This paper addresses the justification for removal of two compressive stress limits, called hereafter Limits 1 and 2, which are currently used in both the ACI 318 Building Code and AASHTO Bridge Specifications to limit concrete stresses due to effective prestress combined with applied loads. Removal of these limits provides relief in sizing of members that have small compression zones that are subjected to large bending moments, such as inverted tee bridge members in the positive moment zones and I-girder bridges made continuous for superimposed loads in the negative moment zone. Limit 1 is $0.6f'_c$, where $f'_c$ is the specified compressive strength at service. It is imposed on concrete stresses due to the effective prestress combined with full dead plus live loads. Limit 2 is $0.45f'_c$. It is imposed on stresses due to the effective prestress combined with dead loads. It is proposed that strength be used as the primary design criterion to determine member capacity in compression at various loading stages, including prestress transfer, lifting, erection, deck weight, superimposed dead load and live load. Various serviceability checks can additionally be used as needed to control camber, deflection, vibration, and cracking. Design steps and numerical examples are given. Also, proposed changes to the ACI 318 Building Code and to the AASHTO-LRFD Bridge Design Specifications are given.
The restrictions on concrete compressive stress at service limit state in the design of prestressed concrete flexural members in the American Concrete Institute Building Code (ACI 318-02), and American Association of State Highway and Transportation Officials (AASHTO) Load and Resistance Factor Design (LRFD) Bridge Design Specifications appear to be based on historical justification. They neither directly address serviceability conditions, such as camber and deflection limits, nor ensure adequate strength under factored loads.

When an inverted tee bridge superstructure is designed and the working stress design compressive stress limits are checked, its capacity is often controlled by the limit imposed by the effective prestress plus dead load, as calculated by the standard elasticity theory without regard to stress-strain nonlinearity or creep redistribution of stresses between the precast beam and the cast-in-place topping.

The same problem sometimes arises in bridge I-girders in integral bridge systems, i.e., when the girders are made continuous with cast-in-place concrete diaphragms and continuity negative moment reinforcement in the deck. Bottom fiber stresses can exceed the allowable limits, even though the strength of the negative moment section meets all code requirements. This situation forces designers to either ignore the code requirements or unnecessarily select a larger member size.

Removal of the unnecessary compression limits could result in creation of a series of more efficient precast, prestressed concrete products. This, coupled with increasing acceptance of self-compacting concrete, would make producers and their engineers develop, for example, efficient inverted double tee members that would be connected in the field with a thin topping, creating a very efficient building floor with a shallow depth and a smooth soffit. Many existing products could be optimized, reducing their weights and increasing their capacities.

As prestressed concrete was being introduced in North America, about 50 years ago, it was believed to be important to double-check its adequacy with both “working stress design,” which was the prevalent design method, and “strength design,” which was getting introduced at that time. Conventionally reinforced concrete, on the other hand, was allowed to be designed starting with the 1963 edition of the ACI 318 Code by either the working stress design method or the strength design method, but not both methods. The overly conservative compressive stress limits in current working stress design practice of prestressed concrete control the design in a number of applications and thus limit the utilization of the full potential of prestressed concrete.

In an earlier paper, a strength-based design method for prestressed concrete flexural members at transfer of prestress was presented as a replacement of working stress design and the compressive stress limit of 0.60\(f'_c\), where \(f'_c\) is the specified compressive strength of concrete at initial prestress transfer.

This paper presents a historical background of compressive stress limits at service load conditions, and provides a justification for total removal of these limits. The two limits being imposed on concrete in compression are Limit 1, 0.60\(f'_c\), on concrete stress due to effective prestress, after allowance for long-term losses, combined with full loads, and Limit 2, 0.45\(f'_c\), on stress due to effective prestress plus dead load. Limits 1 and 2 in the AASHTO-LRFD Specifications are identical to those in the ACI 318 Building Code.

A third limit, unique to bridge design, is 0.40\(f'_c\). It is required to be applied in AASHTO-LRFD to concrete stresses due to 50 percent of effective prestress plus 50 percent of dead load plus 100 percent of live load. It is intended to control fatigue due to live load, which is a repeated cyclic load caused by moving vehicles during the life of a bridge. However, the provisions for that limit should be moved from their current location to the section that deals with design for fatigue effects. In that section, the fatigue truck loading model, rather than the standard truck loading model used for strength calculations, should be used to determine the live load stresses. With these two suggested modifications, this limit should not be removed, and thus the topic is not discussed further in this paper.

The AASHTO Standard Specifications for Highway Bridges (AASHTO-STD) are being phased out. They have similar limits to those in AASHTO-LRFD, except that Limit 2 in AASHTO-STD is 0.40\(f'_c\) rather than 0.45\(f'_c\). Both values were introduced in the 1994 editions of the two sets of AASHTO Specifications. The difference is believed to have occurred by an arbitrary decision by the com-
mittee in charge of maintaining these specifications. For clarity, only the 0.45$f'_c$ value is used in further discussion in this paper.

The three limits of allowable compressive stresses in AASHTO-LRFD can be written as follows:

Limit 1: $f_d + f_l \leq 0.6f'_c$ (1)

Limit 2: $f_d \leq 0.45f'_c$ (2)

Limit 3: $f_d + 2f_l \leq 0.8f'_c$ (3)

where $f_d =$ compressive stresses due to the sum of effective prestress and permanent loads after allowance for time-dependent losses

$f_l =$ compressive stresses due to live load

Fig. 1 shows the domain of limits on $f_d$ and $f_l$ to satisfy each of Limits 1, 2 and 3. For example, when the live load stress, $f_l$, is equal to 0.2$f'_c$ or higher, the maximum dead load plus effective prestress, $f_d$, is controlled by Limit 3 which controls concrete fatigue. When $f_l$ is less than 0.15, Limit 2 controls the maximum allowed $f_d$. Thus, Limit 1 has practically no impact on design by the current LRFD provisions, as seen in Fig. 1.

It can be shown, using a similar graph as in Fig. 1, that Limit 1 never controls design according to the AASHTO-STD Specifications. Therefore, removal of Limit 1 saves an unnecessary design check since either Limit 2 or Limit 3 supersedes it. The proposal in this paper, however, is to remove both Limits 1 and 2.

In the following sections, a historical overview is given. It is followed by a discussion justification for removal of Limits 1 and 2. An alternate strength-based design procedure is presented and illustrated with numerical examples. Changes to the affected sections in the ACI 318 Code and in the AASHTO-LRFD Specifications are proposed.

**HISTORICAL BACKGROUND**

The Prestressed Concrete Institute (PCI) issued the first specifications for bonded pretensioned prestressed concrete on October 7, 1954. In those specifications, the maximum allowable compressive stresses under final dead and live loads was 0.40$f'_c$ for bridge members and 0.44$f'_c$ for building members.

In 1954, the Bureau of Public Roads (BPR) published the first edition of *Criteria for Prestressed Concrete Bridges*, by E. L. Erickson, who at the time was chief of the Bridge Division, Bureau of Public Roads (BPR). That bureau was later renamed the Federal Highway Administration (FHWA).

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At that time, there was a considerable difference of opinion among the various authorities as to how much compression to allow in the extreme concrete fiber under dead, live, and impact load. Several authorities (for example, Hajnal-Konyi, Dobell, Leonhardt, Muller, Billig) at that time had proposed different limits ranging from 0.33$f'_c$ to 0.5$f'_c$ as reported in Reference 6. According to Article 3.4.11 of the 1953 “AASHTO Standard Specifications for Highway Bridges,” an allowable compressive stress limit of 0.4$f'_c$ was given for conventional concrete subjected to dead, live, or impact loads. The Bureau of Public Roads conservatively adopted the same stress limit for prestressed concrete structures.

The allowable compression stress limit of 0.45$f'_c$ appeared in the first published draft of the ACI-ASCE Joint Committee 323 Report “Recommended Practice for Prestressed Concrete” in 1958. The early code writers felt that this limit was conservatively established to decrease the probability of failure of prestressed concrete members due to repeated loads and to preclude excessive creep deformations. In the 1958 report, “Tentative Recommendations for Prestressed Concrete” and 1959 report “PCI Standard Building Code for Prestressed Concrete (Tentative),” the 0.45$f'_c$ limit was retained, but there was no clear commentary as to why this was done.

Although the 0.45$f'_c$ limit was totally removed for non-prestressed concrete at the time of the introduction of strength design, this limit was kept unchanged until 1995 for prestressed concrete. The ACI 318-95, the 1994 AASHTO-LRFD and the 1996 AASHTO-STD Specifications adopted...
changes based on the work by Huo et al. 8 That reference gave justification for a 33 percent increase in the limit to 0.60$f_c'$, and for the introduction of Limit 2, due to the effective prestress plus dead load. It was believed, by some code committee members, that the 0.45$f_c'$ limit should not be exceeded to avoid dealing with the non-linear creep of concrete at higher stress levels.

Since live load is considered transient, it was indicated that a 33 percent additional stress would not be detrimental. Huo et al. 8 indicated that the changes were intended to be transitional, with the eventual goal of total elimination of compressive stress limits. It was further demonstrated that, if working stress had to be performed on a conventionally reinforced rectangular section example that was already designed with strength design, the extreme fiber compressive stress could be as high 0.83$f_c'$.

### FLEXURAL BEHAVIOR OF PRESTRESSED MEMBERS

Two sources of nonlinearity exist in the flexural behavior of prestressed concrete, namely, nonlinearity due to increasing load and nonlinearity due to time-dependent volume change effects. The first part of this section deals with the nonlinearity of the concrete stress-strain relationship and its effect on member condition at time of prestress transfer. It will be shown that the linear elastic calculation of compressive stress is not compatible with the 0.6$f_c'$ limit which falls outside of the linear range of the stress-strain diagram, nor is it representative of a constant safety index against concrete crushing.

The second part of this section demonstrates that continuous stress redistribution occurs due to creep and shrinkage of concrete in a composite member, consisting of two concrete components. When that effect is taken into account, the top fiber stresses of the precast concrete component of the member undergo considerable relief. Therefore, a linear elastic analysis for compressive stress, and limiting it to 0.6$f_c'$, does not correspond to the actual member behavior. On the other hand, methods are available, which will be summarized below, that allow designers to directly check for member strength against concrete crushing, or for member serviceability against excessive deflections.

#### Analysis at Transfer due to Increasing Prestress Level

Huo et al. 9 considered an example of an axially prestressed concrete member, summarized below. The cross section is shown in Fig. 2. Concrete and steel stresses and strains due to a progressively increased number of prestressing strands up to the failure load, are shown in Fig. 3. The strands are assumed to be arranged to produce a concentric load as their number increases.

Two methods of analysis, namely, standard linear analysis and nonlinear analysis, are considered. As shown in Fig. 3, both standard linear analysis and the more accurate nonlinear analysis produce about the same results, a maximum of 26 strands, when the concrete stress is at the 0.6$f_c'$ code limit. The strains are not as close as the stresses between the two models.

Linear analysis appears to indicate that concrete reaches the strength $f_c'$, when 44 strands are used. However, nonlinear analysis more correctly indicates that concrete continues to resist placement of more prestressing up to 62 strands when the strain reaches the assumed ultimate strain of 0.003.

This figure confirms three facts: 1. Concrete crushes when its strain reaches ultimate strain, not when the stress reaches its peak elastic analysis value.

2. As prestressing increases, so does concrete strain and prestress losses. Thus, unlike externally induced compression, prestressing has a self-relieving mechanism.

3. Member failure cannot be accurately predicted without utilization of the entire stress-strain diagram through nonlinear analysis and strength based design.

#### Analysis of Composite Members After Prestress Transfer

In standard practice in current use, linear elastic behavior is employed to check tension and compression limits in concrete and to calculate camber and deflection. Concrete, as well as steel, is assumed to follow Hooke’s Law, i.e., stress equals the product of

<table>
<thead>
<tr>
<th>Table 1. Inverted tee section properties.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Section</td>
</tr>
<tr>
<td>Beam</td>
</tr>
<tr>
<td>Deck</td>
</tr>
<tr>
<td>Composite</td>
</tr>
<tr>
<td>Age-adjusted composite</td>
</tr>
</tbody>
</table>

Note: 1 in. = 25.4 mm; 1 sq in. = 645 mm²; 1 in.⁴ = 16,387 mm⁴; 1 in.² = 416,231 mm⁴.
strain and modulus of elasticity. Pre-
stress losses due to creep and shrink-
age of concrete and relaxation of pre-
stressing steel are calculated sepa-
ately and assumed to be exter-
nally introduced as “negative pre-
stress” in the linear elastic analysis.
When these assumptions are made, true time-
dependent stress redistribu-
tion cannot be determined and design-
ers are left with, at best, nominal stress values that may be highly exagger-
ated. The following four numerical ex-
amples relate to the same bridge beam:
• The first example illustrates the con-
tentional elastic stress analysis for stresses at various stages of loading.
• The second example shows how time-
dependent analysis can be con-
ducted and how much the results dif-
fer from those of the elastic analysis.
• The third example deals with cam-
ber and deflection calculations.
• The fourth example shows how safety can be directly satisfied against failure by conducting a strength analysis of the member. Due to space limitations, only segments of the full example will be given. A more detailed treatment of the topic is given in Reference 10.

**Example 1: Elastic Analysis of an Inverted Tee Bridge Beam**

A precast concrete bridge super-
structure consists of an inverted tee beam, with a span of 82 ft (24.99 m), and a spacing of 2 ft (0.61 m). The beam is made composite with a 6 in. (152 mm) thick cast-in-place slab. Fig. 4 shows the concrete dimensions and steel arrangement. Table 1 gives the section properties of the precast beam and composite section.

Concrete strength of the precast sec-
tion at transfer, \( f'_{ct} = 6 \text{ ksi (} 41.37 \text{ MPa) } \) and at service, \( f'_{ct} = 8 \text{ ksi (} 55.16 \text{ MPa) } \). The deck concrete strength is 6 ksi (41.37 MPa).

Moments at midspan: \( M_g = 2648 \text{ kip-in. (299 kN-m) } \), \( M_d = 1513 \text{ kip-in. (171 kN-m) } \), \( M_l = 5295 \text{ kip-in. (598 kN-m) } \) due to self-weight, deck weight, and live load, respectively.

The stress in the prestressing steel has been determined through time-de-
dependent analysis to be 184.21 ksi (1270 MPa), 173.49 ksi (1196 MPa) and 160.28 ksi (1105 MPa) immediately after transfer, at time of deck placement and at time infinity, respectively. The corresponding prestress force values are: 620.1, 584.0 and 539.5 kips (2758, 2598 and 2400 kN).

The precast concrete top fiber stress, \( f_t \), is calculated using the following elastic stress analysis formula:
where  

\[ f_i = \frac{P}{A} + \frac{P}{S_t} + \frac{M}{S_t} \quad (4) \]

is the stress at location \( i \). The results are shown in Fig. 5 and Table 2.

Both the ACI 318 Code and the AASHTO LRFD Specifications require that the stress due to permanent loads not exceed 0.45 \( f'_{c} \) = 3.6 ksi (24.82 MPa). The stress at erection of 3.817 ksi (26.32 MPa) exceeds that limit. Both codes require that the stress due to full load not exceed 0.60 \( f'_{c} \) = 4.8 ksi (33.09 MPa). The stress of 4.990 ksi (34.41 MPa) exceeds that limit. Thus, according to elastic analysis, the beam fails to meet the code requirements due to both loading combinations.

### Time-Dependent Stress Redistribution in Composite Member

The purpose of this section is to demonstrate two points:

1. If there is a need for serviceability checks, means already exist to calculate the design parameters and to check against required limits. This applies to deflection, camber, cracking, and other parameters.

2. Composite members consisting of precast, prestressed concrete and cast-in-place concrete components undergo continuous stress redistribution that results in significant reduction in the elastic compressive stress at the top fiber of the precast component. Such redistribution cannot be determined through elastic analysis (see, for example, Kamel\(^{11}\)). Thus, the standard practice of elastic analysis, which is based on limiting the top fiber compressive stress, is misleading.

The theory that Kamel used involved an elaborate analysis of time-dependent effects, based on earlier work by Tadros et al.\(^{12}\) The time history, in that analysis, is divided into many small steps. As the size of the time-step decreases, the accuracy of the analysis increases. However, that method is suited for commercial software solutions.

A slightly less accurate, but greatly simplified, method presented by Dilger\(^{13}\) is suitable for hand-calculation and designer-developed spreadsheet solutions. Dilger’s “age-adjusted effective modulus” method is the one adopted in this paper for further discussion.

The age-adjusted effective modulus method is similar to conventional elastic analysis. Instead of using the conventional modulus of elasticity to determine transformed section properties, the age-adjusted effective modulus is used. In addition, deforma-

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**Table 2. Summary of stresses at various loading stages.**

<table>
<thead>
<tr>
<th>Loads</th>
<th>Transfer</th>
<th>Erection</th>
<th>Final</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Top beam (ksi)</td>
<td>Bottom beam (ksi)</td>
<td>Top beam (ksi)</td>
</tr>
<tr>
<td>( P/A )</td>
<td>2.461</td>
<td>2.461</td>
<td>2.317</td>
</tr>
<tr>
<td>( P/e )</td>
<td>–4.133</td>
<td>2.122</td>
<td>–3.892</td>
</tr>
<tr>
<td>( M/e )</td>
<td>3.431</td>
<td>–1.762</td>
<td>3.431</td>
</tr>
<tr>
<td>( M/S )</td>
<td>–</td>
<td>–</td>
<td>1.961</td>
</tr>
<tr>
<td>Total</td>
<td>1.759</td>
<td>2.821</td>
<td>3.817</td>
</tr>
<tr>
<td>Limiting stresses</td>
<td>0.6( f'_{c} )</td>
<td>0.6( f'_{c} )</td>
<td>0.45( f'_{c} )</td>
</tr>
<tr>
<td>Total</td>
<td>3.600</td>
<td>3.600</td>
<td>3.600</td>
</tr>
</tbody>
</table>

The values in the “OK” column indicate that the stresses are within the code requirements. The results are presented in the table as follows:

**Linear elastic analysis**

- **At transfer:**
  - Top beam: 3.817 ksi (26.32 MPa)
  - Bottom beam: 1.547 ksi (10.70 MPa)
- **At erection:**
  - Top beam: 3.817 ksi (26.32 MPa)
  - Bottom beam: 1.547 ksi (10.70 MPa)
- **At final:**
  - Top beam: 3.817 ksi (26.32 MPa)
  - Bottom beam: 1.547 ksi (10.70 MPa)

**Time-dependent analysis**

- **At erection:**
  - Top beam: 3.817 ksi (26.32 MPa)
  - Bottom beam: 1.547 ksi (10.70 MPa)

**Note:** 1 ksi = 6.895 MPa.
tion due to shrinkage and creep of concrete, and relaxation of prestressing steel, are considered during a time interval as “initial” strains, much like pseudo-elastic analysis for effects of temperature change.

To incorporate initial strains into the stress analysis, three steps are followed:

1. Separate various concrete and steel components of the cross section and allow them to deform freely.

2. Restrain the components of the section and calculate the restraining forces to bring the initial strains calculated in Step 1 to zero.

3. Reattach the various components to each other and restore equilibrium by applying equal and opposite forces to those calculated in Step 2.

The sum of stresses in Steps 2 and 3 is the net stress, and the deformation in Step 3 is the net deformation. In Steps 2 and 3, the modulus of elasticity of concrete is adjusted by dividing $E_c$ by $(1 + \chi\psi)$, where $\chi$ is the aging coefficient to allow for gradual introduction of time-dependent stress increments, and $\psi$ is the creep coefficient. To illustrate this procedure, the following example provides a detailed calculation of stress, strain and deformation due to time-dependent effects in a composite section.

Example 2: Time-Dependent Analysis of the Beam of Example 1

This example shows the difference in analysis with the elastic results when time-dependent stress redistribution is accounted for. The input data is the same as in Example 1. Additionally, the following time-dependent properties have been estimated using the formulas in Reference 14 assuming the member is subject to ambient relative humidity of 70 percent. Other sources of prediction of material properties are AASHTO LRFD Specifications, and the PCI Bridge Design Manual (PCI-BMD).15

The modulus of elasticity of the precast concrete at service, $E_b = 5422$ ksi (37.39 GPa). An average compressive strength of the deck of 6 ksi (41.37 MPa) is assumed for the period between the time it starts to act compositionally with the girder and time infinity.

The corresponding modulus of elasticity, $E_d = 4696$ ksi (32.38 GPa).

Creep and shrinkage of precast concrete between erection and time infinity, $\psi_{bdf} = 0.65$, and $\varepsilon_{bsh} = 150 \times 10^{-6}$.

Creep and shrinkage of deck concrete $\psi_{ddf} = 1.3$, and $\varepsilon_{dsh} = 300 \times 10^{-6}$.

The aging coefficient is taken constant for both materials, $\chi_b = \chi_d = 0.7$. For clarity of presentation, the age-adjusted transformed section properties are calculated assuming an area of steel equal to zero.

Note that a more detailed set of calculations of the same example, including the interaction of the prestressing steel may be found in Reference 10. Using the actual area of steel in calculation would automatically account for time-dependent prestress losses. In this example, the prestress losses estimated for Example 1 will be assumed.

Because of the simplified assumptions used in this example, the concrete stress increments due to various actions are identical to those in Example 1, with one exception. Once the deck becomes composite with the girder, differential creep and shrinkage between the two concretes cause continuous redistribution in the concrete stresses. The calculations will, thus, focus on the time interval between erection and time infinity.

The analysis steps explained earlier will be followed.

Step 1

Separate the two section components and allow each to deform freely.

Axial strain in precast beam:

$$\frac{P}{AE} \psi_{bdf} + \varepsilon_{bsh} = \frac{583.97}{252(5422)} (0.65) + 0.00015 = 4.278 \times 10^{-4}$$

Curvature in precast beam:

$$\frac{M}{EI} \psi_{bdf} = \frac{[2648 + 1513 – 583.97(5.14)] (0.65)}{5422(12, 235)} = 1.134 \times 10^{-5} \text{in}^{-1} (4.464 \times 10^{-7} \text{mm}^{-1})$$

Axial strain in deck = 0.0003

Step 2

Calculate the forces and the corresponding stresses in each component that cancel the deformations in Step 1. For this analysis, the age-adjusted effective modulus for each material is used.

Age-adjusted effective modulus for precast beam:

$$E_b^* = \frac{E_b}{1 + \chi_b \psi_{bdf}} = \frac{5422}{1 + (0.7)(0.65)} = 3727 \text{ksi (25.70 GPa)}$$

Age-adjusted effective modulus for deck:

$$E_d^* = \frac{E_d}{1 + \chi_d \psi_{ddf}} = \frac{4696}{1 + (0.7)(1.3)} = 2459 \text{ksi (16.95 GPa)}$$

Axial restraining force in precast beam:

$$- (\text{free axial strain})E_b^*A_b = -4.278(3727)(252)(10^{-4}) = -401.75 \text{ kips} (-1787.08 \text{ kN})$$

Corresponding stress in precast beam:

$$-401.75/252 = -1.594 \text{ ksi} (-10.99 \text{ MPa})$$

The bending restraint force in the beam and the axial restraining force in the deck can be calculated similarly to be –516.97 kips-in. (–2300 kN) and –106.21 kips-in. (–472.46 kN).

The corresponding stresses in the beam are:

Top fibers, –2.264 ksi (–15.61 MPa); Bottom fibers, –1.250 ksi (–8.62 MPa); and in the deck are: top fibers = bottom fibers = –0.738 ksi (–5.09 MPa).

Step 3

Now that the compatibility of the two components has been restored in Step 2, the two components can be
“reattached” and equilibrium restored by applying equal and opposite forces to the forces obtained in Step 2. To undertake the analysis in this step, the age-adjusted composite section properties must be calculated and used. These can be found using a deck-to-beam modular ratio $E_d/E_b = 2459/3727 = 0.66$. The section properties are shown in Table 1.

All the component forces in this step are then combined into an axial force at the centroid of the age-adjusted composite section.

Axial force:
$$= 401.75 + 106.21 = 507.96 \text{ kips (2259.53 kN)}$$

Bending moment:
$$= -102.58 \text{ kip-in. (}-11.59 \text{ kN-m})$$

The corresponding top fiber stress in the beam is:
$$f_t = \frac{507.96}{347} - \frac{102.58(24 - 13.31)}{37,053} = 1.434 \text{ ksi (9.89 MPa)}$$

The stresses at the extreme fibers of the deck and at the bottom fibers of the beam are calculated similarly.

**Step 4**

The summation of all stresses and deformations in Steps 1 through 3 produce the net incremental values due to creep and shrinkage between at time of erection and time infinity. The time-dependent top fiber stress in the beam, for example, equals $-2.264 + 1.434 = -0.830 \text{ ksi (}-5.72 \text{ MPa})$. Other values are given in Table 2.

The additional prestress losses in steel between erection and final time were assumed to be the same as in Example 1. Thus, a concrete stress change due to a steel stress loss of:
$$= 160.28 - 173.49 = -13.21 \text{ ksi (}-91.08 \text{ MPa})$$

needs to be included. As shown in Table 2, the stress change at the top fibers is $0.12 \text{ ksi (0.83 MPa)}$.

The net stresses, including time-dependent effects, are equal to the stresses at the erection plus the time-dependent stress increments. For the beam top fibers, that net stress is:
$$3.817 - 0.830 + 0.12 = 3.107 \text{ ksi (21.42 MPa)}$$

This stress is below the code limit of $0.45f'_c = 3.6 \text{ ksi (24.82 MPa)}$.

Adding live load stresses as shown in Example 1, the beam top fiber stress equals:
$$3.107 + 1.053 = 4.160 \text{ ksi (28.68 MPa)}.$$  

This again is less than the code limit of $0.6f'_c = 4.8 \text{ ksi (33.09 MPa)}$.

A comparison of the results of Examples 1 and 2 leads to the conclusion that a linear elastic analysis does not account for the generally significant stress relief of the compressive stress in the top fibers of the precast component of a composite section, and leads to the false impression that these fibers are overstressed.

It should be emphasized that the value of service load analysis is to check member serviceability, i.e., to check for excessive deformation and cracking, and not to check against concrete crushing which is the function of strength design.

**Example 3: Camber and Deflection Calculations of the Beam of Examples 1 and 2**

This example illustrates the calculation of the camber and deflection at the midspan of the beam of Examples 1 and 2. Numerical integration of the curvature may be used to obtain the deflection.

For this example, the deflection-curvature relationships $\delta = \phi \ell^2/8$ due to prestressing and $\delta = 5\phi \ell^2/48$ due to gravity load, given in Reference 16, are used. The symbols $\delta$, $\phi$, and $\ell$ represent midspan deflection, midspan curvature, and span length, respectively.

(1) **Instantaneous camber at initial prestress transfer**

Design for prestress transfer, using the strength design method proposed in Reference 3, indicates that no draping or debonding of the strands is needed if the concrete strength at prestress transfer, $f'_c$, is set at 5.74 ksi.
(39.58 MPa) and if two No. 4 Grade 60 bonded crack control mild steel reinforcing bars are placed at the top of the member in the end zone. Thus, it can be assumed for curvature and camber calculations that the prestress imposes a constant bending moment over the full span length.

(a) Due to prestress force

\[ \phi_c = \frac{P_e}{EI} = \frac{-620.05(5.14)}{4696 \times 12,235} = -5.550 \times 10^{-5} \text{ in.}^2 \text{ (} -2.185 \times 10^{-6} \text{ mm}^2) \]

\[ \delta = \frac{\phi_c \ell^2}{8} = \frac{-5.550 \times 10^{-4}(82 \times 12)^2}{8} = -6.72 \text{ in.} \text{ (} -171 \text{ mm) } \]

(b) Due to self-weight moment

\[ \phi_s = \frac{M_s}{EI} = \frac{2648}{4696 \times 12,235} = 4.608 \times 10^{-5} \text{ in.}^2 \text{ (} 1.814 \times 10^{-6} \text{ mm}^2) \]

\[ \delta = \frac{5\phi_s \ell^2}{48} = \frac{5(4.608 \times 10^{-5})(82 \times 12)^2}{48} = 4.65 \text{ in.} \text{ (118 mm) } \]

Net initial camber at transfer:

\[ = -6.72 + 4.66 \]

\[ = -2.07 \text{ in.} \text{ (} -53 \text{ mm) } \]

(2) Camber increment between transfer and erection due to initial loads

To account for concrete creep and shrinkage, the age-adjusted modulus of elasticity calculated in Example 3, is used in this step to determine the incremental curvature.

\[ \phi_c = \frac{M_p}{EI} = \frac{(2648 - 620.05 \times 5.14)(0.65)}{4696 \times 12,235} = -6.124 \times 10^{-6} \text{ in.}^2 \text{ (} -2.411 \times 10^{-7} \text{ mm}^2) \]

\[ \delta = \frac{\phi_c \ell^2}{8} = \frac{-6.124 \times 10^{-4}(82 \times 12)^2}{8} = -0.74 \text{ in.} \text{ (} -19 \text{ mm) } \]

(3) Deflection increment due to prestress loss between release and erection

\[ \Delta \psi = (173.49 - 184.21)3.366 = -36.08 \text{ kips} \text{ (} -160.51 \text{ kN) } \]

\[ \phi_e = \frac{\Delta \psi}{EI} = \frac{36.08(5.14)}{5422(12,235)} = 2.797 \times 10^{-6} \text{ in.}^2 \text{ (} 1.101 \times 10^{-7} \text{ mm}^2) \]

\[ \delta = \frac{\phi_e \ell^2}{8} = \frac{2.797 \times 10^{-4}(82 \times 12)^2}{8} = 0.34 \text{ in.} \text{ (9 mm) } \]

(4) Instantaneous deflection due to deck weight

\[ \phi_c = \frac{M_d}{EI} = \frac{1513}{5422(12,235)} = 2.281 \times 10^{-5} \text{ in.}^2 \text{ (} 8.980 \times 10^{-7} \text{ mm}^2) \]

\[ \delta = \frac{5\phi_c \ell^2}{48} = \frac{5(2.281 \times 10^{-5})(82 \times 12)^2}{48} = 2.30 \text{ in.} \text{ (58 mm) } \]

Net camber immediately after deck placement

\[ = -2.47 + 2.30 \]

\[ = -0.17 \text{ in.} \text{ (} -4 \text{ mm) } \]

An accurate estimate of camber just before deck placement allows for a correct prediction of concrete deck thickness directly over the girder (haunch thickness), that would accommodate member camber and produce a smooth roadway profile. An accurate prediction of camber is also an indication of good correlation between the design calculations and actual behavior. There is no code limit on camber at erection. Some designers prefer to have a positive, i.e., upward, camber just after deck placement, for aesthetic purposes.

(5) Time-dependent camber after deck placement to final load

The curvature change due to time-dependent effects can be calculated based on a bending moment obtained from Step 3 of the Example 2:

\[ \Delta \phi_t = \frac{M}{EJ} = \frac{102.58}{3727 \times 37,053} = 7.629 \times 10^{-7} \text{ in.}^2 \text{ (} -2.925 \times 10^{-8} \text{ mm}^2) \]
Deflection due to prestress loss after deck placement

\[
\phi_e = \frac{\Delta Pe}{EI} = \frac{-(160.38 - 173.49)(3.366)(5.14)}{5422(12,235)} = 3.447 \times 10^{-6} \text{ in.}^4 (1.357 \times 10^{-7} \text{ mm}^4)
\]

\[
\delta = \frac{\phi_e \ell^2}{8} = \frac{3.447 \times 10^{-6}(82 \times 12)^2}{8} = 0.42 \text{ in. (11 mm)}
\]

Net deflection before live load application:

\[
\delta = -0.17 - 0.09 + 0.42 = 0.16 \text{ in. (4 mm)}
\]

Deflection due to live load

\[
\phi_e = \frac{M_t}{EI} = \frac{5295}{5422(45,252)} = 2.158 \times 10^3 \text{ in.}^4 (8.496 \times 10^3 \text{ mm}^4)
\]

\[
\delta = \frac{5\phi_e \ell^2}{48} = \frac{5(2.158 \times 10^{-5})(82 \times 12)^2}{48} = 2.18 \text{ in. (55 mm)}
\]

The recommended live load deflection limit for bridge superstructures is:

\[
\varepsilon = \frac{(82 \times 12)}{800} = 1.23 \text{ (31 mm)}
\]

Example 4: Strength Analysis of the Beam of Examples 1 to 3

In addition to limiting service load compressive concrete stresses, the ACI 318 Code and AASHTO Specifications require that the strength of prestressed concrete members due to effective prestress plus full loads be checked. For composite members, the strength of the precast concrete member due to all loads applied to it before it becomes composite with the deck, is not required to be checked.

In both ACI 318 and AASHTO, non-prestressed reinforced concrete is required to be designed for strength only. In this example, strength design will be performed to check the flexural capacity of the precast member alone and the capacity of the composite member. The load and resistance factors of the AASHTO LRFD Specifications are applied.

(a) Strength of the precast member. Loads applied are the member weight and the deck weight. The required moment is:

\[
M_r = 1.25(M_e + M_d) = 1.25(2648 + 1513) = 5201 \text{ kip-in. (587.59 kN-m)}
\]

The steel stress at flexural strength may be obtained using the strain compatibility analysis procedure and the stress-strain power formula given in the PCI Bridge Design Manual:

\[
f_{ps} = \varepsilon_{ps} \left[ 27.613 \left( \frac{887}{1 + (112.4\varepsilon_{ps})^{3/6}} \right)^{1/3} \right]
\]

where \(f_{ps}\) is the stress in seven-wire, 270 ksi (1860 MPa) low relaxation strands, corresponding to a total strain \(\varepsilon_{ps}\), which is the strain in the concrete at the same level as the steel row being considered plus decompression strain (which is approximately taken as the ratio of effective prestress to modulus of elasticity).

The results of the strain compatibility analysis produce a neutral axis depth, \(c = 21.57 \text{ in. (548 mm)}\), compression block depth \(a = \beta_1 c = 0.65(21.57) = 14.02 \text{ in. (356 mm)}\), strain at the centroid of the tension reinforcement = 0.006, and average stress in the prestressing steel:

\[
f_{ps} = \varepsilon_{ps} \left[ 27.613 \left( \frac{887}{1 + (112.4\varepsilon_{ps})^{3/6}} \right)^{1/3} \right]
\]

\[
= 0.006007 \left[ 887 + \frac{27.613}{1 + (112.4 \times 0.006007)^{3/6}} \right]^{1/3}
\]

\[
= 170.0 \text{ ksi (1172 ksi)}
\]
For equilibrium, the tension force must equal the compression force \((T = C)\).

\[
fp_A_p = 170.0(3.366) = 0.85f',ba
\]

\[
= 572 \text{ kips (2544 kN) OK}
\]

The stress level in the prestressing steel is significantly lower than the yield strength of the strand of 0.9(270) = 243 ksi (1675 MPa). Thus, this precast-only section should be considered over-reinforced for this analysis. This is not a surprising outcome as the deck has not yet hardened at this loading stage.

Using Mast’s resistance factor interpolation function given in the PCI Bridge Design Manual:

1. Working stress design using the customary linear elastic analysis assumptions may grossly overestimate compressive stresses at the extreme fibers of the precast concrete component of a composite section.

2. Time-dependent analysis for differential creep and shrinkage between the precast concrete beam and the cast-in-place composite topping is available to designers using existing software and spreadsheets, such as Excel spreadsheet calculations. The analysis shows that the top fiber stress in the precast section is well below the code limits, even though linear elastic analysis would show an unacceptable design.

3. Spreadsheet time-dependent analysis can be effectively used to calculate member camber at various stages of construction and service life of the

Table 3. NU 900 precast beam and composite section properties.

<table>
<thead>
<tr>
<th>Section</th>
<th>Total depth (h) (in.)</th>
<th>Cross-sectional area (A) (sq in.)</th>
<th>Moment of inertia (I) (in.(^4))</th>
<th>Distance between centroid and bottom fibers (y_b) (in.)</th>
<th>Weight of section (lb/ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Precast Beam NU 900</td>
<td>35.4</td>
<td>648.1</td>
<td>110,262</td>
<td>16.1</td>
<td>675.1</td>
</tr>
<tr>
<td>Composite</td>
<td>43.4</td>
<td>1237.9</td>
<td>282,790</td>
<td>27.27</td>
<td>1544</td>
</tr>
</tbody>
</table>

Note: 1 in. = 25.4 mm; 1 sq in. = 645 mm\(^2\); 1 in.\(^4\) = 416,231 mm\(^4\); 1 lb/ft = 0.0146 kN/m.

\[
M_u = 1.25(M_c + M_d) + 1.75(M_i)
\]

\[
= 1.25(2648 + 1513) + 1.75(5295)
\]

\[
= 12,467 \text{ kip-in. (1409 kN-m)}
\]

Using the strain compatibility method, \(c = 8.07\) in. (205 mm), \(a = 5.25\) in. (133 mm), \(f_{ps} = 0.0126, f_{pks} = 254.50\) ksi (1755 MPa), \(\phi = 1.0\), and \(\phi M_n = 20,881 \text{ kip-in. (2359 kN-m)} > M_u\).

The strength analysis of the composite section in this example indicates that the deck slab provides the compression resistance of the flexure couple and that the top of the beam has no impact on the capacity as the compression block depth is within the flange. The compressive stress distribution near member failure is consistent with experimental observations and is quite different from that observed in the working stress analysis of Examples 1 and 2, in which the top fibers of the beam appear to be the most stressed fibers and the slab appears to be lightly stressed (see Fig. 5). Thus, working stress analysis should only be used for serviceability checks under unfactored loads and strength analysis should be used to check strength against overload, using load magnification factors.

**Discussion of Examples 1 to 4**

The solution of Examples 1 to 4 has demonstrated the following:

1. Working stress design using the customary linear elastic analysis assumptions may grossly overestimate compressive stresses at the extreme fibers of the precast concrete component of a composite section.

2. Time-dependent analysis for differential creep and shrinkage between the precast concrete beam and the cast-in-place composite topping is available to designers using existing software and spreadsheets, such as Excel spreadsheet calculations. The analysis shows that the top fiber stress in the precast section is well below the code limits, even though linear elastic analysis would show an unacceptable design.

3. Spreadsheet time-dependent analysis can be effectively used to calculate member camber at various stages of construction and service life of the

Fig. 9. Cross section of bridge.

Fig. 10. Strand pattern at midspan and end of girder.
member. This approach, rather than artificial concrete stress limits, should be the procedure for deflection control.

4. Working stress analysis is only needed for satisfaction of the bottom fiber tensile stress limit. It may also be needed in the rare occasions of fatigue analysis of the compression zone due to live loads. Even in these cases, a realistic time-dependent analysis, rather than the traditional linear elastic analysis, should be used.

Concrete capacity to resist the applied loads with adequate safety against crushing is best handled through strength analysis of both the precast component of the member for loads introduced before the composite topping is hardened, and again for the total section due to full loads. The latter check is a current code requirement.

Fig. 6 shows a comparison between a typical stress diagram based on working stress design and strength design along the beam depth of the non-composite and composite sections. A strength analysis of a composite section indicates that the depth of the equivalent rectangular compression stress block is likely within the thickness of the topping deck. Thus, the impact of high compression in the top fiber of the precast component on member strength is virtually nonexistent. A check of the strength of the precast-only section for applicable loads should be added as a code requirement and the compression limits should be removed as proposed in this paper.

### EXPERIMENTAL PROGRAM

Several full-scale specimens have been tested in this research program. One objective of the experiments was to determine whether camber and deflection can be accurately estimated when the code compression limits are exceeded. A second objective is to verify that no strength or ductility disadvantages result from waiving the code required compression limits. A full description of the experimental program is given in Reference 10. Here, only the results of one inverted tee specimen will be shown.

<table>
<thead>
<tr>
<th>Number of strands and concrete strength</th>
<th>Conventional</th>
<th>Proposed</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of strands at midspan</td>
<td>36</td>
<td>Bottom stress at service</td>
</tr>
<tr>
<td>Number of draped strands at ends</td>
<td>4</td>
<td>Compression limit (0.6(f'_c))</td>
</tr>
<tr>
<td>Precast concrete strength at transfer (ksi)</td>
<td>7.06</td>
<td>Compression limit (0.6(f'_c))</td>
</tr>
<tr>
<td>Precast concrete strength at service (ksi)</td>
<td>7.11</td>
<td>Compression limit (0.45(f'_c))</td>
</tr>
<tr>
<td>Cast-in-place deck concrete strength (ksi)</td>
<td>4.0</td>
<td>Strength under full load</td>
</tr>
</tbody>
</table>

Table 4. Results of proposed versus conventional design of member in Example 5.

A 50 ft (15.24 m) long, 15.75 in. (400 mm) deep IT-400 was pretensioned with 18.05 in. (12.7 mm) diameter bottom strands and two 0.5 in. (12.7 mm) diameter top strands, at the Rinker Materials, Inc. plant in LaPlatte, Nebraska. The beam was shipped to the Structures Laboratory of the University of Nebraska, and placed on supports for a period of 52 days before a 6 in. (150 mm) thick by 48 in. (1220 mm) wide composite topping was placed. The composite beam was tested 20 days after the slab was cast.

The maximum compressive concrete stress at transfer was 4.062 ksi (28 MPa) which was 0.74 of the specified concrete strength of 5.5 ksi (38 MPa) and 0.60 of the actual strength of 6.82 ksi (47 MPa). In both cases, the allowable limit of 0.6\(f'_c\) was exceeded. The maximum compressive stress at the top fiber of the precast member at the time of deck placement, using linear elastic analysis was 0.66 of the specified concrete strength of 8 ksi (55 MPa) and 0.65 of the actual concrete strength of 8.14 ksi (56 MPa).

This was the primary criterion being tested and the calculated stress was much higher than the code limit of 0.45\(f'_c\). Fig. 7 shows the predicted deflection-time diagram, and the measured values at key events. The agreement between experimental and predicted values is excellent.

The beam was then tested to failure. Two hydraulic jacks with a 10 in. (250 mm) stroke were not adequate to accommodate the large deflection of the beam before it failed (see Fig. 8). The load had to be removed and the specimen shimmed several times before it finally collapsed.

The maximum measured deflection and load just before failure were 14.7 in. (373 mm) and 54 kips (240 kN). Obviously, the large deflection shows a tremendous amount of ductility in the beam and no signs of compression-controlled brittle failure. The measured load was close to that predicted by theory, 51.9 kips (231 kN).

### PROPOSED FLEXURAL DESIGN CRITERIA

The following design modifications are proposed for the flexural design of prestressed concrete members:

- Eliminate the requirements for concrete compression limits at prestress transfer, 0.6\(f'_c\), due to effective prestress combined with dead loads, 0.45\(f'_c\), and due to effective prestress combined with full loads, 0.6\(f'_c\).
- Retain the requirement for concrete compression limit due to fatigue loading, 0.4\(f'_c\), for bridge members and other relevant applications.
- Retain service load checks for concrete tensile stresses at transfer and at service. If stress exceeds the modulus of rupture of concrete, generally limited to 6\(\sqrt{f'_c}\) (in pounds per square inch), bonded reinforcement is to be added as per current practice to control flexural cracking.
- Retain all other serviceability criteria such as deflection control, camber control, vibrations, and other parameters. With increasing use of high strength concrete, such criteria...
become increasingly important.

- Add the requirement of strength design for prestress transfer to ensure member capacity against concrete crushing.
- Retain the strength design of full section due to total loads, and use this criterion as the primary design criterion.
- Add the requirement that precast-only sections have adequate flexural strength due to loads applied before the sections become composite with cast-in-place topping.

The steps to be used in design may vary according to designer preference. The following steps may be used for standard precast concrete bridge and building members:

**Step 1:** Select section shape and size based on prior experience.

**Step 2:** Determine area of prestressing steel required for acceptable capacity of the member at final conditions due to full loads. There are two criteria for this step: (a) strength of composite section must be not less than the factored total load moment, and (b) concrete tensile stress due to unfactored service loads must be smaller than the code limit. In this step, the specified compressive strength of the deck concrete can be determined such that the compression block is confined within the deck slab.

**Step 3:** Determine the concrete strength of the precast component of the section, based on strength analysis of the section for the loads applied before it becomes composite with the deck concrete.

**Step 4:** Using strength design, determine the concrete strength at transfer, and debonding and/or draping requirements at the end section. Also, determine if top bonded reinforcement at member end is required to control cracking at transfer.

**Step 5:** Check remaining serviceability criteria, such as deflection and fatigue.

The following example illustrates the proposed design steps.

**Example 5: Flexural Design of a NU-900 I-Beam**

It is required to design an interior beam of the center span of a three-span bridge. The span lengths are 80 - 100 - 80 ft (24.4 - 30.5 - 24.4 m). The superstructure consists of five NU-900 I-beams spaced at 9 ft (2.74 m) on center, as shown in Fig. 9. The beams are designed to act compositely with a 7.5 in. (191 mm) cast-in-place concrete deck slab.

A 0.5 in. (12.7 mm) wearing surface is considered to be an integral part of the slab. Thus, the slab is considered to be 7.5 in. (191 mm) thick for resistance calculations. The design loads and resistance factors of AASHTO LRFD Bridge Design Specifications will be used.

Midspan moment due to weight of the precast member, \( M_{u} = 810.5 \text{ kip-ft} \) (1099 kN-m); due to deck weight, \( M_{d} = 1110.3 \text{ kip-ft} \) (1505 kN-m); due to barrier weight, \( M_{b} = 52.0 \text{ kip-ft} \) (70.5 kN-m); due to future wearing surface, \( M_{ws} = 95.3 \text{ kip-ft} \) (129.2 kN-m); and due to live load-plus-impact, \( M_{LL+I} = 1184.8 \text{ kip-ft} \) (1606.37 kN-m).

Seven-wire, 0.6 in. (15.4 mm) diameter, Grade 270, low-relaxation strands at 2 in. (50.8 mm) spacing are used. The steel stress is assumed to be 202.5 ksi (1396 MPa), just before transfer, 180 ksi (1241 MPa) just after transfer, and 155 ksi (1069 MPa) after allowance for all time-dependent losses.

The following concrete strengths are initially assumed, based on current standard practice in Nebraska: precast concrete strength at prestress transfer = 6.5 ksi (45 MPa), and at service = 8 ksi (55 MPa); cast-in-place deck concrete = 4 ksi (28 MPa). Table 3 shows the properties of the non-composite and composite cross sections.

**Step 1: Section shape and size**

A limited superstructure depth is assumed to be a constraint for this bridge. A relatively shallow, 900 mm (35.4 in.) NU I-girder size is selected.

**Step 2: Required area of reinforcement based on flexural design at midspan**

Initially, select 36 - 0.6 in. (15.24 mm) diameter strands, arranged as shown in Fig. 10. Check the tensile stresses at midspan of the composite section for AASHTO LRFD Service Level III loading combination:

\[
f_b = \frac{P}{A} + \frac{P e}{S_p} - \left(\frac{M_s + M_d}{S_b} - \frac{M_s + M_{ws}}{S_{bc}} - 0.8 \times \frac{M_{LL+I}}{S_{bc}}\right)
\]

\[
= \frac{1210.86}{648.1} + \frac{1210.86 \times 13.1}{6.849} - \frac{(810.5 + 1110.3) \times 12}{6.849} - \frac{(52.0 + 95.3) \times 12}{10,370}
\]

\[
= 0.8 \times \frac{1184.8 \times 12}{10,370}
\]

This stress is within the corresponding tensile stress limit of \(-6 \sqrt{f_c} = -0.537 \text{ ksi} \) (3.70 MPa).

Required strength of composite section:

\[
M_u = 1.25(M_s + M_d + M_b) + 1.5(M_{ws}) + 1.75(M_{LL+I})
\]

\[
= [1.25(810.5 + 1110.3 + 52.0) + 1.5(95.3) + 1.75(1186.6)](12)
\]

\[
= 56.226 \text{ kip-in.} \ (6353 \text{ kN-m})
\]

Strength analysis indicates that the design strength, \( \phi M_u = 77,589 \text{ kip-in.} \) (8766 kN-m), which is significantly larger than the required strength, \( M_u \), indicating that the strength at service is not a critical design criterion. The depth of the equivalent rectangular stress block, \( a = 5.62 \text{ in.} \) (143 mm) which is less than the 7.5 in. (191 mm) slab thickness, indicating that a 4 ksi (28 MPa) deck concrete is adequate.

**Step 3: Required strength of precast section at time of deck placement**

\[
M_u = 1.25(M_s + M_d)
\]

\[
= 28,812 \text{ kip-in.} \ (3255.3 \text{ kN-m})
\]

The analysis for this loading case is done using the strain compatibility method of the PCI Bridge Design Manual which incorporates Mast’s variable strength reduction formula, as described in Example 4. A minimum
factored moment = 0.85

is done for a section subjected to a transfer satisfies the midspan section that the required concrete strength at stress transfer, it can be established Reference 3 for design at time of precast section at transfer.

Step 4: Required strength of precast section at transfer

Using the strength design method of Reference 3 for design at time of prestress transfer, it can be established that the required concrete strength at transfer satisfies the midspan section conditions. For this case, the analysis is done for a section subjected to a factored moment = 0.85Mn = 8267 kip-in. (934 MPa) combined with a factored pretensioning force = 1.15 (36)(0.217)(202.5) = 1819.22 kips (205.54 kN). The corresponding minimum concrete strength is 4.73 ksi (31.61 MPa).

The analysis is repeated at the lifting point, which is assumed to be 5 ft (1.52 m) away from the member end, to determine if any strand draping is required. For that section, the factored moment due to self weight = (1.15)(0.675)(5)(12)/2 = 116 kip-in. (13.11 kN-m). With the full pretensioning force used, i.e., no strands draped, the required concrete strength is 9.8 ksi (67.57 MPa) which is obviously too high.

Now, if four strands are draped, the corresponding strength at release is 6.0 ksi (41 MPa). Because of the requirement of draping, the 0.4f′c section would need to be checked. For that section, the required concrete strength is 4.8 ksi (33.09 MPa) which is below that required for the section at the lifting point.

Step 5: Check deflection and fatigue

An analysis for these design criteria indicates that fatigue stress and deflection are below the recommended AASHTO LRFD limits.

For comparison purposes, an analysis of the same example using the current conventional criteria was undertaken. The results are given in Table 4.

CONCLUSIONS

1. The present restrictions on concrete compressive stress at service limit state in the design of prestressed concrete flexural members neither directly address serviceability conditions, nor ensure adequate strength underfactored loads.

2. In composite members, the current allowable compressive stress of 0.6f′c, due to effective prestress plus full loads, gives the false impression that the highest compressed fibers, and most likely to “crush” under factored load, are the extreme precast concrete fibers. This is in conflict with the strength design method and experimental evidence which indicates that the highest stressed fibers are those in the composite topping.

3. Both the ACI 318 Code and AASHTO Specifications already require that the flexural strength of a member be checked. This applies equally to prestressed and non-prestressed conventionally reinforced members. The compressive stress limits of 0.6f′c, due to full loads, and 0.45f′c due to sustained (dead) loads, are imposed on prestressed concrete members only. Conventionally reinforced members have not been subjected to these limits since the strength design approach was introduced in the codes about 50 years ago, without any related reported deficiencies.

4. Imposing unnecessary compressive stress limits on prestressed concrete members inhibits development and use of efficient sections such as inverted tees and U-shaped precast sections with composite cast-in-place topping slabs.

5. It is proposed that the compressive stress limits of 0.6f′c and 0.45f′c imposed on concrete stresses due to full loads and due to dead loads, respectively, be deleted as design requirements.

6. It is proposed that composite members be checked for strength at all critical loading stages, particularly loads introduced just before the member becomes composite with the topping.

7. Mas’s unified strength design method which provides an interpolation function for transition of the resistance factor from tension-controlled to compression-controlled behavior is the most appropriate for analysis of bridge beams, especially when considering the precast component before composite action takes effect. The method has been adopted as the standard method in ACI 318-02. It is the recommended method in the PCI Bridge Design Manual. The AASHTO LRFD Specifications should be revised to allow for use of this method in bridge design.

8. Strength design should be used for conditions at prestress transfer, and at lifting out of the casting bed, as recommended by Noppakunwijai et al.3

9. Service load limits should remain imposed on such serviceability criteria as camber, deflection, crack control and fatigue as appropriate for the structure being designed. For example, if bridges and some building structures are not desired to be cracked due to effective prestress plus full loads, the tensile stress due to these effects should be limited to 6 f′c as currently required in both ACI and AASHTO.

10. Limiting the compressive stress due to half of the effective prestress and half of the dead loads combined with the full live load is a fatigue check for bridge members. It should be retained. However, it is inconsistent to use a standard truck load. Rather, the “Fatigue Truck,” and other relevant provisions to the fatigue limit state in the AASHTO Specifications should be used.

11. Time-dependent analysis, using the age-adjusted effective modulus method, as described by Dilger, and in the PCI Bridge Design Manual only require modest programming with a standard Excel or similar spreadsheet. It should be used for all serviceability checks, including tensile stress and camber control. Computer software, such as CREEP III17 and CONSPlice PT,18 can also be used to design for service limit states.
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5. PCI’s First Specifications for Pretensioned Bonded Prestressed Concrete Products, First Edition, Prestressed Concrete Institute, Lakeland, FL, 1954.
APPENDIX A — NOTATION

\[ a = \text{depth of equivalent rectangular stress block} \]
\[ A = \text{cross-sectional area} \]
\[ A_b = \text{area of precast concrete beam} \]
\[ A_p = \text{area of pretensioning steel} \]
\[ b = \text{width of compression face of member} \]
\[ c = \text{distance from extreme compression fiber to neutral axis} \]
\[ C = \text{compression force} \]
\[ d_{ext} = \text{depth of extreme steel layer from extreme compression fiber} \]
\[ d_p = \text{distance from extreme compression fiber to centroid of pretensioning tendons} \]
\[ e = \text{eccentricity of prestressing strands} \]
\[ E = \text{modulus of elasticity} \]
\[ E_c = \text{modulus of elasticity of concrete} \]
\[ E_b = \text{modulus of elasticity of beam concrete} \]
\[ E_d = \text{modulus of elasticity of deck} \]
\[ E_{d^*} = \text{age-adjusted, effective modulus of elasticity of precast concrete beam for a gradually applied load at time of deck placement} \]
\[ E_{d^*} = \text{age-adjusted, effective modulus of elasticity of concrete deck for a gradually applied load at time of deck placement} \]
\[ f'_{c^*} = \text{specified compressive strength of concrete at 28 days} \]
\[ f'_{ci} = \text{compressive strength of concrete at time of initial prestress} \]
\[ f_d = \text{compressive stresses due to effective prestress plus dead load} \]
\[ f_L = \text{compressive stresses due to live load} \]
\[ f_{ps} = \text{stress in a given layer of prestressed reinforcement whose strain is } \varepsilon_{ps} \]
\[ f_t = \text{concrete stress at top fiber of precast beam} \]
\[ h = \text{overall depth of member} \]
\[ I = \text{moment of inertia} \]
\[ L = \text{span length} \]
\[ M = \text{moment} \]
\[ M_b = \text{unfactored bending moment due to barrier weight} \]
\[ M_b = \text{unfactored bending moment due to beam self-weight} \]
\[ M_d = \text{unfactored bending moment due to deck weight} \]
\[ M_L = \text{unfactored bending moment due to live load} \]
\[ M_{LL+I} = \text{unfactored bending moment due to live load and impact} \]
\[ M_{WS} = \text{unfactored bending moment due future wearing surface} \]
\[ M_a = \text{nominal flexural resistance} \]
\[ M_{bs} = \text{factored bending moment at a section} \]
\[ P = \text{prestress force} \]
\[ \Delta P = \text{change of prestress force} \]
\[ P_e = \text{effective prestress force after allowing for all losses} \]
\[ S_b = \text{section modulus for extreme bottom fiber of non-composite precast beam} \]
\[ S_{bc} = \text{composite section modulus for extreme bottom fiber of precast beam} \]
\[ S_t = \text{section modulus for extreme top fiber} \]
\[ T = \text{tension force} \]
\[ y_b = \text{distance between section centroid and extreme bottom fiber} \]
\[ \beta_1 = \text{ratio of depth of equivalent uniformly stressed compression zone assumed in strength limit state to depth of actual compression zone} \]
\[ \varepsilon_{ps} = \text{strain in a given layer of reinforcement} \]
\[ \varepsilon_{sh} = \text{shrinkage strain of precast beam} \]
\[ \varepsilon_{dsh} = \text{shrinkage strain of deck} \]
\[ \phi = \text{strength reduction factor} \]
\[ \phi = \text{curvature} \]
\[ \phi_c = \text{curvature at midspan} \]
\[ \Delta \phi = \text{change of curvature} \]
\[ \chi = \text{aging coefficient} \]
\[ \chi_b = \text{aging coefficient for precast beam} \]
\[ \chi_d = \text{aging coefficient for concrete deck} \]
\[ \psi = \text{creep coefficient} \]
\[ \psi_{bsd} = \text{creep coefficient of precast beam at time of deck placement} \]
\[ \psi_{ddf} = \text{creep coefficient of precast beam at final placement} \]
\[ \psi_{ddf} = \text{creep coefficient of cast-in-place slab at final placement} \]
ITEM #1: Add the following paragraph at the beginning of Article 5.5.3.1:

Fatigue of concrete in compression shall be checked using the fatigue load factor specified in Table 3.4.1-1 in combination with truck loading configuration specified in Article 3.6.1.4. Live load stresses shall be computed in accordance with Table 3.3.1-1 and Article 3.6.1.4. The total concrete compressive stress due to live load plus one-half the sum of effective prestress and permanent loads shall not exceed 0.45f′c.

Fatigue of reinforcement need not be investigated for …….(Remainder of Article remains unchanged)

ITEM #2: Revise the first paragraph of Article 5.5.4.1 as follows:

The strength limit state issues to be considered shall be those of strength and stability. For pretensioned concrete members, this method shall be considered as the primary design method for satisfaction of member capacity in compression at various loading stages, including prestress transfer, lifting, erection, deck placement, superimposed dead loads and live loads.

ITEM #3: Add the following paragraph at the end of Article 5.9.4.2.1:

Compressive stress limits specified in Table 5.9.4.2.1-1 may be waived for flexural member if the strength limit state provisions according to Article 5.7.3.2. are satisfied at prestress transfer, at time of cast-in-place deck placement, and at final loading. Further, the deflection and camber at various loading stages must be checked, using conventional analysis methods, for satisfaction of various design and construction limitations.

ITEM #4: In Table 5.9.4.2.1-1, revise the 1st, 2nd, 3rd and 4th bullets, and “Stress Limit” as follows:

- In other than segmentally constructed due to the sum of effective prestress and permanent loads.
- Due to the sum of effective prestress and permanent loads. This limit may be waived if strength design is performed.
- 0.45f′c
- In other than segmentally constructed due to live load and one-half the sum of effective prestress and permanent loads.
- Due to the sum of effective prestress, permanent loads, and transient loads and during shipping and handling. This limit may be waived if strength design is performed. 0.60ϕw,f′c

Other Affected Articles:
None

Background:

The provisions for fatigue should be moved from their current location of Article 5.9.4.2.1, Service Limit State, to Article 5.5.3, Fatigue Limit State. In Article 5.5.3.1, the fatigue truck configuration of Article 3.6.1.4 rather than the standard truck loading used for strength design calculations should be used to determine the live load stresses.

Relative to Item #4, the stress limit for segmentally constructed bridges and “other than segmentally constructed bridges” is the same and the two bullets should be combined into one bullet. The provisions for fatigue limit should be moved from their current location of Article 5.9.4.2.1, Service Limit State, to Article 5.5.3, Fatigue Limit State.

Working stress design should be replaced with the strength design method, consistently with the design of non-prestressed concrete members. Study in 1993, Reference X1, showed that non-prestressed concrete members that are heavily reinforced could experience extreme compressive stress significantly greater than 0.6f′c; if the unfactored load and linear stress-strain theory were employed, with no reported negative effects. Additionally, a study in 1997, Reference X2, showed that the stress-strain relationship for concrete deviates considerably from a linear relationship as the stress in concrete exceeds about 0.5f′c. Therefore the apparent compressive stress using the inaccurate linear analysis is exaggerated. References X3 and X4 shows that strength design in prestressed concrete flexural members is adequate in satisfying the concrete capacity requirements. It shall be noted that members made composite with multistage concrete placement shall be checked using strength design at each stage of concrete placement and the factored loads that exist at each of these stages.

Anticipated Effect on Bridges:

- Consistent design for fatigue for all structural materials and systems.
- Consistent design for conventionally reinforced and prestressed concrete flexural members.
- Improved account for the influence of concrete strength on design.

REFERENCES

APPENDIX C — PROPOSED CHANGES IN ACI CODE

Change Submittal: CG XYZ
Subject: Modifications of ACI 318 Code wording that permit the strength design theory to be the primary method of design of flexural member.

Current Sections Affected: 18.4.3, 18.4.4, R18.4.3, R18.4.4

Reason: Permit the strength design theory to be the primary method of design of flexural members at various loading stages during construction and at service loads after application of full external loads.

Proposed Code Change:
Add a new Section 18.4.3

18.4.3 — Compressive stress limits specified in Section 18.4.2 (a) and (b) may be waived for flexural members if the strength limit state provisions according to Section 18.7 are satisfied at time of cast-in-place deck placement, and at final loading. Further, the deflection and camber at various loading stages must be checked, using conventional analysis methods, for satisfaction of various design and construction limitations.

Renumber Section 18.4.3 to Section 18.4.4

Proposed Commentary Change:
Add Commentary R18.4.3 to read:

R18.4.3 — If creep deformation due to dead load is a concern, it could be evaluated and a satisfactory limit on dead load deflection imposed. For precast members made composite with cast in place topping, the 0.45\(f'_c\) limit is sometimes reached at the top fiber of the precast component of the section immediately after the topping concrete is placed. However, the following two points provide justification for removal of this requirement:

1. Redistribution of stress, due to creep between precast and cast-in-place components of the member, cause gradual reduction of compression in the top fiber of the precast component and a corresponding increase in compression in the cast-in-place component. Therefore, calculated high stress at the time of topping placement is only temporary and gradually decreases upon hardening of topping concrete.

2. Strength analysis of the composite section often indicates that the equivalent rectangular compression stress block is limited within the thickness of the topping and the impact of the high compression in the top fibers of the precast component on member strength is virtually nonexistent.

The 0.6\(f'_c\) limit due to full loads plus effective prestress is only imposed on prestressed members. Study in 1993, Reference 18.X1, showed that non-prestressed concrete members that are heavily reinforced could experience extreme compressive stress significantly greater than 0.6\(f'_c\) if the unfactored load and linear stress-strain theory were employed, with no reported negative effects. Additionally, a study in 1997, Reference 18.X2, showed that the stress-strain relationship for concrete deviates considerably from a linear relationship as the stress in concrete exceeds about 0.5\(f'_c\). Therefore, the apparent compressive stress using the inaccurate linear analysis is exaggerated. References 18.X3 and 18.X4 show that strength design is adequate to satisfy the concrete capacity requirements. It shall be noted that members made composite with multistage concrete placement shall be checked using strength design at each stage of concrete placement and the factored loads that exist at each of these stages.

Renumber Commentary Section 18.4.3 to Commentary Section 18.4.4

R18.4.34

Add new References 18.X1, 18.X2, 18.X3 and 18.X4:

REFERENCES