The Practice of Warping Double Tees

This paper presents the results of a study of warping (twisting) in precast concrete double tees and draws conclusions on what is being successfully done within the industry to minimize or eliminate longitudinal cracking through decks due to warping. Conclusions are given based on the results of an industry survey, an assessment of torsion parameters, theoretical calculation of cracking point, and empirical data from plant controlled testing and field experiences. Data are presented for field-topped tees and pretopped tees of various lengths and widths. Recommendations are given on the best practice to follow in controlling warping in double tees.

Double tees are perhaps the most widely used member in the precast, prestressed concrete industry (see Fig. 1). In general, double tees have performed extremely well over the past 50 years. However, occasionally, a double tee may experience longitudinal cracks at the interface of stem and deck elements. The cracks may range in length from 5 to 30 percent of the tee length.

Under close scrutiny, these cracks appear to develop on top of the deck on one stem and on the underside of the other stem. The cause and significance related to long-term performance of double tees experiencing these cracks have not been universally agreed upon within the industry. Nevertheless, one purpose of this paper is to bring all available knowledge on this subject together so that a consensus can be reached.

A PCI committee report in the January-February 1983 PCI JOURNAL states that the cause of longitudinal cracks
is generally warping or twisting that has been imposed on the double tee member.

The most common occurrence of warping is on decks of parking structures. Therefore, many discussions in this paper are directed toward this application. In general, cracking is caused by:

- Twisting that occurs when the tee is stripped (removed) from its casting bed (e.g., uneven lifting during stripping from binding in the form, uneven sling lengths, or use of two cranes).
- Racking from improper storage in the manufacturing plant storage yard (e.g., dunnage placed on soft or uneven ground).
- Twisting during handling or hauling (e.g., a trailer running over a curb).
- Torsion being a condition of the design (geometry) of the structure containing the erected tee (e.g., sloped/warped for drainage in a parking structure).

Typically, this type of crack is non-structural and has a negligible effect on the load carrying capacity of the tee. If the tee is covered with a composite concrete topping (field-topped), no repair is necessary unless for cosmetic reasons.

A pretopped tee (one in which the entire deck is cast monolithically with the tee stems during manufacture) experiencing cracks may need remedial attention after the tee is erected, depending on the crack width and location. If water penetration is a concern, a wide range of weather resistant materials is available to fill and seal the cracks, such as epoxies, caulking materials, and other sealants.

This paper presents the results and conclusions of a study of warping (twisting) in double tees. Conclusions are based on the results and dissemination of an industry survey, theoretical calculation of cracking point and torsion parameters of the cross section, and empirical data extracted from plant controlled testing and field observations. Data are presented for field-topped tees and pretopped tees of various lengths.

Note that double tees may crack even if the guidelines presented here are followed closely. Recommendations provided in this article, therefore, should be considered with engineering judgment. There is no guarantee that adhering to these recommendations will entirely eliminate the condition, but it should be borne in mind that concrete cracking is not unusual and that not all cracks necessitate remedial action.

The first three bulleted items identified as causes of longitudinal cracking are generally unpredicted, but the fourth is an intentional design condition wherein twisting and warping is known to occur. To achieve durability in parking structures, one of the most important recommendations made in the American Concrete Institute publication ACI 362.1R-97² is to slope decks in order to drain water from the deck surface.*

While direct rain or snow may not enter all areas of a parking structure, wind blown rain and/or vehicles carrying ice/slush/water will distribute

* Other components of durability include (1) low water-cementitious material ratios (0.40 or less), (2) air entrained concrete (6%, percent, 1% tolerance), (3) corrosion inhibiting admixtures, (4) good connections, (5) high quality joint sealants, (6) cover over reinforcement of 1½ to 2 in. (38 to 51 mm), and (7) a well-planned maintenance program. Other less effective measures like surface sealers and traffic bearing membranes are sometimes employed.
water throughout the facility. If the water is contaminated with salts from deicing materials or other sources, corrosion of reinforcement may be accelerated due to chloride penetration. As a result, the life of the structure may be shortened.

Frequent floor wash-downs that are part of a maintenance program are also a source of water throughout the facility. If the floor is not adequately sloped for proper drainage, pools of water or sheets of ice can form, promoting the potential for deterioration of the concrete slab due to freeze-thaw cycles. If a floor is to be sloped, how much slope is necessary? Parking industry guidelines are cited in the following paragraphs.

**Floor Slope Recommendations – Standard References**

Recommendations on the amount of slope necessary to drain a floor has been based mainly upon experience. Many references on the subject are available, and the following excerpts are taken from the most often cited sources:

1. **Parking Structures: Recommended Practice for Design and Construction, PCI publication MNL-129-98:**
   
   "Parking Decks should be pitched for draining. Slopes of 1 1/2 percent are common with 1 percent being the minimum acceptable field limit after construction tolerances are considered."

2. **The Dimensions of Parking, Third Edition, ULI & NPA:**
   
   "The recommended slope to drains is 2 percent minimum in all directions. Normally, in precast parking structures, the main slope (typically in the span direction) is 2 percent and the cross slope (perpendicular to the direction of the main slope) is limited to about 1 percent or even less. If a traffic membrane is used, 1 percent slopes are acceptable, but 2 percent slopes are still preferable."

3. **Guide for the Design of Durable Parking Structures, ACI 362.1R-97:**
   
   "A minimum slope in any direction of 1 1/2 percent is recommended with 2 percent being preferred."

4. **Parking, ENO Foundation for Transportation:**
   
   "Storage floors of cast-in-place floor garages should be sloped about 2 percent (approximately 1/4 in. per ft). Precision manufactured (and installed) precast floors can provide positive drainage with 1 percent (approximately 1/6 in. per ft) slopes, although some designers prefer a 2 percent slope."

5. **Packing Structures, Planning, Design, Construction, Maintenance and Repair, Third Edition:**
   
   "For drainage, the absolute minimum slope should be 1/4 in. per ft or about 1 percent preferred slope is 3/16 in. per ft or about 1 1/2 percent."

   
   "Slopes: 2 percent preferable (1 percent is used typically in precast concrete and is based on the use of quality sealer on surface and the denser type of concrete normally associated with precast concrete)."

**Warping of Double Tees**

The PCI document, Parking Structures: Recommended Practice for Design and Construction, MNL 129-98, presents a cursory discussion of warping of double tees in Chapter 4, Section 4.4. Mention is made of warping for both pretopped and field topped members, and a general comment is made about the flexibility of the members. A sketch indicates how a typical bay of a parking structure could be sloped for drainage, but warping is not shown.

Cross-bay drainage is achieved by lowering one end of the double tee members. Transverse drainage is typically achieved by one of three methods:

1. Pitching the supporting girder or spandrel at the low end only.
2. Sloping the structural topping.
3. Pitching both the high and low ends of the member by using sloped ledges, variable daps in the double tee stems, corbels at varying heights on the supporting spandrel, and/or sloping of the entire supporting element.

The cause of warping (twisting) in double tee members in situ is due to transverse floor slopes (Method No. 1). Many times the other two methods are neither practical nor cost effective. Sloping the field placed topping adds cost for the additional concrete and its weight can require more prestressing force (usually, more strands), thereby adding even more cost to the project.

Using sloped ledges, varying height daps, or placing corbels at varying heights on the face of the spandrel all require particular attention to engineering detailing and to setup within the form so as to ensure proper orientation and dimension. Higher material, labor and forming costs are introduced. It should also be noted that aesthetic considerations often preclude utilization of sloped spandrels on the exterior of a parking deck.

Using varying height daps on the exterior may create a condition wherein the ends of the stems are visible from the outside, which may also be objectionable. Accommodating varying height ledges or corbels on the support spandrel may require that the depth of the spandrel be increased by at least the change in elevation from highest to lowest stem support within a given span.

This may create a problem in that the deck must contain a certain amount of open area (i.e., the space between the top of one spandrel and the bottom of the spandrel above) on its perimeter in order to be considered..."
an open parking structure. Reference should be made to the governing model building code for specific requirements.

For these reasons, designers often maintain a constant elevation for the deck surface at the exterior perimeter of the deck and slope beams or walls on the interior gridline to achieve the transverse slope to the drains. A twist or warp (see Fig. 2) is thereby introduced at the low end of the double tee. This twist propagates along the length of the tee.

The purpose of this paper is to provide a rationale to designers on the limits of warping that can be reasonably expected of typical double tee members currently used in parking deck construction.

As a matter of definition, warping occurs in a double tee when the four stem bearing points are not co-planar and the ends of the tee are rotated with respect to each other. The difference in the slopes at the ends of the tee defines the amount of warp (twist angle). The warp can be expressed as a percent slope (equal to 100 times twist angle) or as a twist with dimensions of in. per ft across the cross section.

INDUSTRY SURVEY

Over the years, the PCI Parking Structures Committee has received numerous questions regarding the process of warping double tees, with particular reference to allowable warp and stresses that will initiate cracking. To provide guidance to the engineering community, the committee conducted a survey of PCI producer members to determine the industry common practice and collect feedback regarding testing or other means of analyzing the limits of twisting and cracking. Fifty-eight surveys were returned to the committee from producers in the United States and Canada.

Survey Results

The survey requested producers to provide section properties, deck reinforcement and the amount of warping they recommend, given various lengths of double tees. Producers were also asked to provide a rationale for their recommendations. All survey responses contained information for tees longer than 50 ft (15.2 m).

The information received (see Tables 1 and 2) regarding the amount of warping ($\Delta$/Span Length) diminished as the double tee lengths decreased. Some respondents reported that short tees are so stiff that they would not settle into their desired (warped) position without forcing the stem down with weights. Blank responses were not considered in the statistical evaluations.

**Producing Comments — Amount of Warping:**

- Double tees with widths of 10 ft (3.05 m) or less and with a 2 in. (51 mm) thick deck have been warped as much as 0.6 in. per ft ($\Delta = 3$ in.). The decks cracked but are functioning well.
- Pretopped double tees with widths of 10 ft (3.05 m) or less with a 4 in. (102 mm) thick deck have been warped as much as 0.4 in. per ft ($\Delta = 2$ in.) and had cracks.
- Three producers reported testing a 10 ft (3.05 m) wide, 60 ft (18.30 m) long double tee. Two found that the tees began to crack at 0.2 in. per ft with a 4 in. (102 mm) thick deck. The third producer conducted a finite element analysis on a 10 ft (3.05 m) wide, 60 ft (18.30 m) long double tee with a $3\frac{1}{4}$ in. (121 mm) thick deck. The tee cracked at 0.175 in. per ft of warp. Test results confirmed their theoretical analysis.

**Producing Comments — Regarding Cracking:**

- Double tees are less susceptible to torsion cracking if the transition from stem to deck is a radius rather than a chamfer; this is because stress concentration is less.
- Using a $\frac{3}{4}$ in. (9.5 mm) diameter longitudinal prestressing strand (parallel to the stems) at the stem locations in pretopped decks helps control cracks. The strand also provides a surface for supporting the deck reinforcement, ensuring proper placement. Typically, the strand is stressed to a nominal value of about 5 kips (22.3 kN).
- It is possible to initiate warping in the plant by shimming one leg higher than the other while the tee is in storage and the shear modulus is smaller. By doing this, the warping is taking place while the concrete is curing and allows the double tee to warp more without developing cracks. Care must be taken to warp the tee in the proper direction.
- Allowing a double tee to settle into its warped position may increase the total amount a double tee can be warped versus forcing the double tee down to make the deck connections.
- There was insufficient data to determine whether or not the type of aggregate in the concrete mix (normal weight or lightweight) influenced the amount double tees can be warped without cracking. However, engineers should keep in mind that, in the end, it is the shear modulus that is important.
- The depth of the tee or the spacing of tee stems appears to have little effect on the amount double tees can be warped without cracking. No reason for this observation was given. It is shown later in this article that the depth of the tee stems has a

| Table 1. Field-topped double tees [2 in. (51 mm) average deck thickness]. |
|--------------------------|----------------|----------------|----------------|
| Span length              | < 20 ft        | 20 to 30 ft     | > 50 ft        |
| Average warp (twist)     | 0.068 in/ft    | 0.100 in/ft     | 0.151 in/ft    |
| Standard deviation       | 0.026 in/ft    | 0.035 in/ft     | 0.043 in/ft    |

Note: 1 ft = 0.3048 m; 1 in. = 25.4 mm.

| Table 2. Pretopped double tees [4 in. (102 mm) average deck thickness]. |
|--------------------------|----------------|----------------|----------------|
| Span length              | < 20 ft        | 20 to 30 ft     | > 50 ft        |
| Average warp (twist)     | 0.076 in/ft    | 0.082 in/ft     | 0.128 in/ft    |
| Standard deviation       | 0.039 in/ft    | 0.040 in/ft     | 0.056 in/ft    |

Note: 1 ft = 0.3048 m; 1 in. = 25.4 mm.
An analysis of standard double tee sections subjected to pure torsion was carried out to isolate the effects of torsion. The complex interaction of stresses caused by gravity and axial loads are not addressed in this paper.

The purposes of the study were to:
1. Compare rotational stiffness and maximum torsion stresses for the various tee sections, and
2. Determine the percentage of total torsion resisted by the deck and by the stems. It was also expected that computer modeling would reveal regions on the section where stress concentrations are likely to occur.

Tee sections and their properties were taken from Chapter 2 of the Fifth Edition of the PCI Design Handbook.\(^9\)

The sections are somewhat "historical" in that tee widths have increased over time. For a brief period, producers manufactured 6 ft (1.83 m) wide members. The 8 ft (2.44 m) wide tee became the industry standard for about two decades, followed by 10 ft (3.05 m) tees, then 12 ft (3.66 m) wide tees. Currently, 15 ft (4.57 m) wide tees are fabricated in some locations. The Design Handbook presents data for 8, 10, and 12 ft (2.44, 3.05, and 3.66 m) sections with varying stem heights and deck thicknesses. Table 3 summarizes the sections used in this analysis.

Two methods were used to calculate the torsion constant \(K\). First, the double tee sections were modeled using low-order finite differences to solve the general partial differential equation for cross sections without openings:

\[
\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = H \tag{1}
\]

where
- \(H = -2G\theta\)
- \(G =\) shear modulus of elasticity
- \(\theta = \) twist angle per unit length

Eq. (1) is derived by using the Prandtl stress function, where \(\phi\) is a function of the orthogonal coordinates \(X\) and \(Y\).

The differential equation accounts for stresses due to pure rotation as well as the out-of-plane warping that occurs for non-circular cross sections. The stress function \(\phi\) for torsion has a dramatic effect on the warping behavior of a double tee.

- There was too much variation in deck reinforcement to conclude how or if this affects cracking or the amount double tees can be warped prior to cracking.
- The data indicate that the thicker the deck, the less double tees can be warped. The reason for this is that the torsion constant is proportional to the cube of thickness. There was insufficient data to determine successful warping criteria for deck thicknesses other than 2 and 4 in. (51 and 102 mm).
- Using rocker-beam assemblies under the front end of hauling trailers can mitigate cracks resulting from transportation of tees within plant storage areas or to jobsites. Fig. 3 shows a trailer fitted with a rocker beam. The double tee stems sat on rubber softeners in the two brackets located on top of the steel tube section. Without the rocker beams, twisting transmitted from the trailer to the tee can be considerably more than the amount designed into the structure to accommodate floor drainage. Truck drivers cannot always avoid curbs and potholes.

**RATIONAL METHOD TO DETERMINE TORSION STIFFNESS**

Described below is a simplified approach for analyzing torsion and a discussion of some practical results obtained from using this methodology.

**Method of Analysis**

Tension and shear stresses in double tee sections that are warped are influenced by a number of stress combinations. Prestress forces and flexure cause tension and compression stresses, whereas flexure and torsion cause shear stresses. An analysis of standard double tee sections subjected to pure torsion was carried out to isolate the effects oftorsion. The complex interaction of stresses caused by gravity and axial loads are not addressed in this paper.

The purposes of the study were to:
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The differential equation accounts for stresses due to pure rotation as well as the out-of-plane warping that occurs for non-circular cross sections. The stress function \(\phi\) for torsion has
the same form as the differential equation describing the deflection of an elastic membrane subjected to pressure. It is also analogous to the shape of the deflected membrane.

The tangent to the membrane parallel to the ZX-plane is equal to the pure torsion shear stress acting in the Y-direction. The tangent to the membrane parallel to the ZY-plane is equal to the torsion shear stress acting in the X-direction. These tangents give the following equations:

\[ T_x = \frac{\partial \phi}{\partial y} \]  

\[ T_y = -\frac{\partial \phi}{\partial x} \]  

The volume under the elastic membrane is exactly equal to one-half of the applied torsion moment. The derivation of Eqs. (1), (2) and (3) is beyond the scope of this paper; however, it can be found in most advanced mechanics of materials textbooks.

A fine mesh size [0.10 in. (2.54 mm)] was utilized so that stress concentrations could be discerned. This resulted in solving more than 125,000 equations for the larger tee cross sections. Stresses and bending moments were numerically integrated over the cross section to check equilibrium requirements and the accuracy of each model.

As a verification, twist angles and stresses were also checked and found to be within 1 percent of known closed form solutions for elliptical, triangular and rectangular cross sections. Once the computer program had generated the membrane surface, the volume was calculated. After the volume was calculated, the torque on the double tee was easily obtained by multiplying the volume by 2.

The relationship between the angle of twist and the torque is taken from Reference 9:

\[ \theta = \frac{M_i}{KG} \]  

in which

- \( \theta \) = twist angle per unit length
- \( M_i \) = calculated torsion moment
- \( G \) = shear modulus of elasticity
- \( K \) = torsion constant for section

In Eq. (4), the only unknown quantity is the torsion constant \( K \).

The second method of calculating \( K \), using a simplified approach, is described in the following paragraphs.

Table 4. Constant \( c \) based on ratio \( b/t \).

<table>
<thead>
<tr>
<th>( b/t ) or ( (h + 2t)/t_{avg} )</th>
<th>Constant ( c )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>0.141</td>
</tr>
<tr>
<td>1.5</td>
<td>0.196</td>
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<td>2.0</td>
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<td>2.5</td>
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<td>4.0</td>
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<td>5.0</td>
<td>0.291</td>
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<tr>
<td>10.0</td>
<td>0.312</td>
</tr>
<tr>
<td>( \infty )</td>
<td>0.333</td>
</tr>
</tbody>
</table>

Fig. 4 shows a cross section of a typical double tee member.

Eq. (5) gives the St. Venant’s torsion constant \( K \) in terms of the thickness \( t \) and width \( b \) for a rectangular section (see Reference 9):

Fig. 5. Torsion stiffness vs. double tee size with 30 in. (762 mm) stem depth.
In this equation, $c$ is a constant whose value depends on the $b/t$ ratio as shown in Table 4.

Table 4 gives the values for the constant $c$ for corresponding $b/t$ ratios.

Double tee sections are subdivided into three rectangular sections, two stems and one deck. For double tees, $b$ and $t$ are the long and short sides of the deck and stem rectangles, respectively. The contribution of the stems to the total torsion constant $K$ was modified by adding two times the deck thickness to the length.

The reason for this modification will be discussed later. By doing so, the simplified value of $K$ matched the more accurate value from the finite difference method. The torsion contribution to constant $K$ for the deck and the stems are calculated using the following equations:

$$K = cbt^3$$  \hspace{1cm} (5)

$$K_{Deck} = c_1bt^3$$  \hspace{1cm} (6)

$$K_{Stem} = c_2(h + 2t)(t_{avg}^3)$$  \hspace{1cm} (7)

The torsion constant for the complete double tee section was obtained by summing the values of $K$ for the deck and the two stems as shown in Eq. (8):

$$K_{Simple} = K_{Deck} + 2K_{Stem}$$  \hspace{1cm} (8)

All of the double tee cross sections listed in the Fifth Edition of the PCI Handbook that are practical to be used for typical parking deck spans were analyzed for stiffness and stresses. A 15 ft (4.57 m) wide tee was also analyzed to evaluate an upper bound.

A shear modulus $G = EI/(2)(1 + \nu)$, in which $E$ is the modulus of elasticity and $\nu$ is Poisson’s ratio, was calculated to be 2006 ksi (13.8 GPa). This value was held constant for all tee sections. [$E = 4696$ ksi (32.4 GPa) for $f’_c = 6000$ psi (41.4 MPa) and $\nu = 0.17$ for concrete].

Two sets of analyses were done for each section, assuming a 60 ft (18.30 m) tee length. The first assumed a $\Delta = 1\frac{1}{2}$ in. (38 mm) warp while the second assumed a 2 in. (51 mm) warp measured over the full width of the tee [i.e., 10 ft (3.05 m)].

Unit twist angles of $\theta = 0.000018$ radians per in. and $\theta = 0.000023$ radians per in. were used for the $1\frac{1}{2}$ and 2 in. (38 and 51 mm) warp cases, respectively. For reference, the twist angle of 0.000023 radians per in. for a 10 ft (3.05 m) wide x 60 ft (18.30 m) long tee corresponds to a warping displacement of:

$$\Delta = (0.000023)(10)(12)(60)(12) = 1.987 \text{ in.} (50.5 \text{ mm})$$

Tables 5 and 6 show the results of the analysis for 8, 10 and 12 ft (2.44, 3.05 and 3.66 m) wide double tees.

Column (2) lists the theoretical twisting moment $M_t$ needed to cause the assumed angle of twist.

Column (3) lists the torsion stiffness constant $K$ calculated by Eq. (4).

Column (4) lists the proposed simple method.

Column (5) shows the margin of error of $K$ by using the simple method.
Table 5. Torsion stiffness of double tee members: 1 1/2 in. warp. $G = 2006$ ksi; Maximum recommended principal tensile stress = $4\sqrt{6000} = 309$ psi; $\theta = 0.000018$ radians per in. (1 1/2 in. warp over 10 ft width for 60 ft span).

<table>
<thead>
<tr>
<th>Double tee size</th>
<th>$M_{act}$ (in.-kips)</th>
<th>$K_{act}$ (in.4/rad)</th>
<th>$K_{Simple}$ (in.4/rad)</th>
<th>$K$ Error (percent)</th>
<th>$K_{Deck}/K_{Sim}$ (percent)</th>
<th>$K_{Stem}/K_{Sim}$ (percent)</th>
<th>$\tau_{max}$ (psi)</th>
<th>$\tau_{max}/\sqrt{6000}$</th>
<th>$K_{act}$ relative to 10DT22+4 (percent)</th>
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<tbody>
<tr>
<td>8DT22+2</td>
<td>64.2</td>
<td>1946.9</td>
<td>1937.0</td>
<td>-0.5</td>
<td>12.7</td>
<td>87.3</td>
<td>199</td>
<td>2.57</td>
<td>40.5</td>
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<tr>
<td>8DT22+4</td>
<td>135.3</td>
<td>4104.4</td>
<td>3979.4</td>
<td>-5.0</td>
<td>49.7</td>
<td>50.3</td>
<td>258</td>
<td>3.33</td>
<td>85.3</td>
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<tr>
<td>8DT30+2</td>
<td>173.4</td>
<td>5257.9</td>
<td>5203.5</td>
<td>-1.0</td>
<td>4.7</td>
<td>95.3</td>
<td>238</td>
<td>3.07</td>
<td>109.3</td>
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<td>8DT30+4</td>
<td>257.4</td>
<td>7804.8</td>
<td>7506.2</td>
<td>-3.8</td>
<td>25.8</td>
<td>74.2</td>
<td>314</td>
<td>4.05</td>
<td>162.3</td>
</tr>
<tr>
<td>10DT22+2</td>
<td>69.1</td>
<td>2095.4</td>
<td>2094.0</td>
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<td>14.8</td>
<td>85.2</td>
<td>205</td>
<td>2.65</td>
<td>109.3</td>
</tr>
<tr>
<td>10DT22+4</td>
<td>135.3</td>
<td>4104.4</td>
<td>3897.4</td>
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<td>50.3</td>
<td>258</td>
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<td>85.3</td>
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<td>10DT30+4</td>
<td>282.4</td>
<td>8564.9</td>
<td>8166.4</td>
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<td>70.2</td>
<td>309</td>
<td>3.99</td>
<td>178.0</td>
</tr>
</tbody>
</table>

Note: 1 ft = 0.3048 m; 1 in. = 25.4 mm; 1 psi = 0.006895 MPa; 1 kip = 4.448 kN; 1 in.-kip = 0.113 kN-m.

Table 6. Torsion stiffness of double tee members: 2 in. warp. $G = 2006$ ksi; Maximum recommended principal tensile stress = $4\sqrt{6000} = 309$ psi; $\theta = 0.000023$ radians per in. (2 in. warp over 10 ft width for 60 ft span).

<table>
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<tr>
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<th>$K$ Error (percent)</th>
<th>$K_{Deck}/K_{Sim}$ (percent)</th>
<th>$K_{Stem}/K_{Sim}$ (percent)</th>
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<th>$\tau_{max}/\sqrt{6000}$</th>
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Note: 1 ft = 0.3048 m; 1 in. = 25.4 mm; 1 psi = 0.006895 MPa; 1 kip = 4.448 kN; 1 in.-kip = 0.113 kN-m.

Column (6) lists the percentage of torsion moment resisted by the deck. Column (7) lists the percentage of the torsion moment resisted by the two stems. Column (8) lists the maximum resultant torsion shear stress found anywhere on the cross section. Column (9) shows how the calculated shear stresses compare to the maximum tensile stress as recommended in ACI 318-99, Section 11.6, Commentary. Column (10) compares the torsion stiffness for each section to the 10DT22+4.

Discussion of Results

The torsion stiffness constant ($K$) of a double tee is dependent upon the two stems and the deck (flange). For all stem heights and deck thicknesses shown in the preceding tables, the contribution of stem and deck thickness for a given tee width is shown.

Fig. 5 shows the variation of the torsion stiffness constant ($K$) for all three double tee widths with a constant 30 in. (762 mm) stem depth. The figure shows two interesting items; first, as the width of the double tee is increased the stiffness is increased moderately and, second, as the deck thickness increases the stiffness increases significantly.

The contribution of the deck in relation to that of the stems varies as the dimensions of the double tees change. Figs. 6 and 7 show the contribution of the stems to the total stiffness of the section. A comparison of the double tee sections with 2 in. (51 mm) decks shows that if the stem depth is held constant, the contribution of the deck decreases as the width increases.

The same trend holds true for 4 in. (102 mm) decks. The difference between 2 and 4 in. (51 and 102 mm) decks is that for the 2 in. (51 mm) decks the contribution of the stems to the total stiffness is between 85 and 95 percent while for the 4 in. (102 mm) decks it is between 45 and 75 percent.

Note that all of the sections in Tables 5 and 6 have fillets. The effect of 3 in. (76 mm) chamfers was also investigated. Table 7 shows a comparison of 10 ft (3.05 m) wide double tees with 3 in. (76 mm) fillets and 3 in. (76 mm) chamfers.

Table 7 shows that both the torsion constant and maximum stress for a
Table 7. Comparison of 3 in. fillet to 3 in. chamfer for 10 ft wide double tee sections: 1\(\frac{1}{2}\) in. warp; \(G = 2006\) ksi; maximum recommended principal tensile stress = 4\(\sqrt{6000}\) = 309 psi; \(\theta = 0.000023\) radians per in. (2 in. warp over 10 ft width for 60 ft span).

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<th>(M_I) actual (in.-kips)</th>
<th>(K_{act}) (in.(^4) per radian)</th>
<th>(K_{simple}) (in.(^4) per radian)</th>
<th>(K) Error (percent)</th>
<th>(K_{deck}/K_{sim}) (percent)</th>
<th>(K_{stem}/K_{sim}) (percent)</th>
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Note: 1 ft = 0.3048 m; 1 in. = 25.4 mm; 1 psi = 0.006895 MPa; 1 kip = 4.448 kN; 1 in.-kip = 0.113 kN-m.

double tee with a chamfer is larger than that of a double tee with a fillet. However, for 10 ft (3.05 m) wide double tees, the torsion constant increased by about 10 percent but the maximum stress increased by over 45 percent.

**Torsion Stress Behavior**

The pure torsion shear stresses were extremely well behaved and consistent. A single discontinuity always occurred in the form of a short "crease" when chamfers were provided at the stem-to-deck interface. Fig. 8 shows a three-dimensional plot of \(\varphi\), the stress membrane that represents the solution to the differential Eq. (1) by the finite difference method.

Torsion stresses are obtained from this membrane. Theory indicates that the steeper the curve, the higher the stress. Maximum shear stresses occur in the deck over each stem and along the exterior edges of the stems just below the deck, i.e., at the fillets or chamfers on the underside of the deck. Fig. 9 shows contour lines of stresses on a double tee section with fillets and chamfers, respectively.

Stresses induced by chamfers were much greater than tees that had fillets. Fillets were substituted for chamfers to provide consistency in modeling and stress comparisons. Fillet sizes
from 2 to 10 in. (51 to 254 mm) were analyzed for a few of the tees but were found to have little impact on the magnitude of the maximum torsion stresses.

The portion of deck outside of the stems behaves much like that of long thin rectangles subjected to torsion. Concentric stress contours are parallel to the deck surfaces with the highest stresses occurring on the exterior surfaces. The stress contours diminish to zero at the interior of the deck. The stems also reveal concentric stress contours that are parallel to the vertical axis rather than parallel to the tapered sides of the tee stems. The overlapping stem and deck regions show a smooth transition of horizontal deck contours and vertical stem contours.

With the exception of the overlapping stem and deck regions, the plotted stress functions show, as expected, that the deck and stems tend to behave as independent torsion resisting components. However, the classical approach of summing the stiffness of component rectangles to calculate the torsion constant of a built-up section was found to consistently underestimate the theoretical stiffness.

This behavior is caused by the increased resistance found in the overlapping stem and deck regions (see Fig. 8). A good correlation with the theoretical stiffness was found by simply increasing the depth of the stems by two times the deck thickness.

It was noted that torsion stiffness increases at a much greater rate than the cross-sectional area. For instance, the torsion stiffness for a 12DT30+4 is nearly twice that of a 10DT22+4. It is also interesting to note that increasing the deck widths from 8 to 10 ft (2.44 to 3.05 m), while holding the stem and deck thickness constant had absolutely no effect on the maximum shear stresses.

The relationship between the torsion constant \( K \) and the torsion stress can be used as a guideline for designers to assess any particular double tee cross section for its potential for cracking. By using the least squares fit theory, the data from Tables 5 and 6 were reduced to the simple curves shown in
Figs. 10 and 11. The first figure uses data for a warp of $1\frac{1}{2}$ in. (38 mm) across the full width of the double tee section, while the second curve is for a 2 in. (51 mm) warp. Note the following assumptions:

1. Elastic analysis assumes 3 in. (76 mm) deck/stem fillets — deck stem chamfers tend to increase stress.

2. Plots are for a unit twist angle of 0.000018 and 0.000023 radians per in. This is equivalent to about $1\frac{1}{2}$ and 2 in. (38 and 51 mm) of warp for a 10 ft (3.05 m) width over a 60 ft (18.30 m) span for Figs. 10 and 11, respectively.

3. Plotted shear stresses are maximum resultant $\tau_{wp}$ and occur on or near the deck/stem chamfers.

4. Linear interpolation is used to calculate $\tau_{max}$ for other twist angles.

The three horizontal lines on Figs. 10 and 11 represent stresses equal to 4, 5, or $6\sqrt{f_c}$. These allow the designer to exercise judgment as to how much stress is acceptable for the particular design conditions.

Stresses are sensitive to the degree of warp. When the warp is $1\frac{1}{2}$ in. (38 mm), all but one of the data points fall under the $4\sqrt{f_c}$ line, but when the tee is warped to 2 in. (51 mm), the stresses shift to the $5\sqrt{f_c}$ state of stress.

**THEORETICAL CALCULATION OF CRACKING POINT**

An in-depth research project on the durability of precast, prestressed parking structures was carried out by PCI in 1995 by the Consulting Engineers Group of San Antonio, Texas. Part of the focus of this research was the determination of warping without cracking of pretopped double tees with 4 in. (102 mm) thick decks (flanges).

To determine the theoretical limits of warping for these members, a finite element analysis was performed using a 1 in. (25.4 mm) grid mesh modeling a system with three supported elements (stems) and the fourth supported on a spring with a constant $K$. The model assumed 5 ft (1.52 m) between the stems [10 ft (3.05 m) overall length] and analyzed tee lengths of 40 and 60 ft (12.2 and 18.3 m).

The model varied the degree of warp to determine the maximum displacement between the stems at the maximum tension stress as defined by the modulus of rupture for a specific concrete. Note that ACI 318	extsuperscript{10} recommends using a value of $f_c$ in the range of $0.1f_c$ to $0.15f_c$.

The maximum tensile stress occurs in the top of the flange near the unwarped (non-spring-supported) stem at the end that is warped. The stress distribution across the flange is similar to a classical bending stress distribution. However, the stress distribution at the top surface is similar to that caused by a concentrated load. Once the maximum tensile stress is reached, the crack length is approximately 3 in. (76.2 mm).

For a high quality concrete with service compressive strength of 6500 psi (44.8 MPa) and corresponding modulus of rupture of 975 psi (6.72 MPa) (equal to $0.15f_c$), the permissible warp (measured between stems) would be on the magnitude of 0.45 in. (11.4 mm) for a 40 ft (12.2 m) tee length and 0.50 in. (12.7 mm) for a 60 ft (18.3 m) length member. Fig. 12 shows the maximum stresses in the deck with respect to stem-to-stem warping. The program write-up cautions that the values chosen for $f_c$ are a function of the acceptability of non-critical cracking.

The finite analysis model was also used to predict crack propagation. The analysis revealed that the crack propagates along the flange rather than through the flange. This is an important point when considering the crack containment afforded by welded wire fabric reinforcement typical in this type of member. For more information regarding this study, see Reference 11.

**EMPIRICAL DATA**

The program referred to in the previous section also included full-scale testing of two double tee members, 10 ft (3.05 m) wide by 28 in. (711 mm) deep with 4 in. (102 mm) thick flanges (10DT24+4). The last 2 ft (0.61 m) of length at each end of the members was held down to a 2 in. (51 mm) thickness as though a pour strip was to be added in the field.

One double tee was 47 ft (14.3 m) long and the other was 60.5 ft (18.4 m) long. Warping was induced in the tee by setting the tee stems on level shim stacks and then removing shims incrementally from beneath one stem. Based on the test results, the following observations can be made:

- The 60.5 ft (18.4 m) long member withstood a warp of $3\frac{1}{4}$ in. (19 mm) between the stems before cracking, while the 47 ft (14.3 m) long member had a very small crack start at $3\frac{1}{8}$ in. (16 mm) differential between its stems. The shorter member was difficult to test because it tended to rise up on opposite corners as shims were removed.

- Cracking in the top flange progressed diagonally across the flange and the bottom cracking remained in the flange to stem intersection. Fig. 13 shows the crack pattern developed in the 60.5 (18.4 m) long member.

- The maximum crack width for $1\frac{7}{8}$ in. (43 mm) differential between the
stems was 0.010 in. (0.25 mm) in the longer member. The crack length extended approximately 6 ft (1.83 m) from the end of the member.

- As the warping increased, cracking occurred at the juncture of the stem and the flange bottom for the leg that was lowered. Cracking occurred in the top of the flange above the adjacent stem on the same end. No cracking occurred at the end with both stems at the same elevation.
- Maximum crack widths increased as warping increased.
- Crack widths were less at the bottom of the flange than at the top.

The load tests substantiated the results predicted by the finite element study. Both studies confirmed that observable cracking occurred when the overall warp exceeded 1 1/2 in. (38 mm).

**SUMMARY OF RESULTS**

Results are summarized for the industry survey and theoretical analysis.

**Industry Survey**

Industry practice and associated experience gained therefrom indicate that warping of twin stemmed tee members in parking structures is not only ongoing, but also commonplace in order to create transverse slopes. When double tees are to be warped, all parties should be well informed regarding the potential for cracks in the members.

The study indicates that a 60 ft (18.3 m) long double tee element can be twisted to the amounts shown in Tables 1 and 2 without cracking. Warping to produce floor slopes of about 1 1/4 percent is routinely done within the industry with no cracking, and as much as 2 1/2 percent with minor cracking.

As previously stated, not all flange cracking is attributable to the in-place condition of the tee. Stripping, handling, and, probably most significantly, hauling the product to the jobsite induce torsion stresses in the element. Because prestressed concrete producers have been facing the challenge of manufacturing plant-cast 4 in. (102 mm) thick flange double tees for parking deck applications for over twenty years, special procedures for handling these members have been developed.

One of the methods successfully used by several precast concrete manufacturers involves using a specially designed rocker beam, which was discussed earlier in the paper (see Fig. 14). Other precautions include providing adequate jobsite access for delivery trucks because the jobsite typically has the worst terrain encountered on route.

**Theoretical Analysis**

The following is a list and discussion of the conclusions that can be drawn from the theoretical analysis.

- Narrower tees have a lower torsion stiffness constant ($K$) than wider tees.

Results from both the finite difference method and simplified method confirm that as the width of the deck decreases, the torsion constant $K$ also decreases.

- Double tees with 2 in. (51 mm) decks are more flexible than those with 4 in. (102 mm) decks.

By reviewing Tables 5 and 6, it is seen that double tees with 2 in. (51 mm) decks have a much lower $K$ value than double tees with 4 in. (102 mm) decks. By looking at the stiffness of a 10DT30+2 and 10DT30+4 it is seen that the contribution of the stems $K_{stem}$ remains constant and the contribution of the deck $K_{deck}$ changes.

- In general, the stems contribute more to the total stiffness than the deck.

As the ratio of the area of the stems to the total area of the double tee increases so does the ratio of the stiffness of the stems to the total stiffness.

- At the junction of the tee stems and the deck, chamfers cause higher stress concentrations in the double tees than do fillets.

The relationship between the torsion stiffness, $K$, and the maximum stresses is not evident from the analysis. However, it is clear that the use of chamfers rather than fillets affects the maximum torsion stress much more than it affects the torsion constant ($K$).

- Results confirm that the simplified method for calculating the torsion constant $K$ is quite accurate when compared to the classical solution.

The torsion constant calculated by the simplified method differs from the more accurate method by only 2 1/2 percent on average for all cross sections.
why proper drainage is imperative in the design of the deck.

3. The information contained herein is informational and not a directive for warping double tee elements. Individual owners and producers need to establish what they are willing to accept regarding the amount double tees should be warped. Also, cracks in double tee elements can originate from other sources such as shipping and handling.

Needs for Future Study

This article presents a comprehensive analysis of double tee sections subjected to torsion only. It excludes other stress combinations. Prestress forces and flexure cause tension and compression stresses, while shear stresses are caused by flexure and torsion. The complex interaction of gravity and axial loads are not addressed here. Further analysis taking these factors into account will help verify the results derived from the torsion only model or may give new information regarding the initiation of cracking at the interface of the stem and the flange as influenced by member geometry and applied loads.

Controlled member testing could be done to provide actual data that could be used to expand the knowledge of double tee warping. A prescribed method of testing and results documentation could be prepared to assist those willing to do the tests and thus create a database for empirically derived warping limits. As of this writing, one producer is performing tests to determine the relation between member width and the initiation of torsion cracking, as well as the implications of a “starter” crack to the development of torsion cracks. This study will be made available to PCI upon completion.

Closer scrutiny of product through its various cycles (i.e., production, cutting of strand, stripping, storage, shipping, and erection) will expand the experience database and provide better correlation of theoretical and observed performance of warped double tees. A number of producers are actively collecting this information. A central database would be helpful in assessing

CONCLUSIONS

Based on the results of this study, the following conclusions can be drawn:

1. Double tees have an inherent ability to sustain a certain amount of twist, thereby providing the designer two-way slope capability utilizing either field-topped or pretopped elements. While surface cracking can be controlled, it should be noted that most surface cracks require no further remedial action than to be covered with a sealer to prevent water penetration. The double tee cast in factory controlled conditions with high performance concrete is one of the best tools in the designer's arsenal for high quality, cost effective parking deck construction. Key aspects of high performance concrete are low water-cementitious material ratio (< 0.40), high strength [> 6000 psi (41.4 MPa)], and sufficient air entrainment (6/2 percent + 1/2 percent).

2. It should not be assumed that the presence of a crack in the top surface of a double tee flange is cause for structural or long-term durability concern. Reports issued by ACI, PCI, and others within the industry indicate that cracking on the magnitude of 0.015 in. (0.38 mm) wide or less is generally insignificant. The depth of the crack, when compared to the depth of reinforcement (as opposed to cracking in line with the reinforcement), is not typically cause for concern. The nature of the cracking described earlier in this paper falls into that category. The significance of a crack in the surface becomes more pronounced when that crack occurs in an area where there is ponded water, perhaps contaminated with chloride ions that lead to corrosion of reinforcement. This is

3. The information contained herein is informational and not a directive for warping double tee elements. Individual owners and producers need to establish what they are willing to accept regarding the amount double tees should be warped. Also, cracks in double tee elements can originate from other sources such as shipping and handling.

Needs for Future Study

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where cracking is most likely to occur based on the highest number of observations available.

ACKNOWLEDGMENT

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The authors are also appreciative of the excellent technical contributions made by the PCI JOURNAL reviewers. They are Alex Aswad, Ken Baur, Suresh Gami, Walter Korkosz, George Nasser, and Kim Seeber.

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11. PCI Research and Development Committee, Durability of Precast Prestressed Concrete Structures, Prepared by the Consulting Engineers Group, Inc. for the Precast/Prestressed Concrete Institute, Chicago, IL, 1995.
APPENDIX A — NOTATION

- \( b \) = long side of deck rectangle
- \( c \) = constant whose value depends on \( b/t \) ratio
- \( E \) = modulus of elasticity
- \( f'c \) = specified compressive strength of concrete
- \( f'c \) = modulus of rupture of concrete
- \( G \) = shear modulus of elasticity
- \( H \) = \(-2G\theta\)
- \( K \) = torsion constant for section
- \( K_{Deck} \) = torsion contribution to constant \( K \) for deck
- \( K_{Stem} \) = torsion contribution to constant \( K \) for stem
- \( K_{Simple} \) = torsion constant for complete double tee section
- \( M_t \) = calculated torsion moment
- \( t \) = short side of stem rectangle
- \( T_{xx} \) = tangent to membrane parallel to \( ZX \) plane
- \( T_{xy} \) = tangent to membrane parallel to \( ZY \) plane
- \( \Delta \) = vertical warp
- \( \phi \) = function of orthogonal coordinates \( X \) and \( Y \)
- \( \mu \) = Poisson's ratio
- \( \theta \) = twist angle per unit length
- \( \tau_{xx} \) = torsional stress in \( XZ \) plane
- \( \tau_{yz} \) = torsional stress in \( YZ \) plane
- \( \tau_{max} \) = maximum torsional stress

APPENDIX B — EXAMPLE OF WARPING FOR DRAINAGE

Fig. B1 shows a typical framing plan for a two-bay, single helix parking structure that has been designed in general conformance with the floor slope recommendations of Section 1. The west bay includes a 5.83 percent ramp, while the east bay is flat (not ramped). Typical east bay double tees are longer than typical west bay tees [64 ft 6 in. and 59 ft 6 in. (19.66 and 18.14 m), respectively] to accommodate a 4 ft (1.2 m) wide walkway needed to satisfy the client’s requirement for Americans with Disabilities Act (ADA) spaces. The shorter tees are 10DT22+2 members while the longer tees are 10DT26+2 and 10DT26+4.

The tees from Gridlines 3 to 8 are pretopped with a 4 in. (102 mm) thick deck. The short tees by stair cores, by the southeast notch, and the tees from Gridlines 2 to 3 and 9 to 10 are field-topped with a constant 3 in. (76 mm) thickness of cast-in-place concrete placed over the precast members.

The rationale for field-topping these tees were to eliminate differential cambers between adjacent short/long tees and the need to slope these tees greater than recommended values for pretopped members.

In the flat (east) bay, drains are located about every 60 ft (18.29 m) along Grid B at the wash line. Tees are set at a constant elevation around the perimeter of the structure (top-of-tee at elevation zero). Typically then stem bearings at the west end of tees are lowered and tees warped in a 30 ft (9.14 m) pattern that allows a ridge/valley arrangement to establish a suitable drainage scheme. Basically, tees are lowered 16 in. (406 mm) at valley lines and 12 in. (305 mm) at ridge lines so that over each 30 ft (9.14 m) bay, a 4 in. (102 mm) rise or fall along Grid B is developed.

Floor slopes resulting from this geometry are 2.07 percent along valley lines and 1.55 percent along ridge lines. These slopes are adequate to drain the floor properly. Because some tees by the elevator/stair core are shorter than typical tees, slopes along the diagonals vary slightly, but are generally about 1.9 percent. Adjustments were necessary in the end bays (turning radius bays), to accommodate notches and not-typical drain locations. On the ramp bay, drains are located in only two places as shown because water will run down the ramp along washes.

To accomplish the drainage pattern shown, tees in the flat bay were warped as shown in Fig. B1. Table B1 shows the anticipated stress created in each section given the specified amount of warp. This example was chosen because it uses double tee sizes that were not included in the analysis. The example shows how the information presented in this paper

<table>
<thead>
<tr>
<th>Double tee section</th>
<th>Location (Grids)</th>
<th>Length (ft)</th>
<th>Warp (in.)</th>
<th>Angle of twist (( \theta ))</th>
<th>( K_{Simple} ) (in. (^4) per radian)</th>
<th>( \tau ) (psi)</th>
<th>Tensile stress range</th>
</tr>
</thead>
<tbody>
<tr>
<td>10DT22+2</td>
<td>1-2 &amp; B-C</td>
<td>45.25</td>
<td>1.90</td>
<td>0.000029</td>
<td>2094</td>
<td>344</td>
<td>310 465</td>
</tr>
<tr>
<td>10DT26+2</td>
<td>2-3 &amp; B-C</td>
<td>64.33</td>
<td>2.90</td>
<td>0.000031</td>
<td>5003</td>
<td>404</td>
<td>310 465</td>
</tr>
<tr>
<td>10DT26+4</td>
<td>3-9 &amp; B-C</td>
<td>64.33</td>
<td>1.33</td>
<td>0.000014</td>
<td>7837</td>
<td>230</td>
<td>310 465</td>
</tr>
<tr>
<td>10DT26+2</td>
<td>9-10 &amp; B-C</td>
<td>64.33</td>
<td>3.00</td>
<td>0.000032</td>
<td>5003</td>
<td>421</td>
<td>310 465</td>
</tr>
</tbody>
</table>

Note: 1 in. = 25.4 mm; 1 psi = 0.006895 MPa.
NOTES:

1. THIS STRUCTURE IS A SINGLE-SHELF DESIGN. THE WEST BAY IS FLAT EXCEPT FOR DRAINAGE SLOPES. THE EAST BAY PLATES ARE INCLINED AT THE SOUTH END OF THE BAY. THE ONLY EGRESS LANE ENDS AT THE NORTH END.

2. DOWNTIME IN THE PLATE ARE ABOUT 4'-1'-1" LONG AND 10'-3'-0" HIGH. THEY ARE PREFABRICATED WITH A CONCRETE BASE AND COMPOSED OF THREE TELLS EACH, CONNECTED TO THE CEILING JOINERS WITH A CONSTANT 3'-THICKNESS OF COMPOSITE TELLS EXCEPT AT THE LIMITS WHERE A 3'-FOOT INTERVAL TELLS A TOWER TELLS 1'-0" TO 1'-5".

3. DOORS ARE PLACED ABOUT 30'-0" APART AND VALLEY LINES ARE LOCATED ABOUT 30'-0" APART. TELLS DROP A BRACE FROM THE EXTERIOR BEARING TO THE INTERIOR BEARING AT EACH END OF THE BUILDING. THE INTERNAL BEARING TELLS ARE 5'-0" APART, DOWN THE RACK 5' BEHIND EACH TELLS TO 5'-0" BEHIND THE ASSOCIATED LINES.

4. BETWEEN GRID LINES (2'-0" DIFFERENTIAL: 3'-0" - 1'-0"

FIGURE B1. TYPICAL FRAMING PLAN

Fig. B1. Typical framing plan.
can be used to help predict warping behavior of various sizes of double tees.

The following is an explanation of how each item in Table B1 was obtained using the 10DT26+2 at Grids 2-3 and B-C as an example.

Double tee section, location, length and warp are obtained from Fig. B1.

Angle of Twist ($\theta$):

$$\theta = \frac{\text{Warp}}{\text{Width} \times \text{Length}} = \frac{2.90 \text{ in.}}{(10 \text{ ft} \times 12 \text{ in.} / \text{ft}) \times (64.33 \text{ ft} \times 12 \text{ in.} / \text{ft})} = 0.000031 \text{ radians per in.}$$

Torsion Constants ($K$):

$K_{\text{Simple}}$ is calculated using the method described in the "Rational Method to Determine Torsion Stiffness" portion of this article. See Fig. 4, Table 4 and Eqs. (6), (7) and (8).

$$A = 577 \text{ sq in.} (645.2 \text{ mm}^2)$$
$$b = 10 \text{ ft} (3.05 \text{ m})$$
$$t = 2 \text{ in.} (51 \text{ mm})$$
$$h = 26 \text{ in.} (660.4 \text{ mm})$$
$$t_{x,\text{avg}} = 6.48 \text{ in.} (164.6 \text{ mm})$$
$$bht = 60$$
$$(h + 2t)/t_{x,\text{avg}} = 4.63$$
$$c_1 = 0.324 \text{ (Table 4)}$$
$$c_2 = 0.287 \text{ (Table 4)}$$
$$K_{\text{Deck}} = 311 \text{ in.}^4 / \text{rad} \text{ (Eq. (6))}$$
$$K_{\text{Stem}} = 2346 \text{ in.}^4 / \text{rad} \text{ (Eq. (7))}$$

$$K_{\text{Total}} = 5003 \text{ in.}^4 / \text{ rad} \text{ (Eq. (8))}$$

Calculated Shear Stress ($\tau$):

From Fig. 11, read off the stress for a 2 in. (51 mm) warp for the given torsion constant ($K_{\text{Simple}} = 5003 \text{ in.}^4 / \text{rad}$) and deck thickness ($t = 2$ in.):

$$\tau_{2^\prime \text{,warp}} = 300 \text{ psi} (2.07 \text{ MPa})$$

Multiply the stress obtained from Fig. 11 by the angle of twist ($\theta = 0.000031$) and divide it by the angle of twist associated with a 2 in. (51 mm) warp ($\theta_{2^\prime \text{,warp}} = 0.000023$).

$$\frac{\tau_{2^\prime \text{,warp}} \times \theta}{\theta_{2^\prime \text{,warp}}} = \frac{300 \text{ psi} \times 0.000031}{0.000023} = 404 \text{ psi} (2.79 \text{ MPa})$$

All the stresses in this example fall between $4\sqrt{f_c}$ and $6\sqrt{f_c}$. This does not mean that the tees will not crack, rather it simply gives a range of stresses for comparison. When using the information presented in this article, it is left to the designer's judgment as to whether the calculated stresses are acceptable for each individual project.

This example shows that setting double tee elevations requires a careful study regarding drainage. This is particularly true when the designer-of-record desires to minimize the number of floor drains by placing them far apart. Particular attention must be given to the camber of the double tees. Designers should not depend on camber of the pre-stressed members to provide drainage slopes.