Fatigue of Deformed Welded-Wire Reinforcement

Using welded-wire reinforcement (WWR) as an alternative to traditional mild steel reinforcing bars has many advantages. WWR has a higher yield strength and is produced under higher quality control standards. Its use also results in fewer labor costs associated with construction. However, many designers are reluctant to use WWR as an alternative to mild steel reinforcing bars due to the unavailability of fatigue design guidance in the American Association of State Highway and Transportation Officials’ AASHTO LRFD Bridge Design Specifications. This paper reports on a fatigue testing program of deformed, high-strength WWR. Based on the results of this testing program, a conservative stress range formula for WWR is presented. This same formula has been adopted for use in the 2007 Interim AASHTO LRFD Bridge Design Specifications. In addition, full monotonic axial tensile stress-strain relationships are presented.

Welded-wire reinforcement (WWR) has many advantages over traditional mild steel reinforcing bars. WWR boasts greater yield strength, is produced under tighter quality control standards, and can significantly lower construction costs associated with on-site workers. However, many design professionals are reluctant to use WWR as a structural reinforcement alternative to mild steel reinforcing bars due to a lack of fatigue design guidance in the American Association of State Highway and Transportation Officials’ AASHTO LRFD Bridge Design Specifications (referred to as “AASHTO specifications” in the remainder of this paper).¹
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The AASHTO specifications do not require the designer to check girder web reinforcement for fatigue. The AASHTO specifications do limit shear capacity, however, such that the web is designed not to crack under service loads. However, as the concrete design community is permitted to use increasingly greater shear capacities by the AASHTO specifications, compared with earlier AASHTO specifications and versions of ACI 318, fatigue may become an issue. When designed according to the AASHTO specifications and with greater concrete compressive strengths, sections may crack at unfactored service loads, making fatigue of the reinforcement an important design issue.

The fatigue limits proposed in this paper have already been adopted in the 2007 Interim AASHTO LRFD Bridge Design Specifications. This paper provides the background and conditions used for developing the new fatigue limits. These limits supplement the limits already in existence for mild reinforcement, prestressing strands, and structural steel members.

BACKGROUND AND PREVIOUS WORK

Structural members fail due to fatigue when cracking develops under repetitive loads that are less than their static load capacity. Three sequential stages lead to fatigue of the member. The process starts with the initiation of cracking, followed by propagation of cracking, in which microcracking gradually takes place in the concrete or cracking grows in a...
Steel element. Slow crack growth is followed by a brief period of quick growth, which leads to the third stage: fracture.

The concrete design philosophy of the AASHTO specifications is to use infinite life loading and stress limits for concrete and steel. This philosophy requires that the endurance limit for the individual components of the structural concrete be established. An endurance limit is the limiting stress range below which the specimen will be able to sustain a virtually unlimited number of loading cycles. The endurance limit is defined in some studies as the stress range corresponding to 2 million cycles, and in others to 5 million cycles. In this study the more conservative approach is adopted. Fatigue limits have been included in the AASHTO specifications for reinforcing bars and strands. The following sections summarize the limited previous research on WWR and the background on how these limits were determined for the AASHTO specifications.

**Fatigue Studies on Mild Reinforcing Bars**

Tilly summarized the factors that influence the fatigue life of reinforcing bars and distinguished between the important factors and the minor factors. Tilly indicated that the important variables included stress range, minimum stress, deformation geometry of a bar, radius of bends, welding, and corrosion. Factors that have minor effects on fatigue strength include bar size, bar orientation, yield strength, and chemical composition.

The current reinforcing bar fatigue formula was developed in a 1976 research project conducted for the National Cooperative Highway Research Program by Hanson et al. Grade 60 (414 MPa), No. 8 (25 mm) reinforcing bars were the primary subjects. Other sizes and grades were also tested. The formula developed by Hanson et al. was adopted in American Concrete Institute (ACI) documents and in the 1994 edition of the AASHTO specifications. The research was performed on single bars embedded in small beams (as flexural reinforcement) that were loaded at two points at the rate of four to eight cycles per second. A total of 353 concrete beams were tested. However, most of the tests were completed in the finite-life region, where the number of cycles was between 10,000 and 1,000,000.

The researchers concluded that 1 million to 5 million cycles (the long-life region) was more important for design purposes and based their conclusions on that region in phase 2 of their work. The formula was, essentially, based on very few tests performed on No. 8 (25 mm), Grade 60 (414 MPa) bars from manufacturer A with a minimum stress of 6 ksi (41 MPa) and a fatigue life in excess of 2 million cycles. The researchers also reported that the fatigue limit of interest to the bridge designer is not sensitive to concrete beam dimensions, concrete material properties, reinforcing bar size, reinforcing steel grade, or reinforcing steel metallurgy.

The formula to determine the allowable steel stress range \( f_r \) for reinforcing bars given in Article 5.5.3.2 of the AASHTO specifications is:

In U.S. customary units:

\[
 f_r = 21 - 0.33 f_{\text{min}} + 8 \left( \frac{r}{h} \right)
\]  

(1)

In SI units:

\[
 f_r = 145 - 0.33 f_{\text{min}} + 55 \left( \frac{r}{h} \right)
\]  

(1)

where

- \( f_r \) = allowable steel stress range (ksi, MPa)
- \( f_{\text{min}} \) = live-load stress combined with the more severe stress from either the permanent loads or the shrinkage- and creep-induced external loads; positive if in tension, negative if in compression (ksi, MPa)
- \( r/h \) = ratio of base radius to height of rolled-on transverse deformations; if the actual value of \( r/h \) is not known, 0.3 may be used. In the fourth edition of the AASHTO specifications (scheduled to be published in 2007), the term \( 21 + 8(r/h) \) will, as recommended in this paper, be replaced with the constant 24 ksi (166 MPa). It is derived from \( 21 + 8(0.3) = 23.4 \), then rounded up to 24 ksi.

A safe fatigue life for cases not reaching the endurance limit was also presented by Hanson et al. as follows:

In U.S. customary units:

\[
 \log N = 6.1 - 4.1 \left(10^5\right) f_r - 1.4 \left(10^5\right) f_{\text{min}} - 0.7 \left(10^5\right) f_u - 0.057 A_3 + 0.32 \left( \frac{r}{h} \right)
\]  

(2)

In SI units:

\[
 \log N = 6.1 - 2.8 \left(10^4\right) f_r - 9.7 \left(10^5\right) f_{\text{min}} + 4.8 \left(10^5\right) f_u - 0.39 A_3 + 0.32 \left( \frac{r}{h} \right)
\]  

(2)

where

- \( N \) = number of cycles to failure
- \( f_u \) = ultimate steel strength (ksi, MPa)
- \( A_3 \) = bar area (in.\(^2\), mm\(^2\))

It was found that with a decrease in the bend-to-bar diameter ratio, the resistance to fatigue is reduced.

Deformations of reinforcing bars are important in developing bond strength between the bars and the concrete. With WWR, however, bond strength is primarily developed with the presence of cross wires. Stress concentrations are typically developed in reinforced bars at the base of a transverse lug, at the intersection of a lug and a longitudinal rib, or at brand mark locations. In tests of reinforcing bars embedded in concrete beams, fatigue fractures usually initiate at these locations that are susceptible to stress concentrations.

Studies have shown that width, height, angle of rise, and base radius of a protruding deformation of a reinforcing bar affect the magnitude of stress concentration. Fatigue strength of reinforcing bars may also be influenced by the orientation of the longitudinal rib. Several studies have also...
indicated that there are small differences between the fatigue strength of bars made with old or new rolls at steel mills.

MacGregor et al. reported fatigue tests on reinforced concrete beams containing No. 5, No. 8, and No. 10 (16 mm, 25 mm, and 32 mm) reinforcing bars with yield strengths of 40 ksi, 60 ksi, and 75 ksi (276 MPa, 414 MPa, and 517 MPa), respectively. They concluded that the fatigue strength of reinforcing bars was relatively insensitive to the tensile strength of the bar metal. However, the fatigue strength of the bars was appreciably lower than that of the base metal. This difference results from the stress concentration at the base of the deformations.

Pasko performed fatigue tests on No. 5 (16 mm), Grade 60 (414 MPa) deformed reinforcing bars conforming to ASTM A615, welded to No. 3 (10 mm) plain transverse bars. The fatigue strength of the reinforcing bars was reduced by one-third when they were tack welded, compared with non-welded bars, while butt welding has been proved to have no effect on fatigue strength.

**Fatigue Studies on Prestressing Strands**

Fatigue failure of prestressing tendons occurs by initiation and propagation of cracks similar to that of reinforcing bars. Prestressed concrete members containing pretensioned or post-tensioned strands are usually designed as noncracked members. In these members, the strand stress range and the minimum stresses are very small for the strands to experience fatigue failure. Prestressing steels do not appear to have an endurance limit. According to Naaman, a fatigue life of 2 million cycles is sufficient for most purposes.

In current AASHTO specifications, Section 5.5.3.3 limits strands to a constant stress of 18 ksi (124 MPa) for radii of curvatures in excess of 30 ft (9.1 m) and 10 ksi (69 MPa) for radii of curvatures less than 12 ft (3.7 m). A linear interpolation of these values may be used for radii between 12 ft and 30 ft. A stress range of 11.6 ksi (78 MPa) is recommended by the FIP Commission on Prestressing Steels for bonded post-tensioned tendons used in the anchorage-tendon system. Full-scale bridge girder tests performed by Rabbat et al. showed that fatigue failure occurred in prestressing strands at their midspan cracks at a stress range of 9 ksi (60 MPa) at 3 million cycles and at a minimum stress of 142 ksi (980 MPa). This value is considerably lower than that contained in the AASHTO and FIP specifications.

Hanson et al. reports that in testing by Warner and Hulsbos at Lehigh University, a pretensioned strand subjected to a 20 ksi (140 MPa) stress range failed after only 570,000 cycles. There appears to be a need for additional studies in this area, especially if cracking is allowed to exist under service loading conditions.

**Fatigue Studies on WWR**

Hawkins and Heaton and Hawkins and Takebe performed fatigue tests with the goal of using WWR as a replacement for deformed reinforcing bars in bridge decks. In 1971, they tested plain (undeformed) wires with cross welds cut from W2 × W2 (W13 × W13) at 6 in. × 6 in. (152 mm × 152 mm) spacing, in six concrete slabs with the same reinforcement and subject to fatigue loading. A conclusion of the WWR testing in air was that the WWR performed equal to or better than the reinforcing bars tested by the Portland Cement Association (PCA) for NCHRP 164. They also indicated that WWR is more desirable than deformed bars for fatigue applications because after first wire fracture, alternate load paths are available through the fabric and multiple fractures have to occur before the performance of the concrete panel is severely affected.

In 1987, Hawkins and Takebe performed another series of tests to evaluate the long-life fatigue of WWR. The mesh was Grade 75 (517 MPa), D16 × D12 (D103 × D77) at 4.875 in. × 5.5 in. (124 mm × 140 mm) spacing. The minimum stress level was approximately 7 ksi (48 MPa). Twelve 2 ft × 7 ft × 7.75 in. (0.61 m × 2.13 m × 197 mm) concrete slabs were tested, six contained mesh with epoxy coating and six without. For a fatigue life of 5 million cycles, acceptable stress ranges observed were 19 ksi (130 MPa) and 20 ksi (140 MPa) for uncoated and coated WWR, respectively.

The Hawkins and Heaton study was the first comprehensive investigation of fatigue to the authors’ knowledge. It successfully challenged work by Bianchini and Kesler that was completed in Illinois in the 1960s, which indicated that WWR had unacceptable fatigue performance in deck slabs. They concluded that the poor performance observed by Bianchini and Kesler was due to the manner in which the WWR was manufactured at that time. Welding heat, penetration, upset time, and other process-related factors were relatively difficult to maintain at a given setting. Hawkins and Takebe recommended that stress ranges of 20 ksi (138 MPa) and 22 ksi (152 MPa) be allowed for uncoated and coated WWR, respectively.

**Testing in Air Versus Concrete**

There has been considerable discussion about whether testing of WWR in air accurately represents its performance in a concrete member. The transverse wires that bear on the concrete to create anchorage may create a potential for secondary bending of the longitudinal wires. Fretting friction between the wires, especially deformed wires, and the surrounding concrete adds to the stress concentrations. Alternatively, because stress concentration in a concrete member only occurs at the location of a crack, probability theory would suggest that a wire would show higher fatigue resistance when embedded in concrete than in the air when tested under uniform stress over its entire length. Reference 23 reports that test results on reinforcing bars in air had less fatigue resistance than bars from the same batch embedded in concrete beams. The NCHRP 164 results showed that testing in concrete beams did not affect the developed fatigue formula.

This paper reports on testing of WWR in air only. A more comprehensive study would include testing in concrete members and various member sizes, reinforcement contents, crack spacings, crack widths, concrete strengths, steel strengths, wire sizes, wire spacings, deformed wires, undeformed wires, and other factors. As will be shown later,
a conservative limit of 16 ksi (110 MPa) for the equation $f_r + 0.33f_{\min}$ is recommended for wires with cross welds as a result of this research. It is lower than the 19 ksi to 22 ksi (130 MPa to 150 MPa) range found in Hawkins et al.’s studies of WWR tested in concrete members. Additional testing in air and concrete is likely to result in greater, more-refined limits.

**TESTING METHOD**

This research is focused on the long-life fatigue resistance of WWR. The endurance limit, as defined earlier, is the limiting stress range below which the specimen does not fail up to a quasi-infinite number of cycles. In the literature, there is no universal value for the number of cycles that corresponds to the endurance limit for steel reinforcing bars and WWR. At the start of testing for this program, the endurance limit was assumed to correspond to 2 million cycles. However, per the advice offered by experts such as Hawkins of the University of Illinois and Rabbat of PCA, the research team decided to change the long-life definition to 5 million cycles. The authors do not believe the difference in the number of cycles will significantly affect test results because specimens that survive 2 million cycles are likely to survive 5 million cycles as well. Also, the greater life limit was felt to render more acceptable results.

To account for variability among WWR producers, three producers, referred to as suppliers A, B, and C, provided the WWR for testing. The work performed in NCHRP 164 demonstrated that if materials met ASTM A497 metallurgical requirements, mechanical properties were adequate for distinction of fatigue properties for design purposes.

Current WWR is produced with yield strengths in the range of 65 ksi to 80 ksi (450 MPa to 550 MPa). Higher strength can be achieved by special order. This study was conducted on WWR with a minimum yield strength of 75 ksi (520 MPa). Stress-strain diagrams were developed in this project to verify that the steel achieved that strength. However, consistent with the findings of the NCHRP 164, the strength factor could be eliminated from the fatigue formula and strength was not taken as a variable parameter in the testing program.

The wire sizes chosen for this research were D12, D18, D20, D28, and D31, (D77, D116, D129, D181, and D200 [metric]), representing the available range of wire sizes in the U.S. market according to the Wire Reinforcement Institute (WRI) Manual of Standard Practice. Note that the letter D designates deformed wire and the number that follows represents 100 times the wire area. Thus, D20 designates a deformed wire with a cross-sectional area of 0.20 in.$^2$ (129 mm$^2$), which is equal to a No. 4 (13 mm) reinforcing bar. Similarly, D31 is equal in diameter and area to a No. 5 (16 mm) bar. These sizes may seem surprisingly large; however, they represent the state-of-the-art in WWR production and have the ability to be used for bridge and other large reinforced concrete applications.

Fatigue testing was performed on an MTS 810 machine with an axial capacity of 55 kip (245 kN) and Test Star control software. The testing system is shown in **Fig. 2**. A sinusoidal, constant axial stress was applied at a frequency of 2.5 cycles per second. Wires with and without welded cross wires were tested. Cross wires had cross-sectional areas that were 40% of those of the primary wires. The gauge length in all tests was 24 in. (610 mm). Only one cross wire per specimen was used.

The methodology of the testing was as follows. First, a series of tests were conducted with various values for $f_r + 0.33f_{\min}$. The number of cycles required to fail the specimens was recorded. The value of $f_r + 0.33f_{\min}$ was progressively increased until the endurance limit was reached. The experience gained from testing, and from previous work, allowed the researchers to establish a conservative limit of 16 ksi (110 MPa) for the equation $f_r + 0.33f_{\min}$ for wires with cross welds and 24 ksi (165 MPa) for wires without cross welds. The testing was then repeated with the established $f_r$ and changing values for $f_{\min}$. The three primary levels of $f_{\min}$ were taken as 6 ksi, 12 ksi, and 18 ksi (40 MPa, 80 MPa, and 120 MPa), representing reasonable levels of dead load plus minimum live loads in actual applications. It is possible, with additional testing, to increase the values of 16 ksi and 24 ksi (110 MPa to 165 MPa) to greater limits and still reach the endurance limit. However, the authors felt that a conservative limit should be tentatively used until additional resources allow for a more comprehensive testing program.

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**Fig. 2.** The MTS 810 fatigue testing machine with Test Star control software.
**Fig. 3.** Fatigue resistance of wires with cross welding.

**Fig. 4.** Fatigue resistance of wires with no cross welding.
Fig. 5. Stress range plus one-third of the minimum stress versus the number of cycles for wires with cross welding.

Fig. 6. Stress range plus one-third of the minimum stress versus the number of cycles for wires with no cross welding.
ANALYSIS OF RESULTS

The research targeted development of a formula for WWR similar to the current formula for reinforcing bars that takes into account two conditions:

- Wires with no cross welds in the high tension zone;
- Wires with cross welds in the high tension zone.

To reduce the amount of testing, only deformed wire was tested. However, the results should be conservative enough to apply to plain wires, which have superior fatigue resistance compared with deformed wires.

Similar to Eq. 1, the proposed formula is expressed as follows:

\[ f_{\text{wwr}} = A + B f_{\text{min}} \]  

(3)

where

- \( f_{\text{wwr}} \) = allowable WWR stress range
- \( f_{\text{min}} \) = minimum live-load stress combined with the more severe stress from either the permanent loads or the shrinkage- and creep-induced external loads; positive if tension, negative if compression (ksi, MPa)
- \( A, B \) = experimental constants

Figure 3 shows a plot of steel stress versus number of cycles for WWR with a cross weld. The testing range for the specimen is shown as a line connecting the two end values. An arrow indicates when testing was stopped. The specimen could have reached a greater number of cycles.

Figure 4 shows the same plot for wires without a cross weld. Figures 5 and 6 show plots of \( f_r + 0.33 f_{\text{min}} \) versus number of cycles. The data points are compared with 16 ksi (110 MPa) in Fig. 5 for wires with cross welds and with 24 ksi (165 MPa) in Fig. 6 for wires without cross welds. The 24 ksi value is comparable to the values already used for bars \([21+8(r/h)] = 23.4\). The points on the left side of the graph that did not meet 2 million cycles represent the results of the early trial and the adjustment process used to determine the appropriate range for the endurance limit. The points below the 2 million mark did not reach the endurance limit because the \( f_r + 0.33 f_{\text{min}} \) value was too great.

The 16 ksi (110 MPa) value for wires with cross welds was established through trial and adjustment. It was believed to be reasonably conservative and consistent with the values for strands and bars. Also, a previous study had suggested that the presence of cross wires in the high-stress zone drops the fatigue resistance by one-third. With the resistance of bars and wires without cross welds assumed to be 24 ksi (165 MPa), the resulting value for wires with cross welds is 16 ksi (110 MPa).

Figure 7 shows a typical stress-strain relationship for the steel from one supplier. The materials from the other two suppliers produced similar graphs except that their yield strengths were closer to 80 ksi (550 MPa). Some tests resulted in yield strengths slightly less than 80 ksi but were accepted for the purposes of fatigue testing in order to avoid repeating two years’ worth of fatigue testing to correct for a small difference in an insignificant fatigue parameter. This experience, however, points to the fact that designers must verify that a
yield strength of 80 ksi is attainable before it is assumed to be available in design. Note that for higher than Grade 60 (414 MPa) steel, yield strength is defined as the strength at a strain of 0.35%, rather than the 1% used for Grade 60 steel.

CONCLUSIONS AND RECOMMENDATIONS

1. Use of WWR in precast concrete products has substantially increased in the past 15 years due to the high quality of WWR and the reduced concrete product fabrication time. It is the standard shear reinforcement in Nebraska and several other states for use in bridge girders. When used for this purpose, the cross wires are only located in the WWR in the top and bottom flanges so the welds are away from the high-stress zones.

2. High-strength concrete is increasingly being used in industry, requiring a corresponding increase in steel strength. WWR offers greater strength without a cost premium over Grade 60 (414 MPa).

3. Greater shear and other stress limits in modern design codes and specifications are likely to result in greater probability of concrete member cracking under service loads and a greater need for fatigue control of steel reinforcement.

4. The research reported herein has resulted in specific recommendations for design of WWR in situations where fatigue limits must be checked.

5. Based on the test results, the proposed fatigue equation for WWR with a cross weld in the high-stress region is:

   In U.S. customary units:
   \[ f_{wwr} = 16 - 0.33f_{\min} \text{ in ksi (4)} \]

   In SI units:
   \[ f_{wwr} = 110 - 0.33f_{\min} \text{ in MPa (4)} \]

   For WWR with no cross weld in the high-stress region, the proposed formula is:

   In U.S. customary units:
   \[ f_{wwr} = 24 - 0.33f_{\min} \text{ in ksi (5)} \]

   In SI units:
   \[ f_{wwr} = 166 - 0.33f_{\min} \text{ in MPa (5)} \]

The definition of the high-stress region for application of Eq. 4 and 5 is:

• For shear reinforcement in I-beams, box beams, and similar members, the clear web height between fillets;

• For shear reinforcement in rectangular beams and other members, the middle two-thirds of the total member depth; and

• For flexural reinforcement, one-third of the span on each side of the section of maximum moment.

Eq. 4 and 5 have recently been approved by the AASHTO specifications for adoption in the fourth edition of the AASHTO specifications, to be published in 2007.

6. If the stress range at any given section exceeds the limit of Eq. 4, the WWR can be fabricated without any cost premium with the cross wires in the high-stress zone eliminated. The missing cross wires can then be replaced at the construction site with loose bars as needed.

7. An effective pattern of WWR is similar to that used for shear reinforcement, where the main wires are welded with two top cross wires and two bottom cross wires. The main function of these cross wires is to improve anchorage of the main vertical wires in the precast concrete member’s web. Wire spacing as small as 2 in. (50 mm), and the high accuracy of computer-controlled spacing, produces structurally effective reinforcement without the labor required to tie individual bars.

8. A more comprehensive testing program with WWR embedded in concrete members would allow evaluation of effects not considered in this study and could result in relaxed limits recommended herein.

ACKNOWLEDGMENTS

Thanks to PCI and WRI for their financial support of this project. Paul Johal, director of Research & Development at PCI, and Roy Reiterman of WRI capably guided the progress of the project and reviewed its results. Basile Rabbat of PCA and Neil Hawkins of the University of Illinois provided invaluable guidance throughout the project. Kelvin Lien, structures laboratory technician at the university, kept the MTS machine well maintained, which allowed it to survive the large combination of stress cycles. Numerous other members of the PCI Research & Development Committee and the WRI Technical Task Force contributed to the success of this project. Tony Naaman provided valuable suggestions for improvement of the manuscript. Thanks to Emily Lorenz, editor-in-chief of the PCI Journal, and Journal editorial assistant Ann Lopez for their tireless efforts in maintaining the Journal as a world-class publication.

REFERENCES


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APPENDIX A: NOTATION

\[ A_t = \text{area of tension reinforcement} \]

\[ A_s = \text{area of transverse reinforcement} \]

\[ A, B = \text{experimental constants} \]

\[ E_c = \text{modulus of elasticity of concrete} \]

\[ f_c = \text{specified concrete compressive strength at 28 days} \]

\[ f_y = \text{bottom fiber tensile stress} \]

\[ f_{num} = \text{stress level due to dead load plus minimum live load} \]

\[ f_r = \text{stress range} \]

\[ f_y = \text{modulus of rupture of concrete} \]

\[ f_s = \text{stress in steel under service loads} \]

\[ f_u = \text{ultimate steel strength} \]

\[ f_{wrr} = \text{allowable steel stress range for straight deformed welded-wire reinforcement} \]

\[ I_v = \text{moment of inertia of cracked section} \]

\[ I_g = \text{moment of inertia of gross concrete section} \]

\[ k_d = \text{distance of the neutral axis from the extreme compression fiber in a cracked transformed concrete section} \]

\[ M = \text{bending moment} \]

\[ M_{DL} = \text{bending moment due to service dead loads} \]

\[ M_{LL} = \text{bending moment due to service live loads} \]

\[ M_{TT} = \text{bending moment due to total service loads} \]

\[ M_s = \text{factored bending moment} \]

\[ n = \text{modular ratio} \]

\[ r/h = \text{ratio of base radius of reinforcing bar to height of rolled-on transverse deformations} \]

\[ s = \text{spacing of rows of ties} \]

\[ V_s = \text{shear resistance provided by concrete} \]

\[ V_p = \text{shear resistance provided by web reinforcement} \]

\[ V_s = \text{factored shear force at the section} \]

\[ w_r = \text{concrete density} \]

\[ WWR = \text{welded-wire reinforcement} \]

\[ y = \text{distance from centroid to section fiber under consideration} \]

\[ y_b = \text{distance from centroid to extreme bottom fiber of non-composite beam} \]

\[ y_{bc} = \text{distance from centroid to extreme bottom fiber of composite section} \]
CONVERSIONS

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Wire designation D31, indicates an area of 0.31 in². The same wire size would be D2000 in SI units, indicating an area of 200 mm².

β = factor used in shear design indicating ability of diagonally cracked concrete to transmit tension

φ = resistance (strength reduction) factor

APPENDIX B: DESIGN EXAMPLES

Example 1: WWR as Primary Tension Reinforcement

A 20-ft-span (6.10 m), 28 ft 4 in.-wide (8.64 m) bridge is designed for its own weight and the live load specified by AASHTO LRFD Bridge Design Specifications. It comprises ten 2 ft 10 in.-wide (0.90 m) plank elements that span in the direction of traffic. A cross section of the element is shown in Fig. 1b and again in Fig. B1. The planks are connected and act together in shear. A 2 in. (51 mm) composite overlay is also placed.

Materials—Precast plank concrete compressive strength at 28 days $f'_{c} = 6$ ksi (41 MPa), overlay concrete compressive strength at 28 days $f'_{c} = 4$ ksi (28 MPa), concrete density $w_{c} = 0.150$ kip/ft³ (24 kN/m³), clear cover to reinforcement = 1.25 in. (32 mm).

**Bending moment**—Weight of plank + overlay = (14.5/12)(34/12)(0.150) = 0.514 k/ft (7.5 kN/m)

Midspan moments:

Dead load moment:

$$M_{DL} = \frac{(0.514)(20)^2}{8} = 25.70 \text{ k-ft (34.8 kN-m)}.$$ 

Live load moment:

$$M_{LL} = 76.90 \text{ k-ft (104 kN-m)}.$$ 

Total load moment:

$$M_{LT} = 25.70 + 76.90 = 102.60 \text{ k-ft (139 kN-m).}$$

**Cross section properties**—For precast concrete plank:

Modulus of elasticity:

$$E_{c} = 33,000(0.150)^{1.5} \sqrt{6} = 4696 \text{ ksi (32,000 MPa)}$$

For cast-in-place overlay:

$$E_{c} = 3834 \text{ ksi (26,000 MPa)}$$

The modular ratio between overlay and plank material = 3834/4698 = 0.82 and between the WWR and the plank material = 29,000/4696 = 6.18.

The gross section moment of inertia of the plank = 34(12.5)/12 = 5534 in.² (2.3 × 10³ m²).

The topping concrete can be transformed to plank concrete by multiplying its width by 0.82. The properties of the resulting transformed inverted-tee section can be calculated using mechanics principles resulting in an area of 464 in.² (0.299 m²), distance between bottom fibers and centroid = 7.09 in. (180 mm), and moment of inertia $I_{g} = 8143$ in.⁴ (3.4 × 10³ m⁴).

**Cracked section**—The neutral axis distance from the extreme compression fiber $kd$ can be determined by setting the first area moment about the neutral axis = 0. In this case, concrete in tension is assumed to be non-existent and steel is transformed to precast concrete using the modular ratio 6.18.

For the precast section, the resulting $kd = 3.27$ in. (83 mm) and for the composite section $kd = 3.96$ in. (101 mm).

The cracked section moment of inertia $I_{c}$ of the precast concrete section = 1782 in.⁴ (0.74 × 10³ m⁴) and of the composite section = 2514 in.⁴ (1.05 × 10³ m⁴).

Check whether the section cracks due to total loading:

Modulus of rupture:

$$0.24\sqrt{f'_{c}} = 0.24\sqrt{6} = 0.588 \text{ ksi (4.1 MPa)}$$

Bottom fiber stress due to total load (assume section to be uncracked):

$$f_{bd} = \frac{My}{I_{g}} = \frac{(25.7)(12)(6.25) + (76.9)(12)(7.09)}{8143}$$

$$f_{bd} = 1.152 \geq 0.588 \text{ ksi}$$

Therefore, the section cracks under total load and the cracked section properties must be used in the computation of steel stress. The cover of the concrete to the steel centroid.


\[
M_{DL \gamma} = \frac{6.18 \cdot (25.70 \cdot (12) \cdot (12.5 - 2.06 - 3.27))}{1782} = 7.669 \text{ ksi (53 MPa)}
\]

\[
M_{TT \gamma} = \frac{6.18 \cdot (102.60 \cdot (12) \cdot (12.5 + 2 - 2.06 - 3.96))}{2514} = 25.658 \text{ ksi (177 MPa)}
\]

The stress range:

\[
f_r = M_{TT \gamma} - n M_{DL \gamma} = 25.658 - 0.33(7.669) = 17.898 \text{ ksi (123 MPa)}
\]

The actual stress range is compared with the allowable stress range as follows. Applying Eq. 4 and 5:

Maximum \( f_r = 25.658 - 7.669 = 17.989 \text{ ksi (123 MPa)} \)

The stress limit with cross welding can be satisfied by solving for the location that satisfies the following relationship:

\[
M_{TT \gamma} - n M_{DL \gamma} = 16 - (0.33) n \frac{M_{DL \gamma}}{I_c r}
\]

Note that \( M_{TT \gamma} = (102.6/25.7)M_{DL \gamma} = 3.99M_{DL \gamma} \), if one reasonably assumes the live-load moment envelope to be a parabola.

Thus, \( M_{DL \gamma} = 20.05 \text{k-ft (27 kN-m)} \)

Using the relationship \( M = (w)(L - x)/2 \), where \( M \) is moment due to a uniform load \( w \) at a location in a simple span defined by a distance \( x \) from left support and distance \( L - x \) from right support, determine the distance \( x \):

\[
(0.5)(0.514)(x)(20 - x) = 20.05
\]

The corresponding \( x = 5.31 \text{ ft (1.6 m)} \). The high-stress zone in the middle of the beam is \( 20 - 5.31 = 14.69 = 9.38 \text{ ft (2.9 m)} \). Thus, using the middle two-thirds as an empirical rule as recommended by the upcoming 2007 AASHTO specifications is conservative.

Assuming that the beam is 1 ft longer than the span length, it is required to terminate the welded D20 at 5.31 + 0.5 = 5.81 ft (1.8 m) from beam end. Number of D20 at 6 in. = \( (5.81)(12)/6 = 12 \). See Fig. B2 for reinforcement details. If shear reinforcement is required at the middle zone where no welding is permitted, supplemental individual bars can be supplied.
The primary transverse strip is designed for an axle load of 32.0 kip (142 kN). The axle consists of two wheels 6 ft (1.8 m) apart. The tire contact area of each wheel is determined according to the AASHTO specifications to be 20 in. (510 mm) in the direction of the slab span. Detailed non-prismatic member analysis is conducted. Figure B4 illustrates how the slab is modeled at the girder support location. A wheel load width of $20 + \frac{8}{3} = 28$ in. (710 mm) is assumed at the mid-thickness of the deck, and a support width of 6 in. (150 mm), represents the web width. According to Article 4.6.2.1.3 of AASHTO specifications, the distribution width for the positive moment is \((26 + 6.6S)\) in. where \(S\) is spacing in feet, or \(\frac{26 + (6.6)(12)}{12} = 8.77\) ft (2.67 m). The corresponding strip width for the interior negative moment design is \(48 + 3.0S\), or 7.00 ft (2.13 m). One lane loading is used as specified in the AASHTO specifications for fatigue analysis. The design section for negative moment is at a distance of one-third of the beam flange width from the centerline of the support but not exceeding the 15 in. (381 mm) beam flange width. Clear concrete cover to reinforcement is 1.0 in. (25 mm) at the bottom and 2.5 in. (64 mm) at the top.

It is the practice of the Nebraska Department of Roads and several other state highway agencies to invoke the empirical design method wherever it is applicable. The empirical method would be valid for this condition. However, because it is empirical, a prescribed amount of steel (No. 5 at 12 in. [16 mm at 305 mm] each way bottom and No. 4 at 12 in. [13 mm at 305 mm] each way top) is specified without calculations. The validity of the method with WWR has not been established. For this reason, the AASHTO specifications’ more detailed strip method will be used to design this deck. It is expected to give larger reinforcement content than that by the empirical method, but it will illustrate the procedure for design of deck slabs of general conditions.

The design section for negative moment is at a distance of one-third of the beam flange width from the centerline of the support but not exceeding the 15 in. (381 mm) beam flange width/3 = 48.2/3 = 16.1 > 15 in. Therefore, the design section for negative moment is at 15 in. from the centerline of the beam. The most critical positive moment section for this deck was found to be Section 1 of Fig. B5, and the most critical negative moment section was Section 2. The critical placement of truck load for interior span is shown in Fig. B5.

The maximum moments were calculated with commercial continuous-beam-analysis software. The moments due to Example 2: Design of Highway Bridge Deck

The bridge cross section shown in Fig. B3 consists of four NU1100 (Nebraska University 1100 mm [43 in.]) girder lines, spaced at 12 ft (3.7 m) on center. The deck is 8.5-in.-thick (216 mm), cast-in-place (CIP) concrete, including a 0.5 in. (13 mm) sacrificial wearing thickness and an 8 in. (203 mm) structural thickness. The slab is continuous and composite with the girders. It is subjected to slab weight, traffic barrier weight of 0.400 kip/ft/side (5.8 kN/m), a future wearing surface of 25 psf (1.2 kN/m²), and a live load as specified by the AASHTO specifications. The haunch over the girder top flange is required to be a minimum of 1 in. (25 mm) along the girder length. Slab concrete strength at 28 days $f'_c = 4$ ksi (27.6 MPa) and density $\omega_c = 0.150$ k/ft³. Clear concrete cover to reinforcement is 1.0 in. (25 mm) at the bottom and 2.5 in. (64 mm) at the top.

It is the practice of the Nebraska Department of Roads and several other state highway agencies to invoke the empirical design method wherever it is applicable. The empirical method would be valid for this condition. However, because it is empirical, a prescribed amount of steel (No. 5 at 12 in. [16 mm at 305 mm] each way bottom and No. 4 at 12 in. [13 mm at 305 mm] each way top) is specified without calculations. The validity of the method with WWR has not been established. For this reason, the AASHTO specifications’ more detailed strip method will be used to design this deck. It is expected to give larger reinforcement content than that by the empirical method, but it will illustrate the procedure for design of deck slabs of general conditions.

The primary transverse strip is designed for an axle load of 32.0 kip (142 kN). The axle consists of two wheels 6 ft (1.8 m) apart. The tire contact area of each wheel is determined according to the AASHTO specifications to be 20 in. (510 mm) in the direction of the slab span. Detailed non-prismatic member analysis is conducted. Figure B4 illustrates how the slab is modeled at the girder support location. A wheel load width of $20 + \frac{8}{3} = 28$ in. (710 mm) is assumed at the mid-thickness of the deck, and a support width of 6 in. (150 mm), represents the web width. According to Article 4.6.2.1.3 of AASHTO specifications, the distribution width for the positive moment is \((26 + 6.6S)\) in. where \(S\) is spacing in feet, or \(\frac{26 + (6.6)(12)}{12} = 8.77\) ft (2.67 m). The corresponding strip width for the interior negative moment design is \(48 + 3.0S\), or 7.00 ft (2.13 m). One lane loading is used as specified in the AASHTO specifications for fatigue analysis.

The design section for negative moment is at a distance of one-third of the beam flange width from the centerline of the support but not exceeding the 15 in. (381 mm) beam flange width/3 = 48.2/3 = 16.1 > 15 in. Therefore, the design section for negative moment is at 15 in. from the centerline of the beam. The most critical positive moment section for this deck was found to be Section 1 of Fig. B5, and the most critical negative moment section was Section 2. The critical placement of truck load for interior span is shown in Fig. B5.

The maximum moments were calculated with commercial continuous-beam-analysis software. The moments due to

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**Fig. B3.** Cross section of the bridge in Example 2, where the deck is designed with welded-wire reinforcement.

**Fig. B4.** Structural analysis model of non-prismatic deck at its connection with supporting girder.

**Fig. B5.** Truck loading position producing maximum moments at critical sections.

**Fig. B6.** Required welded-wire reinforcement of deck. Omitted welded longitudinal wires in the middle zone of each deck span must be replaced with individual wires or bars (not shown).
dead load, live load, and fatigue at Section 1 are 0.53 k-ft/ft, 6.80 k-ft/ft, and 5.6 k-ft/ft (2.36 kN-m/m, 30.25 kN-m/m, and 25.09 kN-m/m), respectively. Note that the negative moment due to railing weight was intentionally ignored in this example in order to demonstrate how to address a case with no cross welds allowed. The corresponding moments in Section 2 are -1.01 k-ft/ft, -7.00 k-ft/ft, and -6.30 k-ft/ft (-4.49 kN-m/m, -31.14 kN-m/m, and -28.02 kN-m/m).

Through trial and adjustment, it has been determined that WWR, Grade 75, D20 at 5 in. (517 MPa, 13 mm at 127 mm) spacing for the bottom reinforcement; WWR, Grade 75, D10 at 6 in. (517 MPa, 9 mm at 152 mm) spacing for the top reinforcement; and WWR, Grade 75, D10 at 6 in. spacing top and bottom for the longitudinal distribution reinforcement would be acceptable. The following calculations demonstrate the adequacy of this reinforcement.

Check whether Section 1 and 2 cracks at total loading:

Moment of inertia of the gross section 
\[ I_e = \frac{(12)(8)}{12} = 512 \text{ in.}^4 \] (6.99 × 10^4 m^4/m) of deck width for the positive moment section and 
\[ (12)(12)/12 = 1728 \text{ in.}^4 \] (2.16 × 10^5 m^4/m) for the negative moment section.
Modulus of rupture \( = 0.24\sqrt{4} = 0.474 \text{ ksi} \) (3.3 MPa)

Bottom fiber stress due to total load in Section 1 = (7.33)(12)(4)/512 = 0.688 > 0.474 ksi

Top fiber stress due to total load in Section 2 = (8.01)(12)(6)/1728 = 0.334 < 0.474 ksi

Therefore, the fatigue need only be investigated for the positive moment zone:

Modulus of elasticity \( E_c = 33,000 \left( 0.150 \right)^{1.5} \sqrt{4} = 3834 \text{ ksi} \) (26,000 MPa).

Modular ratio between reinforcement and deck material = 29,000/3834 = 7.56

Properties of the cracked section are calculated as \( k_d = 1.76 \text{ in.} \) (45 mm) and \( l_o = 112 \text{ in.} \times \text{ft} \) (1.53 \( \times \) 10\(^4 \text{ m}^2\)/m).

The section is assumed to be precracked, and dead load is thus applied using the cracked section properties.

The stress in steel is calculated for both dead loading and total loading as follows:

Total load:
Dead load stress = (7.56)(0.5)(12)(6.75 – 1.76)/112 = 2.158 ksi (14.9 MPa)
Total load stress = (7.56)(5.6)(12)(6.75 – 1.76)/112 = 2.627 ksi (156.0 MPa)

Stress range \( f_s = 22.627 - 2.158 = 20.469 \text{ ksi} \) (141.1 MPa)

Using the current code format, the stress range is compared with the allowable stress range:

Maximum: \( f_s = 24 - 0.33 f_{\text{min}} = 24 - 0.33(2.158) = 23.288 \text{ ksi} \) (160.6 MPa)

Actual \( f_s = 20.469 < 23.288 \text{ ksi} \)

No welding is allowed at this location. Additional calculations at adjacent sections would determine the location where welding would be allowed. This region will be calculated conservatively as 0.75 of the center-to-center span of 12 ft (3.66 m). The area with no welding allowed in the bottom reinforcement mesh is 0.75(12)(2/3) = 6 ft (1.8 m). Thus, the bottom longitudinal steel cannot be welded in the middle 6 ft in each space between girders (Fig. B6). It must be placed loose or replaced with equivalent mild steel reinforcing bars.

A more detailed analysis might give a significantly less restriction of the width over which welding is not permitted.

Example 3: WWR as Shear Reinforcement

Each span of the two-span bridge is 120 ft (36.58 m) long. Four NU1100 girder lines spaced at 12 ft (3.66 m) are used. The bridge has the same cross section as the cross section shown in Fig. B3. The girders are simple span for member weight and slab weight and continuous for barrier weight, live load, and future wearing surface. The girder concrete strength is 8 ksi (55 MPa) and its web width is 5.91 in. (130 mm). Grade 75 ksi deformed WWR is used as shear reinforcement.

Analysis of the critical section in shear using the AASHTO specifications modified compression field theory yields a factored load shear \( V_u \) of 472.8 kip (2103 kN), concrete resistance \( V_c \) of 49.3 kip (219 kN), shear force required to be resisted by the stirrups \( V_s \) of 476.1 kip (2118 kN), and required stirrup reinforcement \( A_s /s = 0.104 \text{ in.} \times \text{in.} \). Using D22 vertical stirrups in pairs, one on each web face, the maximum required spacing is 2(0.22)/0.104 = 4.24 in. Thus, use two D22 (D142) stirrups at 4 in. (102 mm) spacing.

The wires are vertical and are anchored only at the top and bottom flanges with two longitudinal wires at each location. The size of the anchor wire is required to be 0.4(0.22) = 0.88. Use two W10 (W65) top and bottom. This reinforcement should be treated as conventional mild steel reinforcing bars, which are not checked in fatigue. However, should high shear forces in combination with the prestressing acting on the section cause the principal diagonal tension to be in excess of the tensile strength of concrete, the stress range in steel should be calculated, whether bars or wires are used, and checked against the fatigue limits proposed herein.

This type of custom WWR is now common in many areas of the United States for precast bridge I-girders. The sketch in Fig. B7 represents typical I-girder WWR details. Figure B8 shows one of the Platte River East Bridge girders during fabrication.