This paper is a summary of new information that has become available relating to the design of floor systems for vibration. Particular attention is given to the application of this information to precast, prestressed concrete floors. Design recommendations for three different sources of vibration are given, with the reasoning behind the recommendations. The paper will serve as a basis for revising the appropriate section of the Sixth Edition of the PCI Design Handbook and will provide useful information to structural engineers who might be designing structures with potential vibration problems.

Several years ago, the author of this paper was given the task of rewriting Section 9.7, Vibration in Concrete Structures, of the PCI Design Handbook, updating it for the Fifth Edition of that Handbook. At that time, the recommendations in that section seemed like a “black box” to most structural engineers. One had to insert numbers into unfamiliar formulas, and hope that it was done correctly, without an intuitive understanding of the validity of the results.

Since the publication of the Fifth Edition, several new sources of information relating to vibration of floor systems have become available. Two of the best-known authorities on vibration of floors, Allen and Murray, have jointly contributed their knowledge to References 2, 3, and 4. The collaboration of these two authors has helped to “demystify” the design procedures. The new information is based on a resonant vibration model, rather than the older “heel-drop” impact model.

This paper provides a summary of the new information contained in References 2 through 6, as the information relates to the design of precast, prestressed concrete floor systems.

When the natural frequency of a floor system is close to a forcing frequency, and the deflection of the system is significant, motion will be perceptible and perhaps even annoying. Perception is related to the activity of the occupant: a person at rest or en-
engaged in quiet work will tolerate less vibration than a person performing an active function, such as dancing or aerobics. If a floor system dissipates the imparted energy in a very short period of time, the motion is likely to be perceived as less annoying. Thus, the damping characteristics of the system affect acceptability.

Three separate sources of vibrations are discussed, and design recommendations are made for each. The first source relates to vibrations caused by individuals walking, the second relates to vibrations caused by individuals engaged in rhythmic activity, and the third relates to vibrations introduced by machinery.

Limits are stated as a minimum natural frequency of a structural system. These limits depend on the permissible peak accelerations (as a fraction of gravitational acceleration), on the mass affected by an activity, the environment in which the vibration occurs, the effectiveness of interaction between connected structural components, and the degree of damping, among others. Much vibration theory is derived from experience with steel and wood floors. In general, floor vibrations are much less likely to be a problem with stiffer, more massive concrete floors.

Some building types common in precast construction are not dealt with in the references that form the basis for this paper. For example, response to vehicular-induced vibrations in parking structures is not provided, because none of the source references cover this aspect. Choice of limits for usages not listed may be selected, with judgment, by comparison with other types listed here.

It must be emphasized that the calculations presented are very approximate. The actual natural frequency of a floor can be estimated to a reasonable degree of accuracy, but the calculation of the required frequency is based on damping and on human perception, both of which are subject to variation. When in doubt about the acceptability of a proposed floor system, the best way to decide is to compare it, using the same method of analysis, to existing similar systems that are known to be acceptable or unacceptable.

**TYPES OF VIBRATION ANALYSIS**

Three types of vibration analysis are described in this paper. The analyses differ because the inputs that cause the vibration differ.

**Walking**

A walking person's foot touching the floor causes a vibration of the floor system. This vibration may be annoying to other persons sitting or lying in the same area, such as an office, a church, or a residence. Although more than one person may be walking in the same area at the same time, their footsteps are normally not synchronized. Therefore, the analysis is based on the effect of the impact of the individuals walking.

**Rhythmic Activities**

In some cases, more than a few people may engage in a coordinated activity that is at least partially synchronized. Spectators at sporting events, rock concerts, and other entertainment events often move in unison in response to music, a cheer, or other stimuli. In these cases, the vibration is caused by many people moving together, usually at a more or less constant tempo.

The people disturbed by the vibration may be those participating in the rhythmic activity, or those in a nearby part of the structure engaged in a more quiet activity. The people engaged in the rhythmic activity have a higher level of tolerance for the induced vibrations, while those nearby will have a lower level of tolerance.

**Mechanical Equipment**

Mechanical equipment may produce a constant impulse at a fixed frequency, causing the structure to vibrate.

**Analysis Methods**

Because the nature of the input varies for these three types of loads creating vibration, each of the three types requires a somewhat different solution. But, all cases require knowledge of an important response parameter of the floor system, its natural frequency of vibration, and all three analysis methods are based on finding a required minimum natural frequency.

**Avoiding Miscalculation**

When engineers do unfamiliar calculations (and vibration calculations are unfamiliar to engineers used to working with static loads), mistakes are sometimes made when quantities using the wrong units are inserted into formulas. All the equations in this paper are dimensionally correct, in both U.S. customary and SI units. Suggested customary units are kips, inches, and seconds.

When quantities using other dimensions (such as span in feet, weight in psf, and so on) are used, they must be converted to kips, inches, and seconds (or the appropriate SI units). The acceleration due to gravity, g, is taken as 386 in./s², not 32.2 ft/s². In SI units, the equations are also correct, providing consistent units, such as mm, kg, and g = 9800 mm/s² are used.

**NATURAL FREQUENCY OF VIBRATION**

The natural frequency of a floor system is important for two reasons. It determines how the floor system will respond to forces causing vibrations. It also is important in determining how human occupants will perceive the vibrations. It has been found that certain frequencies set up resonance within internal organs of the human body, making these frequencies more annoying to people.

As background information, Fig. 1 shows the human sensitivity over a range of frequencies during various activities. The recommendations that follow were based on levels of human response depicted in Fig. 1. The human body is most sensitive to frequencies in the range of 4 to 8 Hz (cycles per second). This range of natural frequencies is commonly found in typical floor systems.

**Computing the Natural Frequency**

The natural frequency of a vibrating beam is determined by the ratio of its stiffness to its mass (or weight). Text-
Computing the simple-span deflection gives the structural engineer a familiar quantity, which can readily be checked.

Some vibration textbooks use "circular frequency" in radians per second. Circular frequency is $2\pi$ times the frequency in Hertz. Care should be taken not to confuse the two concepts. References 2 through 6 use Hertz, cycles per second.

**Computing Deflection**

The equation for the deflection $\Delta_j$ for a uniformly loaded simple-span beam is:

$$\Delta_j = \frac{5wj^4}{384EI} \quad \text{(2)}$$

where

- $l =$ span length of member
- $I =$ gross moment of inertia, for prestressed concrete members

Many vibration problems are more critical when the mass (or weight) is low. When computing $\Delta_p$, use a minimum realistic live load when computing $w$, not the maximum live load.

For the modulus of elasticity $E$, the dynamic modulus, as measured by the natural frequency, is higher than the static modulus given in the ACI Code (ACI 318-99). It is recommended that the ACI Code modulus be multiplied by 1.2 when computing $\Delta_j$ for use in determining $f_n$.

For continuous spans of equal length, the natural frequency is the same as for simple spans. This may be understood by examining Fig. 2. For static loads, all spans deflect downward simultaneously, and continuity significantly reduces the deflection. But, for vibration, one span deflects downward while the adjacent spans deflect upward. An inflection point exists at the supports, and the deflection and natural frequency are the same as for a simple span.

For unequal continuous spans, and for partial continuity with supports, the natural frequency may be increased by a small amount. References 2 and 3 suggest how this can be done.

**Effect of Supporting Girders**

The deflection of beams or girders supporting the floor system also affects the natural frequency of the floor system. The simple-span deflection $\Delta_g$ of the floor girder may be calculated in the same manner as $\Delta_j$. The natural frequency of the floor system may then be estimated by the following formula:

$$f_n = 0.18 \sqrt{\frac{g}{\Delta_j + \Delta_g}} \quad \text{(3)}$$

For concrete floor systems supported on walls, $\Delta_j$ may be assumed to be zero. For concrete floor systems supported by concrete girders, $\Delta_g$ is normally small, and is often neglected unless the girders are unusually long or flexible. For concrete floor units supported on steel beams, the beam deflection can have a significant effect.
on the behavior, and should usually be included in the computation for $f_n$.

**Minimum Natural Frequency**

Floors with natural frequencies lower than 3 Hz are not recommended because they are subject to "rogue jumping." Rogue jumping occurs when people discover that they can cause vibrations to build up by jumping at the floor's natural frequency. They may then decide to see how large a vibration they can induce. But, it has been found that people have difficulty synchronizing their jumping at frequencies greater than 3 Hz.

**Graphs of Natural Frequency**

Eqs. (1) and (2) may be combined to produce the following Eq. (4) for a floor unit on stiff supports:

$$f_n = \left(\frac{1.58112}{\ell^2}\right)\sqrt{EIg/\omega} \quad (4)$$

For a given concrete floor unit, the quantities under the radical are constant, and the natural frequency is proportional to the reciprocal of the span squared. On a graph of natural frequency versus span, Eq. (4) results in a hyperbolic curve. Fig. 3 shows the relation between span and natural frequency for various topped floor units given in Chapter 2 of the PCI Design Handbook, Fifth Edition.

**DAMPING**

Damping determines how quickly a vibration will decay and die out. This is important because human perception and tolerance of vibration or motion is dependent on how long it lasts. Seasickness is an example of this, in which the duration of motion is much more important than the amplitude or frequency.

Damping usually is expressed as a fraction or percent of critical damping. A critically damped system is so damped that motion slowly returns to zero without ever completing a cycle of motion in the opposite direction. This is a very highly damped system, in which the motion slowly creeps back to the original position asymptotically. Real building structures have damping from 1 percent to a few percent of the critical value.

**Types of Damping**

There are two types of damping used in the literature on building vibrations. Unfortunately, many earlier papers are not clear on which type is used. The two types are:

- **Modal damping** — This is the type...
of damping used in (nearly) steady-state vibration analysis. The damping is caused by energy dissipation from friction and viscous (hysteretic) processes within the system. Textbooks on vibration commonly refer to this type of damping.

**Log-decrement damping** — This type of damping is that observed after a single (nonrepetitive) impulse on the system. The term "log-decrement" refers to the logarithmic decay rate. The decay of motion caused by a single impulse occurs at a faster rate because the energy of the impulse is transferred to other parts of the structure, as well as being absorbed by the loaded element itself. Log decrement damping was commonly used in earlier writings about vibration due to walking, because it seemed more closely related to that caused by individual footfall impact.

The modal damping is approximately half of the log decrement damping. Care should be taken not to confuse the two types of damping. References 2 through 6 are based on modal damping. Larger values of damping (which might be based on log decrement) should not be mixed with damping values used in References 2 through 6.

### Estimation of Damping

Damping of a floor system is highly dependent on the non-structural items (partitions, ceilings, furniture, and other items) present. The modal damping ratio of a bare structure undergoing low-amplitude vibrations can be very low, on the order of 0.01. Non-structural elements may increase this damping ratio, up to 0.05.

It must be appreciated that the results of a vibration analysis are highly influenced by the choice of the assumed damping, which can vary widely. Yet, this choice is based more on judgment than science, because the scientific basis is not any more precise.

### Resonance

Resonance occurs when the frequency of a forcing input nearly matches the natural frequency of a system. A common example is that of a child on a swing being pushed to greater amplitudes of swinging by another person. The swing and child act as a pendulum. If the pushes are well-timed, the child and swing can be swung to large amplitudes by relatively small inputs from the person pushing.

In order to prevent small "pushes" from creating large vibrations, resonance must normally be avoided. Fig. 4 shows the build-up of vibration related to the ratio of the forcing frequency to the natural frequency, and the modal damping ratio. In order to avoid excessive amplification of vibration, the natural frequency must be higher than the frequency of the input forces by an amount related to the damping of the floor system.

### VIBRATIONS CAUSED BY WALKING

Vibrations caused by walking can often be objectionable in lighter construction of wood or steel. Because of the greater mass and stiffness of concrete floor systems, vibrations caused by walking are seldom a problem in these systems. When using all-concrete floor systems of ordinary proportions, it is usually not necessary to check for vibrations caused by walking. When designing concrete floor systems of long span or slender proportions, this section may be used to evaluate their serviceability with respect to vibrations.

### Minimum Natural Frequency

People are most sensitive to vibrations when engaged in sedentary activities while seated or lying. Much more vibration is tolerated by people who are standing, walking, or active in other ways. Thus, different criteria are given for offices, residences, and churches than for shopping malls and footbridges.

An empirical formula, based on resonant effects of walking, has been developed to determine the minimum
natural frequency of a floor system needed to prevent disturbing vibrations caused by walking:4

\[ f_n \geq 2.86 \ln \left( \frac{K}{\beta W} \right) \]  

(5)

where

\( K \) = a constant, given in Table 1
\( \beta \) = modal damping ratio
\( W \) = weight of area of floor panel affected by a point load

The constant 2.86 has the units Hz.

Effective Weight \( W \)

The effect of an impact such as a footfall is strongly influenced by the mass (or weight) of the structure affected by the impact. This weight, \( W \), is normally taken as the dead load (per square foot) of the floor units plus some (not full code) live load, multiplied by the span and by a width \( B \).

For solid or hollow-core slabs, which are stiff in torsion, it is recommended to take \( B \) equal to the span.\(^2\)

For double tees, it is recommended to take \( B \) varying from \( 0.8L \) for 18 in. (457 mm) double tees with 3 in. (76.2 mm) topping to \( 0.6L \) for 32 in. (813 mm) double tees with 3 in. (76.2 mm) topping.\(^5\)

For continuous spans, the effective weight \( W \) may be increased by 50 percent.\(^2,3\) At an unstiffened edge of a floor, the width \( B \) used for estimating floor system weight should be halved.\(^2\)

Recommended Values

The recommended values of \( K \) and \( \beta \) for use in Eq. (5) are listed in Table 1.

**Acceleration**

Eq. (6) is given in References 2, 3, and 4:

\[ \frac{a_p}{g} \leq \frac{P_o e^{-0.35f_n}}{\beta W} \]  

(6)

where

\( P_o \) = constant force representing the walking force
\( f_n \) = natural frequency of the floor structure, Hz
\( \beta \) = modal damping ratio
\( W \) = effective weight of the floor and nominal superimposed load, as previously defined

\( a_p \) = peak acceleration
\( g \) = acceleration due to gravity

When Eq. (6) is inverted and solved for \( f_n \), Eq. (5) results.\(^4\) In Eq. (5), the quantity \( K \) is substituted for \( P_o / (a_p / g) \). Values of the empirical constant \( K \) are taken from Reference 4. Values for \( P_o \) and \( a_p / g \) used to estimate \( K \), are given in References 2 and 3.

**Harmonics**

A harmonic of a frequency is any higher frequency that is equal to the fundamental frequency multiplied by an integer. For instance, if the frequency of an input excitation is 2.5 Hz, the harmonics are 2.5 x 2 = 5 Hz, 2.5 x 3 = 7.5 Hz, and so on. If the fundamental frequency of a floor system is equal to a harmonic of the exciting frequency, resonance may occur.

To understand this concept, refer to Fig. 5. This shows a case in which the forcing frequency is 2.5 Hz and the fundamental frequency of the floor system is 5 Hz. The second harmonic of the forcing frequency is 2 x 2.5 = 5 Hz. In this case, the forcing impulse strikes every other vibration cycle.

This process is less efficient than one which is in resonance striking at each cycle of vibration. Nevertheless, the 2.5 Hz forcing frequency can cause resonance in the 5 Hz fundamental frequency due to the input force striking every second cycle in the fundamental frequency.

Higher harmonics should not be confused with higher modes of vibra-

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**Table 1. Values of \( K \) and \( \beta \) for use in Eq. (4) (based on Table 3 of Reference 4).**

<table>
<thead>
<tr>
<th>Occupancies affected by the vibrations</th>
<th>( K ) kips</th>
<th>( K ) kN</th>
<th>( \beta )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Offices, residences, churches</td>
<td>13</td>
<td>58</td>
<td>0.02</td>
</tr>
<tr>
<td></td>
<td>0.03(^4)</td>
<td></td>
<td>( 0.05)^5</td>
</tr>
<tr>
<td>Shopping malls</td>
<td>4.5</td>
<td>20</td>
<td>0.02</td>
</tr>
<tr>
<td>Outdoor footbridges</td>
<td>1.8</td>
<td>8</td>
<td>0.01</td>
</tr>
<tr>
<td>a. For floors with few non-structural components and furnishings, open work area, and churches.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>b. For floors with non-structural components and furnishings, cubicles.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>c. For floors with full-height partitions.</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: 1 kip = 4.448 kN.

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**Fig. 5. Forcing resonance at second harmonic.**
tion. The second mode of vibration of a simple span beam has a frequency four times the fundamental frequency. This high a frequency is almost never excited. Harmonics refer to the forcing frequency, compared to the fundamental mode of vibration.

Minimum Natural Frequency

The following design criterion for minimum natural frequency for a floor subjected to rhythmic excitation is based on the dynamic response of the floor system to dynamic loading.\(^2^3\)

The objective is to avoid the possibility of being close to a resonant condition:

\[
f_n \geq f \sqrt{1 + \frac{k}{\alpha_i w_p}} \tag{7}
\]

where

- \(f\) = forcing frequency = \((i) f_{\text{step}}\)
- \(f_{\text{step}}\) = step frequency
- \(i\) = number of harmonic = 1, 2, or 3 (see Table 3)
- \(k\) = a dimensionless constant (1.3 for dancing, 1.7 for a lively concert or sports event, 2.0 for aerobics)
- \(\alpha_i\) = dynamic coefficient (see Table 3)
- \(\alpha_i w_p\) = ratio of peak acceleration limit to the acceleration due to gravity (from Table 2)

\(w_p\) = effective distributed weight per unit area of participants

\(w_r\) = effective total distributed weight per unit area (weight of participants plus weight of floor system)

The computation of the natural frequency of the floor system, \(f_n\), is done as discussed previously. If rhythmic activities take place on the upper floors of a tall building, it is sometimes necessary to take the elastic shortening of the columns into effect, in a manner similar to Eq. (3) for girder flexibility. This case is discussed in Reference 7.

Recommended values for all of the parameters on the right side of Eq. (7) are given in Tables 2 and 3, except for \(w_r\), which includes the actual distributed dead weight of the floor system. Note that Eq. (7) uses the distributed weight \(w_p\), not the total weight \(W\) of a panel that was used in Eq. (5).

Higher Harmonics

Eq. (7) will always require a natural frequency \(f_n\) that is higher than the forcing frequency \(f\). Thus, a crucial decision is the determination of whether the forcing frequencies for higher harmonics need be considered. Eq. (8) gives the peak acceleration for a condition of resonance: \(^3\)

\[
a_p = \frac{1.3 \alpha_i w_p}{g} \frac{1}{2 \beta w_r} \tag{8}
\]

In applying Eq. (8), Reference 3 recommends a value for the damping ratio \(\beta\) as follows. “Because participants contribute to the damping, a value of approximately 0.06 may be used, which is higher than ... for walking vibration.”

If the damping ratio \(\beta\) or the total distributed weight \(w_r\) is high enough, the dynamic load \(a_p w_p\) from Table 3 for higher harmonics may result in an acceleration \(a_p g\) within the limits given in Table 2, even though a resonant condition might occur. If this is the case, that harmonic need not be considered.

Most topped or pretopped concrete floors weigh 75 psf or more. For these floors, the weight \(w_r\) is such that the resonant acceleration at the third harmonic frequency will usually be within limits. Usually, only the first and second harmonics need be considered for topped concrete floors.

Adjacent Activities

A space with a quiet activity may be located next to a space with rhythmic activity. In such cases, it is desirable to have a rigid wall between the two spaces, supporting the floor system in each space. If this is not practical, the acceleration limits for the quiet activity should be used in combination with the rhythmic loading for the rhythmic activity. This combination can often be critical for concrete floor systems, requiring a stiffer floor than needed for supporting gravity loads.

<table>
<thead>
<tr>
<th>Activity</th>
<th>Forcing frequency (f), Hz</th>
<th>Weight of participants, (w_p) psf</th>
<th>Dynamic coefficient (\alpha_i)</th>
<th>Dynamic load (a_p w_p), kPa</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dancing:</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>First harmonic</td>
<td>1.5 to 3</td>
<td>12</td>
<td>0.6</td>
<td>6</td>
</tr>
<tr>
<td>Lively concert or sports event</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>First harmonic</td>
<td>1.5 to 3</td>
<td>30</td>
<td>1.5</td>
<td>0.25</td>
</tr>
<tr>
<td>Second harmonic</td>
<td>3 to 5</td>
<td>30</td>
<td>1.5</td>
<td>0.05</td>
</tr>
<tr>
<td>Jumping exercises:</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>First harmonic</td>
<td>2 to 2.75</td>
<td>4</td>
<td>0.2</td>
<td>1.5</td>
</tr>
<tr>
<td>Second harmonic</td>
<td>4 to 5.5</td>
<td>4</td>
<td>0.2</td>
<td>0.6</td>
</tr>
<tr>
<td>Third harmonic</td>
<td>6 to 8.25</td>
<td>4</td>
<td>0.2</td>
<td>0.1</td>
</tr>
</tbody>
</table>

* Based on maximum density of participants on the occupied area of the floor for commonly encountered conditions. For special events, the density of participants can be greater.
VIBRATION ISOLATION FOR MECHANICAL EQUIPMENT

Note: This section is taken from Reference 1. It is included here for the sake of completeness.

Vibration produced by equipment with unbalanced operating or starting forces can usually be isolated from the structure by mounting on a heavy concrete slab placed on resilient supports. This type of slab, called an inertia block, provides a low center of gravity to compensate for thrusts, such as those generated by large fans.

For equipment with less unbalanced weight, a “housekeeping” slab is sometimes used below the resilient mounts to provide a rigid support for the mounts and to keep them above the floor so they are easier to clean and inspect. This slab may also be mounted on pads of precompressed glass fiber or neoprene.

The natural frequency of the total load on resilient mounts must be well below the frequency generated by the equipment. The required weight of an inertia block depends on the total weight of the machine and the unbalanced force. For a long-stroke compressor, five to seven times its weight might be needed. For high pressure fans, one to five times the fan weight is usually sufficient.

A floor supporting resiliently mounted equipment must be stiffer than the isolation system. If the static deflection of the floor approaches the static deflection of the mounts, the floor becomes a part of the vibrating system, and little vibration isolation is achieved. In general, the floor deflection should be limited to about 15 percent of the deflection of the mounts.

Simplified theory shows that for 90 percent vibration isolation, a single resilient supported mass (isolator) should have a natural frequency of about one-third the driving frequency of the equipment. The natural frequency of this mass can be calculated by Eq. (9):

\[ f_n = 0.16 \sqrt{g / \Delta_i} \]

where
\[ f_n = \text{natural frequency of the isolator, Hz} \]
\[ \Delta_i = \text{static deflection of the isolator, in.} \]

Note that the coefficient 0.16 differs from the coefficient 0.18 in Eq. (1). Eq. (9) is for a lump mass; Eq. (1) is for a distributed mass in a beam.

From the above, the required static deflection of an isolator can be determined as follows:

\[ f_n = f_d / 3 = 0.16 \sqrt{g / \Delta_i} \quad \text{or} \]
\[ \Delta_i = 0.23g / f_d^2 \quad (10) \]

and

\[ \Delta_f \leq 0.15 \Delta_i \quad (11) \]

where
\[ f_d = \text{driving frequency of the equipment, Hz} \]
\[ \Delta_f = \text{static deflection of the floor system caused by the weight of the equipment, including inertia block, at the location of the equipment, in.} \]

REAL-WORLD OBSERVATIONS

The author has experienced concern over the apparent flexibility of precast floor units while stored on dunnage. But, he has found the same units to appear much less flexible once they were installed in a building, and interconnected to adjacent members. Concrete members have a high torsional rigidity that enables them to distribute loads laterally more effectively than does bridging in other materials. The author believes this helps prevent serious resonant conditions from occurring in interconnected concrete floors.

A football stadium was constructed using individual 2 ft (0.6 m) wide channel slabs for each row of seats. The natural frequency of the channel slabs was 4.2 Hz. The required minimum natural frequency using Eq. (7) was 4.2 Hz for the first harmonic and 5.8 Hz for the second harmonic. When first installed, connections to adjacent channel slabs at the quarter points of the span were not completed.

Spectators complained about the perceptible movements of the channel slabs. Then, the weld connections at the quarter points were completed. Subsequently, the performance was satisfactory. Apparently, the interconnection suppressed response in the second harmonic, and reduced the response in the first harmonic.

The seat slabs of Example 2 have been used on three major stadiums in service since 1990. The span is 43 ft (13.1 m) in rectangular bays, with a maximum span of 50 ft (15.2 m) in angled bays. Performance is satisfactory, despite the fact that Example 2 shows the maximum span to be 43 ft (13.1 m), and only 36 ft (11.0 m) when the second harmonic is considered.

Apparently, the second harmonic is not being excited, and the first harmonic response is reduced in the angled bays, in which each of the interconnected seat slab sections has a different natural frequency. This helps prevent a resonant response.

Footbridges have little, if any, interconnection with adjacent members. The response of a footbridge resembles that of an individual member stored on temporary supports more than that of an integrated floor system. Footbridges need to be conservatively designed.

The author and the PCI Industry Handbook Committee would greatly appreciate hearing about real-world experiences relating to objectionable vibration (or lack of such) in concrete floor systems.

CONCLUDING REMARKS

Methods of vibration analysis for three sources of vibration have been presented. All three cases are taken from existing publications, and all three depend upon finding the natural frequency of vibration of the floor system.

In addition to showing the reader how the analyses are done, this paper has attempted to show the reader why various aspects of the analysis process are important.

The author once again reminds the reader that the recommendations for acceptability are based on human perception, which is very subjective. Thus, the numerical results cannot be applied as rigidly as those of, say, a strength analysis.
REFERENCES

The following is a brief listing of references, many of which have been published since the writing of the Fifth Edition of the PCI Design Handbook. Reference 3 gives a more comprehensive listing of references on the subject of floor vibrations. A brief description follows below the listing of each reference.

   (This guide covers the design of wood, steel, and concrete floors. The approach given is straightforward, and is compatible with References 3, 4, and 6.)
   (This is a comprehensive reference covering the design of steel floor systems subjected to vibrations from human activity. It is generally applicable to concrete floors as well.)
   (This paper discusses the origin of Eqs. 9.7.3 and 9.7.4.)
   (This paper examined practical spans ranging from 30 to 72 ft (9.1 to 22 m) for topped floor double tees, and evaluated them using four different published criteria. It was concluded that the spans examined have satisfactory performance when subjected to walking inputs, provided the modal damping ratio is at least 0.03.)
   (This article gives practical advice on designing steel floors. The advice also applies to concrete floors.)
   (This is a classic text on vibration.)

APPENDIX A — NOTATION

\[ a_o = \text{acceleration limit} \]
\[ a_p = \text{peak acceleration} \]
\[ B = \text{width of floor affected by a point load} \]
\[ E = \text{dynamic modulus of elasticity} = 1.2 \times \text{static modulus per ACI 318} \]
\[ f = \text{forcing frequency, Hz} = \frac{1}{T}f_{\text{step}}, \text{see Table 3} \]
\[ f_d = \text{driving frequency of the equipment, Hz} \]
\[ f_n = \text{natural frequency in the fundamental mode of vibration, Hz (cycles per second)} \]
\[ f_{\text{step}} = \text{step frequency} \]
\[ g = \text{acceleration due to gravity, 386 in./s}^2 (9800 \text{ mm/s}^2) \]
\[ I = \text{gross moment of inertia for prestressed concrete members} \]
\[ i = \text{number of harmonic (see Table 3)} \]
\[ K = \text{a constant, given in Table 1} \]
\[ k = \text{a dimensionless constant (1.3 for dancing, 1.7 for lively concert or sports event, 2.0 for aerobics)} \]
\[ l = \text{span length} \]
\[ P_o = \text{constant force representing the walking force} \]
\[ W = \text{weight of area of floor panel affected by a point load} \]
\[ w = \text{uniform load, per unit length} \]
\[ w_p = \text{effective distributed weight of participants per unit area} \]
\[ w_t = \text{effective total distributed weight per unit area} \]
\[ \alpha_i = \text{dynamic coefficient (see Table 3)} \]
\[ \beta = \text{modal damping ratio (fraction of critical damping)} \]
\[ \Delta_f = \text{static deflection of the floor system caused by the weight of the equipment, including inertial block, at the location of the equipment} \]
\[ \Delta_s = \text{instantaneous deflection of a supporting girder} \]
\[ \Delta_i = \text{static deflection of isolator} \]
\[ \Delta_j = \text{instantaneous simple-span deflection of a floor panel due to dead load plus actual (not code) live load} \]
Example 1. Vibrations Caused by Walking

Given: 10DT32+2 (see p. 2-15 of Reference 1)
Open office area: 60 ft span

Problem: Check for vibration caused by walking.

Solution: Use Eq. (5) to find minimum required $f_n$:

$$f_n \geq 2.86 \ln \left( \frac{K}{\beta w} \right)$$

Estimate $\beta = 0.02$ (Table 1)
$K = 13$ kips (Table 1)
Estimate effective weight $W$
$w = 89$ psf (p. 2-15 of Reference 1) + 10 psf assumed superimposed load = 99 psf
Estimate effective width $B = 0.6 L = 36$ ft
$W = w(B)(L) = 0.099$ ksf $(36$ ft$)(60$ ft$) = 214$ kips
Note that in this case, it was not necessary to convert ft to in. because the ft units cancel out.

Minimum required $f_n =
2.86 \text{ Hz} \ln \left( \frac{13}{0.02 \times 214} \right) = 3.18 \text{ Hz}$

From Fig. 3, for 10DT32+2 on 60 ft span
Provided $f_n = 3.8 \text{ Hz} > 3.18 \text{ OK}$

Example 2. Stadium Seat

Given: Stadium seat selection shown in Fig. B1.
$f'_c = 5000$ psi, normal weight concrete
$E = 4031$ ksi x $1.2 = 4837$ ksi
$w = 474$ lb/ft
$I_{min} = 12,422$ in.$^4$ on inclined weak axis
(see Fig. B1)

Problem: Find maximum span governed by vibration.

Solution: Use Eq. (7) to find minimum natural frequency:

$$f_n \geq \sqrt{1 + \frac{k}{a_0/g} \frac{w_p}{w_c}}$$

where $k = 1.7$ for sports event
Refer to Tables 2 and 3. For first harmonic:
Assume forcing frequency $f = 2.5$ Hz (Table 3)
Assume acceleration limit $a_0/g = 0.06$ (Table 2)
Weight of participants $w_p = 30$ psf (Table 3)
Dynamic load $a_0 w_p = 8$ psf (Table 3)
Dynamic load component in weak direction =
8 psf $(64/12)$ ft $\times \cos 31.8^\circ = 36.3$ lb/ft
Total weight (a measure of mass) = $474$ psf + $30 \times (64/12)$ ft = 634 lb/ft
Note that the mass is not reduced by $\cos 31.8^\circ$, because mass is the same in all directions.
\[ f_n \geq 2.5 \sqrt{1 + \frac{1.7 \times 36.3 \text{ lb/ft}}{0.06 \times 364 \text{ lb/ft}}} = 4.0 \text{ Hz} \]

Find maximum span from Eq. (4):

\[ f_n = \frac{1.58}{l^2} \sqrt{\frac{EIg}{w}} \]
\[ = \frac{1.58}{l^2} \sqrt{\frac{4837 \text{ k/in.}^2 \times 12,422 \text{ in.}^4 \times 386 \text{ in.} / s^2}{(0.634 \text{ k/ft}) / (12 \text{ in.} / \text{ft})}} \]
\[ f_n = \frac{1,047,000 \text{ in.}^2}{l^2} \]

For \( f_n = 4 \), \( l = 512 \text{ in.} = 43 \text{ ft} \)
\( l_{max} = 43 \text{ ft}, \text{ based on first harmonic} \)

Check second harmonic. Use Eq. (8):

\[ a_o = \frac{1.3}{g} \frac{\alpha w_p}{\beta w_i} \]

Damping \( \beta = 0.06 \)

From Table 3:
Harmonic number \( i = 2 \)
Forcing frequency \( f = 2(2.5) = 5 \text{ Hz} \)
Dynamic load \( \alpha w_p = 1.5 \text{ psf} \)
Dynamic load in weak direction
\[ = 1.5 \text{ psf} (64/12) \cos 31.8^\circ = 6.8 \text{ lb/ft} \]

\[ a_o = \frac{1.3}{g} \left( \frac{6.8 \text{ lb/ft}}{2(0.06) \times 634 \text{ lb/ft}} \right) = 0.116 \]

Maximum permissible \( a_o/g = 0.07 \) (Table 2)

Second harmonic should be considered. Use Eq. (7) with second harmonic:

\[ f_n \geq 5.0 \sqrt{1 + \frac{1.7 \times 6.8}{0.06 \times 634}} = 5.71 \text{ Hz} \]

\( P = 1,047,000 \text{ in.}^2/\text{ft} \) [previously calculated from Eq. (4)]
\( l = 428 \text{ in.} = 36 \text{ ft based on second harmonic} \)
\( l_{max} = 36 \text{ ft} \)

But: Should the second harmonic be considered? See the fourth paragraph under “Real-World Observations.” It is hoped that input from readers of this paper will help to resolve this question.

**Example 3. Vibration Isolation**

Given: A piece of mechanical equipment has a driving frequency of 800 cycles per minute.

Problem: Determine the approximate minimum deflection of the isolator and the maximum deflection of the floor system that should be allowed.

Solution: Use Eq. (10):

\[ \Delta_i = 0.23g f_d^2 \quad f_d = 800 \text{ cpm/60} = 13.33 \text{ Hz} \]
\[ = 0.23 (386 \text{ in./s}^2) / (13.33 \text{ Hz})^2 \]
\[ \Delta_i = 0.50 \text{ in.} \]

From Eq. (11):
\[ \Delta_f \leq 0.15(0.50) = 0.07 \text{ in.} \]

**NOTE:** Metric conversion factors:
1 ft = 0.3048 m; 1 in. = 25.4 mm;
1 kip = 4.448 kN