Tendon Stress in Continuous Unbonded Prestressed Concrete Members — Part 1: Review of Literature

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This paper presents a review of literature relating to prediction of the stress in the tendons at ultimate in continuous concrete members prestressed with unbonded tendons. Laboratory testing of such beams, conducted in Europe and North America in the last 30 years, is reviewed, and a comparison is given between test data and predictions of the tendon stress at ultimate according to the provisions in two codes of practice. Predictions from the equations given in current American and Canadian codes (ACI 318-95 and CSA A23.3-94, respectively) are shown to be inconsistent with the test data, and it is indicated that the pattern of loading is a parameter that needs to be considered in predicting the increase in tendon stress at ultimate in continuous beams. Approaches adopted by recent investigators to account for this factor are outlined and discussed.

Internal unbonded tendons are used widely in prestressing concrete building structures. In the last decade, external prestressing tendons have also found use in the rehabilitation of structures, where they function primarily as unbonded tendons. In addition to the conventional application of unbonded tendons in post-tensioned construction, they are used in precast concrete framing systems, particularly in relation to forming connections for precast structures in seismic zones.[1,2]

Flexural deformation of an unbonded prestressed member subjected to load leads to an increase in the stress level in the tendon. Fig. 1 shows a simply supported, unbonded prestressed concrete beam subjected to a concentrated load. The strain induced in the concrete at the level of the tendon, due to this load, varies according to the bending moment diagram. Compatibility of deformation requires that the tendon elongate by an amount equal to the deformation in the concrete over the length of the tendon, resulting in an increase in the strain in the tendon. The strain increment, and accompanying stress increment, will be uniform over the length of the tendon, assuming that no friction exists between the tendon and its duct.

Determination of this stress increment is necessary to calculate the moment of resistance of a cross section in an unbonded member. Over the past 40 years, numerous experimental and analytical studies have been conducted to identify factors that influence this stress increment. These investigations identified many such parameters, including among others the concrete compressive strength, amounts of prestressed and nonprestressed reinforcement, and the span-to-depth ratio.[3]

Findings from these investigations have served as the basis for the prediction equations adopted by various codes of practice. However, past experimental and analytical work has focused mainly on the behavior of determinate structures, such as simply supported beams. Consequently, the existing body of knowledge on the behavior of continuous beams and slabs, prestressed with unbonded tendons, is limited, with the result that the effects of parameters such as pattern of loading and amount of compression reinforcement at the supports are not considered in code equations.[4]

Part 1 of this two-part paper reviews conclusions drawn from previous studies on continuous concrete beams and one-way slabs prestressed with unbonded tendons. Experimental data from these studies are compared with predictions from two North American codes of practice, ACI 318-95[5] and A 23.3-94,[6] in order to evaluate the validity of provisions in these codes when applied to continuous beams and slabs. In addition, several approaches proposed by recent investigators for calculating tendon stress at ultimate in continuous unbonded members are described.

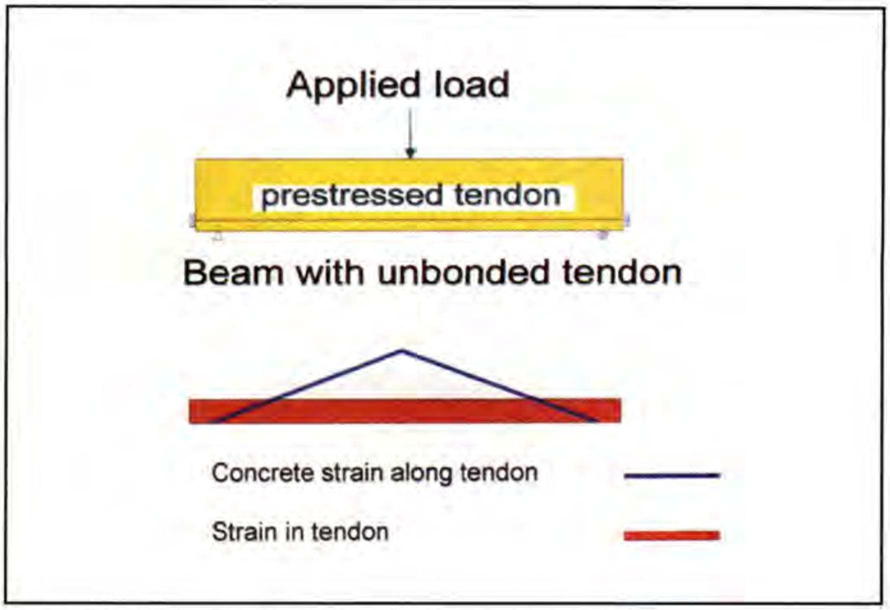

Fig. 1. Diagram showing strain increment in unbonded tendon.

Part 2 of the paper presents a nonlinear numerical model (UBCPB), capable of predicting the response of unbonded, partially prestressed, continuous concrete beams and slabs throughout the entire loading range up to failure. Predictions from UBCPB are compared with available test data in terms of load vs. deflection, and load vs. tendon stress. Results from a parametric study identifying the effects of loading pattern, type of loading and confinement of concrete on the tendon stress at ultimate in unbonded, partially prestressed, continuous concrete beams are presented. Modifications to the Canadian Code equation are suggested.

PREVIOUS EXPERIMENTAL WORK

Experimental work conducted on continuous unbonded prestressed concrete beams and one-way slabs during the last three decades in North America and in Germany is reviewed in this section. The measured increase in tendon stress, Δf_{ps}, at ultimate capacity of the members in a number of these experimental studies is given in Table 1.

Burns and Pierce[7] appear to be the first to include continuous unbonded beams in their test program. Three beams continuous over two equal spans, having a double-tee cross section and a parabolic tendon profile, were tested. The main variables were the amounts of prestressed and nonprestressed reinforcement and the effective level of prestressing (f_{se}).

While the increase in tendon stress at ultimate was not reported, some of their observations and conclusions are of interest: (1) additional bonded reinforcement in the compression zone led to additional rotational capacity and a somewhat longer cracking zone near the support and (2) if proper detailing were done to prevent shear failure, a continuous beam would develop "plastic hinges" at the points of peak moment before reaching ultimate load capacity.

Mattock et al.[8] tested three beams that were continuous over two spans. Two of these beams were prestressed with unbonded tendons while the third had bonded tendons. The beams had a span-to-depth ratio (l/d_p) equal to 33.6, a tee-shaped cross section and a parabolic tendon profile. The beams were subjected to four-point loads on each span in order to approximate a uniformly distributed load. The primary variables were the presence or absence of bond for the prestressed reinforcement, and the amounts of prestressed and nonprestressed reinforcement.

The following conclusions were reported: (1) unbonded continuous beams containing additional nonprestressed bonded reinforcement exhibit ductility and cracking patterns compa-

Table 1. Tendon stress in continuous unbonded prestressed beams and one-way slabs.

Specimen	Type of loading	Measured value of Δf_{ps} MPa	ACI-95 prediction value of Δf_{ps} MPa	CSA-94 prediction value of Δf_{ps} MPa	Remarks
CU-1	Four-point	337	132	205	Two-span T-beams (Mattock et al.[8])
CU-2	loads per span	356	124	207	
S-1	One-point	103*	106	77	Two-span one-way slabs (Hemakom[9])
	load per span	196	106	77	
S-2		169*	107	81	
		239	107	81	
S-3		80*	92	53	
		141	92	53	
Z-I	One-point	192	94	47	Two-span one-way slabs
	load per span	91*	94	47	(Gebre-Michael[10])
Z-II		228	93	43	
		135*	93	43	
C-I	One-point	186	93	76	Two-span one-way slabs (Chen[11])
	load per span	140*	93	76	
C-II		385	110	126	
		175*	110	126	
		163*	110	126	
Slab A	Uniformly	73*	145	86	Three-span one-way slabs
	distributed	108†	145	86	(Burns et al.[12])
	load	135†	145	130	
Slab B		90*	147	74	
		145†	147	74	
		96†	147	110	
VK 2.1	Four-point	547	180	361	Two-span beams (Trost et al.[14])
VK 2.2	loads per span	709	157	308	
VK 2.3		775	157	659	
VK 2.4		752	152	677	
PS-40	Two-point loads per span	420	183	191	Two-span one-way slab (Ivanyi et al.[15])

Note: 1 MPa = 145 psi.
* Single-span loading.
† Two-span loading (three-span slab).

rable to beams with bonded tendons; and (2) a modest amount of compression reinforcement near the center support is required to allow complete redistribution of moment, and consequently a larger increment in tendon stress at ultimate, Δf_{ps}, to develop.

Hemakom[9] and Gebre-Michael[10] tested five one-way slabs continuous over two equal spans with $l/d_p = 60$. Tendon profiles were parabolic and the average prestress level varied from 2.4 to 4.9 MPa (350 to 700 psi). Primary variables were the amount of prestressed reinforcement and the pattern of loading (one-span vs. two-span loading). Loading was by means of a single-point load applied at a distance 0.4 times the clear span length from the center support.

The authors drew the following conclusions: (1) the values of Δf_{ps} for single-span loading were up to 50 percent lower than for double-span loading; (2) the value of Δf_{ps} varied inversely with the percentage of prestressed reinforcement and directly with the compressive strength of the concrete; and (3) the level of effective prestressing did not affect the value of Δf_{ps}.

Chen[11] reported results from tests on two one-way slabs continuous over two equal spans having $l/d_p = 36$ and prestressed with unbonded tendons. Loading was by means of a single-point load applied at a distance 0.4 times the clear span length from the center support. The major variables in this series of tests were the pattern and sequence of loading (one span vs. two-span loading), and the amounts of prestressed and nonprestressed reinforcement.

The following observations were made: (1) the values of Δf_{ps} as predicted by the equations in the 1963 and 1971 ACI 318 Codes [given in subsequent sections as Eqs. (1) and (2)] were found to be conservative; (2) the values of Δf_{ps} for two-span loading were significantly higher than those for single-span loading; and (3) bonded reinforcement served effectively in the distribution of the cracks and enhanced the ultimate moment capacity of the member.

Burns, Charney and Vines[12] tested two half-scale models of a prototype one-way slab, continuous over three equal spans. The specimens, designated as Slab A and Slab B, had span-to-depth ratios of 53. Primary parameters included amount of nonprestressed reinforcement, level of prestressing, and pattern of loading. Each span was loaded with four pairs of point loads in order to approximate a uniformly distributed load. Tests were conducted using various loading patterns to study the behavior of the slabs in the linear and nonlinear ranges.

Test observations and conclusions were as follows: (1) when a slab was loaded to ultimate, the change in tendon stress for the case of two-span loading was nearly double that for single-span loading; (2) the relation between Δf_{ps} and midspan deflection was linear; (3) although the amount of bonded reinforcement (0.12 percent of gross area) provided in Slab A was considerably less than the 0.20 percent required by ACI 318-77, the formation and distribution of cracks in this slab were considered satisfactory; and (4) redistribution of moment was essentially complete at failure (approximately 15 percent).

In a more recent discussion, Burns[13] argued that the value of Δf_{ps} depends on the number of spans being loaded and on the tendon profile in each span. Burns raised the concern that the ACI 318-83 prediction equation [see Eq. (3)] may be unconservative for the case of a continuous member in which only one span is loaded to failure.

Trost et al.[14] tested four two-span, continuous beams having rectangular or T-shaped cross sections. Based on test results, it was concluded that: (1) the main factors influencing Δf_{ps} were the compressive strength of the concrete and the prestressing force applied to the member; (2) the span-to-depth ratio did not affect the value of Δf_{ps}; (3) the presence of nonprestressed bonded reinforcement guaranteed satisfactory crack patterns and ductile behavior; (4) bonded reinforcement must be closely spaced over cracked regions in order to ensure satisfactory load capacity; and (5) the change in tendon stress was proportional to the sum of the deflections at the critical sections.

Ivanyi et al.[15] reported tests on nine two-span, continuous one-way slabs prestressed with unbonded tendons. The parameters studied were load combinations, span-to-depth ratio, level of effective prestressing, tendon profile, and the amounts of prestressed and nonprestressed reinforcement. Test results confirmed the validity of a design approach for partially prestressed, unbonded, one-way slabs suggested for the draft proposal of the German Code, DIN 4227, at that time. The proposed method provided guidelines for proportioning the reinforcement in an unbonded slab to obtain a desired ultimate bending moment capacity and ductility.

Some of the parameters, such as span-to-depth ratio and the amounts of prestressed and nonprestressed reinforcement, identified in the above experimental investigations as affecting the increase in tendon stress at ultimate in continuous members, are similar to those identified in studies conducted on determinate members. However, other parameters unique to continuous members, such as the amount of compression reinforcement at the interior supports where plastic hinging is most likely to initiate, the pattern of loading and the extent of the moment redistribution, have also been identified as having an effect on the value of Δf_{ps}.

PREDICTION EQUATIONS IN NORTH AMERICAN CODES

The evolution of the prediction equations for f_{ps} in the ACI 318 and Canadian A23.3 codes is outlined in this section.

ACI Code

The 1963 version of ACI 318 included, for the first time, a provision for the stress in unbonded tendons at ultimate, f_{ps}. The conservative, and rather simplistic, expression given by Eq. (1) was proposed due to the lack of information at that time regarding the behavior of unbonded members:

$$f_{ps} = f_{se} + 15 \text{ (ksi)}$$
$$f_{ps} = f_{se} + 105 \text{ (MPa)} \qquad (1)$$

where f_{se} is the effective stress in the prestressed reinforcement (after all losses).

Studies conducted by Warwaruk et al.[16] and Mattock et al.[8] showed that Eq. (1) was highly conservative for beams with low reinforcement ratios and unconservative for beams with high reinforcement ratios. Consequently, ACI 318-71 (and subsequently ACI 318-77) adopted a new expression, similar to the one proposed by Mattock et al.,[8] namely:

$$f_{ps} = f_{se} + 10 + \frac{f_c'}{100\rho_p} \text{ (ksi)}$$
$$f_{ps} = f_{se} + 70 + \frac{f_c'}{100\rho_p} \text{ (MPa)} \qquad (2)$$

with the limitations $f_{ps} \le f_{se} + 60$ ksi or $f_{se} + 414$ MPa; $f_{ps} \le f_{py}$ and $f_{se} \ge 0.5 f_{pu}$.

In Eq. (2):

f_c' = compressive strength of concrete

f_{py} = yield strength of prestressed reinforcement

f_{pu} = ultimate strength of prestressed reinforcement

ρ_p = prestressed reinforcement ratio $(= A_{ps}/bd_p)$

A_{ps} = area of prestressed reinforcement

b = width of compression zone

d_p = effective depth of prestressed reinforcement

One of the main criticisms regarding Eq. (2) was that it was unconservative for members with high span-to-depth ratios, as indicated by the findings of Mojtahedi and Gamble.[17] As a result, Eq. (2) was modified in ACI 318-83 to include the effect of the span-to-depth-ratio:

$$f_{ps} = f_{se} + 10 + \frac{f_c'}{\mu\rho_p} \text{ (ksi)}$$
$$f_{ps} = f_{se} + 70 + \frac{f_c'}{\mu\rho_p} \text{ (MPa)} \qquad (3)$$

where $\mu = 100$ for $l/d_p \le 35$; $\mu = 300$ for $l/d_p > 35$; and the same limitations apply as in Eq. (2).

Eq. (3) remained unchanged in ACI 318-89 and ACI 318-95, but has been criticized by several investigators for the following reasons:

1. It is based on a single design parameter f_c'/ρ_p that exhibits poor correlation when plotted against the values of Δf_{ps} from various tests.[18,19]

2. It is discontinuous at $l/d_p = 35$.

3. It does not account for the presence of nonprestressed reinforcement, type of loading, or pattern of loading.

4. It was derived primarily from test data obtained from simply supported, fully prestressed members.

5. Different values are predicted for continuous beams having T-shaped or non-symmetric I-shaped cross sections depending on whether the section being analyzed is subjected to positive or negative bending moment. This is inconsistent with the fact that, neglecting frictional effects, the stress is uniform along the entire length of an unbonded tendon.

Canadian Code

Up to and including the 1977 edition, the Canadian Code (A23.3) adopted the equation recommended by the ACI Code for predicting the stress at ultimate in an unbonded tendon. However, recognizing the deficiencies of Eq. (3), as discussed above, A23.3-M84 embraced a new approach, proposed by Loov,[20] for calculating f_{ps}. The proposed equation, based on the plastic hinge theory initially proposed by Pannell,[21] is as follows:

$$f_{ps} = f_{se} + 5000\frac{\left(d_p - c_y\right)}{l_e} \leq f_{py} \text{ (MPa)}$$

$$f_{ps} = f_{se} + 725\frac{\left(d_p - c_y\right)}{l_e} \leq f_{py} \text{ (ksi)} \quad (4)$$

where

$$c_y = \frac{\phi_p A_{ps} f_{py} + \phi_s A_s f_s - \phi_s A'_s f'_s - 0.85\phi_c f'_c h_f (b - b_w)}{0.85\phi_c \beta_1 f'_c b_w} \quad (5)$$

and l_e is equal to the length of the tendon between the anchors divided by the number of plastic hinges required to develop a failure mechanism in the span under consideration.

In Eq. (5):

A_s = area of nonprestressed tensile reinforcement

A'_s = area of compression reinforcement

f_s = stress in nonprestressed tensile reinforcement

f'_s = stress in nonprestressed compression reinforcement

h_f = flange thickness

b_w = web width

ϕ_p = material reduction factor for prestressed reinforcement (= 0.9)

ϕ_s = material reduction factor for nonprestressed reinforcement (= 0.85)

ϕ_c = material reduction factor for concrete (= 0.6)

β_1 = ratio of depth of equivalent rectangular compression block to depth of neutral axis

The parameter $(d_p - c_y)/l_e$ in Eq. (4) accounts for the amounts and locations of the prestressed and nonprestressed reinforcement, span-to-depth ratio and support conditions. Eq. (4) was also adopted by A23.3-94 except with the constant 5000 increased to 8000, giving:

$$f_{ps} = f_{se} + 8000\frac{\left(d_p - c_y\right)}{l_e} \leq f_{py} \text{ (MPa)}$$

$$f_{ps} = f_{se} + 1160\frac{\left(d_p - c_y\right)}{l_e} \leq f_{py} \text{ (ksi)} \quad (6)$$

where the value of c_y is calculated as before. This change was based on neglecting data corresponding to single-point loading tests considered in the initial derivation, on the assumption that this type of loading is not usually encountered in practice.

Criticism of Eqs. (4) and (6) has been based on the fact that they were derived primarily using data from tests on simply supported members. In addition, both equations fail to account for the type of loading, and, particularly in the case of a continuous tee beam, may give significantly different values depending on whether the midspan or support section is selected.

Campbell and Chouinard[22] found that the values of Δf_{ps} predicted using Eq. (4) for simply supported beams were smaller than those obtained from tests by an average of 50 percent. This under-estimation was reduced to 20 percent when Eq. (6) was used. Naaman and Alkahiri[18,19] showed that, while Eq. (4) was generally on the safe side, poor correlation with test data and inconsistent results were observed in the predicted values for Δf_{ps}. The inconsistency included prediction of negative Δf_{ps} values in some cases.

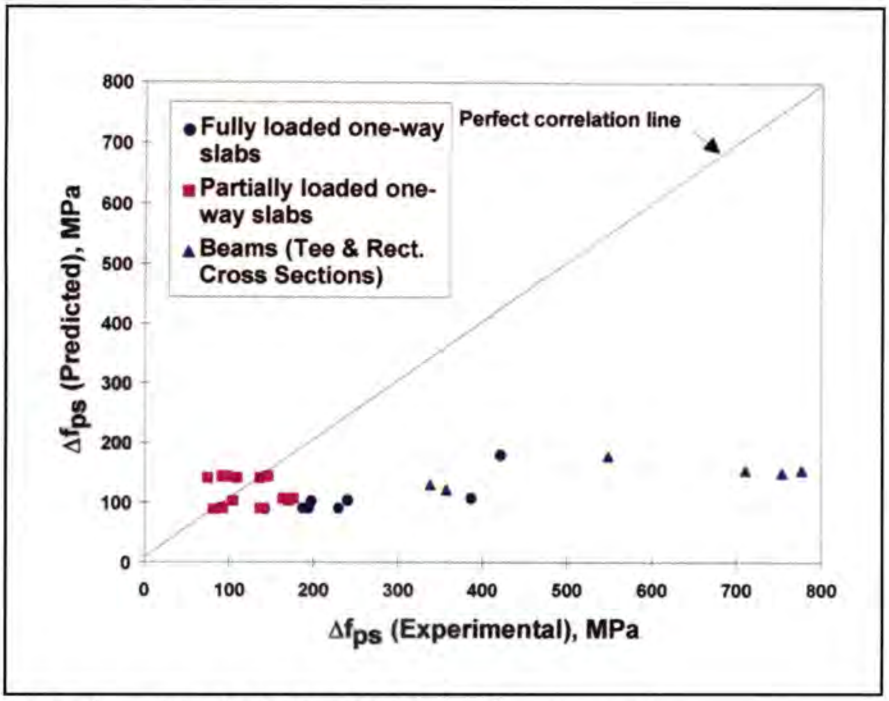

Fig. 2. Δf_{ps} predicted by ACI 318-95 Code vs. Δf_{ps} experimental.

COMPARISON OF TEST DATA WITH CODE PREDICTIONS

Values for Δf_{ps} predicted by Eqs. (3) and (6) are compared with the test data from Table 1 in Figs. 2 and 3. Data plotted above the perfect correlation line indicate an unconservative prediction, while those below the line indicate a conservative prediction.

It can be observed in Fig. 2 that the ACI-95 predictions [(Eq. (3)] for Δf_{ps} show poor correlation with the test data. While predictions for some partially loaded continuous slabs are unconservative, very conservative predictions are given for members with deeper sections (i.e., rectangular and tee beams).

Fig. 3 shows that, although generally conservative, the predictions from A23.3-94 [Eq. (6)] do not follow a consistent trend. In addition, while giving good predictions for partially loaded members, the A23.3-94 equa-

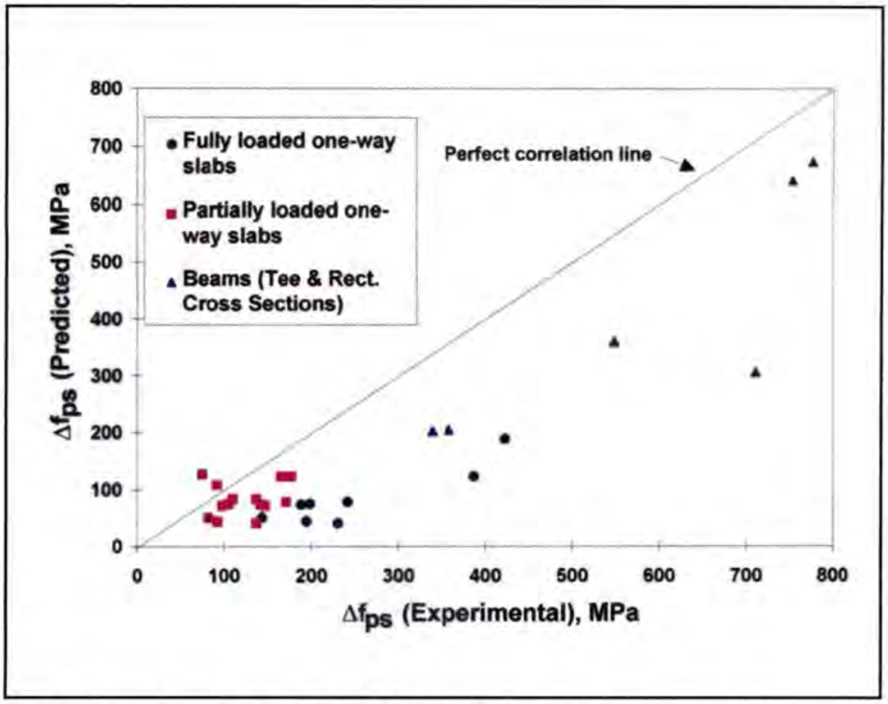

Fig. 3. Δf_{ps} predicted by Canadian A 23.3-94 Code vs. Δf_{ps} experimental.

tion is overly conservative when predicting the increase in tendon stress for continuous members in which more than one span is loaded.

CALCULATION OF f_{ps} FOR CONTINUOUS MEMBERS

The above investigations indicated that the pattern of loading and amount of compression reinforcement over the support had a significant effect on the tendon stress at ultimate. The need to account for the pattern of loading has been recognized by several investigators, and various prediction equations have been proposed.

Naaman and Alkahiri[19,23] proposed the following expression for computing the stress in the prestressing steel at ultimate:

$$f_{ps} = f_{se} + \Omega_u E_p \varepsilon_{cu}\left(\frac{d_p}{c} - 1\right)\frac{L_1}{L} \le 0.94 f_{py} \quad (7)$$

where Ω_u is a bond reduction coefficient, defined as:

$$\Omega_u = \frac{\text{Average change in concrete strain at prestress level in unbonded member}}{\text{Change in concrete strain at critical section of equivalent bonded member}}$$

In Eq. (7), E_p is the modulus of elasticity of the prestressed reinforcement and ε_{cu} is the limiting strain in concrete at ultimate. Eq. (7) accounts for the pattern of loading by means of the ratio L_1/L, which is the length of the loaded portion of the member divided by its total length. It can be seen that Δf_{ps}, as predicted by Eq. (7), increases as the length of the loaded portion of the member increases. The type of loading and span-to-depth ratio is incorporated in the value of Ω_u.

Another approach for dealing with continuous members was proposed by Harajli and Hijazi,[24] who proposed the following equation:

$$f_{ps} = f_{se} + \gamma_s f_{pu}\left(\alpha - \beta\frac{c}{d_p}\right) \le f_{py} \quad (8)$$

$$\text{where } \gamma_s = \left[1 + \frac{1}{\frac{L}{d_p}\left(\frac{0.95}{f} + 0.05\right)}\right]\frac{n_L}{n_0}$$

in which n_L is the number of loaded spans, n_0 is the total number of spans, and the parameters α and β are related to the load geometry factor, f.

Eq. (8) was modified by Harajli and Kanj[25] as follows:

$$f_{ps} = f_{se} + \gamma_0 f_{pu}(1.0 - 3.0q_0) \quad (9)$$

with the limitation $q_0 \le 0.23$ where:

$$q_0 = \frac{A_{ps}f_{se}}{bd_pf'_c} + \frac{A_sf_y}{bdf'_c}$$

$$\gamma_0 = \frac{L_1}{L}\left(0.12 + \frac{2.5}{L/d_p}\right)$$

Thus, the pattern of loading is accounted for in Eqs. (8) and (9) by dividing a measure of the loaded length by a measure of the total length of the member.

Zimmerman and Weller,[26] based on the results of an extensive numerical study, suggested the following equation:

$$f_{ps} = f_{se}(n_1n_2n_3)(n_4n_5n_6) \quad (10)$$

where n_1 to n_6 are factors that depend on the loading pattern, tendon eccentricity, type of cross section, concrete compressive strength and the amounts of prestressed and nonprestressed reinforcement, respectively. The value of n_1, which accounts for the pattern of loading, is a function of the fraction of the loaded length of the beam.

Another expression for predicting the tendon stress at ultimate in continuous unbonded members was proposed by Kordina and Hegger:[27]

$$f_{ps} = f_{se} + \frac{E_p}{L}\sum_1^i k_{bi}k_{vi}k_{si}k_{fi}l_{Gi} \quad (11)$$

where the parameters k_b, k_v, k_s and k_f depend on the concrete strength, percentage of prestressed reinforcement, percentage of nonprestressed reinforcement and cross-sectional shape, respectively. The parameter l_G is the equivalent hinge length, which is a function of the type and distribution of the loading. The subscript i indicates the number of plastic hinges expected to be developed under a given pattern of loading.

The basic assumption behind Eq. (11) is that deformation, which causes the stress increase in the tendons, occurs only in the maximum moment zones (plastic hinges), while the remaining parts of the beam remain undeformed. In contrast to other expressions presented in this review, Eq. (11) accounts explicitly for the individual contributions from each of the hinge zones, in predicting the increase in tendon stress.

SUMMARY OF STUDIES

While many studies have been conducted to investigate the increase in tendon stress at ultimate in a concrete member prestressed with unbonded tendons, only a small portion of these tests has been devoted to continuous members. However, several equations have been proposed recently for prediction of the tendon stress at ultimate in continuous unbonded members.

One parameter addressed by most of these equations is the pattern of loading. Some of the equations use the ratio of the loaded portion of the member to its total length to account for partial loading in a member. This approach has shortcomings when dealing with unequal spans or with spans having different boundary conditions (i.e., an interior span vs. an exterior span).

An alternative method is the consideration of the deformation in the concrete at the level of the tendon along the entire length of the member. Considering the member as a whole is a more theoretically sound approach than using a section analysis, because the increase in tendon stress is a function of the change in concrete strain at the level of the tendon along the entire length of the member, rather than the change in strain at a particular section.

While an exact solution requires a complex iterative approach, a conservative approximation can be obtained. One approach is to estimate the concrete deformation at each of the high moment zones anticipated to develop under a particular pattern of loading, and then sum them up to obtain the average change in concrete strain, and thus the change in tendon stress. This is the approach adopted in Eq. (11), and to a lesser extent in Eq. (6).

Gauvreau[28] and Gilliland[29] have demonstrated that deformation in the concrete due to shear at the level of the tendon is small in comparison to the deformation due to flexure, and thus by estimating the contribution from deformation at each of the high moment zones to the increase in tendon stress, one can obtain a reasonable estimate of the total increase in tendon stress.

Part 2 of this paper describes a mathematical model, which is based on computation of deformation in the concrete over the length of the tendon, developed to predict the increase in tendon stress in simply supported and continuous concrete members prestressed with unbonded tendons.

CONCLUSIONS

Based on the information presented above, the following conclusions may be drawn:

1. There is a lack of experimental data relating to continuous unbonded prestressed concrete beams and slabs.

2. The ACI 318-95 prediction equation shows poor correlation with test data from continuous beams, and may be non-conservative in some cases.

3. The A23.3-94 prediction equation provides good predictions for partially loaded slab members, but may be viewed as overly conservative in the case of beams.

4. A number of parameters associated with continuous members that appear to have a significant influence on the value of Δf_{ps}, are currently not considered in code equations.

5. Using a measure of deformation over the entire length of the member, as a basis for predicting the stress at ultimate of an unbonded tendon, would appear to be a sounder approach than a section-by-section approach.

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APPENDIX — NOTATION

A_{ps} = area of prestressed reinforcement
A_s = area of nonprestressed tensile reinforcement
A_s' = area of compression reinforcement
b = width of compression zone
b_w = web width
c = depth of neutral axis at ultimate
c_y = depth to neutral axis at yield of prestressed reinforcement
d_p = effective depth of prestressed reinforcement
f = load geometry factor
f_{se} = effective stress in prestressed reinforcement (after all losses)
f_{ps} = stress in unbonded tendons at ultimate
f_{py} = yield strength of prestressed reinforcement
f_{pu} = ultimate strength of prestressed reinforcement
f_s = stress in nonprestressed tensile reinforcement
f_c' = compressive strength of concrete
f_s' = stress in the compression reinforcement
Δf_{ps} = increment in tendon stress at ultimate
h_f = flange thickness
k_i = factors used in Eq. (11)
L = length of tendons between anchorages
L_1 = length of loaded portion of member
l = span length
l_e = length of tendon between anchors divided by number of plastic hinges required to develop failure mechanism in span under consideration
l_G = equivalent hinge length
n_i = factors used in Eq. (10)
n_L = number of spans
n_0 = total number of spans

$$q_0 = \frac{A_{ps} f_{se}}{b d_p f_c'} + \frac{A_s f_y}{b d f_c'}$$

β_1 = ratio of depth of equivalent rectangular compression block to depth of neutral axis
ϕ_c = material reduction factor for concrete (= 0.6)
ϕ_p = material reduction factor for prestressed reinforcement (= 0.9)
ϕ_s = material reduction factor for nonprestressed reinforcement (= 0.85)

$$\gamma_0 = \frac{L_1}{L}\left(0.12 + \frac{2.5}{L/d_p}\right)$$

$$\gamma_s = \left[1 + \frac{1}{\frac{L}{d_p}\left(\frac{0.95}{f} + 0.05\right)}\right]\frac{n_L}{n_0}$$

ρ_p = prestressed reinforcement ratio $(= A_{ps}/bd_p)$
Ω = bond reduction coefficient