Flexural Analysis of Prestressed Concrete Sandwich Panels with Truss Connectors

A closed form elastic continuum approach was modified to account for discrete truss connectors to estimate service load deflections and bending stresses of non-loadbearing semi-composite sandwich panels. Predictions of maximum deflections and bending stresses were compared to finite element results and experimental data. Close agreement was found between the two analytical approaches, and both methods produced conservative predictions as compared to experimental data. Sensitivity analyses indicate the presence of unidentified additional shear transfer mechanisms in actual panels that are not fully represented in the elastic models. However, the theory holds promise for design and for understanding the behavior of semi-composite sandwich panels.

Concrete sandwich panels consist of a single layer of insulation sandwiched between two outer concrete faces (wythes). They are often used as wall panels in low-rise construction and are thermally superior to solid concrete panels.

To increase structural efficiency, various types of connectors (sometimes made of steel) can be used to promote composite behavior. These connectors transfer shear across the flexible and weak insulation layer. The resulting degree of composite action is affected by the number and properties of the connectors, resulting in a wide range of possible behavior (from non-composite to fully composite).

The recent PCI state-of-the-art report on sandwich wall panels provides an excellent overview of considerations for sandwich wall panels, including design examples. However, a general design methodology is not available for semi-composite panels. Many sandwich panels in the United States are proprietary, and publicly available experimental data are limited.

Closed form solutions have been developed for sandwich beams to predict elastic deflections and flexural stresses. Solutions usually do not have provisions to account for the presence of discrete interface connectors, such as metal trusses. Finite element methods are also available, but these approaches are cumbersome for design purposes. Notable advances have been made by researchers at the University of Nebraska in applying closed form
and finite element approaches to test results from sandwich beams and panels with a novel interface connector.

An elastic design methodology is desirable for concrete sandwich panels because an important behavioral limitation is the requirement that the panels do not crack during handling or in service. In fact, the desire to limit cracking is the reason panels are often prestressed.

This paper presents a methodology to predict deflections and flexural stresses of non-loadbearing semi-composite panels with truss connectors. Results of the method are compared with finite element results and with test data from panels subjected to loading similar to the service loading condition. The method is then used to illustrate the implications on design and behavior.

CLOSED FORM SOLUTION

This section discusses the methodology used to analyze a simply-supported semi-composite panel under uniform load and how to account for the presence of truss connectors.

Simply Supported Semi-Composite Panel Under Uniform Load

The methodology was originally developed by Allen; a similar approach has also been developed elsewhere. Details of Allen's methodology are presented in Appendix A. This continuum approach is valid for sandwich beams with thick faces (concrete wythes) and a weak core (layer between the wythes).

Further assumptions are that all materials are linear, elastic, and homogeneous, and strain compatibility is maintained between the wythes and the core. The specific relationships derived in the appendices apply to a case where wythe thicknesses are equal. However, the approach can be generalized for differing wythe thicknesses.

The relationships developed are for a simply supported sandwich beam subjected to a uniformly distributed load, as shown in Fig. 1. Panel geometry parameters and elastic material constants for the wythes and core are necessary input for the solution.

Strictly speaking, sandwich panels are not necessarily beams, depending on the loading and support conditions.

However, typical sandwich panels often have aspect ratios of 2:1 or greater, and loading and support conditions in many practical applications produce essentially one-way bending. Therefore, the approach has application in predicting service load behavior of non-loadbearing panels. Further generalization is required for instances where loading and/or support conditions are not symmetric.

Modification to Account for Truss Connectors

Internal truss connectors, being much stiffer than insulation material, increase the shear stiffness of the effective core material. A steel truss connector used in an experimental study conducted by Stine is shown in Fig. 2. Wu computed an effective shear stiffness, $G_{eff}$, that accounted for the change in shear stiffness created by the presence of one-way continuous truss connectors. This modified shear stiffness could then be used for the core shear stiffness in Allen's theory. Details of the derivation are contained in Appendix A.

The following assumptions were used: (1) the inclined truss connector diagonals behave as truss members (ignoring bending stiffness) with ends...
connected at the interfaces between wythes and core materials; (2) the shear connectors are oriented longitudinally and extend the full length of the panel; and (3) the effect of the discrete connectors is “smeared” across the panel width.

A similar approach was examined in a separate study by Salmon and Einea to investigate the effects of thermal bowing on sandwich panels using a novel nonmetallic wythe connector. They examined various boundary conditions for the truss connectors, and recommended that the connector diagonals be treated as truss members. However, they assumed the truss elements were pinned at mid-thickness of the wythes, rather than at the interface between the wythes and insulation.

The decision here to treat the diagonals as truss members was based on the fact that at the inclination shown in Fig. 2, the bending stiffness of the diagonals is less than 1 percent of their axial stiffness. The truss diagonals were assumed to be pinned at the wythe/insulation interface based on experimental observations by Stine. Push-off (direct shear) tests indicated no deformation of the truss diagonals within the length embedded in the concrete wythes.

This differs from behavior observed by Salmon and Einea, where individual nonmetallic connector diagonals were threaded around prestressing strands (which served as the chords) in the wythes. Their particular diagonal truss element did not have a bonded connection between the chord and the diagonals; furthermore, bond between the connectors and the concrete was reduced, permitting some slip of the portion of the truss diagonals embedded within the concrete.

The validity of the assumption of the connectors’ effects being smeared across the panel width should improve with an increasing number of well distributed truss connectors. Comparison with FEM results (later in this paper) verified that errors are not great even when using only two lines of truss connectors across an 8 ft (2.44 m) wide panel.

Application

The closed form solution in the Appendix is readily programmable in a spreadsheet, but cumbersome for hand computations. As an illustration of how this analytical approach could be simplified for design, correction factors can be developed for classical calculations.

Fig. 3 contains correction factors that can be applied to the composite moment of inertia, I, so that conventional calculations can produce the maximum deflections and stresses predicted by the closed form solution. Note that all correction factors are less than one, indicating the adjusted moment of inertia will be less than that computed using the fully composite section. Curves are presented for two, four, and six lines of trusses for 3-2-3 panels with properties presented later in this paper (including insulation stiffness).

The solid curves represent correction factors to be applied when computing stresses, and the dotted curves would be used to compute deflections. The plot legends also identify the curves using the prefixes $C_s$ (for stresses) and $C_d$ (for deflections).

As an example, for a 15 ft (4.57 m) panel with four lines of truss connectors (of the type used in the analyses), the composite moment of inertia correction factors are $C_d = 0.6$ for deflections and $C_s = 0.85$ for stresses. In other words, if 60 percent of the composite moment of inertia is used in a conventional elastic deflection calculation, the deflection predicted by the closed form solution accounting for semi-composite behavior will be obtained ($\delta_{max}$ Appendix B).

Similarly, 85 percent of the composite moment of inertia should be used to compute extreme fiber bending stress ($f_{max}$ Appendix B). Upon verification of the closed form approach with additional experimental data, design curves of a similar type could be generalized for a variety of panel configurations.

Note that for a given number of truss connectors, the correction factors approach unity as the span length increases. This means that longer panels exhibit a larger degree of composite behavior than short panels.

**CLOSED FORM SOLUTION VS. FINITE ELEMENT RESULTS**

The finite element representations of semi-composite sandwich panels tested by Stine were studied, and FEM results were compared to those of the closed form solution described previously. Commercially available software was utilized to produce three-dimensional models of simply supported panels subjected to uniform load. The modeled panels had 3-2-3 construction, that is 3 in. (75 mm) thick concrete wythes with a 2 in. (50 mm) thick insulation layer, and were 8 x 16 ft (2.44 x 4.88 m).
Due to symmetry, only half the panel length was modeled. Eight-noded solid elements were used to represent wythe and insulation materials, and truss elements modeled the diagonals. A schematic FEM mesh for a panel containing two lines of truss connectors is shown in Fig. 4. A crude mesh is shown for clarity; a more refined mesh was employed in the FEM study.

FEM runs were performed for panels with two and three lines of truss connectors similar to the panels tested by Stine. In that experimental study, some material parameters were not directly determined. Therefore, the concrete modulus of elasticity \( E_c = 4400 \text{ ksi} \) \((30.3 \text{ GPa})\) was estimated from a measured compressive strength of 6000 psi \((41.3 \text{ MPa})\) using the ACI equation

\[
57,000 \left( \frac{f_{c'}}{f_{c}} \right)^{0.5} \text{ (psi)}
\]

The shear modulus of the insulation \( G_s = 565 \text{ psi} \) \((3.89 \text{ MPa})\) was selected based on the values for expanded polystyrene observed by Wade et al. For the closed form solution, the effective shear modulus of the core was determined according to Eq. (A11).

Truss forces were computed using Eq. (A12). Comparisons for midspan deflection, extreme fiber bending stresses, and truss forces for the two approaches are shown in Table 1 for panels with two and three lines of truss connectors.

The closed form and FEM predictions are in very good agreement for deflections and stresses, despite the simplification of using an effective core stiffness to account for the trusses in the closed form model. The closed form predictions for deflections and stresses were within 5 percent of those obtained by FEM analysis for both panels.

Larger differences were observed for computed truss forces. The closed form approach predicted truss forces about 20 percent higher than FEM for the two-truss panel; agreement was better (about 10 percent difference) for the three-truss panel. The closer agreement between the two approaches for the three-truss panel is reasonable because the closed form solution’s assumption that the truss stiffness is smeared across the panel width becomes more valid as more trusses are added.

### COMPARISON TO EXPERIMENTAL DATA

Closed form and FEM predictions were compared to the data obtained from tests conducted at the University of Oklahoma. In those tests, full-scale sandwich panels were simply supported and subjected to uniform loading applied by an air bag. The panels were instrumented to measure deflections, wythe strains, and strains in selected truss diagonals. The data used for comparison were obtained from tests of a panel with two lines of truss connectors and a panel with three lines of connectors.

Each panel was carefully fabricated with modified details and techniques to attempt to remove extraneous shear paths during construction. Accidental shear paths can easily be introduced using typical construction details for panel lifting and handling inserts, and by concrete filling voids between insulation sheets and around trusses during fabrication. Comparisons were made within the panels’ range of elastic behavior (prior to any panel cracking or inelastic action in truss diagonals).

### Comparison Using Nominal Input Parameters

As mentioned earlier, the concrete elastic modulus and insulation shear modulus were estimated based on the best available information. The values used to produce the results of Table 1 are thus referred to as nominal values.

Both the closed form and FEM method over-predicted measured deflections, stresses, and truss forces to varying degrees. The two methods over-predicted the midspan deflection by about 60 percent for the two-truss panel and by approximately 50 percent for the three-truss panel. Ratios of predicted to measured bending stresses

### Table 1. Comparison of closed form and finite element results.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Closed form/FEM</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Two-truss</td>
</tr>
<tr>
<td><strong>Midspan deflection</strong></td>
<td>1.05</td>
</tr>
<tr>
<td><strong>Extreme fiber bending stress</strong></td>
<td></td>
</tr>
<tr>
<td>Tension, midspan</td>
<td>0.95</td>
</tr>
<tr>
<td>Compression, midspan</td>
<td>0.95</td>
</tr>
<tr>
<td>Tension, quarter span</td>
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</tr>
<tr>
<td>Compression, quarter span</td>
<td>1.03</td>
</tr>
<tr>
<td><strong>Truss force</strong></td>
<td></td>
</tr>
<tr>
<td>Compression, support</td>
<td>1.18</td>
</tr>
<tr>
<td>Compression, quarter span</td>
<td>1.19</td>
</tr>
</tbody>
</table>
ranged from 1.14 to 1.33 for the two-truss panel and 1.27 to 1.60 for the three-truss panel (the best correlation for each panel was observed at midspan).

Much greater disagreement was observed for forces in the truss diagonals. The prediction models estimated forces six to eight times higher than those determined from strain gauge data from tests. Furthermore, the test data indicated that compression diagonals carried smaller forces than tension diagonals, which is directly opposite to the behavior predicted by the FEM approach.

A number of factors could contribute to the differences between the predicted and measured observations (in addition to experimental scatter or error). Among these are: (1) differences in actual vs. assumed material properties (concrete elastic modulus, insulation shear modulus); (2) differences in boundary (restraint) conditions of the test panels as compared to the models; and (3) the potential presence of extraneous shear paths (in addition to the trusses and insulation). While agreement between the predicted and measured parameters was not as close as desired, use of the nominal input data resulted in conservative predictions of stresses, deflections, and truss forces.

**Sensitivity Analysis**

A sensitivity analysis was conducted to determine the effects of changes in insulation stiffness, concrete modulus, and panel support boundary conditions on predicted deflection, bending stress, and truss forces. The intent was to determine whether uncertainties in the material parameters from the tests could explain the differences between observed and predicted responses. The sensitivity study was performed using the FEM model.

Insulation shear modulus was varied from 260 to 1100 psi (1.79 to 7.58 MPa). The 260 psi (1.79 MPa) value is at the low end of the range for expanded polystyrene, while the 565 psi (3.89 MPa)(nominal) value is at the higher end for that material. The 1100 psi (7.58 MPa) value has been reported for extruded polystyrene. Changing the insulation stiffness over this range had minor effects on predicted quantities. As compared to values obtained using the nominal insulation stiffness, using the highest insulation modulus [1100 psi (7.58 MPa)] reduced deflections by only about 4 to 8 percent. Bending stresses were reduced by about 4 percent also. Predicted truss forces were only reduced by about 15 percent. Therefore, differences caused by uncertainties in insulation stiffness were not large enough to explain the disparity with the test results.

The nominal concrete elastic modulus was computed using the ACI expression 57,000(f′)^0.5. Due to the recognized scatter in the data from which the ACI equation was derived, the nominal concrete modulus was adjusted by 20 percent, and analytical predictions were performed again.

Increasing the nominal concrete modulus by 20 percent (to 5280 ksi (36.4 GPa)) led to a reduction in deflection of 15 to 20 percent as compared to that predicted using the nominal value of 4400 ksi (30.3 GPa). Predicted bending stresses and truss forces were changed on the order of +10 percent and -5 percent, respectively.

Comparisons were also examined using a simultaneous combination of the highest insulation shear modulus [1100 psi (7.58 MPa)] and the largest concrete modulus [5280 ksi (36.4 GPa)]. Using this combination, predicted deflections were approximately 20 to 30 percent higher than those observed in the two tests vs. 50 to 60 percent higher when using the nominal values. Predicted flexural stresses were about 25 percent too large as compared to the 14 to 60 percent range obtained when using nominal values. However, truss forces were still grossly over-predicted.

The ends of Stine’s test panels were actually supported on elastomeric pads, which are not true roller supports. Bearing at the panel ends was through only one wythe. Some longitudinal restraint, applied eccentrically to the panel, could conceivably have been provided at the support locations.

FEM models were examined with roller, pin, and elastic (longitudinally) supports to examine their effects on predicted behavior. While the pin and elastic support conditions considerably reduced predicted deflections (by as much as 65 percent), they also resulted in substantial changes in the flexural stress distribution (due to the eccentrically applied longitudinal reaction) as compared to the roller supported model.

Stine’s test results for the two panels indicated nearly equal extreme fiber compression and tension strains. This is contrary to the FEM predictions found using the pin and elastic supports. Therefore, the roller support condition was felt to be most appropriate for the models, even though it led...
to the largest deflections of the three support conditions examined.

**Additional Shear Transfer Paths**

Another factor that could contribute to the discrepancies between the test results and predictions from the closed form and FEM models could be the presence of extraneous paths of shear resistance. Considerable care was exercised in fabricating Stine's test specimens; however, minor concrete ribs could have formed in some locations (most likely in the vicinity of the trusses) and/or some resistance could have been provided by the regions around the lifting inserts. The test panels were not available for examination during the analytical study, so it was not possible to verify this hypothesis.

With no verifiable cause for additional shear stiffness, the approach taken was to artificially increase the shear stiffness of the insulation material in the analytical models (leaving all other parameters, including truss stiffness, constant) until midspan deflection matched test results. For the two-truss panel, deflection agreement was reached using an insulation shear stiffness approximately 13 times the nominal value used in the previous analyses. Although this increase seems dramatic, it is equivalent to a concrete rib (or the sum of several small ribs) with total thickness of slightly less than 0.4 in. (10 mm) for an 8 ft (2.44 m) wide panel.

Bending stress predictions were improved to within 6 percent of the experimental values. Also, predicted truss forces were overestimated by 60 to 80 percent, instead of the factor of 6 to 8 associated with analyses utilizing the nominal material values. As in the case of the nominal values, agreement between the closed form approach and the FEM model was again very good.

Similarly, agreement of closed form and FEM predictions was improved for the three-truss panel. The increment of artificial insulation stiffness increase used was the 3/2 that applied to the two-truss panel. Deflection was within 5 percent of the measured value, bending stresses were about 10 percent high, and truss forces were over-predicted by a factor of 2 to 2.5.

**DESIGN AND BEHAVIOR IMPLICATIONS**

For the 16 ft (4.88 m) panels examined, predictions from the closed form solution, with modifications to account for the contribution of the truss girder connectors, agree well with predictions from the finite element method. Also, predictions from the closed form solution (and FEM) were found to be conservative with respect to deflections and bending stresses when compared to Stine's experimental data.

The experimental database is limited, and available data suggest that there are behavioral mechanisms not fully captured by either the closed form solution or FEM. Still, the closed form solution has a useful application for design, and for understanding the service load behavior of non-loadbearing semi-composite sandwich panels.

The closed form solution was used to analyze the service load response of simply supported semi-composite sandwich panels. The panels examined used 3-2-3 construction with, and without, trusses similar to those shown in Fig. 2. The following input parameters were used:

- \( b = 96 \text{ in. (2.44 m)} \)
- \( E_c = 4400 \text{ ksi (30.3 GPa)} \)
- \( t = 3 \text{ in. (76 mm)} \)
- \( E_s = 29,000 \text{ ksi (200 GPa)} \)
- \( c = 2 \text{ in. (51 mm)} \)
- \( A_r = 0.047 \text{ in.}^2 \) (30.3 mm²)
- \( G_{ins} = 0.56 \text{ ksi (3.86 MPa)} \)
- \( S = 4 \text{ in. (102 mm)} \)

The number of truss girder lines, \( N \), was varied from zero to six, and panel lengths from 10 to 30 ft (3.05 to 9.15 m) were analyzed. All panels were analyzed with an applied uniform pressure of 30 psf (1.4 kPa). Maximum deflections and bending stresses (i.e., at midspan) were computed. These quantities are solely those caused by service loads; effects of prestress, panel self-weight, and slenderness are not included.

The effects of varying the amount of interface reinforcement on deflections and stresses are shown in Figs. 5 and 6. Panels were analyzed with zero, one, two, four, and six lines of truss connectors over the 8 ft (2.44 m) panel width, and curves can be identified from the legends on the plots (i.e., 2T corresponds to a panel with two truss lines). Also contained in the plots are non-composite (NC) and fully composite (C) response. Results with zero trusses illustrate the contribution due to the insulation only. Results with one truss line are presented to illustrate trends; a single truss line would obviously not be used in an actual panel.

From Fig. 5, the model predicts that the insulation adds significant stiffness as compared to the non-composite response. For a 15 ft (4.57 m) panel, deflection is reduced to 50 percent of the non-composite deflection due to the insulation's contribution to stiffness.
For a 30 ft (9.14 m) long panel, deflection is reduced to 25 percent of the non-composite deflection. As indicated by the curves, adding lines of truss connectors further reduces deflection; however, the benefit of additional lines of trusses quickly diminishes.

The model predicts that increasing from four to six trusses results in only a marginal reduction in deflections. Use of four truss lines in a 15 ft (4.57 m) panel brings deflections to within 65 percent of the fully composite deflection; for a 30 ft (9.14 m) panel, the deflections are within 20 percent of those of a fully composite panel.

However, these deflections are only about 0.15 times those of a non-composite panel. It is apparent that treating a panel with even minimal truss girder reinforcement as non-composite is very conservative with respect to deflections.

Extreme fiber bending stresses at midspan are shown in Fig. 6. As in the case of the deflections, bending stresses are greatly reduced as compared to non-composite response, with larger differences for longer spans. Interestingly, for a given number of truss connectors, the model predicts a nearly constant stress offset when compared to the fully composite curve (C).

As an example, for the four truss panel (4T), maximum bending stress in a semi-composite panel is about 15 psi (103 kPa) higher than for a fully composite panel over the full range of spans analyzed. This means that the percentage difference in predicted bending stresses between a semi-composite and a fully composite panel decreases significantly as the span length increases.

Similar to the deflection observation, there are diminishing returns with regard to stresses as more interface reinforcement is added. Assuming a semi-composite panel to be non-composite is obviously very conservative with regard to stresses caused by external service loads.

Appropriate concern is given to stiffness reduction caused by loss of insulation-to-concrete bond over time. Panels were analyzed with and without the insulation contribution to determine its effect when truss girders were present. A very low value of \( G_{ins} \) (1 \( \times 10^6 \) times the value used for the previous analyses) was input into the prediction model and panels with one, two, and four lines of connectors were analyzed.

Predicted deflections from the analyses are shown in Fig. 7. Curves are shown in pairs, with broken lines identifying the results for panels containing no insulation (the suffix \( G = 0 \) is also added in the plot legend). Even with as few as two lines of trusses, the model predicts that deflections only increase 5 to 12 percent (larger differences for shorter spans) when the effect of the insulation is excluded. Corresponding effects on flexural stresses were slightly less than those for deflections.

This analysis only accounts for the elastic effects of the insulation. Phenomena such as daily cycles of thermal bowing could conceivably also lead to other contributions to stiffness loss, such as deterioration of the concrete locally at the wythe interfaces where truss diagonals are embedded.

In many practical design situations, handling stresses may control the design. Allen's elastic approach can be extended to handle a loading condition similar to that introduced in panel stripping. The authors have developed a closed form solution for a uniformly loaded panel (due to self-weight) with overhangs, but experimental data are needed to verify its suitability. In particular, the assumption of one-way bending may be less appropriate in such a situation.

**CONCLUSIONS**

Based on the results of this investigation, the following conclusions can be drawn:

1. The closed form theory, modified to account for truss connectors, predicted maximum deflections and bending stresses in close agreement with FEM models.

2. As compared to the limited experimental data, the closed form and FEM approaches produced reasonable, but conservative, predictions for panel deflections and bending stresses. Upon verification with more experimental data, the theory holds promise for the elastic design of semi-composite wall panels.

3. A correction was needed to reach close agreement with experimental data, much more so for truss forces (which were still substantially overpredicted). Apparently, there were additional shear transfer mechanisms present in the test panels that were not captured in the closed form and FEM models.

4. The analyses indicate a substantial degree of composite action is provided by truss girders. For a given amount of interface reinforcement, longer panels behave more compositionally than shorter panels.

5. Analysis results predicted that in the presence of truss girders, discount-
ing the insulation stiffness (to represent loss of bond over time) led to only minor increases in predicted stresses and deflections.

**RECOMMENDATIONS**

1. The suitability of the theory should be verified with more test data from panels with other geometries (different wythe thicknesses and panel lengths) and different interface connectors. Additional tests are also needed to categorize the sources of extraneous shear paths and quantify their effects.

2. Experimental data should be obtained to verify the theory's extension to handling and stripping conditions.

3. Tests should be conducted on loadbearing semi-composite panels to establish their behavior and determine if the theory can be modified and extended for these types of panels.

**ACKNOWLEDGMENT**

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APPENDIX A — CLOSED FORM SOLUTION

The continuum approach is valid for sandwich beams with thick faces and a weak core. Further assumptions are that all materials are linear, elastic, and homogeneous, and strain compatibility is maintained between the wythes and the core. The specific relationships derived below are for a case where wythe thicknesses are equal. However, the approach can be generalized for differing wythe thicknesses.

Allen\(^1\) stated that to meet the assumptions of thick faces and a weak core, the following two conditions must be satisfied:

\[
6 \frac{E}{E_{\text{core}}} \left( \frac{d}{c} \right)^2 > 100; \quad 4 \frac{E}{E_{\text{core}}} \frac{t}{d} > 100 \tag{A1}
\]

where

\(E = \) elastic modulus of the wythes (concrete)

\(E_{\text{core}} = \) elastic modulus of the core material (typically insulation)

\(t = \) concrete wythe thickness

\(c = \) core (insulation) thickness

\(d = t + c\) (see Fig. 1)

If these two inequalities are satisfied, the flexural stiffness of the core is less than 1 percent of the flexural stiffness of the concrete wythes, and the maximum core shear stress is no more than 1.01 times the minimum core shear stress (permitting the assumption of constant shear stress over the core depth).

When discrete truss connectors are added to the section, evaluation of the core's elastic modulus is more uncertain. Using an analogy similar to that used for the truss girder's effective shear modulus (described below), the elastic modulus of an inclined diagonal of a truss girder was estimated. This was computed by applying a uniform deformation in the longitudinal direction of the truss (omitting the chords' stiffnesses).

Allen's criteria [Eq. (A1)] would be met if up to 20 truss girders were placed in an 8 ft (2.44 m) wide panel. Thus, it appears that many concrete sandwich panels will meet these criteria, even with inclusion of the effects of truss connectors.

Allen's derivation considers a sandwich beam subjected to a uniformly distributed load, q. By symmetry, only half the beam needs to be considered (origin at midspan, Fig. 1). Due to the load q, the beam undergoes two displacements, \(w_1\) and \(w_2\). The displacement \(w_1\) is due to ordinary bending caused by a shear force \(Q_1\) that is resisted by both the core and the wythes. Displacement \(w_2\) is due to shear deformation of the core under the shear force \(Q_1\).

The displaced shape due to shear deformation of the core is not the same as the bending displacements. To produce compatibility, the stiff concrete wythes participate in the second deflection, \(w_2\), and generate an additional shear force, \(Q_2\), that is associated with non-composite bending (i.e., individual bending of each wythe). The total shear force at a section is:

\[
Q = Q_1 + Q_2 = Q_1 - E_f w_2'''' \tag{A2}
\]

where

\(I_f = bt^3/6 = \) sum of non-composite moments of inertias of the wythes

Derivatives are taken with respect to position, x, along the beam. The term \(w_2''''\) can be related to the shear force, \(Q_1\), leading to the differential equation:

\[
Q_1'' - a^2 Q_1 = -a^2 Q \tag{A3}
\]

where

\[a^2 = \frac{AG}{EI_f \left( \frac{1}{t} - \frac{1}{L} \right)} \tag{A4}\]

\(A = bd^2/c\)

\(G = \) shear modulus of core

\(I = bt^3/6 + bd^2/2\) = total (composite) moment of inertia of the wythes

The parameter \(a^2\) is a measure of the core shear stiffness relative to the local bending stiffness of the wythes. The solution to the differential equation is:

\[
Q_1 = C_1 \cos ax + C_2 \sin ax + Q \tag{A5}
\]

where \(x\) is measured along the longitudinal beam axis (origin at midspan), and \(C_1\) and \(C_2\) are constants.

For a simply-supported beam subjected to uniform load, q (force per unit length), applying the boundary conditions and evaluating the constants, the following equations result for deflections and moments at midspan:

\[
w_1 = \frac{1}{EI} \left[ \frac{5qL^4}{384} - \frac{qL^2}{2} + q - \frac{q}{a^4} \frac{q}{a^4} \frac{q}{a^4} \cosh(aL/2) \right] \tag{A6}
\]

\[
w_2 = \frac{1}{EI} \left[ \frac{qL^2}{8a^2} - \frac{a}{a^2} \frac{a}{a^2} \frac{a}{a^2} \frac{a}{a^2} \cosh(aL/2) \right] \tag{A7}
\]

\[
M_1 = \frac{q}{a^2} \cosh(aL/2) + \frac{qL^2}{8} - \frac{q}{a^2} \cosh(aL/2) \tag{A8}
\]

where

\(M_1 = \) internal moment resisted by the wythes acting compositely

\(M_2 = \) internal moment resisted by both wythes acting non-compositely (\(M_2/2\) for each wythe)

\(L = \) span length

The total midspan deflection can be computed from \(w = w_1 + w_2\), and midspan flexural stresses can be evaluated using superposition of stresses in response to the composite and non-composite bending moments \(M_1\) and \(M_2\) as shown in Eq. (A9):

\[
\sigma_{\text{max}} = \frac{M_1}{I} \left( t + \frac{c}{2} \right) + \frac{M_2}{I_f} \left( \frac{t}{2} \right) \tag{A9}
\]

Modification to Account for Truss Connectors

Internal truss connectors, being much stiffer than the insulation material, increase the shear stiffness of the effective core material. Wu\(^6\) computed an effective shear stiffness, \(G_{\text{eff}}\), that accounted for the change in shear stiffness created
by the presence of one-way continuous truss connectors. This modified shear stiffness could then replace \( G \) in Eq. (A4), and the solution based on Allen’s theory would only be affected by changing the value of the parameter \( a^2 \).

The following assumptions were used: (1) the inclined truss connector diagonals behave as truss members (ignoring bending stiffness) with ends connected at the interfaces between wythes and core materials; (2) the shear connectors are oriented longitudinally and extend the full length of the panels; and (3) the effect of the discrete connectors is smeared across the panel width.

Referring to Fig. A1, the force developed in a single truss diagonal that undergoes a compatible shear deformation, \( \gamma \), of the core is:

\[
F = A_i E_i \gamma \sin \theta \cos \theta \quad \text{(A10)}
\]

where

- \( E_i \) = elastic modulus of steel diagonal
- \( A_i \) = cross-sectional area of steel diagonal

The horizontal component of the truss force is then multiplied by the number of trusses provided across the width of the panel (giving the total horizontal force), and divided by its tributary panel area to produce an equivalent shear stress. Dividing by \( \gamma \) converts this equivalent shear stress to an equivalent shear modulus for the connector diagonals, which is added to the shear modulus of the insulation, resulting in the effective shear modulus of the core:

\[
G_{\text{eff}} = G_{\text{ins}} + G_{\text{truss}} = G_{\text{ins}} + \frac{NE_i A_i \sin^2 \theta \cos \theta}{bS} \quad \text{(A11)}
\]

where

- \( N \) = number of lines of truss connectors provided across panel width
- \( b \) = panel width
- \( S \) = spacing of consecutive diagonals along length of connector (mid-length to mid-length)

The validity of this approach should improve with an increasing number of well distributed truss connectors. A comparison with the FEM results verified that errors are not great even when using only two lines of truss connectors across an 8 ft (2.44 m) wide panel.

Although the approach was not originally intended to predict forces in the truss diagonals, the forces can be estimated by the following procedure. Allen provided an equation to compute the shear stress of the core material, \( \tau = Q E_i d f / (2I) \). The shear force, \( Q \), in this equation should be taken at a section at mid-length of the truss diagonal for which the truss force is to be computed. This shear stress can then be converted to a longitudinal shear force (parallel to the interface) at the section, which is resisted by insulation and trusses in proportion to their relative shear stiffnesses, and the truss force computed as:

\[
F = \frac{G_{\text{truss}} Q b S}{G_{\text{eff}} N \sin \theta} \quad \text{(A12)}
\]
APPENDIX B — NOTATION

- \( a^2 \) = ratio of core shear stiffness to local bending stiffness of wythes
- \( A = bd^2/c \) = shear stiffness parameter
- \( A_1 \) = cross-sectional area of steel diagonal
- \( b \) = panel width
- \( c \) = core thickness
- \( C_d \) = composite moment of inertia correction factor for computing maximum deflection, \( \delta_{\text{max}} \)
- \( C_s \) = composite moment of inertia correction factor for computing maximum bending stress, \( f_{\text{max}} \)
- \( C_1, C_2 \) = constants of integration
- \( d = t + c \) = distance between wythe midfaces
- \( E \) = elastic modulus of wythes
- \( E_c \) = concrete elastic modulus
- \( E_{\text{core}} \) = elastic modulus of core material
- \( E_s \) = elastic modulus of steel diagonal
- \( F \) = force in truss diagonal
- \( f_{\text{max}} = M(t + 0.5c)/(C_s I) \)
- \( G \) = shear modulus
- \( G_{\text{eff}} \) = effective shear modulus of core (including truss connectors)
- \( G_{\text{ins}} \) = insulation shear modulus
- \( G_{\text{truss}} \) = equivalent shear modulus of truss connectors
- \( I =bt^3/6 + btd^2/2 \) = total second moment of areas of wythes about centroid of sandwich beam (composite moment of inertia)
- \( I_1 = bt^3/6 \) = sum of second moments of areas of wythes about their own centroidal axes (non-composite moment of inertia)
- \( L \) = span length
- \( M \) = total bending moment at a section
- \( M_1 \) = internal moment resisted by wythes acting compositely
- \( M_2 \) = internal moment resisted by both wythes acting non-compositely (\( M_2/2 \) for each wythe)
- \( N \) = number of lines of truss connectors provided across panel width
- \( q \) = applied uniformly distributed load (force per unit length)
- \( Q \) = total shear force at section
- \( Q_1 \) = shear force associated with ordinary bending (shared between wythes and core)
- \( Q_2 \) = shear force associated with wythes bending non-compositely due to core shear deformations
- \( S \) = spacing of consecutive diagonals along length of connector (mid-length to mid-length)
- \( t \) = wythe thickness
- \( w \) = total deflection in z direction
- \( w_1 \) = deflection associated with ordinary bending due to \( Q_1 \)
- \( w_2 \) = deflection associated with shear deformation of core due to \( Q_1 \)
- \( x \) = position in span (origin at midspan)
- \( \gamma \) = core shear strain
- \( \Delta \) = axial deformation of truss diagonal
- \( \delta_{\text{max}} = 5qL^4/(384E_c I) \)
- \( \theta \) = inclination angle of truss diagonal, measured from vertical
- \( \sigma_{\text{max}} \) = extreme fiber bending stress at midspan
- \( \tau \) = core shear stress