Most medium-span bridges in the United States are constructed with precast, prestressed concrete girders that support a cast-in-place concrete deck. These girders usually have an upward deflection (camber) that results from the combined effects of the eccentric axial prestressing force and the gravity loads. If the camber is less than expected, the contractor must increase the amount of concrete needed to construct the cast-in-place concrete pad between the top of the girder and the deck. If the camber is greater than expected, the girder may interfere with placement of the deck reinforcement. Differences between expected and actual cambers are even more critical if the bridge owner rejects the girder for being out of compliance with the specifications.

The prediction of girder camber at a particular time is difficult because it depends on the concrete properties, curing conditions, prestress losses, and temperature variations within the girder, all of which vary with time. One contributor to this difficulty is the effect of variations in the profile of internal temperatures over a day. Such variations induce thermal strains, which, if they vary over the height of the girder, result in camber even in the absence of external loads.

Thermal camber typically has the most important consequences during construction. At that time, the girders are

- Field measurements of daily variations in temperature profile and camber for two precast, prestressed concrete girders provided data with which to calibrate models of the effect of temperature variations on camber.
- Camber history was computed using measured temperature profiles and an assumed coefficient of thermal expansion of $5.5 \times 10^{-6}/^\circ F$ ($9.9 \times 10^{-6}/^\circ C$).
- Two practical methods were also developed using 164 observations from 24 girders.

Hang Nguyen, John Stanton, Marc Eberhard, and David Chapman
where

\[ t(Y) = \text{change in temperature at height } Y \]

\[ Y = \text{vertical coordinate, measured upwards from a point located } Y_0 \text{ below the concrete deck surface} \]

\[ Y_0 = \text{distance from top surface at which temperature change is zero, value } = 1200 \text{ mm } = 47.25 \text{ in.} \]

\[ T = \text{temperature at top surface of deck} \]

\[ = 32 - 0.2h_a \text{ (°C) for } h_a \text{ in mm} \]

\[ = 89.6 - 5.08h_a \text{ (°F) for } h_a \text{ in in.} \]

\[ h_a = \text{asphalt thickness} \]

**Figure 1** compares the design positive temperature gradient from Eq. (1) with the one recommended by the AASHTO LRFD specifications for a typical large girder. 3 These specifications define a dimension A that controls the depth to which the temperature change penetrates (Fig. 1). The value of A varies with the superstructure depth and materials, and it is 12 in. (300 mm) for concrete superstructures that are at least 16 in. (410 mm) deep. The change in bottom temperature \( \Delta T_3 \) is 0 unless a site specific study has determined another appropriate value, but it is not to exceed 5°F (3°C). An increase in bottom temperature reduces the thermal camber. A negative design temperature gradient is obtained by multiplying the positive gradient by a factor of -0.2 and -0.3, respectively, for decks with and without asphalt overlays.

A number of researchers have measured temperature gradients and their effects on camber in bridge girders and decks. For example, Shushkewich acquired large amounts of thermocouple data from an instrumented prestressed concrete box girder bridge, the North Halawa Valley Viaduct in Hawaii. 5 The results validated the positive and negative thermal gradients included in the AASHTO LRFD specifications. 3

Barr monitored changes in camber of five simply supported I-girders during a day to evaluate the bridge’s response to daily temperature variations. 6 He found that the camber varied by 0.63 in. (16 mm) between 9:00 a.m. and 9:00 p.m. in one day. He also found that the calculated tensile stresses at the bottom of the girder were larger for the temperature gradient proposed by Priestley than for any of the gradients recommended by the 1994 AASHTO LRFD specifications. 4, 7

Hinkle observed that the cambers of three girders with an average length of 128 ft (39.0 m) varied by up to 0.50 in. (13 mm) between 7:45 a.m. and 1:25 p.m. in the same day. 8 French and O’Neill monitored variations of camber...
girders are typically cast at approximately noon on one day and released the next morning. Weekend girders are typically cast on a Friday and released on the following Monday morning. Typically, the concrete for these weekend girders is more mature at release, even though the curing temperature may have been lower, on average, than for a weekday girder. All of the measurements discussed in this paper were taken approximately two months after the prestressing force had been applied to the girder concrete, by which time the effects of creep, shrinkage, and relaxation were expected to be constant over the course of any one day.

The internal temperatures of the two girders were measured at a single cross section near midspan at one-minute intervals using fifteen type J thermocouples. The changes in camber at midspan of the two girders were measured continuously with a deflection sensor in the fabricator’s storage yard. Figure 2 shows the locations of the thermocouples. More thermocouples were placed in the top flange than in the web, both because the top flange was expected to experience the largest variation in temperature during the day and because the thin flange tips had the largest surface/volume ratio, which increases
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Experimental program

In this study, two large prestressed concrete girders were instrumented to quantify daily temperature variations, develop an understanding of the effects of these variations on camber, and calibrate theoretical and more practical models for predicting thermal camber. The results are summarized here. Nguyen gives details.10

Table 1 lists key properties for the monitored girders. Both girders were Washington State WF100G girders with an I-shaped cross section and a depth of 100 in. (2500 mm) (appendix 5.6-A4-10, Washington State Department of Transportation Bridge Design Manual (LRFD).11 Weekday

Table 1. Girder properties

<table>
<thead>
<tr>
<th>Girder</th>
<th>Girder mark</th>
<th>Cast date</th>
<th>Measurement date</th>
<th>Depth, in.</th>
<th>Length, ft</th>
<th>$f'_c$ at release, psi</th>
<th>$f'_c$ at 28 days, psi</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weekend girder</td>
<td>H6B</td>
<td>May 18, 2012</td>
<td>July 26, 2012</td>
<td>100</td>
<td>172.7</td>
<td>9215</td>
<td>11,260</td>
</tr>
<tr>
<td>Weekday girder</td>
<td>H8A</td>
<td>May 29, 2012</td>
<td>July 25 and 26, 2012</td>
<td>100</td>
<td>164.0</td>
<td>8250</td>
<td>12,430</td>
</tr>
</tbody>
</table>

Note: $f'_c$ = compressive strength of concrete. 1 in. = 25.4 mm; 1 ft = 0.305 m; 1 psi = 6.895 kPa.

Figure 1. Design temperature gradient. $T =$ temperature at top surface of deck; $\Delta T =$ temperature difference over height of girder; $\Delta T_1 =$ temperature change at top surface of girder; $\Delta T_2 =$ temperature change 4 in. below top surface of girder; $\Delta T_3 =$ temperature change at bottom of girder. 1 in. = 25.4 mm; °C = (°F – 32)/1.8.
the possibility of transverse (as well as vertical) variations in temperature.

**Measured temperature histories**

**Figure 3** shows the measured internal temperature histories for girder H8A for 22 hours and for girder H6B for 9 hours. A problem with electrical power supply prevented the data logger from monitoring girder H6B for a longer time.

The data follow expected trends. For both girders, the temperature profile over the height of girders was nearly uniform until about 10:00 a.m. During the late morning and afternoon, the temperature in the top of the flange exposed directly to sunlight increased rapidly until it reached a peak at about 4:00 p.m. At this time, the temperature change in the top flange was approximately equal to 40°F (22°C). In contrast, the temperatures in the web and bottom flange changed less because these locations were shaded by the top flange and by adjacent girders in the storage yard. This situation is similar to that which a girder would experience during installation, at which time the flange, adjacent girders, and deck formwork would similarly shadow the web and bottom flange.

The left side of the web of girder H6B was also exposed to direct sunlight, which might be similar to the exposure condition for an exterior girder in a bridge. **Figure 3** plots that exposure data, in which the temperature during the day, when sunlight was strongest, was higher at the bottom left sensor than at the bottom right or bottom center sensor locations.

In the late afternoon, the internal temperatures in the top of the girder dropped rapidly as the solar radiation became weaker and the air temperature decreased, whereas the temperatures in the web and bottom portions in the girders cooled more slowly. This difference can be explained by the fact that the bottom region of the girders was affected by heat from the ground and was less exposed to wind. During the day, the ground was heated by the sun. However, the ground changes temperature more slowly than does the air, so the bottom flange continued to warm in the evening due to heat from the ground.

**Figure 4** shows the corresponding vertical temperature profiles of the girders at various times. For each elevation that had more than one thermocouple (for example, the flanges), the average temperature is plotted at that elevation. The temperatures varied most over the height of the girder at approximately 4:00 p.m.

**Calculation of thermal camber changes from temperature profiles**

Thermal camber can be predicted from the measured internal temperature profile using the principles of mechanics and an assumed value of the coefficient of thermal expansion. The total strain at any point will consist of
a thermal and a stress-related component (Fig. 5). The curvature at each point along the girder can be computed by finding the profile of total strain that satisfies the following assumptions:

- Plane sections remain plane.
- The free thermal strain is constant across the width at any elevation.
- The concrete and steel are within their linear elastic ranges, and the concrete is uncracked.
- The changes in axial force and bending moment due to thermal effects are both zero for the simply supported girder.

Stresses will develop over the depth of the member if the thermal strain gradient is nonlinear. The total strain $\epsilon_{tot}$ is the sum of the free environmental strain $\epsilon_e$, which consists here of just the thermal strain but more generally may also include other components, such as shrinkage, plus the mechanical strain $\epsilon_m$, which is given by stress $\sigma$ divided by Young’s modulus $E$:

$$\epsilon_{tot} = \epsilon_e + \epsilon_m = \epsilon_e + \sigma / E$$  \hspace{1cm} (2)

If plane sections remain plane, the total strain field must be linear.
where
\( \varepsilon_0 \) = total strain at the origin
\( \phi \) = curvature
\( y \) = vertical coordinate, measured positive downwards from the origin

For this derivation, strains are assumed to be positive in tension, and a positive curvature causes downward deflection. The origin may be taken anywhere. Combining Eq. (2) and (3) gives Eq. (4).

\[
\sigma = E(\varepsilon_0 + \phi y - \varepsilon_t)
\]

For a statically determinate girder, axial-force equilibrium is determined by Eq. (5).

\[
0 = \int \sigma dA = \int E(\varepsilon_0 + \phi y - \varepsilon_t) dA
\]

If the section contains several materials with different properties, they must be taken into account when the integral is evaluated. The two unknowns are \( \varepsilon_0 \) and \( \phi \). The integral can be broken into components, and the axial force equilibrium equation can be rearranged to give a linear equation in \( \varepsilon_0 \) and \( \phi \) (Eq. [6]).

\[
\varepsilon_0 \int E dA + \phi \int Ey dA = \int E\varepsilon_t dA
\]

Similarly, moment equilibrium at the section requires Eq. (7).

\[
0 = \int y \sigma dA = \int y E(\varepsilon_0 + \phi y - \varepsilon_t) dA
\]

Again this equation can be rearranged to give Eq. (8).
Once $\varepsilon_0$ and $\phi$ are known, the total strain field can be calculated from Eq. (3), and the stresses from Eq. (4).

In general, the curvatures at several points along the girder must be computed and then integrated to give the camber. However, if the girder is prismatic and the thermal conditions are the same along the length, the thermal curvature will be constant along the length and may be computed at a single location. The midspan deflection is obtained by integrating this curvature twice. If the member is simply supported, the camber at midspan $\Delta_{\text{camber}}$ is given by Eq. (15).

$$\Delta_{\text{camber}} = -\frac{\phi L^2}{8}$$  \hspace{1cm} (15)

where

$L$ = length of the girder

The negative sign in Eq. (15) results from the fact that the calculated deflection is negative, but in practice, camber is taken as positive upwards.

The measured temperature profiles were used to predict camber changes in girders H6B and H8A over the period that they were monitored. For simplicity, the gross (rather than the transformed) section properties were used in the calculations. According to the Federal Highway Administration, the coefficient of thermal expansion of portland cement concrete ranges from about $4.4 \times 10^{-6}$ to $6.6 \times 10^{-6}/\text{˚F}$ ($7.9 \times 10^{-6}$ to $12.0 \times 10^{-6}/\text{˚C}$). Wight and MacGregor found that the coefficient of thermal expansion for normal-weight concretes made with siliceous aggregates ranges from $5.0 \times 10^{-6}$ to $7.0 \times 10^{-6}/\text{˚F}$ ($9.0 \times 10^{-6}$ to $12.6 \times 10^{-6}/\text{˚C}$). In their calculations, Barr et al. assumed a value of $6.5 \times 10^{-6}/\text{˚F}$ ($11.7 \times 10^{-6}/\text{˚C}$) for this coefficient.

Figure 6 compares the measured camber histories with the values calculated using Eq. (12) to (15) and the measured thermal profiles. Two calculated curves are shown, corresponding to coefficients of thermal expansion of 5.5 $\times 10^{-6}$ (9.9 $\times 10^{-6}/\text{˚C}$) and 6.5 $\times 10^{-6}/\text{˚F}$ (11.7 $\times 10^{-6}/\text{˚C}$). The calculated and measured cambers rise and fall at similar rates and peak at similar times. The calculated curves matched the measured ones most closely when the coefficient of thermal expansion of the concrete was taken as 5.5 $\times 10^{-6}/\text{˚F}$. The small discrepancies can be attributed to the following:

- the assumed value for the coefficient of thermal expansion
For the effective strain profile shown in Fig. 7, the corresponding thermal curvature is Eq. (16).

$$\phi(t) = \frac{e_{TH,eff}(t)}{h}$$

(16)

where

- \( \phi(t) \) = curvature as a function of time
- \( e_{TH,eff}(t) \) = effective free thermal strain \( \alpha \Delta T_{eff} \) at time \( t \)
- \( \Delta T_{eff} \) = effective temperature difference between top and bottom of girder

The corresponding camber at midspan for a simply supported girder can be computed using Eq. (15).

To calculate the camber at a particular time during the day, \( \Delta T_{eff}(t) \) is needed. In the temperature history camber model, the total strain profile is assumed to be linear and is represented by an effective thermal strain gradient that varies linearly from zero at the bottom of the girder to an effective free thermal strain of \( e_{TH,eff} \) at the top (Fig. 7). For simplicity, the thermal difference that leads to \( e_{TH,eff} \) is taken to be proportional to the daily variation in ambient temperature. The constant of proportionality between \( e_{TH,eff} \) and the ambient temperature is then obtained by calibration against measured camber data.

Figure 6 shows that thermal cambers can be computed based on measured internal temperatures. In practice, such internal temperature data are rarely available. Therefore, two practical methods were developed to predict the thermal camber histories that require knowledge of the air temperature alone. In the following two sections, these two methods are referred to as the temperature history camber model and the peak temperature camber model.

**Temperature history camber model**

The temperature history camber model is based on the assumption that the temperature difference between the top and bottom of the girders can be related to variations in the ambient temperature. Site-specific data should be used when available. In their absence, the ambient temperature histories from a nearby meteorological station should be used.

Although the measured temperature data showed that the temperature profiles in the girders were highly nonlinear, the nonlinear temperature profiles nonetheless resulted in a linear total strain profile, in accordance with the Bernoulli hypothesis that plane sections remain plane. In the temperature history camber model, the total strain profile is assumed to be linear and is represented by an effective thermal strain gradient that varies linearly from zero at the bottom of the girder to an effective free thermal strain of \( e_{TH,eff} \) at the top (Fig. 7). For simplicity, the thermal difference that leads to \( e_{TH,eff} \) is taken to be proportional to the daily variation in ambient temperature. The constant of proportionality between \( e_{TH,eff} \) and the ambient temperature is then obtained by calibration against measured camber data.
Eq. (19). The root mean square camber error over time was defined using Eq. (20).

\[ \text{Error}_{\text{rms}} = \sqrt{\frac{1}{n} \sum_{i=1}^{n} (M_i - C_i)^2} \]  

The parameter \( A_0 \) in Eq. (19) was optimized to minimize the root mean square error assuming a coefficient of thermal expansion \( \alpha \) equal to \( 5.5 \times 10^{-6} /\text{F} \) (9.9 \( \times 10^{-6} /\text{C} \)). Figure 8 shows the value of \( A_0 \) optimized for each girder individually. A value of \( A_0 \) of 1.31 gave the optimum fit for the girder population, resulting in root mean square camber error over all girders of 0.11 in. (2.8 mm).

Figure 9 compares the measured thermal camber histories with those predicted by the temperature history camber model (\( A_0 \) equal to 1.31) for the seven Washington girders. The calculated and measured values are represented, respectively, by continuous and dashed lines. The occurrence of several dashed lines in a single plot indicates that the cambers of several nominally identical girders were measured on the same day, so the calculated cambers are the same.

The temperature history camber model reproduced the camber histories well for girders 1A, 1C, 2A, 2B, 2C. The discrepancy between the peak calculated and measured cambers was never more than about 0.15 in. (3.8 mm). The peak discrepancies were greater for girder H6B (0.30 in. [7.6 mm]). One possible explanation for the larger discrepancy for girder H6B is that one side of this girder was exposed to sunlight. Therefore, the girder was heated over its whole height, whereas the other girders were subjected to solar radiation primarily on the top flange. The more uniform temperature gradient of girder H6B should lead to a smaller camber. Another contributing factor is that the Washington girders were wide-flange (WF100G) sections, whereas the five girders from Barr had standard flanges (W74G sections). The WF series have larger top flanges that will tend to heat up more slowly in the same ambient temperature change than a girder with a thinner flange.

Figure 10 compares the measured and calculated thermal cambers using the temperature history camber model (\( A_0 \) equal to 1.31) for the Minnesota and Georgia girders. The cambers predicted by the temperature history camber model were close to the measured values for all of the Minnesota girders. However, the errors were larger for the Georgia girders, for which the environmental conditions (for example, temperature, humidity, and length of day) may have differed from those in Washington and Minnesota.

**Peak temperature camber model**

A second, simpler model, called the peak temperature camber model, was developed. This method does not...
where \( t_0 = \) reference time for counting the thermal camber during that day

\[ \Delta T_{\text{eff}} = A_1 (T_{\text{max}} - T_{\text{min}}) \left( 1 - \cos \left( \frac{(t - t_0)}{24} \frac{2\pi}{24} \right) \right) \]  

(21)

Proposed model

To estimate the camber at any time during the day, an instantaneous value of \( \Delta T_{\text{eff}}(t) \) is needed (Eq. [14]). In the peak temperature camber model described here, that effective temperature is approximated from the maximum and minimum air temperatures of that day using a cosine interpolation function that is a function of time (Eq. [21]):

\[ \Delta T_{\text{eff}}(t) = A_1 (T_{\text{max}} - T_{\text{min}}) \left( 1 - \cos \left( \frac{(t - t_0)}{24} \frac{2\pi}{24} \right) \right) \]  

where

\( t_0 = \) reference time for counting the thermal camber during that day

\( T_{\text{max}} = \) maximum air temperature during the 24-hour period

\( T_{\text{min}} = \) minimum air temperature during the 24-hour period

### Table 2. Girder properties

<table>
<thead>
<tr>
<th>Girder</th>
<th>Girder mark</th>
<th>Girder type</th>
<th>Location</th>
<th>Depth, in.</th>
<th>Length, ft</th>
<th>Length/depth</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1A</td>
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<td>Washington</td>
<td>74</td>
<td>80</td>
<td>13.0</td>
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<tr>
<td>2</td>
<td>1C</td>
<td>W74MG</td>
<td>Washington</td>
<td>74</td>
<td>80</td>
<td>13.0</td>
</tr>
<tr>
<td>3</td>
<td>2A</td>
<td>W74MG</td>
<td>Washington</td>
<td>74</td>
<td>137</td>
<td>22.2</td>
</tr>
<tr>
<td>4</td>
<td>2B</td>
<td>W74MG</td>
<td>Washington</td>
<td>74</td>
<td>137</td>
<td>22.2</td>
</tr>
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<td>5</td>
<td>2C</td>
<td>W74MG</td>
<td>Washington</td>
<td>74</td>
<td>137</td>
<td>22.2</td>
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<tr>
<td>6</td>
<td>H6B</td>
<td>WF100G</td>
<td>Washington</td>
<td>100</td>
<td>173</td>
<td>20.7</td>
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<tr>
<td>7</td>
<td>H8A</td>
<td>WF100G</td>
<td>Washington</td>
<td>100</td>
<td>164</td>
<td>19.7</td>
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<tr>
<td>8</td>
<td>MN54, beam 1</td>
<td>MN54</td>
<td>Minnesota</td>
<td>54</td>
<td>122</td>
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<tr>
<td>9</td>
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<tr>
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<td>Minnesota</td>
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<td>Minnesota</td>
<td>45</td>
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<td>Georgia</td>
<td>79</td>
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<td>Georgia</td>
<td>79</td>
<td>127</td>
<td>19.3</td>
</tr>
</tbody>
</table>

Note: 1 in. = 25.4 mm; 1 ft = 0.305 m.

* MN45 beams 1, 2, and 3 were monitored two days (May 17, 2011, and June 30, 2011).
The time offset $t_0$ reflects several influences, including the time at which the ambient temperature is at a minimum and the lag between the concrete and air temperatures. Then $\Delta T_{\text{eff}}$ has the value of zero when $t$ equals $t_0$ (or $t_0 + 24$ hours) and reaches its maximum of $A_1(T_{\text{max}} - T_{\text{min}})$ at $t$ equal $t_0 + 12$ hours, where $A_1$ is a calibration factor.

The effective thermal strain is calculated using Eq. (22).

$$\varepsilon_{\text{TH,eff}} = A_1 \left( T_{\text{max}} - T_{\text{min}} \right) \left\{ 1 - \cos \left[ \frac{(t - t_0)}{24} \left( \frac{2\pi}{2} \right) \right] \right\}$$ (22)
Figure 9. Temperature history camber model: comparison of measured and calculated thermal camber changes in Washington State girders. Note: 1 in. = 25.4 mm.

Girders 1A and 1C
Girder H8A
Girders 2A, 2B, and 2C
Girder H6B

Figure 10. Temperature history camber model: comparison of measured and calculated thermal camber changes for Minnesota and Georgia State girders. Note: 1 in. = 25.4 mm.

MN54 girders
MN63 girders
MN45 girders (measured on May 17, 2011)
W45 girders, Georgia
Combining Eq. (15) and (22) results in Eq. [23], which gives the thermal camber at midspan for a simply supported girder.

\[
A_{\text{camb}} = \left( \frac{\alpha A_1}{h} \right) \left( T_{\text{min}} - T_{\text{max}} \right) \left[ 1 - \cos \left( \frac{t-t_0}{24} \frac{2\pi}{l} \right) \right] \left( \frac{l^2}{8} \right)
\]  

(23)

Calibration of peak temperature camber model

As was done for the temperature history camber model, the peak temperature camber model was calibrated by optimizing predictions of the cambers. **Table 4** lists the optimal model parameters and individual root mean square errors in Eq. (20) when each of the 24 girders was considered individually. The model was then optimized against the data from all girders at once, and the values \(A_1\) of 1.28 and \(t_0\) of 4.53 (rounded to 4.5) were found to be optimal, with a root mean square camber error of 0.09 in. (2.3 mm).

**Figure 11** compares the measured and calculated thermal camber changes using the peak temperature model with \(A_1\) of 1.28 and \(t_0\) of 4.5 for the Washington girders. As was the case with the temperature history camber model, the greatest error occurred in girder H6B of Washington (0.20 in. [5 mm]). As discussed previously, girder H6B was unusual in that its left side was exposed to the sun. Neither the temperature history camber model nor the peak temperature camber model considered this factor.

**Figure 12** compares the measured and calculated cambers for the Minnesota and Georgia girders. The peak temperature model works well for Minnesota girders. The shapes of the measured and calculated curves were similar, and the average error was less than 0.1 in. (2.5 mm). As was the case in the temperature history camber model, the measured and predicted cambers differed more for the Georgia girders.

**Conclusion**

This research leads to the following conclusions:

- The temperature profile in a precast concrete girder can be highly nonlinear during the afternoon, with a large temperature gradient within the top flange.

- The camber histories computed from the internal temperature profiles of two instrumented girders were close to the measured values. This finding not only verifies the computational method used but it also suggests that the spatial density of measured temperature readings was sufficient for that purpose.

- Both the temperature history camber model and the peak temperature camber model had root mean square average camber errors over time of about 0.1 in. (2.5 mm) when applied to a range of girders, all with I-shaped cross sections. Such small errors would not significantly affect the installation of girders onsite. Since the peak temperature camber model is simpler than the temperature history camber model (and for these data, slightly more accurate), the peak temperature camber model is recommended for practice for use with I-shaped girders on simple spans. It has not

**Table 4. Root mean square errors between measured and calculated thermal camber changes using variables of \(A_1\) and \(t_0\)**

<table>
<thead>
<tr>
<th>Girder</th>
<th>Girder mark</th>
<th>Location</th>
<th>Optimum (A_1)</th>
<th>Optimum (t_0)</th>
<th>Error, in.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1A</td>
<td>Washington</td>
<td>1.11</td>
<td>5.80</td>
<td>0.006</td>
</tr>
<tr>
<td>2</td>
<td>1C</td>
<td>Washington</td>
<td>1.05</td>
<td>6.98</td>
<td>0.014</td>
</tr>
<tr>
<td>3</td>
<td>2A</td>
<td>Washington</td>
<td>1.25</td>
<td>5.21</td>
<td>0.033</td>
</tr>
<tr>
<td>4</td>
<td>2B</td>
<td>Washington</td>
<td>1.19</td>
<td>5.00</td>
<td>0.039</td>
</tr>
<tr>
<td>5</td>
<td>2C</td>
<td>Washington</td>
<td>1.44</td>
<td>5.01</td>
<td>0.018</td>
</tr>
<tr>
<td>6</td>
<td>H6B</td>
<td>Washington</td>
<td>0.78</td>
<td>4.51</td>
<td>0.045</td>
</tr>
<tr>
<td>7</td>
<td>H8A</td>
<td>Washington</td>
<td>1.20</td>
<td>5.53</td>
<td>0.040</td>
</tr>
<tr>
<td>8</td>
<td>MN54, beam 1</td>
<td>Minnesota</td>
<td>1.46</td>
<td>3.35</td>
<td>0.019</td>
</tr>
<tr>
<td>9</td>
<td>MN54, beam 2</td>
<td>Minnesota</td>
<td>1.20</td>
<td>5.13</td>
<td>0.020</td>
</tr>
<tr>
<td>10</td>
<td>MN54, beam 3</td>
<td>Minnesota</td>
<td>1.19</td>
<td>4.04</td>
<td>0.021</td>
</tr>
<tr>
<td>11</td>
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<td>Minnesota</td>
<td>1.26</td>
<td>3.64</td>
<td>0.011</td>
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<tr>
<td>12</td>
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<td>Minnesota</td>
<td>1.24</td>
<td>4.90</td>
<td>0.032</td>
</tr>
<tr>
<td>13</td>
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<td>Minnesota</td>
<td>1.12</td>
<td>3.68</td>
<td>0.025</td>
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<td>14</td>
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<td>1.38</td>
<td>3.70</td>
<td>0.041</td>
</tr>
<tr>
<td>15</td>
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<td>1.00</td>
<td>4.30</td>
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<tr>
<td>16</td>
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<td>1.58</td>
<td>4.23</td>
<td>0.016</td>
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<tr>
<td>17</td>
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<td>1.45</td>
<td>3.88</td>
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<tr>
<td>18</td>
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<td>Minnesota</td>
<td>1.24</td>
<td>4.68</td>
<td>0.039</td>
</tr>
<tr>
<td>19</td>
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<td>Minnesota</td>
<td>1.72</td>
<td>4.24</td>
<td>0.059</td>
</tr>
<tr>
<td>20</td>
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<td>Minnesota</td>
<td>1.40</td>
<td>4.32</td>
<td>0.048</td>
</tr>
<tr>
<td>21</td>
<td>MN45, beam 3</td>
<td>Minnesota</td>
<td>1.18</td>
<td>4.62</td>
<td>0.052</td>
</tr>
<tr>
<td>22</td>
<td>W45, 6</td>
<td>Georgia</td>
<td>3.96</td>
<td>8.23</td>
<td>0.000</td>
</tr>
<tr>
<td>23</td>
<td>W45, 7</td>
<td>Georgia</td>
<td>2.52</td>
<td>8.19</td>
<td>0.000</td>
</tr>
<tr>
<td>24</td>
<td>W45, 8</td>
<td>Georgia</td>
<td>3.71</td>
<td>8.28</td>
<td>0.000</td>
</tr>
</tbody>
</table>

Note: \(A_1\) = calibration factor; \(t_0\) = reference time for counting thermal camber during a day.
Figure 11. Peak temperature camber model: comparison of measured and calculated thermal camber changes for Washington State girders. Note: 1 in. = 25.4 mm.

Girders 1A and 1C

Girder H8A

Girders 2A, 2B, and 2C

Girder H6B

Figure 12. Peak temperature camber model: comparison of measured and calculated thermal camber changes for Minnesota and Georgia State girders. Note: 1 in. = 25.4 mm.

MN45 girders (measured on May 17, 2011)

MN54 girders

MN63 girders

W45 girders, Georgia
been calibrated with other cross-sectional shapes or other boundary conditions.

• The accuracy of the camber prediction depended on the details of the girder’s exposure to the sun. For example, the methods worked less well for Washington girder H6B, which had one side, as well as its top flange, exposed to sunlight. The models also worked better for the Washington and Minnesota girders than for the girders located in Georgia. It is possible that the differences in accuracy arise from differences in temperature, humidity, or solar radiation conditions. It is also possible that the discrepancies are attributable to the paucity of camber observations for the Georgia girders.

References


Notation

\[ A_0 = \text{calibration factor} \]
\[ A_1 = \text{calibration factor} \]
\[ A_{tr} = \text{area of the transformed section} \]
\[ b(y) = \text{girder width} \]
\[ C_i = \text{computed camber values} \]
\[ E = \text{Young's modulus} \]
\[ Error_{rms} = \text{root mean square camber error over time of a girder} \]
\[ f_c = \text{compressive strength of concrete} \]
\[ h = \text{depth of the girder} \]
\[ h_a = \text{asphalt thickness} \]
\[ I_{tr} = \text{moment of inertia of the transformed section} \]
\[ L = \text{length of the girder} \]
\[ M_i = \text{measured camber values} \]
\[ t = \text{time} \]
\[ t_0 = \text{reference time for counting thermal camber during a day} \]
\[ t(Y) = \text{change in temperature at height } Y \]
$$T = \text{temperature at top surface of deck}$$

$$T_{\text{amb}}(t) = \text{ambient temperature at time } t$$

$$T_{\text{amb,eff}}(t) = \text{effective ambient temperature at time } t$$

$$T_{\text{max}} = \text{maximum temperature during a day}$$

$$T_{\text{min}} = \text{minimum temperature during a day}$$

$$y = \text{vertical coordinate, measured positive downwards from origin}$$

$$Y = \text{vertical coordinate, measured upwards from a point located } Y_0 \text{ below the concrete deck surface}$$

$$Y_0 = \text{distance from top surface at which temperature change is zero}$$

$$\alpha = \text{coefficient of thermal expansion}$$

$$\Delta_{\text{camber}} = \text{camber of girder}$$

$$\Delta T = \text{temperature difference over the height of the girder}$$

$$\Delta T_1 = \text{temperature change at top surface of girder in AASHTO LRFD specifications profile}$$

$$\Delta T_2 = \text{temperature change 4 in. (100 mm) below top surface of girder in AASHTO LRFD specifications profile}$$

$$\Delta T_3 = \text{temperature change at bottom of girder in AASHTO LRFD specifications profile}$$

$$\Delta T_{\text{eff}} = \text{effective temperature difference between top and bottom of girder}$$

$$\Delta T_{\text{eff}}(t) = \text{instantaneous effective temperature difference between top and bottom of girder}$$

$$\epsilon_0 = \text{total strain at origin}$$

$$\epsilon_e = \text{free environmental strain}$$

$$\epsilon_m = \text{mechanical strain}$$

$$\epsilon_{\text{tot}} = \text{total strain}$$

$$\epsilon_{\text{TH,eff}} = \text{effective thermal strain}$$

$$\epsilon_{\text{TH,eff}}(t) = \text{effective thermal strain at time } t$$

$$\sigma = \text{stress}$$

$$\phi = \text{curvature}$$

$$\phi(t) = \text{curvature as a function of time}$$
About the authors

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Abstract

It is important to estimate girder camber accurately because differences between expected and actual camber can lead to construction challenges or girder rejection. Field measurements of daily variations in temperature profile and camber for two precast, prestressed concrete girders provided data with which to calibrate models of the effect of temperature variations on camber. Using measured temperature profiles over the height of the girder, the associated camber history was accurately computed, assuming a coefficient of thermal expansion of $5.5 \times 10^{-6}$/°F ($9.9 \times 10^{-6}$/°C). Two practical methods were also developed using 164 observations from 24 girders. To implement the simpler method (peak temperature camber method), the designer needs only to know the girder’s length and depth and to estimate the maximum change in air temperature during the day, which is available from meteorological stations. The errors in the resulting models had root mean square average camber over time of about 0.1 in. (2.5 mm).

Keywords

Bridge, camber, girder, temperature.

Review policy

This paper was reviewed in accordance with the Precast/Prestressed Concrete Institute’s peer-review process.

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