Strengthening of Precast Concrete Girder Bridges by Post-Tensioning for Continuity

Simple-span precast concrete girders have frequently been used in bridge superstructures. However, to accommodate today’s heavy traffic loads, their carrying capacity can be strengthened by providing moment continuity between adjacent spans. This study investigates the use of external tendons to provide partial continuity. It is shown that such a strengthening scheme results in stiffer beam response than individual spans. The effective tendon force governs the beam deflections and opening of the joint between adjacent spans. An analytical method is presented to predict the piece-wise linear response of two-span continuous beams derived from two simple-span beams. Good agreement of both the calculated deflections and widths of joint opening with the observed test results was obtained. Based on a parametric design study, charts are provided for strengthening precast simple-span beams using external prestressing. A design example illustrates the proposed method.

During the last few decades, reinforced concrete bridges have been constructed using a series of precast simple-span girders. Due to today’s heavier traffic loads, progressive structural aging, and changes in code provisions, many old bridges are in need of strengthening. Several strengthening methods, such as section enlargement with additional reinforcement and steel plate bonding, are available. However, compared to such schemes, the use of post-tensioned external tendons to strengthen these girders by providing moment continuity between adjacent spans is a relatively simple technique and economic method to employ.

The feasibility of such an approach has been investigated experimentally by Rao et al. A pair of simply-supported reinforced concrete rectangular beams were converted to a two-span continuous beam by placing continuous external draped tendons through the two spans. The resulting partially continuous beam behaved as though it were a solid continuous beam cast monolithically. In another study by Machida and Bamrungwong, the load-carrying capacity was found to be lower in partially continuous beams.
than in monolithic continuous beams, but the capacity increased with the initial tendon prestress.

The flexural behavior of partially continuous beams is not fully understood due to the unbonded aspect of the external tendons, rotation at the joint between the two adjacent precast spans, and beam response after the opening of the joint. Although recently, Ariyawardena and Ghali\textsuperscript{3} have accounted for the effect of joint openings using a special element in a numerical analysis on the response of precast segmental concrete beams with external tendons, the study has not been extended to continuous span structures.

This study was, therefore, carried out to investigate the flexural response of precast girders (T-beams) rendered partially continuous by using external tendons. Tests were carried out on four partially continuous beams with the tendon type, area and effective prestress as test variables.

An analytical approach is also presented for calculating the width of joint opening, tendon stresses, and flexural capacity of the beams, which were compared with the test results. The approach forms the basis for parametric studies that resulted in the preparation of design charts for flexural strengthening of simple-span precast girders.

**TEST PROGRAM**

Four specimens, each consisting of two simple-span T-beams made partially continuous using external tendons, were fabricated. Each T-beam measured 3 m (9 ft 10 in.) in length and 300 mm (12 in.) in overall depth. To provide stability during testing, the beam web was enlarged into a rectangular section at the intermediate support (see Fig. 1).

The design of the test specimens was carried out using the ACI Code.\textsuperscript{4} The area of internal reinforcement (including shear reinforcement) was selected to provide the same load-carrying capacity as a two-span monolithic continuous beam, designated Beam MCBS-2, and tested earlier by the authors.\textsuperscript{5}

The internal longitudinal reinforcement for both test specimens used in this study and Beam MCBS-2 consisted of two deformed steel bars with a diameter of 16 mm (0.625 in.), designated T16, at the bottom of the web. In the flange, four deformed steel bars with a diameter of 10 mm (0.375 in.), designated T10, were provided. In Beam MCBS-2, six T10 bars formed the negative steel over the intermediate support. The average yield strength was about 540 MPa (78 ksi) for all longitudinal bars.

Shear reinforcement, consisting of 10 mm (0.375 in.) diameter stirrups with an average yield strength of 470 MPa (68 ksi), was provided at spacings of 150, 200 and 100 mm (6, 8 and 4 in.) for Regions I, II and III of each beam, respectively. This assumes that there is adequate shear reinforcement to carry the load-carrying capacity of the partially continuous beam.

The T-beams were cast in wooden molds. The concrete had a mix proportion of 1:1.5:2.1 by weight of cement, sand and coarse aggregates of a maximum size of 10 mm (0.375 in.). The water-cementitious materials ratio was 0.55 and the target cylinder compressive strength of concrete was 30 MPa (4.35 ksi) at 28 days.

After casting, the beams were covered with damp burlap, and the formwork was removed after two days, when the concrete had achieved sufficient compressive strength. The beams were continuously covered with damp burlap until the seventh day, after which they were left in the laboratory under ambient conditions until the day of testing, typically on the 28th day after casting.

To make each specimen continuous over the intermediate support, two T-beams were placed on three supports, with a gap of 30 mm (1.2 in.) in between. Before that, the end surface of each beam was roughened using a punch hammer. Shrinkage compensated grout was then poured to fill the gap after the necessary formwork had been made. The grout was cured for about two days, after which a cylinder...
compressive strength of 30 MPa (4.35 ksi) was achieved. The external tendons were then installed and stressed to join the two T-beams.

Table 1 shows the post-tensioning schemes for the test beams and Table 2 summarizes the type of tendons used and the applied effective prestress, $f_{p,\text{e}}$, measured after anchor set. In Specimen P-1, two seven-wire prestressing steel strands, each with a length of 1420 mm (4 ft 8 in.), diameter of 9.5 mm (0.475 in.) and cross-sectional area of 55.0 mm$^2$ (0.085 sq in.), were installed across the joint over the intermediate support, with one strand on each side of the specimen. Specimen P-2 had the same post-tensioning scheme as Specimen P-1; however, two carbon fiber-reinforced polymer (CFRP) tendons, each with a diameter of 10.5 mm (0.4 in.) and cross-sectional area of 55.7 mm$^2$ (0.086 sq in.) were used instead.

The third specimen, P-3, also had the same post-tensioning scheme as Specimen P-1, except that the steel strands had a length of 1000 mm (3 ft 3 in.), diameter of 12.9 mm (0.51 in.) and cross-sectional area of 100 mm$^2$ (0.155 sq in.). Compared to Specimens P-1 and P-2, Specimen P-3 was subjected to higher post-tensioning forces.

The fourth specimen, P-4, was similar to Specimen P-3, except that the positive moment regions were also post-tensioned using two 9.5 mm (0.375 in.) diameter and 1780 mm (5 ft 10 in.) long steel strands. The shorter tendon length was adopted over the intermediate support in Specimens P3 and P4 so as to avoid anchoring the positive and negative moment tendons at the same section.

The effective tendon depth was 180 mm (7 in.) for tendons over the intermediate support and 200 mm (7.8 in.) for tendons at the positive moment regions. All steel strands had an average ultimate tensile strength of 1900 MPa (276 ksi) and a tensile modulus of 195 GPa (28,275 ksi). The CFRP tendons had a breaking stress of 1868 MPa (270 ksi) and a tensile modulus of 139 GPa (20,155 ksi), according to the manufacturer’s specification.

Specially designed steel brackets were attached to the sides of the beam web using 12.7 mm (0.5 in.) diameter high-strength bolts, so as to anchor the tendons. A jaw-and-barrel system was used to grip the steel strands, while the CFRP tendons, which were provided with threaded sleeves, were anchored with threaded nuts (see Fig. 2).

All beams were tested under four-point loading, placed at third points of each span, to simulate uniformly distributed load (see Fig. 3). The tests were conducted in load control initially and displacement control near the ultimate load. During post-tensioning and loading, the displacements at the maximum positive moment regions under the point loads were measured using 100 mm (4 in.) linear variable differential transducers (LVDTs). Lifting of the beams at the intermediate support during post-tensioning was also monitored using a 100 mm LVDT.

Strain gauges were mounted on the strands and internal tensile reinforcement to monitor strain developments. Flexural crack widths were measured using a hand-held microscope with a resolution of 0.02 mm (0.0008 in.). The section curvature at the joint was measured using aluminum frames designed to hold four 25 mm (1 in.) LVDTs, with two placed above the flange and two below the web. Note that these transducers measured the top and bottom horizontal displace-
ments, which were converted to strains by dividing them by the gauge length. Dividing the strain difference by the depth between the top and bottom LVDTs gives the curvature.

TEST RESULTS AND DISCUSSION

This section discusses load-deflection characteristics, the stress increase in external tendons, the opening of joint and curvature of section, and the crack pattern in positive moment regions.

Load-Deflection Characteristics

The load-deflection response of the partially continuous test beams is shown in Fig. 4. The beam deflection was monitored at the critical sections in the positive moment regions, located under the outer point loads.

For comparison purposes, the predicted response of a control specimen P-0, in which there was no continuity between adjacent precast beams, is presented. The results of the two-span monolithic continuous beam MCBS-2 are also included. Beam MCBS-2 had the same tendon profile and effective tendon prestress as Specimen P-1.

Before the joint opened, all partially continuous beams showed a similar response to those of the monolithic beam. Once the joint started to open, which occurred when the top concrete fiber at the joint attained a stress equal to the modulus of rupture of the grout, the stiffness of the beam was reduced considerably, as can be seen from the change in slope of the load-deflection curve. It was noted that the neutral axis at the intermediate support section shifted downward and the adjacent individual beam rotated about a point close to the neutral axis.

As the load was increased further, the joint opened wider, and the resulting moment was redistributed to the positive moment regions. After the internal tensile reinforcement in the positive moment region yielded, a small increment in the applied load led to large beam deflections. Several drops in the applied load were observed due to the formation of new cracks.

All beams showed a high ductility. Note that the tests were stopped after significant widening of the joint opening and signs of concrete crushing were observed, resulting in the negative moment tendons kinking over the edge of the enlarged web section.

The test results of Specimens P-1 and P-2 indicated that different tendon materials (that is, steel or CFRP) have a minimal effect on the load-deflection response, provided that the tendon length, modulus of elasticity, and

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Intermediate support tendons</th>
<th>Positive moment tendons</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Type</td>
<td>Diameter (mm)</td>
</tr>
<tr>
<td>P-1</td>
<td>Steel</td>
<td>9.5</td>
</tr>
<tr>
<td>P-2</td>
<td>CFRP</td>
<td>10.5</td>
</tr>
<tr>
<td>P-3</td>
<td>Steel</td>
<td>12.9</td>
</tr>
<tr>
<td>P-4</td>
<td>Steel</td>
<td>12.9</td>
</tr>
</tbody>
</table>

Note: 1 MPa = 0.145 ksi.
initial tendon force have similar values. The effect of an increased tendon area with a corresponding higher initial tendon force resulted in a higher joint opening load, stiffer response, higher yield load and ultimate load, with comparable ductility as indicated by the results of Specimen P-3.

It was also observed that, due to the higher prestressing force over the intermediate support, the first crack occurred in the positive moment regions in Specimen P-3 before the joint opened. A substantial increase in load-carrying capacity after yielding of the internal steel reinforcement at the positive moment regions was also observed. This capacity increase is attributed to the larger reserve strength in the external tendons due to a lower stress increase. Strengthening the positive moment as well as the negative moment regions as in Specimen P-4 raised the ultimate load further, to as close as the ultimate load of the monolithic continuous Beam MCBS-2.

All the specimens showed a stiffer response compared to the control Specimen P-0. The maximum beam deflection under service load, assumed as the experimental ultimate load divided by a factor of 1.7, was 6.88, 6.78, 6.75 and 7.3 mm (0.271, 0.267, 0.266 and 0.287 in.) for Specimens P-1, P-2, P-3 and P-4, respectively. All these values are within the allowable maximum immediate deflection due to live load, specified by the ACI Code as Span/360, or 7.9 mm (0.311 in.).

**Stress Increase in External Tendons**

The development of stresses in the external tendons over the joint is shown in Fig. 5. The stress increase was generally negligible before the joint opened because the beam deformation was small. Once the joint opened, the external tendon stress increased considerably. As there was no internal reinforcement crossing the joint, the external tendons, therefore, carried all the tensile forces at the intermediate support.

Specimen P-2 exhibited a lower stress increase compared to Specimen P-1 due to the lower Young’s modulus of CFRP tendons. However, as CFRP tendons do not exhibit plastic deformation, the stress increase needs to be carefully monitored. A larger tendon area in Specimen P-3 resulted in a smaller tendon stress increase compared to Specimen P-1. External tendons in Specimen P-4 exhibited a much lower stress increase compared to all the other test beams due to the strengthening of the positive moment regions which led to stiffer beam behavior and smaller width of the joint.

**Opening of Joint and Curvature of Section**

Fig. 6 shows the curvature of the section at the intermediate support. The curvature was negligible before the opening of the joint. Once the joint opened up, the neutral axis shifted downward and the curvature increased progressively.

Specimens P-1 and P-2 exhibited a similar response, indicating that the tendon type has little influence as long as the initial tendon forces are the same. Increasing the tendon area and, hence, the tendon force in Specimen P-3, and providing tendons in the positive moment region as well in Specimen P-4, resulted in smaller joint curvatures. This is attributed to a larger neutral axis depth and, hence, more concrete area in compression that
helped balance the tensile forces in the tendons over the intermediate support.

Figs. 7(a) and 7(b) show the width of the joint opening, measured at the top concrete surface and at initial tendon level, respectively. Again, a similar pattern was observed in Specimens P-1 and P-2. Due to the smaller joint curvature, the joint opening was smaller in both Specimens P-3 and P-4 than in Specimens P-1 and P-2.

Crack Pattern in Positive Moment Regions

All the test beams showed the same crack propagation in the positive moment regions. Flexural cracks were first observed in these regions under the point loads. As the load was increased, flexural-shear cracks also appeared near the exterior and intermediate supports of the beam. As shown in Fig. 8, the shear cracks were more inclined in Specimen P-4 due to the presence of external tendons in the positive moment regions.

Fig. 9 compares the maximum crack widths measured in the positive moment regions. It was observed that the lower tendon area and, hence, lower tendon forces across the joint in Specimens P-1 and P-2 resulted in larger crack widths in the positive moment regions, because a larger moment was redistributed to the positive moment regions. Strengthening the positive moment regions as in Specimen P-4 resulted in smaller crack widths at all loads.

The measured maximum crack widths at the assumed service load were 0.13, 0.18, 0.10 and 0.12 mm (0.005, 0.007, 0.004 and 0.0047 in.) for Specimens P-1, P-2, P-3 and P-4, respectively, which are all less than 0.3 mm (0.018 in.) specified by the ACI Code for exterior exposure, indicating satisfactory service load behavior as far as maximum crack widths are concerned.

PROPOSED ANALYTICAL METHOD

Based on the test results shown in Fig. 4, the flexural response of partially continuous girders can be characterized by a piece-wise linear relat-
tion consisting of four regimes as shown in Fig. 10.

Initially, in Regime AB, the specimen is uncracked and behaves in an elastic manner similar to a monolithic continuous beam. The maximum negative and positive moments occur, respectively, at sections at the intermediate support and under the outer vertical load (see Fig. 1). The first and second major cracks are expected to form at these critical sections, the order of which depends on the section properties and tendon profiles at these sections. The cracking at the intermediate support refers to the opening of the joint.

After the joint opens, that is, in cracked Regimes BC and CD, the partially continuous beam exhibits lesser stiffness compared to a two-span monolithic continuous beam, but is stiffer than a single-span precast beam. A part of the applied moment is redistributed from the intermediate support to adjacent positive moment regions, the amount of which depends on the degree of continuity provided by the external tendons over the intermediate supports.

A plastic hinge is considered to have formed when the internal tensile steel in the positive moment regions reaches its yield strength. The specimen is assumed to reach the ultimate limit state (Point E) when the concrete in compression at both the joint section and maximum positive moment regions attains a strain of 0.003.

The piece-wise linear flexural response of the beam can be evaluated based on the following assumptions:

1. Plane sections remain plane after bending, so the strain change at any level across the beam section is proportional to the distance from the neutral axis.

2. Only a single crack, that is, due to the opening of joint, occurs at the intermediate support.

3. Second-order effects due to changing tendon eccentricity can be ignored.

4. Secondary moments due to post-tensioning of external tendons are added to the applied moment due to the external load, and are determined on the basis of effective prestress as specified by the ACI Code.\(^4\)

5. After the opening of the joint, the stress increase in external tendons over the joint is assumed to be a function of the opening width. The effect of beam deformation on the stress increase is ignored.

6. The cylinder compressive strength and Young’s modulus of the grout are governed by the strength and modulus of the concrete in the beams.

7. The stress-strain relations of concrete, steel reinforcement, steel strands and CFRP tendons are shown in Fig. 11.

8. The effect of a cast-in-place (CIP) concrete slab on top of the precast girders on continuity between adjacent spans is ignored, as the continuity steel in the CIP slabs is probably small in the original construction. Furthermore, in bridge girders that require strengthening, the concrete slab would have cracked in the vicinity of the joint.

**Uncracked Section Analysis (Regime AB)**

Before the joint opens up, the beam behaves similar to a two-span monolithic continuous beam. It was observed in the tests that cracks occurred within the grout. Thus, the joint can be assumed to open up once the extreme tensile fiber stress at the joint section attains a value equal to the modulus of rupture of the grout, \(f_r\), taken as 0.62 \(\sqrt{f_{c}'}\) (MPa), where \(f_{c}'\) is the cylinder compressive strength of the grout (in MPa). For the positive moment regions, flexural cracks form when the extreme tensile fiber stress reaches the modulus of rupture of concrete.

The cracking moment, \(M_{cr}\), for each
critical section is given by:

\[ M_{cr} = A_{ps} f_{pe} \left( e_o - \frac{M_{sec}}{A_{ps} f_{pe}} + Z_i \right) + f_t Z_t \]

where

- \( A_{ps} \) = area of the external tendons
- \( e_o \) = initial eccentricity of the external tendons, measured from the centroid of the section
- \( f_{pe} \) = effective prestress
- \( M_{sec} \) = secondary moment due to prestressing
- \( Z_i \) = elastic modulus of the critical section with respect to the extreme concrete tensile fiber
- \( A_t \) = transformed area of the beam cross section

Cracked Section Analysis
(Regimes BC and CD)

Once the joint opens up, the section at the intermediate support undergoes rotation and there is a redistribution of moments away from the support section. In effect, the applied moment at the intermediate support section is reduced and the applied moments in the positive moment regions are correspondingly increased. To quantify the reduction in moment at the intermediate support, a reduction factor, termed the degree of continuity, is introduced.\(^2\)

The degree of continuity, \( \lambda \), is the ratio of initial post-tensioning force, \( A_{ps} f_{pe} \), to the ideal post-tensioning force, \( F_i \), that is required to ensure no rotation at the intermediate support up to the ultimate load, that is:

\[ \lambda = \frac{A_{ps} f_{pe}}{F_i} \] (2)

where \( f_{pe} \) is the initial effective prestress.

Note that \( 0 \leq \lambda \leq 1 \), with \( \lambda = 0 \) indicating the case where there is no continuity between adjacent spans, and \( \lambda = 1 \) indicates the case where there is full continuity.

The resulting bending moment due to the applied loads after the opening of the joint is shown in Fig. 12. The bending moment is derived from the case of a two-span monolithic continuous beam, modified with the factor \( \lambda \).

In this study, the ideal post-tensioning force was determined as follows. To ensure zero opening at the top concrete fiber of section at the intermediate support at ultimate condition, that is, when the moment capacity at the
intermediate support is reached, the tensile stress at the top concrete fiber has to be less than or equal to the modulus of rupture of the grout. Thus:

\[
\frac{F_i}{A_y} - \frac{F_i e_z}{Z_{iy}} + \frac{M_{si}}{Z_{iy}} \leq f_y
\]  \hspace{1cm} (3)

where

- \( F_i \) = ideal post-tensioning force
- \( M_{si} \) = corresponding moment capacity of the intermediate support section
- \( A_y \) = cross-sectional area of the concrete
- \( Z_{iy} \) = section modulus of the concrete
- \( f_y \) = tensile strength of the concrete

Eq. (3) gives the ideal post-tensioning force as:

\[
F_i = -f_y + \frac{M_{si}}{A_y} + \frac{e_z}{Z_{iy}}
\]  \hspace{1cm} (4)

Since \( M_{si} = F_i z \), in which \( z \) is the internal moment arm, an iterative procedure is required to solve for \( F_i \). To start with, a value of \( z \) is assumed. By using Eq. (4), \( F_i \) can be obtained. To maintain an equilibrium of forces, the concrete compressive force, \( C \), has to be equal to \( F_i \). Knowing the value of \( C \), the neutral axis depth, \( x \), can be calculated, and the internal moment arm, \( z \), is obtained and compared to the assumed value.

This process is repeated until the calculated value of \( z \) is equal to the assumed value. For the beams in this study, \( F_i \) was found to be 304 kN (68.33 kips).

After the joint opens, the external tendons experience high strain and consequently a stress increase. In order to maintain an equilibrium of forces, the concrete compression zone at the support section needs to carry high stresses. The concrete stresses may exceed \( 0.4f'_c \), which corresponds to the elastic limit for concrete.

The change in external tendon force depends on the width of the joint opening. This width can be calculated by assuming a rigid body rotation as shown in Fig. 13, in which \( x \) is the neutral axis depth and \( d_{ps} \) is the effective tendon depth; both depths are measured from the extreme compressive fiber. The width of the opening at the level of the external tendons, \( \delta \), may be obtained as [see Fig. 13(c)]:

\[
\delta = (d_{ps} - x)(\tan \theta_1 + \tan \theta_2)
\]  \hspace{1cm} (5)

The angles \( \theta_1 \) and \( \theta_2 \) are the rotations of the two spans of the partially continuous beam, and they can be calculated using the conjugate beam method\(^6\) as:

\[
\theta_n = \frac{1}{E_c I_{eff}} \left( \frac{P_n L_n^2}{36} + \frac{M_{sn}}{6} - \frac{\lambda M_{sn}}{2} \right)
\]  \hspace{1cm} (6)

where

- \( P_n \) = applied load on span \( n \) (where \( n = 1, 2 \))
- \( M_{sn} \) = applied moment at the intermediate support section
- \( L_n \) = length of each span
- \( E_c \) = Young’s modulus of concrete
- \( I_{eff} \) = effective moment of inertia of the positive moment section, calculated from Branson’s equation\(^7\) (see Appendix B)

Assuming a small rotation, that is, \( \tan \theta_n = \theta_n \), Eq. (5) can be rewritten in place of Eq. (6) as:
If the spans are equal and are subjected to equal load, then \( \theta_1 = \theta_2 = \theta \), \( P_1 = P_2 = P \), and \( L_1 = L_2 = L \), and Eq. (7) can be simplified as:

\[
\delta = \frac{1}{E_c I_{eff}} \left( d_{ps} - x \right) \sum_{n=1}^{2} \left( \frac{P_n L_n^2}{36} + \frac{M_n L_n}{6} - \frac{\lambda M_n L_n}{2} \right)
\]

(7)

The additional tendon stress due to the opening of the joint can be calculated as:

\[
\Delta f_{ps} = E_{ps} \frac{\delta}{l_t}
\]

(9)

where \( E_{ps} \) and \( l_t \) are Young’s modulus and the length of the external tendons. Substituting Eq. (8) into Eq. (9) gives:

\[
\Delta f_{ps} = \frac{2 E_{ps}}{E_c I_{eff}} \left( d_{ps} - x \right) \left( \frac{P l^2}{36} + \frac{M L}{6} - \frac{\lambda M L}{2} \right)
\]

(10)

The total external tendon stress can, thus, be calculated as:

\[
f_{ps} = f_{pe} + \Delta f_{ps}
\]

(11)

An iterative procedure to calculate \( f_{ps} \) when the plastic hinge starts to develop (Point D in Fig. 10) is shown in Fig. 14. A plastic hinge is assumed to form when the internal longitudinal steel at the sections of maximum positive moment reaches its yield strength; the plastic hinge load, \( P_y \), and the applied moment at the joint, \( M_{sy} \), can be calculated from the bending moment shown in Fig. 12.

Next, the neutral axis depth, \( x \), is assumed. For the first approximation, \( x \) may be taken as one-half the beam depth \( (= h/2) \).

With the values of \( P_y \), \( M_{sy} \), and assumed value of \( x \), the tendon stress, \( f_{ps} \), can be calculated from Eqs. (10) and (11), and correspondingly, the tendon force, \( F \), is determined. To maintain the equilibrium of forces on the joint section, the concrete compressive force, \( C \), has to be equal to \( F \).

Finally, the moment of resistance can be calculated as \( Fz \), and compared to the applied moment, \( M_{sy} \), at this section. If the moment equilibrium is not satisfied, a new value of \( x \) is introduced and the process is re-
The procedure is illustrated by an example in Appendix B.

**Ultimate Limit State (Point E)**

The flexural capacity of the test beams is calculated based on plastic theory in which a collapse mechanism (see Fig. 15) with plastic hinges formed at the critical sections in each span is assumed.

Referring to Fig. 15, which shows the total bending moment for the assumed collapse mechanism of a two-span beam under equal third-point loading, equilibrium between the applied loads and internal resisting moments gives:

\[
\frac{M_{u} + M_{mu}}{3} = \frac{P_{u} L}{12} \quad (12a)
\]

\[
P_{u} = 12(\frac{1}{2}M_{u} + M_{mu})/L \quad (12b)
\]

where

- \(M_{u}\) = ultimate moment capacity of the intermediate support section
- \(M_{mu}\) = ultimate moment capacity of the section under the outer point load

Note that both moments can be calculated using section analysis based on bond reduction coefficients.

For a simply supported beam, Naaman and Alkhairi\(^8\) have proposed the bond reduction coefficient \(\Omega_u\) as:

For one-point loading:

\[
\Omega_u = \frac{2.6}{L_o} \quad (13a)
\]

For third-point or uniform loading:

\[
\Omega_u = \frac{5.4}{L_o} \quad (13b)
\]

where

- \(L_o\) = beam span
- \(d_{ps}\) = effective depth to the external tendons

For the partially continuous beam under investigation, \(L_o\) is taken as the distance between two points of contraflexure for the intermediate support tendons, and as the distance between the outer support and point of contraflexure for positive moment tendons. The point of contraflexure is determined from the elastic bending moment diagram for simplicity.

The external tendon stress at the ultimate limit state, \(f_{ps}\), is then given as:\(^8\)

\[
f_{ps} = E_p \left[ \varepsilon_{ps} + \Omega_u \varepsilon_{ec} \right] + \Omega_u E_p \varepsilon_{ec} \left( \frac{d_{ps}}{\varepsilon_{ps}} - 1 \right) \leq f_{ps}
\]

(14)

where

- \(\varepsilon_{ps}\) = initial effective prestrain in external tendon
\( \varepsilon_{ce} = \text{strain in the concrete at the level of external tendons due to initial effective prestress} \)

\( \varepsilon_{cu} = \text{maximum strain in the extreme concrete compressive fiber} \)

### Deflections

The camber due to prestressing of short tendons over the joint is small compared to the deflection due to vertical loads, and, therefore, can be neglected. The deflection, \( \Delta \), at the section of maximum positive moment of a partially continuous beam under equal third-point loading, can be calculated using the conjugate beam method, and estimated as follows by ignoring higher order terms of \( \lambda \).

(a) For the uncracked and cracked regimes:

\[
\Delta = \left( \frac{1}{16} - \frac{\lambda}{486} - \frac{\lambda}{648(3+5\lambda)} \right) \frac{PL^2}{E_c I}
\]

(b) At the ultimate flexural strength limit state:

\[
\Delta = \left( \frac{2}{27} - \frac{2\lambda}{81} - \frac{\lambda}{54(3+5\lambda)} \right) \left( \frac{\varepsilon_{cu}}{3x_s} + \frac{\varepsilon_{cu}}{x_m} \right)L^2
\]

where \( \lambda \) is taken as 1 before the joint opens, \( E_c \) is the modulus of elasticity of concrete, and \( I \) is the moment of inertia of positive moment section, which should be taken as the transformed moment of inertia, \( I_{tr} \), for the uncracked regime, and as the effective moment of inertia, \( I_{ef} \), for the cracked regime.

The increase in flexural capacity is quantified by the strengthening ratio, \( SR \), defined as the ratio of ultimate load-carrying capacity of the beam after strengthening, \( P_{u,s} \), to that before strengthening, \( P_{u,o} \); that is:

\[
SR = \frac{P_{u,s}}{P_{u,o}}
\]

### PARAMETRIC STUDY

Based on the proposed method, a parametric study was carried out to establish design charts that can be used for strengthening of simple-span beams or girders by rendering them partially continuous. The precast beams considered had a T-section with straight external tendons over the interior supports, and were subjected to third-point loads, as shown in Fig. 18. The internal reinforcement consisted of longitudinal steel reinforcement near the bottom of the web and stirrups at a regular spacing.

Considering a likely situation in practice, the internal longitudinal steel ratio \( (\rho_s = A_s/(bd_s)) \) was varied at 0.50, 0.75, and 1.00 percent, the concrete strength, \( f'c \), was varied at 20, 30, and 40 MPa (3, 4.4 to 6 ksi), and the effective tendon depth, \( d_{ps} \), was varied at 0.6, 0.8, and 1.0 times the overall beam depth. The effective prestress, \( f_{pe} \), was set equal to 0.50f_{puy} as a case study.

Fig. 18. Partially continuous beam for parametric study.

The required area of external tendons at the intermediate support, relative to the existing internal longitudinal steel reinforcement, is defined by the relative prestressing index, \( \chi \), given by:

\[
\chi = \frac{\rho_p f_{py}}{\rho_s f_y}
\]

where

- \( \rho_p = \text{external post-tensioning ratio} \)
- \( \rho_s = \text{internal tensile steel ratio} \)
- \( f_{py} = \text{yield stress of external tendon} \)
- \( f_y = \text{yield stress of internal steel reinforcement} \)

For seven-wire prestressing steel strands, \( f_{py} \) may be taken as the stress corresponding to a strain of 1 percent, or alternatively as stress at 0.2 percent.
offset strain, whereas for FRP tendons, it may be taken as 80 percent of the rupture strength in order to avoid tendon rupture at ultimate.

Fig. 19 shows the strengthening ratio, $SR$, plotted against the relative prestressing index, $\chi$. In general, the strengthening ratio, $SR$, increases with an increase in the relative prestressing index, $\chi$, but at a decreasing rate. Increasing the effective depth of external tendon, $d_{ps}$, leads to a higher strengthening ratio.

The charts can be used to design the strengthening scheme for precast girders or beams, for which the concrete strength and internal tensile steel reinforcement are assumed to be known. That is, corresponding to a desired strengthening ratio, $SR$, the relative prestressing index, $\chi$, and the tendon depth, $d_{ps}$, can be selected from these charts. Using the value of $\chi$, the required area of external tendons is obtained from Eq. (18). A design example illustrating the use of the charts is given in Appendix C.

It should be pointed out that the charts in Fig. 19 were derived for the case of two-span beams under third-point loads, in which the maximum positive and negative bending moments are more critical than that of multi-span beams subjected to uniformly distributed loads on single span, two spans or all spans. Consequently, using the charts for multi-span beams would give conservative results. The charts can also be used for the strengthening design of precast girders with rectangular or I-shaped cross sections.

CONCLUSIONS AND RECOMMENDATIONS

Based on this investigation, the following conclusions can be drawn, along with recommendations for precast beam strengthening using external tendons to provide partial moment continuity:

1. Installing external tendons to provide continuity between two simple-span precast beams has been shown to be feasible in increasing their load-carrying capacity with sufficient ductility at failure.

2. Increasing the effective tendon forces by adopting a larger tendon diameter or area increases the ultimate load, decreases the width of the joint opening, and reduces the beam rotation at the joint.

3. Using FRP tendons resulted in a similar flexural response as that of steel tendons; however, the increase in tendon stresses needs to be more carefully monitored due to the non-ductile characteristic of the material.

4. The provision of additional external tendons in the positive moment regions resulted in higher ultimate load, more controllable deflection and crack widths, less rotation over the interme-

Fig. 19. Charts for flexural strengthening ratio of partially continuous beams.
diate support and, hence, less opening of the support joint.

5. An analytical method to predict the piece-wise linear flexural response and the stress increase in the external tendons of partially continuous beams was proposed and found to be in good agreement with the test results.

6. The strengthening scheme for precast beams by rendering them partially continuous may be selected using design charts.

7. To ensure the effectiveness in strengthening, the beam should fail within the strengthened region under the application of external loads. The required length of external tendons at the intermediate support can be taken as the distance between two points of contraflexure, determined from the elastic bending moment diagram for simplicity.

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REFERENCES


APPENDIX A — NOTATION

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\begin{align*}
A_{ps} &= \text{area of external tendon} \\
A_{tr} &= \text{transformed area of beam cross section} \\
b &= \text{width of beam flange} \\
b_w &= \text{width of beam web} \\
d_{ps} &= \text{effective depth to external tendons} \\
d_t &= \text{effective depth to internal tensile steel} \\
e_o &= \text{initial tendon eccentricity (negative downwards)} \\
E_c &= \text{modulus of elasticity of concrete} \\
E_{ps} &= \text{modulus of elasticity of external tendons} \\
f_c^p &= \text{cylinder compressive strength of concrete} \\
f_{pe} &= \text{initial effective prestress} \\
f_{ps} &= \text{external tendon stresses} \\
f_o &= \text{modulus of rupture of concrete} \\
F_i &= \text{ideal post-tensioning force [Eq. (4)]} \\
h &= \text{total beam height} \\
h_t &= \text{web thickness} \\
I_{eff} &= \text{effective moment of inertia} \\
I_{tr} &= \text{transformed (gross) moment of inertia} \\
L &= \text{effective span between supports} \\
l &= \text{length of external tendon over intermediate support} \\
M_{cr} &= \text{cracking moment} \\
M_{mu} &= \text{ultimate moment capacity of critical section in positive moment region} \\
M_s &= \text{moment capacity of intermediate support section} \\
M_{si} &= \text{ideal moment capacity of intermediate support section} \\
M_{su} &= \text{ultimate moment capacity of intermediate support section} \\
P &= \text{total external load applied on partially continuous beam} \\
P_u &= \text{ultimate load carrying capacity of partially continuous beam} \\
SR &= \text{strengthening ratio [Eq. (17)]} \\
x &= \text{neutral axis depth} \\
\Delta &= \text{deflection due to applied loads} \\
\Omega &= \text{bond reduction coefficient} \\
\chi &= \text{relative prestressing index [Eq. (18)]} \\
\delta &= \text{width of joint opening at level of external tendons} \\
\varepsilon_{ce} &= \text{strain in the concrete at level of external tendons due to initial effective prestress} \\
\varepsilon_{pe} &= \text{initial effective prestrain in external tendons} \\
\lambda &= \text{degree of continuity [Eq. (2)]} \\
\theta &= \text{beam rotation} \\
\rho_p &= \text{external post-tensioning ratio } [A_p/(bd_p)] \\
\rho_s &= \text{internal tensile steel ratio } [A_s/(bd_s)]
\end{align*}
\]
APPENDIX B — WIDTH OF JOINT OPENING AND TENDON STRESS

This appendix shows the computation of width of joint opening and external tendon stress, \( f_{ps} \), at the initiation of plastic hinge (Point D in Fig. 10) for Specimen P-1. Details of the specimen are given in Tables 1 and 2, and Fig. 1.

The following can be computed from section analysis:
Positive moment section: \( I_p = 431,612,823 \text{ mm}^4 \), \( I_c = 149,093,081 \text{ mm}^4 \), \( M_{cr} = 9.74 \text{ kN-m} \).

From Eq. (2):

\[
\lambda = \frac{A_{ps} f_{pe}}{F_i} = \frac{110 \times 914}{304000} = 0.33
\]

A plastic hinge is assumed to form when the internal steel reinforcement at the critical positive moment region (that is, sections under the outer point loads) yields. From structural analysis of positive moment section, the moment \( M_m \) corresponding to the yielding of internal steel can be computed as 58.18 kN-m.

The effective moment of inertia can be calculated from Branson’s equation:

\[
I_{eff} = \left( \frac{M_m}{M_c} \right)^3 I_p + \left[ 1 - \left( \frac{M_m}{M_c} \right)^3 \right] I_c
\]

\[
= \left( \frac{9.74}{58.18} \right)^3 I_p + \left[ 1 - \left( \frac{9.74}{58.18} \right)^3 \right] I_c
\]

\[
= 150,418,656 \text{ mm}^4
\]

The plastic hinge load, \( P \), is then obtained from the equations given in Fig. 12. For beams with equal span subjected to equal third-point loads:

\[
P = \frac{58.18}{(1/12 - 0.33/36) \times 2.85} = 275 \text{ kN}
\]

The intermediate support moment, \( M_s \), is given by:

\[
M_s = 1/12 PL \lambda = 1/12 \times 275 \times 2.85 \times 0.33 = 21.5 \text{ kN-m}
\]

The tendon stress, \( f_{ps} \), is computed following the procedure described in Fig. 14.

1. First, the neutral axis depth, \( x \), is assumed. For the first approximation, \( x \) is taken as one-half the beam depth or 150 mm.
2. The width of joint opening is then computed from Eq. (8):

\[
\delta = \frac{2}{E I_{eff}} (d_{ps} - x) \left( \frac{P L^2}{36} + \frac{M_c L}{6} - \frac{M_{cr} L}{2} \right)
\]

\[
\delta = \frac{2}{29,862 \times 150,418,656} \left( 180 - 150 \right) \left( \frac{275 \times 10^3 \times 2850^2}{36} + \frac{21.5 \times 10^6 \times 2850}{6} - \frac{0.33 \times 21.5 \times 10^6 \times 2850}{2} \right)
\]

\[
= 0.81 \text{ mm}
\]

3. External tendon stress:

\[
f_{ps} = f_{pe} + \Delta f_{ps}
\]

\[
= 914 + (0.81/1420) \times 195,000 = 1025 \text{ MPa}
\]

4. Tendon force:

\[
F = f_{ps} \times A_{ps}
\]

\[
= 1025 \times 110 = 112,750 \text{ N}
\]

5. To maintain equilibrium of forces at the intermediate support section, the concrete compressive force, \( C \), has to be equal to \( F \), or \( C = 112,750 \text{ N} \) (see Fig. B1).

6. The internal moment arm, \( z \), is evaluated by computing the centroid of the concrete compression zone, \( c \). An iterative procedure is required by first assuming the strain in the extreme concrete compressive fiber, \( \varepsilon_{cb} \). Next, calculate the corresponding concrete compressive stress from the stress-strain curve (see Fig. 11), and finally calculate the concrete compressive force, \( C \), by integrating the stress over the compression area. For the derived value of \( x = 150 \text{ mm} \), in order to get \( C = 112,750 \text{ N} \), \( \varepsilon_{cb} \) is found iteratively as 0.00038 and \( c = 51.7 \text{ mm} \). Correspondingly, the internal moment arm is \( z = d_{ps} - c = 180 - 51.7 = 128.3 \text{ mm} \).

7. The moment of resistance of the section can be computed as \( M = C \times z = 112,750 \times 128.3 = 14,465,825 \text{ N-mm} \) or 14.5 kN-m

8. Since \( M \neq M_{cr} \), the process is repeated with a new \( x \) value.

Try a neutral axis depth, \( x = 88 \text{ mm} \), which gives the width of the joint opening as \( \delta = 2.43 \text{ mm} \); additional tendon stress, \( \Delta f_{ps} = 333.6 \text{ MPa} \); tendon stress, \( f_{ps} = 1247.6 \text{ MPa} \); and tendon force, \( F = 137,236 \text{ N} \). To maintain equilibrium of forces, \( C = 137,236 \text{ N} \), which is satisfied when \( \varepsilon_{cb} = 0.00085 \) and \( c = 26 \text{ mm} \). The internal moment arm, \( z = d_{ps} - c = 180 - 26 = 154 \text{ mm} \); and \( M = C \times z = 21.1 \text{ kN-m} = M_{cr} = 21.5 \text{ kN-m} \). Thus, at a plastic hinge load \( P = 275 \text{ kN} \), the width of the joint opening, \( \delta \), is obtained as 2.43 mm and tendon stress, \( f_{ps} = 1247.6 \text{ MPa} \).

Two identical precast simple-span beams are considered with
dimensions and reinforcement details (see Fig. 18) as follows:

\[ L = 7500 \text{ mm (24 ft 7 in.)}, \quad d_s = 450 \text{ mm (1 ft 6 in.)} \]
\[ b = 500 \text{ mm (1 ft 8 in.)}, \quad b_w = 250 \text{ mm (10 in.)} \]
\[ h = 500 \text{ mm (1 ft 8 in.)}, \quad h_f = 100 \text{ mm (4 in.)} \]
\[ A_s = 1650 \text{ mm}^2 (2.56 \text{ sq in.}) \]
\[ A_s/(b d_s) \text{ in span} = 0.75 \text{ percent} \]
\[ f'_{c} = 30 \text{ MPa (4.35 ksi)}, \quad f_y = 460 \text{ MPa (67 ksi)} \]

They are to be made “partially continuous” to increase their flexural capacities by 20 percent (strengthening ratio, \(SR = 1.20\)). Determine the required tendon area to achieve the desired strengthening ratio.

To achieve the desired strengthening ratio, external tendons are installed over the interior support. From the beam cross section, it is observed that the eccentricity of the tendons at the interior support must be less than \(h - h_f = 400 \text{ mm (1 ft 4 in.)}\). Try \(d_{ps} = 300 \text{ mm (1 ft)}\) to give adequate space for placing prestressing jack, giving the ratio \(d_{ps}/h = 0.60\). From the chart in Fig. C1, which is extracted from Fig. 19, for \(f'_{c} = 30 \text{ MPa (4.35 ksi)}, \quad A_s/(b d_s) \text{ in span} = 0.75 \text{ percent}, \quad d_{ps}/h = 0.6\), the required relative prestressing index, \(\chi\), may be obtained as approximately 1.4 for a specified \(SR = 1.20\).

If seven-wire prestressing steel strands (\(f_{pu} = 1900\) MPa, \(f_{py} = 1786 \text{ MPa}\)) are used, then for \(\chi = 1.4\), the required external post-tensioning ratio, \(\rho_p\), can be computed from Eq. (17) as 0.27 percent, and the required tendon area is \(A_{ps} = 506.25 \text{ mm}^2 (0.785 \text{ sq in.})\). The external tendons are to be stressed to 0.5 \(f_{pu}\) (= 950 MPa or 138 ksi) and can be encased in high density polyethylene (HDPE) ducts for corrosion protection.

After the required area of external tendons is determined, a service stress analysis is needed at the intermediate support to ascertain that the allowable stresses are not exceeded.