AASHTO LRFD Bridge Design Specifications provisions for loss of prestress

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- This paper details the time-dependent analysis method for determining loss of prestress in pretensioned bridge girders in the American Association of State Highway and Transportation Officials' *AASHTO LRFD Bridge Design Specifications*.
- This paper aims to make the loss of prestress method more widely understood and better applied in practice.
- This paper clarifies misconceptions related to transformed section properties and prestress gains.

A new method for estimating time-dependent loss of prestress was introduced in the 2005 interim revisions of the American Association of State Highway and Transportation Officials' *AASHTO LRFD Bridge Design Specifications*[1] following the recommendations of the National Cooperative Highway Research Program's (NCHRP) report 496.[2] The primary goal of the research documented in NCHRP report 496 was to update the methodology for estimating prestress loss, extending its applicability to include high-strength concrete girders. The method that was developed (and has since been adopted as AASHTO LRFD specifications article 5.9.5.4) is much more refined and rigorous than previous AASHTO methods. The refinements, while in many ways making the method more technically sound, introduce some nuances that can be confusing to practitioners more familiar with previous approaches. The goal of this paper is to clarify the AASHTO LRFD specifications loss of prestress provisions by demonstrating their foundation in basic mechanics.

This paper follows the organization of the prestress loss provisions currently in AASHTO. Each of the equations appearing in article 5.9.5.4 will be described in detail, followed by a brief discussion of the shrinkage and creep models developed for high-strength concrete as recommended in NCHRP report 496 and currently shown in AASHTO LRFD specifications article 5.4.2. The time-dependent material property models and the methods for estimating prestress loss are independent of each other. In other words, any suitable shrinkage and creep models may be used in the prestress loss method. Explanations of concepts that apply to more than one equation are presented in detail following the comprehensive description of the method. Finally, a discussion is offered to convey the authors' perspective on the current method and recommendations for improvement.

Material property models

Use of the AASHTO LRFD specifications prestress loss provisions requires a method for calculating each of the following material properties:

- concrete modulus of elasticity E_c
- concrete strain due to shrinkage ε_{sh}
- concrete creep strain ε_{cr} via a creep coefficient
- prestressing steel modulus of elasticity E_p
- prestressing steel relaxation Δf_{pR}

The AASHTO LRFD specifications provide guidance on each of these, with the most significant recent changes to the creep and shrinkage model. Those changes were introduced in the 2005 interim revisions following recommendations from NCHRP report 496. [2] The changes also included some slight modifications to the calculation of concrete elastic modulus and steel relaxation.

Detailed information related to the AASHTO LRFD specifications material property model for high-strength concrete, along with a basic definition of the concrete creep coefficient used in the prestress loss provisions, is available in appendix A. Further documentation on the high-strength concrete model is provided by Tadros et al.[2] and Al-Omaishi et al.[3]

Loss of prestress

In the current AASHTO LRFD specifications methodology, loss of prestress is calculated in three stages:

- at transfer
- between transfer and the time of deck placement
- between the time of deck placement and final time

The introduction of time of deck placement in the 2005 interim revisions was a significant change from the previous method. The calculations required for each stage are discussed in detail in the following paragraphs.

Losses at transfer

Losses relative to the initial jacking stress immediately after transfer of prestress to the concrete can be split into four components:

- friction
- anchorage seating
- prestressing steel relaxation
- elastic shortening

Friction and anchorage seating are primarily of concern for posttensioned construction, although precasters must be aware of these two components as part of the pretensioning process. Precasters must also be aware of the relaxation losses that occur between jacking and transfer. While some precasters overjack to compensate for friction and seating losses, many do not overjack to counteract relaxation losses before transfer. Designers should ensure that the steel stress assumed just before transfer is realistic given the precasting environment and relaxation losses that occur during fabrication. Guidelines for estimating relaxation losses before transfer have historically been part of AASHTO LRFD specifications, but they are no longer provided as of the 2005 interim revisions. Relaxation before transfer is not insignificant. In fact, approximately $^1/_4$ of the total relaxation loss happens during the first day that the strand is held in tension as calculated by the intrinsic relaxation equation developed by Magura et al.[4] Because the equation for relaxation before transfer is no longer included by AASHTO, Eq. (5.9.5.4.4b-2) is reproduced from the 2004[5] AASHTO LRFD specifications here for convenience. It applies only to low-relaxation strands.

$$\Delta f_{pR} = \frac{\log(24t)}{40}\left(\frac{f_{pj}}{f_{py}} - 0.55\right) f_{pj} \qquad \text{(AASHTO 5.9.5.4.4b-2)}$$

where

t = time in days from stressing to transfer

f_{pj} = initial stress in the tendon after anchorage seating

f_{py} = specified yield strength of prestressing steel

Elastic shortening losses occur as the concrete responds elastically, and instantaneously, to the compressive load transferred from the bonded prestressing. The designer can consider this effect in two different ways:

- The elastic shortening loss of prestress—the difference in strand tension just after transfer and just before transfer—can be calculated explicitly. The loss of prestress can be determined iteratively using AASHTO LRFD specifications Eq. (5.9.5.2.3a-1) or directly using Eq. (C5.9.5.2.3a-1). Stresses in the concrete are then determined by applying the effective prestress force after transfer (the prestressing force before transfer minus the elastic shortening loss) to the net section concrete properties. The use of gross section properties is typically an acceptable simplification.

- If only the concrete stresses need to be known and a calculation of effective prestressing force immediately after transfer is not needed, the use of transformed section properties may provide a more direct solution. In this case, the concrete stresses can be found by applying the prestressing force before transfer (not calculating any elastic shortening losses explicitly) to the transformed section properties. The elastic shortening losses are accounted for implicitly by use of transformed section properties.

To calculate effective prestress and concrete stresses after transfer, a time-dependent analysis that considers shrinkage and creep of concrete and relaxation of prestressing steel is required.

Time-dependent loss of prestress

The time-dependent analysis method for estimating loss of prestress is independent of the material property model used. No assumptions inherent in the time-dependent analysis method impose concrete material characteristics. Therefore, the prestress loss method in the AASHTO LRFD specifications is not specific to high-strength concrete. Rather, it is a time-dependent analysis approach for a concrete girder with bonded prestressing steel that references the concrete material property models in the AASHTO LRFD specifications developed for high-strength concrete. Any other suitable material property model could be used with the time-dependent analysis approach.

Loss of prestress is calculated by tracing the change in strain in the prestressing steel that occurs with time due to concrete strains caused by elastic, creep, and shrinkage deformations, along with losses due to relaxation. The analysis is based on strain compatibility and equilibrium in the cross section at any time along with prescribed stress-strain relationships for steel and concrete. The time-dependent analysis approach is more straightforward than the complex equations in the AASHTO LRFD specifications suggest. The basic assumptions inherent in the time-dependent analysis method are as follows:

- Pure beam behavior; that is, plane sections remain plane.

- Strain compatibility; that is, perfect bond between the concrete and prestressing steel. Therefore the change in strain in the prestressing steel is equal to the change in strain in the surrounding concrete.

- As required for equilibrium, internal forces equal external forces.

- Superposition is applied for creep strains resulting from different stress increments

- Relationships between stress and strain in concrete and steel are prescribed.

The prestress loss method in the AASHTO LRFD specifications is split into two different periods: before and after deck placement. In both periods, concrete shrinkage, concrete creep, and prestressing relaxation are considered. In this paper, the effects of differential deck shrinkage will be treated separately to avoid the confusion that may follow if prestress gains due to deck shrinkage are combined and superimposed incorrectly with prestress losses due to creep, shrinkage, and relaxation.

For both shrinkage and creep of concrete, the equations for change in prestress are founded on Hooke's law applied to the prestressing steel, which is assumed linear elastic at all times. All equations are the product of the change in concrete strain at the centroid of the prestressing strands and the elastic modulus of prestressing steel. An effort was made to make the fundamental Hooke's law relationship readily apparent in this paper by reformatting equations.

Concrete shrinkage before deck placement

The prestress loss due to shrinkage Δf_{pSR} is given by AASHTO LRFD specifications Eq. (5.9.5.4.2a-1).

$$\Delta f_{pSR} = \varepsilon_{bid} E_p K_{id} \quad \text{(AASHTO 5.9.5.4.2a-1)}$$

where

ε_{bid} = shrinkage strain of girder concrete between time of transfer or end of curing and time of deck placement

K_{id} = transformed section coefficient that accounts for time-dependent interaction between concrete and bonded steel in section being considered for period between transfer and deck placement

The terms in Eq. (5.9.5.4.2a-1) can be regrouped as shown in Eq. (1).

$$\Delta f_{pSR} = E_p \left(\varepsilon_{bid} K_{id}\right) \tag{1}$$

Hooke's law is clearly evident in Eq. (1). The concrete strain at the centroid of the prestressing steel is the product $\varepsilon_{bid}K_{id}$. The K_{id} term is an adjustment to the free shrinkage strain of concrete ε_{bid} during this time to account for the time-dependent interaction between concrete and bonded steel, which provides some internal restraint against shrinkage. It arises from considerations of strain compatibility and equilibrium on the cross section. K_{id} is discussed in detail later in this paper. In addition, a derivation of the equation is provided in appendix B. The subscripts *b*, *i*, and *d* in the shrinkage strain term represent beam (girder), initial time (transfer or start of curing), and deck placement time t_d.

Concrete creep before deck placement The prestress loss due to creep Δf_{pCR} is given by AASHTO LRFD specifications Eq. (5.9.5.4.2b-1).

$$\Delta f_{pCR} = \frac{E_p}{E_{ci}} f_{cgp} \psi_b \left(t_d, t_i\right) K_{id} \qquad \text{(AASHTO 5.9.5.4.2b-1)}$$

where

E_{ci} = modulus of elasticity of concrete at transfer

f_{cgp} = concrete stress at centroid of prestressing tendons due to the prestressing force immediately after transfer and self-weight of the member at section of maximum moment

$\psi_b(t_d,t_i)$ = creep coefficient for girder concrete at the time t_d of deck placement due to loading applied at the time at transfer t_i

The terms in Eq. (5.9.5.4.2b-1) can be regrouped as shown in Eq. (2).

$$\Delta f_{pCR} = E_p \left\{ \left[\left(\frac{f_{cgp}}{E_{ci}} \right) \psi_b \left(t_d, t_i\right) \right] K_{id} \right\} \qquad (2)$$

Hooke's law is again seen in Eq. (2) with the term

$$\left[\left(\frac{f_{cgp}}{E_{ci}} \right) \psi_b \left(t_d, t_i\right) \right] K_{id}$$

as the concrete strain due to creep at the centroid of the prestressing steel. The term

$$\frac{f_{cgp}}{E_{ci}}$$

is the elastic strain in the concrete at the prestressing centroid due to stresses applied at transfer. The product of this elastic strain and the girder creep coefficient at the time of deck placement t_d due to stresses applied at time t_i results in the creep strain in the concrete at the centroid of prestressing. As with the shrinkage strain, the transformed section coefficient K_{id} is used to represent the internal restraint offered by the bonded prestressing steel against concrete creep.

Prestressing steel relaxation before deck placement The AASHTO LRFD specifications provide guidance for a detailed calculation of prestressing steel relaxation, but when low-relaxation strands are used the loss of prestress due to relaxation is small and varies over a small range. Therefore, the specifications recommend assuming a total relaxation from transfer to final time of 2.4 ksi (16.5 MPa), with half of that assumed to occur before deck placement Δf_{pR1}.

Girder concrete shrinkage after deck placement The equation for shrinkage losses after deck placement Δf_{pSD} mirrors that for losses before deck placement and is given by AASHTO LRFD specifications Eq. (5.9.5.4.3a-1).

$$\Delta f_{pSD} = \varepsilon_{bdf} E_p K_{df} \qquad \text{(AASHTO 5.9.5.4.3a-1)}$$

where

ε_{bdf} = shrinkage strain of girder concrete after deck placement

K_{df} = transformed section coefficient that accounts for time-dependent interaction between concrete and bonded steel in section being considered after deck placement

The terms in Eq. (5.9.5.4.3a-1) can be regrouped as shown in Eq. (3).

$$\Delta f_{pSD} = E_p \left(\varepsilon_{bdf} K_{df} \right) \qquad (3)$$

The concrete strain at the centroid of the prestressing steel over this period is the product $\varepsilon_{bdf}K_{df}$. K_{df} is a transformed section coefficient analogous to K_{id}, except that it is developed for use with the properties of the full composite (girder plus deck) cross section. The shrinkage strain from after deck placement ε_{bdf} is calculated most readily when recognizing that it is the difference between final shrinkage strain and that at the time of deck placement (Eq. [4]).

$$\varepsilon_{bdf} = \varepsilon_{bif} - \varepsilon_{bid} \qquad (4)$$

where

ε_{bif} = shrinkage strain of girder concrete between time of transfer or end of curing and final time

Concrete creep after deck placement The equation for creep losses after deck placement Δf_{pCD} is the sum of creep effects due to stresses applied at transfer and stresses applied after transfer. It is given by AASHTO LRFD specifications Eq. (5.9.5.4.3b-1).

$$\Delta f_{pCD} = \frac{E_p}{E_{ci}} f_{cgp} \left[\psi_b \left(t_f, t_i\right) - \psi_b \left(t_d, t_i\right) \right] K_{df} + \frac{E_p}{E_c} \Delta f_{cd} \left[\psi_b \left(t_f, t_d\right) \right] K_{df} \qquad \text{(AASHTO 5.9.5.4.3b-1)}$$

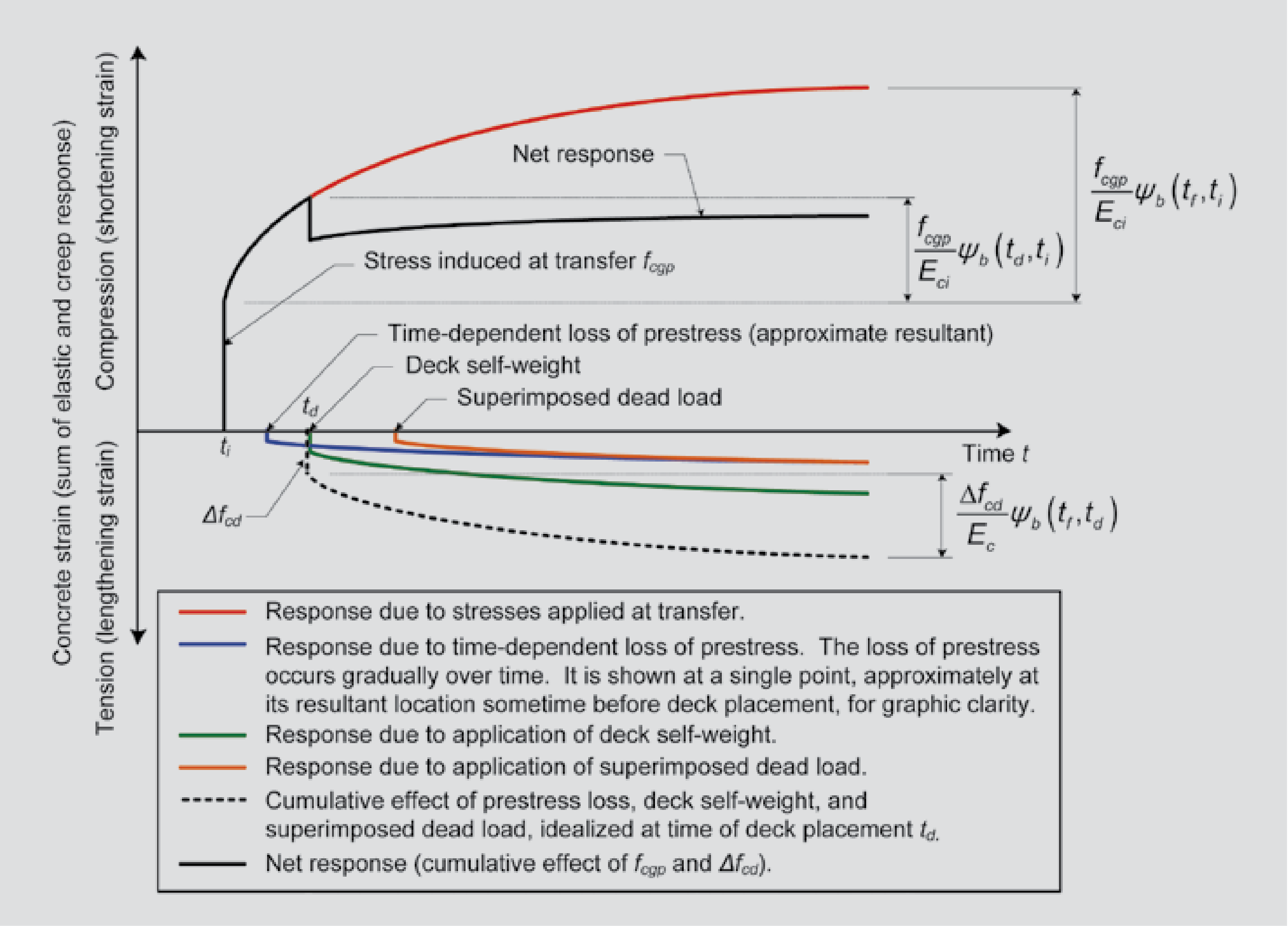

Figure 1. Schematic summary of the components affecting prestress losses due to creep. Note: E_c = modulus of elasticity of concrete at 28 days; E_{ci} = modulus of elasticity of concrete at transfer; f_{cgp} = concrete stress at centroid of prestressing tendons due to prestressing force immediately after transfer and self-weight of member at section of maximum moment; t_d = age of girder concrete at time of deck placement; t_f = age of girder concrete at end of time-dependent analysis; t_i = age of girder concrete at time of prestress transfer; Δf_{cd} = change in concrete stress at centroid of prestressing strands due to time-dependent loss of prestress between transfer and deck placement, combined with deck weight and superimposed loads; $\psi_b(t_d,t_i)$ = creep coefficient for girder concrete at time t_d of deck placement due to loading applied at time t_i of transfer; $\psi_b(t_f,t_d)$ = creep coefficient for girder concrete at final time t_f due to loading applied at time t_d of deck placement; $\psi_b(t_f,t_i)$ = creep coefficient for girder concrete at final time t_f due to loading applied at time of transfer.

where

$\psi_b(t_f,t_i)$ = creep coefficient for girder concrete at final time t_f due to loading applied at the time t_i of transfer

Δf_{cd} = change in concrete stress at centroid of prestressing strands due to time-dependent loss of prestress between transfer and deck placement combined with deck weight and superimposed loads

$\psi_b(t_f,t_d)$ = creep coefficient for girder concrete at final time t_f due to loading applied at time t_d of deck placement

The terms in Eq. (5.9.5.4.3b-1) can be regrouped as shown in Eq. (5).

$$\Delta f_{pCD} = E_p\left\{\left(\frac{f_{cgp}}{E_{ci}}\right)\left[\psi_b(t_f,t_i)-\psi_b(t_d,t_i)\right]\right\}K_{df} + E_p\left\{\left(\frac{\Delta f_{cd}}{E_c}\right)\left[\psi_b(t_f,t_d)\right]K_{df}\right\} \quad (5)$$

The first term accounts for the creep effects, due to the initial prestressing, that continue after deck placement. This is calculated as the difference between the final creep effects and those already considered at the time of deck placement in Eq. (2). The term $[\psi_b(t_f,t_i) - \psi_b(t_d,t_i)]$ is not equal to $\psi_b(t_f,t_d)$ because the creep function is driven largely by the time when the stress changed. The second term in Eq. (5) will generally be opposite in sign relative to the first. The initial stress at transfer f_{cgp} will typically be compression, while the changes in stress due to deck weight, superimposed dead load, and loss of prestress will typically be tensile stress increments. **Figure 1** clarifies the

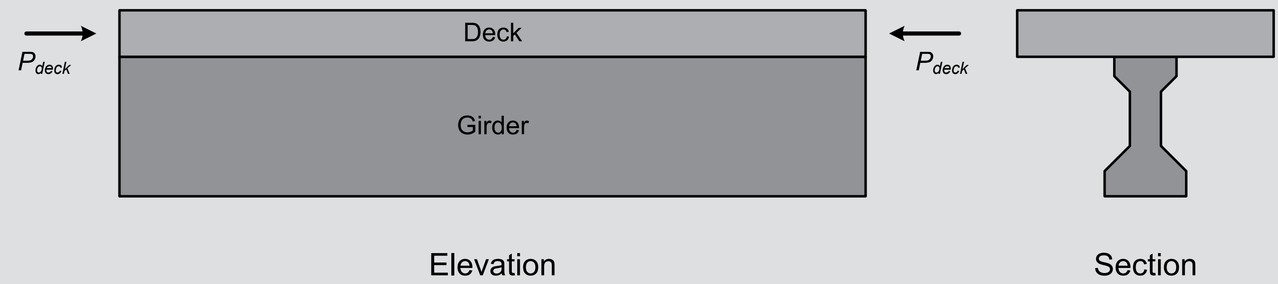

Figure 2. Conceptual model of the effect that deck-girder differential shrinkage has on the composite cross section. Note: P_{deck} = effective force due to deck shrinkage applied to composite section at centroid of deck.

superposition of the effects. Again, the transformed section coefficient K_{df} is used to model the restraint of creep by the bonded prestressing steel. The time-dependent loss of prestress is shown to occur instantaneously for clarity of the graphic. In reality, it would occur gradually.

Prestressing steel relaxation after deck placement If a total loss of prestress due to relaxation of 2.4 ksi (16.5 MPa) is assumed, as recommended by the AASHTO LRFD specifications, half of that value should be taken between deck placement and final time Δf_{pR2}.

Shrinkage of the deck concrete

Starting with the 2005 interim revisions, the AASHTO LRFD specifications method for estimating loss of prestress recognized the interaction between a cast-in-place deck and a precast concrete girder when they are compositely connected. The shrinkage differential exists primarily because much of the total girder concrete shrinkage occurs before the system is made composite. Therefore, while the girder and deck behave compositely, the deck has greater potential shrinkage. Strain compatibility at the deck-girder interface, however, requires that the two elements behave as one unit; therefore, an internal redistribution of stresses is necessary.

The effect of differential shrinkage can be modeled as an effective force at the centroid of the deck (**Fig. 2**).

In typical construction, the deck is above the neutral axis of the composite section and the centroid of the prestressing steel is below the neutral axis at the critical section. The effective compressive force due to differential shrinkage causes a tensile strain on the opposite face (bottom) of the girder. Assuming strain compatibility between the prestressing steel and the surrounding concrete, the tensile strain leads to an increase in the effective force in the prestressing steel. The AASHTO LRFD specifications method calls this a prestressing gain. This prestressing gain, however, does not increase the compressive stress in the surrounding concrete. Rather, the concrete experiences a tensile stress increment, too. A further discussion of elastic gains is provided later in this paper.

An explanation of the article 5.9.5.4.3d equations related to shrinkage of deck concrete is warranted. First, the prestress gain due to deck shrinkage Δf_{pSS} is calculated by AASHTO LRFD specifications Eq. (5.9.5.4.3d-1).

$$\Delta f_{pSS} = \frac{E_p}{E_c} \Delta f_{cdf} K_{df} \left[1 + 0.7\psi_b\left(t_f, t_d\right)\right] \qquad \text{(AASHTO 5.9.5.4.3d-1)}$$

where

Δf_{cdf} = change in concrete stress at centroid of prestressing strands due to shrinkage of deck concrete

The terms in AASHTO Eq. (5.9.5.4.3d-1) can be regrouped as shown in Eq. (6).

$$\Delta f_{pSS} = E_p \left(\frac{\Delta f_{cdf}}{\left\{ \frac{E_c}{\left[1 + 0.7\psi_b\left(t_f, t_d\right)\right]} \right\}} \right) K_{df} \qquad (6)$$

The term in the outer parentheses in Eq. (6) is the strain at the prestressing steel centroid due to deck shrinkage. The strain is found through division of the stress change by the age-adjusted effective modulus (rather than the elastic modulus of concrete) because the effective force due to deck shrinkage builds up over time and is partially relieved by creep. A detailed description of age-adjusted effective modulus is provided later in this paper.

The AASHTO LRFD specifications recommend calculating the stress change caused by the effective deck shrinkage force at the centroid of the prestressing steel using

Eq. (5.9.5.4.3d-2), which is split into its components in Eq. (7) and (8) for clarity.

$$\Delta f_{cdf} = P_{deck}\left(\frac{1}{A_c} - \frac{e_{pc}e_d}{I_c}\right) \tag{7}$$

where

P_{deck} = effective force due to deck shrinkage applied to composite section at centroid of deck; P_{deck} is defined for the purposes of this paper only and is not a variable used in AASHTO LRFD specifications

A_c = area of composite cross section

e_{pc} = eccentricity of prestressing force with respect to centroid of composite section, positive where centroid of prestressing steel is below centroid of composite section

e_d = eccentricity of deck with respect to gross composite section, positive where deck is above girder

I_c = moment of inertia of composite cross section

The effective force due to deck shrinkage can be calculated as the product of shrinkage strain, elastic modulus, and effective deck area (Eq. [8]). Because the gradual buildup of stress will be partially relieved by simultaneous creep of the deck concrete, an age-adjusted effective modulus is used in place of the concrete elastic modulus.

$$P_{deck} = \varepsilon_{ddf}\left\{\frac{E_{cd}}{\left[1+0.7\psi_d\left(t_f,t_d\right)\right]}\right\}A_d \tag{8}$$

where

ε_{ddf} = shrinkage strain of deck concrete between time of deck placement or end of deck curing and final time

E_{cd} = modulus of elasticity of deck concrete

$\psi_d(t_f,t_d)$ = creep coefficient for deck concrete at final time t_f due to loading applied at time t_d of deck placement

A_d = effective cross-sectional area of composite deck concrete

The AASHTO LRFD specifications guide the user to calculate a stress change at the centroid of the prestressing steel and the prestressing gain due to deck shrinkage but stop short of providing a prescriptive equation to calculate concrete stress at the extreme fiber. Equation (9), published by Al-Omaishi et al.,[6] calculates extreme bottom fiber concrete stress Δf_{cbSS} using the effective force defined in Eq. (8).

$$\Delta f_{cbSS} = P_{deck}\left(-\frac{1}{A_c} + \frac{y_{bc}e_d}{I_c}\right)K_{df} \tag{9}$$

where

y_{bc} = eccentricity of concrete extreme bottom fiber with respect to centroid of composite cross section

Maintaining a consistent sign convention for Eq. (7) and (9) is a challenge. In typical construction, P_{deck} will be an effective compression force and Δf_{cbSS} will be a tensile stress increment in the extreme bottom concrete fiber.

The effective force P_{deck}, will be present on the composite cross section even if the deck concrete is cracked because the force will be transferred by reinforcement across the cracks and will eventually be carried into the girder via the composite connection.

Transformed section coefficients K_{id} and K_{df}

The transformed section coefficients represent the fact that steel restrains the creep and shrinkage of concrete. Because the two materials are bonded, the differences in time-dependent behavior lead to an internal redistribution of stress. In addition, the difference in elastic response between steel and concrete must be considered in the stress redistribution.

Consider shrinkage as an example. **Figure 3** shows ε_{sh} as the shrinkage expected of a concrete specimen that has no restraint against shortening (that is, no reinforcement). Such an idealized type of specimen was used in developing the model used to predict shrinkage strains. In a prestressed concrete girder, however, there is restraint against shrinkage because the prestressing steel bonded to the concrete does not shrink. Because strain compatibility must be satisfied, equilibrium requires a redistribution of stresses to accommodate the differences in time-dependent and elastic behavior of the two materials. In Fig. 3, ε_{res} is used to denote the amount by which the free shrinkage is reduced at the level of the prestressing centroid by bonded steel. The net shortening at the centroid of prestressing is ε_{net}. The transformed section coefficient is derived to quantify the effect of the prestressing steel's restraint. It can be thought of as the simple ratio given in Eq. (10).

$$K_{id} = \frac{\varepsilon_{net}}{\varepsilon_{sh}} \tag{10}$$

K_{id} and K_{df} represent the same phenomenon. K_{id} is derived

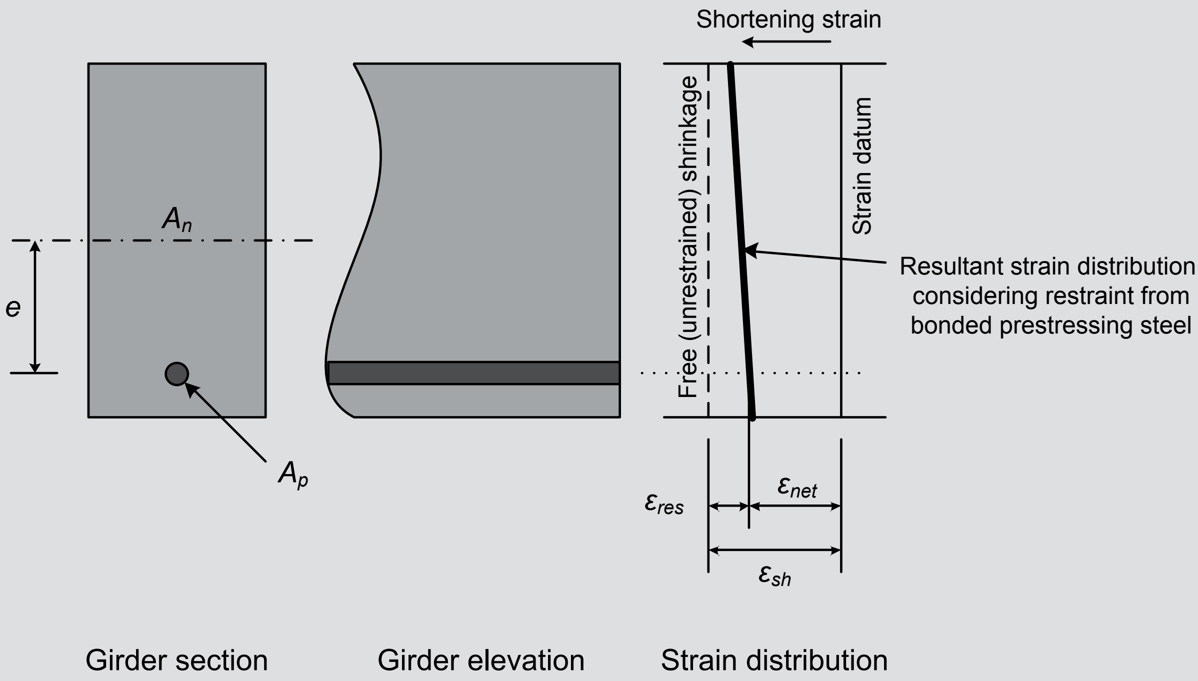

Figure 3. Bonded prestressing steel partially restrains the free shrinkage strain inherent to the concrete. The effect is represented mathematically by the transformed section coefficient. Note: A_n = area of net cross section; A_p = area of prestressing steel; e = eccentricity between the centroid of the girder concrete and the centroid of the prestressing steel; ε_{net} = net shrinkage strain of section at centroid of prestressing steel considering shrinkage of concrete and restraint against shrinkage from bonded prestressing steel; ε_{res} = portion of free shrinkage of concrete effectively restrained by bonded prestressing steel at centroid of prestressing steel; ε_{sh} = concrete strain due to shrinkage.

with respect to the girder only and is used for calculations before deck placement, while K_{df} is derived for the girder-deck composite system. A detailed derivation is in appendix B.

Age-adjusted effective modulus

The AASHTO LRFD specifications account for the creep of concrete by using an age-adjusted effective modulus in some instances. The method approximates the principle of creep superposition attributed to McHenry[7] for a time varying stress history. **Figure 4** shows the general relationship between elastic modulus, effective modulus, and age-adjusted effective modulus. When a change in stress in concrete Δf_c is applied instantaneously, the short-term strain can be calculated according to the elastic modulus E_c for the concrete. If the stress is maintained, the strain will increase gradually as concrete creeps (from point A to B in Fig. 4). The net behavior of the concrete can be approximated by defining an effective elastic modulus (Eq. [11]) to describe the cumulative stress-strain behavior from the origin to point B.

$$E_{c,eff} = \frac{E_c}{1+\psi\left(t,t_i\right)} \tag{11}$$

where

$E_{c,eff}$ = effective modulus of elasticity of concrete used to describe the response of concrete to an instantaneous stress increment considering both elastic and creep effects

$\psi(t,t_i)$ = creep coefficient for concrete at time t due to stresses applied at time of a stress change t_i

If the stress change is not instantaneous, the creep response of concrete is slightly less. Figure 4 shows the difference schematically, where the stress change has been split into three equal increments (rather than a truly gradual increase) for the purpose of clarity in the graphic. If the stress builds over time, the total creep strain is less than when the same stress change is instantaneous. Equation (11) is adjusted slightly to yield Eq. (12) to represent an age-adjusted effective modulus of concrete $E_{c,AAEM}$.

$$E_{c,AAEM} = \frac{E_c}{1+\chi\left[\psi\left(t,t_i\right)\right]} \approx \frac{E_c}{1+0.7\left[\psi\left(t,t_i\right)\right]} \tag{12}$$

where

χ = aging coefficient

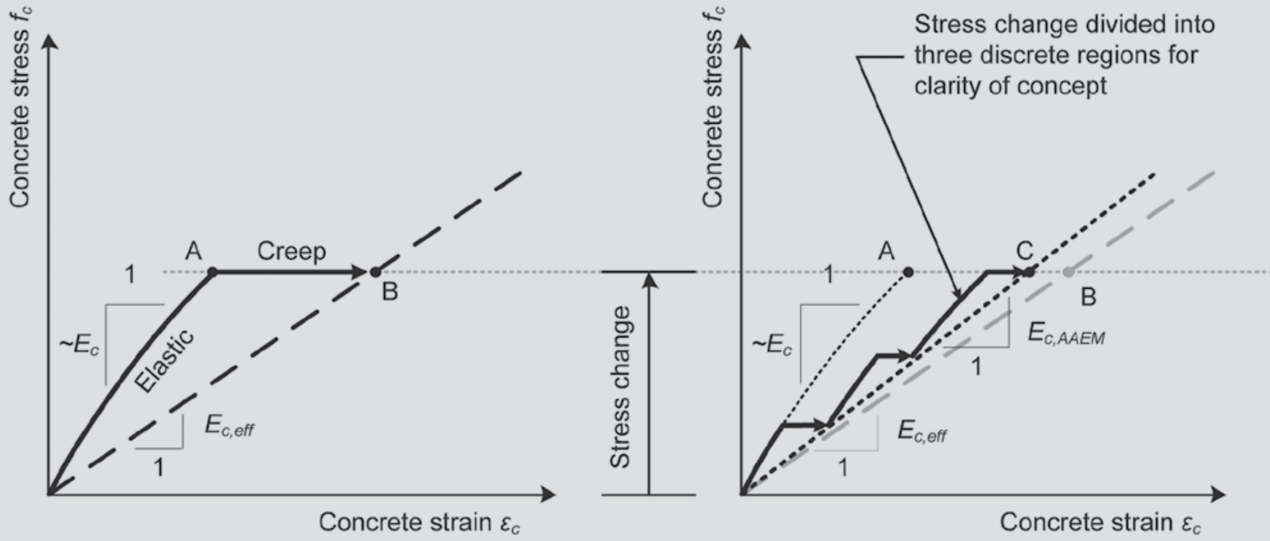

Figure 4. Schematic definition of an effective modulus and age-adjusted effective modulus as used to account for the creep response of concrete. Note: E_c = modulus of elasticity of concrete; $E_{c,AAEM}$ = age-adjusted effective elastic modulus of concrete used to describe response of concrete to gradual stress increment considering both elastic and creep effects; $E_{c,eff}$ = effective modulus of elasticity of concrete used to describe the response of concrete to an instantaneous stress increment considering both elastic and creep effects.

The aging coefficient χ was first proposed by Trost[8,9] in 1967, then refined by Bazant[10] and Dilger.[11] The AASHTO LRFD specifications adopt a constant value of 0.7 for χ. A description of the method is also available in Collins and Mitchell.[12]

Elastic losses and gains

Prestress loss occurs when the strain in the steel decreases to match the shortening of concrete at the same level as the steel. The application of load, however, such as the deck weight at time of deck placement, causes an elongation in the prestressing steel and a corresponding increase in steel stress. The application of load also causes an increment of tensile stress in the concrete at the level of the steel. Use of the term *prestress gain* to describe this elastic response to load causes confusion. The following discussion attempts to clarify the matter.

The elastic shortening loss at transfer is well understood, and the equation for estimating that loss did not change in 2005 with the implementation of a new time-dependent loss calculation method. This is a loss of prestress (relative to the stress in the strands just before transfer) that follows concrete's elastic response to the applied load from pretensioned strands. The concrete and prestressing strands must reach a point of equilibrium. The concrete girder is holding the prestressing steel at a length greater than its zero-stress length. In turn, the concrete experiences compressive stress and a consequent shortening based on the elastic properties of the concrete material. Because the concrete girder has shortened, the prestressing steel bonded to the concrete is allowed to get closer to its zero-stress length than it was after jacking. Therefore, the tension is less than at the time of jacking. This loss of stress in the prestressing steel due to shortening of the concrete at transfer results in less precompression of the concrete. This point will be contrasted with the idea of an elastic gain.

An analogy to reinforced (nonprestressed) concrete helps to clarify the concept. When a reinforced concrete beam is loaded (**Fig. 5**), tension stresses and eventually cracking would be expected on the bottom face of a simply supported beam. As load is applied to the beam, the steel undergoes an elongation and therefore takes on a tensile force. As a result of the applied load, the steel experiences a gain in tension. This force would not be considered to be precompressing the concrete in this region or acting to resist the formation of cracks. The tension gain in the steel follows an elongation that is also experienced by the surrounding concrete, rendering a tensile stress increment in the concrete as well as the steel.

Applied to reinforced concrete, the concept seems quite straightforward. With respect to prestressed concrete, however, the term *gain* is misleading. Based on the terminology alone, it seems reasonable to sum all prestress loss and gain terms to arrive at an effective prestressing force. If the intent is to estimate the stress in the prestressing steel, a summation of all prestress loss and gain terms is appropriate. Such an estimate may be desirable when checking steel stresses against a specified limit. When checking concrete stresses against a tension limit, however, the approach is problematic. Prestress gains can be caused by the application of deck weight, superimposed dead load, and live load. In addition, prestress gains result from deck

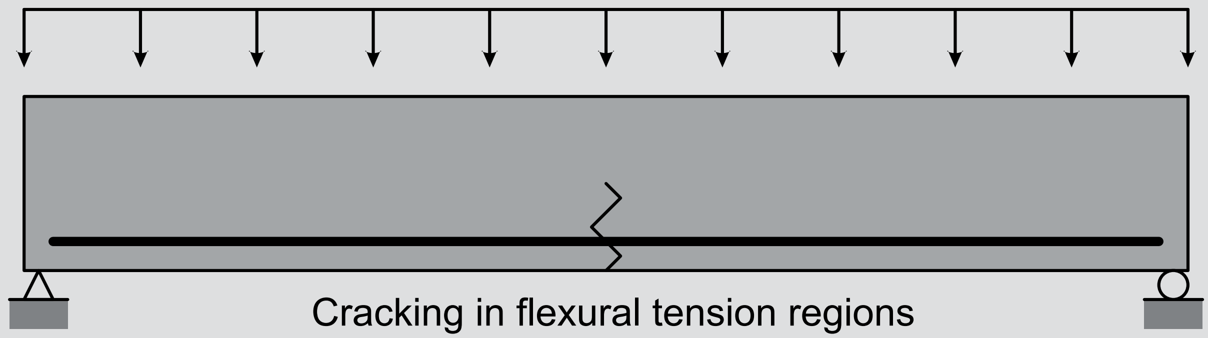

Figure 5. Typical flexural response of a simply supported reinforced concrete beam.

shrinkage. In each of these cases, the elongation that leads to a prestress gain (increase in tension) is coupled with an elongation in the concrete that leads to tensile stress (just as in the reinforced concrete beam). Therefore, it would be a gross error to assume that the prestress gains due to the application of external loads or deck shrinkage act to further precompress the surrounding concrete. Such an error is likely to result if the true effective prestress force, found by summation of all losses and gains, is used to calculate concrete stress by a traditional combined stress formulation.

The plot of effective prestress over time in **Fig. 6** (red line) was presented in NCHRP report 496 and has been reproduced in numerous settings since. The idea is to summarize the components influencing the effective prestressing force. It is fundamentally correct, but possibly misleading, in its treatment of prestress gains. To clarify the point, plots for the extreme fiber concrete stresses have been added by the authors. The plots assume a simply supported precast concrete girder with the centroid of prestressing below the neutral axis with a cast-in-place deck on top. Stresses are shown, conceptually, at the midspan section. While Fig. 6 is intended to be realistic, in some cases scale has been sacrificed for clarity.

Transformed section properties: Prestressing effects

The idea of using transformed section properties to calculate stresses is now mentioned in the AASHTO LRFD specifications (C5.9.5.2.3a) and in much of the literature related to the NCHRP report 496 recommendations. Although the issue has been addressed by many others,[6,13–15] further clarification may still be necessary. First, the various ways of idealizing a cross section will be summarized, with reference to a simple, concentrically prestressed cross section (**Fig. 7**):

- Net cross section: A net girder section treats the concrete and steel as separate components (though strain compatibility still applies). The net cross section is made of only the concrete component A_n, where the area of prestressing steel A_p has been subtracted.
- Transformed cross section: A transformed girder cross section accounts for the differences in the elastic response of concrete and steel. If they are assumed to be perfectly bonded, the steel experiences the same strain as the surrounding concrete. The steel has a much stiffer response per unit area because it has a higher elastic modulus than the concrete. The difference is accounted for by transforming the steel to an equivalent area of concrete, found as the product of the steel area and the modular ratio between steel and concrete n, where n is equal to $\dfrac{E_p}{E_c}$.
- Gross cross section: A gross girder section disregards the differences in elastic response between concrete and steel. The area of the steel is treated no differently from concrete, and the total gross area A_g of the cross section is used in calculating the properties.

Of the three, the use of transformed section properties and net section properties to calculate stresses are most accurate and, in fact, numerically equal. Gross section properties are used most commonly in practice for simplicity. The error involved with the use of gross section properties is typically negligible because steel makes up a small percentage of the cross-sectional area.[13]

First, a proof will be offered to demonstrate that the transformed and net section properties offer identical results. Consider the pretensioning arrangement defined in **Fig. 8**.

The theory under inspection is that the use of net section and transformed section properties will yield exactly the same stress in the concrete σ_c. Correct application of both techniques is presented in Eq. (13), followed by a proof.

$$\sigma_c = \frac{P'}{A_n} = \frac{P}{A_{TR}} \tag{13}$$

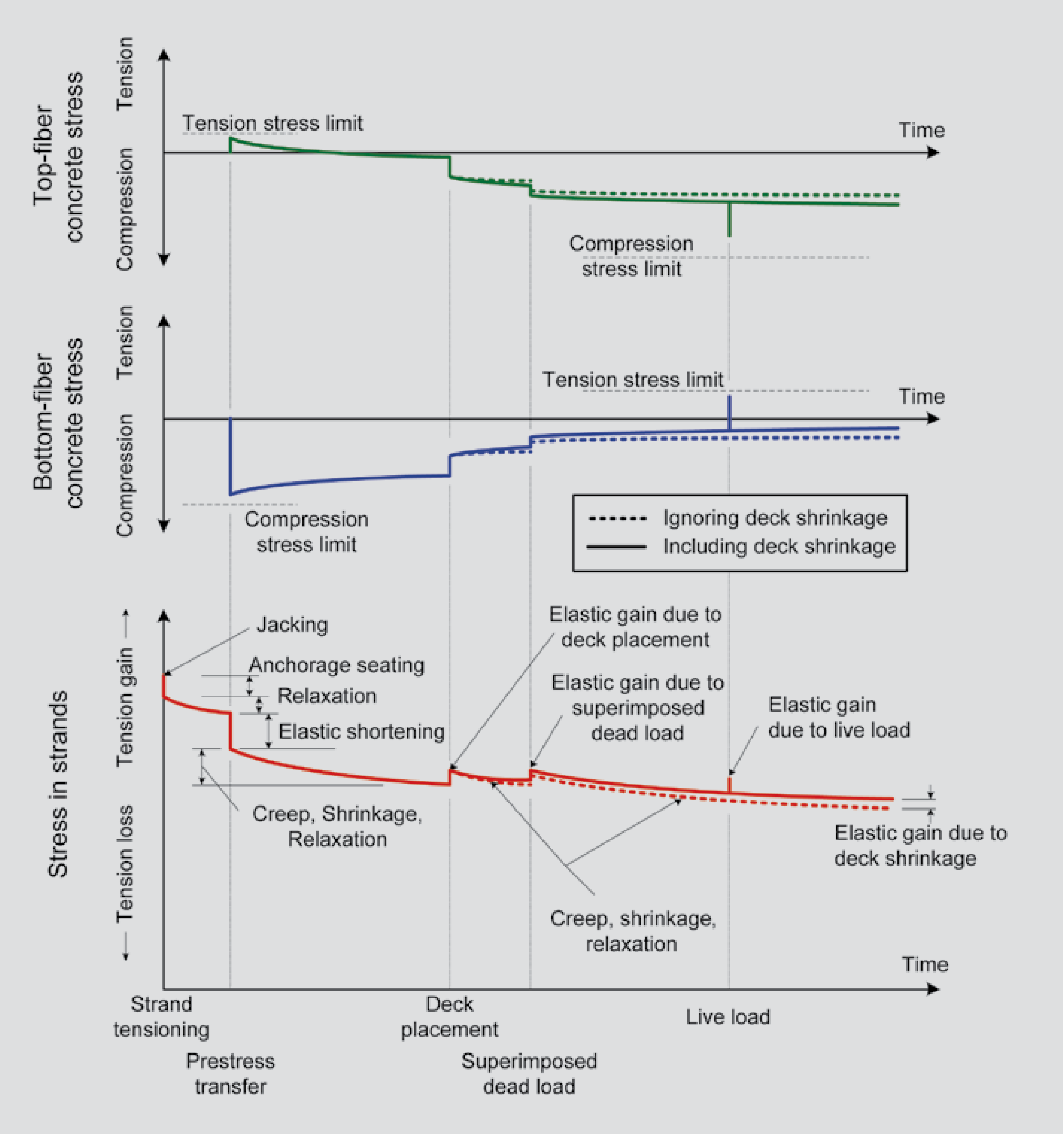

Figure 6. Schematic summary of the time-dependent response of a prestressed concrete girder with respect to the effective prestressing force and the extreme fiber concrete stresses at the top and bottom of the girder. Source: Data from Tadros, Al-Omaishi, Seguirant, and Gallt (2003).

where

P' = effective tensile stress in prestressing strands after transfer, considering elastic shortening losses

P = effective tensile stress in prestressing strands just before transfer

A_{TR} = area of transformed section, where the area of steel has been transformed to an equivalent area of concrete based on the modular ratio

Equation (13) could be rewritten as Eq. (14).

$$\frac{P'}{P} = \frac{A_n}{A_{TR}} \tag{14}$$

Equation (15) was developed as an expression for P' (Fig. 8).

$$P' = E_p A_p \varepsilon'_p = E_p A_p \left(\varepsilon_p - \frac{L - L'}{L} \right) \tag{15}$$

where

ε'_p = tensile strain in prestressing steel after transfer, considering effects of elastic shortening

ε_p = tensile strain in prestressing steel before transfer

L = length of concrete member just before transfer

L' = length of concrete member after transfer, considering elastic shortening due to prestress

The change in length of the concrete component upon force transfer can be approximated by Hooke's law equation for change in length of an axially loaded member [Eq. (16)].

$$\left(L - L' \right) = \frac{P' L}{A_n E_c} \tag{16}$$

Substituting Eq. (16) into Eq. (15) yields Eq. (17).

$$P' = A_p E_p \left(\varepsilon_p - \frac{P'}{A_n E_c} \right) \tag{17}$$

Solving for P' yields Eq. (18).

$$P' = \frac{E_p \varepsilon_p A_p}{1 + \left(\frac{E_p}{E_c} \right) \left(\frac{A_p}{A_n} \right)} \tag{18}$$

From Fig. 8, the force in the strands before transfer is given by Eq. (19).

$$P = E_p \varepsilon_p A_p \tag{19}$$

Substituting Eq. (18) and (19) into the the left side of Eq. (14) yields the expression in Eq. (20).

$$\frac{P'}{P} = \frac{\dfrac{E_p \varepsilon_p A_p}{1 + \left(\frac{E_p}{E_c} \right) \left(\frac{A_p}{A_n} \right)}}{E_p \varepsilon_p A_p} = \frac{1}{1 + \left(\frac{E_p}{E_c} \right) \left(\frac{A_p}{A_n} \right)} = \frac{1}{1 + n \left(\frac{A_p}{A_n} \right)} \tag{20}$$

Recalling the definition of transformed section properties

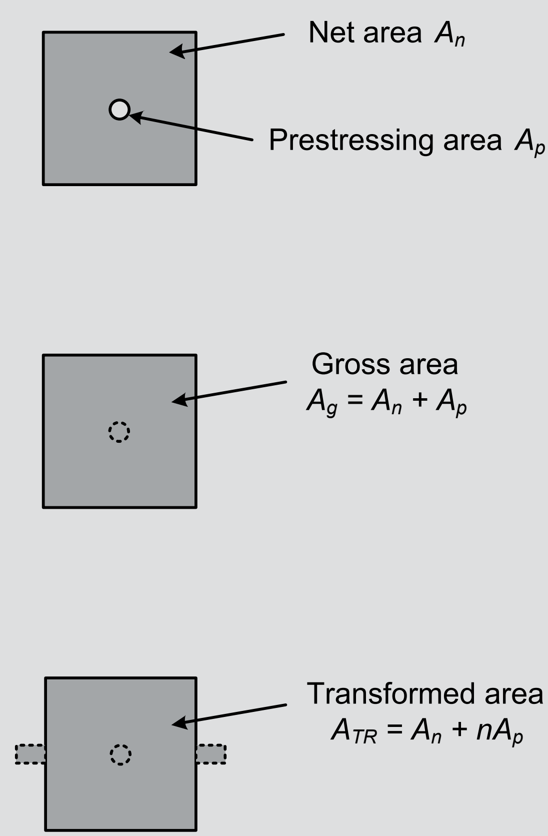

Figure 7. Comparison of net section, gross section, and transformed section representations. Note: A_g = gross area of cross section, including both steel and concrete; A_n = area of net concrete section; A_p = area of prestressing steel; A_{TR} = area of transformed section, where the area of steel has been transformed to an equivalent area of concrete based on the modular ratio; E_c = modulus of elasticity of concrete; E_p = modulus of elasticity of prestressing steel; n = modular ratio E_p/E_c.

(Fig. 7), the right side of Eq. (14) becomes the expression in Eq. (21).

$$\frac{A_n}{A_{TR}} = \frac{A_n}{A_n + nA_p} = \frac{1}{1 + n \left(\frac{A_p}{A_n} \right)} \tag{21}$$

It is clear that Eq. (20) and (21) are identical, thus proving the relationships shown in Eq. (13) and (14). Therefore, the use of net section properties and transformed section properties produces identical results, but loss of prestress during transfer due to elastic shortening of concrete Δf_{pES} must be calculated explicitly to determine the effective prestressing force needed to use net section properties [Eq. (22)].

$$P' = P - A_p \Delta f_{pES} \tag{22}$$

A more detailed derivation of the relationship between net section and transformed section properties, including treat-

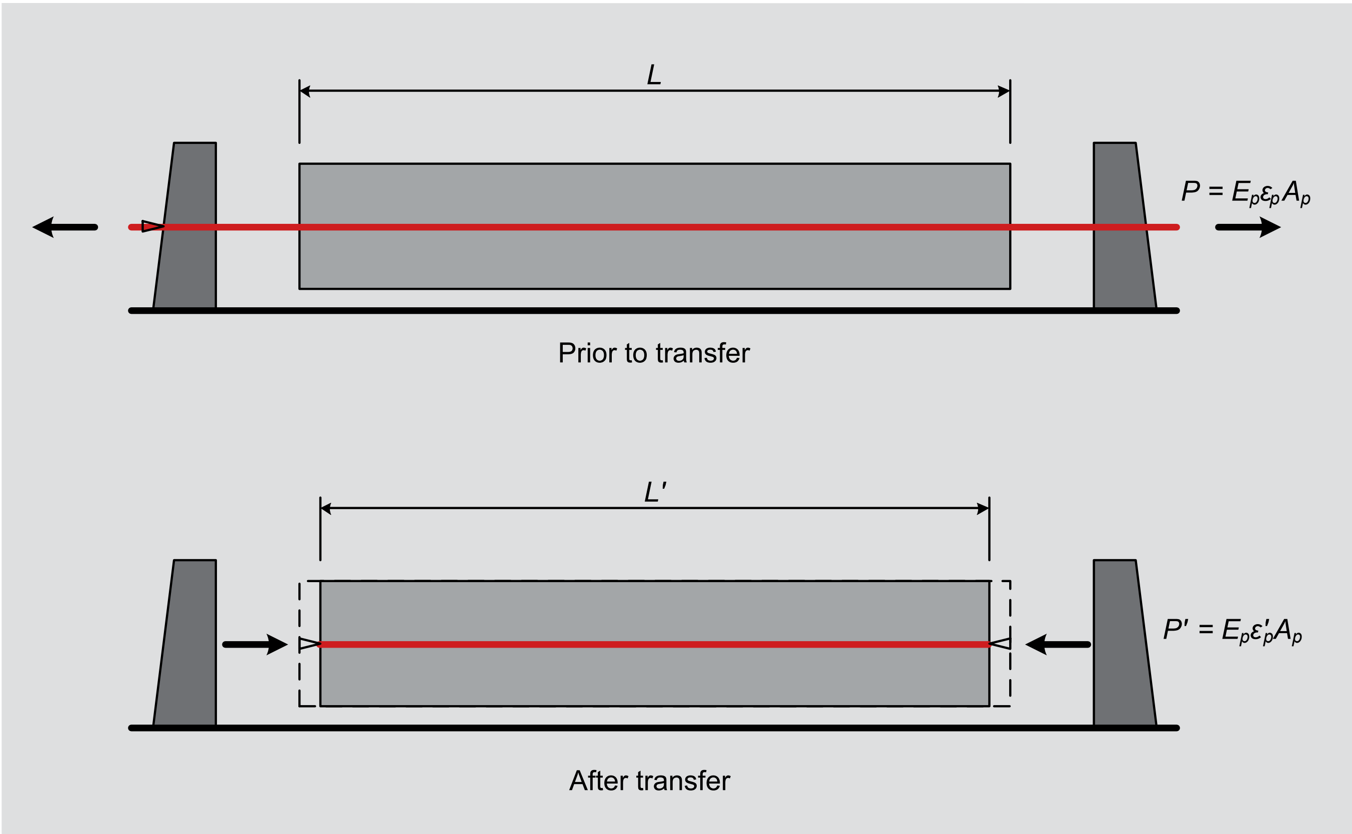

Figure 8. Typical pretensioning sequence. Note: A_p = area of prestressing steel; E_p = modulus of elasticity of prestressing steel; L = length of concrete member just before transfer; L' = length of concrete member after transfer, considering elastic shortening due to prestress; P = effective tensile stress in prestressing strands just before transfer; P' = effective tensile stress in prestressing strands after transfer, considering elastic shortening losses; ε_p = tensile strain in prestressing steel before transfer; ε'_p tensile strain in prestressing steel after transfer, considering effects of elastic shortening.

ment of eccentric prestressing, is available in Huang.[14]

For now, the following observations can be made:

- The use of net section properties and transformed section properties produces identical results for stress in the concrete.
- The use of transformed section properties is the most direct method for calculating concrete stress because it does not require an independent calculation of elastic shortening losses.
- A separate calculation of effective prestress would be needed when transformed section properties are used if effective prestress must be quantified.
- Gross section properties can be used as a direct replacement for net section properties with minimal error when the area of prestressing steel is small relative to the area of concrete.[13]

Transformed section properties: Applied loads

It is also appropriate to use transformed section properties when calculating stresses caused by externally applied loads. If perfect bond is assumed, then the prestressing steel will experience the same strain as the surrounding concrete when the cross section undergoes the combined effects of curvature and axial strain but will exhibit a stiffer response because of its higher elastic modulus. The additional stiffness is accounted for by use of transformed section properties.

Net section properties can be used with equal accuracy but additional computational effort. This approach would require calculation of stresses due to the combination of external load on the net section and force in the prestressing steel as it resists elongation. A closed form solution for this approach is cumbersome. The most practical approach may be a series of iterations until strain compatibility is satisfied.

Again, gross section properties are often used to simplify stress calculations (ignoring the force in the prestressing steel as it resists elongation mentioned for net section properties). The use of gross section properties introduces a technical error, but it is generally negligible. In addition, the simplification error is often conservative because gross section properties underestimate the true stiffness of the cross section.

Discussion

The previous sections have provided a description of the prestress loss calculation method presented in the current AASHTO LRFD specifications, including the bases for the various steps in the calculations. The increased complexity in the provisions relative to previous versions has caused some concern among practitioners and has introduced the potential for errors in application of the provisions because of a lack of familiarity with the methodology. The following sections provide a discussion of some of the issues involved.

Uncertainty in time-dependent analysis

The complexity and rigor of the AASHTO LRFD specifications method, relative to previous methods, suggests an improved refinement and precision in the estimate of prestress losses and the determination of time-dependent stresses in the concrete. However, time-dependent behavior of prestressed concrete elements is difficult to predict because of the uncertainty in many dependent variables, such as the following:

- the actual compressive strength of concrete
- the elastic modulus, shrinkage strain, and creep strain of concrete
- initial jacking force in the prestressing strands, which is only required to be within 5% of the target force according to PCI's *Manual for Quality Control for Plants and Production of Structural Precast Concrete Products*[16]
- construction practices and sequence of construction (that is, time of deck placement)
- effective area of the deck behaving compositely with the girder
- magnitude of applied loads
- as-built geometry of the finished structure and location of the prestressing strands

Given the many sources of uncertainty, one must have reasonable expectations for any time-dependent analysis method for prestressed concrete. It is possible that the increased complexity of the AASHTO LRFD specifications method could reduce its accuracy if engineers do not understand the provisions and apply them incorrectly. This paper has sought to explain the fundamental principles and subsequently reduce the number of instances where the provisions are applied in error.

Stages for analysis

The AASHTO LRFD specifications method recognizes the placement of a cast-in-place deck as a significant action in the life of a prestressed girder. The self-weight of the deck decompresses the concrete precompression region, and the action of the composite system changes the way that the girder responds to loads. In fact, the AASHTO method requires the user to define a variable t_d that is the age of the girder concrete at the time of deck placement. This type of detail in the construction sequence is beyond the designer's control and in many cases beyond the designer's ability to estimate. Therefore, it is important to understand the sensitivity of the method to this variable.

Figure 9 was developed to show the sensitivity of the prestress loss calculations to t_d over a range of realistic values. The plot is based on example 9.4 in the *PCI Bridge Design Manual.*[17] Elastic shortening losses have not been included in the plot because they are not affected by the time of deck placement. In addition, the prestress gain due to deck shrinkage is not included in the plots for prestress loss. Deck shrinkage is included, however, in the extreme fiber concrete stress results presented later. Figure 9 shows plots for the total time-dependent prestress loss (sum of shrinkage, creep, and relaxation effects) and for the division of those losses before and after deck placement. Relaxation losses are assumed to be divided evenly between the two periods, regardless of the time of deck placement, as recommended by the AASHTO LRFD specifications method. The relaxation losses are small relative to the other components, so this assumption does not have a significant effect on the plot in Fig. 9.

Figure 9 shows that the sensitivity of total prestress losses to the time of deck placement is insignificant, especially compared with the inherent uncertainty in the time-dependent analysis of a prestressed concrete girder. The same conclusion can also be reached by a more rigorous time-step analysis method.[18] If there are particular reasons to have an accurate estimate of prestress losses at the time of deck placement, such as a more reliable estimate of camber, then the division of the time periods at deck placement may be necessary. For the design and layout of prestressing, however, stress calculations at transfer and at service are likely to control and the numerical value chosen for the time of deck placement is of little consequence.

For the same example, the sensitivity of bottom-fiber concrete stresses to the time of deck placement is also of interest. The bottom-fiber concrete stress at service, for this example, varies over a range of 37 psi (0.26 MPa)

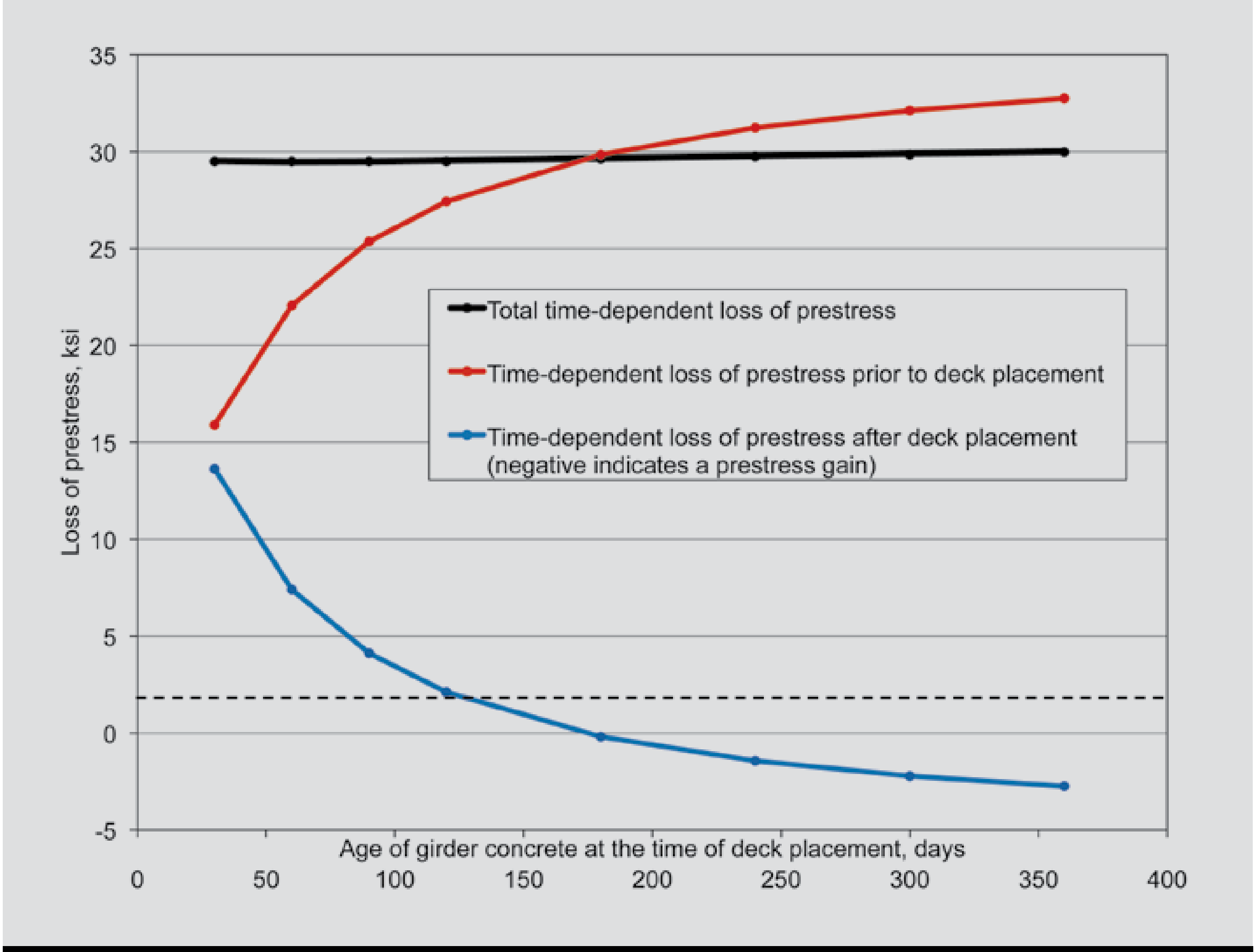

Figure 9. Plot of the American Association of State Highway and Transportation Officials prestress loss method's sensitivity to the time of deck placement input variable as applied to Example 9.4 in the *PCI Bridge Design Manual.* Note: 1 ksi = 6.895 MPa.

for deck placement times between 30 days and 365 days, with the tensile stress increasing as the girder age at deck placement increases. To put that range into perspective, the removal of one $^1/_2$ in. (12 mm) diameter prestressing strand at the centroid of prestressing would change the bottom fiber stress at service by approximately 60 psi (0.4 MPa). In other words, the time of deck placement is practically insignificant for final time estimates of effective prestress or extreme fiber concrete stress.

Deck shrinkage

The AASHTO LRFD specifications method appropriately recognizes the effective force that develops as a result of deck shrinkage. The magnitude of that force, however, is a function of the shrinkage differential between the deck and the girder, not the total deck shrinkage. Therefore, it would be more correct to replace Eq. (8) with Eq. (23). The 2011 edition of the *PCI Bridge Design Manual*[19] recommends using half the value obtained by Eq. (8) in design due to the likelihood of deck cracking and reinforcement reducing this effect.

$$P_{deck} = \left(\varepsilon_{ddf} - \varepsilon_{bdf}\right)\left\{\frac{E_{cd}}{\left[1+0.7\psi_d\left(t_f,t_d\right)\right]}\right\}A_d \qquad (23)$$

Transformed section coefficients

As detailed previously, the transformed section coefficients K_{id} and K_{df} represent the restraint that the bonded prestressing steel offers against shrinkage and creep in the concrete. There are, however, some inconsistencies between the final formulation of those equations and the behavior they represent. The equations from the AASHTO LRFD specifications method are given below.

$$K_{id} = \frac{1}{1+\left(\frac{E_p}{E_{ci}}\right)\left(\frac{A_p}{A_g}\right)\left(1+\frac{A_g e_{pg}^2}{I_g}\right)\left[1+0.7\psi_b\left(t_f,t_i\right)\right]}$$

(AASHTO 5.9.5.4.2a-2)

where

e_{pg} = eccentricity of prestressing steel centroid with respect to gross concrete section

I_g = moment of inertia of the gross concrete section

$$K_{df} = \frac{1}{1+\left(\frac{E_p}{E_{ci}}\right)\left(\frac{A_p}{A_c}\right)\left(1+\frac{A_c e_{pc}^2}{I_c}\right)\left[1+0.7\psi_b\left(t_f,t_i\right)\right]}$$

(AASHTO 5.9.5.4.3a-2)

The K_{id} coefficient is intended to represent the concrete-steel interaction in the girder before deck placement. Therefore, the age-adjusted effective modulus used in the formulation would be better defined by the creep coefficient before deck placement $\psi_b(t_d,t_i)$, rather than the creep coefficient at final time $\psi_b(t_f,t_i)$.

The equation for the transformed section coefficient for the composite section K_{df} has a similar inconsistency. Because K_{df} represents behavior in response to loads applied at the time of deck placement, the age-adjusted effective modulus should be defined using the creep coefficient for loads applied at deck placement $\psi_b(t_f,t_d)$.

Last, the transformed section coefficients are relatively insensitive to the input variables for typical cross sections. For common bridge girders, values will often be in the 0.80 to 0.90 range. Given the complexity of the calculation and the relatively steady value of the result, it may be reasonable to adopt a constant value for standard bridge types.

Conclusion

The AASHTO LRFD specifications' method for loss of prestress is a refined approach to the time-dependent analysis of prestressed girders that remains independent of any particular material property model. The computational intensity can be overwhelming for designers exposed to the method for the first time, and the apparent complexity of the equations can be intimidating. However, a thorough understanding of the fundamental concepts involved with the method's development provides clarity and improves the designer's ability to apply the provisions correctly. Despite the rigor of the method, one should remain mindful of the inherent uncertainty involved with time-dependent analysis of prestressed members. Opportunities to simplify the provisions and reduce the complexity of the equations should be explored further.

Acknowledgments

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Notation

A_c = area of composite cross section

A_d = effective composite cross-sectional area of deck concrete

A_g = gross area of cross section, including both steel and concrete

A_n = area of net concrete section

A_p = area of prestressing steel

A_{TR} = area of transformed section where the area of steel has been transformed to an equivalent area of concrete based on the modular ratio

e = eccentricity between the centroid of the girder concrete and the centroid of the prestressing steel

e_d = eccentricity of deck with respect to gross composite section, positive where deck is above girder

e_{pc} = eccentricity of prestressing force with respect to centroid of composite section, positive where centroid of prestressing steel is below centroid of composite section

e_{pg} = eccentricity of prestressing steel centroid with respect to gross concrete section

e_{pn} = eccentricity of prestressing steel centroid with respect to net concrete section

E_c = modulus of elasticity of concrete at 28 days

$E_{c,AAEM}$ = age-adjusted effective elastic modulus of concrete used to describe response of concrete to gradual stress increment considering both elastic and creep effects

E_{cd} = modulus of elasticity of deck concrete

$E_{c,eff}$ = effective modulus of elasticity of concrete used to describe the response of concrete to an instantaneous stress increment considering both elastic and creep effects

E_{ci} = modulus of elasticity of concrete at transfer

E_{ct} = modulus of elasticity of concrete at time t under consideration

E_p = modulus of elasticity of prestressing steel

f_c = stress in the concrete

f_c' = specified compressive strength of concrete

f_{cgp} = concrete stress at centroid of prestressing tendons due to prestressing force immediately after transfer and self-weight of member at section of maximum moment

f_{pj} = initial stress in the tendon after anchorage seating

f_{py} = specified yield strength of prestressing steel

I_c = moment of inertia of composite cross section

I_g = moment of inertia of gross concrete section

I_n = moment of inertia of net concrete section

k_f = adjustment factor for specified concrete compressive strength at time of transfer or end of curing

k_{hc} = adjustment factor for average ambient relative humidity in creep coefficient calculations

k_{hs} = adjustment factor for average ambient relative humidity in shrinkage calculations

k_s = adjustment factor for member size, specifically volume–to–surface area ratio

k_{td} = adjustment factor for time development that sets the rate at which shrinkage strain asymptotically approaches ultimate value (set equal to 1.0 when determining final shrinkage)

K_1 = correction factor for source of aggregate to be taken as 1.0 unless determined by physical test and as approved by authority of jurisdiction

K_{df} = transformed section coefficient that accounts for time-dependent interaction between concrete and bonded steel in section being considered after deck placement

K_{id} = transformed section coefficient that accounts for time-dependent interaction between concrete and bonded steel in section being considered for period between transfer and deck placement

L = length of concrete member just before transfer

L' = length of concrete member after transfer, considering elastic shortening due to prestress

n = ratio of elastic moduli of prestressing steel and girder concrete

P = effective tensile stress in prestressing strands just before transfer

P' = effective tensile stress in prestressing strands after transfer, considering elastic shortening losses

P_c = restraint force applied to concrete by bonded prestressing steel

P_{deck} = effective force due to deck shrinkage applied to composite section at centroid of deck

P_p = effective compression force applied to prestressing steel by shrinkage of concrete

RH = ambient relative humidity

t = time in days from stressing to transfer for relaxation calculations; time of interest after application of stress for creep calculations

t_d = age of girder concrete at time of deck placement

t_f = age of girder concrete at end of time-dependent analysis

t_i = age of concrete when load is initially applied

V/S = ratio of volume to surface area

w_c = unit weight of concrete

y_{bc} = eccentricity of concrete extreme bottom fiber with respect to centroid of composite cross section

α_n = variable representing $1+\dfrac{A_n e_{pn}^2}{I_n}$

χ = aging coefficient

Δf_c = change of stress in concrete

Δf_{cbSS} = stress increment at bottom concrete fiber due to differential shrinkage between precast concrete girder and cast-in-place composite deck

Δf_{cd} = change in concrete stress at centroid of prestressing strands due to time-dependent loss of prestress between transfer and deck placement combined with deck weight and superimposed loads

Δf_{cdf} = change in concrete stress at centroid of prestressing strands due to shrinkage of deck concrete

Δf_{pCD} = loss of prestress between time of deck placement and final time due to creep of girder concrete

Δf_{pCR} = loss of prestress between time of transfer and deck placement due to creep of girder concrete

Δf_{pES} = loss of prestress during transfer due to elastic shortening of concrete

Δf_{pR} = loss of prestress before transfer due to relaxation

Δf_{pR1} = loss of prestress between transfer and deck placement due to relaxation

Δf_{pR2} = loss of prestress after deck placement due to relaxation

Δf_{pSD} = loss of prestress between time of deck placement and final time due to shrinkage of girder concrete

Δf_{pSR} = loss of prestress between time of transfer and deck placement due to shrinkage of girder concrete

Δf_{pSS} = increase of effective prestressing force due to differential shrinkage between a precast concrete girder and cast-in-place composite deck

ε_{bdf} = shrinkage strain of girder concrete after deck placement

ε_{bid} = shrinkage strain of girder concrete between time of transfer or end of curing and time of deck placement

ε_{bif} = shrinkage strain of girder concrete between time of transfer or end of curing and final time

ε_c = strain in the concrete

ε_{cr} = creep strain

ε_{ddf} = shrinkage strain of deck concrete between time of deck placement or end of deck curing and final time

ε_{el} = elastic strain

ε_{net} = net shrinkage strain of section at centroid of prestressing steel considering shrinkage of concrete and restraint against shrinkage from bonded prestressing steel

ε_p = tensile strain in prestressing steel before transfer

ε_p' = tensile strain in prestressing steel after transfer, considering effects of elastic shortening

ε_{res} = portion of free shrinkage of concrete effectively restrained by bonded prestressing steel at centroid of prestressing steel

ε_{sh} = shrinkage strain

ε_{total} = total strain

σ_c = concrete stress

ψ = creep coefficient

$\psi(t,t_i)$ = creep coefficient for concrete at time t due to stresses applied at time t_i

$\psi_b(t_d,t_i)$ = creep coefficient for girder concrete at time t_d of deck placement due to loading applied at time t_i of transfer

$\psi_b(t_f,t_d)$ = creep coefficient for girder concrete at final time t_f due to loading applied at time t_d of deck placement

$\psi_d(t_f,t_d)$ = creep coefficient for deck concrete at final time t_f due to loading applied at time t_d of deck placement

$\psi_b(t_f,t_i)$ = creep coefficient for girder concrete at final time t_f due to loading applied at time of transfer

Appendix A: AASHTO LRFD specifications model for high-strength concrete properties

National Cooperative Highway Research Program report 496[2] recommended new material property models to better characterize the behavior of high-strength concrete. The models were developed empirically by testing representative concrete mixtures from four different states. New models were proposed for elastic modulus, shrinkage, and creep. Detailed information related to the development of the model is available elsewhere,[2,3] so only a brief summary is provided here.

Elastic modulus

American Association of State Highway and Transportation Officials' *AASHTO LRFD Bridge Design Specifications* Eq. (5.4.2.4-1) is used to predict concrete elastic modulus.

$$E_c = 33{,}000 K_1 w_c^{1.5} \sqrt{f_c'} \quad \text{(AASHTO 5.4.2.4-1)}$$

where

K_1 = correction factor for source of aggregate to be taken as 1.0 unless determined by physical test and as approved by authority of jurisdiction

w_c = unit weight of concrete

f_c' = specified compressive strength of concrete

The model for determining elastic modulus is largely unchanged from previous versions of AASHTO, still based largely on density and compressive strength. The K_1 factor has been added as an adjustment to the elastic modulus of concrete based on the stiffness of the specific coarse aggregate in the mixture. In the absence of data to calibrate K_1, a value of 1.0 is used. **Figure A1** is a schematic of the model used to predict elastic modulus, including a conceptual representation of its sensitivity to key input variables.

Shrinkage of concrete

Shrinkage of concrete is a decrease in volume primarily due to the loss of excess water over time. The AASHTO LRFD specifications model is shown in Eq. (5.4.2.3.3-1).

$$\varepsilon_{sh} = k_s k_{hs} k_f k_{td} \left(0.48 \times 10^{-3}\right) \quad \text{(AASHTO 5.4.2.3.3-1)}$$

where

k_s = adjustment factor for member size, specifically volume–to–surface area ratio

k_{hs} = adjustment factor for average ambient relative humidity in shrinkage calculations

k_f = adjustment factor for specified concrete compressive strength at time of transfer or end of curing

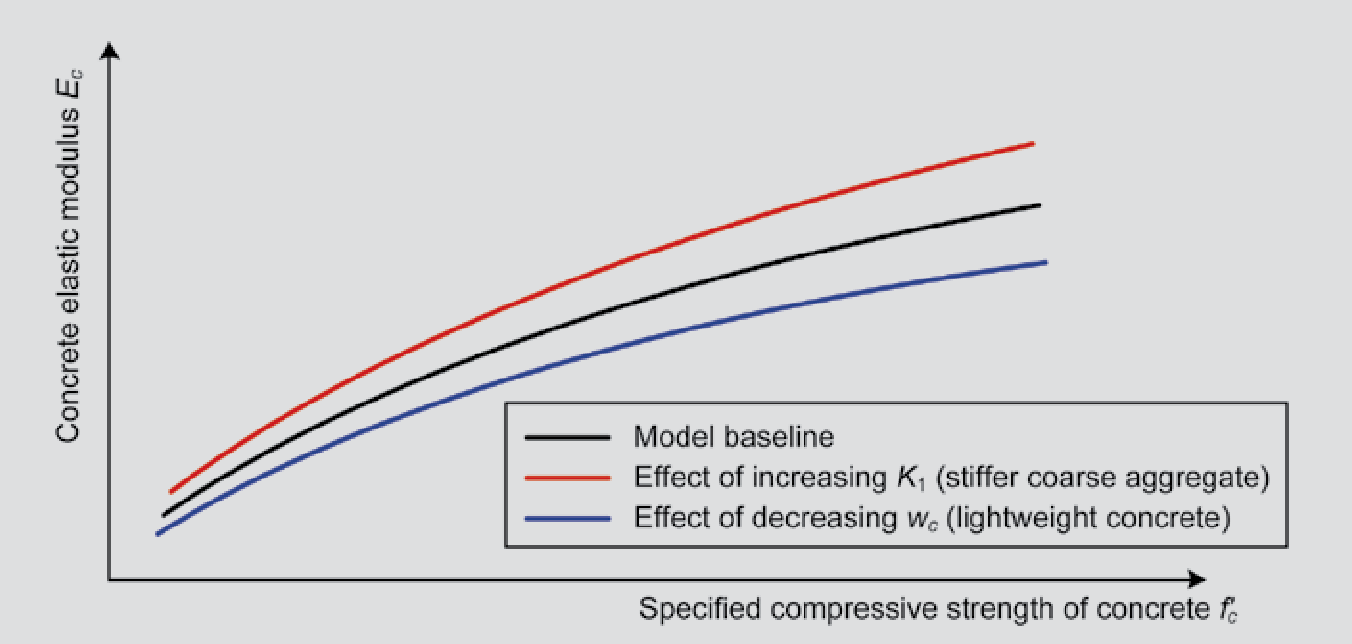

Figure A1. Schematic of the model for estimating the concrete elastic modulus demonstrating the effects of key variables. Note: K_1 = correction factor for source of aggregate to be taken as 1.0 unless determined by physical test and as approved by authority of jurisdiction; w_c = unit weight of concrete.

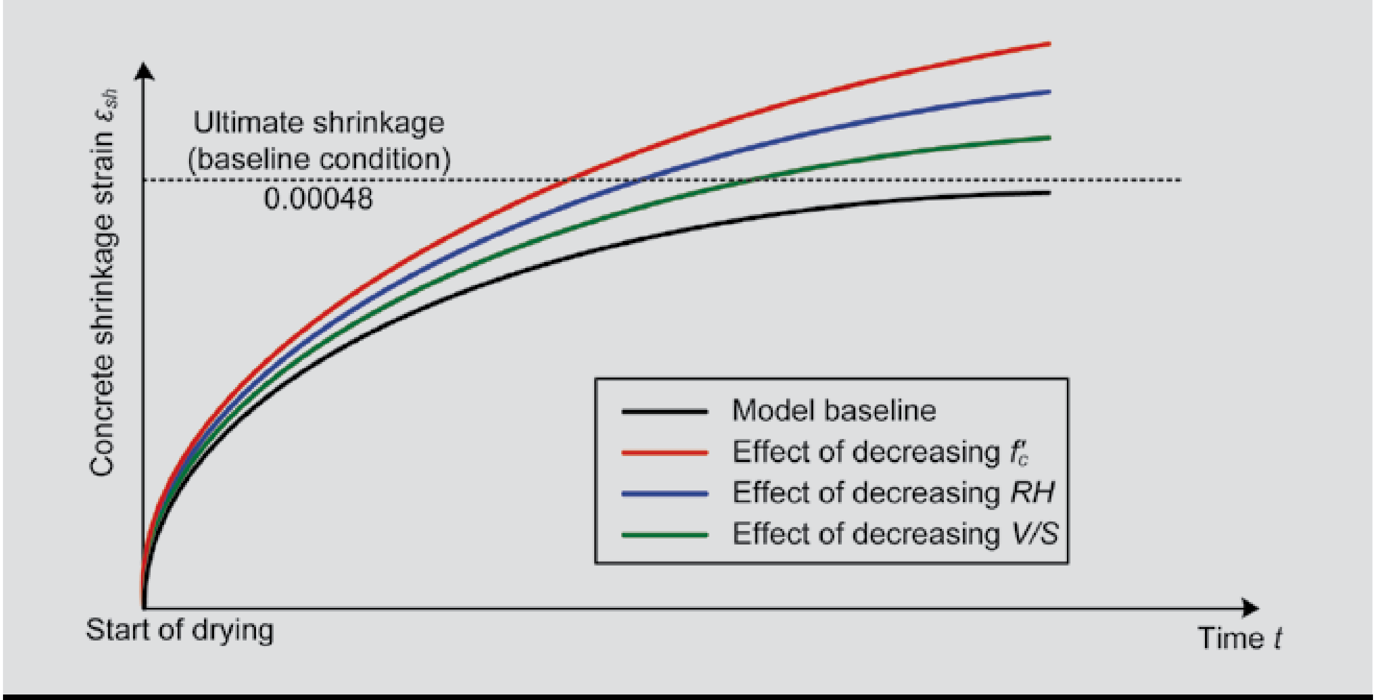

Figure A2. Schematic of the model for estimating the shrinkage strain of concrete demonstrating the effects of key variables. Note: f'_c = specified compressive strength of concrete; RH = ambient relative humidity; V/S = ratio of volume to surface area.

k_{td} = adjustment factor for time development that sets the rate at which shrinkage strain asymptotically approaches ultimate value (set equal to 1.0 when determining final shrinkage)

The equation asymptotically approaches an ultimate shrinkage value experimentally determined as 0.00048 for the baseline specimen. Adjustment factors alter the ultimate shrinkage for conditions that differ from the baseline test specimen. The time-development factor k_{td} sets the rate at which the ultimate shrinkage value is approached. **Figure A2** shows a schematic of the model used to predict shrinkage strains, including a conceptual representation of its sensitivity to key input variables.

Creep of concrete

Creep is an increase in strain due to sustained loads on the concrete (expressed schematically in **Fig. A3**).

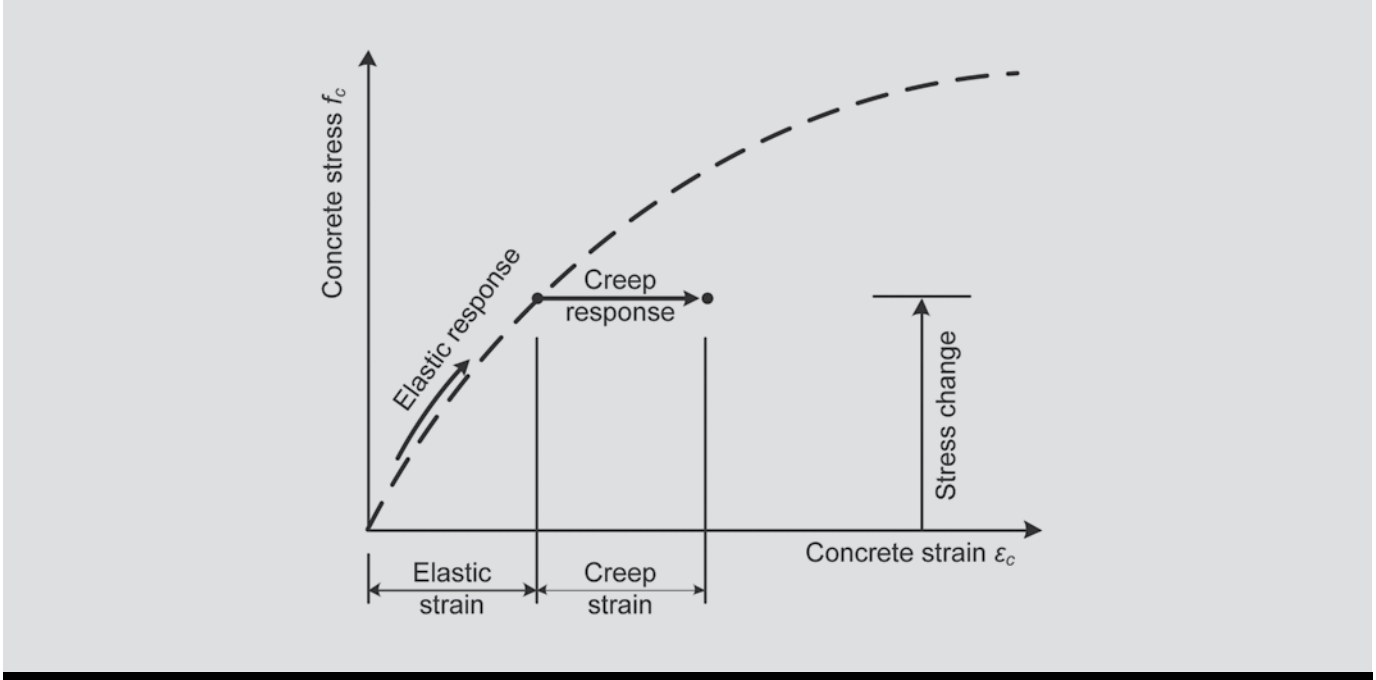

Figure A3. Schematic definition of concrete creep behavior relative to its elastic response. Note: f_c = stress in the concrete; ε_c = strain in the concrete

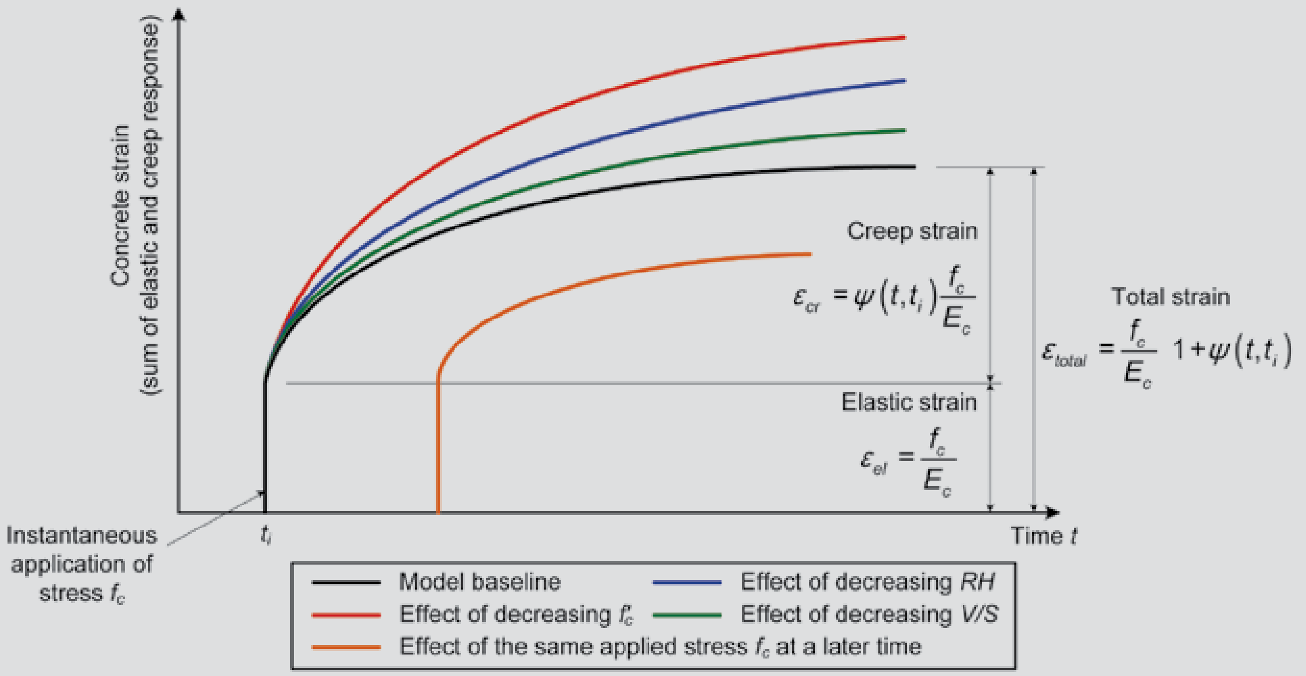

Figure A4. Schematic of the model for estimating the creep strain of concrete demonstrating the effects of key variables and defining the creep coefficient relative to the elastic strain. Note: E_c = modulus of elasticity of concrete; f_c = stress in the concrete; f_c' = specified compressive strength of concrete; *RH* = ambient relative humidity; t_i = age of girder concrete at time of prestress transfer; *V/S* = ratio of volume to surface area; ε_{cr} = creep strain; ε_{el} = elastic strain; ε_{total} = total strain; $\psi(t,t_i)$ = creep coefficient for concrete at time *t* due to stresses applied at time t_i.

In the AASHTO LRFD specifications method, the creep strain due to a given stress increment is expressed in terms of the elastic strain caused by the same stress. The ratio of the creep strain to the elastic strain is termed the creep coefficient. The creep coefficient ψ at time t due to a stress change that occurs at time t_i is determined by the following equation.

$$\psi(t,t_i) = 1.9 k_s k_{hc} k_f k_{td} t_i^{-0.118} \quad \text{(AASHTO 5.4.2.3.2-1)}$$

where

k_{hc} = adjustment factor for average ambient relative humidity in creep coefficient calculations

t_i = age of concrete when load is initially applied

Similar to the shrinkage model, the equation for the creep coefficient asymptotically approaches an ultimate value that is initially set for baseline conditions. A series of correction factors adjusts the ultimate values for conditions other than those used in developing the baseline equation. The creep coefficient is also based largely on the age of the concrete when the load is applied. Stress applied at a later age will lead to smaller creep strains than the same stress change at an earlier age. **Figure A4** shows a schematic of the model used to predict creep strains, including a conceptual representation of its sensitivity to key input variables. Although the model was developed using specimens under sustained compressive stress, AASHTO inherently assumes that the same model can be used to represent time-dependent strains following a tension stress increment.

Appendix B: Transformed section coefficient (derivation)

To aid understanding of the transformed section coefficients K_{id} and K_{df}, the K_{id} equation will be derived. The derivation of K_{df} is similar but with respect to the composite (girder plus deck) section properties rather than the girder section properties. The development of the equation is similar for both shrinkage and creep. For the sake of consistency throughout the presentation, only shrinkage will be discussed. The main ideas of this derivation are documented by Tadros et al.[2]

The derivation references Fig. 3. The shrinkage strain distribution across the girder section is affected by the presence of bonded prestressing steel. ε_{sh} is the free shrinkage of concrete that would exist without any internal restraint. ε_{res} denotes the reduction in shrinkage at the centroid of the prestressing caused by the steel's restraint. The net change in strain at the centroid of the prestressing is given by ε_{net}. The transformed section coefficient is the ratio of the net strain to the free strain expressed in Eq. (24).

$$K_{id} = \frac{\varepsilon_{net}}{\varepsilon_{sh}} \tag{24}$$

Three assumptions are made in developing an equation for K_{id}:

- The shrinkage strain is uniform over the cross section.
- The concrete would undergo a free shrinkage ε_{sh} in the absence of prestressing steel.
- Compatibility requires the same strain in the steel as in the surrounding concrete.

Figure 3 shows that the shrinkage of the surrounding concrete imposes a strain ε_{net} on the prestressing steel. The effective compressive force imposed on the steel by concrete shrinkage P_p can be calculated using Eq. (25).

$$P_p = A_p E_p \varepsilon_{net} \tag{25}$$

For equilibrium within the cross section, an equal and opposite force must be applied to the concrete. Moreover, the effective force on the concrete causes a strain ε_{res} at the centroid of the prestressing steel. Equation (26) can be written recognizing that the stress in the concrete at the centroid of the prestressing must be equal to the product of the strain ε_{res} and the modulus of elasticity of the concrete. Furthermore, the stress is caused by the combined axial and eccentricity effects of the effective tensile force P_c applied to the concrete from the prestressing steel.

$$\varepsilon_{res} E_{ct} = \frac{P_c}{A_n} + \frac{P_c e_{pn}^2}{I_n} \tag{26}$$

where

E_{ct} = modulus of elasticity of concrete at time t under consideration

e_{pn} = eccentricity of prestressing steel centroid with respect to net concrete section

I_n = moment of inertia of net concrete section

Equation (26) can be solved for the effective force P_c applied to the concrete shown in Eq. (27).

$$P_c = \frac{\varepsilon_{res} E_{ct} A_n}{1 + \dfrac{A_n e_{pn}^2}{I_n}} = \frac{\varepsilon_{res} E_{ct} A_n}{\alpha_n} \tag{27}$$

where

α_n = variable representing $1 + \dfrac{A_n e_{pn}^2}{I_n}$

The force P_c will accumulate gradually, so it is appropriate to substitute an age-adjusted effective modulus for the elastic modulus if creep effects are also being considered. Equation (12) defines the age-adjusted effective modulus. Replacing E_{ct} in Eq. (27) with $E_{c,AAEM}$ as defined in Eq. (12) ($E_{c,AAEM}$ is replaced with $E_{ct,AAEM}$ and E_c with E_{ct}) yields Eq. (28).

$$P_c = \frac{\varepsilon_{res} E_{ct} A_n}{\alpha_n \left\{1 + \chi\left[\psi\left(t, t_i\right)\right]\right\}} \tag{28}$$

The restraint strain ε_{res} in Eq. (28) can be represented as the difference between the free shrinkage strain and the net strain given in Eq. (29).

$$\varepsilon_{res} = \varepsilon_{sh} - \varepsilon_{net} \tag{29}$$

Requiring equilibrium of forces, the terms in Eq. (28) and (25) can be set equal to one another. Also, Eq. (29) will be substituted into Eq. (28) to remove the unknown ε_{res}. Solving for the ratio

$$\frac{\varepsilon_{net}}{\varepsilon_{sh}},$$

which is the definition of the transformed section coefficient K_{id} (Eq. [24]), produces Eq. (30).

$$K_{id} = \frac{\varepsilon_{net}}{\varepsilon_{sh}} = \frac{1}{1+\left(\frac{A_p}{A_n}\right)\left(\frac{E_p}{E_{ct}}\right)\alpha_n\left\{1+\chi\left[\psi\left(t,t_i\right)\right]\right\}} \tag{30}$$

Equation (30) closely resembles the formulation given in the American Association of State Highway and Transportation Officials' *AASHTO LRFD Bridge Design Specifications*. The following assumptions are made to arrive at the AASHTO LRFD equation:

- Gross section properties are substituted for net section properties.
- The time of interest for the concrete elastic modulus is assumed to be the time of transfer. Therefore E_{ct} will be replaced with E_{ci}.
- The aging coefficient χ will be assigned a constant value of 0.7 as supported by the work of Dilger[11] and explained previously.
- The time of interest in the creep coefficient used in the age-adjusted effective modulus term is assumed to be the final time t_f.

The assumptions, when substituted in Eq. (30), produce the following AASHTO equation for K_{id}:

$$K_{id} = \frac{1}{1+\left(\frac{E_p}{E_{ci}}\right)\left(\frac{A_p}{A_g}\right)\left(1+\frac{A_g e_{pg}^2}{I_g}\right)\left\{1+0.7\left[\psi\left(t_f,t_i\right)\right]\right\}}$$

(AASHTO 5.9.5.4.2a-2)

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Abstract

This paper details the American Association of State Highway and Transportation Officials' *AASHTO LRFD Bridge Design Specifications* time-dependent analysis method used for determining the loss of prestress in pretensioned bridge girders. The fundamental mechanics on which the method relies are explained in detail. New concepts introduced as part of the 2005 interim revisions are clarified. This paper aims to make the loss of prestress method more widely understood and more accurately applied in practice.

Keywords

Creep, LRFD, prestress gain, prestress loss, relaxation, shrinkage, transformed section.

Review policy

This paper was reviewed in accordance with the Precast/Prestressed Concrete Institute's peer-review process.

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