interface, the lower compressive strength is reported. The reinforcement ratio ρ is computed as A_{vf}/A_{cr}, where A_{vf} is the area of reinforcement crossing the shear plane, and A_{cr} is the area of the shear interface. Only specimens with no. 3 or 8 mm diameter reinforcing bars and larger were included in this database. The reinforcement yield strength f_y is the reported yield strength of the reinforcing bars, while clamping stress $\rho_{fy,limited}$ is computed considering the limitation on the value of f_y of 60,000 psi (420 MPa).[1,6] While most researchers report the peak measured shear force using the notation V_u, the peak measured shear force in the database is denoted as V_{test} to avoid confusion between it and the ultimate (factored) design shear force, which is also denoted as V_u in design provisions.[1,6] The peak measured shear stress v_{test} is defined as V_{test}/A_{cr}. The tables are organized by interface condition, which is given as concrete to concrete cast monolithically (referred to in

Table 2. Shear friction tests of push-off specimens with monolithic uncracked interface

Researcher(s)	Specimen	f'_c, psi	A_{cr}, in.^{2}	ρ	f_y, psi	$\rho f_{y,limited}$, psi*	V_{test}, lb	v_{test}, psi	μ_{test}	V_{test}/V_{calc} PCI Eq. (5-32b)	V_{test}/V_{calc} PCI Eq. (5-32a)	V_{test}/V_{calc} ACI 318-14 Eq. (22.9.4.2)
Normalweight concrete												
Hofbeck, Ibrahim, and Mattock (1969)	1.1A	3920	50	0.004	50,700	223	37,500	750	3.4	1.34	2.40	2.40
	1.1B	4340	50	0.004	48,000	211	42,200	844	4.0	1.55	2.85	2.85
	1.2A	3840	50	0.009	50,700	446	50,000	1000	2.2	1.27	1.60	1.60
	1.2B	4180	50	0.009	48,000	422	49,000	980	2.3	1.27	1.66	1.66
	1.3A	3840	50	0.013	50,700	669	55,000	1100	1.6	1.14	1.17	1.43
	1.3B	3920	50	0.013	48,000	634	53,500	1070	1.7	1.14	1.21	1.36
	1.4A	4510	50	0.018	50,700	892	68,000	1360	1.5	1.36	1.36	1.62
	1.4B	3855	50	0.018	48,000	845	64,000	1280	1.5	1.28	1.28	1.66
	1.5A	4510	50	0.022	50,700	1115	70,000	1400	1.3	1.40	1.40	1.67
	1.5B	4065	50	0.022	48,000	1056	69,200	1384	1.3	1.38	1.38	1.72
	1.6A	4310	50	0.026	50,700	1338	71,600	1432	1.1	1.43	1.43	1.74
	1.6B	4050	50	0.026	48,000	1267	71,000	1420	1.1	1.42	1.42	1.77
Mattlock, Li, and Wang (1976)	M1	4180	50	0.004	50,900	224	38,000	760	3.4	1.36	2.42	2.42
	M2	3900	50	0.009	52,700	464	49,000	980	2.1	1.22	1.51	1.51
	M3	3995	50	0.013	52,300	690	55,500	1110	1.6	1.13	1.15	1.39
	M4	4150	50	0.018	50,900	896	57,000	1140	1.3	1.14	1.14	1.40
	M5	3935	50	0.022	52,700	1159	64,000	1280	1.1	1.28	1.28	1.63
	M6	4120	50	0.026	52,700	1391	66,000	1320	0.9	1.32	1.32	1.63
Kahn and Mitchell (2002)	SF-4-1-U	6805	60	0.004	69,500	220	57,880	965	4.4	1.74	3.13	3.13
	SF4-2-U	6805	60	0.007	69,500	440	80,080	1335	3.0	1.70	2.17	2.17
	SF-4-3-U	6805	60	0.011	69,500	660	85,830	1431	2.2	1.49	1.55	1.55
	SF-7-1-U	11,734	60	0.004	83,000	220	87,550	1459	6.6	2.63	4.74	4.74
	SF-7-2-U	12,410	60	0.007	83,000	440	118,110	1969	4.5	2.51	3.20	3.20
	SF-7-3-U	13,103	60	0.011	83,000	660	138,430	2307	3.5	2.40	2.50	2.50
	SF-7-4-U	12,471	60	0.015	83,000	880	149,090	2485	2.8	2.49	2.49	2.02
	SF-10-1-U-a	12,053	60	0.004	83,000	220	100,090	1668	7.6	3.01	5.42	5.42
	SF-10-1-U-b	14,326	60	0.004	83,000	220	91,880	1531	7.0	2.76	4.97	4.97
	SF-10-2-U-a	14,776	60	0.007	83,000	440	130,650	2178	5.0	2.78	3.54	3.54
	SF-10-2-U-b	14,804	60	0.007	83,000	440	124,050	2068	4.7	2.62	3.36	3.36
	SF-10-3-U-a	16,170	60	0.011	83,000	660	144,820	2414	3.7	2.51	2.61	2.61
	SF-10-3-U-b	13,934	60	0.011	83,000	660	147,900	2465	3.7	2.56	2.67	2.67
	SF-10-4-U-a	15,468	60	0.015	83,000	880	156,030	2601	3.0	2.60	2.60	2.11
	SF-10-4-U-b	16,476	60	0.015	83,000	880	160,040	2667	3.0	2.67	2.67	2.16
	SF-14-1-U	17,957	60	0.004	83,000	220	94,950	1583	7.2	2.85	5.14	5.14
	SF-14-2-U	17,362	60	0.007	83,000	440	108,460	1808	4.1	2.30	2.94	2.94
	SF-14-3-U	16,255	60	0.011	83,000	660	146,230	2437	3.7	2.54	2.64	2.64
	SF-14-4-U	16,059	60	0.015	83,000	880	155,970	2600	3.0	2.60	2.60	2.11

Table 2 (continued). Shear friction tests of push-off specimens with monolithic uncracked interface

Researcher(s)	Specimen	f'_c, psi	A_{cr}, in.^{2}	ρ	f_y, psi	$\rho f_{y,limited}$, psi*	V_{test}, lb	v_{test}, psi	μ_{test}	V_{test}/V_{calc} PCI Eq. (5-32b)	V_{test}/V_{calc} PCI Eq. (5-32a)	V_{test}/V_{calc} ACI 318-14 Eq. (22.9.4.2)
Sneed, Krc, Wermager, and Meinheit (2016)	N-MO-U-1	4840	49.5	0.013	72,200	780	63,410	1281	1.6	1.28	1.28	1.48
	N-MO-U-2	4840	49.5	0.013	72,200	780	62,200	1257	1.6	1.26	1.26	1.45
									Average	1.86	2.34	2.39
									Maximum	3.01	5.42	5.42
								Normalweight concrete	Minimum	1.13	1.14	1.36
									STD	0.655	1.179	1.098
									COV	0.351	0.503	0.459
Sand-lightweight concrete												
Mattock, Li, and Wang (1976)	A1	3740	50	0.004	47,700	210	37,900	758	3.6	1.65	3.03	3.03
	A2	4095	50	0.009	53,600	472	45,700	914	1.9	1.32	1.63	1.63
	A3	3910	50	0.013	53,200	702	51,000	1020	1.5	1.21	1.22	1.30
	A4	4100	50	0.018	50,900	896	55,000	1100	1.2	1.29	1.29	1.38
	A5	3960	50	0.022	50,900	1120	59,500	1190	1.1	1.40	1.40	1.50
	A6	4250	50	0.026	51,800	1368	67,200	1344	1.0	1.58	1.58	1.68
Sneed, Krc, Wermager, and Meinheit (2016)	S-SH-MO-U-1	4770	49.5	0.013	72,200	780	55,430	1120	1.4	1.32	1.32	1.40
	S-SH-MO-U-2	4770	49.5	0.013	72,200	780	56,590	1143	1.5	1.34	1.34	1.43
									Average	1.39	1.60	1.67
									Maximum	1.65	3.03	3.03
								Sand-lightweight concrete	Minimum	1.21	1.22	1.30
									STD	0.149	0.596	0.566
									COV	0.107	0.372	0.339
All-lightweight concrete												
Mattock, Li, and Wang (1976)	E1	4150	50	0.004	52,300	230	39,000	780	3.4	1.83	3.23	3.23
	E2	4030	50	0.009	52,300	460	43,600	872	1.9	1.45	1.80	1.80
	E3	4065	50	0.013	52,300	690	48,000	960	1.4	1.30	1.32	1.32
	E4	4040	50	0.018	53,200	936	57,500	1150	1.2	1.53	1.53	1.44
	E5	4115	50	0.022	50,500	1111	60,000	1200	1.1	1.60	1.60	1.50
	E6	4050	50	0.026	52,300	1381	62,500	1250	0.9	1.67	1.67	1.56
	G1	4145	50	0.004	52,300	230	41,000	820	3.6	1.93	3.39	3.39
	G2	3880	50	0.009	50,500	444	42,300	846	1.9	1.43	1.81	1.81
	G3	4100	50	0.013	51,800	684	53,000	1060	1.6	1.44	1.48	1.48
	G4	4420	50	0.018	53,200	936	57,500	1150	1.2	1.53	1.53	1.44
	G5	4005	50	0.022	51,800	1140	57,000	1140	1.0	1.52	1.52	1.43
	G6	4005	50	0.026	51,800	1368	59,500	1190	0.9	1.59	1.59	1.49
Sneed, Krc, Wermager, and Meinheit (2016)	A-SH-MO-U-1	4700	49.5	0.013	72,200	780	52,030	1051	1.3	1.40	1.40	1.31
	A-SH-MO-U-2	4700	49.5	0.013	72,200	780	52,550	1062	1.4	1.42	1.42	1.33
									Average	1.55	1.81	1.75
									Maximum	1.93	3.39	3.39
								All-lightweight concrete	Minimum	1.30	1.32	1.31
									STD	0.170	0.653	0.679
									COV	0.110	0.361	0.387

Note: A_{cr} = area of concrete shear interface; A_{vf} = area of shear reinforcement across shear plane; COV = coefficient of variation; f'_c = concrete compressive strength; f_y = yield strength of reinforcement; $f_{y,limited}$ = yield strength of reinforcement, limited to a maximum value of 60,000 psi; STD = standard deviation; v_{test} = peak measured shear stress; V_{calc} = calculated value of the nominal shear strength; V_{test} = peak measured shear strength; μ_{test} = effective coefficient of friction calculated using the measured shear stress and the yield stress of reinforcement, limited to a maximum value of 60,000 psi = $v_t/\rho f_{y,limited}$; ρ = shear friction reinforcement ratio = A_{vf}/A_{cr}; $\rho f_{y,limited}$ = clamping stress. 1 in.^{2} = 645.2 mm^2; 1 lb = 4.448 N; 1 psi = 6.895 kPa.

* $\rho f_{y,limited}$ is computed using the actual yield strength but is not greater than 60,000 psi.

Table 3. Shear friction tests of push-off specimens with monolithic precracked interface

Researcher(s)	Specimen	f_c', psi	A_{cr}, in.^{2}	ρ	f_y, psi	$\rho f_{y,limited}$, psi*	V_{test}, lb	v_{test}, psi	μ_{test}	V_{test}/V_{calc} PCI Eq. (5-32b)	V_{test}/V_{calc} PCI Eq. (5-32a)	V_{test}/V_{calc} ACI 318-14 Eq. (22.9.4.2)
						Normalweight concrete						
Hofbeck, Ibrahim, and Mattock (1969)	2.1	3100	50	0.004	50,700	223	29,500	590	2.6	1.06	1.89	1.89
	2.2	3100	50	0.009	50,700	446	34,000	680	1.5	0.86	1.09	1.10
	2.3	3900	50	0.013	50,700	669	42,000	840	1.3	0.87	0.90	1.08
	2.4	3900	50	0.018	50,700	892	50,000	1000	1.1	1.00	1.00	1.28
	2.5	4180	50	0.022	50,700	1115	65,000	1300	1.2	1.30	1.30	1.60
	2.6	4180	50	0.026	50,700	1338	69,250	1385	1.0	1.39	1.39	1.70
	3.3	3100	50	0.009	50,700	446	34,000	680	1.5	0.86	1.09	1.10
	3.4	4040	50	0.016	47,200	741	51,400	1028	1.4	1.03	1.03	1.28
	3.5	4040	50	0.025	42,400	1039	57,600	1152	1.1	1.15	1.15	1.43
	4.1	4070	50	0.004	66,100	264	35,200	704	2.7	1.16	1.90	1.90
	4.2	4070	50	0.009	66,100	528	49,000	980	1.9	1.14	1.33	1.33
	4.3	4340	50	0.013	66,100	792	59,000	1180	1.5	1.18	1.18	1.43
	4.4	4340	50	0.018	66,100	1056	70,000	1400	1.3	1.40	1.40	1.69
	4.5	3390	50	0.022	66,100	1320	66,00	1320	1.0	1.32	1.32	1.95
	5.1	2450	50	0.004	50,700	223	25,500	510	2.3	0.91	1.63	1.63
	5.2	2620	50	0.009	50,700	446	35,000	700	1.6	0.89	1.12	1.34
	5.3	2385	50	0.013	50,700	669	40,500	810	1.2	1.13	1.13	1.70
	5.4	2580	50	0.018	50,700	892	39,750	795	0.9	1.03	1.03	1.54
	5.5	2620	50	0.022	50,700	1115	50,500	1010	0.9	1.28	1.28	1.93
Mattock, Li, and Wang (1976)	N1	4180	50	0.004	50,900	224	23,000	460	2.1	0.82	1.47	1.47
	N2	3900	50	0.009	52,700	464	39,000	780	1.7	0.97	1.20	1.20
	N3	3995	50	0.013	52,300	690	48,000	960	1.4	0.98	0.99	1.20
	N4	4150	50	0.018	50,900	896	57,500	1150	1.3	1.15	1.15	1.42
	N5	3935	50	0.022	50,900	1120	58,750	1175	1.0	1.18	1.18	1.49
	N6	4120	50	0.026	50,000	1320	59,500	1190	0.9	1.19	1.19	1.47
Kahn and Mitchell (2002)	SF-4-1-C	6805	60	0.004	69,500	220	35,000	583	2.7	1.05	1.89	1.89
	SF-4-2-C	6805	60	0.007	69,500	440	55,690	928	2.1	1.18	1.51	1.51
	SF-4-3-C	6805	60	0.011	69,500	660	71,130	1186	1.8	1.23	1.28	1.28
	SF-7-1-C	11,734	60	0.004	83,000	220	41,680	695	3.2	1.25	2.26	2.26
	SF-7-2-C	12,410	60	0.007	83,000	440	51,730	862	2.0	1.10	1.40	1.40
	SF-7-3-C	13,103	60	0.011	83,000	660	71,510	1192	1.8	1.24	1.29	1.29
	SF-7-4-C	12,471	60	0.015	83,000	880	62,730	1046	1.2	1.05	1.05	0.85
	SF-10-1-C-a	12,053	60	0.004	83,000	220	25,780	430	2.0	0.77	1.40	1.40
	SF-10-1-C-b	14,326	60	0.004	83,000	220	29,970	500	2.3	0.90	1.62	1.60
	SF-10-2-C-a	14,676	60	0.007	83,000	440	50,780	846	1.9	1.08	1.37	1.37
	SF-10-2-C-b	14,804	60	0.007	83,000	440	48,110	802	1.8	1.02	1.30	1.30
	SF-10-3-C-a	16,170	60	0.011	83,000	660	64,650	1078	1.6	1.12	1.17	1.17
	SF-10-3-C-b	13,924	60	0.011	83,000	660	63,360	1056	1.6	1.10	1.14	1.14
	SF-10-4-C-a	15,468	60	0.015	83,000	880	74,160	1236	1.4	1.24	1.24	1.00
	SF-10-4-C-b	16,476	60	0.015	83,000	880	76,280	1271	1.4	1.27	1.27	1.03
	SF-14-1-C	16,015	60	0.004	83,000	220	24,880	415	1.9	0.75	1.35	1.35
	SF-14-2-C	15,496	60	0.007	83,000	440	40,180	670	1.5	0.85	1.09	1.09
	SF-14-3-C	15,392	60	0.011	83,000	660	55,500	925	1.4	0.96	1.00	1.00
	SF-14-4-C	15,982	60	0.015	83,000	880	73,270	1221	1.4	1.22	1.22	0.99

Table 3 (continued). Shear friction tests of push-off specimens with monolithic precracked interface

Researcher(s)	Specimen	f_c', psi	A_{cr}, in.²	ρ	f_y, psi	$\rho f_{y,limited}$, psi*	V_{test}, lb	v_{test}, psi	μ_{test}	V_{test}/V_{calc} PCI Eq. (5-32b)	V_{test}/V_{calc} PCI Eq. (5-32a)	V_{test}/V_{calc} ACI 318-14 Eq. (22.9.4.2)
Mansur, Vinayagam, and Tan (2008)	AN-2	5831	55.8	0.009	76,870	523	66,180	1186	2.3	1.39	1.62	1.62
	AN-4	5831	55.8	0.017	76,870	1046	82,305	1475	1.4	1.48	1.48	1.56
	AN-6	5831	55.8	0.026	76,870	1570	104,570	1874	1.2	1.87	1.87	1.98
	AM-2	10,008	55.8	0.009	76,870	523	60,710	1088	2.1	1.27	1.49	1.49
	AM-3	10,008	55.8	0.013	76,870	785	93,075	1668	2.1	1.67	1.67	1.52
	AM-4	10,008	55.8	0.017	76,870	1046	113,555	2035	1.9	2.04	2.04	1.59
	AH-2	12,618	55.8	0.009	76,870	523	62,940	1128	2.2	1.32	1.54	1.54
	AH-3	12,618	55.8	0.013	76,870	785	100,050	1793	2.3	1.79	1.79	1.63
	AH-4	12,618	55.8	0.017	76,870	1046	114,670	2055	2.0	2.06	2.06	1.40
	B1-4	10,618	55.8	0.011	43,511	486	54,460	976	2.0	1.18	1.43	1.43
	B2-2	12,314	55.8	0.006	43,511	243	41,850	750	3.1	1.29	2.20	2.20
	B2-4	12,314	55.8	0.011	43,511	486	59,260	1062	2.2	1.29	1.56	1.56
	B2-5	12,314	55.8	0.014	43,511	608	66,460	1191	2.0	1.29	1.40	1.40
	B2-6	12,314	55.8	0.017	43,511	730	74,215	1330	1.8	1.33	1.33	1.30
	B3-4	13,808	55.8	0.011	43,511	486	64,505	1156	2.4	1.40	1.70	1.70
	B4-2	15,432	55.8	0.006	43,511	243	48,660	872	3.6	1.49	2.56	2.56
	B4-4	15,432	55.8	0.011	43,511	486	68,245	1223	2.5	1.48	1.80	1.80
	B4-5	15,432	55.8	0.014	43,511	608	74,770	1340	2.2	1.45	1.57	1.57
	B4-6	15,432	55.8	0.017	43,511	730	80,630	1445	2.0	1.45	1.45	1.41
Sneed, Krc, Wermager, and Meinheit (2016)	N-MO-P-1	4840	49.5	0.013	72,200	780	61,070	1234	1.6	1.23	1.23	1.42
	N-MO-P-2	4840	49.5	0.013	72,200	780	56,970	1151	1.5	1.15	1.15	1.33
									Average	1.21	1.42	1.48
									Maximum	2.06	2.56	2.56
							Normalweight concrete		Minimum	0.75	0.90	0.85
									STD	0.272	0.342	0.320
									COV	0.225	0.241	0.216
					Sand-lightweight concrete							
Mattock, Li, and Wang (1976)	B1	3740	50	0.004	49,600	218	22,500	450	2.1	0.96	1.73	1.73
	B2	3360	50	0.009	50,900	448	32,600	652	1.5	0.97	1.22	1.22
	B3	3910	50	0.013	50,900	672	42,000	840	1.3	1.02	1.05	1.07
	B4	4100	50	0.018	49,100	864	47,000	940	1.1	1.11	1.11	1.18
	B5	3960	50	0.022	50,500	1111	50,000	1000	0.9	1.18	1.18	1.26
	B6	4250	50	0.026	51,800	1368	57,700	1154	0.8	1.36	1.36	1.44
	C1	2330	50	0.004	49,600	218	18,200	364	1.7	0.77	1.40	1.40
	C2	2330	50	0.009	53,600	472	25,700	514	1.1	0.87	0.92	1.10
	C3	2000	50	0.013	50,900	672	26,300	526	0.8	1.03	1.03	1.32
	C4	2050	50	0.018	52,300	920	28,000	560	0.6	1.07	1.07	1.37
	C5	2330	50	0.022	53,600	1179	32,000	640	0.5	1.08	1.08	1.37
	C6	2330	50	0.026	49,600	1309	37,000	740	0.6	1.25	1.25	1.59
	D1	5995	50	0.004	51,800	228	18,500	370	1.6	0.77	1.36	1.36
	D2	5995	50	0.009	52,300	460	33,400	668	1.5	0.98	1.22	1.22
	D3	5710	50	0.013	52,300	690	38,600	772	1.1	0.92	0.94	0.97
	D4	5710	50	0.018	52,300	920	51,100	1022	1.1	1.20	1.20	1.28
	D5	5600	50	0.022	52,300	1151	51,100	1082	0.9	1.27	1.27	1.35
	D6	5600	50	0.026	51,800	1368	61,000	1220	0.9	1.44	1.44	1.53

Table 3 (continued). Shear friction tests of push-off specimens with monolithic precracked interface

Researcher(s)	Specimen	f_c', psi	A_{cr}, in.²	ρ	f_y, psi	$\rho f_{y,limited}$, psi*	V_{test}, lb	v_{test}, psi	μ_{test}	V_{test}/V_{calc} PCI Eq. (5-32b)	V_{test}/V_{calc} PCI Eq. (5-32a)	V_{test}/V_{calc} ACI 318-14 Eq. (22.9.4.2)
Hoff (1993)	1 LWC1	8490	84	0.005	53,600	281	24,108	287	1.0	0.54	0.86	0.86
	2 LWC1	8510	84	0.005	53,600	281	30,660	365	1.3	0.68	1.09	1.09
	3 LWC1	8290	84	0.005	53,600	281	34,692	413	1.5	0.78	1.24	1.24
	4 LWC1	8490	84	0.010	68,000	571	63,924	761	1.3	1.00	1.12	1.12
	5 LWC1	8510	84	0.010	69,000	571	57,120	680	1.2	0.89	1.00	1.00
	6 LWC1	8290	84	0.010	68,000	571	61,068	727	1.3	0.96	1.07	1.07
	1 LWC2	9270	84	0.005	53,600	281	41,076	489	1.7	0.92	1.46	1.46
	2 LWC2	8760	84	0.005	53,600	281	28,140	335	1.2	0.63	1.00	1.00
	3 LWC2	8730	84	0.005	53,600	281	25,116	299	1.1	0.56	0.89	0.89
	4 LWC2	9270	84	0.010	68,000	571	62,076	739	1.3	0.97	1.09	1.09
	5 LWC2	8760	84	0.010	68,500	571	58,128	692	1.2	0.91	1.02	1.02
	6 LWC2	8730	84	0.010	68,500	571	57,120	680	1.2	0.89	1.00	1.00
	1 HSLWC	10,310	84	0.005	72,100	314	56,112	668	2.1	1.18	1.79	1.79
	2 HSLWC	10,910	84	0.005	72,100	314	46,032	548	1.7	0.97	1.47	1.47
	3 HSLWC	11,020	84	0.005	72,100	314	49,140	585	1.9	1.04	1.56	1.56
	4 HSLWC	10,310	84	0.010	66,800	571	73,080	870	1.5	1.14	1.28	1.28
	5 HSLWC	10,910	84	0.010	66,800	571	73,080	870	1.5	1.14	1.28	1.28
	6 HSLWC	11,020	84	0.010	66,800	571	75,096	894	1.6	1.18	1.31	1.31
Sneed, Krc, Wermager, and Meinheit (2016)	S-SH-MO-P-1	4770	49.5	0.013	72,200	780	50,593	1022	1.3	1.20	1.20	1.28
	S-SH-MO-P-2	4770	49.5	0.013	72,200	780	51,880	1048	1.3	1.23	1.23	1.31
									Average	1.00	1.20	1.26
									Maximum	1.44	1.79	1.79
									Minimum	0.54	0.86	0.86
									STD	0.209	0.216	0.220
									COV	0.209	0.180	0.174
All-lightweight concrete												
Mattock, Li, and Wang (1976)	F1	4150	50	0.004	53,200	234	22,500	450	1.9	1.05	1.83	1.83
	F2	4030	50	0.009	52,300	460	26,500	530	1.2	0.88	1.10	1.10
	F2A	3970	50	0.009	50,900	448	31,000	620	1.4	1.04	1.32	1.32
	F3	4065	50	0.013	52,300	690	36,700	734	1.1	1.00	1.01	1.01
	F3A	3970	50	0.013	51,400	678	35,100	702	1.0	0.96	0.99	0.99
	F4	4040	50	0.018	50,900	896	43,500	870	1.0	1.16	1.16	1.09
	F5	4115	50	0.022	51,800	1140	46,000	920	0.8	1.23	1.23	1.15
	F6	4050	50	0.026	53,200	1404	49,100	982	0.7	1.31	1.31	1.23
	H1	4145	50	0.004	49,800	219	20,000	400	1.8	0.96	1.74	1.74
	H2	3880	50	0.009	51,800	456	31,000	620	1.4	1.03	1.30	1.30
	H3	4100	50	0.013	51,800	684	43,300	866	1.3	1.18	1.21	1.21
	H4	4420	50	0.018	51,800	912	47,000	940	1.0	1.25	1.25	1.18
	H5	3950	50	0.022	50,500	1111	49,500	990	0.9	1.32	1.32	1.25
	H6	4080	50	0.026	49,800	1315	52,100	1042	0.8	1.39	1.39	1.30

Table 3 (continued). Shear friction tests of push-off specimens with monolithic precracked interface

Researcher(s)	Specimen	f'_c, psi	A_{cr}, in.[2]	ρ	f_y, psi	$\rho f_{y,limited}$, psi*	V_{test}, lb	v_{test}, psi	μ_{test}	V_{test}/V_{calc} PCI Eq. (5-32b)	V_{test}/V_{calc} PCI Eq. (5-32a)	V_{test}/V_{calc} ACI 318-14 Eq. (22.9.4.2)
Sneed, Krc, Wermager, and Meinheit (2016)	A-SH-MO-P-1	4700	49.5	0.013	72,200	780	46,120	932	1.2	1.24	1.24	1.16
	A-SH-MO-P-2	4700	49.5	0.013	72,200	780	52,690	1064	1.4	1.42	1.42	1.33
									Average	1.15	1.30	1.26
									Maximum	1.42	1.83	1.83
All-lightweight concrete									Minimum	0.88	0.99	0.99
									STD	0.166	0.225	0.230
									COV	0.144	0.173	0.182

Note: A_{cr} = area of concrete shear interface; A_{vf} = area of shear reinforcement across shear plane; COV = coefficient of variation; f'_c = concrete compressive strength; f_y = yield strength of reinforcement; $f_{y,limited}$ = yield strength of reinforcement, limited to a maximum value of 60,000 psi; STD = standard deviation; v_{test} = peak measured shear stress; V_{calc} = calculated value of the nominal shear strength; V_{test} = peak measured shear strength; μ_{test} = effective coefficient of friction calculated using the measured shear stress and the yield stress of reinforcement, limited to a maximum value of 60,000 psi = $v_n/\rho f_{y,limited}$; ρ = shear friction reinforcement ratio = A_{vf}/A_{cr}; $\rho f_{y,limited}$ = clamping stress. 1 in.[2] = 645.2 mm[2]; 1 lb = 4.448 N; 1 psi = 6.895 kPa.
* $\rho f_{y,limited}$ is computed using the actual yield strength but is not greater than 60,000 psi.

this paper as *monolithic uncracked*) (Table 2), concrete to concrete cast monolithically and precracked before testing (referred to in this paper as *monolithic precracked*) (Table 3), concrete to hardened concrete with roughened surface (referred to in this paper as *cold-joint roughened*) (Table 4), or concrete placed against hardened concrete not intentionally roughened (referred to in this paper as *cold-joint smooth*) (Table 5). Within each table, specimens are grouped in terms of concrete type. Concrete type is given as normalweight, sand-lightweight, or all-lightweight, where each type is designated by its aggregate composition. For the purposes of this database, the unit weight of concrete and aggregate source are not included because most studies did not report these values.

The database includes 302 specimens from nine studies.[12,14–21] **Figure 1** shows the data distribution in terms of concrete type, interface condition, compressive strength of concrete f'_c, reinforcement ratio ρ, clamping stress $\rho f_{y,limited}$, and area of shear interface A_{cr}. The following sections include additional discussion of data distribution.

Analysis of database

This section compares the results from the experiments summarized in Tables 2 through 5 to the values computed using the *PCI Design Handbook*[1] and ACI 318-14[6] shear friction design provisions. Load factors and strength reduction factors ϕ were taken as 1.0 for all test values.

Shear strength

This section compares the peak measured shear force V_{test} with the calculated shear strength V_{calc} computed using PCI Eq. (5-32b) (μ_e approach), PCI Eq. (5-32a) (μ approach), and ACI 318-14 Eq. (22.9.4.2) (μ approach) for specimens with different interface conditions (monolithic uncracked, monolithic precracked, cold-joint roughened, and cold-joint smooth) and different concrete types (normalweight, sand-lightweight, and all-lightweight). Because PCI Eq. (5-32b) (μ_e approach) is not applicable for shear friction design of case 3 interface conditions (Table 1), it is not compared for cold-joint smooth interface specimens. In this evaluation, the shear strength V_{calc} is computed using $f_{y,limited}$, corresponding to the actual reported yield strength of the reinforcement f_y but not taken more than 60,000 psi (420 MPa) per the *PCI Design Handbook*[1] and ACI 318-14.[6] The ratio V_{test}/V_{calc} is reported in Tables 2 through 5 for each test specimen and each of the three design equations (where applicable). In addition, the mean, standard deviation, coefficient of variation, and maximum and minimum values of V_{test}/V_{calc} are reported in Tables 2 through 5 for each group of specimens with the same interface condition and concrete type.

Normalweight concrete **Figure 2** shows the ratio V_{test}/V_{calc} for the normalweight concrete specimens. The vertical axis for each graph ranges from 0 to 4.0 for each of the three equations evaluated. For clarity of presentation in the graphs, values of V_{test}/V_{calc} larger than 4.0 are not plotted, but they are reported in Tables 2 through 5. These points are denoted with arrow symbols in the graphs to show their corresponding values on the horizontal axis. All values larger than 4.0 were high-strength, normalweight concrete with either a monolithic uncracked or cold-joint roughened interface tested by Kahn and Mitchell.[16]

For the monolithic uncracked normalweight concrete tests, f'_c ranges from 3840 to 17,957 psi (26.48 to 123.81 MPa) and $\rho_{fy,limited}$ ranges from 211 to 1391 psi (1.45 to 9.591 MPa). All three design provisions produce conservative values of shear strength (that is, V_{test}/V_{calc} larger than 1.0) for all specimens for the entire ranges of f'_c and $\rho_{fy,limited}$ tested and especially for large values of f'_c. PCI Eq. (5-32b) tends to be the most accurate (that is, mean value closest to 1.0).

For the precracked monolithic normalweight concrete

Table 4. Shear friction tests of push-off specimens with roughened interface

Researcher(s)	Specimen	f_c', psi	A_{cr}, in.²	ρ	f_y, psi	$\rho f_{y,limited}$, psi*	V_{test}, lb	v_{test}, psi	μ_{test}	V_{test}/V_{calc} PCI Eq. (5-32b)	V_{test}/V_{calc} PCI Eq. (5-32a)	V_{test}/V_{calc} ACI 318-14 Eq. (22.9.4.2)
						Normalweight concrete						
Mattock (1976)	B1	5840	50	0.004	51,270	226	24,350	487	2.2	1.03	2.16	2.16
	B2	5840	50	0.009	50,550	445	35,000	700	1.6	1.05	1.57	1.57
	B3	6055	50	0.013	51,270	677	52,700	1054	1.6	1.28	1.56	1.56
	B4	6055	50	0.018	53,820	947	63,800	1276	1.3	1.31	1.35	1.35
	B5	5895	50	0.025	49,250	1262	78,500	1570	1.3	1.57	1.57	1.65
	B6	5895	50	0.032	49,250	1576	85,000	1700	1.1	1.70	1.70	1.79
	D1	3770	50	0.004	51,270	226	29,500	590	2.6	1.24	2.62	2.62
	D2	3770	50	0.009	51,270	451	46,000	920	2.0	1.37	2.04	2.04
	D3	2940	50	0.013	56,000	739	50,500	1010	1.4	1.37	1.37	1.72
	D4	2940	50	0.018	56,000	986	50,100	1002	1.0	1.36	1.36	1.70
	D4A	2495	50	0.018	54,000	950	49,700	994	1.0	1.59	1.59	1.99
	D5	2955	50	0.025	46,360	1150	60,500	1210	1.1	1.64	1.64	2.05
	D5A	2795	50	0.025	46,200	1146	62,500	1250	1.1	1.79	1.79	2.24
	D6	2955	50	0.032	48,500	1552	73,500	1470	0.9	1.99	1.99	2.49
Kahn and Mitchell (2002)	SF-7-1-CJ	11,734	60	0.004	83,000	220	54,000	900	4.1	1.92	4.09	4.09
	SF-7-2-CJ	11,734	60	0.007	83,000	440	82,100	1368	3.1	2.06	3.11	3.11
	SF-7-3-CJ	12,471	60	0.011	83,000	660	110,300	1838	2.8	2.26	2.78	2.78
	SF-7-4-CJ	12,471	60	0.015	83,000	880	132,680	2211	2.5	2.36	2.51	2.51
	SF-10-3-CJ	12,953	60	0.011	83,000	660	113,910	1899	2.9	2.34	2.88	2.88
	SF-10-4-CJ	12,953	60	0.015	83,000	880	126,040	2101	2.4	2.24	2.39	2.39
	SF-14-1-CJ	14,756	60	0.004	83,000	220	90,910	1515	6.9	3.23	6.89	6.89
	SF-14-2-CJ	14,756	60	0.007	83,000	440	99,190	1653	3.8	2.49	3.76	3.76
	SF-14-3-CJ	15,218	60	0.011	83,000	660	134,710	2245	3.4	2.76	3.40	3.40
	SF-14-4-CJ	15,218	60	0.015	83,000	880	153,120	2552	2.9	2.72	2.90	2.90
Harries, Zeno, and Shahrooz (2012)	615-3A	5800	160	0.004	67,300	246	112,500	701	2.9	1.41	2.85	2.85
	615-3B	5800	160	0.004	67,300	246	96,500	591	2.4	1.19	2.40	2.40
	615-4A	5800	160	0.007	61,500	438	114,500	694	1.6	1.05	1.58	1.58
	615-4B	5800	160	0.007	61,500	444	129,000	794	1.8	1.19	1.79	1.79
	1035-3A	5800	160	0.004	130,000	252	90,000	571	2.3	1.14	2.27	2.27
	1035-3B	5800	160	0.004	126,000	246	105,000	653	2.7	1.32	2.66	2.66
	1035-4A	5800	160	0.007	140,000	444	135,700	835	1.9	1.25	1.88	1.88
	1035-4B	5800	160	0.008	131,300	450	113,500	706	1.6	1.05	1.57	1.57
Shaw and Sneed (2014)	N-5-R-4	4860	49.5	0.013	66,200	780	59,060	1193	1.5	1.35	1.53	1.53
	N-5-R-5	4860	49.5	0.013	66,200	780	53,420	1079	1.4	1.22	1.38	1.38
	N-5-R-6	4860	49.5	0.013	66,200	780	53,440	1080	1.4	1.22	1.38	1.38
	N-8-R-1	7550	49.5	0.013	66,200	780	74,040	1496	1.9	1.69	1.92	1.92
	N-8-R-2	7550	49.5	0.013	66,200	780	56,090	1133	1.5	1.28	1.45	1.45
	N-8-R-3	7550	49.5	0.013	66,200	780	64,140	1296	1.7	1.47	1.66	1.66
Normalweight concrete									Average	1.65	2.25	2.31
									Maximum	3.23	6.89	6.89
									Minimum	1.03	1.35	1.35
									STD	0.556	1.048	1.019
									COV	0.338	0.467	0.440

Table 4 (continued). Shear friction tests of push-off specimens with roughened interface

Researcher(s)	Specimen	f'_c, psi	A_{cr}, in.²	ρ	f_y, psi	$\rho f_{y,limited}$, psi*	V_{test}, lb	v_{test}, psi	μ_{test}	V_{test}/V_{calc} PCI Eq. (5-32b)	V_{test}/V_{calc} PCI Eq. (5-32a)	V_{test}/V_{calc} ACI 318-14 Eq. (22.9.4.2)
Sand-lightweight concrete												
Shaw and Sneed (2014)	S-5-R-1	4580	49.5	0.013	66,200	780	51,430	1039	1.3	1.38	1.57	1.57
	S-5-R-2	4580	49.5	0.013	66,200	780	50,400	1018	1.3	1.36	1.54	1.54
	S-5-R-3	4580	49.5	0.013	66,200	780	63,900	1291	1.7	1.72	1.95	1.95
	S-8-R-1	7200	49.5	0.013	66,200	780	72,040	1455	1.9	1.94	2.20	2.20
	S-8-R-2	7200	49.5	0.013	66,200	780	67,380	1361	1.7	1.81	2.05	2.05
	S-8-R-3	7200	49.5	0.013	66,200	780	66,720	1348	1.7	1.80	2.03	2.03
Sneed, Krc, Wermager, and Meinheit (2016)	S-SL-CJ-09-R-1	5380	49.5	0.009	72,200	540	49,340	997	1.8	1.60	2.17	2.17
	S-SL-CJ-09-R-2	5380	49.5	0.009	72,200	540	50,480	1020	1.9	1.63	2.22	2.22
	S-SL-CJ-13-R-1	5570	49.5	0.013	72,200	780	63,170	1276	1.6	1.70	1.92	1.92
	S-SL-CJ-13-R-2	5570	49.5	0.013	72,200	780	59,370	1199	1.5	1.60	1.81	1.81
	S-SL-CJ-17-R-1	4950	49.5	0.017	72,200	1020	62,380	1260	1.2	1.48	1.48	1.58
	S-SL-CJ-17-R-2	4950	49.5	0.017	72,200	1020	65,150	1316	1.3	1.55	1.55	1.65
	S-SL-CJ-22-R-1	5000	49.5	0.022	72,200	1320	64,460	1302	1.0	1.53	1.53	1.63
	S-SL-CJ-22-R-2	5000	49.5	0.022	72,200	1320	57,590	1163	0.9	1.37	1.37	1.45
	S-CL-CJ-9-R-1	4770	49.5	0.009	72,200	540	37,100	749	1.4	1.20	1.63	1.63
	S-CL-CJ-9-R-2	4770	49.5	0.009	72,200	540	42,900	867	1.6	1.39	1.89	1.89
	S-CL-CJ-13-R-1	4640	49.5	0.013	72,200	780	50,800	1026	1.3	1.37	1.55	1.55
	S-CL-CJ-13-R-2	4640	49.5	0.013	72,200	780	46,900	947	1.2	1.26	1.43	1.43
								Sand-lightweight concrete	Average	1.54	1.77	1.79
									Maximum	1.94	2.22	2.22
									Minimum	1.20	1.37	1.43
									STD	0.204	0.287	0.267
									COV	0.133	0.162	0.149
All-lightweight concrete												
Shaw and Sneed (2014)	A-5-R-1	6080	49.5	0.013	66,200	780	48,440	979	1.3	1.48	1.67	1.67
	A-5-R-2	6080	49.5	0.013	66,200	780	52,800	1067	1.4	1.61	1.82	1.82
	A-5-R-3	6080	49.5	0.013	66,200	780	51,410	1039	1.3	1.57	1.78	1.78
	A-8-R-1	7843	49.5	0.013	66,200	780	61,770	1248	1.6	1.88	2.13	2.13
	A-8-R-2	7843	49.5	0.013	66,200	780	63,940	1292	1.7	1.95	2.21	2.21
	A-8-R-3	7843	49.5	0.013	66,200	780	64,130	1295	1.7	1.96	2.21	2.21
Sneed, Krc, Wermager, and Meinheit (2016)	A-SL-CJ-13-R-1	4380	49.5	0.013	72,200	780	46,500	939	1.2	1.42	1.61	1.61
	A-SL-CJ-13-R-2	4380	49.5	0.013	72,200	780	46,900	947	1.2	1.43	1.62	1.62
	A-CL-CJ-13-R-1	4460	49.5	0.013	72,200	780	41,800	844	1.1	1.27	1.44	1.44
	A-CL-CJ-13-R-2	4460	49.5	0.013	72,200	780	43,800	885	1.1	1.34	1.51	1.51
								All-lightweight concrete	Average	1.59	1.80	1.80
									Maximum	1.96	2.21	2.21
									Minimum	1.27	1.44	1.44
									STD	0.254	0.288	0.288
									COV	0.160	0.160	0.160

Note: A_{cr} = area of concrete shear interface; A_{vf} = area of shear reinforcement across shear plane; COV = coefficient of variation; f'_c = concrete compressive strength; f_y = yield strength of reinforcement; $f_{y,limited}$ = yield strength of reinforcement, limited to a maximum value of 60,000 psi; STD = standard deviation; v_{test} = peak measured shear stress; V_{calc} = calculated value of the nominal shear strength; V_{test} = peak measured shear strength; μ_{test} = effective coefficient of friction calculated using the measured shear stress and the yield stress of reinforcement, limited to a maximum value of 60,000 psi = $v_n/\rho f_{y,limited}$; ρ = shear friction reinforcement ratio = A_{vf}/A_{cr}; $\rho f_{y,limited}$ = clamping stress. 1 in.² = 645.2 mm²; 1 lb = 4.448 N; 1 psi = 6.895 kPa.
* $\rho f_{y,limited}$ is computed using the actual yield strength but is not greater than 60,000 psi.

Table 5. Shear friction tests of push-off specimens with smooth interface

Researcher(s)	Specimen	f_c', psi	A_{cr}, in.^{2}	ρ	f_y, psi	$\rho f_{y,limited}$, psi*	V_{test}, lb	v_{test}, psi	μ_{test}	V_{test}/V_{calc} PCI Eq. (5-32b)	V_{test}/V_{calc} PCI Eq. (5-32a)	V_{test}/V_{calc} ACI 318-14 Eq. (22.9.4.2)
						Normalweight concrete						
Mattock (1976)	C1	5870	50	0.004	50,910	224	10,500	210	0.9	n/a	1.56	1.56
	C2	5870	50	0.009	50,910	448	18,000	360	0.8	n/a	1.34	1.34
	C3	5980	50	0.013	50,550	667	21,400	428	0.6	n/a	1.07	1.07
	C4	5980	50	0.018	51,640	909	30,000	600	0.7	n/a	1.10	1.10
	C5	6165	50	0.022	52,730	1160	39,000	780	0.7	n/a	1.12	1.12
	C6	6165	50	0.032	45,250	1448	44,100	882	0.6	n/a	1.10	1.10
	G1	5870	50	0.004	50,910	224	8000	160	0.7	n/a	1.19	1.19
	G2	5870	50	0.009	50,910	488	13,200	264	0.6	n/a	0.98	0.98
	G3	5980	50	0.013	50,550	732	19,200	384	0.6	n/a	0.96	0.96
	G4	5980	50	0.018	51,640	944	25,000	500	0.6	n/a	0.92	0.92
	G5	6165	50	0.022	52,730	1216	29,300	586	0.5	n/a	0.84	0.84
	G6	6165	50	0.032	45,250	1498	38,900	778	0.5	n/a	0.97	0.97
	H1	5825	50	0.004	55,450	210	9400	188	0.8	n/a	1.28	1.28
	H2	6080	50	0.009	55,450	480	16,100	322	0.7	n/a	1.10	1.10
	H3	6080	50	0.013	55,450	720	23,000	460	0.6	n/a	1.05	1.05
	H4	6075	50	0.018	53,640	960	25,500	510	0.5	n/a	0.90	0.90
	H5	6180	50	0.025	46,800	1157	32,700	654	0.6	n/a	0.94	0.94
	H6	5900	50	0.032	46,800	1488	38,000	760	0.5	n/a	0.95	0.95
Kahn and Mitchell (2002)	SF-10-1-CJ†	14,326	60	0.004	83,000	220	31,730	529	2.4	n/a	2.00	2.00
	SF-10-2-CJ†	12,053	60	0.007	83,000	440	49,290	822	1.9	n/a	1.04	1.04
Shaw and Sneed (2014)	N-5-S-4	4860	49.5	0.013	66,200	780	30,850	623	0.8	n/a	1.33	1.33
	N-5-S-5	4860	49.5	0.013	66,200	780	34,680	701	0.9	n/a	1.50	1.50
	N-5-S-6	4860	49.5	0.013	66,200	780	39,150	791	1.0	n/a	1.69	1.69
	N-8-S-1	7550	49.5	0.013	66,200	780	65,560	1324	1.7	n/a	2.83	2.83
	N-8-S-2	7550	49.5	0.013	66,200	780	53,300	1077	1.4	n/a	2.30	2.30
	N-8-S-3	7550	49.5	0.013	66,200	780	55,330	1118	1.4	n/a	2.39	2.39
									Average	n/a	1.33	1.33
									Maximum	n/a	2.83	2.83
								Normalweight concrete	Minimum	n/a	0.84	0.84
									STD	n/a	0.518	0.518
									COV	n/a	0.391	0.391

Table 5 (continued). Shear friction tests of push-off specimens with smooth interface

Researcher(s)	Specimen	f_c', psi	A_{cr}, in.²	ρ	f_y, psi	$\rho f_{y,limited}$, psi*	V_{test}, lb	v_{test}, psi	μ_{test}	V_{test}/V_{calc} PCI Eq. (5-32b)	V_{test}/V_{calc} PCI Eq. (5-32a)	V_{test}/V_{calc} ACI 318-14 Eq. (22.9.4.2)
Sand-lightweight concrete												
Shaw and Sneed (2014)	S-5-S-1	4580	49.5	0.013	66,200	780	38,530	778	1.0	n/a	1.96	1.96
	S-5-S-2	4580	49.5	0.013	66,200	780	34,110	689	0.9	n/a	1.73	1.73
	S-5-S-3	4580	49.5	0.013	66,200	780	39,800	804	1.0	n/a	2.02	2.02
	S-8-S-1	7200	49.5	0.013	66,200	780	67,030	1354	1.7	n/a	3.40	3.40
	S-8-S-2	7200	49.5	0.013	66,200	780	57,880	1169	1.5	n/a	2.94	2.94
	S-8-S-3	7200	49.5	0.013	66,200	780	58,860	1189	1.5	n/a	2.99	2.99
Sneed, Krc, Wermager, and Meinheit (2016)	S-SL-CJ-09-S-1	5380	49.5	0.009	72,200	540	26,950	544	1.0	n/a	1.98	1.98
	S-SL-CJ-09-S-2	5380	49.5	0.009	72,200	540	32,590	658	1.2	n/a	2.39	2.39
	S-SL-CJ-13-S-1	5570	49.5	0.013	72,200	780	39,490	798	1.0	n/a	2.01	2.01
	S-SL-CJ-13-S-2	5570	49.5	0.013	72,200	780	48,770	985	1.3	n/a	2.48	2.48
	S-SL-CJ-17-S-1	4950	49.5	0.017	72,200	1020	49,810	1006	1.0	n/a	1.93	1.93
	S-SL-CJ-17-S-2	4950	49.5	0.017	72,200	1020	56,530	1142	1.1	n/a	2.20	2.20
	S-SL-CJ-22-S-1	5000	49.5	0.022	72,200	1320	49,810	1006	0.8	n/a	1.49	1.49
	S-SL-CJ-22-S-2	5000	49.5	0.022	72,200	1320	56,530	1142	0.9	n/a	1.70	1.70
	S-CL-CJ-9-S-1	4770	49.5	0.009	72,200	540	31,900	644	1.2	n/a	2.34	2.34
	S-CL-CJ-9-S-2	4770	49.5	0.009	72,200	540	37,900	766	1.4	n/a	2.78	2.78
	S-CL-CJ-13-S-1	4640	49.5	0.013	72,200	780	41,000	828	1.1	n/a	2.08	2.08
	S-CL-CJ-13-S-2	4640	49.5	0.013	72,200	780	40,400	816	1.0	n/a	2.05	2.05
	S-CL-CJ-17-S-1	4550	49.5	0.017	72,200	1020	43,100	871	0.9	n/a	1.67	1.67
	S-CL-CJ-17-S-2	4550	49.5	0.017	72,200	1020	48,900	988	1.0	n/a	1.90	1.90
Sand-lightweight concrete									Average	n/a	2.20	2.20
									Maximum	n/a	3.40	3.40
									Minimum	n/a	1.49	1.49
									STD	n/a	0.498	0.498
									COV	n/a	0.226	0.226
All-lightweight concrete												
Shaw and Sneed (2014)	A-5-S-1	6080	49.5	0.013	66,200	780	41,470	838	1.1	n/a	2.39	2.39
	A-5-S-2	6080	49.5	0.013	66,200	780	40,080	810	1.0	n/a	2.31	2.31
	A-5-S-3	6080	49.5	0.013	66,200	780	39,250	793	1.0	n/a	2.26	2.26
	A-8-S-1	7843	49.5	0.013	66,200	780	46,090	931	1.2	n/a	2.65	2.65
	A-8-S-2	7843	49.5	0.013	66,200	780	48,040	970	1.2	n/a	2.76	2.76
	A-8-S-3	7843	49.5	0.013	66,200	780	51,740	1045	1.3	n/a	2.98	2.98
Sneed, Krc, Wermager, and Meinheit (2016)	A-SL-CJ-13-S-1	4380	49.5	0.013	72,200	780	37,800	764	1.0	n/a	2.18	2.18
	A-SL-CJ-13-S-2	4380	49.5	0.013	72,200	780	38,800	784	1.0	n/a	2.23	2.23
	A-CL-CJ-13-S-1	4460	49.5	0.013	72,200	780	36,900	745	1.0	n/a	2.12	2.12
	A-CL-CJ-13-S-2	4460	49.5	0.013	72,200	780	37,300	754	1.0	n/a	2.15	2.15
All-lightweight concrete									Average	n/a	2.40	2.40
									Maximum	n/a	2.98	2.98
									Minimum	n/a	2.12	2.12
									STD	n/a	0.294	0.294
									COV	n/a	0.122	0.122

Note: A_{cr} = area of concrete shear interface; A_{vf} = area of shear reinforcement across shear plane; COV = coefficient of variation; f_c' = concrete compressive strength; f_y = yield strength of reinforcement; $f_{y,limited}$ = yield strength of reinforcement, limited to a maximum value of 60,000 psi; STD = standard deviation; v_{test} = peak measured shear stress; V_{calc} = calculated value of the nominal shear strength; V_{test} = peak measured shear strength; μ_{test} = effective coefficient of friction calculated using the measured shear stress and the yield stress of reinforcement, limited to a maximum value of 60,000 psi = $v_n/\rho f_{y,limited}$; ρ = shear friction reinforcement ratio = A_{vf}/A_{cr}; $\rho f_{y,limited}$ = clamping stress. 1 in.² = 645.2 mm²; 1 lb = 4.448 N; 1 psi = 6.895 kPa.

* $\rho f_{y,limited}$ is computed using the actual yield strength but is not greater than 60,000 psi.

† Specimens were reported as having a smooth interface, so they are included in this table.

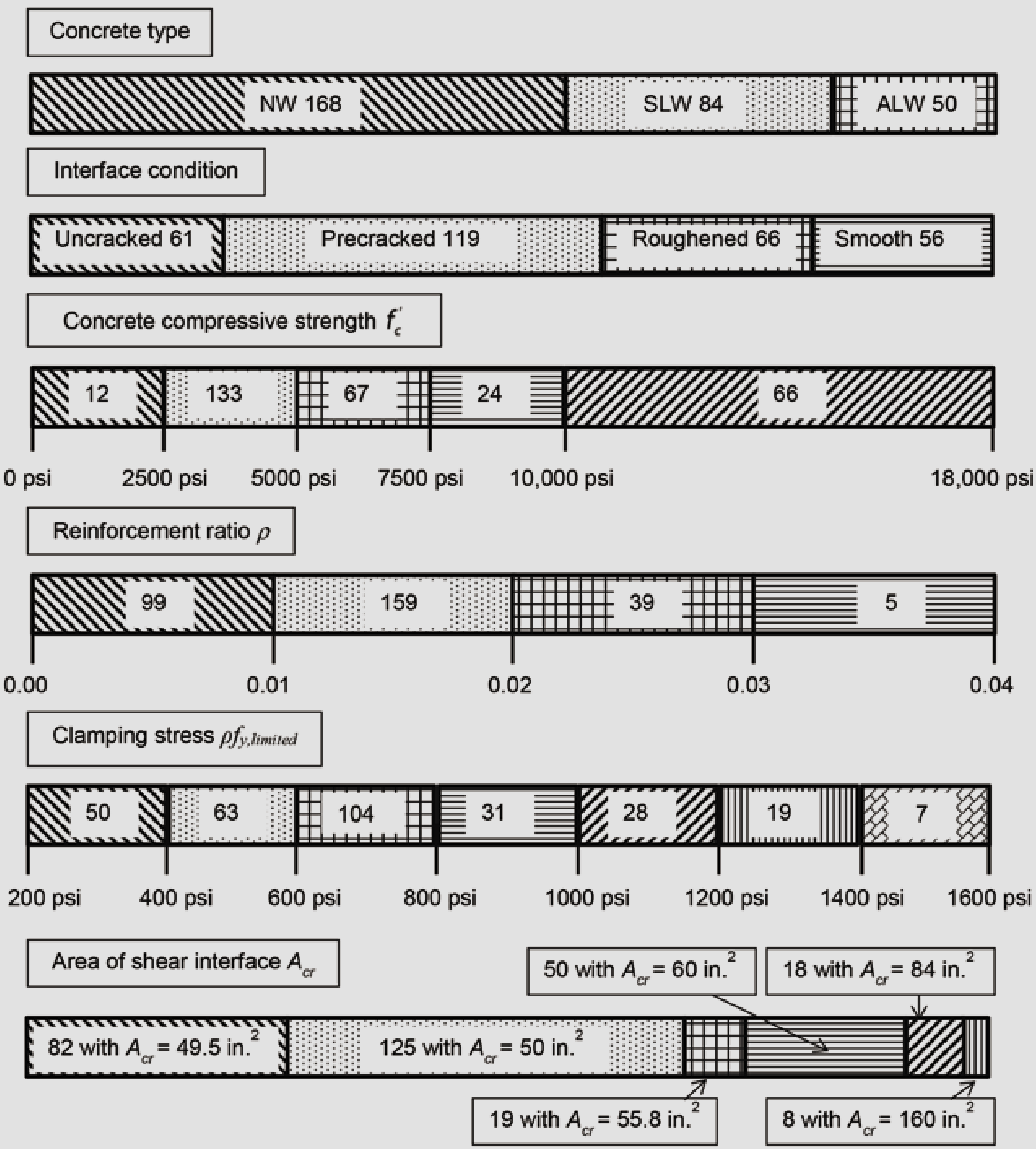

Figure 1. Distribution of data in terms of concrete type, interface condition, compressive strength of concrete f_c', shear reinforcement ratio ρ, clamping stress $\rho_{fy,limited}$, and area of concrete shear interface A_{cr}. Note: ALW = all-lightweight; NW = normalweight; SLW = sand-lightweight. 1 in.^{2} = 645.2 mm^2; 1 psi = 6.895 kPa.

specimens, f_c' ranges from 2385 to 16,475 psi (16.44 to 113.59 MPa), and $\rho_{fy,limited}$ ranges from 223 to 1570 psi (1.54 to 10.82 MPa). All three design provisions produce some V_{test}/V_{calc} values less than 1.0. Figure 2 and Table 2 show that for ACI 318-14 Eq. (22.9.4.2), the values less than 1.0 are associated with specimens made with higher strength concrete, while for PCI Eq. (5-32a) the values less than 1.0 occur for low values of f_c' (that is, less than 4000 psi [28 MPa]). PCI Eq. (5-32b) tends to have values less than 1.0 for low values of $\rho_{fy,limited}$. PCI Eq. (5-32b) tends to be the most accurate.

For cold-joint roughened normalweight concrete specimens, f_c' ranges from 2495 to 15,218 psi (17.20 to 104.92 MPa), and $\rho_{fy,limited}$ ranges from 226 to 1576 psi (1.56 to 10.87 MPa). All three design provisions produce conservative values of shear strength for all specimens for the entire ranges of f_c' and $\rho_{fy,limited}$ tested. PCI Eq. (5-32b) tends to be the most accurate, and PCI Eq. (5-32a) and ACI 318-14 Eq. (22.9.4.2) are especially conservative for large values of f_c'.

For cold-joint smooth normalweight concrete specimens,

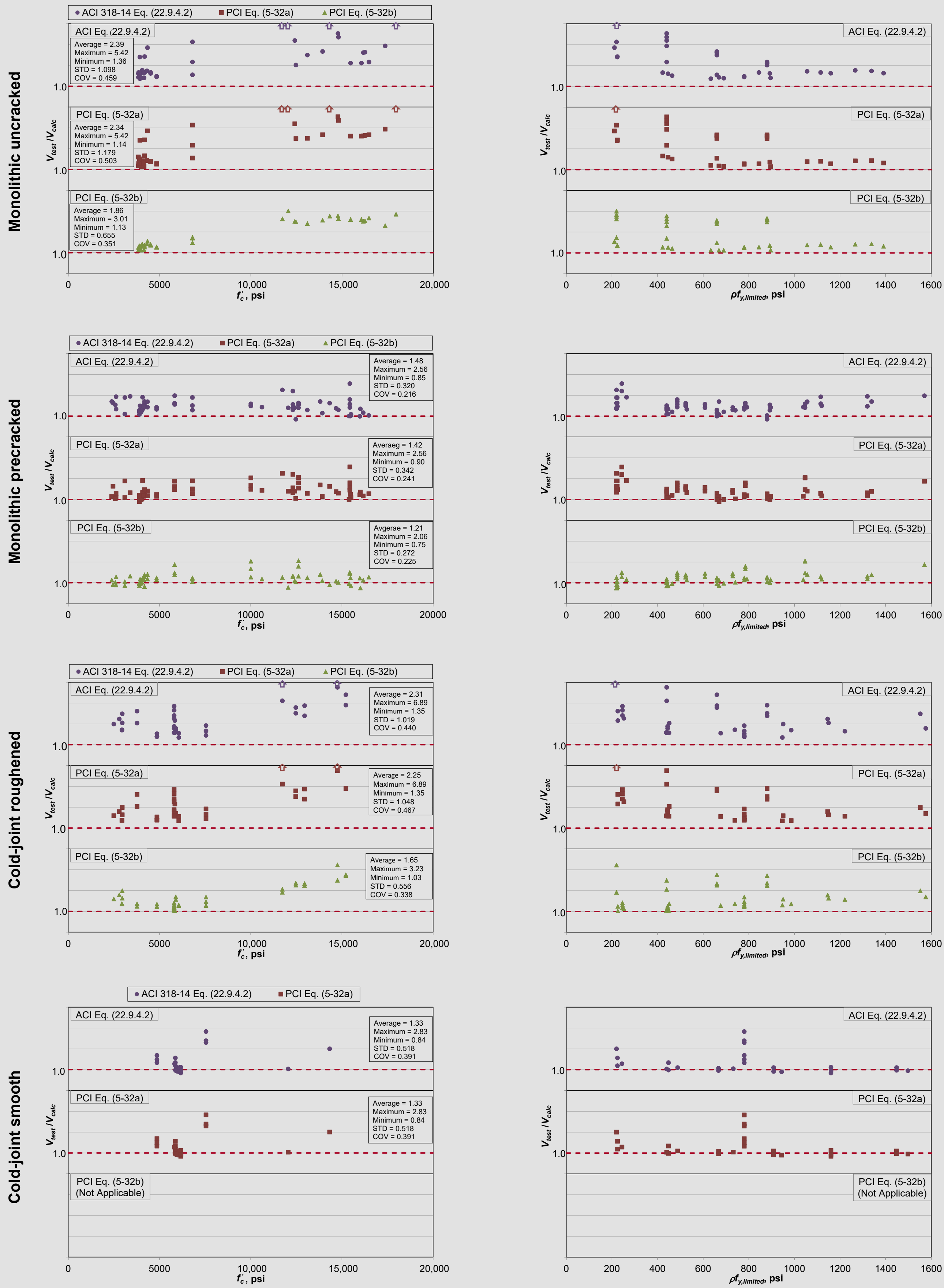

Figure 2. Concrete compressive strength f'_c and clamping stress $\rho_{fy,limited}$ versus the ratio of peak measured shear strength V_{test} to the nominal shear strength V_{calc} computed using the seventh edition *PCI Design Handbook* Eq. (5-32a), second edition *PCI Design Handbook* Eq. (5-32b), and ACI 318-14 Eq. (22.9.4.2) for normal-weight concrete specimens with different interface conditions. Note: COV = coefficient of variation; STD = standard deviation. 1 psi = 6.895 kPa.

f_c' ranges from approximately 4860 to 14,326 psi (33.51 to 98.77 MPa), and $\rho f_{y,limited}$ ranges from 224 to 1498 psi (1.54 to 10.33 MPa). ACI 318-14 Eq. (22.9.4.2) and PCI Eq. (5-32a) produce some V_{test}/V_{calc} values less than 1.0 throughout the range of $\rho f_{y,limited}$. No trends are apparent with respect to compressive strength. Because PCI Eq. (5-32b) is not applicable for the cold-joint smooth condition, it is omitted from the graph.

Lightweight concrete **Figure 3** shows the ratio V_{test}/V_{calc} for the combined sand-lightweight and all-lightweight concrete specimens.

For sand-lightweight concrete specimens with a monolithic uncracked interface, f_c' ranges from approximately 3740 to 4770 psi (25.79 to 32.89 MPa) and $\rho f_{y,limited}$ ranges from 210 to 1368 psi (1.45 to 9.43 MPa). All three design equations produce conservative values of shear strength for all specimens, and PCI Eq. (5-32b) tends to be the most accurate.

For the precracked monolithic sand-lightweight concrete specimens, f_c' ranges from 2000 to 11,020 psi (13.79 to 75.98 MPa), and $\rho f_{y,limited}$ ranges from 218 to 1368 psi (1.50 to 9.43 MPa). The Mattock et al.[15] series C specimens have values of f_c' that are lower than 2500 psi (17 MPa), corresponding to the minimum values for structural concrete in accordance with the *PCI Design Handbook*[1] and ACI 318-14,[6] but they are included in Fig. 3 for completeness. All three design equations produce some V_{test}/V_{calc} values less than 1.0. Figure 3 and Table 3 show that PCI Eq. (5-32b) tends to have values less than 1.0 for low values of $\rho f_{y,limited}$, and for the entire range of f_c' tested, PCI Eq. (5-32b) tends to be the most accurate.

For cold-joint roughened sand-lightweight concrete specimens, f_c' ranges from 4580 to 7200 psi (31.58 to 49.64 MPa), and $\rho f_{y,limited}$ ranges from 540 to 1320 psi (3.72 to 9.10 MPa). All three design provisions produce V_{test}/V_{calc} values larger than 1.0 for all specimens. PCI Eq. (5-32b) tends to be the most accurate.

For cold-joint smooth sand-lightweight concrete specimens, f_c' ranges from approximately 4580 to 7200 psi (31.58 to 49.64 MPa), and $\rho f_{y,limited}$ ranges from 540 to 1320 psi (3.72 to 9.10 MPa). ACI 318-14 Eq. (22.9.4.2) and PCI Eq. (5-32a) produce V_{test}/V_{calc} values larger than 1.0 throughout the range of $\rho f_{y,limited}$ tested.

For monolithic uncracked all-lightweight concrete specimens, f_c' ranges from approximately 3880 to 4700 psi (26.75 to 32.41 MPa), and $\rho f_{y,limited}$ ranges from 230 to 1381 psi (1.59 to 9.52 MPa). Figure 3 shows that all three design equations produce conservative values of shear strength for all monolithic uncracked specimens, and PCI Eq. (5-32b) tends to be the most accurate.

For precracked monolithic all-lightweight concrete specimens, f_c' ranges from 3880 to 4700 psi (26.75 to 32.41 MPa), and $\rho f_{y,limited}$ ranges from approximately 219 to 1404 psi (1.51 to 9.68 MPa). All three design equations produce some V_{test}/V_{calc} values less than 1.0. Figure 3 and Table 2 show that PCI Eq. (5-32b) tends to have values less than 1.0 for low values of $\rho f_{y,limited}$. PCI Eq. (5-32b) tends to be the most accurate.

For cold-joint roughened all-lightweight concrete specimens, f_c' ranges from approximately 4380 to 7843 psi (30.20 to 54.08 MPa), and all $\rho f_{y,limited}$ values are 780 psi (5.38 kPa). All three design equations produce V_{test}/V_{calc} values larger than 1.0 for the ranges of f_c' and $\rho f_{y,limited}$ tested. PCI Eq. (5-32b) tends to be the most accurate.

For cold-joint smooth all-lightweight concrete specimens, f_c' ranges from 4380 to 7843 psi (30.20 to 54.08 MPa), and all $\rho f_{y,limited}$ values are 780 psi (5.38 MPa). ACI 318-14 Eq. (22.9.4.2) and PCI Eq. (5-32a) produce V_{test}/V_{calc} values larger than 1.0 for all data, with minimum values equal to or larger than 2.12 for both equations.

Effective coefficient of friction

This section compares the effective coefficient of friction μ_{test} associated with the measured shear strength V_{test} (or v_{test}) calculated using Eq. (4) with the value of μ_e computed using PCI Eq. (5-33) for specimens with different interface conditions (monolithic uncracked, monolithic precracked, and cold-joint roughened) and different concrete types (normalweight, sand-lightweight, and all-lightweight). As mentioned previously, PCI Eq. (5-32b) is not applicable to cold-joint smooth interface conditions, and therefore PCI Eq. (5-33) is also not applicable for the cold-joint smooth interface case.

$$\mu_{test} = \frac{v_{test}}{\rho f_{y,limited}} \tag{4}$$

In this evaluation, μ_{test} is computed with Eq. (4) using the actual yield strength of the reinforcement taken equal to or less than 60,000 psi (420 MPa), $f_{y,limited}$, per the *PCI Design Handbook*[1] and ACI 318-14,[6] for direct comparison with the design provisions. The value of μ_e computed using PCI Eq. (5-33) is plotted against v_n considering V_n equals V_u/ϕ, where ϕ is 1.0 and v_n is V_n/A_{cr}. The maximum values of μ_e and v_n specified by the *PCI Design Handbook* are considered in the evaluation. Because the maximum value of v_n is a function of the concrete type, normalweight, sand-lightweight, and all-lightweight concrete specimens are presented separately.

Normalweight concrete **Figure 4** plots the values of μ_{test} associated with v_{test} for the normalweight concrete specimens for the monolithic uncracked, monolithic precracked, and roughened interface conditions. (Note

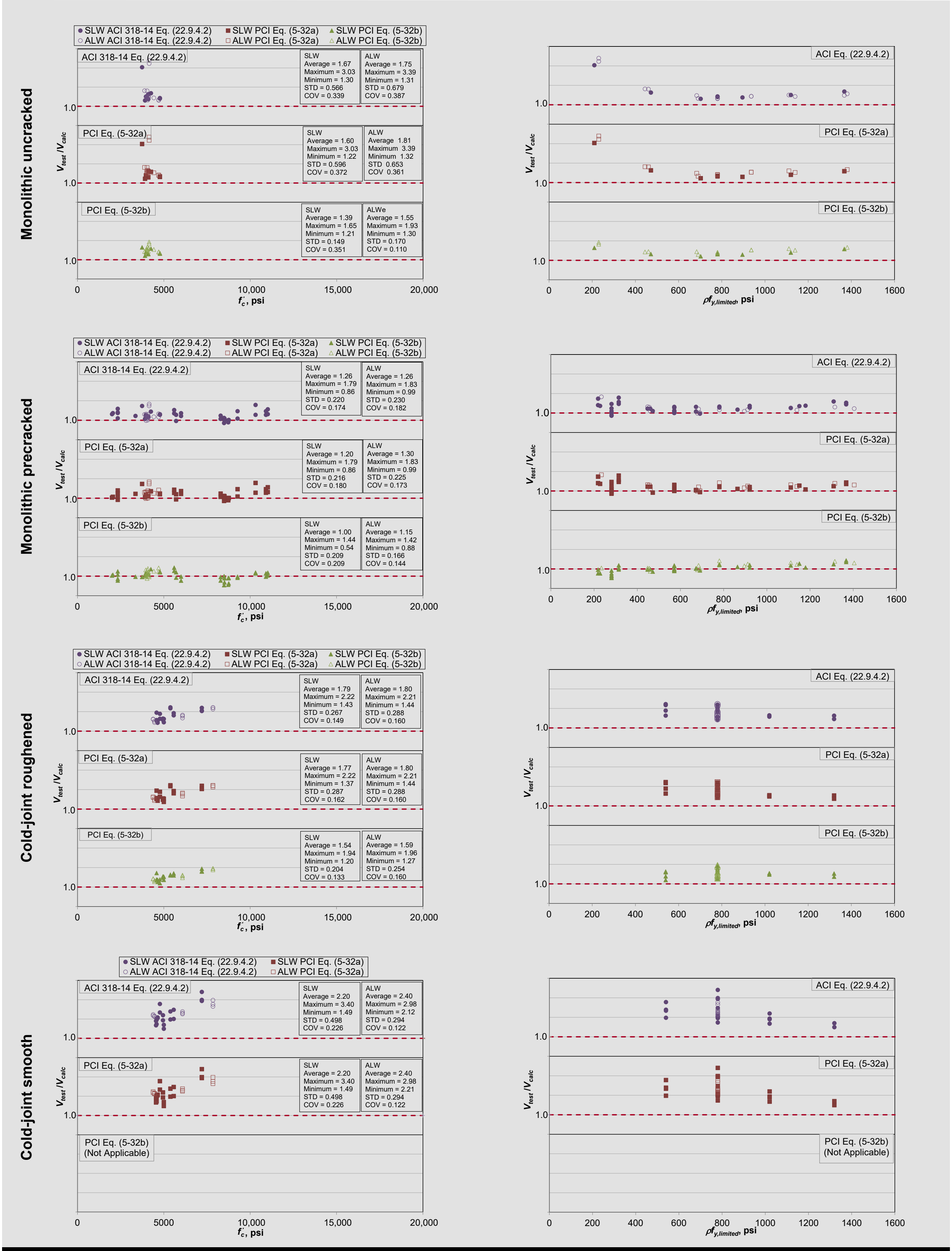

Figure 3. Concrete compressive strength f_c' and clamping stress $\rho_{fy,limited}$ versus the ratio of peak measured shear strength V_{test} to the nominal shear strength V_{calc} computed using the seventh edition *PCI Design Handbook* Eq. (5-32a), second edition *PCI Design Handbook* Eq. (5-32b), and ACI 318-14 Eq. (22.9.4.2) for lightweight concrete specimens with different interface conditions. Note: ALW = all lightweight; COV = coefficient of variation; SLW = sand lightweight; STD = standard deviation. 1 psi = 6.895 kPa.

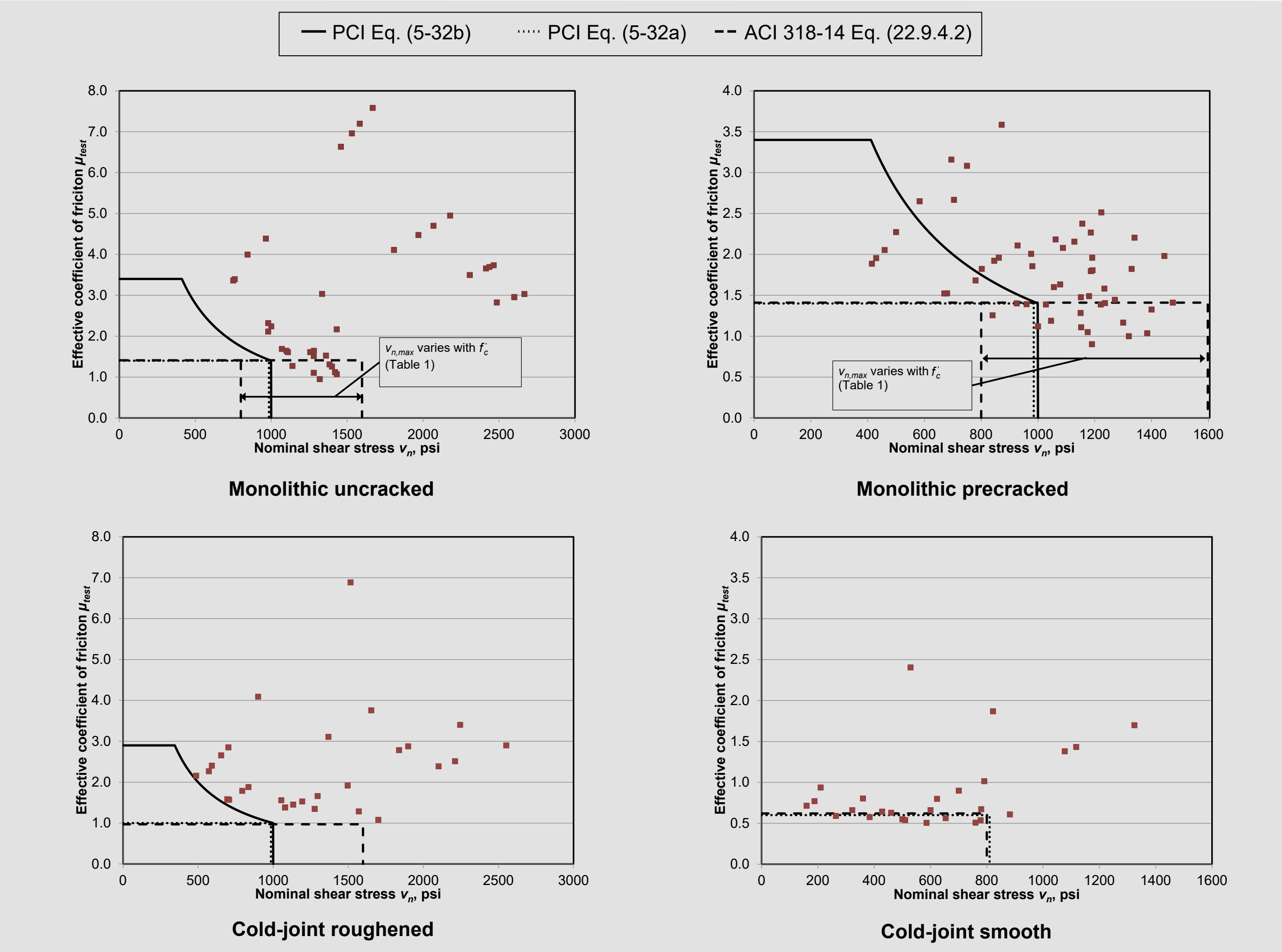

Figure 4. Effective coefficient of friction μ_{test} for normalweight concrete specimens with different interface conditions. Note: Values of effective coefficient of friction μ_e and coefficient of friction μ from the seventh edition *PCI Design Handbook* and ACI 318-14 are shown for comparison. $v_{n,max}$ = maximum nominal shear stress. 1 psi = 6.895 kPa.

that smooth interface specimens are discussed later in this section.) For the monolithic uncracked and precracked interface conditions, data from Tables 2 and 3 with $f_c^{'}$ greater than or equal to 3333 psi (22.98 MPa) are plotted in the figures for consistency with the limits on v_n plotted in the graph (Table 1). The only specimens that did not meet this criterion are specimens 2.1 and 2.2 and 5.1 to 5.5 by Hofbeck et al.[12] in Table 3. For the roughened interface condition, data with $f_c^{'}$ greater than or equal to 4000 psi (28 MPa) are plotted in the figure for the same reason, which included all specimens in Table 4 except for series D by Mattock.[14] Figure 4 shows that PCI Eq. (5-33) is conservative (all values of μ_{test} plotted to the right and above the equation) for the monolithic uncracked specimens. For the monolithic precracked specimens, there were several unconservative values with respect to PCI Eq. (5-33). PCI Eq. (5-33) is conservative for the roughened interface conditions.

For comparison, the coefficient of friction μ specified by the *PCI Design Handbook*[1] and ACI 318-14[6] and used in PCI Eq. (5-32a) and ACI 318-14 Eq. (22.9.4.2), respectively, is also plotted in Fig. 4 for the monolithic uncracked, monolithic precracked, and roughened interface conditions, as well as for the cold-joint smooth interface condition, including the limitations on v_n (Table 1). For the smooth interface condition, data with $f_c^{'}$ greater than or equal to 4000 psi (28 MPa) are plotted in the figure, which included all normalweight concrete specimens in Table 5. Because the coefficient of friction μ is the lower bound of the effective coefficient of friction μ_e, Fig. 4 shows that the value of μ specified by the *PCI Design Handbook*[1] and ACI 318-14[6] is generally conservative with respect to the test results for each concrete type and interface condition when the limit for the maximum shear stress is also considered (with the exception of a few normalweight concrete monolithic precracked specimens and some normalweight concrete specimens with a cold-joint smooth interface). This can also be observed from Tables 2 through 5, where PCI Eq. (5-32a) and ACI 318-14 Eq. (22.9.4.2) produce values of V_{test}/V_{calc} larger than 1.0 for nearly all specimens, with no value being less than 0.84.

Lightweight concrete Figure 5 plots the values of

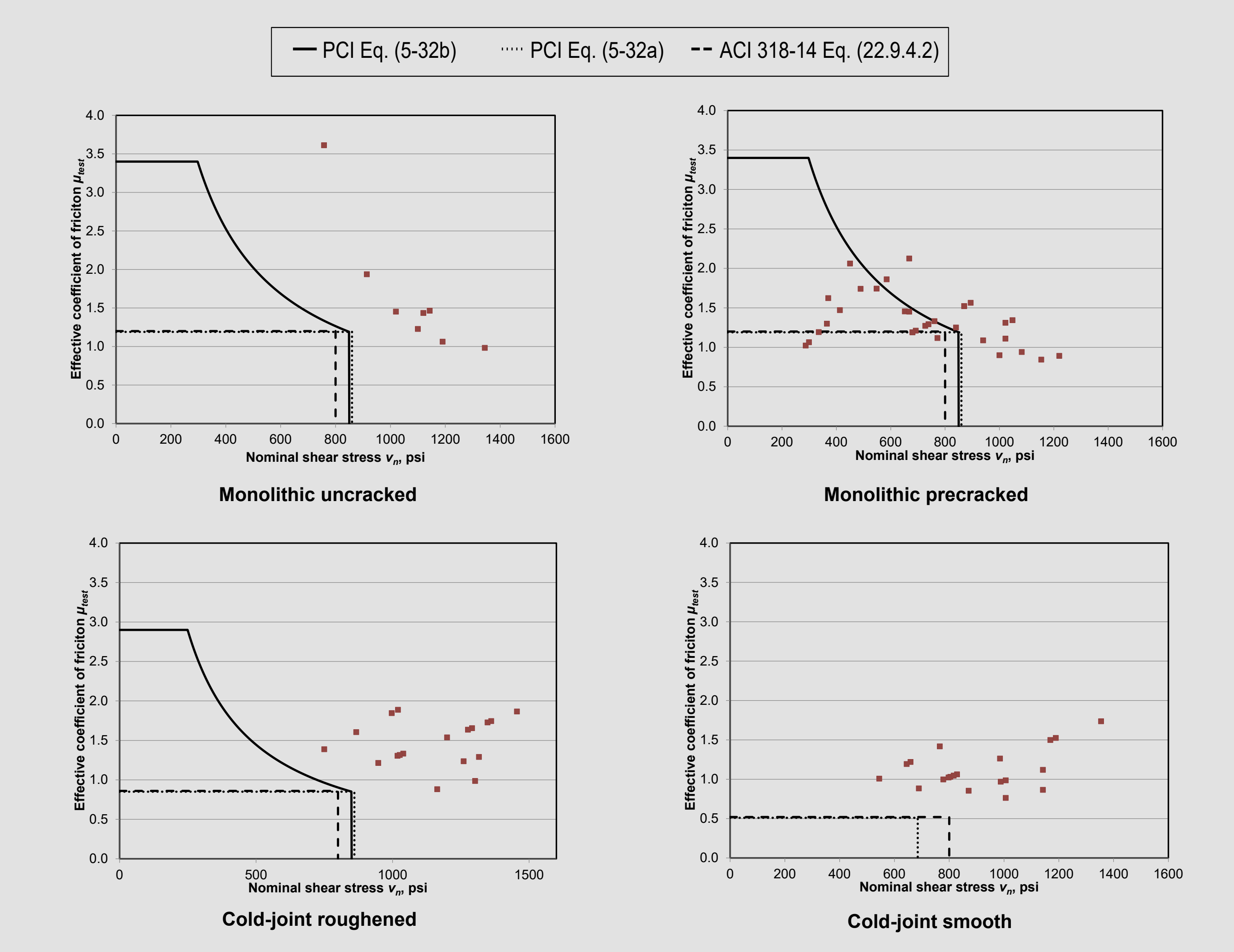

Figure 5. Effective coefficient of friction μ_{test} for sand-lightweight concrete specimens with different interface conditions. Note: Values of effective coefficient of friction μ_e and coefficient of friction μ from the seventh edition *PCI Design Handbook* and ACI 318-14 are shown for comparison. 1 psi = 6.895 kPa.

μ_{test} associated with v_{test} for the sand-lightweight concrete specimens. Series C by Mattock et al.[15] in Table 3 is omitted from the graph because the values of f_c' were lower than values corresponding to the limits on v_n plotted in the graphs (Table 1). All values of μ_{test} for the monolithic uncracked and the roughened interfaces were conservative compared with PCI Eq. (5-33). Several of the monolithic precracked specimens, however, were unconservative.

Figure 6 shows the value of μ_{test} for the all-lightweight concrete specimens. All values of μ_{test} for the monolithic uncracked and the roughened interfaces were conservative compared with PCI Eq. (5-33). Values of μ_{test} for the monolithic precracked specimens were closely predicted by PCI Eq. (5-33); however, there were a few unconservative values.

For comparison, the coefficient of friction μ specified by the *PCI Design Handbook*[1] and ACI 318-14[6] and used in PCI Eq. (5-32a) and ACI 318-14 Eq. (22.9.4.2), respectively, is also plotted in Fig. 5 and 6 for the monolithic uncracked, monolithic precracked, and roughened interface conditions, as well as for the cold-joint smooth interface condition, including the limitations on v_n (Table 1). For the smooth interface condition, data with f_c' greater than or equal to 4000 psi (28 MPa) are plotted in the figure, which included all lightweight concrete specimens in Table 5. Because the coefficient of friction μ is the lower bound of the effective coefficient of friction μ_e, Fig. 5 and 6 show that the value of μ specified by the *PCI Design Handbook*[1] and ACI 318-14[6] is generally conservative with respect to the test results for each concrete type and interface condition when the limit for the maximum shear stress is also considered (with the exception of a few monolithic precracked lightweight concrete specimens). This can also be observed from Tables 2 through 5, where PCI Eq. (5-32a) and ACI 318-14 Eq. (22.9.4.2) produce values of V_{test}/V_{calc} larger than 1.0 for nearly all specimens, with no value being less than 0.86.

Discussion

Comparison of shear friction design equations

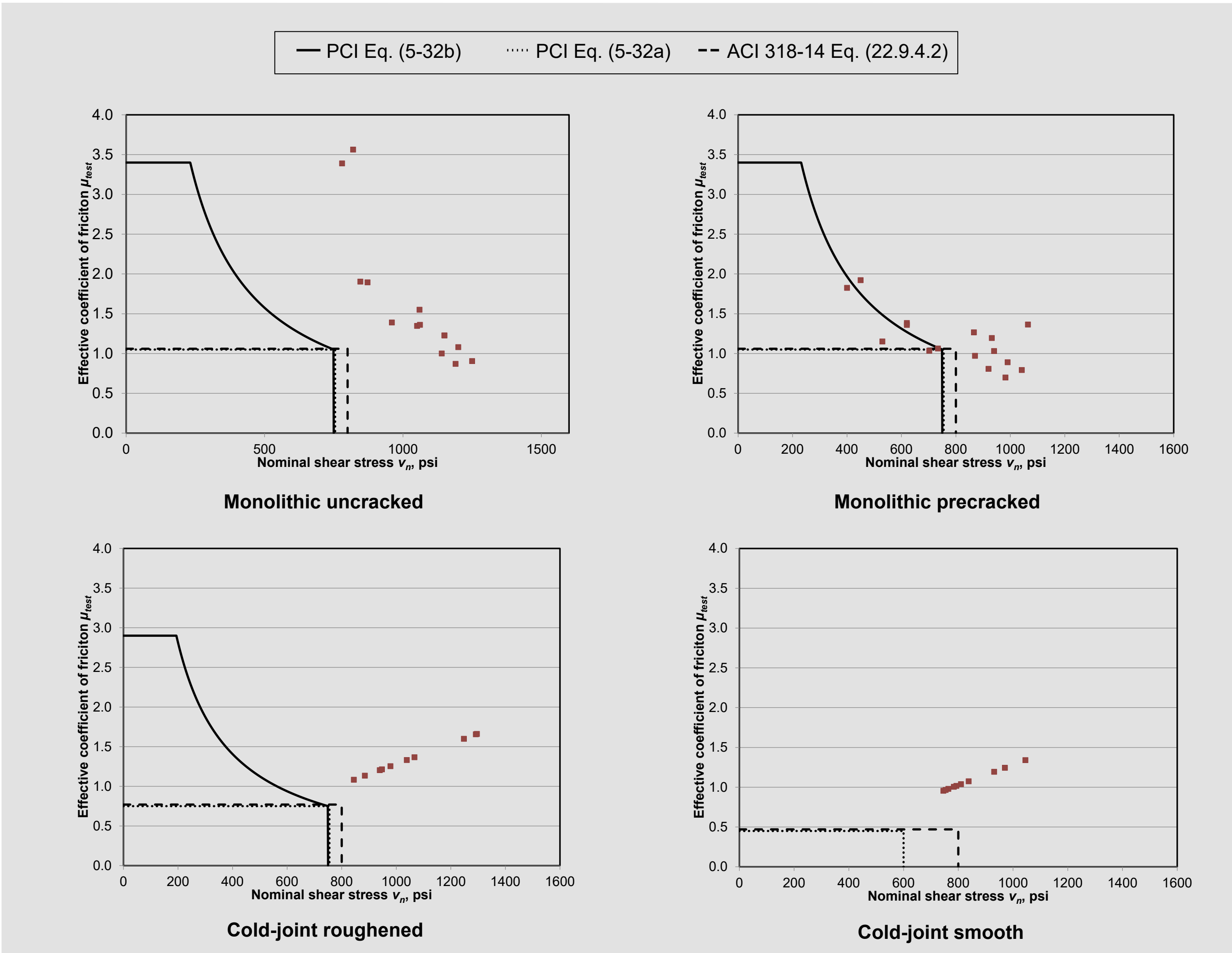

Figure 6. Effective coefficient of friction μ_{test} for all-lightweight concrete specimens with different interface conditions. Note: Values of effective coefficient of friction μ_e and coefficient of friction μ from the seventh edition PCI Design Handbook and ACI 318-14 are shown for comparison. 1 psi = 6.895 kPa.

Values of V_{test}/V_{calc} summarized in Tables 2, 3, and 4 indicate that PCI Eq. (5-32b) (μ_e approach) is more accurate (that is, mean value closest to 1.0) and has a lower standard deviation than both PCI Eq. (5-32a) and ACI 318-14 Eq. (22.9.4.2) (μ approach) for normalweight, sand-lightweight, and all-lightweight concrete with monolithic uncracked, monolithic precracked, and cold-joint roughened interface conditions. Values of V_{test}/V_{calc} computed using PCI Eq. (5-32a) and ACI 318-14 Eq. (22.9.4.2) are more conservative (that is, larger mean value) than values computed using PCI Eq. (5-32b). For PCI Eq. (5-32a) and ACI 318-14 Eq. (22.9.4.2), no values of V_{test}/V_{calc} are lower than 0.75, which is the value of the strength reduction factor ϕ for shear.[1,6] For PCI Eq. (5-32b), the only values of V_{test}/V_{calc} less than 0.75 are precracked sand-lightweight concrete specimens tested by Hoff[20] with low values of $\rho_{fy,limited}$ (281 psi [1.94 MPa]). However, the shear strength of these specimens exhibited a large degree of scatter, and specimens tested by Mattock et al.[15] with lower values of $\rho_{fy,limited}$ and lower concrete compressive strength (specimens B1, C1, and D1 in Table 3) had higher shear strengths. Therefore, the cause of these low values is unknown.

For specimens with a smooth interface condition, Table 5 shows that average values and coefficients of variation of V_{test}/V_{calc} using PCI Eq. (5-32a) and ACI 318-14 Eq. (22.9.4.2) are similar for normalweight, sand-lightweight, and all-lightweight concrete. Because PCI Eq. (5-32b) is not applicable for this case, it is not compared. For both equations, no values of V_{test}/V_{calc} are lower than 0.75.

Distribution of data

With regard to the distribution of data, Fig. 2 shows that there is a gap in the data for normalweight concrete specimens with 7000 psi (48 MPa) < f_c' <11,000 psi (76 MPa) for all interface conditions. A comparison of Fig. 2 and 3 indicates that the available sand-lightweight and all-lightweight concrete test data have a much smaller range of compressive strength than the available normalweight concrete test data for all interface conditions. Figures 2 and 3 show a consistent range of available test data with respect to clamping stress with the exception of sand-lightweight and all-lightweight with cold-joint roughened and smooth interfaces, where data are lacking for low values of $\rho_{fy,limited}$. With respect to shear interface area, Fig. 1 shows that most specimens (91%) were of similar size, that is, A_{cr} of approximately 50 or 60 in.^{2} (32,000 or 39,000 mm^2); 6% had an A_{cr} of approximately 84 in.^{2} (54,000 mm^2), and 3% had an A_{cr} of approximately 160 in.^{2} (103,000 mm^2).

Use of λ in the coefficient of friction μ for a cold-joint smooth interface condition

For the cold-joint smooth interface condition, Fig. 4, 5, and 6 show that the value of the coefficient of friction μ (0.6λ) specified by the *PCI Design Handbook*[1] and ACI 318-14[6] for normalweight concrete is in good agreement with values determined from the test results using Eq. (2), whereas values of μ specified for sand-lightweight and all-lightweight concrete are conservative with respect to the test results. This is in part because the modification factor λ in the coefficient of friction μ (Table 1) reduces the value of μ for sand-lightweight and all-lightweight concrete by a factor of 0.85 and 0.75, respectively. In fact, values of μ_{test} determined for the sand-lightweight and all-lightweight concrete specimens with a cold-joint smooth interface were higher than those of the normalweight concrete specimens with a smooth interface in most cases. This can be explained by the fact that the normalweight concrete specimens included in Table 5 by Mattock[14] had a broken bond, were precracked, or both, whereas the sand-lightweight and all-lightweight specimens by Shaw and Sneed[17] and Sneed et al.[18] were cast with a smooth cold joint and were not precracked. In his 2001 paper, Mattock pointed out that the shear strength of these normalweight concrete specimens was equal to the shear yield strength of the reinforcement perpendicular to the interface (hence the value of 0.6 in the coefficient of friction μ equal to 0.6λ, Table 1) and that true shear friction across a smooth interface cannot be developed in the absence of interfacial roughness. In addition, because there is no aggregate crossing the shear interface, the strength of the aggregate should not influence the shear transfer strength. Given this reasoning and the results in Fig. 5 and 6, there does not appear to be a justification for including the modification factor λ in the coefficient of friction μ for the smooth interface condition in the *PCI Design Handbook*[1] and ACI 318-14.[6] Thus, the authors recommend removing the modification factor λ in the coefficient of friction μ for a smooth interface condition (case 3) to provide more accurate and economical designs.

Conclusion and recommendations

This paper presents a database of shear friction test results collected from the literature and analyzes the results in terms of the effective coefficient of friction μ_e approach used in the *PCI Design Handbook*[1] and the coefficient of friction μ approach used in the *PCI Design Handbook* and ACI 318-14.[6] Gaps in the literature are identified and discussed. Results of the analysis led to the following conclusions:

- Values of V_{test}/V_{calc} from the database indicate that PCI Eq. (5-32b) (μ_e approach) is more accurate and has a lower standard deviation than both PCI Eq. (5-32a) and ACI 318-14 Eq. (22.9.4.2) (μ approach) for normalweight, sand-lightweight, and all-lightweight concrete with monolithic uncracked, monolithic precracked, and cold-joint roughened interface conditions. For PCI Eq. (5-32a) and ACI 318-14 Eq. (22.9.4.2), no values of V_{test}/V_{calc} were lower than 0.75. For PCI Eq. (5-32b), the only values lower than 0.75 were for the precracked sand-lightweight concrete specimens tested by Hoff[20] with low values of $\rho_{fy,limited}$ (281 psi [1.94 MPa]). The cause of these low values is unknown.

- Values of V_{test}/V_{calc} from the database show that PCI Eq. (5-32a) and ACI 318-14 Eq. (22.9.4.2) provide an accurate estimation of the shear transfer strength for normalweight concrete with a cold-joint smooth interface condition. PCI Eq. (5-32a) and ACI 318-14 Eq. (22.9.4.2) provide conservative estimations of the shear transfer strength for sand-lightweight and all-lightweight concrete with a cold-joint smooth interface condition.

- There does not appear to be a justification for including the modification factor λ in calculating the coefficient of friction μ as 0.6λ for the cold-joint smooth (that is, concrete placed against hardened concrete not intentionally roughened) interface condition in the *PCI Design Handbook*[1] and ACI 318-14.[6] The authors recommend removing the modification factor λ term in the coefficient of friction μ for a smooth interface condition (case 3) to provide more accurate and economical designs.

Acknowledgments

This research was conducted with the sponsorship of PCI and the American Concrete Institute Concrete Research Council. The authors wish to thank Neal Anderson of Simpson Gumphertz & Heger Inc., Roger Becker of PCI, Reid Castrodale of Castrodale Engineering Consultants PC, Harry Gleich of Metromont Precast, Neil Hawkins, and Larbi Sennour of Consulting Engineering Group Inc., who served as advisors to this project. Their assistance and input are greatly appreciated.

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Notation

A_c = area of concrete shear interface (ACI 318-14)

A_{cr} = area of concrete shear interface (*PCI Design Handbook*)

A_{vf} = area of shear reinforcement across shear plane

f_c' = concrete compressive strength

f_y = yield strength of reinforcement

$f_{y,limited}$ = yield strength of reinforcement, limited to a maximum value of 60,000 psi (420 MPa)

v_n = nominal shear stress

$v_{n,max}$ = maximum nominal shear stress

v_{test} = peak measured shear stress

V_{calc} = calculated value of the nominal shear strength

V_n = nominal shear strength

V_{test} = peak measured shear force

V_u = factored shear force

λ = modification factor reflecting the reduced mechanical properties of lightweight concrete relative to normalweight concrete of the same compressive strength

μ = coefficient of friction

μ_e = effective coefficient of friction

μ_{test} = effective coefficient of friction calculated using the measured shear stress and the yield stress of reinforcement, limited to a maximum value of 60,000 psi (420 MPa) = $v_n/\rho_{fy,limited}$

ρ = shear friction reinforcement ratio = A_{vf}/A_{cr}

$\rho_{fy,limited}$ = clamping stress

ϕ = strength reduction factor

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Abstract

Since the introduction of the effective coefficient of friction μ_e approach to the *PCI Design Handbook: Precast and Prestressed Concrete*, several studies have provided additional test results that can be used to compare and validate the shear friction design provisions. This paper presents a database of shear friction test results collected from the literature that was analyzed for the effective coefficient of friction approach used in the *PCI Design Handbook* (Eq. [5-32b]), and the coefficient of friction approach used in the *PCI Design Handbook* (Eq. [5-32a]) and the ACI *Building Code Requirements for Structural Concrete (ACI 318-14) and Commentary (ACI 318R-14)* (Eq. [22.9.4.2]).

The database was limited to push-off specimens subjected to monotonic loading and without external normal forces. The data were categorized in terms of concrete type, interface condition, compressive strength of concrete, clamping stress, and area of shear interface to help identify gaps in the literature. Analysis of the database showed that PCI Eq. (5-32b) is more accurate and has a lower standard deviation than both PCI Eq. (5-32a) and ACI 318-14 Eq. (22.9.4.2) for normalweight, sand-lightweight, and all-lightweight concrete with monolithic uncracked, monolithic precracked, and cold-joint roughened interface conditions. For the cold-joint smooth interface condition, the authors recommend removing the modification factor λ in the coefficient of friction μ to provide more accurate and economical designs.

Keywords

All-lightweight concrete; coefficient of friction; cold joint; interface condition; modification factor; monolithic; monotonic loading; roughened; sand-lightweight concrete; shear friction; uncracked.

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This paper was reviewed in accordance with the Precast/Prestressed Concrete Institute's peer-review process.

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