A Probabilistic Comparison of Prestress Loss Methods in Prestressed Concrete Beams

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Camber and long-term deflections are directly linked to the loss of prestressing force, a characteristic of all prestressed concrete members. In this investigation, the effects of the inherent variability of the parameters used to estimate prestress loss was studied for two typical bridge beam cases using several prestress loss predictive methods at final service conditions. A parametric study was conducted to assess the general effect of single parameter variation on the calculation of prestress loss. Monte Carlo simulations were used to assess the distribution of prestress loss considering the variability of all parameters simultaneously. Results of the Monte Carlo simulations were also used to evaluate the impact of loss variability on deflection and cracking moment calculations. Results show that the variability of parameters has a notable effect on prestress loss variation and that this variation significantly influences the estimated deflection and cracking moment of prestressed concrete beams. Indeed, the variation in camber between beams cast on the same prestressing bed can be partially explained through the variability of materials and predictive methods used.

Loss of prestress is a characteristic of all prestressed concrete members wherein the level of prestress force first applied to the member is reduced over time due to short- and long-term conditions.

Many methods have been used to estimate the prestress losses in prestressed concrete members. These methods produce discrete values representing the expected losses, and vary considerably in both length and complexity of calculations. The methods consider many factors in their respective calculations but do not consider the effect of variability due to the parameters such as concrete strength, strand stress, strand area, dimensional properties, and environmental conditions.

The purpose of this study was to investigate the effect of input variation on the computed losses from six common methods used to estimate prestress losses. To assess these effects, two composite bridge systems [a 70 in. (1780 mm) I-beam and a 21 in. (535 mm) spread box beam] used in simple span configurations were considered. A parametric study was conducted to independently assess the influence of each parameter on the total prestress loss.

A second assessment was performed using a Monte Carlo simulation that randomly selected values for each parameter used by the loss calculation methods based on the known or estimated variability of each parameter. The result was a more inclusive look at the variability of total prestress losses. The resulting variability was then used to assess variability in expected deflection and cracking moment.

BACKGROUND

An accurate prediction of the prestress loss in a prestressed concrete member is important in the design phase to assess the expected behavior of the member over its life. Calculated losses are used to predict service conditions such as expected concrete stress levels, camber, deflection, and cracking loads. However, an accurate prediction of prestress loss is difficult because of the complex interaction between the various sources of losses and the inherent non-homogeneity of concrete members.

In general, short-term losses include elastic shortening of the member at the time of prestress application and any steel relaxation that occurs prior to transfer of the prestress force. Long-term prestress losses include creep and shrinkage of concrete and any additional steel relaxation that occurs after prestress transfer and during the life of the member.

There are numerous methods used to estimate prestress losses. Some methods require more knowledge about the member properties and loading conditions than other methods. It is important to realize that all such methods are approximations.

Steinberg[1] studied the effects of statistical variability in the parameters used to calculate prestress losses. Using three common precast shapes – a rectangular beam (12RB16), a double tee (10LDT32), and an inverted T-beam – he showed that prestress losses are consistently calculated lower (by about 33 percent) when using deterministic or nominal parameters than when including statistical variability. Based on the analysis using the PCI General Method,[2] Steinberg recommended that nominal losses computed by this method be increased by 25 percent when checking calculated stresses at final conditions due to full load plus effective prestress against allowable stresses. Further work was recommended to determine the effect of variability on prestress losses for additional member types.

The primary objective of the research presented herein was to compare the variability of total prestress losses computed by several methods while accounting for parameter variability through a probabilistic assessment. Two bridge types — an I-beam superstructure and a spread box beam superstructure — were utilized for prestress loss comparisons and estimating the range of deflections and loads to cause cracking based on variability of total prestress losses.

The methods used to estimate prestress losses can be divided into the following three primary categories:

1. Lump sum of total losses
2. Lump sum of individual losses
3. Time-dependent cumulative losses

The lump sum of total losses method (Category 1) is considered to be the least accurate due to an oversimplification of the parameters involved in calculating prestress losses. The lump sum of individual losses (Category 2) is the most common category for prestress loss methods used in design. These methods account for more variables than the Category 1 approach and are relatively straightforward in computation. Category 3, the time-step method, is the most accurate, but is also fairly lengthy and involves knowledge of member loading over its life duration.

Table 1. Summary of methods used to estimate prestress losses.

Calculation method	Lump sum total	Lump sum individual	Time dependent
AASHTO Standard Specifications[3]		X	
AASHTO LRFD Specifications[4]		X	
Time-Step Method[5]			X
PCI General Method[2]			X
PCI Simplified Method[6]	X		
ACI 318-99[7] and Zia[8]		X	

Recommended calculation methods published by AASHTO, PCI, ACI, and others[2-8] were considered. Each method and the respective category are listed in Table 1.

The AASHTO LRFD[4] prestress loss method is based on that provided in the AASHTO Standard Specifications,[3] although steel relaxation is presented differently in the estimate of elastic shortening. While additional methods can be found (e.g., CEB-FIP Model 90[10]), the study was limited to the six methods in Table 1. The reader is directed to each respective reference for details in calculating losses according to AASHTO LRFD[4] and Standard Specifications.[3] The Appendix contains details on the time-step method,[5] PCI General[2] and Simplified[6] methods, and the Zia method.[8]

The losses computed in this study are long-term, and include the effects of steel relaxation prior to transfer and elastic shortening. Anchorage losses at the jack prior to seating of the strands are not considered, as this value is known to the precaster and taken into consideration during the initial stressing operation.

Prestressed Concrete Bridge Sections

Prestress losses were computed for composite interior beams in two bridge cross sections that represent sections typically used by the Michigan Department of Transportation

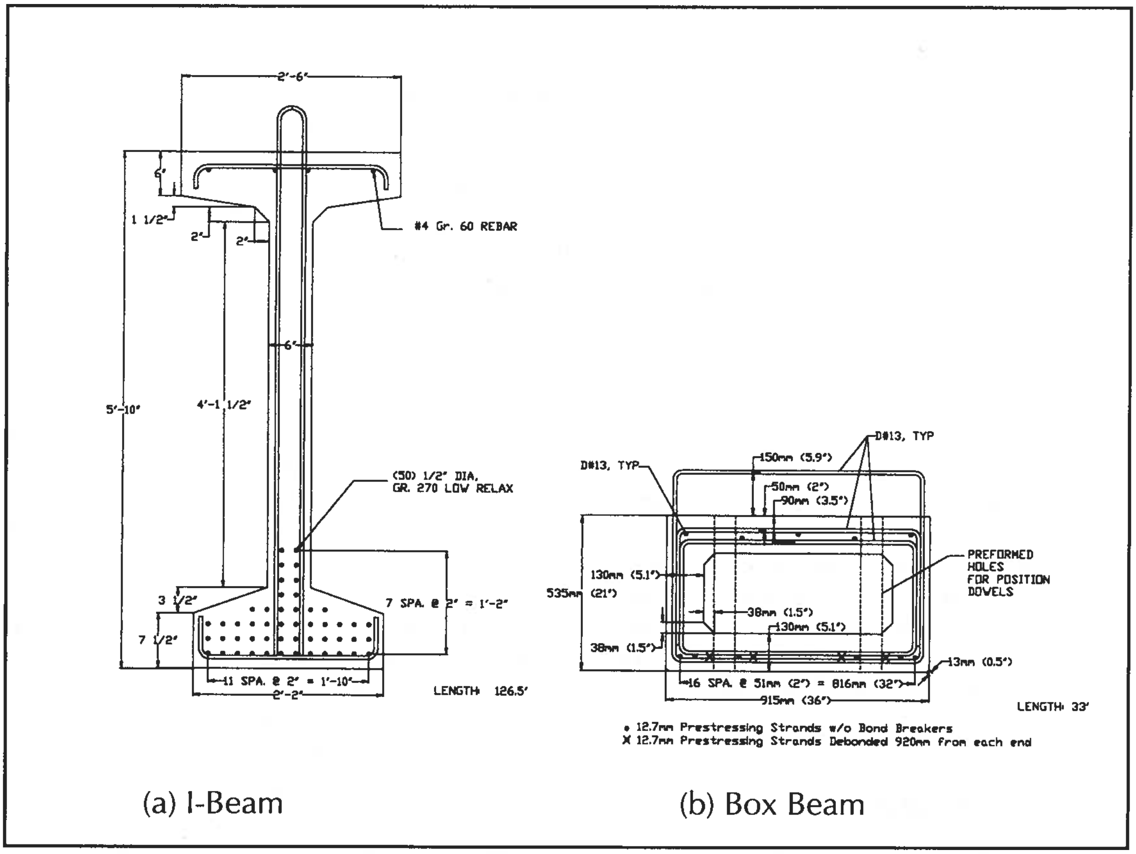

Fig. 1. Cross sections of beams considered showing dimensions and reinforcing details.

(MDOT) in new construction of simple-span bridges. Case I studied a 70 in. (1780 mm) I-beam system used for a composite bridge in Mt. Pleasant, Michigan (MDOT Bridge #37021). The 9 in. (230 mm) composite deck is supported by nine I-beams spaced at 7.5 ft (2.3 m) on center.

Each beam was prestressed with fifty $^1/_2$ in. (12.7 mm) diameter low-relaxation prestressing strands and had an overall length of 126.5 ft (38.6 m). The beams spanned 125.5 ft (38.3 m) center-to-center of bearings. Fig. 1 shows the dimensions and reinforcing details of the non-composite I-beam cross section.

Case II studied a 21 in. (535 mm) box beam used for a bridge in Southfield, Michigan (MDOT Bridge #63101). The 87 ft (26.5 m) wide bridge required ten box beams spaced at 9 ft (2.7 m) on center, achieving a spread box system.

Each 34 ft (10.4 m) beam spanned 33 ft (10 m) between bearing centers, supported a 9 in. (230 mm) composite deck, and was prestressed with ten $^1/_2$ in. (12.7 mm) diameter strands in the bottom flange. The dimensions and reinforcing details of the non-composite box beam are shown in Fig. 1 (right).

Statistical Description of Parameters

The nominal (design) properties of the two prestressed concrete bridge systems were taken primarily from the bridge plans. Table 2 contains a summary of the nominal beam properties used to estimate prestress losses. Values for steel and concrete strength, jacking stress, sectional area, perimeter, and moment of inertia came directly from the bridge plans.

The unit weight of concrete was estimated as 150 lb per cu ft (2400 kg/m^3), and relative humidity was based on a site location in Lower Michigan using the AASHTO LRFD Specifications[4] Fig. 5.4.2.3.3-1. Dead load moments from the girder self-weight and the deck dead load were calculated using bridge plan details. The water content of the concrete was estimated from relative humidity data[11] and the time between initial stressing of the strand and release of the member was determined from discussions with PCI certified plant managers that had produced the beams.[12]

To proceed with a probabilistic assessment and include the variability of each parameter associated with prestress losses, statistical data were gathered from many sources. Data presented by several references were traced back to the original source and are presented in detail by Gilbertson.[13] Table 3 lists the referenced statistical data used in this study.

Several random variables were used for the prestress loss calculations for which no statistical information could be found. Most of the missing data were specific to the bridge systems and beams used in this study. Table 4 summarizes each non-referenced statistical variable, the nominal value, estimated high-to-low values, and the estimated statistical values (mean, coefficient of variation, and type of distribution).

Non-referenced statistical properties were estimated by applying allowable tolerances or ranges to the known nominal value. This produced high and low values with a reasonable range. (For example, the allowable PCI design tolerances[6] were applied to the cross sections studied and new section properties were calculated to determine the high and low values.)

The mean, μ_x, represents the value expected from an actual measurement, taken here as the average between high (maximum) and low (minimum). The mean may be higher or lower than the nominal value depending on the skew of the range for the maximum and minimum estimated values. Statistical z-tables were then used to determine the standard deviation, σ_x, and coefficient of variation, V_x, as follows.

In a standard normal distribution, the mean, μ_x, is zero and the standard deviation, σ_x, is one. A standard normal curve produces a bell shape centered at zero, with an area under the curve of one. The z-value is on the bottom scale of this curve and is a random variable. The z-tables indicate the integral of the normal curve from minus infinity ($-\infty$) to z; thus, when $z = 0$, 50 percent of the data fall to the left of zero.

This property can be applied to any normal curve by Eq. (1), which transforms a normal random variable to allow z-tables to be used on the data. Eq. (1) can be used to solve for a standard deviation given a mean and a range of values that encompass a determined amount of the data represented by the integral from $-\infty$ to z.

$$z = \frac{x_{max} - \mu_x}{\sigma_x} \qquad (1)$$

in which

z = random variable for standard normal distribution

x_{max} = maximum value of x, calculated by multiplying percent difference allowed by nominal value of x

μ_x = mean value of x determined from x_{max} and x_{min}

σ_x = standard deviation of x

The z-value used in Eq. (1) may be obtained from a statistical z-table found in most statistics and probability textbooks.[17] For this study, it was assumed that the probability of a random variable falling between the values of x_{max} and x_{min} was 90 percent; thus, a z-value of 1.65, which corresponds to an area under the normal curve of 0.95, was selected.

Table 2. Nominal design values used to estimate prestress losses.

Beam properties	Case I 70 in. I-beam	Case II 21 in. box beam
Ultimate strand strength, f_{pu} (ksi)	270	270
Yield strength of strand, f_{py} (ksi)	243	243
Jacking strand stress, f_{pj} (ksi)	202.6	202.6
Concrete strength at release, f'_{ci} (psi)	5,800	3,046
Concrete strength at 28 days, f'_c (psi)	7,000	5,076
Unit weight of concrete, γ_c (pcf)	150	150
Relative humidity, RH (percent)	75	75
Modulus of elasticity of strand, E_s (ksi)	28,500	28,500
Gross area of cross section, A_g (sq in.)	774	467
Perimeter of cross section (in.)	229.1	183.3
Area of prestressing steel, A_s (sq in.)	7.65	1.53
Gross moment of inertia, I_g (in.^{4})	511,000	24,600
Strand eccentricity, e (in.)	28.71	8.50
Moment due to girder self-weight, M_g (in.-kips)	19,346	794
Moment due to superimposed dead load, M_{sdl} (in.-kips)	23,216	2,385
Water content (lb/yd^3)	315	315
Time, t (hours)	36	36

Note: 1 ksi = 6.89 MPa; 1 pcf = 16.02 kg/m^3; 1 in. = 25.4 mm; 1 in.-kip = 0.113 kN-m; 1 lb/yd^3 = 0.593 kg/m^3.

Table 3. Referenced statistical data.

Variable	Nominal value	Mean	Coefficient of variation	Distribution type	Reference number
f_{pu}	270 ksi	281 ksi	0.025	Normal	14
f_{py}	240 ksi	1.027f_{pyn}	0.022	Normal	15
γ_c	γ_{cn}	γ_{cn}	0.030	Normal	15
E_s		28400 ksi	0.020	Normal	14
A_s	0.153 sq in.	0.1548 sq in.	0.0125	Normal	15
e		0 to + $^1/_{16}$ in.	0.04 to 0.68	Normal	15
Dead load	DL_n	1.05(DL_n)	0.1	Normal	15

Note: 1 ksi = 6.89 MPa; 1 pcf = 16.02 kg/m^3; 1 in. = 25.4 mm; 1 in.-kip = 0.113 kN-m; 1 lb/yd^3 = 0.593 kg/m^3.

Table 4. Estimated statistical data.

Variable	Nominal value	High/Low (x_{max}/x_{min})	Mean, μ_x	Coefficient of variation, V_x	Distribution type
f_{pj}	202.6 ksi	212.7/192.5	202.6 ksi	0.030	Normal
f'_c	f'_c	—	1.10f'_{cn}	0.174	Normal
f'_{ci}	f'_{ci}	—	1.10$f'_{ci\,n}$	0.200	Normal
RH	75 percent	89/60	75 percent	0.118	Normal
A_g-I	774 in.	812.0/745.4	779 sq in.	0.026	Normal
A_g-II	467 in.	481.3/452.8	467 sq in.	0.018	Normal
Perimeter -I	229.1 in.	231.2/227.9	229.5 in.	0.004	Normal
Perimeter -II	183.3 in.	184.3/182.3	183.3 in.	0.003	Norma^4
I_g-I	511,000 in.^{4}	528392/493854	511,123 in.^{4}	0.020	Normal
I_g-II	24,600 in.^{4}	25,804/23,396	24,600 in.^{4}	0.030	Normal
Water content	315 lb/yd^3	325.0/305.0	315 lb/yd^3	0.014	Normal
t, time	36 hrs	48/24	36 hrs	0.202	Normal

Note: Variable notation I and II refer to girder cross sections I) I-beam and II) box beam.
Note: 1 ksi = 6.89 MPa; 1 pcf = 16.02 kg/m^3; 1 in. = 25.4 mm; 1 in.-kip = 0.113 kN-m; 1 lb/yd^3 = 0.593 kg/m^3.

The σ_x of the random variable was calculated by rearranging Eq. (1) and applying the mean and maximum values from Table 4. The V_x (coefficient of variation) was calculated by taking the ratio of σ_x to μ_x. This process, along with the PCI Design Handbook[6] tolerances, was used to approximate

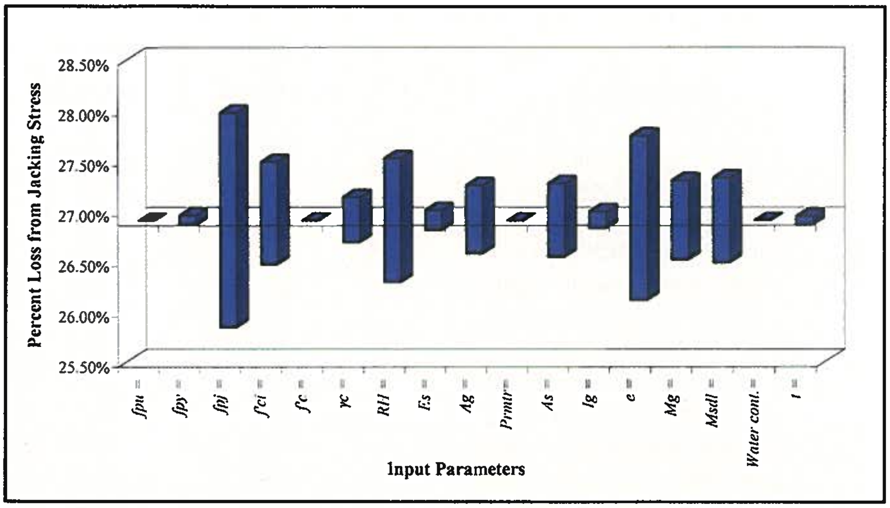

Fig. 2. Parametric study results using AASHTO LRFD method for I-beam example.

σ_x and V_x for the jacking stress, f_{pj}, relative humidity, *RH*, gross section area, A_g, perimeter, moment of inertia, I_g, water content, and time from jacking to release of strands, *t*.

The statistical parameters for concrete strength (V_x and distribution type) were selected from Ellingwood[16] because the published values fit within the range presented by many others, the definitions of the nominal and mean values were clear, and the source is used in many other studies. However, the mean value used to represent concrete strength was modified from the referenced value because the original data were for reinforced cast-in-place concrete beams.

When constructing cast-in-place beams, it is important to meet a required strength once, usually specified at 28 days. Thus, the actual concrete strengths are relatively close to the required compressive strengths, as reflected by a 1 percent increase from the nominal to the mean values given by Ellingwood. Similar data for concrete strengths of precast, prestressed members remain unpublished.

The production of precast, prestressed concrete sections often requires two strengths, one at release (f'_{ci}), and the other usually at 28 days (f'_c). Prestressed beams are typically cast with a higher target compressive strength to ensure that the required release strength is reached early and the beam can be removed from the casting bed.

Discussions with PCI certified plant managers provided an estimation of the actual concrete strength being 20 percent higher than the nominal strength at both the time of release and at 28 days. A 10 percent increase from nominal to mean strength (see Table 4) was used as a conservative estimate because actual test/field data were unavailable. The V_x value was referenced from Ellingwood[16] for the 28-day compressive strength, but was increased slightly for release strengths to reflect more variation at early stages of concrete curing.

It was found that correlation among the variables exists. For example, the area of a precast section is directly linked to the cross-sectional perimeter and the moment of inertia. Initially, all variables were assumed mutually independent.

A test was performed to assess the effects of correlating several of the variables using two of the six prestress loss methods for four cases: (1) mutually independent variables, (2) mutually dependent correlation between area, perimeter, and moment of inertia, (3) release concrete strength as a function of 28-day strength, and (4) a combination of the second and third cases.

The analysis showed that the V_x changed slightly among the four cases but showed little difference in the mean total prestress losses, and general trends remained. Therefore, for simplicity, all variables were assumed independent of each other for comparison of total losses from the six prestress loss estimation methods.

PARAMETRIC STUDY

The influence of each loss input parameter (each variable) was investigated through a parametric study to determine the individual effect on the total prestress loss for each method. Final losses were computed for each of the methods (listed in Table 1) from a common set of input data representing nominal (design) values (see Table 2) for the base case.

Not all of the data shown in Table 2 were used by each of the predictive loss methods. The PCI Simplified Method,[6] for example, does not require many of the parameters used by the more complex time-step[5] and individual lump sum methods.[2,3,4,7] Individual loss components were calculated for each base case as well as the total loss of prestressing steel stress in the member at final service conditions.

Final prestress losses (or losses estimated at the end of service life) were also computed plus or minus one standard deviation from the nominal base case. A program was written to iterate through each variable, calculating the loss using the full range of the variable. The individual component losses and total losses were stored for each changing variable.

The variable was then returned to its base case value, and the next variable was changed to calculate losses for plus or minus one standard deviation. The process was repeated through all of the parameters used in the various prestress loss methods.

The results calculated were prestress losses due to elastic shortening, steel relaxation, creep of concrete, concrete shrinkage, and total final loss for each of the methods using individually varying input parameters. A graphical result is shown in Fig. 2 for total losses using the AASHTO LRFD method with the I-beam example. Similar results were found for other prestress loss estimation methods and when using the box beam example.[13]

From Fig. 2, the base case (horizontal line) represents using only nominal (design) values and does not vary from parameter to parameter. In this

Table 5. Prestress losses from Monte Carlo Simulation.

Parameter	AASHTO Standard Specifications[3]	AASHTO LRFD[4]	Time-Step[5]	PCI General[2]	PCI Simplified[6]	ACI 318-99[7]
	Case I (I-beam) / Case II (box beam)					
Elastic Shortening (ksi)						
Nominal	18.4 / 10.3	18.4 / 10.3	18.4 / 10.3	18.4 / 10.3	—	18.4 / 9.73
Mean	18.1 / 10.2	18.1 / 10.2	18.1 / 10.2	18.1 / 10.2	—	18.1 / 9.63
Coefficient of Variation	0.119 / 0.124	0.119 / 0.123	0.118 / 0.123	0.117 / 0.100	—	0.135 / 0.133
Shrinkage (ksi)						
Nominal	5.75 / 5.75	5.75 / 5.75	10.1 / 11.0	9.91 / 12.7	—	4.66 / 4.95
Mean	5.74 / 5.76	5.74 / 5.76	10.1 / 11.0	10.1 / 12.3	—	4.64 / 4.96
Coefficient of Variation	0.232 / 0.229	0.229 / 0.231	0.144 / 0.144	0.047 / 0.083	—	0.358 / 0.357
Creep (ksi)						
Nominal	26.8 / 8.68	26.7 / 8.66	13.4 / 7.92	16.3 / 5.69	—	19.1 / 4.20
Mean	26.9 / 8.71	26.8 / 8.69	13.0 / 7.72	16.3 / 6.20	—	18.1 / 3.82
Coefficient of Variation	0.084 / 0.109	0.085 / 0.109	0.128 / 0.148	0.105 / 0.253	—	0.172 / 0.359
Steel Relaxation (ksi)						
Nominal	1.54 / 3.25	4.09 / 6.14	3.97 / 4.77	3.95 / 4.90	—	3.31 / 4.24
Mean	1.56 / 3.26	3.94 / 5.97	3.61 / 4.39	3.57 / 4.48	—	3.37 / 4.26
Coefficient of Variation	0.182 / 0.049	0.092 / 0.054	0.188 / 0.169	0.183 / 0.166	—	0.602 / 0.027
Total Losses (ksi)						
Nominal	52.4 / 27.9	54.9 / 30.8	45.8 / 34.0	48.5 / 33.6	63.3 / 34.6	45.5 / 23.1
Mean	52.3 / 27.9	54.7 / 30.6	44.7 / 33.3	48.1 / 33.1	63.3 / 34.8	44.2 / 22.7
Coefficient of Variation	0.069 / 0.074	0.067 / 0.071	0.094 / 0.093	0.071 / 0.091	0.054 / 0.036	0.110 / 0.119

Note: 1 ksi = 6.89 MPa.

example, the base prestress loss of 27.1 corresponds to the percent loss from initial jacking, or 54.9 ksi (380 MPa) as listed in Table 5 for the AASHTO LRFD Method.[4] (While Table 5 lists the results of the Monte Carlo simulation, the nominal losses using MCS match those computed in the parametric study for the nominal base case.)

The upper and lower bounds of the histogram in Fig. 2 represent plus and minus one standard deviation above and below the nominal respective parameter values. A variation of zero percent indicates that the parameter is not used in the loss estimation method presented. Mean values, representing field data values, were not used by the parametric study because the purpose of the parametric study was to show the significance of variation of the individual design parameters.

The total prestress loss is affected by the parameters mentioned above. However, it is important to note that the individual parameters may have a strong influence on the individual loss components, and may not have as significant of an effect on the total loss. For example, if the low value of a parameter causes one form of loss to increase but another to decrease, the result of summing these effects may be reduced. Parameters f'_{ci} (concrete strength at strand release), RH (relative humidity) and e (eccentricity of strand) were found to have a high V_x (coefficient of variation) and thus a greater impact on the variability of the total loss. The jacking stress, f_{pj}, did not necessarily have a high V_x but had a strong influence because the final stress is directly affected by this parameter.

Changes in parameters such as f_{pj}, f'_{ci}, RH, and e caused very large changes in the total prestress loss. However, these changes are small compared to some of the effects they had on the individual loss components summed to achieve the total loss. The remaining parameters influenced total loss to a lesser degree.

MONTE CARLO SIMULATION

All analytical systems contain a certain degree of variability. When these systems are formed by a combination of random variables, the resulting variability of the system generally cannot be found in a closed form approach. An alternative approach that allows the estimation of variability in a system given the variability of its components is Monte Carlo Simulation (MCS).[17] MCS was used in this study to determine the total variability of prestress loss estimated by each method (and its components) considering all variables simultaneously, as opposed to the parametric study, which considered the prestress loss influence of only one parameter at a time.

The general process of an MCS involves selecting a random number and using it to generate a random variable that falls within the statistical limitations of each of the parameters or variables involved. The random values produced for each parameter in the loss calculation are then used to calculate a prestress loss based on a given estimation method. Repeating the process many times produces a matrix of values that can be used to calculate the mean (μ_x), coefficient of variation (V_x), distribution type, or other desired statistical parameters of the output.

In this study, 10,000 simulations were used to estimate the prestress loss at final service conditions for

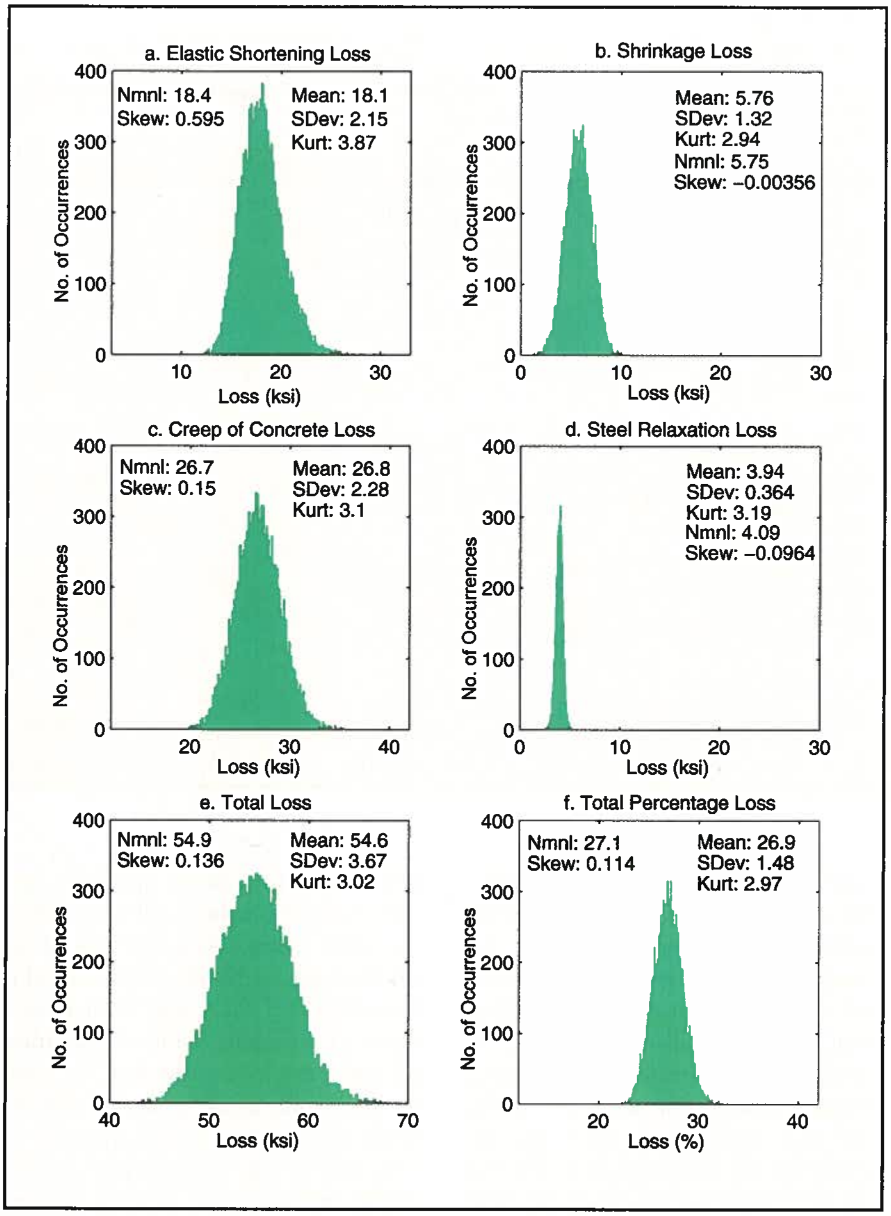

Fig. 3. Monte Carlo Simulation output distributions using AASHTO LRFD individual loss components for I-beam example.

each of the methods considered. Samples were conducted with 100,000 simulations; however, the results were similar to samples with 10,000 simulations. Therefore, 10,000 simulations were performed to conserve computation time in this study.

The MCS program also calculated losses using the nominal (design) values so as to establish a base case. Base case values for nominal loss calculated by all of the methods for both the I-beam and box beam examples were consistent between the MCS program and the parametric study, thus providing a check balance against programming errors.

The loss of prestress at final service conditions due to individual components, as well as the total losses, are presented in Table 5 for all six predictive methods for both the I-beam and box beam examples. Nominal stresses were computed using the design parameters. The mean and coefficient of variation computed from the MCS are also listed for each simulated distribution.

Fig. 3 graphically depicts a histogram of each individual loss component using the AASHTO LRFD Method[4] for predicting losses in the I-beam example.

Values given on each distribution include 'Nmnl' (using nominal parameters), 'Skew' (coefficient of skewness, a measure of whether a tail exits to the right or left of the distribution, where a value of 0.0 describes a symmetrical distribution about the mean), 'Mean' (the calculated mean of the distribution), 'SDev' (standard deviation), and 'Kurt' (coefficient of kurtosis, a measure of the peakedness of the distribution, where a value of 3.0 describes a normal distribution).

Fig. 3a shows that the predicted elastic shortening losses can vary from about 12 to 28 ksi (83 to 193 MPa) for the I-beam example. Fig. 3e shows the distribution of the combined stress loss in ksi (MPa). A mean of 54.6 ksi (375 MPa) corresponds to the mean total loss listed in Table 5 for this case. Fig. 3f displays the percent loss of prestress relative to the initial jacking force. Similar distributions were found for individual component losses when using the other predictive methods.[13]

Fig. 4 shows the resulting histograms of total losses computed for each method using the I-beam example. Fig. 5 shows total losses for the box beam example. Total losses are a summation of the individual loss components and can vary greatly between the predictive methods.

The scale on the loss axis of each graph is equal, thus allowing an easy comparison between the general distributions. However, the graphs do not necessarily begin at the same value; thus, some losses are distributed similarly even though the distributions are about a different mean.

Results from the two AASHTO methods[3,4] are nearly the same for the I-beam example, while the time-step method,[5] PCI General Method,[2] and ACI 318-99[7,8] are similar to each other. The PCI Simplified Method[6] produced the highest estimation of total loss, followed by the AASHTO methods.

The highest standard deviation occurred using ACI 318-99, though all standard deviations, σ_x, were between 3.40 and 4.87 ksi (24 to 34 MPa). The smallest standard deviation occurred in the PCI Simplified Method. However, the standard deviation alone should not be used to determine the resolution of one method over the other methods.

The PCI Simplified Method required far less input than the other methods, and if input random variables do not vary significantly, the output will not vary appreciably, as shown here. In addition, all distributions are approximately normal based on the coefficients of skewness (about zero) and kurtosis (about 3), except for the PCI General Method.

The latter method is most likely not normally distributed. A normal distribution would be expected if all parameters were summed to achieve the result. The complexity and nonlinearity of the loss equations allow for the possibility of other distribution types as found in the PCI General Method.

The results of the 21 in. (535 mm) box beam example were somewhat different than the 70 in. (1780 mm) I-beam example. For the smaller box beam cross section, the smallest predicted mean strand stress loss was computed by the ACI 318-99 method, which was approximately 30 percent smaller than the typical value of around 33 ksi (228 MPa). The next smallest mean loss was computed using the AASHTO Standard Method followed by AASHTO LRFD.

The time-step and PCI General Method results were nearly the same, both in mean values and standard deviation. The largest mean loss was computed using the PCI Simplified Method, though it was not much larger than the time-step method or PCI General Method [34.8 ksi versus 33.3 ksi (240 MPa versus 230 MPa)]. The largest standard deviation was found in the time-step and PCI General Methods. The range of values for standard deviations fell between 1.25 and 3.11 ksi (8.6 and 21 MPa).

Careful observation shows that the order of prestress losses, from least to largest mean loss is not consistent between the two cross sections used in this study. Figs. 6 and 7 show the spread of the distributions of loss calculation methods on the same graph for the two examples considered. Note that these figures are merely a combined representation of Figs. 4 and 5, respectively.

The time-step and PCI General Methods are near the lower end of the range on the I-beam, but near the upper range for the box beam. This change of order is a direct result of the cross-sectional properties and concrete strength differences in the two examples.

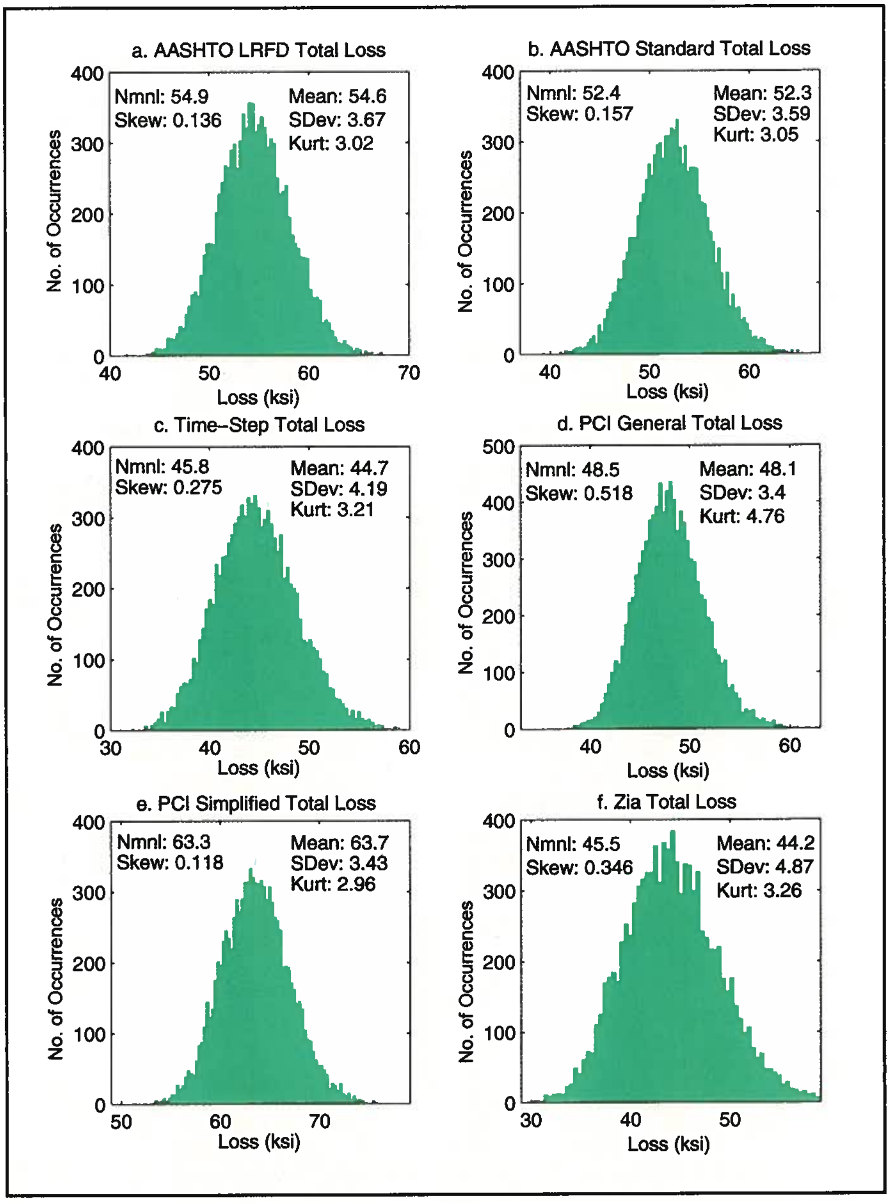

Fig. 4. Monte Carlo Simulation output distributions showing total prestress losses for all methods (I-beam example).

A second observation from Figs. 6 and 7 is that depending upon the variability present and method used, the total mean loss can vary from 30 to 75 ksi (207 to 517 MPa) for the I-beam example, and 15 to 42 ksi (103 to 290 MPa) for the box beam example. When considering the percent prestress loss for each case, results from the Monte Carlo Simulation method indicate total losses in the I-beam range from 14.8 percent to 37.0 percent and in the box beam from 7.4 to 20.7 percent.

The above is a very large spread considering the fact that a designer has some flexibility in selecting a method for prestress loss determination but may not understand the impact of each method or the corresponding variability. As a result, deflection and cracking moment estimations will vary and depend on the prestress loss method selected, as well as the geometric, material, and environmental conditions related to the beam.

DEFLECTIONS AND CRACKING MOMENT

The results of the probabilistic comparison of prestress loss methods have

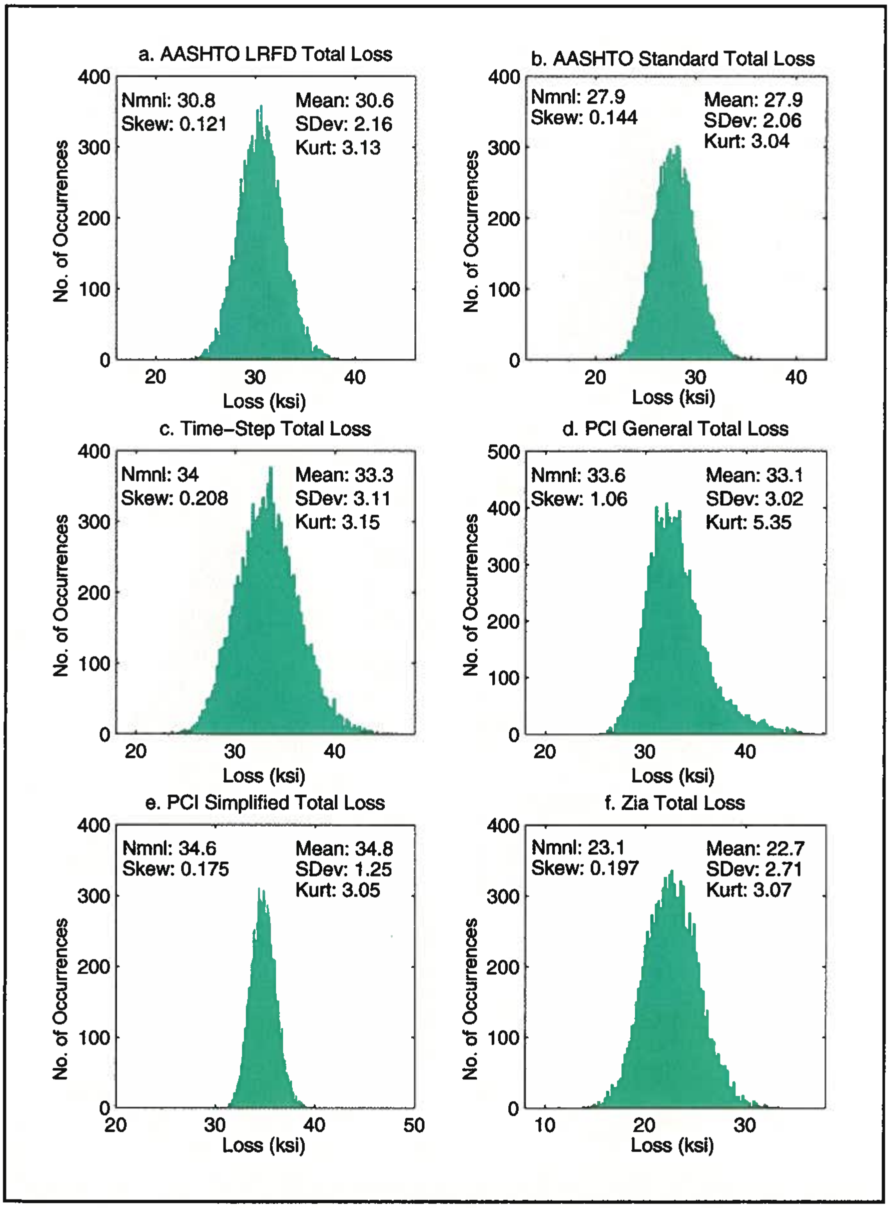

Fig. 5. Monte Carlo Simulation output distributions showing total prestress losses for all methods (box beam example).

a significant impact on the expected deflection and cracking moment of the beams. The variation in prestress losses contributes to the variation in beam deflection both immediately and over time. The higher the prestress loss, the lower the level of compressive stress in the bottom fiber at the midspan, and hence a greater net deflection.

Also, the additional applied moment needed to cause beam cracking (over and above the self-weight and deck weight) is affected by prestress loss variation. A lower compressive state of stress in the bottom of the member means the member is closer to the tensile cracking load, thereby requiring a lower additional moment to cause the member to crack.

It is important to realize that like prestress losses, deflection and camber values are not deterministic. They do, in fact, differ with the inherent variations in material and geometric properties, and environmental conditions. Variations in deflection and cracking moment for both bridge examples were considered using the simulated prestress loss distributions from the AASHTO LRFD[4] and time-step[5] methods.

The AASHTO LRFD method is commonly used in estimating deflection and cracking moment during the bridge design process, and the time-step method has been considered a more accurate method when compared to measured losses.[18] Table 6 presents upper and lower bounds for the simulated total loss distributions based on a 95 percent confidence level for the two loss estimation methods considered. In addition, nominal design properties were used with the AASHTO LRFD method to represent design estimates for comparison.

These prestress loss ranges were then used to estimate expected ranges for deflection and additional applied moment to cause cracking. The other design parameters used in the calculation of the deflection and cracking moment were taken as the nominal values and were not treated as variables. This was done to produce a parametric study indicating the degree in which estimated losses affect deflection and cracking moment.

The final deflection in the beam reflects the deflection due to all prestress losses occurring in the prestressing steel and was calculated based on the PCI Design Handbook[6] equivalent load estimations (with a sign convention of positive for camber and negative for deflection). The variation of estimated deflection around the nominal deflection has the potential to be large. For example, the I-beam has an estimated nominal long-term camber of 1.84 in. (47 mm), but due to the variability of estimated prestress losses using the AASHTO LRFD method, the camber could fall between 1.59 to 2.14 in. (40 to 54 mm).

When using the time-step method to estimate prestress losses, the predicted long-term cambers are even larger than nominal predictions. For the box beam example, the nominal design predictions for final deflection indicate a negative camber of –0.112 in. (–2.8 mm). Based on a 95 percent confidence level with simulated losses using the two methods, the final deflection could be expected to fall between –0.101 to –0.133 in. (–2.5 to –3.4 mm).

The range of estimated deflection due to variability appears somewhat large for final conditions calculated for these two examples, and similar results can be expected for short-term or initial camber. Indeed, precast manufacturers have seen large ranges in initial cambers and short-term deflections in the precast plant for beams cast at the same time on the same bed. If a more accurate estimation of deflection is warranted, then a more in-depth approach may be needed beyond the discrete value estimation that is typically used today with nominal design properties.

The additional moment applied to cause cracking over and above the self-weight and deck weight was also determined for each bridge example using prestress losses based on the 95 percent confidence level as above, and are listed in Table 6. Values for the I-beam example indicate that a beam may crack from an applied moment 11.5 percent lower than nominally estimated using AASHTO LRFD "high" losses, or may be able to withstand 31 percent more moment using the time-step "low" losses.

Because of the variability in prestress losses, the estimated variation of additional moment to cause cracking implies that load limits imposed on a bridge based on nominal design values may not be accurate. In fact, a much smaller moment may cause cracking if the prestress losses are near the distribution's upper limit of the 95 percent confidence level.

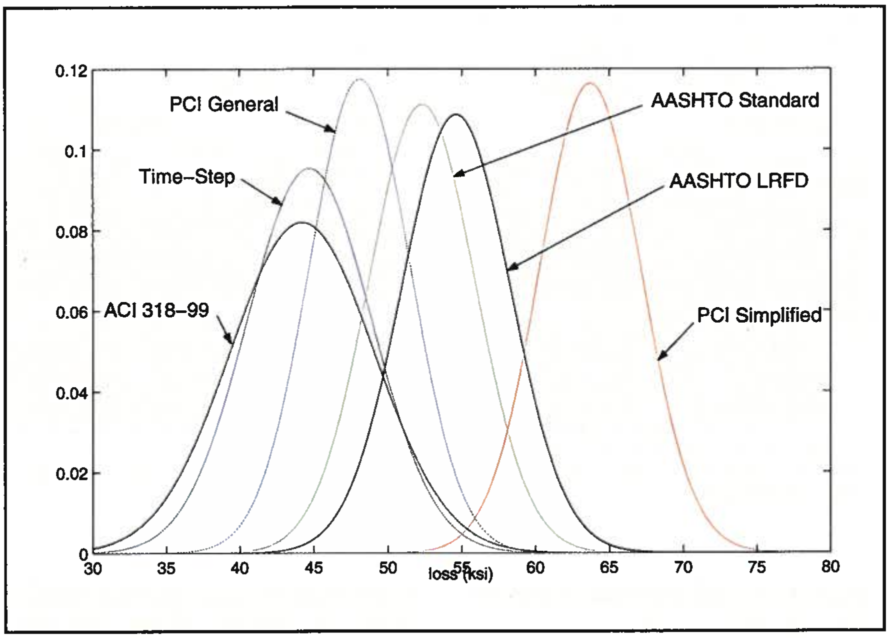

Fig. 6. Total prestress losses for all methods (I-beam example).

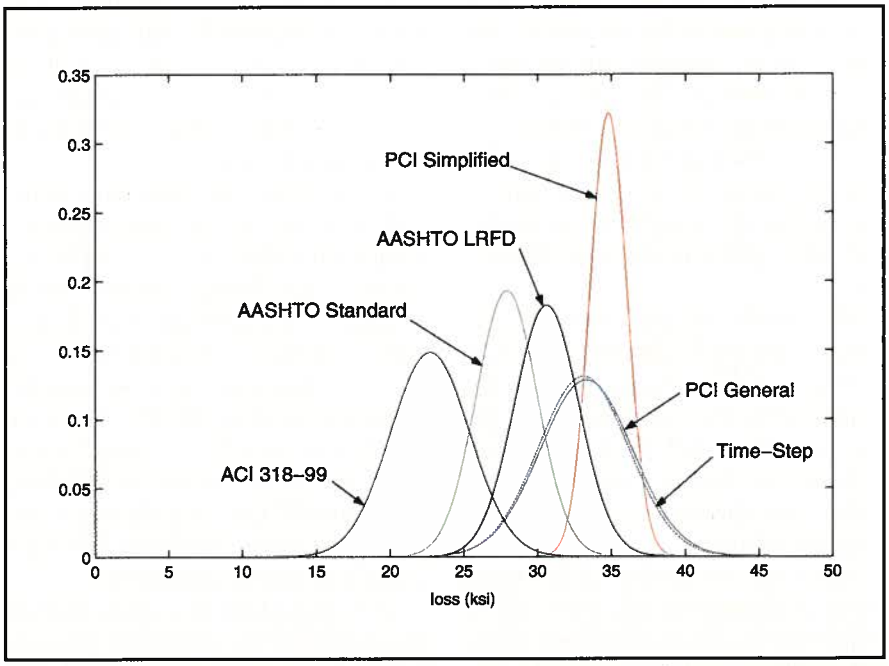

Fig. 7. Total prestress losses for all methods (box beam example).

DISCUSSION OF RESULTS

The purpose of this study was to show the variability found in several of the current methods used to estimate prestress loss in prestressed concrete members. Two practical bridge examples were used to estimate losses by six methods at final service conditions. These methods predict prestress losses and, it must be emphasized, are not exact procedures.

Prestress losses are typically calculated as discrete values in time when in reality the short-term and long-term losses are highly variable due to material, geometric, and environmental variations. However, the designer uses discrete nominal values to estimate deflections and safe loads which can be applied to the beams. Variability of the loss methods was investigated in two ways, first through a parametric study and then through a Monte Carlo simulation technique.

A parametric study varying single input parameters for prestress loss estimation showed that there is a degree of uncertainty in the calculation of prestress loss for all methods. The variation of each parameter had an impact on the individual components of prestress loss. Those parameters having the greatest effect on total losses were initial strand stress, initial concrete strength at release, relative humidity, and strand eccentricity. However, the summation of the loss components resulted in a lower impact on the total loss.

Other parameters may have had a high positive effect on one component and a high negative effect on another, thus causing the sum to be small and the result on total loss to be minor. The parametric study was limited in that the combined effect due to the

Table 6. Deflection and cracking moment summary.

Parameter	Nominal (AASHTO LRFD[4])	AASHTO LRFD[4]		Time step[5]	
		Low*	High*	Low*	High*
Case I: 70-in. I-girder					
Total losses	27.1 percent	23.3 percent	30.5 percent	18.1 percent	26.2 percent
Deflection	1.84 in.	1.59 in.	2.14 in.	1.92 in.	2.56 in.
Additional moment	2564 ft-kips	2896 ft-kips	2268 ft-kips	3363 ft-kips	2652 ft-kips
Case II: 21-in. Box Beam					
Total losses	15.2 percent	13.0 percent	17.2 percent	13.4 percent	19.4 percent
Deflection	–0.112 in.	–0.101 in.	–0.122 in.	–0.104 in.	–0.133 in.
Additional moment	327.9 ft-kips	346.5 ft-kips	310.8 ft-kips	342.8 ft-kips	292.1 ft-kips

Note: 1 in. = 25.4 mm; 1 ft-kip = 1.36 kN-m.
* Low/High Prestress Losses based on 95 percent confidence interval about the Monte Carlo Simulation mean loss for each predictive method.

variability of each parameter could not be considered.

The Monte Carlo Simulation allowed for the interaction of variation between loss prediction parameters to be studied, thus providing a more precise description of the variability that should be expected in total prestress loss calculations. The Monte Carlo Simulation provided distribution descriptors including mean, μ_x, and standard deviation, σ_x, for the prestress loss output representing the combined variability effects of the input parameters.

The results showed that a large range of the predicted total loss could be expected for both bridge examples considered. The outcome, though meaningful in itself, has a greater impact on the designer when put into terms of expected deflection and cracking moment.

While the variability of prestress losses in themselves may seem like a minor detail, the implications reach beyond theory and into reality. Calculations of long-term deflections and cracking moments based on a 95 percent confidence interval about the simulated mean prestress loss show large differences from the nominal predictions that would be estimated in the design stage of these structures. The variation in deflection is important for constructibility and long-term serviceability.

During construction, the deck and wearing surface are placed such that the existing beam camber and the deflection due to additional dead loads in the form of deck, wearing surface, diaphragms, traffic barriers, and additional permanent structures produces a smooth vertical profile on the bridge, thus reducing or eliminating "humps" in the vertical profile. Over time, long-term loss of prestress can cause these "humps" to form. In this case, these occur above the piers and abutments and can be unfavorable to the serviceability of the bridge.

An accurate approximation of the losses allows the designer to predict long-term deflections that can be accounted for during construction to minimize the likelihood of the formation of "humps." Cracking moment is also very important in determining the allowable loading that can be placed safely on a structure. If nominal prestress loss values are used to determine the allowable load, and the actual loss values are higher, the beam will crack under a lesser load application.

It is important to realize that an exact method for predicting prestress losses has not been established. But more importantly, variability of the many factors used in predicting prestress losses plays a large role in the designer's ability to accurately estimate losses, deflection, and cracking moment.

While this study provides insight into the variability that can be expected when predicting prestress losses, additional simulation techniques could be implemented for deflections and cracking moment calculations to refine the results. Further study to more accurately define variability of some of the parameters could also refine the procedures. For example, the ultimate creep coefficient used was a discrete value and very likely contributes to the system variability. Variation in the shrinkage function is also very likely.

Note that the modulus of elasticity of concrete was estimated from the concrete compressive strength. In addition, the variability of concrete strengths of the actual plant cast members should be investigated. Furthermore, consideration needs to be placed on the variability of functions used to estimate the geometric properties for use in estimating prestress losses.

While this study compared the variability of losses using loss estimation methods, it did not consider the actual field measurements. Those reporting test results or field measurements with comparisons to predicted values need to be aware of the impact of material and geometric properties, and environmental conditions on the reported results. Future design specifications need to include such variability so that designers and owners can better understand the actual expected behavior of prestressed concrete structures.

As the precast industry continues toward more efficient use of prestressed concrete members through higher concrete strengths, smaller sections and larger ratios of prestressing steel in members, deflections and cracking moments may become more critical such that variability will play a larger role in these future designs. Several more elements of the Monte Carlo Simulation can be refined. However, the information presented in this paper gives a more reasonable understanding as to the variability involved with predictions for several prestress loss methods, as well as deflections and cracking moments.

CONCLUDING REMARKS

Prestress losses, as well as camber and deflection, are affected by several parameters with inherent variability. This study compares predictive methods for prestress losses and the impact of losses on predicted camber and deflection due to the variability of parameters used for such predictions.

Using referenced and estimated variability in conjunction with Monte Carlo simulations, it is confirmed that the variability of material, geometric, and environmental factors leads to large variations in predicted losses and subsequent camber and deflection predictions using common predictive methods. Such variations have been noted in practice for similar prestressed concrete members, including members cast on the same bed.

For the two typical bridge systems studied herein, the primary influencing parameters were jacking stress (f_{pj}), compressive strength of concrete at strand release (f'_{ci}), relative humidity (RH), and eccentricity of strand (e). The inherent variability of these and all prestress loss parameters, compounded by the complex interactions of prestress loss predictions, helps to justify camber and deflection variations often seen in the field.

Lastly, the authors hope that this study will provide designers with some insight into the relative importance of the various factors that influence the determination of prestress losses.

ACKNOWLEDGMENT

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The methods for estimating prestress loss contained within this appendix are those which may not be readily available to most engineers. The time-step method, PCI General and Simplified methods, and the Zia method are summarized herein. The AASHTO methods should be available to most engineers and, therefore, have not been reproduced in this article.

Time-Step Method [ACI 209 (1982)][5]

The total prestress loss at final service conditions under this method is computed by the following equation:

Total Loss:

$$\Delta f_{pT} = \Delta f_{pES} + \Delta f_{pR} + \Delta f_{pS} + \Delta f_{pC} \quad \text{(A1)}$$

where

Δf_{pT} = total loss of prestress in pretensioned tendons, ksi
Δf_{pE} = loss due to elastic shortening of concrete at transfer, ksi
Δf_{pR} = loss due to relaxation of prestressed tendons before and after transfer, ksi
Δf_{pS} = loss due to shrinkage of concrete, ksi
Δf_{pC} = loss due to creep of concrete after transfer, ksi

Elastic Shortening:

$$\Delta f_{pES} = \left(\frac{E_{ps}}{E_{ci}}\right)\left(f_{cgs(Fi+G)}\right) \quad \text{(A2)}$$

where

E_{ps} = modulus of elasticity of prestressing tendons, ksi
E_{ci} = modulus of elasticity of concrete at time prestress is applied, ksi
$f_{cgs\,(Fi+G)}$ = stress in concrete at centroid of prestressing steel due to initial prestressing force F_i and self-weight of member, ksi

$$f_{cgs\,(Fi+G)} = \frac{F_i}{A_c} + \frac{F_i e_o^{\,2}}{I} - \frac{M_G e_o}{I} \quad \text{(A3)}$$

where

F_i = $A_{ps}\,(f_{pi} - \Delta f_{pES} - RE_i)$
A_c = gross area of concrete section considered, sq in.
e_o = eccentricity of center of gravity of tendons with respect to center of gravity of concrete at section considered, in.
I = moment of inertia of gross concrete section, in.^{4}
M_G = bending moment due to dead weight of member and any other permanent loads in place at time of transfer, in.-kips
A_{ps} = area of prestressing strands, sq in.
RE_i = loss due to steel relaxation prior to transfer, ksi

Steel Relaxation:
Stress Relieved Strand:

$$\Delta f_{pR} = \frac{f_{ps}(t_i)}{10}\left(\frac{f_{ps}(t_i)}{f_{py}} - 0.55\right)\log\left(\frac{t_j}{t_i}\right) \quad \text{(A4)}$$

where

t_i = time at beginning of time step, hours
t_j = time at end of time step, hours
f_{ps} = stress in tendon at beginning of time-step, ksi
f_{py} = stress resulting in yielding of tendon, ksi

In Eq. (A4), f_{ps}/f_{py} must not be less than 0.55.

Low-Relaxation Steel:
Use a denominator of 45 instead of 10 in Eq. (A4).

Note that the time-step method should be used over several intervals, and the cumulative results used to estimate prestress loss. One time-step, from the beginning to the end of the member life will result in a poor estimate of total loss.

Shrinkage of Concrete:
Composition of concrete mix, characteristics of the aggregates, relative humidity, and curing history affect the losses due to shrinkage between any number of time steps.

The shrinkage strain is assumed to be uniform throughout the member:

$$\Delta f_{pS} = E_{ps}\varepsilon_{SU} K_{SH} K_{SS} \frac{b(t_j - t_i)}{(b+t_i)(b+t_j)} \quad \text{(A5)}$$

where

ε_{su} = ultimate shrinkage strain of concrete material
K_{SH} = corrective factor which depends on the average relative humidity of the environment where structure is built (Table A1)
K_{SS} = corrective factor which depends on size and shape of member (Table A2)
b = 35 for moist-cured concrete and 55 for steam-cured concrete

$$\varepsilon_{SU} = \left[2 + \frac{11}{230}(w - 220)\right]10^{-4} \quad \text{(A6)}$$

where

w = water content, lb per cu yd

In place of Table A2, the following equation may be used to find K_{CS} and K_{SS}:

$$K_{CS} = K_{SS} = 1.14 - 0.09\left(\frac{V}{S}\right) \quad \text{(A7)}$$

where

V = volume of the member
S = surface area of the member

Table A1. K_{SH} values, type of curing influence.

Moist-cured concrete		Steam-cured concrete
40 percent < H < 80 percent $K_{SH} = 1.40 - 0.01H$	$t \geq 7$ days	$t \geq 1$ to 3 days $K_{SH} = 1.40 - 0.01H$
80 percent < H < 100 percent $K_{SH} = 3 - 0.03H$	$t \geq 7$ days	$t \geq 1$ to 3 days $K_{SH} = 3 - 0.03H$

Table A2. K_{SS} and K_{CS} values, volume-to-surface ratio influence.

V/S ratio (in.)	K_{CS} (Creep)	K_{SS} (Shrinkage)
1	1.05	1.04
2	0.96	0.96
3	0.87	0.86
4	0.77	0.77
5	0.68	0.69
6	0.68	0.6

Note that V/S is commonly taken as the cross-sectional area divided by the perimeter.

Creep of Concrete:

$$\Delta f_{pC}(t_i, t_j) = n_p C_{CU} K_{CH} K_{CA} K_{CS} f_{cgs}(t_i)\left[g(t_j) - g(t_i)\right] \quad \text{(A8)}$$

where

n_p = E_{ps}/E_c (n_{pi} can replace n_p in the early stages, where $n_{pi} = E_{ps}/E_{ci}$)
C_{CU} = ultimate creep coefficient (Table A3)
K_{CH} = correction factor depending on average relative humidity of environment where structure is built
K_{CA} = age at loading factor
K_{CS} = shape and size factor (Table A2)

$$f_{cgs}(t_i) = \frac{f_{ps}(t_i)A_{ps}}{A_c}\left(1 + \frac{e_0^{\,2}}{r^2}\right) - \frac{M_D e_o}{I} \quad \text{(A9)}$$

where $f_{cgs}(t_i)$ is taken at the beginning of a time-step, not at the end.

The time function suggested by ACI Committee 209 is:

$$g(t) = \frac{t^{0.60}}{10 + t^{0.60}} \quad \text{(A10)}$$

where t is the time, measured in days.

By using the equations for $f_{cgs}(t_i)$, the values of the stress in the prestressing steel at time t_i, $f_{ps}(t_i)$ depend on the total losses including creep, over all time intervals preceding the interval considered, such that:

$$f_{ps}(t_i) = f_{pi} - \sum \Delta f_{pT}(t_i, t_j) \quad \text{(A11)}$$

Table A3. C_{CU} values.

Compressive strength (psi)	C_{CU}
3000	3.1
4000	2.9
5000	2.65
6000	2.4
7000	2.2
8000	2

PCI COMMITTEE ON PRESTRESS LOSSES: GENERAL METHOD (1975)[2]

The total loss is calculated by the following procedure:
Total Loss:

$$TL = ANC + DEF + ES + \sum(CR + SH + RET) \quad \text{(A12)}$$

where

TL = total loss of prestress, psi
ANC = loss of prestress due to anchorage of prestressing steel, psi
DEF = loss of prestress due to deflecting device in pretensioned construction, psi
ES = loss of prestress due to elastic shortening, psi
CR = loss of prestress due to creep of concrete over time interval t_1 to t, psi
SH = loss of prestress due to shrinkage of concrete over time interval t_1 to t, psi
RET = loss of prestress due to steel relaxation over time interval t_1 to t, psi

Elastic Shortening:

$$ES = \frac{E_s}{E_{ci}} f_{cr} \quad \text{(A13)}$$

where

E_s = modulus of elasticity of steel, psi
E_{ci} = modulus of elasticity of concrete at time of initial prestress, psi
f_{cr} = concrete stress at center of gravity of prestressing force immediately after prestress force is applied minus stress due to all dead loads acting at that time, psi:

$$f_{cr} = \frac{P_i}{A_c}\left(1 + \frac{e^2}{r^2}\right) - \frac{M_o e}{I} \quad \text{(A14)}$$

where

P_i = $A_s(f_t - ES - RE_i)$
f_t = stress at which tendons are anchored in prestressing bed, psi
RE_i = loss due to steel relaxation prior to transfer, psi
A_c = gross area of concrete section, sq in.
e = eccentricity of center of gravity of tendons with respect to center of gravity of concrete at the section considered, in.

Table A4. Minimum time increments.

Step	Beginning time, t_1	End time, t
1	Pretensioned anchorage of prestressing steel	Age at prestressing of concrete
2	End of Step 1	Age = 30 days, or time when a member is subjected to load in addition to its own weight
3	End of Step 2	Age = 1 year
4	End of Step 3	End of service life

r^2 = I/A_c, sq in.

M_o = bending moment due to dead weight of member and any other permanent loads in place at transfer, in.-kips

I = moment of inertia of gross concrete section, in.^{4}

A_s = area of prestressing strands, sq in.

Time-Dependent Losses:

Note that Table A4 reflects the minimum time increments recommended by the original reference. The authors of this article feel that even more time steps are required.

Creep of Concrete (loss over each time step):

$$CR = (UCR)(SCF)(MCF)(PCR)(f_c) \quad \text{(A15)}$$

where

UCR = ultimate loss of prestress due to creep of concrete, psi per psi of compressive stress in concrete

SCF = factor that accounts for effect of size and shape of a member on creep of concrete (Table A5)

MCF = factor that accounts for the effect of age at prestress and length of moist cure on creep of concrete. Note: this should not be included for accelerated cured concrete (Table A6)

PCR = amount of creep over time interval t_1 to t

f_c = concrete compressive stress at center of gravity of prestressing steel at time t_1, taking into account loss of prestress force occurring over preceding time interval, psi

Normal weight concrete:

For moist cure not exceeding seven days:

$$UCR = 95 - \frac{20E_c}{10^6} \geq 11 \quad \text{(A16)}$$

For accelerated cure:

$$UCR = 63 - \frac{20E_c}{10^6} \geq 11 \quad \text{(A17)}$$

where E_c is the modulus of elasticity of concrete, psi.

Table A5. *SCF* values (size and shape factor for creep).

V/S	*SCF*
1	1.05
2	0.96
3	0.87
4	0.77
5	0.68
>5	0.68

Table A6. *MCF* values (moist cure factor for creep).

Age of prestress transfer (days)	Period of cure (days)	*MCF*
3	3	1.14
5	5	1.07
7	7	1.00
10	7	0.96
20	7	0.84
30	7	0.72
40	7	0.60

Table A7. *AUC* values (ultimate creep factor).

Time after transfer (days)	*AUC*
1	0.08
2	0.15
5	0.18
7	0.23
10	0.24
20	0.30
30	0.35
60	0.45
90	0.51
180	0.61
365	0.74
End of service	1.00

$$PCR = (AUC)_t - (AUC)_{t_1} \quad \text{(A18)}$$

where

AUC = portion of ultimate creep at time after prestress transfer (Table A7)

t_1 = time at beginning of time interval, days

t = time at end of time interval, days

Shrinkage of Concrete (loss over each time step):

$$SH = (USH)(SSF)(PSH) \quad \text{(A19)}$$

where

USH = ultimate loss of prestress due to shrinkage of concrete, psi

SSF = factor that accounts for the effect of size and

Table A8. *SSF* values (size and shape factor for shrinkage).

V/S	*SSF*
1	1.04
2	0.96
3	0.86
4	0.77
5	0.69
6	0.60

Table A9. *AUS* values (ultimate shrinkage factor).

Time after end of curing (days)	*AUS*
1	0.08
3	0.15
5	0.20
7	0.22
10	0.27
20	0.36
30	0.42
60	0.55
90	0.62
180	0.68
365	0.86
End of service	1

shape of a member on concrete shrinkage (Table A8)

PSH = amount of shrinkage over time interval t_1 to t

Normal weight concrete:

$$USH = 27000 - \frac{3000E_c}{10^6} \geq 12000 \qquad \text{(A20)}$$

$$PSH = (AUS)_t - (AUS)_{t_1} \qquad \text{(A21)}$$

where AUS is the portion of ultimate shrinkage at time after end of curing (Table A9).

Steel Relaxation:

Low-relaxation strand:

$$RET = f_{st}\left(\frac{\log 24t - \log 24t_1}{45}\right) \times \left(\frac{f_{st}}{f_{py}} - 0.55\right) \qquad \text{(A22)}$$

Stress-relieved strand:

$$RET = f_{st}\left(\frac{\log 24t - \log 24t_1}{10}\right) \times \left(\frac{f_{st}}{f_{py}} - 0.55\right) \qquad \text{(A23)}$$

where

f_{st} = stress in prestressing steel at time t_1, psi

f_{py} = stress at 1 percent elongation of prestressing steel, psi

f_{pu} = guaranteed ultimate tensile strength of prestressing steel, psi

PCI COMMITTEE ON PRESTRESS LOSSES: SIMPLIFIED METHOD (1975)[6]

The PCI Simplified Method (1975) requires the calculation of concrete stress at transfer and the concrete stress after the application of additional permanent dead loads. Concrete stress at transfer is calculated from the stress in the steel at transfer, which is estimated by reducing the jacking stress by a reduction factor. These values are then entered into an equation based on concrete weight, strand type, and type of tensioning. The result is an estimate for total prestress loss in which

f_{cr} = concrete stress existing immediately after prestress has been applied to concrete, psi

f_{cds} = stress due to all permanent (dead) loads not used in computing f_{cr}, psi

$$f_{cr} = \frac{A_s f_{si}}{A_c} + \frac{A_s f_{si} e^2}{I_c} - \frac{M'e}{I_c} \qquad \text{(A24)}$$

where

f_{si} = $0.90 f_t$ for stress-relieved steel
= $0.925 f_t$ for low-relaxation steel

f_{si} = stress in tendon at critical location immediately after prestressing force has been applied to concrete, psi

f_t = stress at which tendons are anchored in pretensioning bed, psi

M' = moment due to loads including weight of member at time prestressing is applied to concrete, in.-lb.

A_s = cross-sectional area of prestressing tendons, sq in.

A_c = gross cross-sectional area of the non-composite concrete member, sq in.

e = tendon eccentricity measured from center of gravity of concrete section to center of gravity of tendons, in.

I_c = moment of inertia of gross cross section of concrete member, in.^{4}

$$f_{cds} = \frac{M_{sd}e}{I} \qquad \text{(A25)}$$

where

f_{cds} = concrete stress at center of gravity of tendons due to all superimposed permanent dead loads that are applied to the member after it has been prestressed, psi

M_{sd} = moment due to superimposed permanent dead loads and sustained loads applied after prestressing, in.-lb

Adjustments: Equations are based on $V/S = 2.0$.
See Tables A10 and A11.

ZIA METHOD (1979)[8]

Elastic shortening:

Members with bonded tendons:

$$ES = K_{es}E_s \frac{f_{cir}}{E_{ci}} \qquad \text{(A26)}$$

Table A10. PCI Committee, Simplified Method equations.

Concrete weight		Type of tendon			Tensioning		
Normal	Light	Stress Relieved	Low Relaxation	Bar	Pre	Post	Equations
X		X			X		$TL = 33.0 + 13.8f_{cr} - 4.5f_{cds}$
	X	X			X		$TL = 31.2 + 16.8f_{cr} - 3.8f_{cds}$
X			X		X		$TL = 19.8 + 16.3f_{cr} - 5.4f_{cds}$
	X		X		X		$TL = 17.5 + 20.4f_{cr} - 4.8f_{cds}$
X		X				X	$TL = 29.3 + 5.1f_{cr} - 3.0f_{cds}$
	X	X				X	$TL = 27.1 + 10.1f_{cr} - 4.9f_{cds}$
X			X			X	$TL = 12.5 + 7.0f_{cr} - 4.1f_{cds}$
	X		X			X	$TL = 11.9 + 11.1f_{cr} - 6.2f_{cds}$
X				X		X	$TL = 12.8 + 6.9f_{cr} - 4.0f_{cds}$
	X			X		X	$TL = 12.5 + 10.9f_{cr} - 6.0f_{cds}$

Note: The above equations are only valid when $f_{cr} > f_{cds}$.

Table A11. Adjustments to the Simplified Method.

V/S	1	2	3	4
Percent adjustment	+3.2	0	-3.8	-7.6

Table A12. K_{re} and J values.

Type of tendon	K_{re}	J
270 Grade stress-relieved strand or wire	20000	0.15
250 Grade stress-relieved strand or wire	18500	0.14
240 or 235 Grade stress-relieved wire	17600	0.13
270 Grade low-relaxation strand	5000	0.04
250 Grade low-relaxation wire	4630	0.037
240 or 235 Grade low-relaxation wire	4400	0.035
145 or 160 Grade stress-relieved bar	6000	0.05

Table A13. C-values.

(f_{pi}/f_{pu})	Stress-relieved strand or Wire C	Stress-relieved bar or low relaxation strand or Wire C
0.80	—	1.28
0.79	—	1.22
0.78	—	1.16
0.77	—	1.11
0.76	—	1.05
0.75	1.45	1.00
0.74	1.36	0.95
0.73	1.27	0.90
0.72	1.18	0.85
0.71	1.09	0.80
0.70	1.00	0.75
0.69	0.94	0.70
0.68	0.89	0.66
0.67	0.83	0.61
0.66	0.78	0.57
0.65	0.73	0.53
0.64	0.68	0.49
0.63	0.63	0.45
0.62	0.58	0.41
0.61	0.53	0.37
0.60	0.49	0.33

where

K_{es} = 1.0 for pretensioned members

E_s = modulus of elasticity of prestressing tendons, usually 28×10^6 psi

E_{ci} = modulus of elasticity if concrete at time prestress is applied, psi

f_{cir} = net compressive stress in concrete at center of gravity of tendons immediately after prestress has been applied to concrete, psi

$$f_{cir} = K_{cir}\left(\frac{P_{pi}}{A_c} + \frac{P_{pi}e^2}{I_c}\right) - \frac{M_G e}{I_c} \quad \text{(A27)}$$

where

K_{cir} = 0.9 for pretensioned members, adjustment factor because P_{pi} does not include losses for elastic shortening or steel relaxation prior to transfer

P_{pi} = prestressing force in tendons at critical location on span after reduction for losses due to friction and seating loss at anchorages but before reduction for *ES*, *CR*, *SH* and *RE*, psi

A_c = area of gross concrete section at cross section considered, sq in.

e = eccentricity of center of gravity of tendons with respect to center of gravity of concrete at cross section considered, in.

I_c = moment of inertia of gross concrete section at cross section considered, in.^{4}

M_G = bending moment due to dead weight of member being prestressed and to any other permanent loads in place at time of prestressing, in.-lb.

Creep of concrete:

For members with bonded tendons:

$$CR = K_{cr}\frac{E_s}{E_c}\left(f_{cir} - f_{cds}\right) \quad \text{(A28)}$$

where

K_{cr} = 2.0 for pretensioned members. For members made of sand lightweight concrete, the foregoing value of K_{cr} should be reduced by 20 percent.

E_c = modulus of elasticity of concrete at 28 days, psi

f_{cds} = stress in concrete at center of gravity of tendons due to all superimposed permanent dead loads that are applied to member after it has been prestressed, psi

$$f_{cds} = \frac{M_{sd}e}{I_c} \quad \text{(A29)}$$

where

M_{sd} = moment due to superimposed permanent dead loads and sustained loads applied after prestressing, in.-lb

Shrinkage of concrete:

$$SH = 8.2 \times 10^{-6} K_{sh} E_s \left(1 - 0.06\frac{V}{S}\right)(100 - RH) \quad \text{(A30)}$$

where

K_{sh} = 1.0 for pretensioned members

V/S = volume-to-surface ratio. Usually taken as gross cross-sectional area of concrete member divided by its perimeter, in.

RH = average relative humidity surrounding the concrete member, percent

Steel relaxation:

$$RE = \left[K_{re} - J(SH + CR + ES)\right]C \quad \text{(A31)}$$

where K_{re}, J, and C are taken from Tables A12 and A13.