In this study, 18 development length tests were carried out on single strand rectangular and multiple strand T-shaped semi-lightweight beams having design compressive strengths of 7000 psi (48 MPa). In the rectangular beam tests, the design moment capacity was exceeded in all specimens. However, in the T-beam tests, bond failures at loads below the design capacity occurred in some specimens immediately after the formation of a flexure-shear crack near the loading point. Additional T-beam tests showed that the bond failure associated with flexure-shear cracking could be prevented by increasing the transverse reinforcement near the point of maximum moment. The shift in the tension force that occurs when flexural cracks turn diagonally needs to be considered when determining if sufficient anchorage of strands is provided. Therefore, the authors recommend that the current AASHTO and ACI requirements for strand development should be enforced at a “critical section” that is located a distance $d_p$ from the point of maximum moment towards the free end of the strand, where $d_p$ is the distance from the extreme compression fiber to the centroid of the prestressed reinforcement.
The use of higher concrete strengths has reduced the cost of bridge structures by increasing the maximum span lengths that can be bridged using standard girder cross sections. However, with longer span lengths the self weight of the prestressed concrete sections has become an increasingly larger portion of the total design load for the bridge. Therefore, in order to reduce the dead load of the concrete girder, lightweight aggregate is often used in the concrete mix.

In the state of Indiana, lightweight aggregate (consisting mostly of expanded shale) has been used to produce semi-lightweight (SLW) prestressed concrete for selected projects since 1991. The SLW concrete used in these projects has weighed around 130 lb per cu ft (2080 kg/m³) compared to 145 lb per cu ft (2320 kg/m³) for normal weight concrete. The SLW concrete is obtained by partially replacing the gravel or limestone coarse aggregate with a lightweight substitute.

PROBLEM STATEMENT

In a recent study sponsored by the Federal Highway Administration (FHWA), the applicability of the current AASHTO equation for calculating development lengths of prestressing strands in pretensioned lightweight concrete beams was evaluated. The unit weight of the concrete was less than 120 lb per cu ft (1920 kg/m³) indicating that the coarse aggregate had been replaced in full with a lightweight ingredient. In the FHWA study, the current AASHTO development length equation was found to be unconservative for estimating development lengths in girders when lightweight concrete was used.

In view of these findings, the applicability of the current AASHTO development length equation for strands in SLW concrete girders was also questioned. Therefore, the research presented in this paper, which was co-sponsored by the Indiana Department of Transportation (INDOT) and the FHWA, focused on determining the development length of prestressed strand when SLW concrete is used. This was necessary in order to determine the adequacy of existing structures and to provide recommendations for the design of future projects using SLW concrete.

All of the SLW prestressed girder bridges built in Indiana prior to this study had a concrete design strength of 7000 psi (48 MPa) and are reinforced with 1/2 in. special (13.3 mm) diameter strand. Therefore, this study focused primarily on the development length determination of 1/2 in. special (13.3 mm) prestressing strand in SLW members with a similar concrete design strength.

BACKGROUND

Discussed below is background information on validation of prestressing strand, assessment of strand surface condition, and review of existing strand transfer and development length equations.

Strand Validation (Moustafa Test Method)

Logan, in a special report published in the March-April 1997 PCI JOURNAL, concluded that there is a significant difference in bond performance in pretensioned concrete beams among strands produced by different strand manufacturers. The report recommended that all 0.5 in. (12.7 mm) diameter strand used in pretensioned applications be required to have a minimum average pullout capacity of 36 kips (160 kN), with a standard deviation of 10 percent for a six-sample group, when embedded 18 in. (460 mm) in concrete test blocks. This test procedure has become known as the Moustafa method, named after Dr. Saad Moustafa who first conducted pullout tests on similar specimens in the 1970s.

The first task of this study presented herein, therefore, involved the fabrication and testing of a similar pullout specimen to determine if the strand used in this study would meet the minimum average pullout capacity recommended by Logan. Following discussions with Logan, the transverse reinforcement used in the pullout specimens in this study was modified from that shown in Ref. 3 to provide a transverse tie next to each strand (see Fig. 1). Since 1/2 in. special strand has a nominal diameter of 0.522 in. (13.3 mm) instead of 0.5 in. (12.7 mm), the corresponding minimum average pullout capacity for the 1/2 in. special strand (assuming a similar average bond stress at ultimate load) is 37.6 kips (167 kN).

The girders for all of the semi-lightweight (SLW) girder bridges in Indiana were manufactured at CSR Hydro-Conduit, Lafayette, Indiana. During the last ten years, Hydro-Conduit has used strand primarily from two suppliers. Therefore, at the outset of this experimental program, it was decided that the test specimens would be fabricated using prestressing steel from those same two suppliers. References to strand supplied from these suppliers were modified to maintain anonymity. The extracted data regarding the strands and the testing equipment used in this study is presented in Table 1.
companies will be denoted by the letters “A” and “B” throughout this paper. Therefore, the pullout specimen used to “validate” the strands in this study contained nine strands from each strand supplier.

**Surface Condition Assessment**

Many observers have noted differences in appearance, color, and drawing lubricant residue of strand from different manufacturers. Therefore, attempts were made to document the initial surface condition of the strand used in this study. Visual appearance of the strand, in terms of color and signs of weathering, were noted for the strand used in the pullout and beam specimens.

In addition, every piece of strand used in the pullout specimen was wiped with a white paper towel prior to tying it into the reinforcing bar cage to remove residue and aid in the visual assessment of the initial surface condition. This process was also performed by Logan prior to casting his pullout specimens.

**Transfer and Development Lengths**

The “transfer length” is defined as the distance required to transfer the fully effective prestress force in the strand to the concrete. The transfer length is not a quantity specified in either the ACI Building Code or the AASHTO Specifications. However, both codes suggest a transfer length of 50 strand diameters when checking shear provisions. The ACI Commentary to the Building Code (Section 12.9) provides a formula for calculating the transfer length that is based on the expression for development length.

According to this formula, the transfer length ($L_t$) is given by:

$$L_t = \frac{f_{se}}{3} d_b \quad (1)$$

where $f_{se}$ is the effective stress (ksi) in the strand after all losses, and $d_b$ is the nominal diameter of the strand in inches.

The “development length” is the bond length required to anchor the strand as it resists external loads on the member. As external loads are applied to a flexural member, the member resists the increased moment demand through increased internal tensile and compressive forces. The increased tension in the strand is achieved through anchorage to the surrounding concrete.

In the current specifications, it is assumed that the development length is equal to the长度 required to transfer the effective prestress force (transfer length) plus an additional length required to develop the increase in strand tension produced by the external load demand. This additional length required to develop the maximum stress in the strand is often referred to as the “flexural bond length.” The development length is specified by both the ACI and AASHTO Codes as:

$$L_d = \left(\frac{f_{ps}}{3} - \frac{2}{3} f_{se}\right) d_b \quad (2)$$

where $f_{ps}$ is the stress in the prestressed strand at nominal strength of the member (in ksi), and $f_{se}$ and $d_b$ are the same as in Eq. (1).

Transfer lengths affect structural design considerations in two ways. First, current code provisions for shear design of prestressed concrete members are based on the amount of pre-compression in the member. Since the effective prestress has been observed to vary approximately linearly from zero at the end of the member to be fully effective at the end of the transfer zone, significant deviations in the actual transfer length from the code suggested 50 strand diameters could lead to inadequate estimates of the member’s shear strength.

The transfer length can also have a significant impact on the flexural behavior of prestressed concrete members. Kaufman and Ramirez, and Russell and Burns, have found that anchorage failures were likely when diagonal shear cracking propagated through the transfer zone of a pretensioned strand. Beams with debonded strands are especially susceptible to this phenomenon. Therefore, the value of the transfer length is important in determining whether flexural cracks will likely propagate into this zone prior to the member reaching its nominal capacity.

In practice, development length requirements are typically checked, rather than designed for. When a prestressed concrete member is designed, required longitudinal reinforcement quantities are based on service load stresses as well as calculations of nominal capacities. Both the ACI and AASHTO Codes prescribe reinforcement ratio limits to ensure that ductility is provided through ample yielding of the prestressed reinforcement at ultimate loads.

Thus, for flexural considerations, the designer calculates a nominal moment capacity of the prestressed section by estimating a final level of

![Fig. 2. Whittemore points mounted on transfer length specimen.](image-url)
stress that will be achieved by the strand ($f_{ps}$). Based on the estimate of $f_{ps}$, the designer calculates a development length ($L_d$) by Eq. (2). A check is then made to ensure that the strand will have a large enough embedment length ($L_e$) in the concrete to obtain the estimated stress at nominal capacity ($f_{ps}$).

The embedment length is defined as the bonded length of the prestressed strand from the beginning of bond to the critical section. In most design applications, and in the literature, the critical section is interpreted as the point of maximum moment. ACI Section 12.10.2 states that “critical sections for development of reinforcement in flexural members are at points of maximum stress and at points within the span where adjacent reinforcement terminates...” Both the ACI and AASHTO Codes imply that if the embedment length is greater than the development length ($L_e > L_d$), then the beam should be able to reach its nominal moment capacity in the absence of shear failure.

Conversely, if the embedment length is less than the development length ($L_e < L_d$), then bond failure may occur prior to the beam reaching its nominal moment capacity and the design is unsatisfactory. However, research has shown that bond failures may still occur when ($L_e > L_d$) if web shear cracking occurs and propagates into the transfer zone. Russell and Burns recommended design procedures which take this into consideration when normal weight concrete is used.

### TEST PROGRAM

While considerable research has been published on the experimental determination of transfer and development lengths in members utilizing normal weight concrete, with emphasis on structural behavior and implications for design, similar work for members made of semi-lightweight concrete is essentially absent from the literature. Therefore, the initial objective of this experimental program was to determine the transfer and development length of prestressed strand in semi-lightweight girders, and to assess the adequacy of current code provisions for the design of such members.

#### Measurement of Transfer Lengths

Transfer lengths were experimentally determined by measuring concrete surface strains at the ends of test specimens. Stainless-steel points were secured to the specimens at 2 in. (51 mm) spacings prior to detensioning the strands. The points were mounted using a five-minute epoxy and were located at the depth of the strand.

Distances were measured between points using a Whittemore gauge that had a 10 in. (254 mm) gauge length and had a differential reading capability of 0.00254 mm, with a perceived accuracy of twice this amount. Therefore, the resolution of the gauge was about 20 $\mu$e. Surface strain readings were taken prior to detensioning, immediately after detensioning, and periodically during the first month after stripping.

Two specimens were fabricated specifically for measuring transfer lengths. These specimens had a cross section that measured 4 x 6 in. (100 x 150 mm) and contained two concentric strands. One of the specimens contained “A” strands while the other specimen contained “B” strands. Whittemore readings were taken on both sides and at both ends of each specimen, which was approximately 7 ft 10 1/2 in. (2400 mm) long. Fig. 2 shows the Whittemore points mounted on one of the transfer length specimens.

#### Evaluation of Development Lengths

Development lengths must be evaluated, rather than determined, in experimental programs. This is typically done using test specimens that are loaded such that the maximum moment occurs at the point in the beam where the provided embedment length $L_e$ is equal to the calculated development length $L_d$.

This point is commonly referred to as the “critical section.”

Development length evaluation in this experimental program consisted of testing six single-strand specimens and six multiple-strand specimens. The specimens in this investigation had fully-bonded straight strands and were tested by applying loads from a hydraulic actuator that was located at a distance $L_d$ from the end of the specimen. Loads were applied incrementally until failure of the members occurred.

Interpretation of the test results is straightforward. A flexural failure indicates that the embedment length is adequate to develop the strand, while a bond failure indicates that the embedment length is not sufficient and that the actual development length is larger than the calculated value.

### Calculation of Development Lengths for Test Specimens

The ACI and AASHTO development length equation [Eq. (2)] considers the development length to be a function of three variables, namely:

- $f_{se}$: effective stress in strand after all losses (ksi)
- $d_b$: nominal diameter of strand in inches
- $f_{ps}$: stress in strand at nominal strength of member (ksi)

Thus, the code-prescribed development length is not a single value that can be evaluated for a given strand. Instead, it is a function of both the strand properties and the properties of the member in which it is cast. Interestingly, for a given strand size and member geometry, the development length may be calculated to be different values by different designers, depending on the assumptions which are made in calculating $f_{se}$ and $f_{ps}$. From Eq. (2), it can be seen that the calculated development length is largest when $f_{ps}$ is maximized and $f_{se}$ is minimized.

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**Table 1. Single-strand beam parameters.**

<table>
<thead>
<tr>
<th>Number of beams</th>
<th>Strand producer</th>
<th>Embedment length</th>
<th>Concrete strength</th>
<th>Strand size</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>A</td>
<td>6 ft 1/2 in.</td>
<td>7000 psi</td>
<td>1/2 in. special</td>
</tr>
<tr>
<td>3</td>
<td>B</td>
<td>6 ft 1/2 in.</td>
<td>7000 psi</td>
<td>1/2 in. special</td>
</tr>
</tbody>
</table>

Note: 1 in. = 25.4 mm; 1 ft = 0.305 m; 1 psi = 0.006895 MPa.
In other words, if the designer overestimates \( f_p \) while underestimating \( f_{se} \) (i.e., by overestimating prestress losses), then the calculated development length will be “longest.” While there may be other implications on design (i.e., member sizing, stress and camber calculations, etc.), the result of calculating excessively long development lengths (in terms of actual bond performance) is conservative since it will result in longer required embedment lengths.

However, the opposite case may not be true. If the designer underestimates \( f_p \) while at the same time overestimating \( f_{se} \) (by underestimating prestress losses), then the calculated development length will be minimized. This calculation will, in turn, lead to shorter required embedment lengths and less conservative designs (since bond failure could occur if the actual development length is larger than the provided embedment length).

Since this research was aimed at evaluating the validity of the current code equations for development lengths when semi-lightweight concrete is used, it was determined that the “worst-case” scenario should be tested. With this in mind, the experimental tests in this study were designed so that the “shortest” development lengths that might realistically be calculated by designers would be tested.

As noted above, the “shortest” development lengths are calculated when \( f_p \) is minimized and \( f_{se} \) is maximized. The stress in the strand at nominal strength of the member, \( f_{p0} \), is typically estimated by either direct calculation from code equations or by a strain compatibility analysis. While the strain compatibility analysis is generally considered to be more accurate, especially when more than one layer of steel is provided, the code equations typically yield a lower estimate of \( f_{p0} \).

Using the ACI Code, the stress in the strand at member nominal capacity may be estimated by the equations in Section 18.7.2 of the Code. For members with bonded prestressing tendons and no compression reinforcement, the formula for \( f_{ps} \) reduces to:

\[
 f_{ps} = f_{pu} \left( 1 - \frac{\gamma_p}{\beta_1} \left( \frac{f_{pu}}{f_c} \right) \right)
\]

where
- \( f_{pu} \) = specified tensile strength of prestressed tendons in ksi
- \( \gamma_p \) = factor for the type of prestressing tendon used (= 0.28 for low-relaxation strand)
- \( \beta_1 \) = factor used to enable ultimate flexural capacity calculations to be made by representing the concrete in compression by an equivalent rectangular stress block
- \( \rho_p \) = ratio of prestressed reinforcement = \( A_p/bd_p \), where \( A_p \) is the area of the prestressed reinforcement in the tension zone, \( b \) is the width of the compression face of the member, and \( d_p \) is the distance from the extreme compression fiber to the centroid of the prestressed reinforcement.

Thus, in order to test the most severe condition, the stress in the strand at nominal capacity of the test specimens was calculated using Eq. (3). In addition, the effective stress after all losses, \( f_{se} \), was calculated by assuming only 8 percent total losses. This value was also used by Logan and is a practical minimum value that might be calculated by design engineers.

**Single-Strand Development Length Specimens**

Six single-strand development length specimens were fabricated and tested in this investigation. The purpose of the single-strand specimens was to provide an economical means to conduct multiple development length tests with the same concrete and strand supplier combinations. Table 1 lists the concrete and strand parameters of the single-strand test beams.

The single-strand beams were used for two development length tests each – one test per end. Shear reinforcement was provided only in the central
portion of the beams to ensure that this region would be intact after the first end of the beam was tested. The central portion of the beam was part of the loaded span for the testing of both ends of the single-strand beams.

The shear reinforcement in the center region of the beams was purposefully not centered in the beam. This was done to provide the ability to increase the embedment length at one end of the test specimens in the event that initial beam tests based on the calculated development lengths would experience bond failure.

The nomenclature used for the single-strand specimens is the following: "[Concrete Strength & Type]-[Strand Type]-[Beam # within Series][Test End]"

Thus, the name “7SLW-A-2L” would refer to a test specimen utilizing 7000 psi Semi-LightWeight concrete that had a single ½ in. special (13.3 mm) strand produced by Manufacturer “A”, and was the test at the “Long” end of specimen number 2 (of 3). The “long” end refers to the beam end with the greatest distance to the shear reinforcement located in the central region.

Although all of the development length beams in this study\(^4\) contained semi-lightweight concrete, the study was carried out concurrently with another study that utilized normal weight concrete. Therefore, the term “SLW” was (unnecessarily) used in the naming of all specimens in this study. All of the test data presented in this paper are from prestressed beams with a design concrete strength of 7000 psi (48 MPa).

The single-strand specimens had a rectangular cross section measuring 8 x 12 in. (200 x 305 mm), and contained a single prestressing strand located at a depth of 10 in. (255 mm) (see Fig. 3). The width of 8 in. (200 mm) was slightly larger than the 6½ in. (165 mm) width used by Logan\(^3\) for his single-strand specimens, in order to minimize the depth of concrete in compression and maximize the strain in the prestressed strand at the ultimate flexural capacity of the specimens. The increased specimen width was needed because ½ in. special (13.3 mm) diameter strands were tested in this study, whereas Logan tested specimens containing ½ in. (12.7 mm) diameter strands.

Using the 8 in. (200 mm) width, the strain in the strands at nominal flexural capacity of the test beams in Table 1 was estimated at 2.7 percent based on a strain compatibility analysis. Although this value was lower than the 3.5 percent value recommended by Buckner\(^9\) for minimum strand strains in development length specimens, it was larger than the 2.0 percent value calculated by Logan for his single-strand beams that failed by strand rupture.

Fig. 4 shows the loading arrangements corresponding to the 7000 psi (48 MPa) single-strand rectangular beams. Loads were incrementally applied to the beams using a hydraulic ram powered by an electronically controlled power unit. Values of load, deflection at the applied load, and strand slip at the beam end were recorded throughout the entire loading sequence of all 12 tests (two tests per beam for six beams). Fig. 5 shows the test setup used for the single-strand beam tests.

Multiple-Strand Development Length Specimens

The purpose of the single-strand specimens in this investigation was to provide an inexpensive means to conduct numerous development length tests on beams having the same concrete and strand supplier combination. To study the effect of multiple strands (at close spacing) on development length, full-scale specimens containing multiple strands were tested in addition to the six single-strand specimens. These specimens were designed based on the analysis of test data from the single-strand rectangular specimens.

Results from tests on single-strand specimens indicated that the current ACI and AASHTO equations were appropriate for use with semi-lightweight concrete. Therefore, the first group of three multiple strand specimens was designed with an embedment length based on the current code provisions. These specimens each contained five bottom strands.
and had a T-shaped cross section with an overall depth of 21 in. (535 mm) and a compression flange width of 36 in. (915 mm) (see Fig. 6).

Two of the three T-beam specimens contained ½ in. special (13.3 mm) diameter “A” strands while the third T-beam had a similar strand pattern but utilized “B” strands. All three T-beams had a design compressive strength of 7000 psi (48 MPa). This initial group of three specimens were cast at the same time and in the same prestressing bed (see Fig. 7).

This was enabled by splicing all five strands in the span between the bulkheads of the beams using “A” and “B” strands (see Fig. 6). Splicing of the strands ensured that all beams had the same initial tension in the strands. These beams are referred to as T-Beam A1, T-Beam A2, and T-Beam B (see Table 2).

Unlike the single-strand specimens, which were each tested at both ends, the multiple-strand T-beams were designed with a length that was approximately equal to twice the calculated development length, so that a point load applied at midspan would effectively test the anchorage at both ends simultaneously. The actual length of the T-beams was 6 in. (150 mm) longer than twice the calculated development length, because the load was applied to the beam through a 6 in. (150 mm) wide steel plate.

This beam length increase ensured that the length of embedment of the strand (from the free end of the beam to the edge of the loading plate) coincided with the development length calculated based on the principles discussed earlier in this paper. Fig. 9 shows the dimensions and loading arrangement for the multi-strand T-beams.

Design of shear reinforcement using both the AASHTO and ACI Code provisions showed that #4 (13 mm) stirrups at 15 in. (305 mm) spacings would provide sufficient shear reinforcement for all T-beams in this study. However, the transverse reinforcement provided in T-beams A1, A2, and B was #4 (13 mm) stirrups at 6 in. (150 mm) spacings, or more than twice the code-required amount. Fig. 10 shows the vertical stirrup spacing for the first group of three T-beams.

The T-beams were tested in the Ketelhut Structural Engineering Laboratory at Purdue University. Loads were incrementally applied to the beams through a 6 in. wide x 24 in. long (150 x 610 mm) steel plate using a 220 kip (978 kN) capacity MTS hydraulic actuator. Values of load, midspan deflection, and strand slip for all five strands at both ends of the beam were recorded during the testing of each T-beam. Figs. 11 and 12 show the test setup and strand slip measuring device used for the multi-strand T-beams.

(1) ½-Special (13.3 mm) strand to control stresses @ release. This strand was bonded only at member ends and was cut prior to testing.
Importance of Stirrup Spacing on Longitudinal Steel Stress

The results of the development length tests will be presented later in this paper. However, some of the findings are mentioned at this point because they affected the fabrication of the second group of three T-beam specimens.

T-Beam B experienced bond failure prior to reaching the nominal moment capacity. A careful review of a videotape of this load test (in slow motion) showed that an inclined flexure-shear crack occurred immediately prior to strand slip and subsequent web-shear cracking. It has been observed\(^9\) that the initiation of inclined cracks in simply supported beams will cause an increase in the tension demand closer to the support.

The ACI Code accounts for this tension force shift in flexural members with non-prestressed reinforcement (ACI 12.10.3) by requiring that longitudinal bars in tension be extended for a distance equal to the effective depth of the member beyond the point where they are required to resist flexure.

The behavior of T-Beam B suggested that this shift in tension demand in the prestressed reinforcement resulted in a bond failure. This observed behavior also suggests that the "critical section" for prestressed concrete members referred to in ACI 12.9.1 may not correspond to the location of the maximum moment, but rather a section at some distance closer to the free end of the strand. This hypothesis is discussed in the following paragraphs.

Fig. 13 shows an idealized bilinear representation of the stress capacity in bonded prestressed tendons versus the distance from the free end of the strand. For the T-beam specimens, development length tests were conducted so that the maximum stress in the strands \(f_s\) was produced at a distance equal to the ACI prescribed development length \(L_d\).

In Fig. 13, the stress in the prestressed concrete strand must lie below the bilinear curve at all locations in order for bond requirements to be satisfied. Note that at a distance \(x\) closer to the free end from the development length \(L_d\), the maximum permissible stress in the strand is equal to \(f_s - \Delta f\).

Fig. 14 (from MacGregor\(^9\)) shows the internal forces in a cracked beam without stirrups. For wide cracks, the aggregate interlocking force \(V_d\) disappears, along with \(V_d, V_{ce}\) and \(C_i\), and \(T_2 = T_1\). In other words, the inclined crack has made the tensile force at Point C a function of the moment at Section A-B-D-E,\(^9\) and Point C can now be assumed to be the "critical section."

Table 2. Multi-strand T-beam parameters.

<table>
<thead>
<tr>
<th>T-beam</th>
<th>Strand producer</th>
<th>Embedment length</th>
<th>Concrete strength</th>
<th>Strand size</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>A</td>
<td>6 ft 1/2 in.</td>
<td>7000 psi</td>
<td>1/2 in. special</td>
</tr>
<tr>
<td>A2</td>
<td>A</td>
<td>6 ft 1/2 in.</td>
<td>7000 psi</td>
<td>1/2 in. special</td>
</tr>
<tr>
<td>B</td>
<td>B</td>
<td>6 ft 1/2 in.</td>
<td>7000 psi</td>
<td>1/2 in. special</td>
</tr>
</tbody>
</table>

Note: 1 in. = 25.4 mm; 1 ft = 0.305 m; 1 psi = 0.006895 MPa.
For a beam without stirrups, having its nominal moment capacity (and corresponding stress $f_{pu}$) demanded at a distance ($L_d$) from the free end of the strand, the onset of flexure-shear cracking will produce a strand stress (at the crack location) that lies above the bilinear curve in Fig. 13. If the ACI expression for strand development length represents the actual length required to develop the strand stress ($f_{pu}$) corresponding to nominal capacity for the beam, then inclined cracking may lead to bond failure.

Fig. 15 (from MacGregor10) shows the internal forces in a cracked beam with stirrups. In this case, the presence of stirrups will ensure that there will always be a compression force $C_f$ and a shear force $V_{cz}$ acting on the part of the beam below the crack, and therefore $T_2$ will be less than $T_1$. However, even though the tension force at Point C will be less than the tension at Section A-B-D-E, the strand stress may still lie above the bilinear curve in Fig. 13 and failure by bond can still occur.

The ACI Commentary to Section 12.10.3 notes that “A diagonal tension crack in a flexural member without stirrups may shift the location of the calculated tensile stress approximately a distance $d$ towards a point of zero moment. When stirrups are provided, this effect is less severe, although still present to some extent.” Thus, if a flexure-shear crack forms, some degree of shift in the location of maximum tensile stress will occur.

Fig. 13 illustrates that a change in strand stress of $(\Delta f)$ equal to $(x/d_b)$ ksi must occur over the distance $x$ from the point of maximum moment so that the strand stress (at distance $x$ from the maximum moment) will not lie above the bilinear curve. Therefore, when an inclined crack occurs, extending from the point of maximum moment to a point “$x$” closer to the free end of the strand, a reduction in strand stress of $(x/d_b)$ ksi must occur over the horizontal projection of the crack to preclude bond failure. This can be accomplished by providing extra transverse reinforcement across the crack.

The amount of required transverse reinforcement to cause the appropriate reduction in strand stress may be estimated using the model in Fig. 16. This model, which assumes the inclined crack can be represented by a linear crack, is the basis for the calculation of required transverse reinforcement.
Model Assumptions:
- The flexure-shear crack can be represented by a linear crack as shown.
- Dowel action is conservatively ignored.
- The weight of the beam is negligible.
- The line of action of the sum of all aggregate-interlock forces ($V_c$) passes approximately through point O. Therefore, the moment due to this force about point O is small and can be ignored.

The force in the stirrups crossing the crack can be determined by summing moments about point O.

$$\Sigma M_O = 0 \quad V_c \left(\frac{x}{2}\right) + \frac{Rb}{j_d} \left(\frac{x}{2}\right) A_{mu} (j_d) - Rb = 0$$

Solving...

$$V_c = \frac{2A_{mu} jd}{d_b}$$

Assuming all stirrups crossing the crack are yielding, then the force in the stirrups ($V_c$) is equal to the total area of the stirrups ($A_{mu}$) multiplied the yield stress ($f_{yu}$).

Therefore,

$$A_{mu} f_{yu} = \frac{2A_{mu} jd}{d_b} \rightarrow A_{mu} = \frac{2A_{mu} jd}{f_{yu} d_b}$$

For the 7000 psi (48 MPa) T-beams...

$$A_{mu} = 5(0.167) = 0.835 \text{ sq in.}$$

$$a = \beta c = 0.987 \text{ in.} \quad \text{from flexural analysis of section}$$

$$jd = d_b - a/2 = 19 - (0.987)/2 = 18.51 \text{ in.}$$

$$f_{yu} = 60 \text{ ksi}$$

$$d_b = 0.522 \text{ in.} \quad \text{for 1/2"-Special strand}$$

$$A_{mu} = \frac{2(0.835)(18.51)}{(60)(0.522)} = 0.99 \text{ sq in.}(640 \text{ sq mm})$$

Therefore, using #4 (13 mm) stirrups ($A_{mu} = 0.40 \text{ in}^2/$stirrup), 3 stirrups are required to cross the crack.

Fig. 16. Calculation of transverse reinforcement required to reduce the tension force across an inclined crack by the amount $\Delta T$. 
This calculation indicates that, for T-Beam B, at least three #4 (13 mm) stirrups would have needed to cross the diagonal crack in order to cause the appropriate reduction in strand stress and preclude possible bond failure. A post-failure inspection of T-Beam B indicated that only one (or perhaps two) stirrup crossed the inclined crack that preceded failure.

### Second Group of Additional Three T-Beams

Based on the observed behavior (flexure-shear cracking followed by bond failure) of T-Beam B, three additional 7 ksi (48 MPa) T-beam specimens were fabricated and tested in this experimental program. These T-beams each contained strand from Producer “B” and were identical in length, cross section, and longitudinal reinforcement to the other three T-beams presented in this paper. However, the three additional specimens differed from the original three T-beams in the amount of transverse reinforcement near midspan. Fig. 17 shows the varying amounts of transverse reinforcement used in the additional three T-beams.

One T-beam had #4 (13 mm) stirrups at 6 in. (152 mm) on center throughout its entire length. The stirrup spacing in this beam was identical to the spacing in T-Beam B (that failed by bond). A second T-beam had #4 (13 mm) stirrups at 3 in. (75 mm) on center in the middle portion and was used to test the hypothesis explained in the previous paragraphs, namely, that increased stirrup spacing can reduce the tension shift that supposedly resulted in bond failure of T-Beam B.

The third additional T-beam had #4 (13 mm) stirrups at 15 in. (375 mm) on center in the middle portion of the beam. This corresponded to the ACI Code-required amount of transverse reinforcement for shear. The additional three T-beams were cast end-to-end, as in Fig. 7, which ensured that they each had identical levels of prestress.

The reinforcement in the flanges of the additional T-beams was similar to that in the original three T-beams. Table 3 lists the parameters of the additional three T-beams.

### TEST RESULTS

Presented below is a summary of the strand pullout, strand surface condition, strand transfer length, and single-strand and multi-strand development length test results.

#### Pullout Test Results

The pullout specimen was cast and tested at two days and at four days. The strands were pulled out of the block using a hydraulic ram furnished by CSR Hydro-Conduit. The load was recorded using a load cell that was placed between two steel plates. The lower plate had two steel angles welded to the bottom side to allow the same contact area with the concrete block that was specified by Logan. Fig. 18 shows the load cell configuration used to determine the pullout values.

The maximum load occurring during a given pullout test was stored automatically by a data acquisition system. Also, the load at “first slip” was ob-

---

### Table 3. Parameters of additional three T-beams.

<table>
<thead>
<tr>
<th>T-beam name</th>
<th>Strand producer</th>
<th>Embedment length</th>
<th>Concrete strength</th>
<th>Strand size</th>
</tr>
</thead>
<tbody>
<tr>
<td>B-3&quot;</td>
<td>B</td>
<td>6 ft 1½ in.</td>
<td>7000 psi</td>
<td>½ in. special</td>
</tr>
<tr>
<td>B-6&quot;</td>
<td>B</td>
<td>6 ft 1½ in.</td>
<td>7000 psi</td>
<td>½ in. special</td>
</tr>
<tr>
<td>B-15&quot;</td>
<td>B</td>
<td>6 ft 1½ in.</td>
<td>7000 psi</td>
<td>½ in. special</td>
</tr>
</tbody>
</table>

Note: 1 in. = 25.4 mm; 1 ft = 0.305 m; 1 psi = 0.006895 MPa.
Fig. 18. Load cell arrangement used to measure pullout force.

Figure 18. Load cell arrangement used to measure pullout force.

...tained by placing a small piece of tape on the strand at the point where it entered the concrete. As soon as a motion of the tape was detected, the corresponding value of load was recorded.

Tables 4 and 5 show the pullout loads and "first slip" values, while Fig. 19 shows the positioning of the strands in the pullout specimen. As indicated, the average pullout capacity for each strand type tested exceeded the minimum value of 37.6 kips (167 kN), indicating that the bond quality of both strands was acceptable. Tables 4 and 5 show that the pullout and first-slip values were slightly higher at four days than at two days, as expected, due to the higher concrete strength and presumed higher modulus of elasticity. These tables also show that, although both strands had similar ultimate pullout capacities, the "B" strand began to slip at a significantly lower load.

Surface Condition Assessment Results

From the beginning of this study it was noted by several observers that the two 1/2 in. special (13.3 m m) strands used had a markedly different appearance. The "A" strand had a bluish hue about it that might best be described as a "gun-steel blue" color. The "B" strand, on the other hand, had a yellow/brassy tint.

Prior to casting the pullout specimens, the strand samples were wiped with a paper towel to remove residue and help assist in the visual assessment. Fig. 20 shows the towels used to wipe the strands for the pullout specimen. This figure shows there is a distinct difference between the residue on the "B" strands than the "A" strands, as there was much more of a tendency to bind or tear the towels on the "A" strands when applying equal pressure. In general, the residue corresponding to the "A" strands was brown or rust colored while the residue corresponding to the "B" strands was black. The chemical composition of this residue was not determined.

It should be noted that all "towel wipes" in this study were performed by the same person. Also, the two strand rolls in this study (one roll of "A" strand and one roll of "B" strand) were placed indoors when they were received from the producers in an attempt to minimize weathering during the course of the study. Both strand rolls were received at the precasting plant during the summer of 1997.

Table 4. Results from pullout tests at two days (concrete compressive strength was 4800 psi).

<table>
<thead>
<tr>
<th>Strand designation</th>
<th>Max. pullout force (lb)</th>
<th>Load at first slip (lb)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>36,450</td>
<td>31,300</td>
</tr>
<tr>
<td>8</td>
<td>37,300</td>
<td>29,500</td>
</tr>
<tr>
<td>7</td>
<td>41,000</td>
<td>32,000</td>
</tr>
<tr>
<td>6</td>
<td>41,000</td>
<td>36,000</td>
</tr>
<tr>
<td>Average</td>
<td>38,940</td>
<td>32,200</td>
</tr>
<tr>
<td>Std. Dev.</td>
<td>2,080</td>
<td>2,380</td>
</tr>
</tbody>
</table>

Table 5. Results from pullout tests at four days (concrete compressive strength was 5200 psi).

<table>
<thead>
<tr>
<th>Strand designation</th>
<th>Max. pullout force (lb)</th>
<th>Load at first slip (lb)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>39,350</td>
<td>33,800</td>
</tr>
<tr>
<td>4</td>
<td>41,450</td>
<td>31,600</td>
</tr>
<tr>
<td>3</td>
<td>43,200</td>
<td>36,700</td>
</tr>
<tr>
<td>2</td>
<td>42,450</td>
<td>41,600</td>
</tr>
<tr>
<td>1</td>
<td>41,100</td>
<td>37,200</td>
</tr>
<tr>
<td>Average</td>
<td>41,510</td>
<td>36,180</td>
</tr>
<tr>
<td>Std. Dev.</td>
<td>1,310</td>
<td>3,390</td>
</tr>
</tbody>
</table>

Note: 1 lb = 4.448 N; 1 psi = 0.006895 MPa.
Values of surface strains were measured on specimens similar to those in Fig. 2. The transfer length can then be inferred directly from the recorded values of strain. For a concentrically prestressed member supported along the entire length, such as the one in Fig. 2, the member will be under simple axial loading when the prestress force is transferred to the concrete. Thus, the compressive stresses (and hence strains) in the member will vary from zero at the free end to a near-constant value corresponding to the \( P/A \) stress. The length it takes for the strand to bond to the concrete and “transfer” the full tension in the tendon to the concrete is called the transfer length.

Fig. 21 shows the results of surface strain measurements taken for the 7000 psi (48 MPa) SLW transfer-length specimen with two \( \frac{1}{2} \) in. special (13.3 mm) “B” strands. Results from the specimen containing “A” strands were similar. Measurements were taken immediately after transfer of prestress, and at 3, 14, 36, and 66 days after transfer.
days thereafter. The vertical dashed lines are drawn at the approximate breaks between the “sloping” portion of the curves where strains are increasing and at the “flat” portion corresponding to the region under constant stress of $P/A$. The distance from the end of the transfer length specimen to the dashed line is the transfer length.

Results from both transfer length specimens indicated that the measured transfer lengths were less than 50 strand diameters (the amount suggested by both AASHTO and ACI when checking shear provisions) except in the case of one end where splitting of the concrete was noted. In this case, the measured transfer length was close to 70 strand diameters.

**Single-Strand Development Length Test Results**

A total of twelve single-strand development length tests were conducted in this study (six beams tested at both ends). As discussed previously, the single-strand beam specimens provided a cost effective means of conducting several load tests. The failure loads and deflections corresponding to the maximum sustained load are shown in Table 6.

Table 6 shows the test results for the single-strand rectangular beams containing “A” and “B” strands. This table lists the maximum moment in the beams, occurring at a distance $L_d$ from the end of the beam and determined from the measured values of applied load. In every case, the maximum moment exceeded the AASHTO nominal moment capacity ($M_n$) indicating the beams’ strands were adequately developed at the point of maximum moment (which occurred at a distance equal to the code-prescribed development length $L_d$ from the beam end).

This finding is consistent with the results from measurements of strand end-slip during testing, which revealed that slip did not occur in all but one specimen, namely, 7SLW-A-1S. In this specimen, a strand slip of 0.051 in. (1.3 mm) was recorded on the data scan prior to failure. However, this minimal slip occurred after the nominal moment capacity had been exceeded by over 10 percent.

Table 6 also indicates that all failures occurred after considerable deflections had occurred. Each of the 7000 psi (48 MPa) beams deflected more than 1½ in. (40 mm) in a 15 ft 3 in. (4.65 m) span prior to reaching its ultimate capacity. Nine of the twelve beams failed in flexure when the strands ruptured. The other three specimens failed in shear. While these shear failures occurred when the shear stress on the section $V/2bd_p$ was less than 92 psi (631 MPa), or $1.1\sqrt{f_y}$, they occurred well after yielding of the prestressing steel had occurred and after considerable ductility had been exhibited. It is possible that when the prestressing steel yielded, the effects of dowel action diminished and the lightly reinforced beams (without stirrups) became susceptible to shear.

In summary, test results from the twelve single-strand development length specimens indicated that the code-required development lengths were sufficient to develop the capacity of a single prestressed strand in a member cast with 7000 psi (48 MPa) semi-lightweight (SLW) concrete. Therefore, as previously mentioned, the T-beam specimens containing multiple strands were also designed using the ACI and AASHTO development length expressions.

**Development Length Test Results on Multi-Strand T-Beams**

Table 7 summarizes the test data, including the failure load, deflection corresponding to the maximum sustained load, and maximum moment for each T-beam. The first group of three T-beam specimens cast, namely, T-Beams A1, A2 and B were identical. T-Beams A1, A2 and B were identical to A2) exceeded the AASHTO nominal moment capacity for the section and failed by strain rupture. Thus, the AASHTO and ACI development length expressions were conservative for these beams.

Both of these failures were ductile, as midspan deflections exceeded 1½ in. (36 mm) prior to the ultimate capacity being attained. T-Beam B had essentially the same ultimate capacity as T-Beam A1 of 120.5 kips (536 kN). Interestingly, though, this load corresponded to a much lower deflection for the B beam [0.82 in. versus 2.02

<table>
<thead>
<tr>
<th>Beam</th>
<th>Max. load (lb)</th>
<th>Max. moment (kip-ft)</th>
<th>Deflection at max. load (in.)</th>
<th>Failure mode</th>
</tr>
</thead>
<tbody>
<tr>
<td>7SLW-A-1S</td>
<td>10,740</td>
<td>40.0</td>
<td>2.1</td>
<td>Shear</td>
</tr>
<tr>
<td>7SLW-A-1L</td>
<td>10,370</td>
<td>39.8</td>
<td>3.1</td>
<td>Shear</td>
</tr>
<tr>
<td>7SLW-A-2S</td>
<td>10,600</td>
<td>39.5</td>
<td>2.7</td>
<td>Flexure — Strand rupture</td>
</tr>
<tr>
<td>7SLW-A-2L</td>
<td>10,000</td>
<td>38.5</td>
<td>&gt;3.0</td>
<td>Flexure — Strand rupture</td>
</tr>
<tr>
<td>7SLW-A-3S</td>
<td>10,540</td>
<td>39.2</td>
<td>2.6</td>
<td>Flexure — Strand rupture</td>
</tr>
<tr>
<td>7SLW-A-3L</td>
<td>10,350</td>
<td>39.8</td>
<td>2.4</td>
<td>Flexure — Strand rupture</td>
</tr>
<tr>
<td>7SLW-B-1S</td>
<td>11,000</td>
<td>40.9</td>
<td>2.2</td>
<td>Flexure — Strand rupture</td>
</tr>
<tr>
<td>7SLW-B-1L</td>
<td>10,680</td>
<td>40.9</td>
<td>&gt;3.0</td>
<td>Flexure — Strand rupture</td>
</tr>
<tr>
<td>7SLW-B-2S</td>
<td>10,980</td>
<td>40.8</td>
<td>1.7</td>
<td>Flexure — Strand rupture</td>
</tr>
<tr>
<td>7SLW-B-2L</td>
<td>10,530</td>
<td>40.4</td>
<td>2.6</td>
<td>Shear, then Strand rupture</td>
</tr>
<tr>
<td>7SLW-B-3S</td>
<td>11,180</td>
<td>41.5</td>
<td>2.0</td>
<td>Flexure — Strand rupture</td>
</tr>
<tr>
<td>7SLW-B-3L</td>
<td>10,350</td>
<td>39.7</td>
<td>2.9</td>
<td>Flexure — Strand rupture</td>
</tr>
</tbody>
</table>

Note: 1 lb = 4.448 N; 1 kip-ft = 1.357 kN-m; 1 in. = 25.4 mm.

* AASHTO Nominal Moment Capacity ($M_n$) = 34.6 kip-ft ($f_y = 260$ ksi).
Table 7. Results from the multi-strand T-beam tests.

<table>
<thead>
<tr>
<th>T-beam</th>
<th>Maximum load (kips)</th>
<th>Maximum moment* (kip-in.)</th>
<th>Deflection at max. load (in.)</th>
<th>Failure mode</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>120.5</td>
<td>362.9</td>
<td>2.02</td>
<td>Flexure – Strand rupture</td>
</tr>
<tr>
<td>A2</td>
<td>124.0</td>
<td>373.1</td>
<td>1.42</td>
<td>Flexure – Strand rupture</td>
</tr>
<tr>
<td>B</td>
<td>120.5</td>
<td>362.9</td>
<td>0.82</td>
<td>Bond</td>
</tr>
<tr>
<td>B-3&quot;</td>
<td>129.9</td>
<td>390.5</td>
<td>1.39</td>
<td>Flexure – Strand rupture</td>
</tr>
<tr>
<td>B-6&quot;</td>
<td>110.0</td>
<td>332.0</td>
<td>0.31</td>
<td>Bond/Web shear failure</td>
</tr>
<tr>
<td>B-15&quot;</td>
<td>100.0</td>
<td>302.6</td>
<td>0.24</td>
<td>Bond/Web shear failure</td>
</tr>
</tbody>
</table>

Note: 1 lb = 4.448 N; 1 kip-ft = 1.357 kN-m; 1 in. = 25.4 mm.
*AASHTO Nominal Moment Capacity (M) = 340.3 kip-ft.

in. (20 mm versus 51 mm)) and it failed by bond, as the strands slipped with respect to the surrounding concrete and pulled in from the end of the beam (see Fig. 22).

After reviewing videotape of the failure of T-Beam B, the investigators noted that a flexure-shear crack developed just prior to collapse (see Fig. 22). This observation led to the hypothesis that the flexure-shear crack caused an increase in tension nearer the beam end and effectively shifted the “critical section” from the section at the point load to the place where the flexure-shear crack intersected the strand. This was discussed earlier in this paper.

It is important to note that the cracking that occurred in T-Beam B prior to failure was not dissimilar to the crack patterns that developed in T-Beams A1 and A2, yet significantly different modes of failure occurred. It is likely (based partly on pullout specimen behavior and towel-wipe tests) that the bond quality of the “A” strand may cause it to develop over a shorter distance than the equivalent-sized “B” strand.

If this were true, then a flexure-shear crack which shifts the tension demand closer to the support may not be critical in the case where “A” strand was used. In general, this tension shift would only lead to sudden collapse upon cracking if the actual distance required to develop a strand lies between the point of maximum moment and the point where the diagonal crack intersects the strand.

In order to test the hypothesis outlined above, three additional 7000 psi (48 MPa) T-beam specimens utilizing 1/2 in. special (13.3 mm) “B” strand were fabricated and tested, each having the same dimensions and test configuration as the original three T-beams. Table 7 shows the results from the additional three 7000 psi (48 MPa) T-beam tests.

T-Beam B-6" was tested first. This beam had a constant 6 in. (152 mm) stirrup spacing which was identical to that in T-Beam B. As Table 7 indicates, T-Beam B-6" failed by bond/shear at a load of 110.0 kips (489.2 kN), corresponding to 97.6 percent of the AASHTO nominal moment capacity for the section. Strand slip data showed that all strands had small values of slip prior to reaching the maximum sustained load.

At the time of failure, the load was held constant and crack patterns were being recorded. Therefore, it is likely that additional slip of the strands occurred during the time period when the load was held constant (but end-slip readings were not continuously recorded). While it cannot be proven, it is plausible that additional slip of the strands resulted in a reduced prestress.
force and, therefore, a loss in shear capacity. Fig. 24 shows the failure cracks for T-Beam B-6”.

T-Beam B-3” was the next beam tested. This beam had #4 (13 mm) stirrups at 3 in. (75 mm) on center in the middle portion of the beam. Table 7 shows that this beam failed by strand rupture at a load of 129.9 kips (577.7 kN). This load corresponded to a maximum moment in the beam that was 14.7 percent larger than the AASHTO nominal moment capacity for the section.

Review of strand-slip data for this beam showed that slip was essentially zero at the time of failure. Nine of the ten ends measured had a recorded slip at failure that was less than 0.001 in. (0.03 mm). The other strand had a recorded slip of 0.046 in. (0.12 mm).

T-Beam B-3” had the same strand and concrete batch used in T-Beam B-6”, which experienced bond failure at a load of only 110.0 kips (489.2 kN). In other words, with stirrups spaced at 3 in. (75 mm) on center, T-Beam B-3” was able to withstand an applied load that was 18.1 percent larger than the failure load for T-Beam B-6”. Fig. 25 shows the failure crack and corresponding strand rupture for T-Beam B-3”.

T-Beam B-15” was the last beam tested in the series. This beam had a stirrup spacing in the central region of the beam of 15 in. (375 mm), which corresponded to the AASHTO and ACI Code minimum amount required for shear. As expected, this beam experienced bond/shear failure at only 100.0 kips (444.9 kN), the lowest load for all the 7000 psi (48 MPa) T-beam specimens tested (see Table 7). Fig. 26 shows the shear failure that occurred after strand slip initiated in the member.

In summary, both of the 7000 psi (48 MPa) T-beams containing “A” strand (A1 and A2) experienced flexural failures (by strand rupture). Each had #4 (13 mm) stirrups at 6 in. (152 mm) throughout the entire length of the beam. Three of the four T-beams utilizing “B” strand (B, B-6”, and B-15”) experienced bond failure and, in two cases (believed subsequent), shear failure when loaded at a distance from the end of the beam equal to the
AASHTO and ACI development lengths.

These failures occurred suddenly, and without much warning, at significantly smaller deflections (see Table 7). Flexural failure (by strand rupture) was achieved in a T-beam using "B" strand when #4 (13 mm) stirrups were provided at 3 in. (75 mm) centers in the middle portion of the beam. This spacing provided a stirrup area that was five times greater than the amount required by the AASHTO and ACI shear provisions.

**MAJOR FINDINGS**

This study has yielded credible evidence that there is an interaction between the shear carried by a prestressed concrete member near the point of maximum moment and the length required to sufficiently anchor the longitudinal reinforcement. Although the findings of this study were made in the context of tests on members with semi-lightweight concrete, which typically have a lower modulus of rupture and would thus be more susceptible to flexure-shear cracking, the principles discussed herein should also be applicable for members cast with normal-weight concrete.

Measurements of concrete surface strains indicated that the transfer lengths associated with both "A" and "B" strand in 7000 psi (48 MPa) SLW concrete were less than the AASHTO and ACI Codes' assumed 50 strand diameters in the absence of longitudinal splitting at transfer. In the end where splitting occurred, the measured transfer length was close to 70 strand diameters. These measurements also showed that the transfer lengths remained essentially unchanged during the first 60 days following transfer of prestress.

Tests on single-stranded rectangular beams and multiple-stranded T-beams revealed that the length required to develop the tensile capacity of a strand in concrete is, in some cases, dependent on the member geometry and loading configuration. Strand rupture (associated with flexural failure) occurred for both "A" and "B" strands when cast in single-strand rectangular specimens containing 7000 psi (48 MPa) concrete.

When the same combinations of strand and concrete were tested in the multi-stranded T-beams, however, the results were mixed. In particular, the combination of 1/2 in. special (13.3 mm) "B" strands and a 7000 psi (48 MPa) concrete mix resulted in bond failures for three of the four T-beam specimens tested. Two other T-beam specimens, which had the same concrete mix and the same diameter "A" strands, resulted in flexural failures by strand rupture.

The review of the failure of T-beam B in this study showed that the bond failure was preceded by a flexure-shear crack. It is postulated that the onset of cracking resulted in a shift of the maximum tensile stress (i.e., the critical section) in the strand from the point of maximum moment towards the free end of the strand.

Although similar crack patterns were noted for the T-beams containing the 1/2 in. special (13.3 mm) "A" strands, that failed by strand rupture, it is surmised that the actual development length for the "A" strand may have been considerably less than the code value that was tested. In this case, then, a shift in the "critical section" would still result in an embedment length (to the critical section) that is larger than the actual development length for the strand, and collapse would not occur.

On the other hand, if the actual development length of the "B" strand was close to the code-determined value that was tested, then a shift in the critical section could lead to collapse. The above hypothesis is consistent with the earlier slip measured for the "B" strand in the Moustafa pullout specimen tests.

To test this theory, three additional T-beam specimens, containing the same concrete mix and strand combination as T-Beam B, were fabricated and tested. Each of these additional T-beams had different amounts of transverse reinforcement near midspan. For the T-beam with the largest amount of transverse reinforcement at the point of maximum moment, namely, T-Beam B-3", bond failure was prevented (presumably by minimizing the shift in the location of the critical section) and the mode of failure was flexure by strand rupture.

The amount of transverse reinforcement required to prevent bond failure represents between 2.5 to 5 times the amount required by shear design (Note: T-Beam B-6" failed by bond while T-Beam B-3" failed by strand rupture). While such increases in the amount of transverse reinforcement are not practical for most design situations, the designer must consider that the critical section may shift in the event of inclined cracking.

**CONCLUSIONS**

Based on the work carried out in this investigation, the following conclusions can be drawn:

1. Both of the 1/2 in. special (13.3 mm) strands used in this study met the requirement for minimum average pullout force (37.6 kips) according to the Moustafa procedure. Therefore, according to the Moustafa test, both strands had acceptable bond characteristics.

2. Twelve load tests on rectangular single-strand beams indicated that the AASHTO and ACI development lengths of:

\[
L_d = \left( f_{ps} - \frac{2}{3} f_{se} \right) d_o
\]

provided sufficient embedment to develop the full capacity of a single strand in the SLW concrete mixes used.

3. The onset of flexure-shear cracking resulted in a shift of the maximum tensile stress (i.e., the critical section) in the strand from the point of maximum moment towards the free end of the strand.

**RECOMMENDATIONS**

The following recommendations are made based on the results of this study.

1. Since a shift in the location of the critical section may occur due to flexure-shear cracking, the authors recommend that the current AASHTO and ACI requirements for strand development length be enforced at a critical section that is located a distance \( d_o \) from the point of maximum moment.
towards the free end of the strand, where $d_p$ is the distance from the extreme compression fiber to the centroid of the prestressed reinforcement.

2. For beam sections other than the point of maximum moment, strand anchorage should be provided so that the required prestress force at a given section may be theoretically "developed" (based on linear interpolation of Fig. 13 or equivalent) at a section located a distance $d_p$ from the point of interest towards the free end of the strand.

3. In place of the above recommendations, the designer may elect to provide enough transverse reinforcement to minimize the shift in tensile demand that will occur in the event of diagonal cracking. The amount of required transverse reinforcing steel may be calculated using the model presented in Fig. 16.

**Discussion**

Recommendations 1 and 2 may appear to be too conservative at first glance. However, the implications for most design situations will be small. For shallow members, the effective depth is clearly small, so checking development length requirements at a small distance of $d_p$ from the current point will not be overtaxing on design. For larger members with fully bonded strands, the issue of development length is seldom a critical factor in design.

The implications for the design of members containing blanketed strands is beyond the scope of this work. However, the more stringent development check would certainly be conservative. Since the termination of blanketing is usually staggered, then a shift in the location of maximum tensile stress may not be as critical as in this study due to the inherent redundancy provided by strands with different debonded regions.

Finally, the recommendation to check development length requirements at a distance $d_p$ from the point of interest towards the free end of the strand is based on the assumption that the initiating crack will have an orientation of 45 degrees with respect to the longitudinal member axis. While more sophisticated models may be used to estimate the most likely crack angle (considering beam geometry and reinforcing ratios), the authors believe the simple and conservative assumption of 45 degrees is highly desirable from a design viewpoint.

**NEED FOR ADDITIONAL RESEARCH**

Since this study involved only bonded strand in SLW concrete, additional research would be beneficial to quantify the:

1. Effect of flexure-shear cracking on members with unbonded tendons.
2. Effect of different concrete mixes, and corresponding aggregate interlock, on the tension shift noted in this study. The model used to calculate required transverse reinforcement (Fig. 16) in this paper conservatively ignored the role of aggregate interlock by assuming the resultant force passed through the point about which moments were summed.

**ACKNOWLEDGMENTS**

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APPENDIX — NOTATION

\( a \) = depth of equivalent rectangular stress block
\( A_{pt} \) = area of prestressed reinforcement in tension zone
\( A_{total} \) = area of total shear reinforcement crossing crack
\( b \) = width of compression face of member [calculation of \( \rho_p \) in Eq. (3)]; also, horizontal distance from beam reaction force to section of interest (see Fig. 16)
\( C_r \) = resultant compressive force in concrete above inclined crack (see Figs. 14, 15, 16)
\( C_f \) = resultant compressive force in concrete below inclined crack (see Figs. 14, 15, 16)
\( d_b \) = nominal diameter of strand (in.)
\( d_p \) = distance from extreme compression fiber to centroid of prestressed reinforcement
\( f'c \) = specified compressive strength of concrete (ksi)
\( f_{ps} \) = stress in strand at nominal strength of member (ksi)
\( f_{pe} \) = specified tensile strength of prestressed tendons (ksi)
\( f_{se} \) = effective stress in strand after all losses (ksi)
\( f_{sy} \) = yield strength of shear reinforcement
\( j_d \) = “lever arm” between resultant tension and compression forces at nominal flexural capacity of section
\( L_d \) = development length
\( L_e \) = embedment length
\( L_t \) = transfer length
\( R \) = vertical reaction force at end of simply supported beam (see Fig. 16)

\( V_a \) = resultant of all aggregate interlock forces along inclined crack
\( V_{ax} \) = horizontal component of aggregate interlock forces along inclined crack
\( V_{ay} \) = vertical component of aggregate interlock forces along inclined crack
\( V_s \) = force in stirrups crossing crack = \( A_{fy} \)
\( V_{ct} \) = resultant shear force in concrete above inclined crack (see Figs. 14, 15, 16)
\( V_{ct} \) = resultant shear force in concrete below inclined crack (see Figs. 14, 15, 16)
\( x \) = horizontal projection of inclined crack
\( \beta \) = factor used to enable ultimate flexural capacity calculations to be made by representing the concrete in compression by an equivalent rectangular stress block
\( A_f \) = change in strand stress along horizontal projection of inclined crack
\( D_f \) = change in prestress force along horizontal projection of inclined crack
\( \gamma_p \) = factor for type of prestressing tendon used (= 0.28 for low-relaxation strand)
\( \rho_p \) = ratio of prestressed reinforcement = \( A_{pt} / d_p \)