Analysis of the Flexural Strength of Prestressed Concrete Flanged Sections

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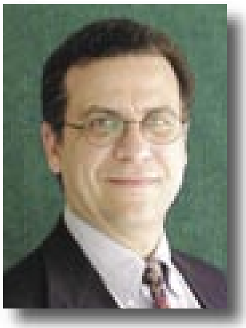

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Inconsistencies in the sectional response of prestressed concrete flanged sections predicted by the AASHTO LRFD and AASHTO Standard Specifications, including the maximum reinforcement limits, may arise due to different interpretations of the equivalent rectangular compressive stress block idealization. Strain compatibility analyses with nonlinear material properties were performed for a variety of non-rectangular prestressed concrete sections to identify the inconsistencies between the two specifications. Responses of the sections obtained using nonlinear strain compatibility analyses were compared to those predicted by the AASHTO LRFD and Standard Specifications. Modifications to the AASHTO LRFD procedure are proposed to correct for errors in determining the contribution of the top flange overhangs. Improvements in the accuracy of predicted sectional response are demonstrated through a parametric study. Comparisons of measured flexural strengths of prestressed concrete I-beams found in the literature to strengths predicted by the specifications, the proposed modified LRFD procedure, and the strain compatibility analyses are also included.

Differences between the procedures in the AASHTO Standard Specifications[1] and the AASHTO LRFD Specifications[2] for the calculation of nominal bending resistance of reinforced and prestressed concrete T-sections have been the subject of much discussion.[3-9] The differences arise due to conflicting interpretations of the equivalent rectangular stress block approximation for the case of T-sections. As a result, the depth of the neutral axis, c, determined according to the two specifications can be significantly different at nominal flexural strength of the section, which leads to inconsistencies in the provisions that depend on c, such as whether or not the section is over-reinforced, as well as differences in nominal moment capacity.

In this paper, provisions of the AASHTO Standard and LRFD Specifications are compared to results from strain compatibility analyses of reinforced and prestressed concrete non-rectangular sections using realistic stress-strain models for the constituent materials in an effort to identify design procedures that more accurately represent the actual behavior of such members. Modifications to the LRFD procedure are proposed to increase the accuracy of computed moment capacities for typical sections. Computed flexural capacities are compared to measured flexural capacities obtained from existing tests to validate the proposed modifications.

BACKGROUND

Reinforcement Limits

In order to provide adequate ductility of prestressed concrete sections, the ACI Code,[10] AASHTO Standard Specifications,[1] and AASHTO LRFD Specifications[2] use the amount of tensile reinforcement in the section as the control variable. For prestressed concrete, the three specifications define the maximum reinforcement limit in terms of different parameters; the AASHTO Standard Specifications control the maximum amount of tensile reinforcement by placing a limit on the reinforcement index, ω.

This limitation can be expressed by the following relationship:

$$\omega_w = \frac{A_{sr} f_{ps}}{b_w d f_c'} \leq 0.36\beta_1 \tag{1}$$

Versions of the ACI Code up to 2002 utilized a similar expression in order to limit the maximum tensile reinforcement. The AASHTO LRFD Specifications, on the other hand, limit the maximum amount of tensile reinforcement by imposing the following limit on the ratio of neutral axis depth, c, to effective depth, d_e:

$$\frac{c}{d_e} \leq 0.42 \tag{2}$$

Even though the Standard and LRFD Specifications define the maximum reinforcement limit using different criteria, the two parameters (the reinforcement index, ω, and the c/d_e ratio) are related to each other through equilibrium of internal tensile and compressive forces. This relation can be shown by substituting the compression force capacity of the web, $0.85 f_c' \beta_1 c b_w$, for the tensile force in the portion of the prestressing steel, $A_{sr} f_{ps}$, equilibrating the compressive force in the web [see Eq. (4b)] into Eq. (1):

$$\frac{0.85 f_c' \beta_1 c b_w}{b_w d f_c'} \leq 0.36\beta_1 \tag{3a}$$

and after simplification:

$$\frac{c}{d} \leq \frac{0.36}{0.85} = 0.42 \tag{3b}$$

Eq. (3b) is identical to Eq. (2) when d in Eq. (3b) equals d_e [as in Eq. (2)]—in other words, when the section does not contain any non-prestressed tension reinforcement.

In the AASHTO Specifications, the sections with tensile reinforcement exceeding the maximum reinforcement limit are termed “over-reinforced.” By preventing the use of the full flexural capacity of over-reinforced sections, the specifications impose an additional safety margin to account for the limited ductility of those sections. In other words, the specifications permit the use of prestressed concrete sections with steel amounts exceeding the maximum limit, but with a usable flexural strength that is less than the actual strength of the section.

The reduction in usable flexural strength is achieved in the Standard Specifications and LRFD Specifications by placing an upper limit on the ultimate moment capacity of over-reinforced sections. In the 2002 version of the ACI Code, on the other hand, the reduction in usable flexural strength of over-reinforced sections is achieved through the use of strength reduction factors that decrease with decreasing strand strain at ultimate capacity.

Limiting the maximum tensile reinforcement in flexural members dates back to the 1971 edition of the ACI Code, which placed an upper limit of 0.30 on the reinforcement index, ω, which is directly proportional to the amount of tensile reinforcement. The first appearance of a limit on the reinforcement index equal to 0.30 is found in the report titled “Tentative Recommendations for Prestressed Concrete” by the ACI-ASCE Joint Committee 323.[11] The justification for such a limit was expressed as the need “to avoid approaching the condition of over-reinforced beams for which the ultimate flexural strength becomes dependent on the concrete strength…”

Ductility Considerations

Warwaruk, Sozen, and Siess[12] conducted an extensive experimental and analytical study on the flexural strength of prestressed concrete beams. It was stated that for smaller amounts of longitudinal tensile reinforcement, small variations in the reinforcement ratio or concrete strength do not cause significant changes in strand stress at ultimate load. However, if the section has a large amount of tensile reinforcement, then the strand stress, and hence the moment capacity, are affected significantly by marginal changes to the concrete strength or the reinforcement ratio.

The idea behind a limitation on the maximum amount of longitudinal reinforcement was explained as follows: “…a beam should be proportioned to have a low value of longitudinal reinforcement to concrete strength ratio primarily because changes in the concrete strength will then affect the strength only negligibly; a policy which has long been followed in the design of ordinary reinforced concrete beams.” Based on the strand strain at ultimate equal to 0.01, and a maximum usable concrete strain equal to 0.003, the authors proposed the limit $\rho(f_{ps}/f_c') \leq 0.25$ for sections with a rectangular compression zone and without any non-prestressed reinforcement.

This limit was obtained in a similar way as the limits on the maximum reinforcement index in the ACI-ASCE Joint

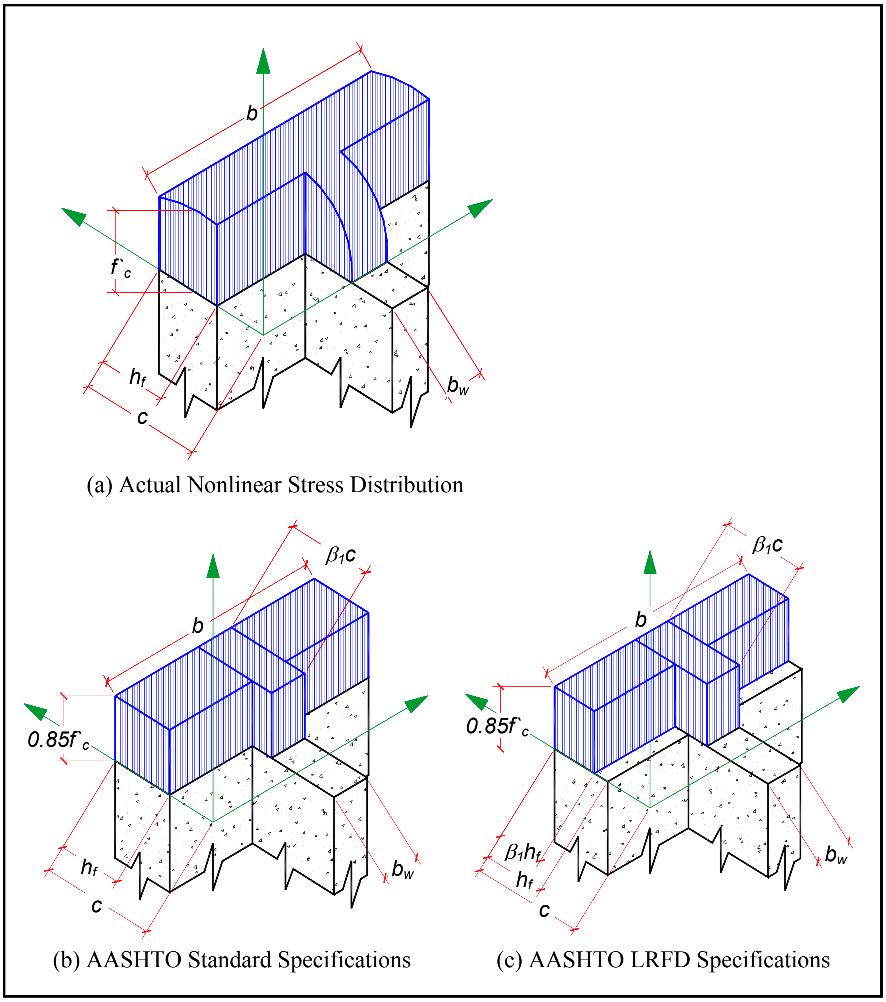

Fig. 1. Compressive stress distributions for T-section.

Committee 323[11] report and the AASHTO Standard Specifications. When computing the 0.30 limit on the reinforcement index in the ACI-ASCE Joint Committee 323 report, the maximum usable concrete strain was assumed to be 0.004 with an assumed strand strain at ultimate of 0.01.

The maximum longitudinal reinforcement limits recommended by ACI-ASCE Joint Committee 323 and by Warwaruk, Sozen, and Siess were changed to $0.36\beta_1$ by Mattock[13] in order to account for variations in concrete compressive strength. Mattock also used the condition $f_{ps} = f_{py}$ to define the limit on the maximum tensile reinforcement. The 1999 edition of the ACI Code indicates that the ASTM-specified yield strength of low-relaxation Grade 270 strands is $f_{py} = 0.90 f_{pu}$, which corresponds to a strain of 0.01 for strands with a typical stress-strain behavior. In addition, the AASHTO Standard Specifications state that "Prestressed concrete members shall be designed so that the steel is yielding as ultimate capacity is approached."

Mast[14] recommended using the net tensile strain in the extreme tension steel at nominal strength in order to determine if the section is compression-controlled or tension-controlled. Mast proposed a steel strain greater than or equal to 0.005 for tension-controlled sections, and a steel strain less than or equal to 0.0025 for compression-controlled sections. It should be noted that these limits apply to the net steel strain, i.e., they do not include the steel strain due to the effective prestress. This approach is currently used by ACI 318-02[15] with the strain limits of 0.005 and 0.002, respectively, for the tension- and compression-controlled sections.

For the type of sections that are typically used for prestressed concrete bridge girders (e.g., rectangular, I, T, and box), regular designs include a composite deck that carries the major part of the internal compressive force. For these cases, the maximum tensile reinforcement limits are easily met. But, for the cases in which neither a composite deck nor a wide top flange exists to help carry the compressive component of the internal couple, the neutral axis may be located in the web of the section. For these cases, the increased neutral axis depth, c, can lead to over-reinforced sections.

Strength Considerations

Both the AASHTO Standard and the AASHTO LRFD Specifications approximate the actual nonlinear concrete compressive stress distribution at nominal capacity (Fig. 1a) with the Whitney rectangular stress block that has an average compressive stress of $0.85f_c'$ "uniformly distributed over an equivalent compression zone bounded by the edges of the cross section and a line parallel to the neutral axis at a distance $a = \beta_1 c$ from the extreme compression fiber." However, in the implementation of the equivalent rectangular compression block, the LRFD Specifications neglect the contribution of the bottom of the top flange overhangs (i.e., the area below $\beta_1 h_f$) to the internal compression force. The differences in the way the two specifications treat the overhanging portions of the top flange of non-rectangular sections lead to inconsistencies in the determination of whether or not the section is considered over-reinforced, as well as in the resulting flexural capacity.

The AASHTO Standard Specifications use the procedure that has been used by the ACI Building Code since the 1983 Edition.[16,17] These standards assume that T-section behavior exists when the depth of the equivalent rectangular compressive stress block, $a = \beta_1 c$, drops below the top flange of the section, and that once T-section behavior begins, the full depth of the top flange carries compressive stress with an assumed uniformly constant stress intensity of $0.85f_c'$. In this case, the depth of the neutral axis of flanged sections can be calculated using the expression:

$$a = \beta_1 c = \frac{A_{sr} f_{ps}}{0.85 f_c' \, b_w} \tag{4a}$$

which can be rewritten as:

$$0.85f_c' \beta_1 c b_w = A_{sr} f_{ps} \tag{4b}$$

where A_{sr} is defined as:

$$A_{sr} = A_{ps} - A_{sf} = A_{ps} - \frac{0.85f_c'(b - b_w)h_f}{f_{ps}} \tag{4c}$$

Combining Eqs. (4b) and (4c):

$$(0.85f_c')(\beta_1 c)b_w + (0.85f_c')h_f(b - b_w) = A_{ps}f_{ps} \tag{4d}$$

Eq. (4d) is the expression for the internal equilibrium of the compressive force in the concrete and tensile force in the prestressing steel. As seen in the second term of the left hand side of Eq. (4d), the Standard Specifications allow the full flange depth of h_f to contribute to the total compressive force carried by the section. In this case, the contribution of the top flange overhangs to the total internal compressive force is $0.85f_c'(b - b_w)h_f$ as illustrated in Fig. 1b.

The AASHTO LRFD Specifications, on the other hand, consider the section to be a T-section if the depth of the neutral axis, c, exceeds the depth of the top flange, h_f. The LRFD Specifications use the following equation to determine the depth of the neutral axis of prestressed concrete sections with T-section behavior and without any mild reinforcement:

$$c = \frac{A_{ps}f_{pu} - 0.85f_c' \beta_1 h_f(b - b_w)}{0.85f_c' \beta_1 b_w + kA_{ps}\dfrac{f_{pu}}{d_p}} \tag{5a}$$

which can be rewritten as:

$$\begin{aligned}(0.85f_c')(\beta_1 c)b_w &+ (0.85f_c')(\beta_1 h_f)(b - b_w)\\ &= A_{ps}f_{pu}\left(1 - k\frac{c}{d_p}\right)\end{aligned} \tag{5b}$$

where the term following A_{ps} on the right hand side is the expression given in the LRFD Specifications for the predicted average stress in the prestressing steel.

As a result, Eq. (5b) is the expression for the internal equilibrium of the compressive force in the concrete and tensile force in the prestressing steel. As seen in the second term of the left hand side of the equilibrium expression, the LRFD Specifications limit the depth of the equivalent rectangular stress block acting on the overhanging portions of the flange of a T-section to $\beta_1 h_f$ (Fig. 1c).

This implicitly means that, in the case of a T-section, the full depth of the top flange never contributes to the total compressive force when the equivalent rectangular stress block assumption is used, regardless of the magnitude of c. This assumption results in an overestimation of the neutral axis depth, c, for the LRFD Specifications in comparison with the Standard Specifications, as the web contribution must increase to compensate for the portion of the top flange that is being neglected.

Due to the differences in these two interpretations of the Whitney equivalent rectangular stress block assumption, there can be cases for which the AASHTO LRFD Specifications indicate that the section is over-reinforced while the AASHTO Standard Specifications indicate that the same section is not over-reinforced.

Strand Stress

The LRFD and Standard Specifications use different procedures to predict the stress in the prestressing steel at nominal capacity. In the procedure used by the Standard Specifications, the strand stress is predicted by Eq. (6), which is independent of the neutral axis depth:

$$f_{ps} = f_{pu}\left[1 - \frac{\gamma}{\beta_1}\left(\rho\frac{f_{pu}}{f_c'}\right)\right] \tag{6}$$

In the LRFD procedure, on the other hand, the location of the neutral axis is determined first with Eq. (5a), which implicitly includes an assumed value for the strand stress. With this estimate of neutral axis location, strand stress is computed using Eq. (7):

$$f_{ps} = f_{pu}\left(1 - k\frac{c}{d_e}\right) \tag{7}$$

Nominal Moment Capacity

Both the AASHTO Standard and AASHTO LRFD Specifications use formulas for computing the flexural strength of over-reinforced sections that differ from those used for under-reinforced sections. The AASHTO Standard and LRFD Specifications use Eqs. (8) and (9), respectively, for the calculation of moment capacity of under-reinforced prestressed concrete sections:

$$\begin{aligned}M_n = A_{sr}f_{ps}d&\left(1 - 0.6\frac{A_{sr}f_{ps}}{b_w d f_c'}\right)\\ &+ 0.85f_c'(b - b_w)h_f(d - 0.5h_f)\end{aligned} \tag{8}$$

$$\begin{aligned}M_n = A_{ps}f_{ps}&\left(d_p - \frac{a}{2}\right)\\ &+ 0.85f_c'(b - b_w)(\beta_1 h_f)\left(\frac{a}{2} - \frac{h_f}{2}\right)\end{aligned} \tag{9}$$

For the moment capacity of over-reinforced sections, on the other hand, the Standard and LRFD Specifications recommend using Eqs. (10) and (11), respectively:

$$\begin{aligned}M_n = (0.36\beta_1 - 0.08\beta_1^2)&f_c' b_w d^2\\ &+ 0.85f_c'(b - b_w)h_f(d - 0.5h_f)\end{aligned} \tag{10}$$

$$\begin{aligned}M_n = (0.36\beta_1 - 0.08\beta_1^2)&f_c' b_w d_e^2\\ &+ 0.85f_c'(b - b_w)(\beta_1 h_f)(d_e - 0.5h_f)\end{aligned} \tag{11}$$

The last two equations are obtained by substituting into Eqs. (8) and (9), respectively, the maximum amount of tensile reinforcement allowed by Eqs. (1) and (2). Through the use of Eqs. (10) and (11), in effect, the flexural strength of over-reinforced sections is limited to the value of the moment capacity corresponding to the maximum limit of tensile reinforcement. Any additional capacity that may be provided by having more steel than allowed by the reinforcement limits is neglected. This limitation on moment capacity is intended to ensure that sections with limited ductility have reserve moment capacity.

Even though the specifications penalize the use of over-reinforced sections by making a trade-off between the ductility and strength, Article 5.7.3.3.1 of the AASHTO LRFD Specifications states that "Over-reinforced sections may be used in prestressed and partially prestressed members only if it is shown by analysis and experimentation that sufficient ductility of the structure can be achieved." This statement effectively penalizes the design and use of prestressed and partially prestressed over-reinforced sections more severely than provisions that simply limit the flexural resistance, as is done in the Standard Specifications.

The discrepancies described above concerning the two AASHTO specifications, as well as those of the ACI 318 approach, should be rectified to ensure that consistent levels of safety are achieved regardless of the choice of design procedure.

RESEARCH METHODOLOGY

This paper summarizes some of the findings of a study conducted at the University of Minnesota to investigate precast concrete T-section behavior at nominal strength. The study included sectional analyses of several reinforced and prestressed concrete non-rectangular sections following the procedures given in the AASHTO Standard Specifications, the AASHTO LRFD Specifications, and a nonlinear strain compatibility analysis.

The sections were also analyzed with a modified LRFD procedure that is proposed in this paper, and the results were compared with those from the specifications as well as results from the strain compatibility analysis. A comparison of predicted and measured flexural strengths was also done using the limited experimental data available in the literature.

Strain compatibility analyses were conducted using RESPONSE-2000, a sectional analysis program by Bentz and Collins[18] incorporating a nonlinear stress-strain material model. The model used for the concrete was proposed by Popovics, Thorenfeldt, and Collins,[19] and the steel model was the modified Ramberg-Osgood method as implemented by Mattock.[20] The tensile strength of the concrete was neglected in the analyses. Typical stress-strain relations used in the strain compatibility analyses for the concrete and prestressing steel are shown in Fig. 2.

A computer code utilizing internal force equilibrium and strain compatibility between steel and concrete was also developed to verify the results obtained using RESPONSE-2000. The material models used in the code for concrete and steel were the same as those used in RESPONSE-2000. The code developed by the authors uses an iterative numerical solution procedure to determine the sectional response.

The nonlinear analysis procedure includes incremental changes (increasing from zero) in the top fiber concrete strain of the section. For each value of top fiber concrete strain, internal force equilibrium is satisfied by changing the location of the neutral axis, and for each equilibrium point, the neutral axis depth, the strand stress, and the bending moment values are calculated and stored in a matrix. This procedure is repeated until the maximum value of the bending moment is reached.

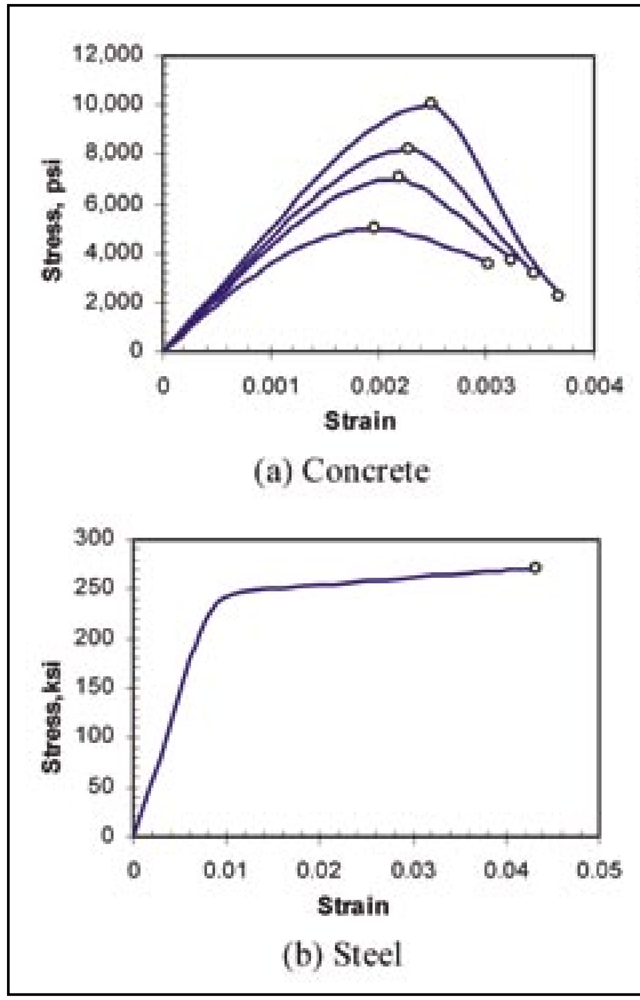

Fig. 2. Material stress-strain relations, using nonlinear material model. Note: 1 psi = 0.006895 MPa; 1 ksi = 6.895 MPa.

In this paper, results of sectional analyses performed using RESPONSE-2000 were used to make comparisons between the sectional responses predicted by the AASHTO Specifications. Thus, when the term "strain compatibility" is used, it refers to the strain compatibility analyses performed using RESPONSE-2000 with nonlinear material models for the concrete and the steel. A comparison of the sectional responses predicted by RESPONSE-2000 and the nonlinear code developed by the authors is also presented later in the paper.

Four non-rectangular sections were studied in an attempt to illustrate the discrepancies that exist in the response of the sections predicted by the specifications, including the neutral axis locations, ultimate flexural capacity, and the strand stress at ultimate capacity. Predictions of these quantities were obtained from the AASHTO Standard Specifications, AASHTO LRFD Specifications, and the strain compatibility analyses. In addition, analytical results were compared to the test results of prestressed concrete I-beams found in the literature, which were identified as over-reinforced and as having neutral axis depths within the web at nominal strength.

The amounts of prestressing steel that would just cause the sections to be over-reinforced according to the strain compatibility analyses were compared to those given by the limits in the specifications. The criterion used to determine the amount of steel from the strain compatibility analyses that would produce an over-reinforced section was the amount associated with a total steel strain (including strain due to effective prestress) of 0.01 at the centroid of the strands when the ultimate moment capacity of the section was reached, as was done previously by other researchers.[11-13]

ANALYSIS OF SECTION BEHAVIOR

Limits of T-Section Behavior

Analyses were conducted to reproduce the results given in Fig. C5.7.3.2.2-1 of the LRFD Specifications (see Fig. 3). This chart was originally developed by Naaman,[21] and is used to indicate the difference in neutral axis depth with the amount of steel for the AASHTO LRFD and AASHTO Standard (and ACI) Specifications for the T-section shown. Results obtained from the strain compatibility analyses are also superimposed on the chart using triangle symbols. (Note that there is an interpretation of the neutral axis depth for the ACI and AASHTO Standard Specifications, part of which is incorrect.)

The plot has been divided along the *x*-axis into three regions to compare the neutral axis depths calculated according to the specifications with those from the strain compatibility study. Region I represents those cases for which c is always smaller than h_f, and both the LRFD and Standard Specifications indicate rectangular section behavior. As shown, in this region the neutral axis depth values calculated according to both specifications with the equivalent rectangular stress block assumption are in good agreement with the values from the strain compatibility analyses using nonlinear stress-strain behavior for the concrete and bilinear stress-strain behavior for the steel.

Region II in Fig. 3 covers those cases for which the neutral axis begins to drop below the flange. In this region, the LRFD Specifications indicate T-section behavior while the Standard Specifications still indicate that the section behaves as a rectangular section (i.e., equivalent rectangular stress block depth $a = \beta_1 c$ still lies within the flange).

The change from rectangular section to T-section behavior is associated with the change in the slope of the line for the LRFD Specifications representing the relationship between c and A_s. For T-section behavior, as compared to rectangular section behavior, the reduced width of the web requires a deeper stress block to generate an equivalent compressive force. This "larger depth" requirement is indicated by the steeper slope of the line for the LRFD Specifications in Region II than in Region I.

For this particular example, because the width of the web is one-quarter the width of the flange, the slope of the line for the LRFD Specifications is four times greater in Region II than it is in Region I. As seen in Region II, the values calculated for c according to the Standard Specifications are between h_f and h_f/β_1. In Region II, the Standard Specifications approximate the location of the neutral axis with acceptable accuracy compared to the strain compatibility analyses while the procedure in the LRFD Specifications grossly overestimates the neutral axis depth.

In Region III, T-section behavior is predicted by both the LRFD and Standard Specifications. As shown in Fig. 3, for a specific value of steel area, both specifications overestimate the neutral axis depth compared to the strain compatibility analyses, but with much larger errors for neutral axis depth calculations using the LRFD Specifications. For example, for a steel area of 8 sq in. (5161 mm^2), the error in c relative to the strain compatibility approach is 20 percent with the Standard Specifications and 93 percent with the LRFD Specifications.

Overestimation of the neutral axis depth can result in erroneous computations of strand stresses and moment capacities. Another impact of erroneous c values is on the use of the provisions of specifications that depend on the c/d value (e.g., definition of over-reinforced sections). Because the LRFD Specifications lead to overestimates in the depth of the neutral axis, c, for Regions II and III, these specifications have the tendency to prematurely classify sections as over-reinforced.

The red dashed line in Fig. 3 is used in Article C5.7.3.2.2 of AASHTO LRFD Specifications to show the inconsistency between the AASHTO LRFD Specifications, AASHTO Standard Specifications, and ACI Code. This dashed line is actually a misinterpretation of the neutral axis depth in the ACI Code and AASHTO Standard Specifications. The plot is based on the assumption that T-section behavior begins when $c \geq h_f$, taking the total depth of the flange as effective in compression at that point and, thereby, indicating a negative neutral axis depth in the web for equilibrium. The correct assumption, denoted by the blue solid line in Fig. 3, is that T-section behavior begins when the depth of the equivalent rectangular stress block exceeds the flange depth (i.e., $a = \beta_1 c \geq h_f$).

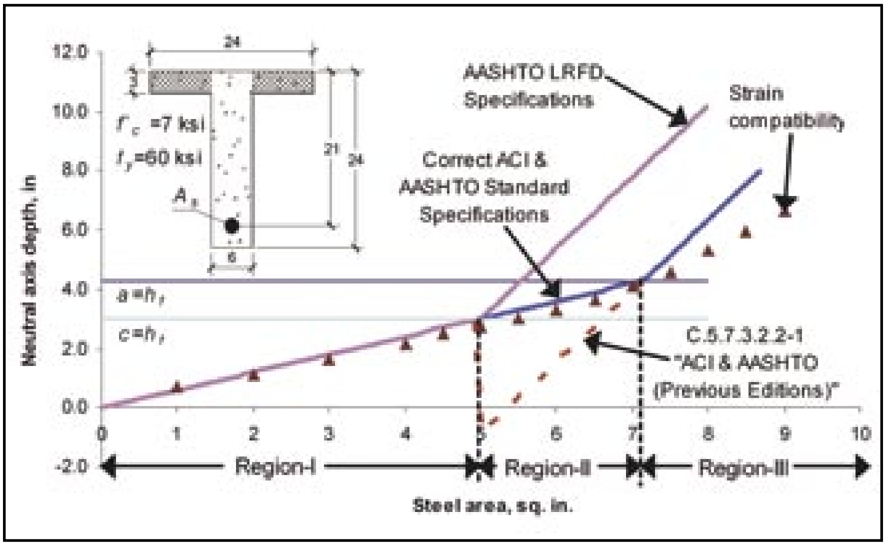

Fig. 3. Change in neutral axis depth with amount of steel in a T-section.[21]
Note: 1 in. = 25.4 mm; 1 sq in. = 645.2 mm^2; 1 ksi = 6.895 MPa.

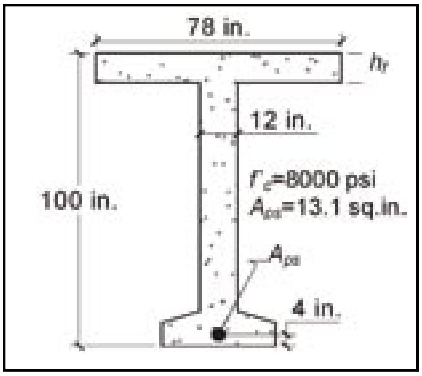

Fig. 4. 100 in. (2540 mm) deep T-section. Note: 1 in. = 25.4 mm; 1 sq in. = 645.2 mm^2; 1 psi = 0.006895 MPa.

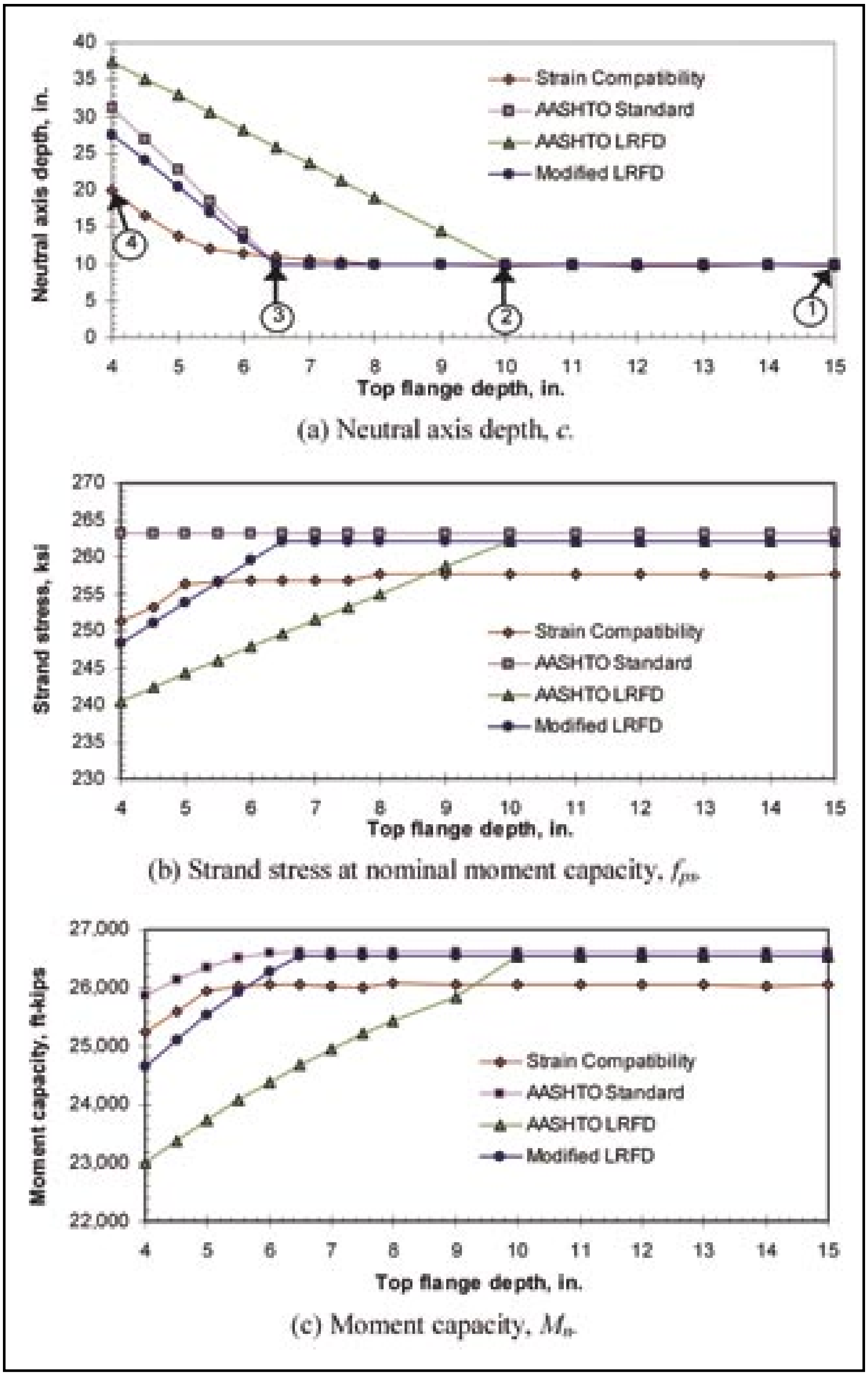

Fig. 5. Effect of top flange depth on section response. Note: 1 in. = 25.4 mm; 1 ksi = 6.895 MPa; 1 ft-kip = 1.356 kN-m.

Influence of Top Flange Depth

In order to investigate the change in neutral axis depth in relation to the depth of the top flange, the section shown in Fig. 4 (similar to the example provided in Reference 6) has been analyzed. The depth of the top flange was gradually decreased from 15 to 4 in. (381 to 102 mm). The location of the neutral axis, the steel stress and the internal moment at the ultimate limit state were determined according to the LRFD Specifications, the Standard Specifications, and strain compatibility analyses.

Note that while decreasing the top flange depth, the total depth of the section was kept constant at 100 in. (2540 mm) (i.e., concrete was removed from the bottom of the top flange overhangs to reduce h_f). The area of the prestressing steel was taken as constant at 13.1 sq in. (8452 mm^2) so that the initial neutral axis depth was 10 in. (254 mm) for a 78 in. (1981 mm) wide compression flange. The effective prestress was taken as 189 ksi (1303.2 MPa), which corresponds to $0.70f_{pu}$ for Grade 270 prestressing strands.

The change in the neutral axis depth is shown in Fig. 5a. Calculations according to both AASHTO Specifications and strain compatibility analyses indicate that the neutral axis position remains unchanged for top flange depths between 15 and 10 in. (381 and 254 mm) due to rectangular section behavior and because the concrete tensile strength was neglected. Four points along the c versus h_f plot identified from the strain compatibility analyses are labeled 1 through 4. Point 1 corresponds to the case when the top flange depth was 15 in. (381 mm). In this case, the LRFD Specifications, Standard Specifications, and the strain compatibility analysis indicate that the section behaves as a rectangular section.

At Point 2, the depth of the top flange is 10 in. (254 mm). As evident from Fig. 5a, both AASHTO Specifications still indicate the neutral axis depth to be at 10 in. (254 mm). However, T-section behavior begins at this point for the LRFD Specifications as noted by the change in slope of the c versus h_f relation. This slope change occurs because the LRFD Specifications consider c, not the depth of the equivalent rectangular stress block ($a = \beta_1 c$), as the limiting value for T-section behavior, whereas the Standard Specifications consider that rectangular section behavior will continue until $a = h_f$, which occurs at a flange depth of 6.5 in. (165 mm) [i.e., when $h_f = a = \beta_1 c = 0.65(10) = 6.5$ in.].

The point at which T-section behavior begins for the Standard Specifications (i.e., when $a = h_f = 6.5$ in.) corresponds to Point 3 in Fig. 5a. According to the LRFD Specifications, the neutral axis depth for this value of flange depth is approximately 26 in. (660 mm), and the rectangular stress block acts on an approximately 17 in. (432 mm) deep portion of the web section (i.e., $a = \beta_1 c = 0.65(26) \approx 17$ in.). This means that according to the LRFD Specifications, a large part of the web, in addition to the flanges, is under compression.

At Point 4, h_f is 4 in. (102 mm) and the neutral axis depths determined by both the LRFD and Standard Specifications exceed h_f. Fig. 5a shows that the difference between the neutral axis depths calculated according to the LRFD and Standard Specifications has lessened at Point 4. This occurs because the contribution of the web to the total concrete compressive force increases, which, in turn, reduces the effect of the difference in flange contributions according to the two specifications. However, there is still a difference in c values, because the Standard Specifications take the full flange depth as effective, whereas the LRFD Specifications limit the flange overhang contribution to a depth of $\beta_1 h_f$.

In the example in Fig. 5a, the LRFD Specifications overestimated the neutral axis depth by as much as 149 percent as compared to the results of the strain compatibility analyses, and the LRFD results began to diverge from the compatibility analysis results at a flange depth of 10 in. (254 mm), whereas the maximum error for the Standard Specifications was 65 percent and the results did not begin to diverge until the flange depth was reduced to 6.5 in. (165 mm).

MODIFICATION OF LRFD PROCEDURE

Proposed Changes

As noted earlier, there are two reasons that the LRFD Specifications overestimate the neutral axis depth: the first reason is the use of $c = h_f$ as the limit for T-section behavior, and the second reason is the use of $\beta_1 h_f$ limit for the maximum flange overhang contribution to the internal compressive force once the T-section behavior begins (see Figs. 1b and 1c). A modification that overcomes both of these problems is proposed to the procedure outlined in the LRFD Specifications to indicate T-section behavior begins when $a = \beta_1 c = h_f$ rather than when $c = h_f$. This modification also solves the second problem mentioned above by enabling the entire flange depth to become effective when $a \geq h_f$.

Because the neutral axis depth and moment capacity of under-reinforced sections and the moment capacity of over-reinforced sections depend on the amount of flange overhang contribution to the internal compressive force, the corresponding equations in the LRFD Specifications (Eqs. 5.7.3.1.1-3, 5.7.3.2.2-1, and C5.7.3.3.1-2) were also modified to remove the $\beta_1 h_f$ limit on the contribution of the flange overhangs. After the modification, Eqs. 5.7.3.1.1-3, 5.7.3.2.2-1, and C5.7.3.3.1-2 in the LRFD Specifications [Eqs. (5), (9), and (11), respectively, in this paper] take the forms of Eqs. (12), (13), and (14), respectively:

$$c = \frac{A_{ps} f_{pu} - 0.85 f'_c (b - b_w) h_f}{0.85 f'_c \beta_1 b_w + k A_{ps} \dfrac{f_{pu}}{d_p}} \tag{12}$$

$$M_n = A_{ps} f_{ps} \left(d_p - \frac{a}{2} \right) + 0.85 f'_c (b - b_w) h_f \left(\frac{a}{2} - \frac{h_f}{2} \right) \tag{13}$$

$$M_n = (0.36\beta_1 - 0.08\beta_1^2) f'_c b_w d_e^2 + 0.85 f'_c (b - b_w) h_f (d_e - 0.5 h_f) \tag{14}$$

The procedure in the LRFD Specifications using Eqs. (12), (13), and (14) instead of Eqs. 5.7.3.1.1-3, 5.7.3.2.2-1, and C5.7.3.3.1-2 is referred to as the "modified LRFD procedure" in the remainder of this paper.

The neutral axis depth values for the preceding example were computed with the modified LRFD procedure and are shown in Fig. 5a. The figure reveals that there is better agreement between the values computed with the modified LRFD approach and the strain compatibility results than there is with either the Standard or the LRFD Specifications. As can be seen, with the modified LRFD procedure, the T-section behavior starts at $h_f = 6.5$ in. (165 mm), similar to the AASHTO Standard Specifications, and for the h_f values investigated, a maximum error of 48 percent occurred compared to the strain compatibility analysis results. This compares with maximum errors in neutral axis depths calculated according to the AASHTO Standard and LRFD Specifications of 65 and 149 percent, respectively.

Plots of the change in strand stress at nominal moment capacity of the section with increasing h_f are shown in Fig. 5b. Note that the strand stress values in the y-axis starts from 230 ksi (1586 MPa) to better show the difference between the values from different methods. Results obtained with the modified LRFD procedure are also superimposed on the plot. For h_f values between 10 and 15 in. (254 and 381 mm), the LRFD and Standard Specifications indicate almost the same strand stress, with values being approximately 2 percent more than the value obtained from strain compatibility analyses.

The difference between the strand stress values computed according to the LRFD and Standard Specifications becomes more evident for h_f values smaller than 10 in. (254 mm). When h_f is less than 10 in. (254 mm), the LRFD Specifications assume that the section behaves as a T-section and overestimates of c begin to occur. Overestimation of c leads to smaller stresses calculated in the prestressing strand at nominal capacity. However, using the modified LRFD procedure, the prediction accuracy improves considerably over both the LRFD and Standard Specifications, as seen in Fig. 5b.

As mentioned earlier, the AASHTO Standard Specifications use Eq. (6) to predict the strand stress. In contrast to the one used by the AASHTO LRFD Specifications, this equation is independent of the neutral axis depth. It includes the ratio of prestressed reinforcement, $\rho = A_{ps}/bd$, and since this ratio remains constant for this example, the plot of the strand stress versus top flange depth for the Standard Specifications procedure indicates no change with top flange depth, as shown in Fig. 5b.

As evident from Fig. 5b, the modified LRFD procedure yields a more realistic estimate of the strand stress than the Standard Specification procedure. For flanged section behavior at ultimate moment capacity, any decrease in top flange depth causes an increase in the neutral axis depth in the web. At ultimate capacity, the increased neutral axis depth results in a reduction in the strand strain, and hence in the strand stress.

As noted above, the procedure used by the Standard Specifications to determine the strand stress cannot represent this behavior. The LRFD procedure and, in particular, the modified LRFD procedure, on the other hand, satisfactorily take the changes in neutral axis location into account when determining the strand stress at ultimate capacity (see Fig. 5b).

Fig. 5c shows the changes in nominal bending resistance with variation in top flange depth. In the region between h_f = 10 in. (254 mm) and h_f = 15 in. (381 mm), where both the LRFD and the Standard Specifications indicate rectangular section behavior, the nominal capacities calculated according to the specifications differ by approximately 2 percent from the nominal capacity obtained using the strain compatibility analysis. The LRFD Specifications show a marked and consistent decrease in the moment capacity as h_f is reduced below 10 in. (254 mm), whereas the Standard Specifications indicate that the moment capacity is nearly constant until h_f drops below approximately 6 in. (152 mm).

With the modified LRFD procedure, the strength of the section is predicted to be constant until h_f = 6.5 in. (165 mm). As a result, for smaller values of h_f, for which the section behaves as a T-section, the flexural capacities calculated according to the Standard Specifications and the modified LRFD procedure are in better agreement with the strain compatibility results compared to those calculated according to the LRFD Specifications. For example, for h_f = 4 in. (102 mm), the LRFD Specifications underestimate the flexural capacity by approximately 9 percent, whereas the Standard Specifications overestimate the capacity by 2 percent and modified LRFD procedure underestimates it by 2 percent.

It is evident from the above observations that the modified LRFD procedure provides a better estimate of the response of the section than does the LRFD and Standard Specifications. The two reasons for the improved accuracy of the modified LRFD procedure are: (1) rectification of the error associated with the contribution of flange overhangs in the LRFD Specifications, and (2) better strand stress characterization in the LRFD procedure (as it takes into account the changes in neutral axis location due to changes in both the flange depth and the strand area) than in the Standard Specifications.

Verification Using Seguirant's Analyses[4]

As noted earlier, due to the difference in interpretation of the equivalent rectangular stress block assumption between the two specifications, a section that is not over-reinforced according to the Standard Specifications could be classified as over-reinforced according to the LRFD Specifications. To investigate this inconsistency between the two specifications, the section shown in Fig. 6 was analyzed using two amounts of reinforcement, namely, 62 and 70 strands.

These sections were the same as those used by Seguirant[4] to illustrate the tendency for the LRFD Specifications to underpredict the nominal bending resistance. In the analysis, 7 ksi (48.3 MPa) concrete and ½ in. (12.7 mm) diameter

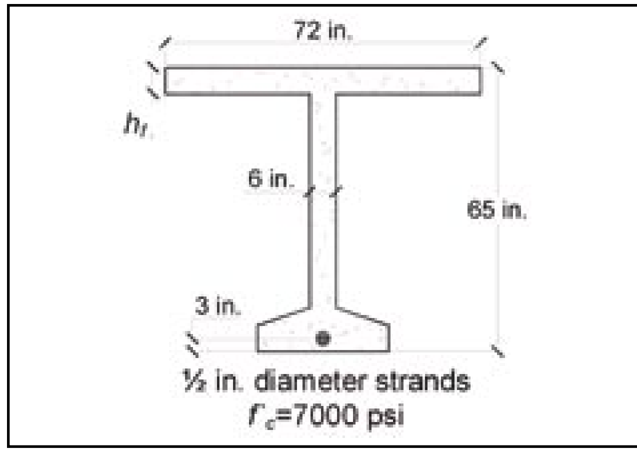

Fig. 6. Section analyzed by Seguirant.[4] Note: 1 in. = 25.4 mm; 1 psi = 0.0069 MPa.

strands with 162 ksi (1117 MPa) effective prestress ($0.60f_{pu}$) were used as was done by Seguirant.[4] Results of the analyses for a 62- and 70-strand case are summarized in Tables 1 and 2, respectively.

For the 62- and 70-strand cases, the Standard Specifications predicted the depth of the neutral axis at nominal flexural resistance to be 106 and 176 percent of the values predicted by strain compatibility analyses, respectively (see Tables 1 and 2). On the other hand, the LRFD Specifications gave neutral axis depths that were 319 and 335 percent of those obtained from strain compatibility analyses for both cases, respectively. The modified LRFD procedure provided neutral axis depth estimates that were 105 and 144 percent of those predicted by the strain compatibility analyses, respectively, for the 62- and 70-strand cases.

Results in Tables 1 and 2 indicate the errors in the moment capacities calculated according to the LRFD Specifications, relative to the strain compatibility analyses, were 10 and 18 percent for the 62- and 70-strand cases, respectively, whereas the errors were within 1 percent for the Standard Specifications and within 2 percent for the modified LRFD procedure.

Note that all three methods indicated that the section with 62 strands was not over-reinforced. When the number of strands was increased to 70, the Standard Specifications and modified LRFD procedure still indicated that the section was not over-reinforced. Based on the limit of a minimum total steel strain of 0.01 at the centroid of the strands at the ultimate capacity (similar to the criteria used earlier by other researchers[11-13]), the strand strain values calculated from the strain compatibility analyses indicate that the sections are not over-reinforced for both the 62- and 70-strand cases.

Note that the relationships between the maximum reinforcement limits according to the specifications and that according to the strain compatibility analyses are investigated in detail later in the paper. Nonetheless, overestimation of the neutral axis depth by the LRFD Specifications resulted in a c/d_e value that was larger than the limit of 0.42. As a result, the LRFD Specifications classify the section as over-reinforced and require a different formula [Eq. (11)] to calculate the nominal bending capacity rather than the one used for sections that are under-reinforced [Eq. (9)].

Table 1. Comparison of AASHTO Specifications' procedures to values predicted by strain compatibility analyses (in percent) for 62-strand case.*

Method	Over-reinforced?	c	Location of resultant C	f_{ps}	M_n
Strain compatibility	—	7.81 in. (100 percent)	2.62 in. (100 percent)	257.3 ksi (100 percent)	144,708 in.-kips (100 percent)
Standard Specifications	No	8.26 in. (106 percent)	2.89 in. (110 percent)	261.2 ksi (102 percent)	146,284 in.-kips (101 percent)
LRFD Specifications	No	24.94 in. (319 percent)	3.92 in. (150 percent)	239.6 ksi (93 percent)	130,517 in.-kips (90 percent)
Modified LRFD	No	8.22 in. (105 percent)	2.88 in. (110 percent)	260.0 ksi (101 percent)	145,800 in.-kips (101 percent)

* Percentages are taken with respect to results from the strain compatibility analyses.

Note: 1 in. = 25.4 mm; 1 ksi = 6.895 MPa; 1 in.-kip = 0.113 kN-m.

Table 2. Comparison of AASHTO Specifications' procedures to values predicted by strain compatibility analyses (in percent) for 70-strand case.*

Method	Over-reinforced?	c	Location of resultant C	f_{ps}	M_n
Strain compatibility	—	9.76 in. (100 percent)	2.91 in. (100 percent)	254.9 ksi (100 percent)	161,171 in.-kips (100 percent)
Standard Specifications	No	17.15 in. (176 percent)	3.46 in. (119 percent)	260.0 ksi (102 percent)	162,958 in.-kips (101 percent)
LRFD Specifications	Yes	32.65 in. (335 percent)	5.19 in. (178 percent)	230.2 ksi (90 percent)	131,667 in.-kips (82 percent)
Modified LRFD	No	14.07 in. (144 percent)	3.25 in. (112 percent)	252.8 in. (99 percent)	157,728 in.-kips (98 percent)

* Percentages are taken with respect to results from the strain compatibility analyses.

Note: 1 in. = 25.4 mm; 1 ksi = 6.895 MPa; 1 in.-kip = 0.113 kN-m.

As mentioned earlier, both the AASHTO LRFD and Standard Specifications introduce an additional "factor of safety" for sections that do not satisfy the maximum reinforcement limits by limiting the nominal moment capacity, as these sections are deemed not to have sufficient ductility. This penalty applied to sections that are considered to be over-reinforced is, to a great extent, the reason that—relative to the strain compatibility analysis—the LRFD Specifications underpredict the flexural resistance of the section with 70 strands by 18 percent, while the error is 10 percent for the case with 62 strands.

Some researchers[5] believe that although the LRFD Specifications overestimate the depth of the neutral axis, the effect on the nominal flexural resistance is not as significant. Regarding this issue, the LRFD Specifications state in Article C5.7.3.2.2 that "Neither treatment of flanged sections (referring to the procedures followed by the LRFD and Standard Specifications) has a significant effect on the value of the nominal flexural resistance, because it is primarily controlled by the steel; however, each significantly affects provisions based on c/d_e, such as the limit of maximum reinforcement, moment redistribution, and ductility requirements."

This statement is partly correct, because the LRFD and Standard Specifications predict similar moment capacities only when the section is considered as under-reinforced in both specifications. When the section becomes over-reinforced for the case of the LRFD Specifications and not the Standard Specifications, then the difference between the moment capacities predicted by the two specifications becomes greater due to the previously explained upper limit on moment capacity.

In addition to this, after the section becomes over-reinforced according to both specifications, and both specifications use the capacity equations for over-reinforced sections, there will still be a difference between the moment capacities predicted by the LRFD and Standard Specifications. This occurs because the LRFD Specifications use the β_1 factor in order to reduce the moment contribution of the top flange overhangs.

Validation with Experimental Data

Thirty-eight 12 in. (305 mm) deep prestressed concrete I-beams tested in flexure by Hernandez[22] were used to validate the analysis results. Twenty-one of the beams were reported to have failed in flexure. The measured flexural strengths were compared to those predicted by the AASHTO Specifications and the strain compatibility analyses. Reported mate-

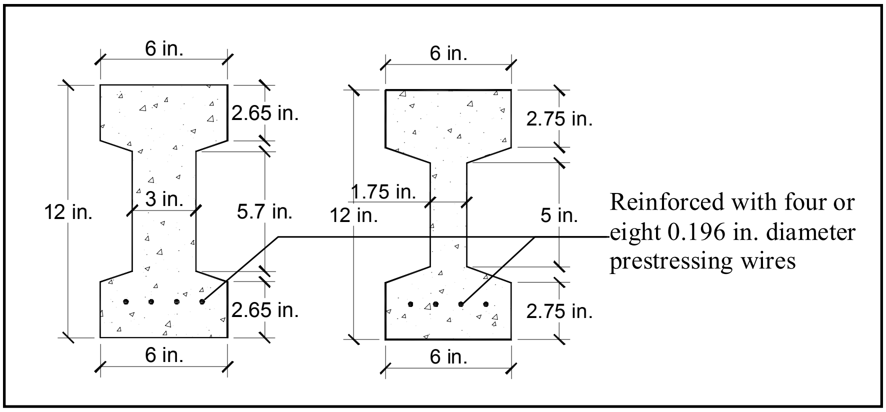

Fig. 7. Nominal dimensions of beam sections tested by Hernandez.[22]
Note: 1 in. = 25.4 mm.

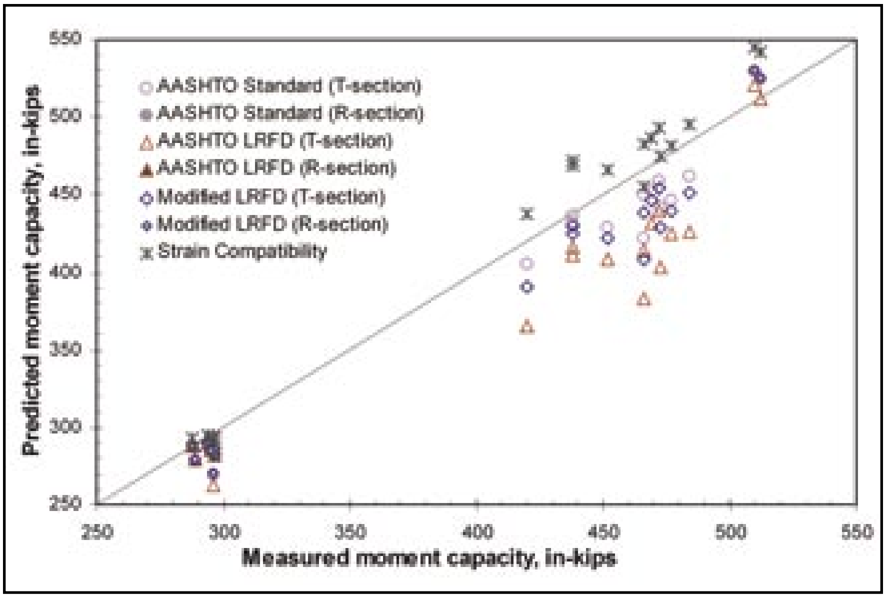

Fig. 8. Comparison of predicted and measured[22] moment capacities without maximum reinforcement limit. Note: 1 in.-kip = 0.113 kN-m.

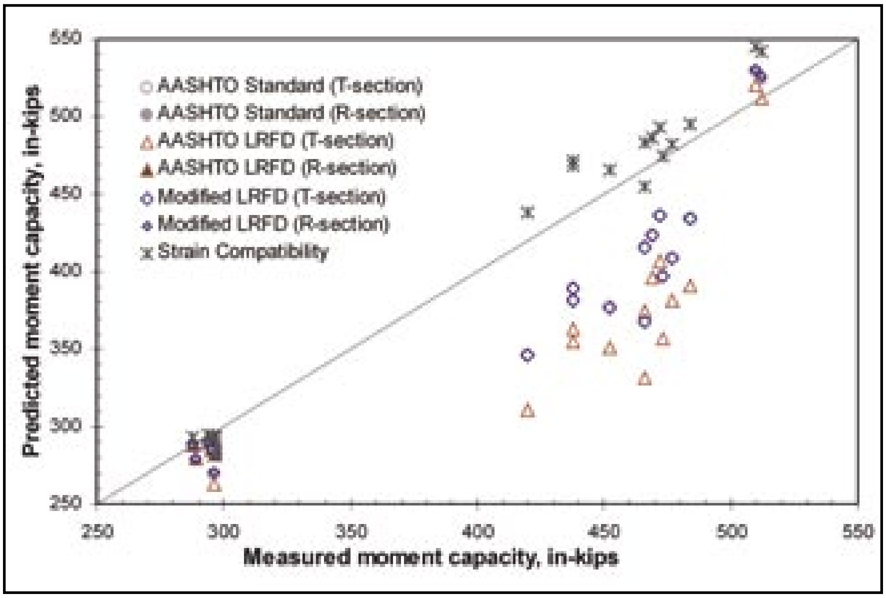

Fig. 9. Comparison of predicted and measured[22] moment capacities with maximum reinforcement limit imposed. Note: 1 in.-kip = 0.113 kN-m.

rial properties were used for each beam and the tensile strength of the concrete was neglected. In the experimental study, two different reinforcement ratios were used with two different cross sections, as shown in Fig. 7. Even though the nominal dimensions of all of the beams were the same, the web and flange dimensions varied slightly. In the analyses, the reported actual dimensions were used.

Figs. 8 and 9 provide a comparison of the predicted and measured moment capacities that are tabulated in Table 3. The moment values plotted in Fig. 8 were computed according to the specifications with no consideration of maximum reinforcement limits. In other words, in computing the moment capacities, the equations provided in the specifications for under-reinforced sections [Eqs. (8), (9), and (13) for Standard Specifications, LRFD Specifications, and the modified LRFD procedure, respectively] were used, even though some of the beams would be classified as over-reinforced according to some of the specifications. In this way, the relation between the measured moment capacities[22] and the capacities predicted by the specifications can be investigated without considering the artificial limitation placed on sections that were considered over-reinforced.

Moment capacities predicted by the strain compatibility analysis were in good agreement with the test results, as shown in Fig. 8. The mean value of the ratio of the predicted to measured values was 1.02 with a coefficient of variation of 0.03. Such close agreement between the predicted and measured capacities was expected because (a) nonlinear material properties were used in the strain compatibility analysis, and (b) even though the tensile strength of concrete was neglected in the analysis, the contribution of the concrete tensile strength to the moment capacity is insignificant.

There is also good agreement between the moment capacities predicted by the AASHTO Standard Specifications and the measured capacities. The mean value of the predicted to measured moment capacities for the Standard Specification results was 0.97 with a coefficient of variation of 0.03.

Table 3. Measured[22] and predicted moment capacities.*

Beam	Measured	AASHTO Standard			AASHTO LRFD			Modified LRFD			Strain compatibility	
	M in.-kips	c in.	ω_p	M in.-kips	c in.	c/d_e	M in.-kips	c in.	c/d_e	M in.-kips	c in.	M in.-kips
G1	296	2.2	0.15	287	2.2	0.21	287	2.2	0.21	287	2.1	294
G2	294	2.1	0.15	290	2.1	0.20	290	2.1	0.20	290	2.0	295
G3	420	8.2	0.59	405 (346)	7.6	0.75	365 (311)	6.9	0.68	391 (346)	6.1	438
G4	466	7.3	0.52	422 (368)	7.0	0.69	383 (331)	6.2	0.62	408 (368)	5.6	455
G8	296	2.9	0.20	270	3.2	0.31	263	2.9	0.28	271	2.6	281
G9	297	2.2	0.16	286	2.2	0.21	286	2.2	0.21	286	2.1	293
G11	295	2.2	0.16	285	2.3	0.22	285	2.3	0.22	285	2.1	292
G12	477	5.2	0.37	446 (409)	5.4	0.53	425 (381)	5.0	0.50	440 (409)	4.7	482
G16	438	5.8	0.41	432 (381)	5.8	0.57	411 (355)	5.5	0.54	425 (381)	5.2	469
G17	452	5.8	0.41	429 (377)	5.9	0.58	408 (351)	5.5	0.55	422 (376)	5.2	466
G18	297	2.4	0.17	282	2.4	0.23	282	2.4	0.23	282	2.2	290
G19	438	5.6	0.40	437 (389)	5.7	0.56	416 (363)	5.3	0.53	430 (383)	5.0	472
G22	484	5.3	0.38	462 (434)	5.6	0.55	426 (391)	4.9	0.48	451 (434)	4.5	495
G23	289	2.6	0.18	278	2.5	0.24	279	2.5	0.24	279	2.3	288
G24	473	6.5	0.46	442 (397)	6.4	0.63	403 (357)	5.7	0.56	429 (397)	5.1	475
G25	288	2.1	0.15	288	2.2	0.21	288	2.2	0.21	288	2.0	293
G27	512	2.9	0.19	525	3.0	0.30	512	2.8	0.28	526	2.7	542
G30	510	2.7	0.18	529	2.8	0.28	521	2.7	0.27	530	2.6	545
G31	469	4.9	0.35	452 (424)	5.1	0.50	432 (396)	4.7	0.47	446 (424)	4.5	486
G32	466	5.6	0.41	450 (416)	5.8	0.58	414 (375)	5.1	0.51	439 (416)	4.7	483
G37	472	4.8	0.34	459 (436)	5.0	0.49	440 (407)	4.6	0.46	454 (436)	4.4	493

* Shaded cells indicate beams with T-section behavior at ultimate. For over-reinforced sections, the moment capacity with no maximum reinforcement limit is reported followed in parentheses by the capacity with reinforcement limit imposed.

Note: 1 in. = 25.4 mm; 1 in.-kip = 0.113 kN-m.

The LRFD Specifications underestimated the moment capacity of the beams compared to the measured capacities, as seen in Fig. 8. The mean value of the predicted to measured moment capacities in the case of LRFD Specifications was 0.93 with a coefficient of variation of 0.06. When the modified LRFD procedure was used, the predicted moment capacities approached the measured values, with a mean value of 0.96 and a coefficient of variation of 0.04. As indicated in Table 3, there were three beams (G8, G27, G30) for which the LRFD Specifications indicated T-section behavior at ultimate moment capacity while the Standard Specifications and the strain compatibility analyses indicated rectangular section behavior. Modifying the LRFD procedure, as explained previously, resolves this inconsistency.

It should be noted that the data analyzed included beams with rectangular section behavior at ultimate moment capacity (i.e., depth of the compression block was within the flange) as well as beams with flanged section behavior. Even though the response of the sections that behaved as rectangular sections was accurately predicted by both the Standard and LRFD Specifications (i.e., error in T-beam definition does not come into play), these data were included for the sake of comparison. The computed values for mean and coefficient of variation also include the results of the beams with rectangular section behavior at ultimate bending moment. The behavior type (rectangular versus T-section) can be distinguished by the symbols plotted in Figs. 8 and 9.

In Fig. 9, moment capacities predicted by the specifications subjected to the maximum reinforcement limit provisions [using Eqs (10), (11), and (14)] are plotted against the measured moment values. As shown, the data points for sections with rectangular compression zones at ultimate moment capacity did not change as compared to Fig. 8. Because these beams were lightly reinforced (reinforced with four wires), they were under-reinforced according to both the LRFD and Standard Specifications, and were unaffected by the maximum reinforcement provisions.

The beams reinforced with eight wires were considered over-reinforced according to both specifications. Fig. 9 shows the safety margin that the specifications place on the computed moment capacity of the over-reinforced sections by limiting the maximum tensile reinforcement to be used in computing the moment capacity. In this case, the LRFD Specifications still predict smaller capacities than the Standard Specifications. On the other hand, moment capacities predicted by the Standard Specifications and the modified LRFD procedure for the over-reinforced beams were identical. This agreement in predicted values occurred because when the β_1 factor used in the LRFD Specifications for the flange overhangs is removed, the equations used to compute the moment capacity of over-reinforced sections in the AASHTO LRFD and Standard Specifications [Eqs. (10) and (11)] become identical.

A comparison of Figs. 8 and 9 suggests that the maximum tensile reinforcement limits are inappropriate from the perspective of moment capacity prediction; the purpose of these limits is to compensate for lower ductility, which is investigated later in this paper.

PARAMETRIC STUDY

Possible differences among the responses predicted by the LRFD and Standard Specifications, strain compatibility analyses, and the modified LRFD procedure were investigated over a wider variation of cross section properties.

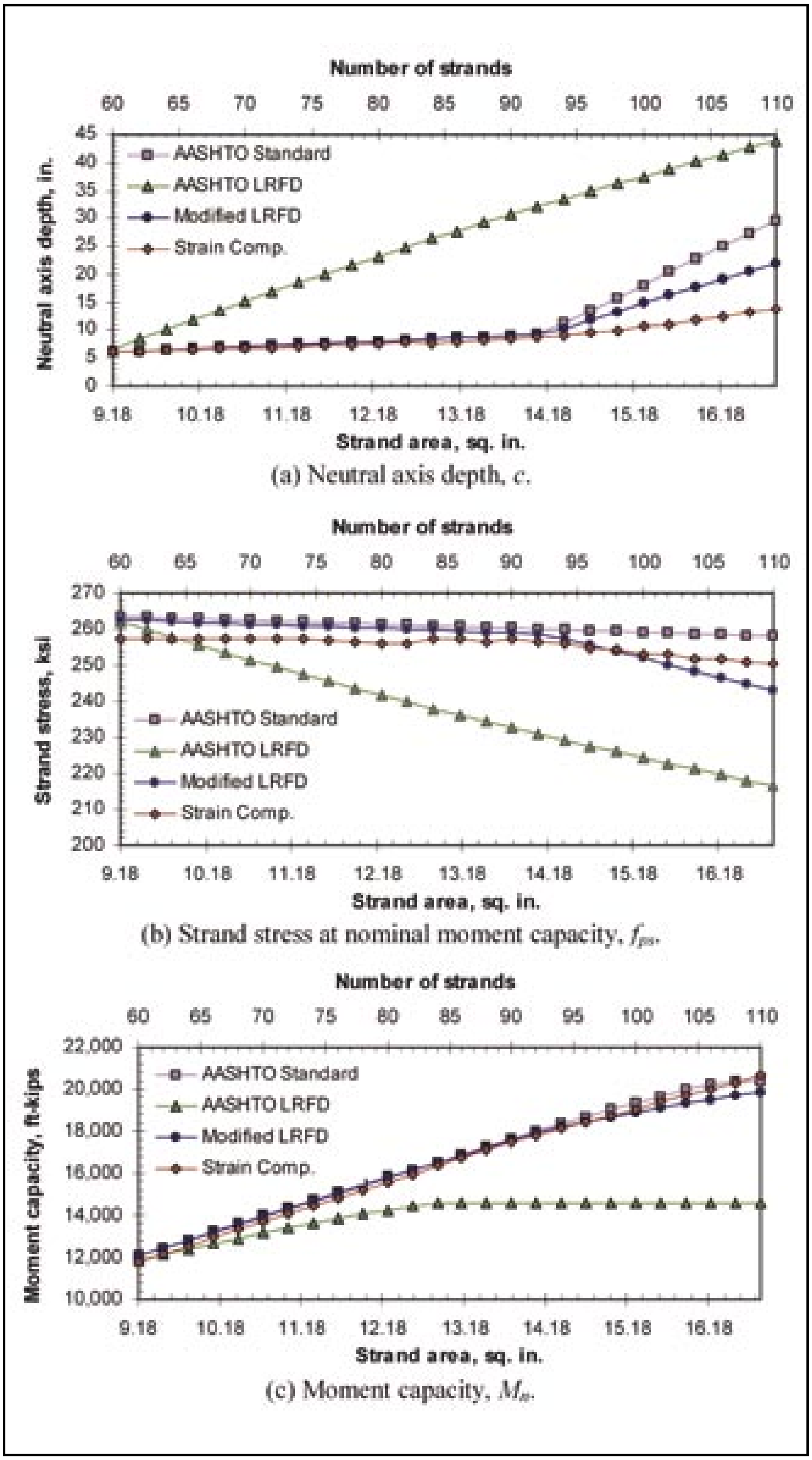

Fig. 10. Variation in section response with amount of prestressing steel [65 in. (1651 mm) deep section]. Note: 1 in. = 25.4 mm; 1 ksi = 6.895 MPa; 1 ft-kip = 1.356 KN-m.

Influence of Total Strand Area

The same section shown in Fig. 6 and used previously by Seguirant[4] was analyzed for various amounts of prestressing steel. However, some differences in reinforcement quantities and material properties were adopted. As in Seguirant's study, the steel was provided by ½ in. (12.7 mm) diameter strands with an effective prestress of 162 ksi (1117 MPa) ($0.60f_{pu}$). Unlike Seguirant's study, the number of strands was varied between 60 and 110 while the center of gravity of the strands was assumed constant, and the concrete strength was taken as 10 ksi (69 MPa). The results are shown in Fig. 10.

Fig. 10a is similar to Fig. 3, which was for a 24 in. (610 mm) deep reinforced concrete T-section. For the present case, the LRFD Specifications assume that T-section behavior starts when c exceeds h_f, and, as shown in the figure, the LRFD Specifications begin to overestimate the neutral axis depth when there are 60 strands in the section. The Standard Specifications, on the other hand, indicate almost the same neutral axis location as the strain compatibility analysis until the number of strands is increased to 92.

For this amount of prestressing steel, the Standard Specifications begin to treat the section as a T-section, and overestimate c as compared to the strain compatibility analysis. Similar results were obtained earlier by Badie and Tadros[3] using strain compatibility analysis with the Whitney equivalent rectangular stress block assumption for the concrete.

The neutral axis depth values computed with the modified LRFD procedure are also shown in Fig. 10a. As for the Standard Specifications, T-section behavior starts at 92 strands for the modified LRFD procedure. The modified LRFD procedure had better agreement with the strain compatibility results.

For the section shown in Fig. 3, which had mild steel reinforcement, the curves for the LRFD and Standard Specifications had the same slope once T-section behavior began (i.e., in Region III). This was not the case for the sections represented in Fig. 10a, which had prestressing steel. The difference between these two figures arises because mild reinforcement has a well-defined yield stress that is assumed to be constant, whereas prestressing strand does not have a well-defined yield point. The LRFD and Standard Specifications compute stress in prestressing steel at nominal resistance using different equations [Eqs. (7) and (6), respectively], which results in different strand stresses according to the two specifications. These differences cause unequal rates of strand stress change with respect to total strand area as noted by the different slopes in Fig. 10b.

The change in strand stress at ultimate capacity of the section is shown in Fig. 10b. As illustrated, the LRFD Specifications underestimate the strand stress compared to the strain compatibility analysis, while the Standard Specifications slightly overestimate it. When the LRFD Specifications are modified as mentioned previously, the results fall into close agreement with those obtained by the strain compatibility analysis. It is also good to note that the strand stress values in y-axis starts from 200 ksi (1379 MPa) to better indicate the difference between the values predicted by different methods.

Fig. 10c shows how the nominal bending resistance calculated according to the LRFD, Standard Specifications, modified LRFD procedures, and strain compatibility analysis change with amount of prestressing steel. Once T-section behavior begins (i.e., when there are 60 strands according to the LRFD Specifications), the LRFD Specifications begin to underestimate bending capacity. As the number of strands increases, the depth of the web participating in the internal compressive force increases until the section becomes over-reinforced at 84 strands.

Subsequently, Eq. (11) is used to compute LRFD nominal bending resistance. As is evident in the plot, this equation is independent of the amount of steel, and the LRFD Specifications predicted a constant value of bending capacity when there were more than 84 strands in the section. According to the Standard Specifications, the maximum reinforcement limit was reached at 108 strands. For the number of strands between 84 and 108, the LRFD Specifications indicate that the section is over-reinforced while the Standard Specifications indicate otherwise.

The inconsistency described above severely limits practitioners' choices, as the LRFD Specifications penalize the use of these so-called "over-reinforced" sections in two ways: (1) by placing a conservative limit on nominal bending resistance, and (2) by requiring additional analyses and experimentation to show that there is sufficient ductility. As shown in Fig. 10, modifying the LRFD procedure as described earlier minimizes the inconsistency between the moment capacities calculated according to the LRFD and Standard Specifications.

Influence of Flange Width and Strand Distribution

The Mn/DOT (Minnesota Department of Transportation) Type 63 section shown in Fig. 11 was analyzed to further investigate the relation between the sectional response predicted by the LRFD and Standard Specifications, and strain compatibility analyses. The Mn/DOT Type 63 section is currently being used in Minnesota for prestressed concrete through-girder pedestrian bridge construction with typical spans on the order of 135 ft (41.2 m). Because of the through-girder type construction, no composite deck exists on top of

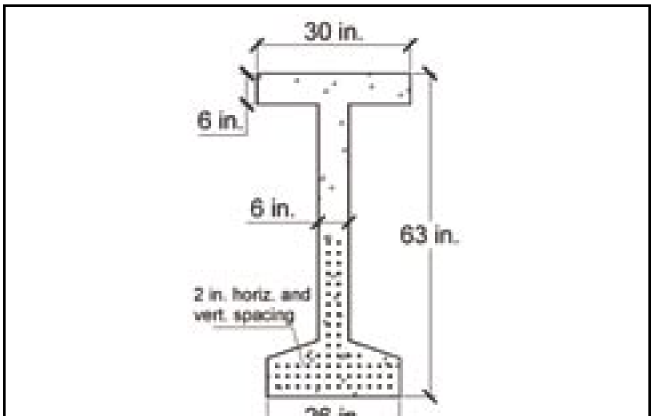

Fig. 11. Mn/DOT Type 63 section. Note: 1 in. = 25.4 mm.

the girders, and there is interest in a more accurate evaluation of the strength and ductility of these girders.

The large span length requires the use of a large number of strands to control deflections. In addition, the section has a narrow top flange ($b_{flange}/b_{web} = 5$). Because neither a composite deck nor a wide top flange is provided to help carry the compressive part of the internal couple, the neutral axis is located within the web of the section, causing the section to be over-reinforced according to the LRFD Specifications and, consequently, causing it to fail to meet the required strength. With the narrow top flange, the difference between the response of the section predicted by the LRFD and Standard Specifications is less significant than it would be for a section with a wider top flange. From this aspect, this section provides a lower bound for the difference between the sectional quantities predicted by the LRFD and Standard Specifications.

The Mn/DOT Type 63 section was analyzed assuming 8.2 ksi (56.5 MPa) concrete strength and ½ in. (12.7 mm) diameter strands with an effective prestress of 162 ksi (1117 MPa) ($0.60f_{pu}$). The number of strands varied from 20 to 60, and the strands were placed in the typical pattern used for this type of section, that is, spaced 2 in. (51 mm) on center in the horizontal and vertical direction. Thus, the depth to the center of gravity of strands was lowered as the number of strands increased, which was not the case for the other sections studied in this paper. The results of the analyses are shown in Fig. 12.

The behavior of this section, which has a narrow top flange, was similar to the section responses shown in Fig. 10 for the 65 in. (1651 mm) deep section, which had a wider top flange. Once T-section behavior begins at 20 strands, the AASHTO LRFD Specifications start to overestimate the neutral axis depth (Fig. 12a) and underestimate the strand stress (Fig. 12b) compared to the strain compatibility results. On the other hand, in the T-section region, the AASHTO Standard Specifications overestimate the neutral axis depth and the strand stress, which is independent of the neutral axis depth for the Standard Specifications. The results from the modified LRFD procedure are in close agreement with those predicted by the nonlinear strain compatibility analysis.

In contrast to the previous example (see Fig. 10), the moment capacity of the section in Fig. 12c decreased after the section became over-reinforced (i.e., at 44 strands according to the Standard Specifications, at 38 strands according to the LRFD Specifications, and at 46 strands according to the modified LRFD procedure). This occurred because the LRFD and Standard Specifications use the distance from the extreme

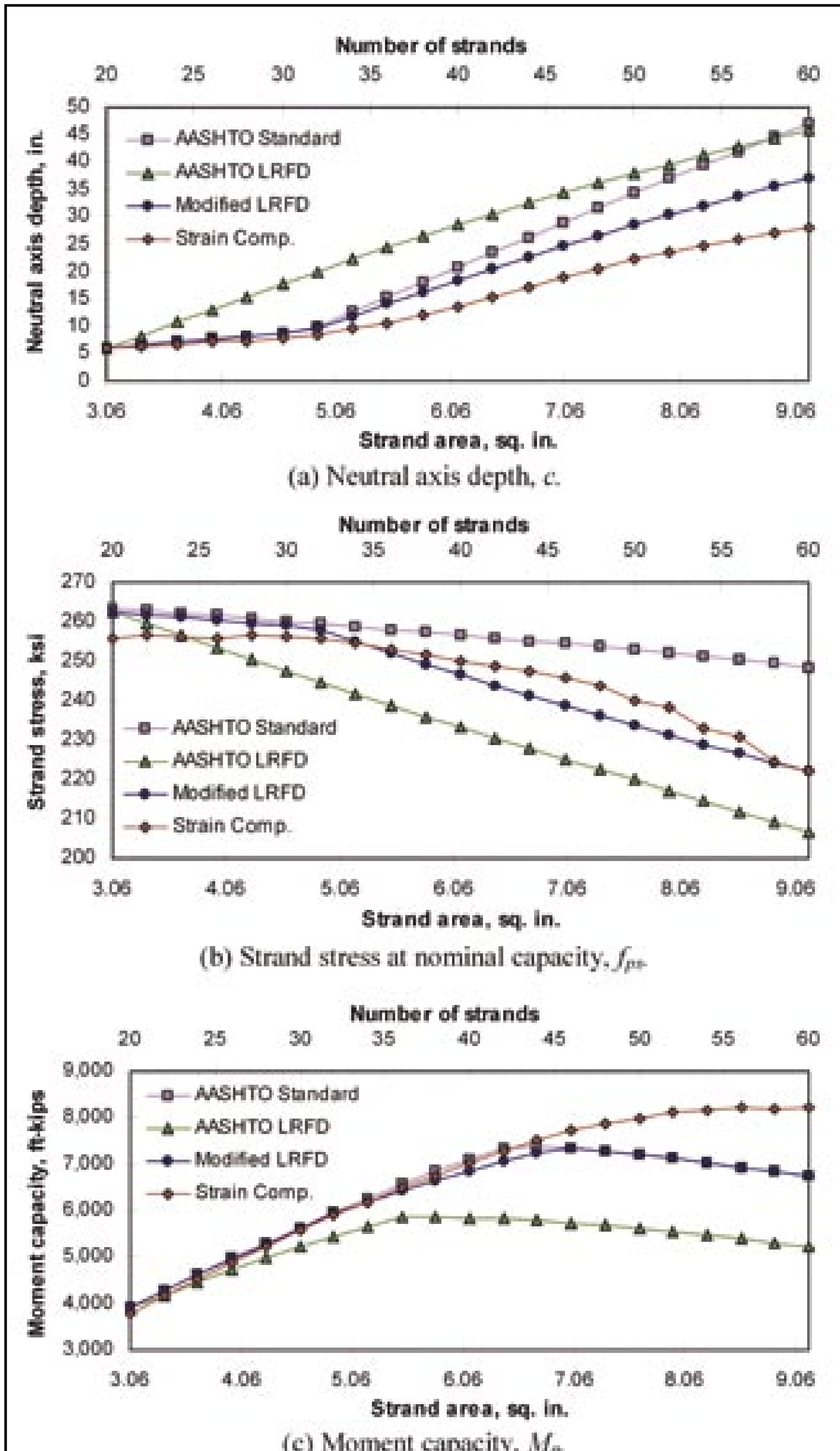

Fig. 12. Variation in section response with amount of prestressing steel [63 in. (1600 mm) deep section]. Note: 1 in. = 25.4 mm; 1 ksi = 6.895 MPa; 1 ft-kip = 1.356 kN-m.

compression fiber to the centroid of the strands to compute the moment capacity of over-reinforced sections, and this distance decreases with increasing number of strands as the strands were distributed through the depth of the section in this example.

Note that once the section became over-reinforced according to the Standard Specifications and the modified LRFD procedure, both specifications (modified LRFD and Standard) indicate the same moment capacity values for increasing number of strands even though strand stresses according to the two specifications are different. This is so because, according to both procedures, moment capacity of over-reinforced sections is computed based solely on the compressive portion of the internal couple.

Verification of RESPONSE-2000 Results

As mentioned earlier, RESPONSE-2000 was used to obtain the strain compatibility sectional analysis results used in the comparisons with the predictions by the LRFD Specifications. To verify the results obtained using RESPONSE-2000, the authors developed their own nonlinear sectional analysis code. A brief description of this sectional analysis code is given earlier in the paper.

Fig. 13 provides a comparison of neutral axis depth, c, strand stress at nominal capacity, f_{ps}, and moment capacity, M_n, of the Mn/DOT Type 63 section, predicted by RESPONSE-2000 and the authors' code. As shown, the results from the two methods agree well, especially for the neutral axis location and the nominal moment capacity. It should be noted here that, among the sections mentioned in this paper, the Mn/DOT Type 63 section had the largest difference between the sectional response predicted by RESPONSE-2000 and the authors' analysis code. For the other sections studied, the difference predicted by the two nonlinear analysis methods was smaller.

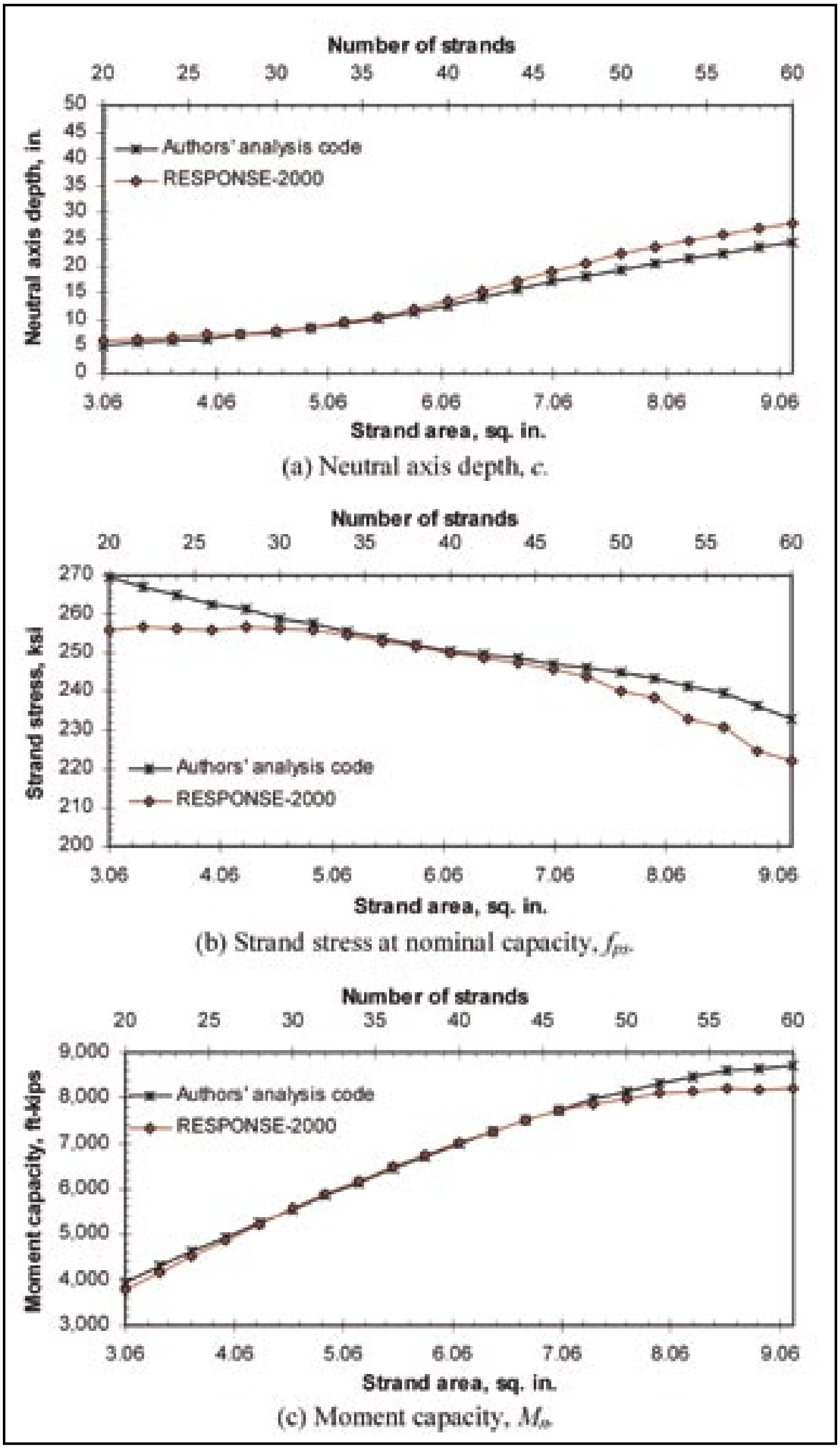

Fig. 13. Comparison of section response predicted by RESPONSE-2000 and authors' code [63 in. (1600 mm) deep section]. Note: 1 in. = 25.4 mm; 1 ksi = 6.895 MPa; 1 ft-kip = 1.356 kN-m.

The reason that there is a small discrepancy between the strand stress values predicted by RESPONSE-2000 and the authors' code is related to the maximum concrete compressive strain values used. When performing the analyses with RESPONSE-2000, the authors did not have full control on the value of the top fiber concrete strain at failure; consequently, the point of failure as predicted by RESPONSE-2000 was not always clear. In some of the cases studied, the RESPONSE-2000 analyses stopped with very low top fiber concrete strain values (as low as $\varepsilon_c = 0.002$), whereas for the same cases, maximum top fiber strain values obtained from the authors' code were in excess of $\varepsilon_c = 0.003$.

As is evident in Fig. 13, the difference in values for top fiber concrete strains at failure between REPONSE-2000 and the authors' code did not have a significant effect on the predicted neutral axis depth and the nominal moment capacity, and the results obtained using REPONSE-2000 can be used with confidence to make comparisons between the sectional responses predicted using the LRFD Specifications. Analyses indicated that a similar observation was valid for the other sections reported in this paper.

Table 4. Number of strands for over-reinforced sections.

Method	65 in. deep T-section	63 in. deep T-section
AASHTO Standard	108 strands	44 strands
AASHTO LRFD	84 strands	38 strands
Modified LRFD procedure	118 strands	46 strands
Strain compatibility	124 strands	48 strands

Note: 1 in. = 25.4 mm.

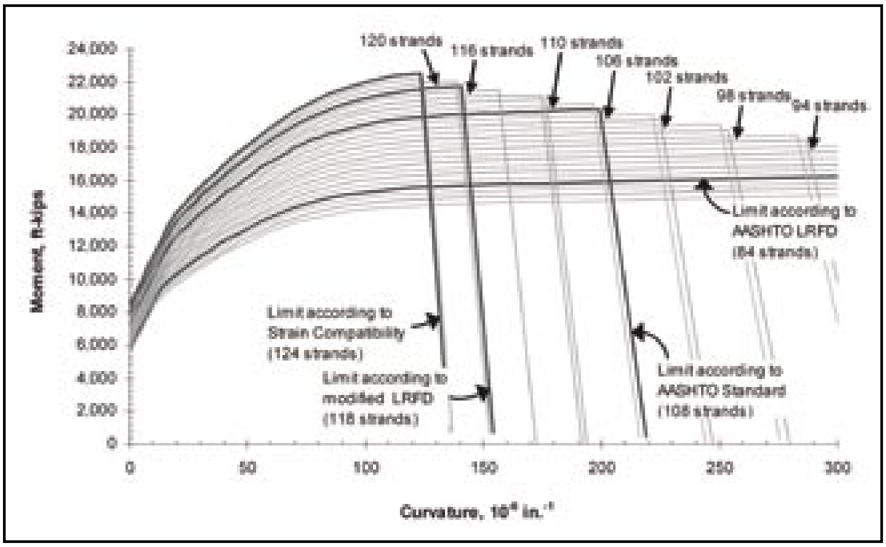

Fig. 14. Moment-curvature relations for 65 in. (1651 mm) deep section. Note: 1 in. = 25.4 mm; 1 ft-kip = 1.356 kN-m.

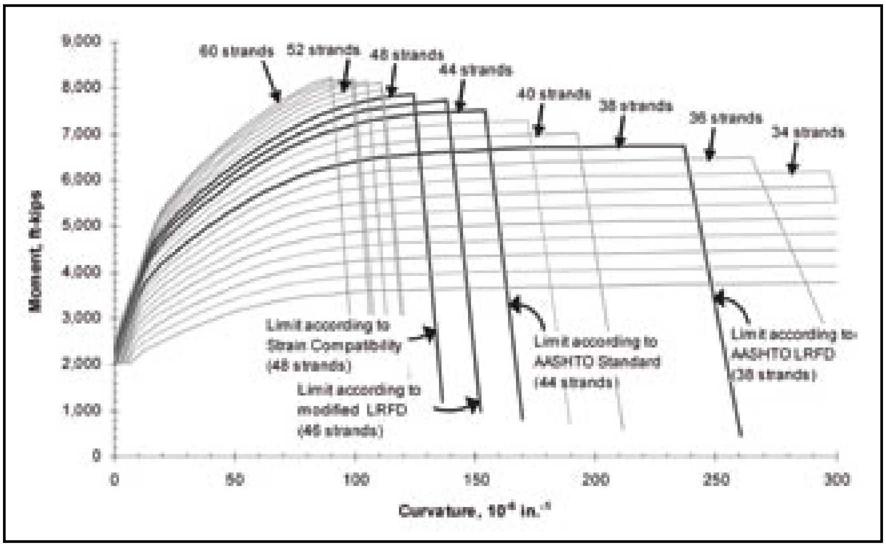

Fig. 15. Moment-curvature relations for 63 in. (1600 mm) deep section. Note: 1 in. = 25.4 mm; 1 ft-kip = 1.356 kN-m.

OTHER DESIGN CONSIDERATIONS

Defining Maximum Reinforcement Limits

In order to investigate the relation between the maximum reinforcement limits according to the specifications and strain compatibility analyses, the sections shown in Figs. 6 and 11 were further studied. For each case, the number of prestressing strands was determined at the limit of over-reinforced behavior.

The criteria used in this study to determine the maximum amount of strands from the strain compatibility analysis was a minimum total steel strain of 0.01 at the centroid of the strands at the ultimate capacity. As explained earlier, similar values for strand strain at ultimate capacity were used previously by other researchers[11-13] in order to define maximum reinforcement limits used in various design specifications.

The number of strands needed to make the 65 and 63 in. (1651 and 1600 mm) deep sections (see Figs. 6 and 11) over-reinforced according to the Standard Specifications, LRFD Specifications, modified LRFD procedure, and the strain compatibility analysis are tabulated in Table 4 with the corresponding moment-curvature plots given in Figs. 14 and 15.

The results indicate that the AASHTO LRFD Specifications are grossly conservative, and the limits according to the modified LRFD procedure are in best agreement with those from strain compatibility analyses. It is also evident from Table 4 that the difference between the limits predicted by the AASHTO Standard and AASHTO LRFD Specifications is larger for the 65 in. (1651 mm) deep section than for the 63 in. (1600 mm) deep section. As explained previously, the 65 in. (1651 mm) deep section had a wider top flange, which makes the discrepancy between the two specifications more significant.

The good agreement between the maximum reinforcement limits predicted by the modified LRFD procedure and those from strain compatibility analyses, which used a criterion similar to what has been used previously by other researchers, proves the accuracy of the modified LRFD procedure in predicting not only the flexural response of prestressed concrete sections, but also the limit at which the section should be considered over-reinforced.

A Comment Regarding Strand Stress Prediction According to the ACI Code

The ACI Code[15] uses the same procedure as the AASHTO Standard Specifications to predict the neutral axis location and the strand stress at ultimate. As noted earlier, the formula used in this procedure to predict the strand stress at ultimate load [Eq. (6)] was originally proposed by Mattock,[13] and is independent of neutral axis depth. Regarding the accuracy of this equation in predicting the actual strand stress at ultimate, Mattock stated that "for values of $\rho_p f_{pu}/f_c'$ greater than the value at which the depth of the equivalent rectangular stress block, *a,* becomes equal to the flange thickness, h_f, Eq. (D) [referring to Eq. 18-3 in the ACI Code, which is given in Eq. (6)] rapidly becomes unconservative."

However, regarding this equation, Section R18.7.2 of the Commentary to the ACI Code states that "Eq. 18-3 (referring to the equation used to predict the strand stress at ultimate) may underestimate the strength of beams with high percentages of reinforcement and, for more accurate evaluations of their strength, the strain compatibility and equilibrium method should be used." For flanged sections, this latter statement from the ACI Code Commentary appears to conflict with the previously quoted passage from Mattock.[13]

The strand stress and the moment capacity of a prestressed concrete T-beam with h_f/d_p = 1/10 and b_w/b = 1/12, and with 5 ksi (34.5 MPa) concrete strength are shown in Figs. 16 and 17, respectively. These plots were reproduced from the plots by Mattock.[13] The strain compatibility analysis done by Mattock included nonlinear stress-strain behavior for steel, and the Whitney stress block assumption for concrete. Superimposed on the figures are the results from the present strain compatibility analysis using RESPONSE-2000 with nonlinear material behavior for both the steel and the concrete.

Also shown in Fig. 17 are the moment capacity results using previous editions of the ACI Code,[10] which limited the maximum tension reinforcement to be used in determining the moment capacity. The ACI 318-99 Code equations yielded conservative results for the moment capacity of the sections with high percentages of reinforcement, while Eq. (18-3) of the ACI Code overestimated the strand stress at the ultimate moment, f_{ps}, for the sections with large amounts of strand (see Fig. 16).

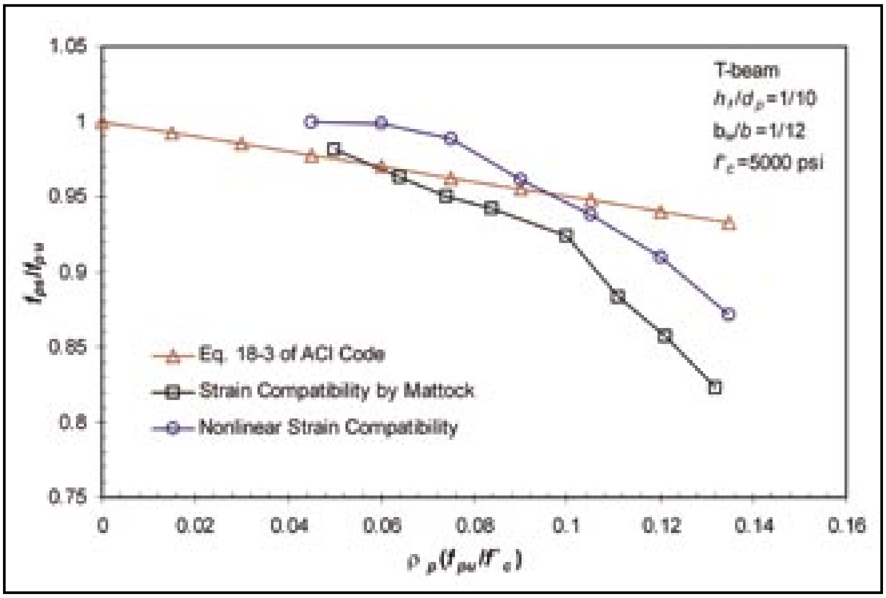

Fig. 16. Comparison of strand stresses at nominal moment capacity. Note: 1 psi = 0.006895 MPa.

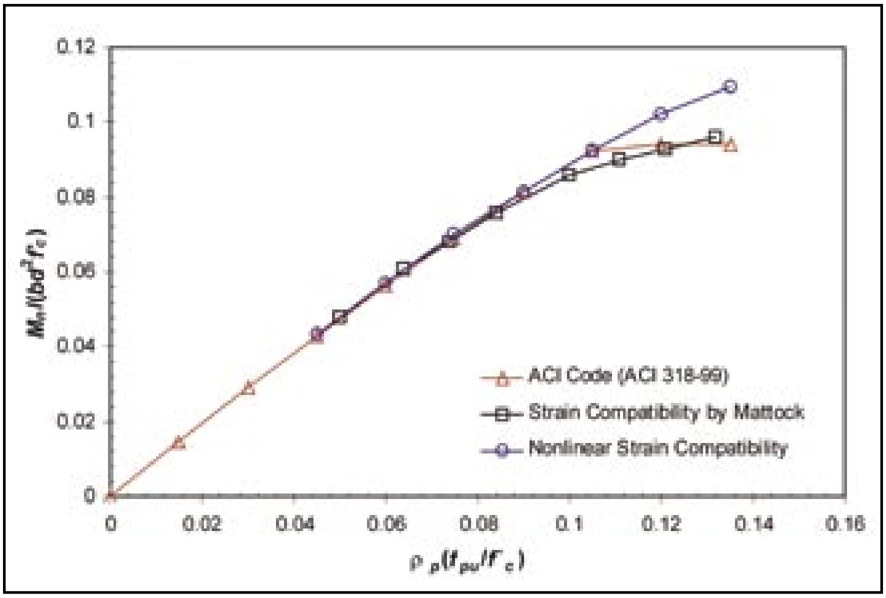

Fig. 17. Comparison of moment capacities. Note: 1 psi = 0.006895 MPa.

Consequently, the previously quoted statement in Section R18.7.2 of the Commentary to the ACI Code was misleading, considering that Eq. 18-3 does not itself compute the flexural strength, and given that it overestimates the strand stress at ultimate moment capacity of flanged sections with high percentages of reinforcement. It was, in fact, the limit on moment capacity for over-reinforced sections that made previous editions of the ACI Code procedure for determining M_n conservative.

In the 2002 version of the ACI Code,[15] sections with large amounts of tensile reinforcement are defined as compression-controlled or transition sections, as opposed to the term "over-reinforced sections" used in the previous editions of the Code, and the reduction in usable flexural strength of these sections is achieved through the use of strength reduction factors that become more severe with decreasing strand strain at ultimate capacity.

Consequently, the statement in the ACI Code[15] should be rewritten to more accurately portray the performance of ACI Eq. 18-3 for flanged sections with large amounts of tensile reinforcement. A suggestion for alternative wording is the following: "Eq. 18-3 may overestimate strand stress at ultimate load for flanged sections with high percentages of reinforcement, and for more accurate evaluations of their strength, the strain compatibility and equilibrium method should be used."

SUMMARY, CONCLUSIONS, AND RECOMMENDATIONS

Strain compatibility analyses with nonlinear material properties were performed for a variety of prestressed concrete sections. The objectives of the study were to identify the inconsistencies between the AASHTO LRFD and AASHTO Standard Specifications procedures, and to investigate the accuracy of both specifications in predicting the flexural response of prestressed concrete sections. Sectional responses from strain compatibility analyses were compared to those predicted by the AASHTO LRFD Specifications, the AASHTO Standard Specifications, and a procedure obtained by modifying the equations in the AASHTO LRFD Specifications to rectify errors in the determination of the flange contribution.

Measured flexural strengths of prestressed concrete I-beams found in the literature were also used for comparison. A parametric study of the flexural behavior of non-rectangular sections was used to demonstrate the shortcomings of the AASHTO LRFD Specifications in the determination of neutral axis depth and flexural capacity, as well as to present the improvements in accuracy of the proposed modifications.

The following conclusions are drawn from this study:

1. Inconsistencies exist between the AASHTO LRFD and AASHTO Standard Specifications in predicting the response of reinforced and prestressed concrete sections that are non-rectangular. The differences are related to the selection of (a) the neutral axis depth at which transition from rectangular section behavior to T-section behavior occurs, and (b) the amount of the top flange overhang contribution to the internal compressive force.

2. The AASHTO LRFD Specifications overestimate the neutral axis depth of T-sections compared to the AASHTO Standard Specifications and nonlinear strain compatibility analysis.

3. Overestimation of the neutral axis depth according to the AASHTO LRFD Specifications leads to the section being considered as over-reinforced at reinforcement ratios for which the AASHTO Standard Specifications and the strain compatibility analyses indicate otherwise.

4. The tendency to prematurely classify some sections as over-reinforced results in large differences between the moment capacities predicted by the AASHTO LRFD Specifications and the other methods.

5. Limiting the maximum amount of tensile reinforcement to be used in determining the moment capacity, as used in the AASHTO LRFD and Standard Specifications, is a means of providing an additional safety margin to account for the poor flexural ductility of sections with large amounts of tensile reinforcement. The provision in the AASHTO Standard Specifications yields similar maximum reinforcement limits as the strain compatibility analyses considering a limiting total prestressing steel strain of 0.01 at nominal capacity. Results of the AASHTO LRFD Specifications are grossly conservative compared to those of the Standard Specifications and the strain compatibility analyses.

6. The inconsistencies between the sectional response and the maximum reinforcement limits predicted by the AASHTO LRFD Specifications and the other methods (AASHTO Standard Specifications and the strain compatibility analyses) can be reduced by modifying the procedure of the AASHTO LRFD Specifications by changing the T-section limit from $c = h_f$ to $a = h_f$. With this modification, the $\beta_1 h_f$ maximum limit for the depth of the top flange overhang contribution to the internal compressive force in the LRFD Specifications is automatically removed.

7. The procedure outlined in the AASHTO Standard Specifications to determine the stress in prestressing steel at ultimate moment does not take into account the effect of changes in the neutral axis location caused by changes in top flange depth. In this respect, the LRFD procedure for strand stress provides more realistic sectional response. Thus, it is proposed that the LRFD strand stress relation be used with the modified procedure.

Based on the findings of this investigation, it is recommended that the procedure currently used by the AASHTO LRFD Specifications should be modified as explained above in order to more accurately predict the response of flanged prestressed concrete sections at ultimate capacity. This modification will also reduce the inconsistencies currently existing between the AASHTO LRFD and AASHTO Standard Specifications.

Validation of numerical results with experimental data was limited to small-scale flanged specimens tested by Hernandez[22] due to parcity of data in the literature. There is a need for additional large-scale flexural tests on over-reinforced prestressed concrete flanged sections, especially as more applications of untopped flanged cross sections with large amounts of prestressing strand may develop.

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REFERENCES

1. AASHTO, *Standard Specifications for Highway Bridges*, 17th Edition, American Association of State Highway and Transportation Officials, Washington, DC, 2002.
2. AASHTO, *LRFD Bridge Design Specifications*, Second Edition, American Association of State Highway and Transportation Officials, Washington, DC, 1998.

3. Badie, S. S., and Tadros, M. K., "Flexural Strength According to AASHTO LRFD Specifications," Open Forum: Problems and Solutions, PCI JOURNAL, V. 44, No. 4, July-August 1999, pp. 122-127.
4. Seguirant, S. J., "Effective Compression Depth of T-Sections at Nominal Flexural Strength," Open Forum: Problems and Solutions, PCI JOURNAL, V. 47, No. 1, January-February 2002, pp. 100-105. See also: Author's Closure, PCI JOURNAL, V. 47, No. 3, May-June 2002, pp. 112-113.
5. Naaman, A. E., Reader Comments on: "Effective Compression Depth of T-Sections at Nominal Flexural Strength," Open Forum: Problems and Solutions, PCI JOURNAL, V. 47, No. 3, May-June 2002, pp. 107-111.
6. Naaman, A. E., "Rectangular Stress Block and T-Section Behavior," Open Forum: Problems and Solutions, PCI JOURNAL, V. 47, No. 5, September-October 2002, pp. 106-112. See also: Author's Closure, PCI JOURNAL, V. 48, No. 2, March-April 2003, pp. 118-119.
7. Rabb, A. P., Reader Comments on: "Rectangular Stress Block and T-Section Behavior," Open Forum: Problems and Solutions, PCI JOURNAL, V. 48, No. 2, March-April 2003, p. 117.
8. Girgis, A., Sun, C., and Tadros, M. K., "Flexural Strength of Continuous Bridge Girders—Avoiding the Penalty in the AASHTO LRFD Specifications," Open Forum: Problems and Solutions, PCI JOURNAL, V. 47, No. 4, July-August 2002, pp. 138-141.
9. Weigel, J. A., Seguirant, S. J., Brice, R., and Khaleghi, B., "High Performance Precast, Pretensioned Concrete Girder Bridges in Washington State," PCI JOURNAL, V. 48, No. 2, March-April 2003, pp. 28-52.
10. ACI Committee 318, "Building Code Requirements for Structural Concrete (318-99) and Commentary (318R-99)," American Concrete Institute, Farmington Hills, MI, 1999.
11. ACI-ASCE Joint Committee 323, "Tentative Recommendations for Prestressed Concrete," *ACI Journal*, V. 54, No. 7, January 1958, pp. 545-578.
12. Warwaruk, J., Sozen, M. A., and Siess, C. P., "Investigation of Prestressed Reinforced Concrete for Highway Bridges: Part III—Strength and Behavior in Flexure of Prestressed Concrete Beams," *Bulletin No. 464*, Engineering Experiment Station, University of Illinois, Urbana, IL, 1962, 105 pp.
13. Mattock, A., "Modification of ACI Code Equation for Stress in Bonded Prestressed Reinforcement at Flexural Ultimate," *ACI Journal*, V. 81, No. 4, July-August 1984, pp. 331-337.
14. Mast, R. F., "Unified Design Provisions for Reinforced and Prestressed Concrete Flexural and Compression Members," *ACI Structural Journal*, V. 89, No. 2, March-April 1992, pp. 185-199.
15. ACI Committee 318, "Building Code Requirements for Structural Concrete (318-02) and Commentary (318R-02)," American Concrete Institute, Farmington Hills, MI, 2002.
16. ACI Committee 318, "Building Code Requirements for Reinforced Concrete (318-83)," American Concrete Institute, Farmington Hills, MI, 1983.
17. ACI Committee 318, "Commentary on Building Code Requirements for Reinforced Concrete (318-83)," American Concrete Institute, Farmington Hills, MI, 1983.
18. Bentz, E., and Collins, M., *User Manual for RESPONSE-2000 Reinforced Concrete Sectional Analysis Using the Modified Compression Field Theory,* University of Toronto, Toronto, Ontario, Canada, 2001.
19. Collins, M. P., and Mitchell, D., *Prestressed Concrete Structures*, Prentice Hall, Englewood Cliffs, NJ, 1991.
20. Mattock, A., "Flexural Strength of Prestressed Concrete Sections by Programmable Calculator," PCI JOURNAL, V. 24, No. 1, January-February 1979, pp. 32-54.
21. Naaman, A. E., "Unified Design Recommendations for Reinforced, Prestressed, and Partially Prestressed Concrete Bending and Compression Members," *ACI Structural Journal*, V. 89, No. 2, March-April 1992, pp. 200-210.
22. Hernandez, G., "Strength of Prestressed Concrete Beams with Web Reinforcement," Ph.D. Thesis, University of Illinois, Urbana, IL, June 1958.

APPENDIX A — NOTATION

a = depth of equivalent rectangular stress block, in.
A_{ps} = area of prestressing steel, sq in.
A_s = area of non-prestressed tension reinforcement, sq in.
A_{sf} = prestressing steel area required to develop ultimate compressive strength of overhanging portions of top flange, sq in.
A_{sr} = prestressing steel area required to develop ultimate compressive strength of web of section, sq in.
b = width of compression face of member, in.
b_w = web width, in.
c = distance from extreme compression fiber to neutral axis, in.
C = resultant of internal compressive force carried by concrete at ultimate, kips
d = distance from extreme compression fiber to centroid of prestressing force, in.
d_e = effective depth from extreme compression fiber to centroid of tensile force in tensile reinforcement, in.
d_p = distance from extreme compression fiber to centroid of prestressing tendons, in.
f'_c = specified compressive strength of concrete, ksi
f_{ps} = average stress in prestressing steel at ultimate load, ksi
f_{pu} = specified tensile strength of prestressing steel, ksi
f_{py} = yield strength of prestressing steel, ksi
f_y = specified minimum yield strength of reinforcing bars, ksi
h_f = compression flange depth, in.
k = factor for type of prestressing tendon
= 0.28 for low-relaxation steel
= 0.38 for stress-relieved steel
= 0.48 for bars
M_n = nominal flexural resistance, in.-kips
β_1 = ratio of depth of equivalent uniformly stressed compression zone assumed in strength limit state to depth of actual compression zone
γ, γ_p = factor for type of prestressing tendon
= 0.28 for low-relaxation steel
= 0.40 for stress-relieved steel
= 0.55 for bars
ρ = prestressed reinforcement ratio = A_{ps}/bd
ω_w = reinforcement index considering web of flanged sections

$$= \frac{A_{sr} f_{ps}}{b_w d f'_c}$$

Note: For common terms, AASHTO LRFD notation is used even for the AASHTO Standard Specification equations.