A practical approach for finite-element modeling of transfer length in pretensioned, prestressed concrete members using end-slip methodology

J. Chris Carroll, Thomas E. Cousins, and Carin L. Roberts-Wollmann

The analysis and design of pretensioned, prestressed concrete members is becoming more complex with advances in material technology. In many cases, advanced forms of analyses are required, such as strut-and-tie models and numerical models, especially to predict member behavior in the end zone. Unfortunately, the computational costs associated with these advanced methods can quickly escalate. There is a need for a practical approach to accurately simulate the behavior of pretensioned, prestressed concrete members, including the effects of prestress transfer. This paper presents a modeling approach that accounts for slip between prestressing strands and the surrounding concrete within the transfer zone and accurately predicts member behavior based on specified transfer lengths.

The modeling approach uses a matrix of solid elements to represent the concrete truss elements for the prestressing strand and nonlinear springs for the interface between prestressing strand and concrete.

Comparison with experimental data confirms the practicality and accuracy of the model in predicting member behavior based on specified transfer lengths.

Background

Pretensioned, prestressed concrete members rely on precompression applied through internal bonded strands, which are tensioned before casting. The tensile force is transferred from the prestressing strand to the surrounding concrete.
Prestressing strands elongate in the direction of the applied load during tensioning, causing a reduction in the cross-sectional area due to the Poisson effect. The concrete is then placed in the formwork. When the strands are released, the now-hardened concrete restrains them from returning to their original length and cross section. As the strands retract, adhesion between the prestressing strands and surrounding concrete is broken. The strand tries to expand laterally, giving rise to circumferential stresses along its length. Mechanical interlock and friction due to the Hoyer effect generate bond stresses between the strand and the surrounding concrete. The distance required to transfer the force in the prestressing strand to the surrounding concrete is the transfer length, defined by the American Concrete Institute’s (ACI’s) *Building Code Requirements for Structural Concrete (ACI 318-11)* and Commentary (ACI 318R-11) as the distance over which the strand should be bonded to the concrete to develop the effective prestress in the prestressing steel. The effective prestress is assumed to vary linearly from zero at the end of the member to full development at the transfer length, where the strand ceases to slip and adhesion again contributes to the bond between the two materials.

ACI 318-11 and the American Association of State Highway and Transportation Officials (AASHTO) have similar criteria for calculating transfer length. In equations for the development length of prestressing strands, the first component accounts for the transfer length, while the second accounts for the flexural bond length. Equation (1) shows the expression for transfer length \( l_t \), used in the determination of development lengths for both ACI 318-11 and AASHTO LRFD *Bridge Design Specifications*,\(^3\) which was derived based on research performed in the late 1950s and early 1960s. Janney\(^1\) first concluded that the stress along a concrete prism was directly proportional to the tensile stresses in the pretensioned wire, though there was no strain compatibility within the transfer zone as a result of slip. Hanson and Kaar\(^4\) later found the average bond stress to be 400 psi (2.75 MPa), based on average transfer length measurements for 7/16 in. (6 mm), 1/2 in. (10 mm), and 3/8 in. (13 mm) diameter strands pretensioned to 150 ksi (1030 MPa). Based on an average bond stress of 400 psi, Eq. (2) was derived to estimate transfer lengths for 250 ksi (1720 MPa) stress-relieved strand. Subsequently, Janney\(^5\) found no significant increase in transfer length, regardless of a 26% higher jacking stress using 270 ksi (1860 MPa) strand. Thus Eq. (2) was simplified to Eq. (1)\(^6\) and was first implemented in the 1963 ACI *Building Code Require-

\[ l_t = \left( \frac{f_{se}}{3} \right) d_s \]  
\[ l_t = \left( \frac{f_{se}}{2.94} \right) d_s \]

In addition to the code provisions, a number of transfer length equations have been proposed over the course of the past 60 years,\(^7\)\(^--\)\(^22\) taking into consideration the stress in the strand, strand diameter, concrete strength, method of release, and vertical location of the strand. For the most part, transfer length studies have relied on measurements of concrete surface strains, which can be cumbersome and time-consuming. Although mechanical bond does resist the tendency of the strand to slip, some slip, which has been shown to be directly proportional to transfer length, occurs within the transfer length. Figure 1 shows the theoretical relationship of the strain in the concrete and steel with respect to the distance from the end of the member.

An alternative method for measuring transfer lengths is to measure the slip between the end of the strand and the end

![Figure 1. Concrete and steel strain distribution within transfer zone. Note: \( l_t \) = transfer length, \( \epsilon_c(x) \) = strain in concrete along transfer length, \( \epsilon_{co} \) = strain in concrete just after transfer, \( \epsilon_s(x) \) = strain in strand along transfer length, \( \epsilon_{so} \) = strain in strand just before transfer, \( \epsilon_{si} \) = strain in strand just after transfer.](image-url)
of the member. Assuming a linear distribution of strains (Fig. 1), the elastic shortening occurring from the end of the specimen to the end of the transfer length in the specimen itself equals the area under the curve for the strain in the concrete along the transfer length $\varepsilon_c(x)$ calculated by Eq. (3), while the shortening of the strand equals the area between the curve for the strain in the strand along the transfer length $\varepsilon_s(x)$ and the initial steel-strain plateau calculated by Eq. (4). The end slip $L_{es}$ is equal to the difference between the shortening of the strand within the transfer zone (sum of elastic shortening and end slip) $\delta_s$ and the elastic shortening of concrete within the transfer zone $\delta_c$ (Eq. [5]) as derived by Cousins et al.\textsuperscript{23} based on the assumption of linear variations for both concrete and steel strains. By substituting the sum of $\varepsilon_{so}$ and $\varepsilon_{cs}$ (where $\varepsilon_{so}$ is the strain in the concrete just after transfer, and $\varepsilon_{cs}$ is the strain in the strand just after transfer) for the strain in the strand just before transfer $\varepsilon_s$, along with the stress in the strand just after transfer $f_{so}$ and modulus of elasticity of the strand $E_{so}$, for the steel strain $\varepsilon_{so}$ into Eq. (5), the transfer length can be solved by Eq. (6).

$$\delta_c = \int_0^l \varepsilon_c(x) \, dx = \frac{\varepsilon_{so} l}{2}$$  \hspace{2cm} \text{Eq. (3)}

$$\delta_s = \varepsilon_{so} l - \int_0^l \varepsilon_s(x) \, dx = \varepsilon_{so} l - \frac{\varepsilon_{cs} l}{2}$$  \hspace{2cm} \text{Eq. (4)}

$$L_{es} = \varepsilon_{so} l - \frac{\varepsilon_{cs} l}{2} - \frac{\varepsilon_{so} l}{2}$$  \hspace{2cm} \text{Eq. (5)}

$$l_i = \frac{2L_{es} E_{so}}{f_{so} + E_{so} \varepsilon_{so}}$$  \hspace{2cm} \text{Eq. (6)}

Guyon\textsuperscript{24} derived Eq. (7), accounting for the possibility of a constant bond stress distribution (linear strain distribution) or a linear bond stress distribution (parabolic strain distribution), to calculate end slip. Equation (7) includes a factor $\alpha$ that varies with bond stress distribution. With a constant bond stress distribution, $\alpha$ equals 2 and the calculated transfer length will equal that calculated by Eq. (6). However, a linear bond stress distribution results in $\alpha$ equal to 3. Studies by Martí-Vargas\textsuperscript{25} and Balazs\textsuperscript{26} found $\alpha$ to be 2.44 and $2/(1 - b)$, respectively, where $b$ is the power of the bond stress relationship.

$$l_i = \frac{\varepsilon_{so} l}{\varepsilon_{si}}$$  \hspace{2cm} \text{Eq. (7)}

Transfer length is an important aspect of pretensioned, prestressed concrete, especially with respect to shear design and serviceability requirements within end-zone regions at transfer. Both ACI 318-11 and AASHTO LRFD specifications include provisions for shear design but give less guidance for end-zone analysis. Service stress limits are provided for the ends and other locations throughout a member, but the method of analysis is determined by the complexity of the end zone. In most cases, simple bending stress calculations are sufficient; in others, more advanced techniques such as strut-and-tie models, neural networks, and even numerical models may be required. Neural networks, for example, can provide accurate results but can be complex.\textsuperscript{27–29} Likewise, numerical modeling can also provide accurate results but can be somewhat cumbersome, and the complexity can increase almost exponentially with the slightest modifications.

In many cases, numerical models of pretensioned, prestressed concrete members neglect the effects of slip within the transfer length. Two common approaches apply the prestressing force throughout the entire member length.\textsuperscript{30} The first method considers equivalent loads applied externally to the member, while the second uses steel segments perfectly bonded to the member combined with the application of initial strains. While these approaches may be sufficient in some applications, they fail to account for slip between the concrete and prestressing strands, a relationship sometimes needing consideration for model accuracy. Kannel et al.\textsuperscript{31} used numerical models to simulate the effects of unreleased strands on girders during transfer. The numerical models in the study used solid elements to represent the concrete and truss elements to represent the prestressing strands and applied the prestressing force using two methods: a ramped area and approximate rigid-plastic springs, both accounting for the transfer length. Ayoub and Filippou\textsuperscript{32} and Arab et al.\textsuperscript{33} also used numerical models to predict the behavior of pretensioned, prestressed concrete members accounting for transfer length. Ayoub and Filippou used a combination of beam-column, tendon, and bond elements to represent the behavior of the concrete, prestressing steel, and transfer of forces, respectively. The modeling technique used an initial strain applied to the tendon element to simulate the prestressing force, while the bond elements were based on the bond stress-slip relationship by Eligehausen et al.\textsuperscript{34} for anchored reinforcing bars. Like Kannel et al., Arab et al. used solid elements to represent the concrete. The transfer of the prestressing force was simulated using two methods: an extrusion technique or friction-based model and an embedment technique using strain compatibility, both also accounting for transfer length. Each of the three studies accounting for transfer length compares well with experimental results.

As designs continue to incorporate stronger, more advanced materials, more advanced analytical methods are becoming necessary. However, the associated costs and complexity cannot outweigh the benefits of such methods. Numerical analysis of pretensioned, prestressed concrete members can accurately predict their behavior, but the
The smallest of changes can increase the cost and complexity exponentially. Advanced methods of analysis are needed to predict the behavior of pretensioned, prestressed concrete members accounting for bond slip; more important, a practical method of advanced analysis is needed.

Test specimens

This study includes data from two types of test specimens: T beams and top-strand blocks (Fig. 2 and 3), respectively. The T beams were 24 ft (7.3 m) long and had three different cross sections corresponding to the size of prestressing strand used. The small, medium, and large beams each contained 1/2 in. (13 mm) diameter regular, 1/4 in. diameter super, and 0.6 in. (15 mm) diameter strands, respectively. In each cross section, three strands were placed 2 in. (50 mm) from the bottom of the formwork with a lateral center-to-center spacing of 2 in. The size and shape of the cross section were also influenced by the desired tensile strain in the strand anticipated to occur at the ultimate flexural capacity of the member. Tests have shown the development length of prestressing strands to be dependent on the strain in the strand at the time of failure. Thus as recommended by Buckner, the cross sections were designed such that the strain at ultimate flexural capacity would exceed the minimum required elongation of 3.5%. Taking into consideration the strand size and desired flexural behavior at failure, the T beam test specimens used are considered to be representative of other shapes and sizes of pretensioned, prestressed concrete beams.

The top-strand block test specimens were 12 ft (3.66 m) long and also had three different cross sections, which were used to evaluate the effect of vertical position on transfer length. While 1/2 in. (13 mm) diameter regular and 1/4 in. diameter super strands were both used in various groups of the top-strand blocks, each individual set of blocks cast always included the same size strand in the

Figure 2. T beam cross sections. Note: 1 in. = 25.4 mm.

Figure 3. Top-strand blocks. Note: 1 in. = 25.4 mm.
three cross sections. The sizes of the blocks were selected based on the desired comparisons for strand vertical location but were selected with surface strain magnitudes taken into consideration. While six different cross sections were used to obtain the transfer length and end-slip data, only the small and medium T beams and the 4 × 4 in. (100 × 100 mm) prism were modeled and used for the comparisons between numerical models and experimental results presented in this paper.

### End-slip reliability

A total of 119 transfer lengths were calculated based on concrete surface strains, while 57 of the 119 were also estimated using end-slip measurements. Concrete surface strains are typically measured using a demountable mechanical (DEMEC) strain gauge and surface-mounted gauge points. The DEMEC gauge has a gauge length of 8 in. (200 mm), and the gauge points are approximately 1/4 in. (6 mm) in diameter with a small, fine point indentation in the center. The points were placed on the test specimens at the elevation of the strands at spacings of 2 in. (50 mm) and 4 in. (100 mm) and attached using a five-minute epoxy. A spacing of 2 in. was used in areas expected to be within the anticipated transfer zone, ensuring a defined ascending branch of the strain plot, while the remaining points located beyond the anticipated transfer zone, corresponding to the strain plateau, were spaced at 4 in. Before transfer, initial measurements were obtained by taking the average of three successive measurements. Immediately after transfer, the gauge points were measured once more. The difference between the two readings at any one location provides the change in length from the point at which zero prestress force is applied to the point at which the entire prestress force is applied. The change in length is then divided by the gauge length, resulting in the average strain across the gauge length for that location. Based on each set of concrete surface strains, strain profiles were created for each transfer zone and the 95% average maximum strain of concrete surface strains, while 57 of the 119 were also estimated using end-slip measurements. Concrete surface strains are typically measured using a demountable mechanical (DEMEC) strain gauge and surface-mounted gauge points. The DEMEC gauge has a gauge length of 8 in. (200 mm), and the gauge points are approximately 1/4 in. (6 mm) in diameter with a small, fine point indentation in the center. The points were placed on the test specimens at the elevation of the strands at spacings of 2 in. (50 mm) and 4 in. (100 mm) and attached using a five-minute epoxy. A spacing of 2 in. was used in areas expected to be within the anticipated transfer zone, ensuring a defined ascending branch of the strain plot, while the remaining points located beyond the anticipated transfer zone, corresponding to the strain plateau, were spaced at 4 in. Before transfer, initial measurements were obtained by taking the average of three successive measurements. Immediately after transfer, the gauge points were measured once more. The difference between the two readings at any one location provides the change in length from the point at which zero prestress force is applied to the point at which the entire prestress force is applied. The change in length is then divided by the gauge length, resulting in the average strain across the gauge length for that location. Based on each set of concrete surface strains, strain profiles were created for each transfer zone and the 95% average maximum strain method was used to estimate the transfer lengths.35

The end-slip measurements were taken at the dead end of the test specimens using a depth micrometer and U brackets attached directly to the strands. The U brackets were placed approximately 1 in. (25 mm) from the face of the test specimen on each strand with a hose clamp providing a stationary reference point for measurements before and after transfer. Measurements were not taken at the live end of the test specimens as a result of the U brackets bending and hose clamps breaking during transfer. A small aluminum flat bar was also embedded in the concrete during casting to provide a smooth, flat surface for measuring. As with the DEMEC points, three initial measurements were taken before transfer and were averaged to ensure an accurate reference point. Following transfer, measurements were again taken with the end slip equal to the distance between the initial and final measurements minus the elastic shortening of the protruding strand. There is an approximately linear relationship between the transfer lengths calculated from concrete surface strains and their corresponding end slips (Fig. 4). Furthermore, transfer lengths were calculated from end-slip measurements using Eq. (6) and compared with the transfer lengths calculated from concrete surface strains. Figure 5 shows the relationship of transfer lengths calculated using concrete surface strains with respect to transfer lengths calculated using Eq. (6), showing a good correlation. Eq. (6) was used for the development of the numerical models in this study because of its theoretical accuracy.

### Numerical modeling approach

The analytical models presented in this study are constructed using GT STRUDL structural design and analysis software. More complex finite element packages have existing contact elements, but the goal of this modeling approach is to limit model complexity. Each model comprises three components:

- a matrix of solid brick elements representing the concrete
- truss elements representing the prestressing strands
- nonlinear spring elements accounting for the interface between the prestressing strands and the surrounding concrete

Both the solid elements and truss elements use a linear elastic material and are assumed to remain elastic. The nonlinear springs have a bilinear force-versus-displacement relationship and are used to account for the slip between strand and concrete. The solid elements and truss elements are not directly connected. Instead, a series of duplicate joints is generated along the line corresponding to the desired location of the prestressing strand(s), to which the truss elements are attached. Joint constraints are applied to the duplicate joints, restraining their movement in all directions to that of their counterpart except in the direction of the truss elements. In the direction of the truss elements, the movement of the duplicate joints relative to their counterpart is controlled by the nonlinear spring elements. Contrary to the modeling techniques used in other studies, the force-versus-displacement curves for the nonlinear spring elements are based solely on the relationship between end slip and transfer length.

The effective prestress is assumed to vary linearly from zero at the end of the member to full development at the transfer length, where the strand ceases to slip. However, the transfer length would be better represented by two regions34 (Fig. 6), a plastic zone and a much shorter elastic zone as defined by Cousins et al.11 Within the plastic zone, the stress in the strand increases with a linear variation.
Figure 4. Correlation of transfer lengths from concrete surface strains with end-slip measurements. Note: $R^2$ = coefficient of determination.

$y = 1.01x$

$R^2 = 0.78$

Figure 5. Correlation of transfer lengths from concrete surface strains with transfer lengths calculated using Eq. (6). Note: $E_{ps}$ = modulus of elasticity of prestressing strands; $f_{so}$ = stress in strand just after transfer; $L_{es}$ = end slip; $R^2$ = coefficient of determination; $\varepsilon_{co}$ = strain in concrete just after transfer.
transfer length (boxed region in Fig. 8), the rate of force increase in the strand varied with respect to the percentage of slip. Similar to the 95% average maximum strain method, to accurately model the application of prestress, at least 95% of the prestress force should be developed in the strand before reaching the specified transfer length. Therefore, of the four percentages evaluated, a slip equal to 2.5% of the total end slip consistently allows for the force in the strand to reach 95% of its maximum before the specified transfer length (Fig. 9).

Following are the steps used to generate the nonlinear spring curves for a 4 × 4 in. (100 × 100 mm) prism.

1. Identify the jacking stress $f_{sj}$, typically taken as 0.75$f_{pu}$, where $f_{pu}$ is the ultimate tensile strength of the strand.
2. Estimate loss of prestress due to relaxation $RET$ (include shrinkage if not moist cured).
3. Calculate the stress in the strand just after transfer $f_{si}$.
4. Estimate loss of prestress due to elastic shortening $ES$.

Within this region, a gradual buildup of force occurs along the interface between the prestressing strand and the surrounding concrete, creating a constant bond stress. Before the force in the strand reaches its maximum, the increase in strand stress must gradually decrease to zero, likewise the bond stress, occurring within the elastic zone.

As previously discussed, prestressing strands slip during transfer throughout the transfer length, tapering to zero at the end. Thus, a bond-slip relationship can be determined to represent the interaction between the strand and surrounding concrete. This study creates a bond-slip relationship based on the assumed bond stresses occurring in the plastic and elastic zones. Within the plastic zone the bond stress is constant; therefore, the bond-slip relationship is also constant. Likewise, within the elastic zone, the bond stress and the bond-slip relationship both taper to zero. Because the numerical modeling approach used in this study uses nonlinear springs to represent the bond-slip behavior, a force-versus-slip curve is needed rather than a bond-versus-slip curve. Each nonlinear spring is a bilinear curve (Fig. 7). The plateau of each curve is determined by multiplying the stress in the strand just after transfer (including losses due to elastic shortening) by its area to obtain the maximum force. The rate of force increase in the strand is then determined by dividing the maximum force by a specific transfer length, resulting in units of kip/in. (kN/mm), which is then multiplied by the desired nonlinear spring spacing converting the units back to kip (kN). Therefore, the force in the strand will increase at a constant rate for slip values between $0.025L_{es}$ and $L_{es}$.

An iterative process was used to determine the slip ($0.025L_{es}$) at the intersection between the ascending portion and the plateau of the nonlinear spring curves. Various percentages of slip were used in the models, and the strand force was plotted with respect to the distance from the end of the member (Fig. 8). In the transition zone around the transfer length (boxed region in Fig. 8), the rate of force increase in the strand varied with respect to the percentage of slip. Similar to the 95% average maximum strain method, to accurately model the application of prestress, at least 95% of the prestress force should be developed in the strand before reaching the specified transfer length. Therefore, of the four percentages evaluated, a slip equal to 2.5% of the total end slip consistently allows for the force in the strand to reach 95% of its maximum before the specified transfer length (Fig. 9).

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2. Estimate loss of prestress due to relaxation $RET$ (include shrinkage if not moist cured).
3. Calculate the stress in the strand just after transfer $f_{si}$.
4. Estimate loss of prestress due to elastic shortening $ES$.

$$RET = f_{sj} \left( \frac{\log 24t - \log 24t_1}{45} \right) \left( \frac{f_{py}}{f_{py} - 0.55} \right)$$

where

$t = \text{time at end of time step}$

$t_1 = \text{time at beginning of time step}$

$f_{py} = \text{specified yield strength of the strand}$

$$ES = \frac{(f_{si} - ES_{assumed})A_{ps}}{A_{e}} \left[ \frac{E_{ps}}{E_{es}} \right]$$

where
Figure 8. Nonlinear spring calibration curves. Note: $l_t$ = transfer length.

Figure 9. Detailed view of transition zone of nonlinear spring calibration curves. Note: $l_t$ = transfer length.


\[ ES_{\text{assumed}} = \text{assumed loss of prestress due to elastic shortening} \]

\[ A_{ps} = \text{area of prestressing strand} \]

5. Calculate the stress in the strand just after transfer \( f_{so} \).

6. Calculate the stress in the concrete at the elevation of the steel due to the prestress force only \( f_{cgs} \).

7. Estimate end slip using specified transfer length using Eq. (8).

\[ I_{es} = \frac{l_t}{2E_{ps}} \left( f_{so} + \frac{E_{ps}}{E_{ci}} f_{cgs} \right) \]

where

\[ E_{ci} = \text{modulus of elasticity of concrete at time of transfer} \]

8. Develop a force-versus-displacement curve.

The prestress force is simulated by a change in temperature applied to the truss elements, resulting in an equivalent strain equal to that generated by the desired prestress force. While a temperature load is used, other programs may also allow for the use of an initial strain condition. In either case, the desired loading is based on the stress in the strand immediately before transfer, while losses due to elastic shortening are accounted for by deformation of the solid elements. Equations (9) and (10) are used to calculate the equivalent temperature load or strain, respectively. To further reduce computation time, symmetry was used at the midspan of each model and for the T beam models an additional plane of symmetry was used throughout their length.

\[ \Delta T_{\text{equiv}} = \frac{f_{so}}{E_{ps} \alpha_{\text{steel}}} \]

where

\[ \Delta T_{\text{equiv}} = \text{equivalent change in temperature in prestressing strand} \]

\[ \alpha_{\text{steel}} = \text{coefficient of thermal expansion for prestressing strand} \]

\[ e_{\text{equiv}} = \frac{f_{so}}{E_{ps}} \]

where

\[ e_{\text{equiv}} = \text{equivalent strain in prestressing strand} \]

**Numerical model verification**

Two models were created for verification before creating the full models and comparing with experimental data (Table 1). The first model was a 4 × 4 in. (100 × 100 mm) prism, 12 ft (3.7 m) long with a concentric prestress force. The second model was a 6 in. (150 mm) wide × 12 in. (300 mm) deep rectangular beam, 12 ft (3.7 m) long with an eccentric prestress force applied 4 in. below the neutral axis. The compressive strength of the concrete was assumed to be 4000 psi (287 MPa). The modulus of elasticity of the prestressing strands was assumed to be 28,500 ksi (196 GPa). Each model contained one \( \frac{1}{2} \) in. (13 mm) diameter, 270 ksi (1860 MPa), low-relaxation prestressing strand, initially stressed to 0.75 \( f_{pu} \) and having a transfer length of 25 in. (635 mm). For each model, a 7-day moist cure was assumed along with relaxation losses occurring over the same period, similar to the casting conditions of the test specimens included in this study. Furthermore, symmetry was used at the midspan of each model to reduce computation time.

Axial displacement, camber (beam only), strand force development, transfer lengths, and the maximum concrete surface strains were compared with calculated values. For the calculated values, the force in the strand was assumed to vary linearly from zero to the end of the specified transfer length and remain constant thereafter, including losses due to elastic shortening. Each displacement was calculated using virtual work. The longitudinal displacement due to application of prestress force \( \delta_p \) for the prism and beam were calculated using Eq. (11) and (12), respectively, while the camber due to application of prestress force \( \delta_c \) of the beam was calculated using Eq. (13). The transfer length was determined based on the distance from the end of the member for 95% of the force in the strand to be developed. Furthermore, the maximum strain was also calculated for each model. For initial comparisons, the self-weight was not taken into consideration, and the values in Table 1 only include the effects due to the application of prestress.

**Table 1. Numerical model verification summary**

<table>
<thead>
<tr>
<th>Model</th>
<th>Axial displacement, in. Calculation</th>
<th>Analysis</th>
<th>Camber, in. Calculation</th>
<th>Analysis</th>
<th>Strand force, kip Calculation</th>
<th>Analysis</th>
<th>Maximum strain, 10^-6 Calculation</th>
<th>Analysis</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prism</td>
<td>0.0289</td>
<td>0.0300</td>
<td>n/a</td>
<td>n/a</td>
<td>28.4</td>
<td>28.4</td>
<td>492</td>
<td>493</td>
</tr>
<tr>
<td>Beam</td>
<td>0.0157</td>
<td>0.0164</td>
<td>0.0939</td>
<td>0.0936</td>
<td>29.4</td>
<td>29.4</td>
<td>264</td>
<td>264</td>
</tr>
</tbody>
</table>

Note: n/a = not applicable. 1 kip = 4.45 kN; 1 in. = 25.4 mm.
where

\[ P_0 = \text{prestress force just after transfer} \]

\[ x = \text{distance from each member} \]

\[ A_g = \text{gross cross-sectional area} \]

\[ L = \text{length of numerical model} \]

\[ \delta_x = \int_0^l \frac{P_0 x}{l_t A_g E_{ct}} \, dx + \int_0^l \frac{P_0}{l_t A_g E_{ct}} \, dx + \frac{P_0 l_t}{2 A_g E_{ct}} + \frac{P_0 (L - l_t)}{A_g E_{ct}} \]  

\[ = \frac{P_0 l_t}{2 A_g E_{ct}} + \frac{P_0 l_t e^2}{2 E_{cs} I_g} + \frac{P_0 (L - l_t)}{A_g E_{ct}} + \frac{P_0 (L - l_t) e^2}{E_{cs} I_g} \]  

which

\[ e = \text{strand eccentricity} \]

\[ I_g = \text{gross moment of inertia} \]
based on the measured transfer lengths and end-slip values calculated using Eq. (8). Each model also used the measured concrete compressive strength from the day of transfer and the material properties for the prestressing strands provided by the manufacturer. A nonlinear analysis was performed for each model, most of which proved sensitive to the convergence tolerance. Figure 12 shows a typical T beam with camber and distribution of longitudinal stresses resulting from the applied prestress and self-weight using the modeling approach presented in this study. Also, Table 2 shows a summary of the results from each of the numerical models, including comparisons for end slip, transfer length, and maximum concrete surface strains.

The analytical end-slip values between the prestressing strands and surrounding concrete were calculated by the difference of the longitudinal displacements between the duplicate joints and their counterparts located at the ends of the test specimens. Figure 13 compares the experimental end-slip values with the analytical end-slip values based on the numerical models. The data show a good correlation between calculated and experimental values, with a coefficient of determination $R^2$ of 0.84 and a slope of nearly 1 for the line of regression. Likewise, the transfer lengths were calculated based on the distance from the end...
approaching the calculated maximum around the transfer length. The maximum value of force developed in the strand for each model had a difference of less than 0.1% compared with the calculated force in the strand, including the losses due to elastic shortening. It was equally accurate when the self-weight was applied and the force in the strand slightly increased. To evaluate the force increase with respect to transfer length, the force in the strand for each model was normalized with respect to its maximum and plotted along with the normalized experimental concrete surface strains.

Figures 15 and 16 show typical comparative plots for a prism model and T beam model, respectively. Figure 15 shows the strand force and surface of the member required for the force in the strand to reach 95% of the maximum. Figure 14 compares the analytical and experimental transfer lengths. The data again showed an almost perfect correlation, with an $R^2$ of 1.00 and a slope of 1 for the line of regression. Such close correlation verifies the ability of the model to consistently produce the user-specified transfer lengths by simple modifications to nonlinear spring curves.

The force increase in the strand was also evaluated for each model. The force in the strand increased at a rate equal to the plateau of the nonlinear spring force–versus–displacement curve for each model and began to level off, approaching the calculated maximum around the transfer length. The maximum value of force developed in the strand for each model had a difference of less than 0.1% compared with the calculated force in the strand, including the losses due to elastic shortening. It was equally accurate when the self-weight was applied and the force in the strand slightly increased. To evaluate the force increase with respect to transfer length, the force in the strand for each model was normalized with respect to its maximum and plotted along with the normalized experimental concrete surface strains. Figures 15 and 16 show typical comparative plots for a prism model and T beam model, respectively. Figure 15 shows the strand force and surface strains.

### Table 2. Summary of results

<table>
<thead>
<tr>
<th>Specimen</th>
<th>End slip, in.</th>
<th>Transfer length, in.</th>
<th>Maximum surface strain, 10⁻⁶</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Analysis</td>
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Note: 1 in. = 25.4 mm.
**Figure 13.** Correlation between experimental end-slip values and analytical end-slip values from numerical models.

\[ y = 0.99x \]
\[ R^2 = 0.84 \]

**Figure 14.** Correlation between transfer lengths from concrete surface strain measurements with analytical transfer length values from numerical models. 

Note: \( R^2 = \) coefficient of determination.
strains for a prism specimen to increase at the same relative rate and level off around the transfer length as expected. Likewise, Fig. 16 shows the strand force and surface strains for a T beam specimen to increase at the same relative rate and also level off around the transfer length. The only observable difference was the slight increase in concrete surface strains just beyond the transfer length, which is typical in prestressed concrete beams. Figures 15 and 16 also show the force in the strand to develop at least 95% of the maximum calculated value at a distance from the end of the member less than or equal to the specified transfer length.

Figure 17 compares the surface strains at the elevation of the strand with the experimental surface strains for a prism model. The analytical values increased within the transfer zone and began to level off near the transfer length, approaching the calculated maximum strain. While the experimental values also increased within the transfer zone and leveled off near the transfer length, the maximum strain was approximately 20% less than the calculated maximum strain. Figure 18 compares the surface strains for a T beam model. In this case, the values again increased within the transfer zone and began to level off near the transfer length. However, the strains exceeded the calculated maximum before gradually decreasing to the calculated maximum, a typical trend in prestressed concrete beams. The experimental values also ascended above the average maximum and then descended but are still approximately 15% to 20% higher than the values obtained from the model. In both models the strain profiles were similar with variations between the maximums, which is likely due to variations in the concrete modulus of elasticity of the numerical models versus that of the experimental test specimens. The modulus of elasticity was estimated from the measured concrete strengths using the ACI 318-11 provisions.

**Conclusion**

A practical modeling approach is presented for finite element analysis of pretensioned, prestressed concrete members accounting for the transfer of prestress. The modeling approach uses existing knowledge of the relationship between transfer length and end slip, creating a force-versus-displacement curve representing the interface between the prestressing strand and surrounding concrete. The prestress force is simulated using equivalent temperature loads, and the materials are assumed to remain elastic. Twenty-three analytical models were constructed and compared with experimental data. The model accounts for bond-slip behavior and compares well with the experimental data for end slip, strand force development, transfer length, and concrete surface strains. The comparisons confirm the practicality and accuracy of the model.
Acknowledgments

The authors gratefully acknowledge the support provided by the Federal Highway Administration Innovative Bridge Research and Deployment Program and the Virginia Center for Transportation Innovation and Research. The authors express their gratitude to the technical staff of the Structural Engineering and Materials Laboratory at Virginia Polytechnic Institute and State University for their assistance in the preparation and testing of the specimens. The authors also express their thanks to Michael Swanger for his assistance during model development.

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Figure 17. Comparison of experimental concrete surface strains and analytical concrete surface strains for a typical prism model. Note: $l_t$ = transfer length.

Figure 18. Comparison of experimental concrete surface strains and analytical concrete surface strains for a typical T beam model. Note: $l_t$ = transfer length.


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**Notation**

- $A_g$ = gross cross-sectional area
- $A_{ps}$ = area of prestressing strand
- $b$ = power of bond stress relationship
- $d_b$ = strand diameter
- $e$ = strand eccentricity
- $E_{ci}$ = modulus of elasticity of concrete at transfer
- $E_{ps}$ = modulus of elasticity of prestressing strands
- $ES$ = loss of prestress due to elastic shortening
- $ES_{assumed}$ = assumed loss of prestress due to elastic shortening
- $f'_{cl}$ = concrete compressive strength at transfer
- $f_{cgs}$ = stress in concrete at elevation of strands due to prestress
- $f_{pu}$ = ultimate tensile strength of strand
- $f_{py}$ = specified yield strength of strand
- $f_{se}$ = effective prestress in strand after all losses
- $f_{si}$ = stress in strand just before transfer
- $f_{sj}$ = jacking stress
- $f_{so}$ = stress in strand just after transfer
- $I_g$ = gross moment of inertia
- $l_t$ = transfer length
- $L$ = length of numerical model
- $L_{es}$ = end slip
- $P_o$ = prestress force just after transfer
- $R^2$ = coefficient of determination
- $RET$ = loss of prestress due to relaxation
- $s$ = nonlinear spring spacing
- $t$ = time at end of time step
- $t_i$ = time at beginning of time step
- $U_i$ = plastic transfer bond stress
- $x$ = distance from end of member
- $\alpha$ = bond stress distribution factor
- $\alpha_{steel}$ = coefficient of thermal expansion for prestressing strand
- $\delta_c$ = elastic shortening of concrete within transfer zone
- $\delta_s$ = sum of elastic shortening and end slip
- $\delta_t$ = longitudinal displacement due to application of prestress force
- $\delta_y$ = camber due to application of prestress force
- $\Delta T_{equiv}$ = equivalent change in temperature in prestressing strand
- $\varepsilon_c(x)$ = strain in concrete along transfer length
- $\varepsilon_{c,o}$ = strain in concrete just after transfer
- $\varepsilon_{equiv}$ = equivalent strain in prestressing strand
- $\varepsilon_{f,(x)}$ = strain in strand along transfer length
- $\varepsilon_{ij}$ = strain in strand just before transfer
- $\varepsilon_{so}$ = strain in strand just after transfer
Nonlinear spring force-versus-displacement curve and equivalent load calculation example

Given:

4 × 4 in. (100 × 100 mm) concrete prism

Concrete compressive strength at transfer \( f_{ct} = 5700 \) psi (39.3 MPa)

\[ E_{ct} = 57 \sqrt{f_{ct}} = 57 \sqrt{5700} = 4303 \text{ ksi (29.67 MPa)} \]

\[ E_{ps} = 28,500 \text{ ksi (196,500 MPa)} \]

\[ \alpha_{steel} = 6.5 \times 10^{-6} \text{ in./in.}/°F (1.17 \times 10^{-5} \text{ mm/mm}/°C) \]

\[ A_s = 16 \text{ in.}^2 (10,300 \text{ mm}^2) \]

\[ A_{ps} = 0.153 \text{ in.}^2 (98.7 \text{ mm}^2) \]

\[ l_t = 19.59 \text{ in.} (497.6 \text{ mm}) \]

\[ ES_{assumed} = 11.89 \text{ ksi} \]

nonlinear spring spacing \( s = 1 \text{ in.} (25.4 \text{ mm}) \)

1. Identify jacking stress \( f_{sj} \), typically taken as \( 0.75f_{pu} \)

\[ f_{sj} = (0.75)(270) = 202.5 \text{ ksi (1396 MPa)} \]

2. Estimate loss of prestress due to relaxation (include shrinkage if not moist cured)

\[ RET = f_s \left( \frac{\log 24t - \log 24(1/24)}{45} \right) \left( \frac{f_s}{f_{ps}} - 0.55 \right) \]

\[ = 202.5 \left( \frac{\log 24(7) - \log 24(1/24)}{45} \right) \left( \frac{202.5}{243} - 0.55 \right) \]

\[ = 2.84 \text{ ksi (20 MPa)} \]

3. Calculate stress in strand just before transfer \( f_{si} \)

\[ f_{si} = 202.5 - 2.84 = 199.66 \text{ ksi (1377 MPa)} \]

4. Estimate loss of prestress due to elastic shortening

\[ ES = \left[ \frac{(f_{si} - ES_{assumed}) A_{ps}}{A_s} \right] \left[ \frac{E_{ps}}{E_{ci}} \right] \]

5. Calculate stress in strand just after transfer \( f_{so} \)

\[ f_{so} = 199.66 - 11.89 = 187.77 \text{ ksi (1295 MPa)} \]

6. Calculate stress in concrete at elevation of steel due to prestress force only \( f_{cgs} \)

\[ f_{cgs} = \frac{f_{ps} A_{ps}}{A_s} = \frac{(187.77)(0.153)}{16} \]

\[ = 1.80 \text{ ksi (12.4 MPa)} \]

7. Estimate end slip using transfer length

\[ L_{es} = \frac{l_t}{2E_{ps}} \left( f_{ps} + \frac{E_{ps}}{E_{ci}} f_{cgs} \right) \]

\[ = \frac{19.59}{2(28,500)} \left[ 187.77 + \left( \frac{28,500}{4303} \right)(1.80) \right] \]

\[ = 0.068631 \text{ in. (1.743 mm)} \]

8. Develop force-versus-displacement curve

\[ f_{ms} A_{ms} = \frac{(188.77)(0.153)(1)}{19.59} \]

\[ = 1.467 \text{ kip (6.525 kN)} \]

\[(2.5%)L_{es} = (0.025)(0.068631) = 0.001716 \text{ in. (0.04358 mm)}\]

Finally, calculate equivalent temperature load and equivalent strain.
\[ \Delta T_{\text{equiv}} = \frac{f_{st}}{E_p \alpha_{\text{st}}} = \frac{199.66}{(28,500)(6.5 \times 10^{-6})} = 1078^\circ F (581^\circ C) \]

\[ \epsilon_{\text{equiv}} = \frac{f_{st}}{E_p} = \frac{199.66}{28,500} = 0.007006 \]

About the authors

J. Chris Carroll, PhD, is an assistant professor in the Department of Civil Engineering at the University of Louisiana at Lafayette.

Thomas E. Cousins, PhD, PE, is a professor in the Department of Civil and Environmental Engineering at Virginia Polytechnic Institute and State University in Blacksburg, Va.

Carin L. Roberts-Wollmann, PhD, PE, is a professor in the Department of Civil and Environmental Engineering at Virginia Polytechnic Institute and State University.

Abstract

This paper presents a numerical modeling approach that accounts for slip between prestressing strands and the surrounding concrete within the transfer zone. The objective of the study was to develop a modeling approach that can accurately and efficiently predict member behavior based on specified transfer lengths. The modeling approach uses three components: a matrix of solid elements to represent the concrete, truss elements to represent the prestressing strand, and nonlinear springs accounting for the interface between the prestressing strand and surrounding concrete. The results of 23 numerical models are included and compared with experimental data from corresponding test specimens. The study confirms the practicality and accuracy of the modeling approach to predict member behavior based on specified transfer lengths.

Keywords

Bond-slip behavior, bond stress, end slip, end-zone behavior, finite element modeling, nonlinear springs, transfer length.

Review policy

This paper was reviewed in accordance with the Precast/Prestressed Concrete Institute’s peer-review process.

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