

# Calculating shear friction using an effective coefficient of friction

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Since the second edition of the *PCI Design Handbook*<sup>1</sup> was published in 1978, the use of an effective coefficient of friction  $\mu_e$  has been promoted by PCI instead of the coefficient of friction  $\mu$  per *Building Code Requirements for Structural Concrete (ACI 318-05) and Commentary (ACI 318R-05)*.<sup>2</sup> PCI's basis for the use of  $\mu_e$  was justified by research and publications by A. Fattah Shaikh<sup>3</sup> and relationships presented by Charles H. Raths.<sup>4</sup> These works followed Allan H. Mattock's earlier research and articles proposing the methodology and concepts pertaining to shear friction.<sup>5</sup>

Many engineers have used shear friction, as proposed by the *PCI Design Handbook*, to design a variety of precast concrete connections. Connections such as beam bearings, daps, and corbels are some of the most critical connections in precast concrete construction because they support permanent gravity loads with little redundancy.

Considering the sixth edition of the *PCI Design Handbook*<sup>6</sup> and the new load factors and  $\phi$  factors as presented in ACI 318, this paper intends to discuss several mathematical anomalies in the *PCI Design Handbook* equations for  $\mu_e$  that have been exacerbated by the new codes. These anomalies, as presented in the sixth edition of *PCI Design Handbook*, produce substantially less conservative results than originally intended in Shaikh's research.

This paper will focus on the mathematical relationships to produce results per the original data and findings of Shaikh

## Editor's quick points

- This paper examines the current formulas used for calculating the effective coefficient of friction  $\mu_e$  in the *PCI Design Handbook*.
- Original test data are compared with these equations and formulas as developed by A. Fattah Shaikh and Charles Raths.
- Mathematical modifications and clarifications are recommended for the *PCI Design Handbook* formulas in the calculation of  $\mu_e$ .

and others. All notations follow those provided in chapter 2 of ACI 318-05.

## Shear-friction formulas using $V_n$ or $V_u$

At the time Shaikh and Mattock were conducting their research, the nomenclature that was used to define loads and capacities was in transition to today's format in which the subscripts  $n$  and  $u$  represent capacity (or strength) and applied factored loads, respectively. At the time their research was published, the term  $V_u$  might have referred to strength or to a load, depending on the wording and usage. However, the terms can only be interchanged with regard to the basic equation  $\phi V_n \geq V_u$ , where  $\phi$  is the strength-reduction factor. This is the basis for the first mathematical anomaly.

Shaikh presented his test data in a graph of  $\mu_e$  compared with nominal shear stress  $v_u$  (Figure 3 of his article).<sup>3</sup> However, per the wording and the usage, the plotted data are nominal shear stresses  $v_n$ . The calculations of  $\mu_e$  as proposed by Birkeland,<sup>7</sup> Raths, and Mattock were also calculated using the nominal shear stress values  $v_n$ . The equation, as presented by Raths, was concluded to represent the best reasonable lower bound to the test data. Note that for Raths's equation,  $\mu_e = 1.4$  at  $v_{n,max} = 1000$  psi (6.9 MPa). Therefore, the original plotted equation should have been written as Eq. (1).

$$\mu_e = 1000\lambda^2 A_{cr} \mu / V_n \quad (1)$$

where

$A_{cr}$  = area of shear-crack interface

$\lambda$  = modification factor related to unit weight of concrete

For normalweight concrete,  $\lambda^2 = 1$  as used in the original test data. Therefore, it does not affect the results for normalweight concrete.

The maximum value for  $V_n$  was set to  $1000\lambda^2 A_{cr}$  or  $0.3\lambda^2 f'_c A_{cr}$ , where  $f'_c$  is the specified compressive strength of concrete. With concrete having a strength of 3333 psi (23.0 MPa) or greater, the  $1000A_{cr}$  term governed, which is the governing case for most precast concrete strengths. This was the equation in the fourth and fifth editions of the *PCI Design Handbook*.<sup>8,9</sup> However, in the sixth edition, the term  $V_n$  was changed to  $V_u$ . With the change in  $\phi$  factors from 0.85 to 0.75, the equations deviate from the original as graphed by Shaikh by an even greater margin.

The values of  $V_n$  should not change from one *PCI Design Handbook* edition to the next or from ACI 318-99 to ACI 318-02 unless the underlying methodology in the calcu-

lation of  $V_n$  changes. **Figure 1** shows the effects of this interchanging  $V_n$  for  $V_u$  without maintaining the basic relationship of  $\phi V_n \geq V_u$ . The graphs are for monolithic normalweight concrete with  $f'_c > 3333$  psi (23.0 MPa). This eliminates the issue of the  $\lambda^2$  term and concrete strengths. Figure 1 shows  $\mu_e$  per ACI 318-05, Shaikh research, *PCI Design Handbook* fourth and fifth editions, and the current *PCI Design Handbook*, sixth edition. With the new  $\phi$  factor of 0.75 for shear,  $V_n$  has increased 33.33% from the original intent and the amount of shear-friction steel has been reduced 25% to get the same nominal capacity. These are significant variations as indicated in the following example calculation.

Given:

$$V_d = 100 \text{ kip}$$

$$V_l = 35 \text{ kip}$$

$$f'_c = 5000 \text{ psi}$$

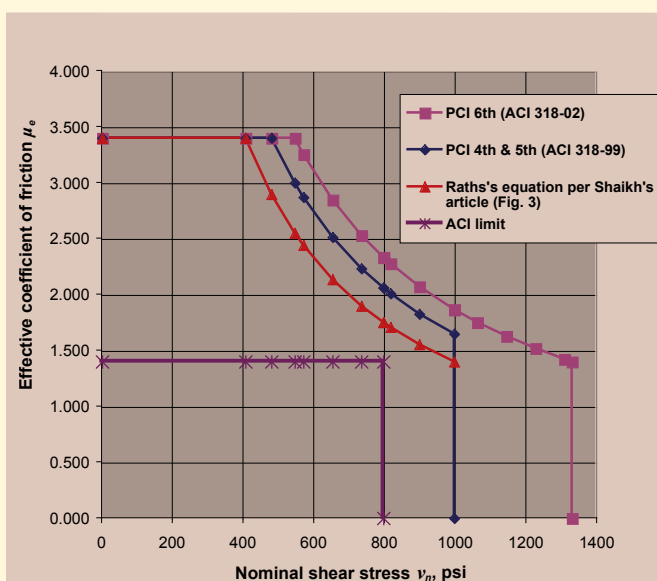
$$A_{cr} = 250 \text{ in.}^2$$

$$\text{concrete density} = 150 \text{ lb/ft}^3$$

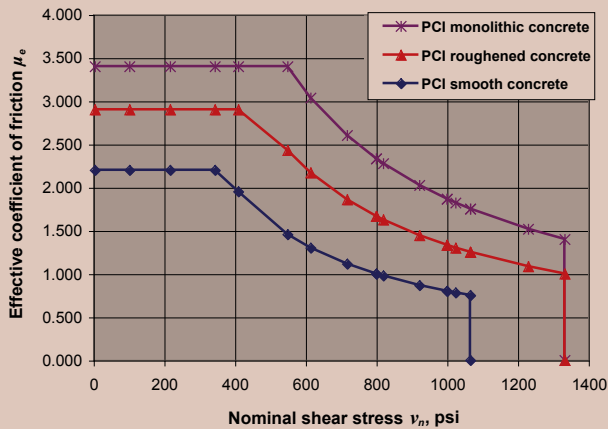
$$\lambda = 1.0$$

$$\mu = 1.4\lambda = 1.4(1.0) = 1.4$$

$$V_n = 234.7 \text{ kip}$$



**Figure 1.** This graph compares the results from four sources for the effective coefficient of friction  $\mu_e$  with nominal shear stress  $v_n$  for monolithic normalweight concrete. Sources: Data from *PCI Design Handbook* 1992, 1999, and 2004; Shaikh 1978; Raths 1977; and *Building Code Requirements for Structural Concrete (ACI 318-05) and Commentary (ACI 318R-05)* 2005. Note: 1 psi = 6.895 kPa.



**Figure 2.** This graph compares various crack-interface conditions represented in the sixth edition of the *PCI Design Handbook* for effective coefficient of friction  $\mu_e$  with nominal shear stress  $v_n$  for normalweight concrete. Note: 1 psi = 6.895 kPa.

Using test data:

$$V_n = 234.7 \text{ kip (1044 kN)}$$

$$\mu_e = 1.4(1000)(250)/234,700 = 1.491$$

$$A_{vf} = \frac{234.7}{1.49(60)} = 2.62 \text{ in.}^2 \text{ (1690 mm}^2\text{)}$$

According to the fourth and fifth editions of the *PCI Design Handbook*:

$$V_u = 1.4(100) + 1.7(35) = 199.5 \text{ kip (887 kN)}$$

$$V_n \geq 199.5/0.85 = 234.7 \text{ kip (1044 kN)}$$

$$\mu_e = 1.4(1000)(250)/199,500 = 1.754$$

$$A_{vf} = \frac{199.5}{1.754(0.85)(60)} = 2.23 \text{ in.}^2 \text{ (1439 mm}^2\text{)}$$

According to the sixth edition of the *PCI Design Handbook*:

$$V_u = 1.2(100) + 1.6(35) = 176 \text{ kip (783 kN)}$$

$$V_n \geq 176/0.75 = 234.7 \text{ kip (1044 kN)}$$

$$\mu_e = 1.4(1000)(250)/176,000 = 1.989$$

$$A_{vf} = \frac{176}{1.989(0.75)(60)} = 1.97 \text{ in.}^2 \text{ (1270 mm}^2\text{)}$$

where

$A_{vf}$  = area of shear-friction reinforcement

$V_l$  = shear force at section due to unfactored live load

$V_d$  = shear force at section due to unfactored dead load

Note that  $V_n$  is the same value in all cases; however, the amount of shear-friction steel  $A_{vf}$  decreased.

## $\mu_e$ should equal $\mu$ at $V_{n,max}$

In the original plots of  $\mu_e$  to the test data, all assumed  $\mu_e$  should approach and be equal to  $\mu$ , per ACI, as the capacity of the interface was reached. **Figure 2** shows the current sixth-edition *PCI Design Handbook* calculations of  $\mu_e$  for varying crack interfaces. The concrete strength is assumed to be greater than 4000 psi (27.6 MPa) and the concrete density is about 150 lb/ft<sup>3</sup> (2400 kg/m<sup>3</sup>). This again eliminates the  $\lambda^2$  term and  $f'_c$  from the discussion.

Figure 2 illustrates the second mathematical anomaly in that  $\mu_e$  never gets to  $\mu$ . The graph represents the current *PCI Design Handbook* equation  $\mu_e = 1000\lambda A_{cr}\mu/V_u$ . The plots continue beyond the  $V_n$  limit of  $1000A_{cr}$  until the value of  $\mu_e = \mu$ . Note that for all interface conditions, for  $\mu_e$  to equal  $\mu$ ,  $v_n = V_n/A_{cr}$  must be greater than the 1000 psi (7 MPa) limit.

The equation for  $\mu_e$  in terms of  $V_u$  results in non-conservative values for  $\mu_e$  for all crack interfaces, even when  $V_n$  is limited to  $1000A_{cr}$ . When  $V_n = 1000A_{cr}$ ,  $\mu_e = 1.867$  for monolithic conditions and  $\mu_e = 1.333$  for concrete-to-roughened concrete interface, both values of  $\mu_e$  are greater than  $\mu$  of 1.4 and 1.0, respectively.

The solution is to make  $\mu_e$  a function of  $\mu V_{n,max}/V_n$ . Therefore, as  $V_n$  approaches  $V_{n,max}$ ,  $\mu_e$  will approach  $\mu$ . The  $1000A_{cr}$  term should be replaced by the maximum nominal shear capacity  $V_{n,max}$ .

## The $\lambda^2$ term

Since the inception of the term  $\mu_e$  in the second edition of the *PCI Design Handbook*, there has been confusion as to the actual equation for  $\mu_e$  concerning the  $\lambda$  term. The equation is written by PCI as  $\mu_e = 1000\lambda A_{cr}\mu/V_u$ . The confusion comes from the term  $\mu$ —is it 1.4 for monolithic concrete or  $1.4\lambda$ ? As directed by the statement to use the value in *PCI Design Handbook* table 4.3.6.1, you would substitute  $1.4\lambda$  for  $\mu$  and get a  $\lambda^2$  term. Over the years, the *PCI Design Handbook* has shown calculations for  $\mu_e$  in both formats. For example, in the fifth edition on pages 4-55 and 4-62, the calculation of  $\mu_e$  used a single  $\lambda$  term, and on page 4-36,  $\mu_e$  is calculated with  $\lambda^2$ . However, the current edition does not show any calculations of  $\mu_e$  using the  $\lambda^2$  term (pages 4-55, 4-78, 4-82, and 6-49). In all of the examples cited,  $\lambda$  is 1.0 and does not factor into the results.

Some of the confusion starts with ACI writing  $\mu$  as  $1.4\lambda$ ,  $1.0\lambda$ ,  $0.7\lambda$ , or  $0.6\lambda$ . A better way might have been for ACI Eq. (11-25) to be written as Eq. (2).

$$V_n = A_{vf} f_y \mu \lambda \quad (2)$$

where

$\mu$  and  $\lambda$  are separate variables:

$\mu = 1.4, 1.0, 0.6, \text{ or } 0.7$

$\lambda = 1.0, 0.85, \text{ or } 0.75$

$f_y = \text{specified yield strength of reinforcement}$

The  $\lambda^2$  term is an interesting occurrence for the calculation of  $V_{n,max}$ . In all other chapters of ACI 318 and the *PCI Design Handbook*, the nominal shear capacities are a function of  $\lambda$ , not  $\lambda^2$ . The comments in chapter 11 of ACI 318-05 concerning lightweight concrete do not indicate this magnitude of strength reduction.

The  $\lambda^2$  term as part of the calculation of  $\mu_e$  seems to fit the data well. The following mathematics is presented to eliminate some of the confusion. It is proposed that the equation for  $\mu_e$  be written as Eq. (3).

$$\mu_e = \mu V_{n,max} / V_n \text{ or } \mu_e = \mu \phi V_{n,max} / V_u \quad (3)$$

where

$$\begin{aligned} V_{n,max} &= 0.30 \lambda f'_c A_{cr} \leq 1000 \lambda A_{cr} \text{ for monolithic interface} \\ &= 0.25 \lambda f'_c A_{cr} \leq 1000 \lambda A_{cr} \text{ for roughened concrete-to-concrete interface} \\ &= 0.20 \lambda f'_c A_{cr} \leq 800 \lambda A_{cr} \text{ for either concrete-to-concrete or concrete-to-steel interface} \end{aligned}$$

$\mu = 1.4\lambda, 1.0\lambda, 0.7\lambda, \text{ or } 0.6\lambda$ , depending on the crack interface

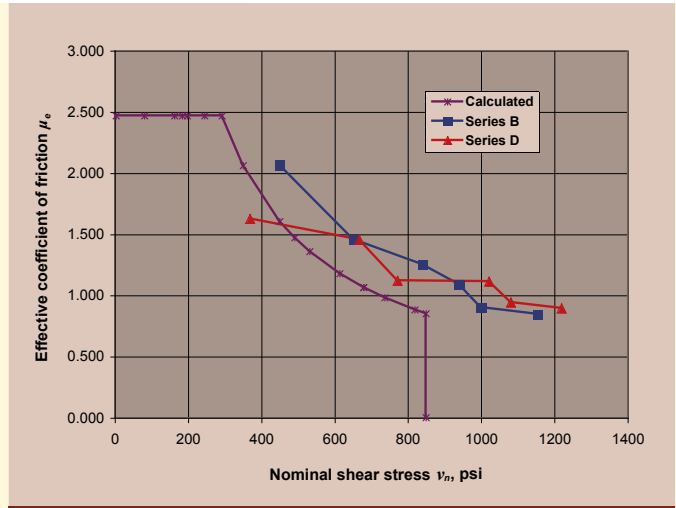
Note that  $\lambda$  is part of  $\mu$ .

Therefore, the equation for  $\mu_e$ , using a monolithic interface and strengths greater than 3333 psi (23.0 MPa), becomes Eq. (4).

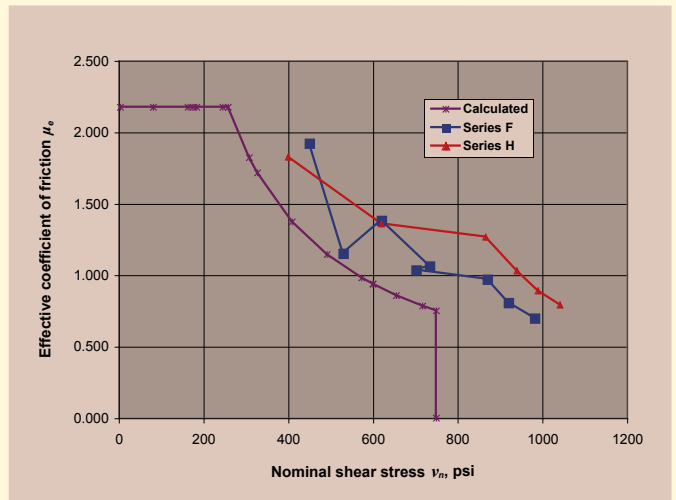
$$\mu_e = 1.4 \lambda (\phi 1000 \lambda A_{cr} / V_u) = \phi 1.4 \lambda^2 1000 A_{cr} / V_u \quad (4)$$

This mathematically incorporates the  $\lambda^2$  term into the calculation of  $\mu_e$  and a single  $\lambda$  term in the calculation  $V_{n,max}$ .

**Figures 3 and 4** are graphs comparing the data from Mattock's work<sup>5</sup> and the proposed formula for roughened concrete-to-concrete interface. Figure 3 represents the data from Mattock's test series B and D, representing data for the 4000 psi (28 MPa) sanded, lightweight concrete and an initially cracked interface. Figure 4 represents the data from Mattock's test series E and F, representing data



**Figure 3.** This graph compares Mattock's (1976) test series B and D research data with the author's proposed modifications to the effective coefficient of friction  $\mu_e$  with the resulting nominal shear stress  $v_n$  for sanded, lightweight concrete. Note:  $f'_c > 4000$  psi;  $\lambda = 0.85$ ;  $\mu = 1.0$  as defined by roughened concrete-to-concrete interface for initially cracked. 1 psi = 6.895 kPa.



**Figure 4.** This graph compares Mattock's (1976) test series E and F research data with the author's proposed modifications to the effective coefficient of friction  $\mu_e$  with the resulting nominal shear stress  $v_n$  for all lightweight concrete. Note:  $f'_c > 4000$  psi;  $\lambda = 0.75$ ;  $\mu = 1.0$  as defined by roughened concrete-to-concrete interface for initially cracked. 1 psi = 6.895 kPa.

for the 4000 psi all-lightweight concrete and an initially cracked interface. These data are shown in Fig. 9 of Mattock's article. The data, per Mattock's notation, were plotted in terms of  $\mu_e = \rho f_y / v_u$  compared with  $v_u$ . However, the term  $v_u$  is actually  $v_n$  because it is the nominal shear stress as defined by the article.

These graphs indicate a good correlation with the limited test data and the proposed equation. The maximum value for  $V_n$  with the  $\lambda^2$  does not seem warranted. However, the calculation of  $\mu_e$  does get the  $\lambda^2$  as part of mathematics and Raths' original equation.

**Table 1.** PCI Design Handbook, sixth edition, proposed Table 4.3.6.1 modifications

Crack-interface condition	Recommended $\mu$	Maximum $\mu_e$	$V_{n,max}$
Concrete to concrete, cast monolithically	$1.4\lambda$	$3.4\lambda$	$0.30\lambda f'_c A_{cr} \leq 1000\lambda A_{cr}$
Concrete to hardened concrete, with roughened surface	$1.0\lambda$	$2.9\lambda$	$0.25\lambda f'_c A_{cr} \leq 1000\lambda A_{cr}$
Concrete to concrete	$0.6\lambda$	$2.2\lambda$	$0.20\lambda f'_c A_{cr} \leq 800\lambda A_{cr}$
Concrete to steel	$0.7\lambda$	$2.4\lambda$	$0.20\lambda f'_c A_{cr} \leq 800\lambda A_{cr}$

Note:  $A_{cr}$  = area of shear-crack interface;  $f'_c$  = specified compressive strength of concrete;  $V_{n,max}$  = maximum nominal shear capacity;  $\lambda$  = modification factor related to density of concrete;  $\mu$  = coefficient of friction;  $\mu_e$  = effective coefficient of friction.

### $\mu_e$ maximum is a function of $\lambda$

There is a final query as to the values of  $\mu_e$  maximum and the effects of lightweight concrete. Per ACI 318-05, if  $\mu$  is a function of  $\lambda$ , should  $\mu_e$  maximum also be a function of  $\lambda$ ? It is the author's opinion that it would also be prudent to limit  $\mu_{e,max}$  by  $\lambda$ , if only to be conservative.

### Conclusion

Shear friction and  $\mu_e$  are widely used for a variety of critical connections in precast concrete construction. Many of these connections support sustained gravity loads with little or no redundancy.

In the spirit of the original data as presented by Shaikh, the author would like to present the following mathematical modifications to the sections of the *PCI Design Handbook* pertaining to shear friction and the calculation of  $\mu_e$ . Equation (5) and **Table 1** are the proposed modifications to the sixth-edition *PCI Design Handbook* Eq. (4.3.6.2) and

Table 4.3.6.1, respectively.

Modifications to Eq. (4.3.6.2) are as follows:

$$\mu_e = \mu V_{n,max}/V_n \text{ or } \mu_e = \mu \phi V_{n,max}/V_u \quad (5)$$

where

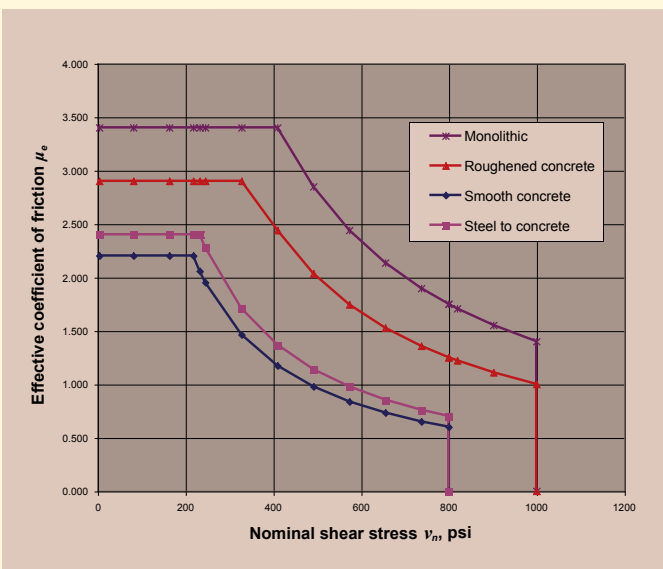
$\lambda = 1.0$  for normalweight concrete

$\lambda = 0.85$  for sanded, lightweight concrete

$\lambda = 0.75$  for all-lightweight concrete

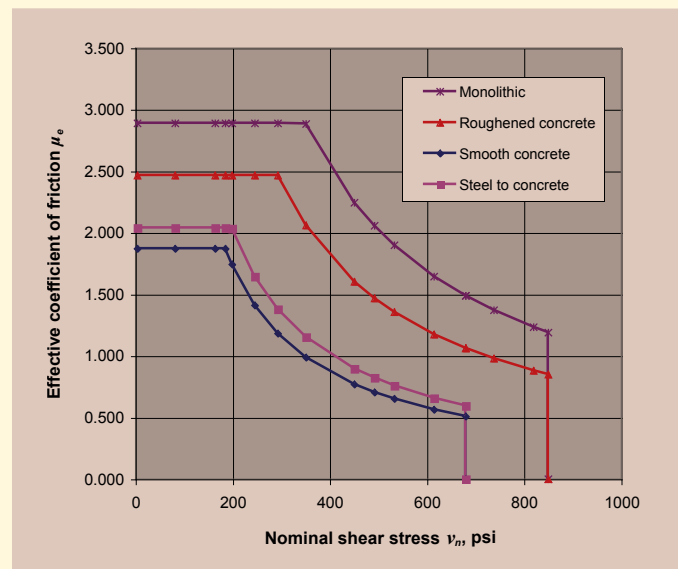
$\mu = 1.4\lambda, 1.0\lambda, 0.7\lambda, \text{ or } 0.6\lambda$ , depending on the crack interface

**Figures 5, 6, and 7** illustrate the proposed modifications for the various crack interfaces and concrete densities for concrete strengths in the normal precast concrete range of 4000 psi (28 MPa) or greater.



**Figure 5.** This graph compares the proposed modifications with the effective coefficient of friction with the resulting nominal shear stress  $v_n$  for normalweight concrete with various crack interfaces and concrete densities.

Note:  $f'_c > 4000$  psi;  $\lambda = 1.0$ . 1 psi = 6.895 kPa.



**Figure 6.** This graph compares the proposed modifications to the effective coefficient of friction  $\mu_e$  with the resulting nominal shear stress  $v_n$  for sanded, lightweight concrete with various crack interfaces and concrete densities.

Note:  $f'_c > 4000$  psi;  $\lambda = 0.85$ . Note: 1 psi = 6.895 kPa.

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## Notation

$A_{cr}$  = area of shear-crack interface

$A_{vf}$  = area of shear friction

$f_y$  = specified yield strength of reinforcement

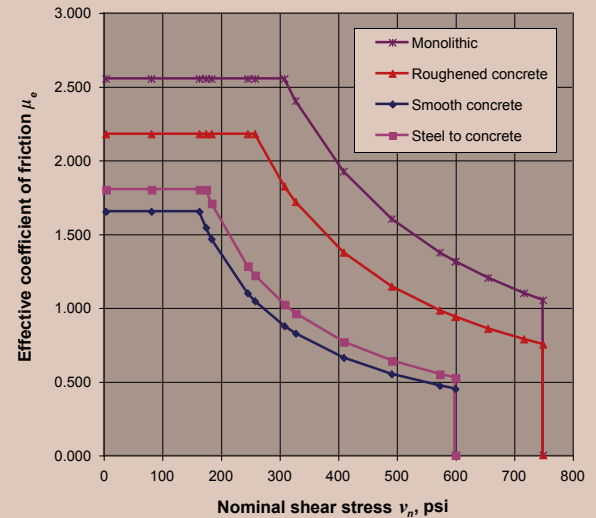
$f'_c$  = specified compressive strength of concrete

$v_n$  = nominal shear stress

$v_{n,max}$  = maximum nominal shear stress

$v_u$  =  $v_n$ , as defined by Mattock<sup>5</sup>

= nominal ultimate shear stress



**Figure 7.** This graph compares the proposed modifications to the effective coefficient of friction  $\mu_e$  with the resulting nominal shear stress  $v_n$  for all lightweight concrete with various crack interfaces and concrete densities. Note:  $f'_c > 4000$  psi;  $\lambda = 0.75$ . 1 psi = 6.895 kPa.

$$= V_u / \phi A_{cr}$$

$V_l$  = shear force at the section due to unfactored live load

$V_d$  = shear force at the section due to unfactored dead load

$V_n$  = nominal shear strength

$V_{n,max}$  = maximum nominal shear capacity

$V_u$  = factored shear force at section

$\lambda$  = modification factor related to density of concrete

$\mu$  = coefficient of friction

$\mu_e$  = effective shear-friction coefficient

$$\mu_{e,max} = A_s / bd$$

= ratio of nonprestressed compression reinforcement

$\phi$  = strength-reduction factor

## About the author



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## Synopsis

This paper examines the current formulas used for the calculation of the effective coefficient of friction  $\mu_e$  in *PCI Design Handbook: Precast and Prestressed Concrete*. It is not a commentary on the underlying methodology, but an examination of the original test data compared with these equations. Several mathematical modifications and clarifications are recommended to the formulas in the calculation of  $\mu_e$ . These modifica-

tions are consistent with the spirit of the original test data and formulas developed by A. Fattah Shaikh and Charles Raths.

## Keywords

Effective coefficient of friction, factored shear force, nominal shear strength, shear friction.

## Review policy

This paper was reviewed in accordance with the Precast/Prestressed Concrete Institute's peer-review process.

## Reader comments

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