Recent developments in seismic force–resisting systems for buildings\textsuperscript{1-6} and bridges\textsuperscript{7-16} use unbonded posttensioning. Unbonded posttensioning tendons have the capacity to distribute strains caused by locally imparted member deformations over greater tendon lengths. As a result, the deformation capacity of these members increases considerably and their self-centering capabilities improve significantly. Reliable design of seismically resistant posttensioned structures requires knowledge of the mechanical properties of the posttensioning system.

Typical posttensioning systems consist of two major components, namely, the posttensioning tendons, which can be bonded or unbonded, and the anchorage hardware, which transfers the posttensioning forces to the structure. Each tendon may consist of a single monostrand (monostrand tendons) or multiple monostrands (multistrand tendons). The design of posttensioning systems typically includes selection of the tendon geometric and material properties, selection of the initial posttensioning forces (accounting for all losses), and design of the anchorage zones so that all posttensioning forces applied by the anchorage hardware can be safely transferred to the structure.

Currently, no specific consideration is given to the design of the anchorage hardware. Anchorage hardware designs are typically provided by the tendon manufacturers and are

- Cyclic tensile testing was conducted for a set of 0.5 in. (13 mm) and 0.6 in. (15 mm) diameter Grade 270 (1860 MPa) monostrand-anchor systems to complete failure of all wires.

- A numerical model was developed that is capable of capturing the basic characteristics of the observed experimental response. The predictions of the numerical results are in good agreement with the experimental data.

- Based on the research findings, recommendations for the design of unbonded monostrand-anchor systems are provided.

Effects of anchorage hardware on the cyclic tensile response of unbonded monostrands

Petros Sideris, Amjad J. Aref, and Andre Filiatrault
response of unbonded monostrands.

• Develop a numerical model for the response of monostrand-anchor systems.

• Indicate design implications for posttensioned structures due to the effects of anchorage setups on the response of unbonded monostrands.

Experimental investigation

Specimen description

The test specimens were unbonded monostrands incorporating barrel anchors with two-piece wedges at both ends, (Fig. 1). Monostrands with diameters of 0.5 in. (13 mm) and 0.6 in. (15 mm) were considered along with the appropriate-sized wedges and barrels provided by the manufacturer. Figure 2 and Table 1 indicate the dimensions of the barrels and wedges. All monostrands, barrels, and wedges were provided by a single manufacturer. The monostrands were Grade 270 (1860 MPa), cold-drawn, seven-wire, low-relaxation cables conforming to ASTM A416/A416M. They had a nominal ultimate strength $f_{pu}$ of 270 ksi and a nominal yield strength $f_{py}$ of 243 ksi (1675 MPa) (0.9 $f_{pu}$, per the American Association of State Highway and Transportation Officials’ AASHTO LRFD Bridge Design Specifications). They had a nominal ultimate strength $f_{pu}$ of 270 ksi and a nominal yield strength $f_{py}$ of 243 ksi (1675 MPa) (0.9 $f_{pu}$, per the American Association of State Highway and Transportation Officials’ AASHTO LRFD Bridge Design Specifications). Table 2 presents the mechanical properties of the monostrands considered in the experimental study, including the nominal ultimate force strength $F_{pu}$, nominal yield force strength $F_{py}$, and effective force $F_{peff}$ (per AASHTO LRFD specifications). The anchor-to-anchor length for all specimens was 53 in. (1350 mm) to 54 in. (1370 mm).

Test setup, loading system, and instrumentation

The test specimens were mounted to the top and bottom crosshead of a uniaxial testing machine (Fig. 3). Loading was applied by moving the bottom crosshead upward or downward (at a constant rate of approximately 0.01 in./sec [2.5 mm/sec]) assuming to meet prescribed design standards. These standards typically require that the anchorage hardware components remain elastic under the ultimate tendon loading.

Anchorage hardware normally includes a cylindrical barrel with a tapered interior surface and a two-piece or three-piece wedge. Anchorage of the tendons is achieved by sliding the wedge pieces into the tapered interior surface so that the tendon is compressed into the barrel. The anchorage forces are transferred to the structure through bearing contact of the barrel with the anchorage zone. Depending on the type of anchorage hardware, intermediate contact plates may be used to transfer the bearing force over a larger contact area.

During seismic loading of concrete structures incorporating bonded posttensioning tendons, no additional loading is applied to the anchorage hardware components because localized deformations (for example, at the base of bridge cantilever columns) cause localized strains in the tendons. For unbonded posttensioning tendons, though, large forces can be transferred to the anchorages, imparting additional demands on the anchorage hardware and the concrete in the anchorage zones. Moreover, because the force variations in the anchorage setups are large, the mechanical properties of the tendons, including the effects of their anchorage hardware, should be considered in the design and modeling of unbonded posttensioning systems.

Limited studies focusing on the effects of anchorage hardware on the posttensioning tendon response are available in the literature. Walsh and Kurama investigated the response of seven-wire monostrand-anchor systems (in other words, monostrands with their anchorage hardware) and observed that single monostrand wires prematurely fractured at the location of the anchors (as opposed to the nearly simultaneous fracture of all wires of anchor-free monostrands). Walsh and Kurama investigated the effects of loading and boundary conditions on the premature fracture strength and strain of multistrand- and monostrand-anchor systems. MacDougall and Bartlett investigated the strain distribution, affected length, and stress distribution along unbonded posttensioning tendons with fractures of symmetric and nonsymmetric wires. In their studies, MacDougall and Bartlett did not consider the effects of anchorage hardware.

Research scope and objectives

This paper focuses on the cyclic tensile behavior of unbonded monostrand-anchor systems and presents the findings of an experimental study. A simplified numerical model for monostrand-anchor systems capable of reproducing the experimental results with reasonable accuracy is also described. The objectives of this study are the following:

• Identify the effects of the anchorage hardware on the treatment of unbonded monostrands.

• Develop a numerical model for the response of monostrand-anchor systems.

• Indicate design implications for posttensioned structures due to the effects of anchorage setups on the response of unbonded monostrands.

Figure 1. Barrel with two-piece wedge for 0.5 in. (13 mm) diameter monostrand.
tween the load cell and the top crosshead to safely transfer the applied pressure. A steel plate was also used between the bottom crosshead and the anchorage setup. Each plate had a thickness of 1 in. (25 mm) or larger. The string potentiometer was attached to the steel plates that were in contact with the top and bottom anchorage setups; hence, the deformation/sliding of the anchorage hardware was incorporated in the displacement measurements.

No preloading was applied to the monostrand-anchor system before testing. The anchorage hardware components were set by hand and were stressed at the beginning of each test by slightly lowering the bottom crosshead. At the end of this process, the monostrand force was still small (<0.5 kip [2 kN]) so that its bottom end could be slightly moved by hand. All string potentiometers were initialized at that time. Also, the original (anchor-to-anchor) length $L_0$ of the monostrand-anchor system was measured at that time.

### Test program and load protocol

Five 0.5 in. (13 mm) and five 0.6 in. (15 mm) diameter monostrand-anchor systems were tested. The five tests with the 0.5 in. diameter monostrand-anchor systems were identified as 05-1, 05-2, 05-3, 05-4, and 05-5. The five tests with the 0.6 in. diameter monostrand-anchor systems were identified as 06-1, 06-2, 06-3, 06-4, and 06-5.

A mixed force/displacement-controlled loading protocol was considered. In the first cycle, the specimen was monotonically loaded approximately to the effective strength of the monostrand ($0.72 F_{pu}$ or about 30 kip [130 kN] and 40 kip [180 kN] for the 0.5 in. [13 mm] and 0.6 in. [15 mm] diameter monostrands, respectively [Table 3]) and then unloaded to 5 kip (22 kN) and 10 kip (44 kN) for the 0.5 in and 0.6 in. diameter monostrands, respectively.

---

**Table 1. Dimensions of anchorage hardware**

<table>
<thead>
<tr>
<th>Hardware</th>
<th>Dimensions</th>
<th>0.5 in.</th>
<th>0.6 in.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Barrel</strong></td>
<td>Height of barrel $H_b$, in.</td>
<td>1.44</td>
<td>1.78</td>
</tr>
<tr>
<td></td>
<td>Exterior diameter of barrel $D_b$, in.</td>
<td>1.625</td>
<td>1.82</td>
</tr>
<tr>
<td></td>
<td>Interior diameter of barrel at top end $d_{bt}$, in.</td>
<td>1</td>
<td>1.13</td>
</tr>
<tr>
<td></td>
<td>Interior diameter of barrel at bottom end $d_{bb}$, in.</td>
<td>0.646</td>
<td>0.694</td>
</tr>
<tr>
<td><strong>Two-piece wedge</strong></td>
<td>Height of wedge $H_w$, in.</td>
<td>1.19</td>
<td>1.59</td>
</tr>
<tr>
<td></td>
<td>Exterior diameter of wedge at top end $D_{wt}$, in.</td>
<td>1</td>
<td>1.125</td>
</tr>
<tr>
<td></td>
<td>Exterior diameter of wedge at bottom end $D_{wb}$, in.</td>
<td>0.735</td>
<td>0.762</td>
</tr>
<tr>
<td></td>
<td>Interior diameter of wedge $d_w$, in.</td>
<td>0.4725</td>
<td>0.575</td>
</tr>
<tr>
<td></td>
<td>Distance between the two parts of a wedge $\Delta d$, in.</td>
<td>0.062</td>
<td>0.062</td>
</tr>
</tbody>
</table>

Note: 1 in. = 25.4 mm.
In the first loading cycle, the force-displacement curve was nonlinear. This nonlinearity, which is not observed in anchor-free monostrands, is attributed to the seating of the wedge into the anchor as well as the slippage of the monostrand into the wedge before the lateral compression applied by the wedge to the monostrand became significant. This nonlinearity ceases after a posttensioning force of approximately 15% of $F_{pu}$ is reached. The effect of seating and slippage was removed from the recorded displacement signals by considering a linear loading branch in the force-versus-displacement response. This linear branch was obtained by interpolating between the values of $0.35F_{pu}$ and $0.72F_{pu}$ (effective force [Table 2]), as shown in Fig. 4. The intersection of this branch with the horizontal axis indicated the seating and slippage length $ΔL_s$. A corrected estimate $L_{o,c}$ of the original length $L_o$ was calculated by adding $ΔL_s$ to $L_o$. Moreover, $ΔL_s$ was removed (as an offset) from the (measured) applied displacement history $u_{ap}$.

The next cycles were conducted by reloading to a maximum force $F_{max}$ and unloading always to the minimum force of 5 kip or 10 kip for the 0.5 in. and 0.6 in. diameter monostrands, respectively. For both monostrand diameters, this maximum force $F_{max}$ started from 10 kip for the second cycle and increased with increments of 10 kip per cycle for the subsequent cycles until the nominal yield force was reached. Following yielding of the monostrand, the next cycles were conducted based on displacement readings, with displacement increments $Δu$ of 0.5 in. to 0.65 in. (17 mm) for the 0.5 in. diameter monostrands, or 0.6 in. to 0.75 in. (19 mm) for the 0.6 in. diameter monostrands (from the maximum achieved displacement at the previous cycle). Unloading was always performed to the minimum force of 5 kip or 10 kip for the 0.5 in. and 0.6 in. diameter monostrands, respectively. Cycles were applied until fracture failure of all wires was achieved. Table 3 presents the loading protocols for the 0.5 in. and 0.6 in. diameter monostrand-anchor systems.

During testing, the force-versus-displacement response was plotted on-screen (in real time). Because the test machine was manually operated, the loading protocol was approximately followed based on the plotted measurements.

### Test results

**Global response** Figure 4 shows a typical force-displacement response for the second test with the 0.5 in. (13 mm) diameter monostrand-anchor systems (identified as 05-2). Before the nominal yield strength of the monostrand $F_{py}$ was reached, the monostrand-anchor system exhibited a loading stiffness that was smaller than the unloading stiffness and reloading stiffness (for reloading up to the maximum force/displacement of the loading history). The monostrand-anchor systems failed prematurely by fracture of a single monostrand wire in the vicinity of the anchorage hardware. Due to their premature failure, monostrand-anchor systems did not reach the nominal strength (per ASTM A416/A416M) and deformation capacity of anchor-free monostrands. Wires fractured, one at a time, with the applied displacement, in contrast to the nearly simultaneous fracture of all wires observed by Walsh and Kurama for anchor-free monostrands.

### Table 2. Mechanical properties of seven-wire low relaxation monostrand

<table>
<thead>
<tr>
<th>Property</th>
<th>0.5 in. monostrand</th>
<th>0.6 in. monostrand</th>
</tr>
</thead>
<tbody>
<tr>
<td>Original monostrand area before fracture of wires $A_{PT,o}$ in.$^2$</td>
<td>0.153</td>
<td>0.217</td>
</tr>
<tr>
<td>Nominal ultimate force strength of monostrand $F_{pu}$ kip</td>
<td>41.3</td>
<td>58.6</td>
</tr>
<tr>
<td>Nominal yield force strength of monostrand $F_{py} = 0.9F_{pu}$ kip</td>
<td>37.2</td>
<td>52.7</td>
</tr>
<tr>
<td>Effective force strength of monostrand $F_{p,eff} = 0.8F_{py} = 0.72F_{pu}$ kip</td>
<td>29.7</td>
<td>42.2</td>
</tr>
</tbody>
</table>

Note: 1 in. = 25.4 mm; 1 kip = 4.448 kN.
The difference between the loading and the unloading/reloading stiffness—prior to yielding of the monostrand—resulted from the anchorage setups. The increase of the axial load in the monostrand-anchor system caused sliding of the wedges into the tapered interior surface of the barrels, providing additional extension to the monostrand-anchor system and further compressing (laterally) the monostrand. This additional extension translated into lower stiffness for the loading branch in the force-displacement response. The data in Table 3, which presents the loading protocol for the monostrand-anchor systems, illustrates these effects.

### Table 3. Loading protocol for the monostrand-anchor systems

<table>
<thead>
<tr>
<th>Cycle</th>
<th>Force, kip</th>
<th>Displacement</th>
<th>Force, kip</th>
<th>Displacement</th>
<th>Force, kip</th>
<th>Displacement</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>30*</td>
<td>n/a</td>
<td>40*</td>
<td>n/a</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>10</td>
<td>n/a</td>
<td>10</td>
<td>n/a</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>20</td>
<td>n/a</td>
<td>20</td>
<td>n/a</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>30</td>
<td>n/a</td>
<td>30</td>
<td>n/a</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>37</td>
<td>(u_y)</td>
<td>40</td>
<td>n/a</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>n/a</td>
<td>(u_y + \Delta u)</td>
<td>5</td>
<td>52</td>
<td>(u_y)</td>
<td>10</td>
</tr>
<tr>
<td>7</td>
<td>n/a</td>
<td>(u_y + 2\Delta u)</td>
<td>5</td>
<td>n/a</td>
<td>(u_y + \Delta u)</td>
<td>10</td>
</tr>
<tr>
<td>8</td>
<td>n/a</td>
<td>(u_y + 3\Delta u)</td>
<td>5</td>
<td>n/a</td>
<td>(u_y + 2\Delta u)</td>
<td>10</td>
</tr>
<tr>
<td>9</td>
<td>n/a</td>
<td>(u_y + 4\Delta u)</td>
<td>5</td>
<td>n/a</td>
<td>(u_y + 3\Delta u)</td>
<td>10</td>
</tr>
<tr>
<td>10</td>
<td>n/a</td>
<td>(u_y + 5\Delta u)</td>
<td>5</td>
<td>n/a</td>
<td>(u_y + 4\Delta u)</td>
<td>10</td>
</tr>
</tbody>
</table>

* This step was omitted in the first test.

Note: n/a = not applicable; \(u_y\) = nominal yield extension of monostrand-anchor system; \(\Delta u\) = displacement/extension increment during testing; \(\Delta u = 0.5\) in. to 0.65 in. for 0.5 in. diameter monostrand-anchor system; \(\Delta u = 0.6\) in. to 0.75 in. for 0.6 in. diameter monostrand-anchor system. 1 in. = 25.4 mm; 1 kip = 4.448 kN.
curve of monostrand-anchor systems (Fig. 4). Due to the friction in the interface between the wedge and the barrel, subsequent unloading did not result in reverse sliding of the wedge. Thus, the unloading branch in the force-versus-displacement curve (Fig. 4) was solely controlled by the monostrand. On reloading, additional sliding of the wedges occurred only when the applied load exceeded the previously reached maximum load.

Figure 5 presents the measured loading stiffness $K_l$ and unloading/reloading stiffness $K_{un}$ for all tests with the 0.5 in. (13 mm) and 0.6 in. (15 mm) diameter monostrand-anchor systems. These results are compared with the nominal stiffness of the anchor-free monostrands obtained by assuming an unbonded monostrand elastic modulus $E_{PT}$ of 28,500 ksi (197,000 MPa) per the PCI Bridge Design Manual24 (because the elastic modulus of anchor-free monostrands was not measured during testing). For all tests, the unloading/reloading stiffness of monostrand-anchor system $K_{un}$ was lower than the nominal stiffness of the anchor-free monostrands, except for the test designated as 06-3. The difference between the unloading/reloading stiffness of monostrand-anchor systems and the nominal stiffness of the anchor-free monostrand was in the range of 1% to 7% for both monostrand diameters, except for tests 05-1 and 06-5, for which this difference reached 15% and 11%, respectively. The loading stiffness was smaller than the nominal stiffness of the anchor-free monostrand by 38% to 44% for the 0.5 in. (13 mm) diameter monostrand-anchor systems and by 32% to 50% for the 0.6 in. (15 mm) diameter monostrand-anchor systems.

**Ultimate strength and fracture strain** The ultimate strength and fracture strain of the monostrand-anchor systems was controlled by the fracture of the first monostrand wire. All tested monostrand-anchor systems failed prematurely, thereby demonstrating lower deformation capacity and strength than that expected. Fracture of the first wire was observed for strains in the range of 1.5% to 4% (Fig. 6), which was lower than the expected fracture strain of 6% to 7%.17,25 The corresponding strength was in the range of 243 to 270 ksi (1670 to 1860 MPa) (Fig. 6), not reaching the nominal strength of 270 ksi except in one test (Fig. 6). Similar premature failures of monostrand-anchor systems were also reported by Walsh and Kurama.17–19

The 0.5 in. (12.7 mm) diameter monostrand-anchor systems exhibited superior deformation capacity and strength than that of the 0.6 in. (15.2 mm) diameter monostrand-anchor systems. On average, the ultimate strength of the 0.5 in. and 0.6 in. diameter monostrand-anchor systems was 260 and 255 ksi (1790 and 1760 MPa), respectively. Also, on average, the fracture strain of the 0.5 in. and 0.6 in. diameter monostrand-anchor systems was 2.93% and 2.41%, respectively. All monostrand-anchor system strains $\varepsilon$ were calculated using the following equation.

$$\varepsilon = (u_{ap} - \Delta L_{o})/L_{o,c}$$

where

$u_{ap}$ = applied displacement of the monostrand-anchor system (including the contribution of the monostrand and the two anchorage hardware setups)
The seating and slippage length $\Delta L_s$ was removed (as an offset) from the applied displacement $u_{ap}$.

A seven-wire monostrand consists of an inner wire and six outer wires that form a spiral around the inner wire. Except for the interwire contact and friction forces, when an axial load is applied to a monostrand, the inner wire is subjected to tension, while the outer wires are subjected to the combined effects of tension and bending. At the anchorage setups, the outer wires are further subjected to localized contact/friction tractions imparted by the wedge. As a result of the localized combined loading, the outer wires always fractured first at the point of contact with the wedge.

**Successive wire fracture** The response of monostrand-anchor systems beyond the fracture of the first mono-

stranded wire is of high importance because it controls the residual strength of systems incorporating unbonded monos-

strands. Upon fracture of a single (outer) wire, the lateral compres-

sion applied by the wedges on the wires is relieved. However, the application of further axial extension to the mono-

strand-anchor system increases the applied axial load and lateral compression to the monostrand wires until the localized tractions (at the point of contact with the wedge) become critical in another outer wire leading to its fracture. This phenomenon is repeated until most of the outer wires fracture. After fracture of three or more outer wires, the wedge loses the capability to apply large lateral compres-

sion. Thus, the fracture of the remaining wires is practi-

cally controlled only by the applied axial load and any of the remaining wires (inner or outer) can fracture. In many tests, the inner wire was the last one to break, which was attributed to the relative sliding of this wire with respect to its adjacent outer wires. Moreover, in some tests, more than one wire broke almost simultaneously.

Most wires were fractured at a single anchorage hardware setup (Fig. 7). In some anchorage hardware setups, cracks on the wedges were also observed.

**Figure 8** shows the strains at successive fracture of all wires for all tested 0.5 in. (13 mm) and 0.6 in. (15 mm) di-

ameter monostrand-anchor systems. Mean strain values are also included in these figures as well as in Table 4. After fracture of the first wire, the 0.6 in. diameter monostrandanchor systems exhibit slightly better deformation capacity.
than the 0.5 in. diameter monostrand-anchor systems.

**Numerical modeling**

A simplified mechanics-based (as opposed to phenomenological) numerical model for the monostrand-anchor systems is first formulated and then calibrated based on the experimental data. The objective of this model was to predict the response of the monostrand-anchor systems and provide insight on the response of the anchorage hardware.

**Formulation**

The numerical model consists of three coupled nonlinear springs in series (Fig. 9). The two end springs represent the anchorage hardware setups, while the central spring represents the monostrand itself. For simplicity, the barrel and the wedges are assumed to be rigid, while the part of the monostrand that is in contact with the anchorage hardware components is assumed to be axially rigid. The equilibrium equations for the end springs are derived based on a two-dimensional free-body diagram (Fig. 10). From equilibrium of the monostrand (in the vicinity of the anchorage setup [Fig. 10]), the following equation is obtained:

\[ F_{w} = \frac{F_{r}}{2} \]

where

\[ F_{r} = \text{axial force of the monostrand in the vicinity of anchorage hardware setup} \]
\[ F_{w} = \text{friction force at the monostrand-to-wedge interface} \]

From the equilibrium of the wedge in the lateral and longitudinal/axial direction, Eq. (1) and (2) are obtained:

\[ F_{wn} = F_{bn}\sin\theta - F_{bf}\cos\theta \] (1)
\[ F_{wf} = F_{bn}\cos\theta + F_{bf}\sin\theta \] (2)

where

\[ F_{wn} = \text{normal force at the monostrand-to-wedge interface} \]
\[ F_{bn} = \text{normal force at the wedge-to-barrel interface} \]
\[ F_{bf} = \text{tangential (friction) force at the wedge-to-barrel interface} \]
\[ \theta = \text{inclination angle of the tapered interior surface of the barrel} \]

Compatibility of component displacements results in

<table>
<thead>
<tr>
<th>Table 4. Mean strains at successive fracture of monostrand wires</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Monostrand diameter</strong></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>0.5 in.</td>
</tr>
<tr>
<td>0.6 in.</td>
</tr>
</tbody>
</table>

Note: 1 in. = 25.4 mm.
where

\( \mu_B = \text{coefficient of friction at the wedge-to-barrel interface} \)

\( Z_B = \text{elastoplastic hysteretic function of the sliding } u_{Bf} \) (Fig. 11), with \(|Z_B| \leq 1\)

\( u_{B(\gamma)} = \text{yield sliding displacement of } Z_B \) (Fig. 11)

The relative sliding of the monostrand on the wedge \( u_{Wf} \) is neglected (in other words, \( u_{Wf} = 0 \)) because it is small for forces exceeding 15% to 20% of the nominal ultimate monostrand force strength \( F_{pu} \) (Fig. 4).

The lateral force–versus–lateral displacement response of the monostrand due to the compression applied by the wedge pieces (Fig. 10) is given by Eq. (6).

\[
F_{wa} = K_w \langle 2u_{wa} \rangle
\]

where

\( K_w = \text{lateral compressive stiffness of the monostrand} \)

\( \langle \cdot \rangle = \text{Macaulay bracket defined for a parameter } u \text{ as:} \)

\( \langle u \rangle = u \text{ for } u > 0 \text{ and } \langle u \rangle = 0 \text{ otherwise} \)

The factor of 2 accounts for the fact that the monostrand is pushed from two opposite sides by the two wedge pieces.

By taking the ratio of the two expressions of Eq. (1) and (2) and then substituting Eq. (5) and (6), Eq. (7) provides \( F_T \).

\[
F_T = 2 \left( \frac{\cos \theta + \mu_B Z_B \sin \theta}{\sin \theta - \mu_B Z_B \cos \theta} \right) F_{wa}
\]

where

\( \mu_B = \text{coefficient of friction at the wedge-to-barrel interface} \)

\( Z_B = \text{elastoplastic hysteretic function of the sliding } u_{Bf} \) (Fig. 11), with \(|Z_B| \leq 1\)

\( u_{B(\gamma)} = \text{yield sliding displacement of } Z_B \) (Fig. 11)
For monotonic tensile loading \((Z_B > 0)\), \(F_{ws}\) and \(F_{Bn}\) should always be positive. Thus, from Eq. (7), the following condition for the inclination angle \(\theta\) of the barrel and the coefficient of friction \(\mu_B\) at the wedge-to-barrel interface is calculated.

\[
\sin \theta - \mu_B Z_B \cos \theta > 0 \Rightarrow \tan \theta > \mu_B
\]

This condition ensures that the anchorage hardware setup has the capacity of locking itself and remaining locked after removal of the tensile loading. Equations (1), (2), (5), and (6) form the constitutive model of the end springs in Fig. 9.

The response of the central spring (Fig. 9), which represents the monostrand, can be captured by a tension-only hysteretic spring that follows the smooth skeletal curve from Mattock.\(^{26}\)

\[
\sigma_{PT,sk} = E_{PT} \varepsilon_{PT} \left[ r_{PT} \left( 1 - r_{PT} \right) \left( 1 + \left( \frac{E_{PT} \varepsilon_{PT}}{K f_{py}} \right)^{\frac{l}{l}} \right) \right]
\]

where

- \(\sigma_{PT,sk}\) is the axial tensile stress of the skeletal curve obtained from monotonic loading
- \(\varepsilon_{PT}\) is the tensile strain of the monostrand
- \(r_{PT}\) is the ratio of the postelastic to elastic modulus of a monostrand
- \(K\) is the ratio of the actual yield stress to the nominal yield stress \(f_{py}\) of a monostrand
- \(R\) is the smoothness factor that controls the transition from the elastic to the inelastic range

The axial stress of the monostrand \(\sigma_{PT}\) is given by the solutions of the following constitutive equation in rate form:

\[
\dot{\sigma}_{PT} = E_{PT} \dot{\varepsilon}_{PT} \text{ with } \sigma_{PT} \leq \sigma_{PT,sk}
\]

where

- \(\dot{\sigma}_{PT}\) is the monostrand axial stress rate
- \(\dot{\varepsilon}_{PT}\) is the monostrand axial strain rate

The axial force \(F_{PT}\) and extension \(u_{PT}\) of the central spring are given as

\[
F_{PT} = \sigma_{PT} A_{PT}
\]

where

\[ A_{PT} = \text{current monostrand area incorporating effects of wire fracture} \]

\[ u_{PT} = \varepsilon_{PT} L_{PT} \]

where

\[ L_{PT} = \text{initial monostrand length} \]

**Incorporation of successive wire fracture effects**

Due to the fracture of monostrand wires, the area of the monostrand reduces. The current area \(A_{PT}\) of the monostrand depends on the maximum strain and can be obtained from Eq. (8).

\[
A_{PT} = \frac{N_w \left( \varepsilon_{PT,max} \right)}{7} A_{PT,o}
\]

where

\[ A_{PT,o} = \text{original monostrand area before fracture of wires initiates} \]

\[ N_w = \text{current number of available wires, which is a function of the maximum monostrand strain in the history of loading } \varepsilon_{PT,max} \]

Specific values of \(N_w\) are obtained from Table 4 or Fig. 8, assuming that the displacement at the anchorage hardware setups is much smaller than the extension of the monostrand.

This effect of successive wire fracture on the response of the anchorage setups is accounted for by modifying the lateral compressive force \(F_{ws}\) in Eq. (6), resulting in Eq. (9).

\[
F_{ws} = \left( 1 - r_{ws} \frac{A_{PT,o} - A_{PT}}{A_{PT,o}} \right) K_{wr} \left\{ 2 u_{ws} \right\}
\]
where

\[ r_{W_n} = \text{factor that represents the total reduction of } F_{W_n} \text{ if all} \]
\[ \text{wires fracture, with } 0 < r_{W_n} < 1 \]

The factor \( r_{W_n} \) never equals unity (complete loss of compression) because even after complete fracture of all wires, the wedge pieces remain tight and in contact with the remaining part of the broken wires (Fig. 7). Based on the experimental observations, it is assumed that all wires fracture at a single anchorage setup. Thus, Eq. (9) is used instead of Eq. (6) only for one of the two end springs.

### Solution strategy

Equilibrium requires that all springs have the same force, while compatibility requires that the total applied extension \( u_{ap} \) to the monostrand-anchor system equals the sum of the extensions of the individual springs (Fig. 9). These requirements result in the following set of nonlinear algebraic equations:

\[
\begin{align*}
F_{PT} &= F_{T}^{(1)} \\
F_{PT} &= F_{T}^{(2)} \\
F_{PT} &= F_{T}^{(1)} + F_{T}^{(2)} = u_{ap}
\end{align*}
\]

where

\[ F_{T}^{(1)} = \text{axial force of first end spring of the anchorage hardware setup} \]
\[ F_{T}^{(2)} = \text{axial force of second end spring of the anchorage hardware setup} \]
\[ u_{T}^{(1)} = \text{total displacement contributed to monostrand-anchor system by the first end spring} \]
\[ u_{T}^{(2)} = \text{total displacement contributed to monostrand-anchor system by the second end spring} \]

Only the first spring (having the superscript 1) uses Eq. (9) instead of Eq. (6). For a given applied extension \( u_{ap} \), a Newton-Raphson iterative scheme is employed to solve this system of equations.

### Comparison with experimental data

Table 5 presents the model parameters \( K, R, r_{PT}, \mu_B, u_{Bf,y}, K_{np}, \) and \( r_{W_n} \) that were calibrated using the experimental data from all tests. Calibration was performed by trial and error (rather than using an optimization scheme). The angle \( \theta \) was taken from the geometric properties of the barrels. The effects of wire fracture (Eq. [8] and [9]) were modeled using the mean fracture strains from Table 4, while the monostrand elastic modulus was assumed to be equal to 28,500 ksi (197,000 MPa). Figure 12 presents a comparison between experimental results and numerical predictions (using the values in Table 5) for the second test with the 0.5 in. (13 mm) diameter monostrand-anchor systems (identified as 05-2). The numerical and experimental force-versus-displacement responses were in good agreement until the first wire fractured (Fig. 12). The model then followed the general characteristics of the response
of structures and the corresponding seismic forces and displacement demands. Based on the experimental results obtained in this study (Fig. 6) for both the 0.5 in. (13 mm) and 0.6 in. (15 mm) diameter monostrand-anchor systems, the stress at fracture of the first wire considered for design $f_{pf}$ should not exceed the nominal yield stress $f_{py}$ $(0.9 f_{pu}$ or 243 ksi [1670 MPa]). The strain at fracture of the first wire considered for design $\varepsilon_{pf}$ should not exceed 1.5% (Fig. 7).

The loading stiffness of monostrand-anchor systems $K_{l}$ is given by Eq. (10).

$$K_l = \frac{K_{PT} K_{Aj}}{2K_{PT} + K_{Aj}}$$

where

$K_{PT}$ = stiffness of the individual monostrand = $E_{PT}A_{PT}/L_{PT}$

$K_{Aj}$ = loading stiffness of a single anchorage hardware setup

Based on Eq. (3), (4), (6), and (7), $K_{Aj}$ is computed:

$$K_{Aj} = \frac{4K_{w}}{\tan \theta} \left( \frac{\cos \theta + \mu_b \sin \theta}{\sin \theta - \mu_b \cos \theta} \right)$$

Using the calibrated values of Table 5, $K_{Aj}$ is computed to be 265 and 322 kip/in. (46,400 and 56,400 kN/m) for the anchorage hardware used with the 0.5 in. (13 mm) and 0.6 in. (15 mm) diameter monostrand-anchor systems, respectively. The unloading/reloading stiffness of the monostrand-anchor systems $K_{un}$ is given by an expression similar to that of Eq. (10) by substituting $K_{Aj}$ with $K_{un}$, where $K_{un}$ is the unloading/reloading stiffness of the anchorage hardware setup. Using Eq. (3), (4), (6), and (7), $K_{un}$ is computed:

Figure 12. A comparison between experimental and numerical results for second test with 0.5 in. diameter monostrand-anchor systems (identified as 05-2). Note: 1 in. = 25.4 mm; 1 kip = 4.448 kN.
0.5 in. and 0.6 in. diameter monostrand-anchor systems, respectively. The effect of the anchorage hardware on the loading and unloading/reloading stiffness of monostrand-anchor systems $K_{un}$ diminishes with the length of the monostrand, as demonstrated in Fig. 14, which shows the ratios of the loading and unloading/reloading stiffness over the stiffness of the anchor-free monostrand, $K_{l}/K_{PT}$ and $K_{un}/K_{PT}$, respectively. For practical applications, the effect of the anchorage hardware on the unloading/reloading stiffness is negligible for monostrands longer than 50 in. (1270 mm) and for both the 0.5 in. and 0.6 in. diameter monostrand-anchor systems because $K_{un}/K_{PT}$ is greater than 0.97 for both the 0.5 in. and 0.6 in. diameter monostrand-anchor systems. However, for monostrand-anchor systems

Unlike $K_{A,l}$, $K_{A,un}$ is not constant. Using the calibrated values in Table 5, an average value of $Z_B$ equal to 0.5, and $u_{bf}$ equal to 0.15 in. (3.8 mm) and 0.17 in. (4.3 mm) for the 0.5 in. (13 mm) and 0.6 in. (15 mm) diameter monostrand-anchor systems, respectively, estimates of the unloading/reloading stiffness $K_{A,un}$ were obtained. These estimates are $K_{A,un}$ of 6875 kip/in. (1204 MN/m) and 9554 kip/in. (1673 MN/m) for the anchorage hardware used with the 0.5 in. and 0.6 in. diameter monostrand-anchor systems, respectively. The effect of the anchorage hardware on the loading and unloading/reloading stiffness of monostrand-anchor systems $K_{un}$ diminishes with the length of the monostrand, as demonstrated in Fig. 14, which shows the ratios of the loading and unloading/reloading stiffness over the stiffness of the anchor-free monostrand, $K_{l}/K_{PT}$ and $K_{un}/K_{PT}$, respectively. For practical applications, the effect of the anchorage hardware on the unloading/reloading stiffness is negligible for monostrands longer than 50 in. (1270 mm) and for both the 0.5 in. and 0.6 in. diameter monostrand-anchor systems because $K_{un}/K_{PT}$ is greater than 0.97 for both the 0.5 in. and 0.6 in. diameter monostrand-anchor systems. However, for monostrand-anchor systems

\[
K_{A,un} = \frac{4K_{un}}{\tan \theta} \left[ \frac{\cos^2 \theta + \mu_B Z_B^2 \sin^2 \theta + \mu_B \left( \frac{u_{bf}}{u_{bf}} \right)}{\left( \sin \theta - \mu_B Z_B \cos \theta \right)^2} \right]
\]
shorter than 700 in. (18,000 mm), $K/K_{PT}$ does not exceed 0.95 (Fig. 14); therefore, the actual stiffness of the monostrand-anchor system, given by Eq. 10, should be considered for design.

**Conclusion**

This paper investigated the cyclic tensile response of monostrand-anchor systems composed of unbonded monostands with their anchorage hardware. Cyclic tensile testing was conducted for 0.5 in. (13 mm) and 0.6 in. (15 mm) diameter monostrand-anchor systems until all wires had fractured. All monostands were Grade 270 (1860 MPa), cold-drawn, seven-wire, low-relaxation cables. A simplified numerical model was formulated to capture the basic characteristics of the observed cyclic response until fracture of the monostands. The response of monostrand-anchor systems after fracture of the first wire provided information on the residual strength of monostrand-anchor systems, which is of interest for collapse analysis of structures incorporating unbonded posttensioning. Moreover, design implications associated with the effects of the anchorage hardware on the monostrand-anchor systems were identified. Major findings of this paper include the following:

- Failure of monostrand-anchor systems occurred prematurely in the form of fracture of a single monostand wire. Stresses at fracture of the first wire varied between the yield stress and the ultimate stress. The corresponding strains were in the range of 1.5% to 4%, much smaller than the fracture strain of 6% to 7% reported for anchor-free monostands. These observations are in agreement with the findings of previous investigations.

- After the first wire fractured, the remaining wires fractured one at a time with the applied extension (unlike the nearly simultaneous fracture of all wires of anchor-free monostands). The fracture of the last one or two wires occurred at strains exceeding 6% to 7%. Wire fractures resulted in sudden drops in the axial tensile resistance of monostrand-anchor systems.

- Premature wire fracture was driven by the localized contact/friction tractions imparted to the monostand by the wedge (in the vicinity of the anchorage hardware). As a result, outer wires always fractured first at the point of contact with the wedge. No wires fractured away from the anchorage hardware setups, while most wires fractured at a single anchorage hardware setup.

- The anchorage setups resulted in a loading stiffness of the monostrand-anchor systems that was different from their unloading/reloading stiffness. Both the loading stiffness and the unloading/reloading stiffness were smaller than the stiffness of the anchor-free monostand. This difference diminishes with the length of the monostand. For monostands exceeding 50 in. (1270 mm), the unloading/reloading stiffness exceeds 97% of the stiffness of the anchor-free monostand. However, the loading stiffness of monostrand-anchor systems exceeds 95% of the stiffness of the anchor-free monostands only for monostand lengths exceeding 700 in. (17,800 mm). Thus, for practical applications, the effect of the anchorage hardware on the unloading/reloading stiffness of monostrand-anchor systems can be neglected for monostrand-anchor systems longer than 50 in. However, the effect of the anchorage hardware on the loading stiffness of monostrand-anchor systems of length not exceeding 700 in. should be taken into account for monostands.

- The numerical model developed as part of this study satisfactorily captured the response of the monostrand-anchor system up to the fracture of the first wire. After the first wire fractured, the model captured the stepwise reduction of strength but with limited predictive capability of the strains at which successive wires fractured. The numerical model helped identify the effect of the anchorage setups on the response of the monostrand-anchor systems.

- The reduced strength and ductility of monostrand-anchor systems compared with anchor-free monostands should be considered in the design of systems incorporating unbonded posttensioning. Based on the experimental results generated in this study, the design of unbonded monostrand-anchor systems should take into account a fracture stress that does not exceed the yield strength (243 ksi [1670 MPa]) of the monostands and a fracture strain that does not exceed 1.5%. New anchorage hardware that alleviates stress/strain concentration at the monostrand wires should be developed so that the full strength and ductility of monostands is considered for design.

- The loading stiffness of each anchorage setup was estimated to be 265 kip/in. (46.4 MN/m) and 322 kip/in. (56.5 MN/m) for the 0.5 in. (13 mm) and 0.6 in. (15 mm) diameter monostrand-anchor systems, respectively. An average value of the unloading/reloading stiffness of each anchorage setup was estimated to be 6875 kip/in. (1204 MN/m) and 9554 kip/in. (1671 MN/m) for the 0.5 in and 0.6 in. diameter monostrand-anchor systems, respectively.

**Acknowledgments**

The authors would like to acknowledge the Federal Highway Administration of the U.S. Department of Transportation for providing funding for this research through the Multidisciplinary Center for Earthquake Engineering Research of the University at Buffalo–SUNY. The Bodossaki
Foundation and the Alexander S. Onassis Public Benefit Foundation are also acknowledged for providing partial financial support to the first author. The Structural Engineering and Earthquake Simulation Laboratory personnel at the University at Buffalo are also acknowledged for providing help and guidance throughout the experimental investigation. The findings of this study presented in this paper represent those of the authors and not the sponsors of the research.

References


**Notation**

- $A_{PT}$ = current monostrand area incorporating effects of wire fracture
- $A_{PT,o}$ = original monostrand area before fracture of wires
- $d_{bt}$ = interior diameter of barrel anchor at top end
- $d_{bb}$ = interior diameter of barrel anchor at bottom end
- $d_w$ = interior diameter of wedge
- $D_b$ = exterior diameter of barrel anchor
- $D_{wb}$ = exterior diameter of wedge at bottom end
- $D_{bt}$ = exterior diameter of wedge at top end
- $E_{PT}$ = elastic modulus of unbonded monostrand
- $f_{pf}$ = stress at fracture of first wire
- $f_{pu}$ = nominal ultimate strength of monostrand
- $f_{py}$ = nominal yield strength of monostrand
- $F_{BF}$ = tangential (friction) force at wedge-to-barrel interface
- $F_{BN}$ = normal force at wedge-to-barrel interface
- $F_{max}$ = maximum force of monostrand
- $F_{p,eff}$ = effective force of monostrand
- $F_{pu}$ = nominal ultimate force strength of monostrand
- $F_{py}$ = nominal yield force strength of monostrand
- $F_{PT}$ = monostrand axial force, referring to central spring of numerical model
- $F_T$ = axial force of monostrand in vicinity of anchorage hardware setup, refers to end springs of numerical model
- $F_T^{(3)}$ = axial force of first end spring of anchorage hardware setup
- $F_T^{(2)}$ = axial force of second end spring of anchorage hardware setup
- $F_{Bf}$ = tangential (friction) force at monostrand-to-wedge interface
- $F_{Bn}$ = normal force at monostrand-to-wedge interface
- $H_b$ = height of barrel anchor
- $H_w$ = height of wedge
- $K$ = ratio of actual yield stress to nominal yield stress of monostrand
- $K_l$ = loading stiffness of monostrand-anchor system
- $K_{A,l}$ = loading stiffness of single anchorage hardware setup
- $K_{A,un}$ = unloading/reloading stiffness of single anchorage hardware setup
- $K_{PT}$ = stiffness of individual monostrand
\[ K_u = \text{unloading/reloading stiffness of monostrand-anchor system} \]
\[ K_w = \text{lateral compressive stiffness of monostrand} \]
\[ L_o = \text{original anchor-to-anchor length of tested monostrand-anchor system} \]
\[ L_{o,c} = \text{corrected length of tested monostrand-anchor system accounting for seating and slippage at anchorage hardware} \]
\[ L_{PT} = \text{initial monostrand length, referring to central spring of numerical model} \]
\[ N_w = \text{current number of available (not fractured) wires} \]
\[ r_{PT} = \text{ratio of postelastic to elastic modulus of monostrand} \]
\[ r_w = \text{factor representing total reduction of } F_w \text{ if all wires fracture} \]
\[ R = \text{smoothness factor that controls transition from elastic to inelastic range} \]
\[ u_{ap} = \text{applied displacement history of monostrand-anchor system} \]
\[ u_{bf} = \text{relative sliding of wedge on barrel} \]
\[ u_{bf,y} = \text{yield sliding displacement of } Z_B \]
\[ u_{PT} = \text{monostrand axial extension, referring to central spring of numerical model} \]
\[ u_r = \text{total displacement contributed to monostrand-anchor system by a single anchorage hardware setup} \]
\[ u_r^{(1)} = \text{total displacement contributed to monostrand-anchor system by first end spring} \]
\[ u_r^{(2)} = \text{total displacement contributed to monostrand-anchor system by second end spring} \]
\[ u_{Wf} = \text{sliding of monostrand on wedge} \]
\[ u_{W0} = \text{horizontal displacement of wedge (positive when compressing strand)} \]
\[ u_y = \text{nominal yield extension of monostrand-anchor system} \]
\[ Z_B = \text{elastoplastic hysteretic function of sliding } u_{bf} \]
\[ \Delta d = \text{distance between the two parts of a wedge} \]
\[ \Delta L_s = \text{seating and slippage length} \]
\[ \Delta u = \text{displacement/extension increment during testing} \]
\[ \varepsilon = \text{strain of monostrand-anchor system} \]
\[ \varepsilon_{pf} = \text{strain at fracture of first wire} \]
\[ \varepsilon_{PT} = \text{tensile strain of monostrand} \]
\[ \varepsilon_{PT,max} = \text{maximum monostrand strain in history of loading} \]
\[ \dot{\varepsilon}_{PT} = \text{monostrand axial strain rate} \]
\[ \theta = \text{inclination angle of tapered interior surface of barrel} \]
\[ \mu_B = \text{coefficient of friction at wedge-to-barrel interface} \]
\[ \sigma_{PT} = \text{axial stress of monostrand} \]
\[ \sigma_{PT,sk} = \text{axial tensile stress of skeletal curve obtained from monotonic tensile loading of monostrand} \]
\[ \dot{\sigma}_{PT} = \text{monostrand axial stress rate} \]
**About the authors**

Petros Sideris, PhD, is an assistant professor in the Department of Civil, Environmental, and Architectural Engineering at the University of Colorado–Boulder.

Amjad J. Aref, PhD, is a professor in the Department of Civil, Structural, and Environmental Engineering at the University at Buffalo–SUNY.

Andre Filiatrault, PhD, Eng, is a professor in the Department of Civil, Structural, and Environmental Engineering at the University at Buffalo–SUNY.

**Abstract**

This paper presents an investigation of the cyclic tensile response of monostrand-anchor systems, that is, unbonded monostrands including their anchorage hardware. Cyclic tensile testing was conducted for 0.5 in. (13 mm) and 0.6 in. (15 mm) diameter monostrand-anchor systems to fracture of all wires. All monostrands were Grade 270 (1860 MPa), cold-drawn, seven-wire, low-relaxation cables. Major findings of this experimental study include the following: premature failure of monostrand-anchor systems (for strains in the range of 1.5% to 4%) in the form of fracture of a single wire; successive fracture of monostrand wires (one at a time) with the applied extension, as opposed to the nearly simultaneous fracture of all wires in anchor-free monostrands; and differences between the loading and unloading/reloading stiffness of monostrand-anchor systems due to the seating and sliding of the wedges of the anchorage hardware. A numerical model was developed that is capable of capturing the basic characteristics of the observed experimental response. The numerical results are in good agreement with the experimental data.

**Keywords**

Anchorage, cyclic testing, slippage, strain, unbonded monostrand, wire.

**Review policy**

This paper was reviewed in accordance with the Precast/Prestressed Concrete Institute’s peer-review process.

**Reader comments**

Please address and reader comments to journal@pci.org or Precast/Prestressed Concrete Institute, c/o PCI Journal, 200 W. Adams St., Suite 2100, Chicago, IL 60606.