

## Vibration Characteristics of Double Tee Building Floors



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*Precast, prestressed concrete floors in office, residential and retail buildings are usually found to be satisfactory from a vibration performance viewpoint. Nevertheless, such floors may occasionally undergo detectable oscillations. Furthermore, with advances in concrete technology, the strength and stiffness of floors are optimized, leading to more flexible structures. It is possible that vibration criteria may then govern floor design. Before determining performance and floor acceptability, human tolerance of vibration must be defined and the human comfort factor must be related to some predictable dynamic parameters of the system. These relationships are examined with regard to acceleration, oscillation frequency, amplitude and damping. It is concluded that the double tee cross sections in common use today are not susceptible to vibration problems under normal conditions.*



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**T**he major criteria governing the design of composite precast concrete floors made of double tees are stress, strength and stiffness under service loads. Currently, there are no specific requirements to ensure that floors do not exhibit excessive vibrations. ACI 318-89<sup>1</sup> and most building codes contain limits on static floor deflections under live loads (such as span/360). However, floor vibrations depend on many parameters in addition to floor stiffness, including material density, cross section, floor plan configuration, superimposed loads, type of forcing (loading) function and damping.

Existing specifications and codes of practice give very little guidance, if any, on dynamic floor excitation and

vibration response. Thresholds of acceptable vertical vibrations are totally absent from model codes in the United States. The study of human response to vibration has been, and is still, an important area of concern to engineers. Although a person could be subjected to vibrations from various sources, this paper looks at vibration transmitted through an oscillating building used for office, residential or retail purposes.

Many tests have been conducted to measure human response to oscillations. Reiher and Meister<sup>2</sup> are credited with the first systematic investigation on the effects of both horizontal and vertical whole-body harmonic vibrations on humans. They noted that the psychological identification of per-

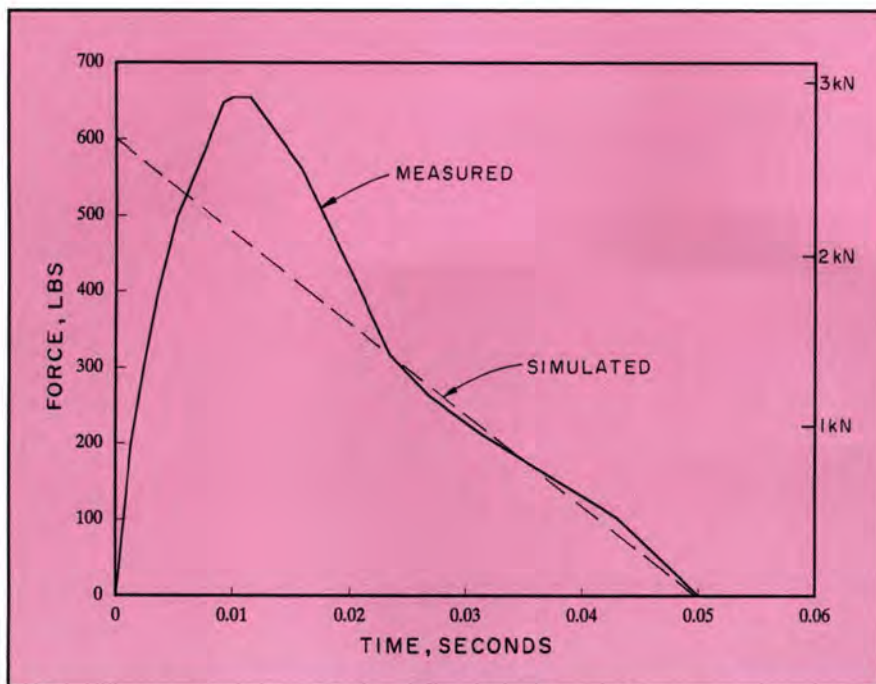


Fig. 1. The single heel impact (drop), based on studies by Murray (Refs. 5 to 8).

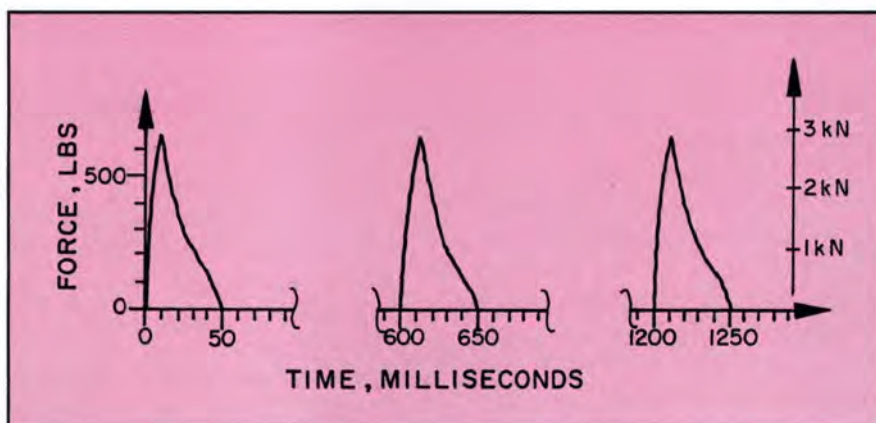


Fig. 2. Multiple heel drop impact history by Tolaymat (Ref. 15).

sonal comfort varies widely among individuals, and also depends on sex, health, age and education. Another psychological aspect worthy of consideration is that occupants of office and residential buildings do not expect to experience perceptible motions in such structures. Therefore, they tend to express deeper concern over any such vibration, even if the level is much below accepted standards.

Finally, when comparing test results from different sources, careful attention should be given to the types of vibrations and test durations. A comprehensive literature review on the subject of vibrations is available in Ref. 3. The following section summarizes recent research and practice on

vibration response of building floors for offices, residences and similar structures.

The objective of this study is to determine whether currently used double tee cross sections would perform satisfactorily based on state-of-the-practice standards for human tolerance of floor vibrations in office environments.

## RECENT RESEARCH AND PRACTICE

Following a comprehensive study of forcing functions, Ellingwood and Tallin<sup>4</sup> reported that the normal rate of walking is about 112 steps per minute, corresponding to a forcing frequency of about 1.86 Hz. They correctly

pointed out the major difference between walking forces and rhythmic activity forces, with the latter having negligible spatial variation in comparison to the temporal variation. They also concluded that the fundamental mode response is the major contributor to human discomfort, and that the use of realistic forcing functions is important in assessing floor sensitivity caused by disturbing dynamic motion.

Between 1981 and 1991, Murray<sup>5-8</sup> published several papers focusing on a revised approach related to the acceptability of steel beam/concrete slab floors subjected to a single heel drop of a person weighing 190 lbs (86 kg). The heel drop excitation used in Murray's studies is based on Ohmart's linear (decreasing) ramp function, having a maximum value of 600 lbs (273 kg) and a duration of 0.050 seconds, as shown in Fig. 1.<sup>9</sup>

Murray recognized the essential importance of the floor damping properties and recommended the following formula for determining the minimum required damping ratio  $D$ :

$$D \geq 35 A_1 f_1 + 2.5 \quad (1)$$

where

$f_1$  = fundamental floor frequency  
 $A_1$  = maximum amplitude under single heel drop impact

According to Murray,  $A_1$  in Eq. (1) can be found from the following equation:

$$A_1 = A/N_{eff} \quad (2)$$

where

$A$  = maximum initial amplitude  
 $N_{eff}$  = number of effective beams, which depends on beam spacing and span, effective slab depth and transformed moment of inertia. This value is determined by an empirical formula.

Murray's approach appears to be satisfactory for structural floors in the frequency range of 5 to 8 Hz. However, the use of Eq. (1) for frequencies above 10 Hz is not recommended.

Foschi and Gupta<sup>10</sup> formulated a reliability-based recommendation for residential floor assemblies consisting of stiffened plates. A finite strip analysis was applied to transient vibrations set up by a foot-fall impact consider-

Table 1. Loading function for walking.  
 $F = P[1.0 + \sum a_i \cos 2\pi ift]$ , where  
 $P = 0.7 \text{ kN (158 lbs)}$

Harmonic $i$	Frequency range $if$	Dynamic load factor $a_i$
1	1.5 to 2.5	0.5
2	3.5 to 4.5	0.2
3	5 to 7	0.1
4	7 to 10	0.05

ing two persons on the floors (an impactor and a receiver). The floor response at the receiver's position is utilized in conjunction with the human tolerance to vibration rating as suggested by Wiss and Parmelee.<sup>11</sup>

Foschi and Gupta<sup>10</sup> recommended that the static deflection under a concentrated load of 225 lbs (1 kN) be limited to 0.04 in. (1 mm), and be independent of the joist span. A serviceability reliability index ( $\beta$ ) of 2.0 was chosen, which means that 2 percent of the floors may exceed Wiss and Parmelee's rating ( $R$ ) of 3, corresponding to distinctly perceptible vibrations.

Smith and Chui<sup>12</sup> investigated the dynamic behavior of wood joist floors attached to plywood or a similar sheathing material and subjected to a rectangular heel drop impulse function. They proposed that the root-mean-square (rms) acceleration be designated as the criterion for judging an occupant's perception tolerance of a residential building. For design purposes, they recommended a frequency larger than 8 Hz.

Osborne and Ellis<sup>13</sup> studied some long span, composite steel floors by using various modeling methods to predict their service performance. Impact tests were conducted to evaluate the vibration parameters for both bare and finished floors. They concluded that frequencies for the bare system were 4.9 Hz for main girders and 8.2 Hz for secondary beams. They also determined that the floors in service would be acceptable by British Standard BS6472, which allows a multiplier of 8 to the ISO (International Standards Organization) basic curve for office use.<sup>14</sup> Although the frequency was within the calculated range, measured damping was below

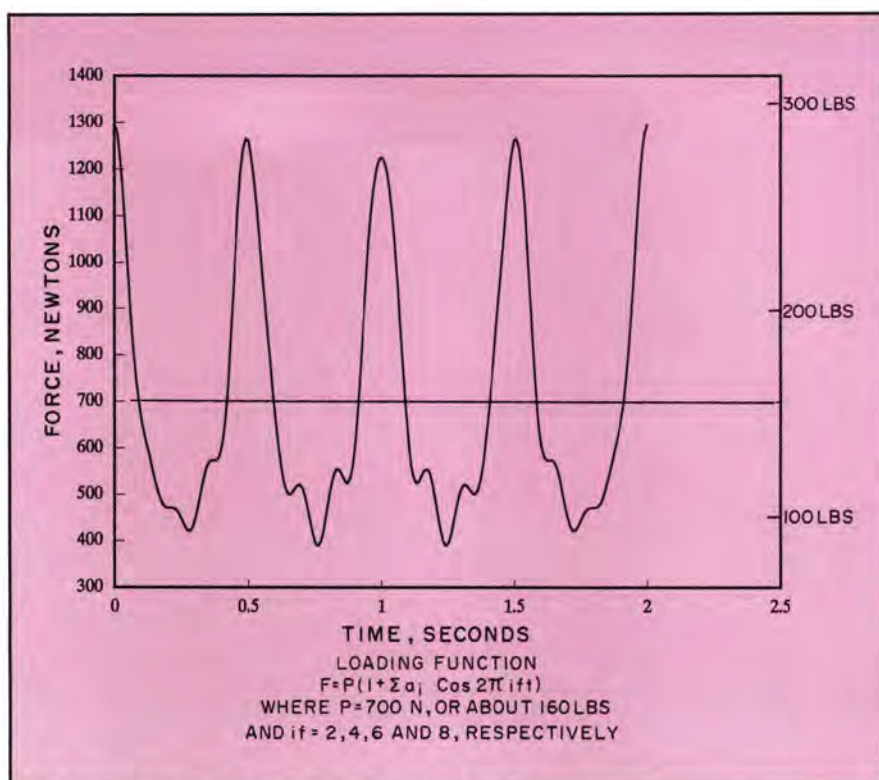


Fig. 3. Loading function for walking by Allen (Ref. 16).

the expected value (2 to 3 percent by Canadian standards).

In 1988, Tolaymat<sup>15</sup> investigated 96 known cases representing a wide range of composite steel systems with spans between 23 and 95 ft (7 and 29 m) and beam spacings from 2 to 24 ft (0.6 to 7.3 m). Half of the cases were found to have acceptable vibrations. He advocated the use of multiple heel drop impulses, as shown in Fig. 2, at intervals of approximately 0.6 seconds, where each impulse is similar to that shown in Fig. 1.

The emphasis of his study was tied to the premise that human perceptibility of vibration is a function of the amplitude rate of decay, and disturbing vibrations are values with amplitudes which do not decay rapidly. Tolaymat's method was shown to disagree with field results only 8 percent of the time.

Allen<sup>16-18</sup> has extensively investigated the subject of floor vibrations. With regard to walking vibrations, he recommended the use of a continuous multiple harmonic expression as a loading function:

$$F = P \{1 + \sum a_i \cos 2\pi ift\} \quad (3)$$

where

$P$  = weight (taken as 0.7 kN for design)

$f$  = stepping frequency

$i$  = harmonic multiplier

$a_i$  = dynamic load factor for the harmonic expression

$t$  = time

Values of the product  $if$  and coefficient  $a_i$  are listed in Table 1, and a typical loading function curve is shown in Fig. 3.

With regard to human tolerance of vibration, Allen recommended the use of the ISO baseline curve for rms acceleration<sup>14</sup> (see Fig. 4), with an adequate multiplier which depends on the occupancy. He conservatively estimated that the "crest ratio" of peak to rms acceleration is 1.70 for typical walking vibrations.<sup>16</sup>

Allen concluded that a multiplier of 5 to 8 (average =  $\pm 6$ ) applied to the ISO base curve is reasonable for office buildings. This converts the ordinate axis to peak values using a factor of 10 ( $= 6 \times 1.7$ ), and obtains an allowable peak acceleration of 0.5 percent ( $= 0.05 \times 10$ ) of gravity for office floor frequencies in the range of 4 to 8 Hz. Shopping malls and footbridges are allowed higher multipliers. Allen's suggested tolerance limits are also dis-

played in Fig. 4 and should be considered conservative because of the approximations in both the crest ratio and multiplier.

Menn<sup>19</sup> cited a formula recommended by Rausch<sup>20</sup> for estimating human sensitivity to structural vibration. According to Rausch, human sensitivity is primarily a function of acceleration, but is commonly quantified in terms of amplitude and frequency. Rausch's approach relies on calculating a sensitivity factor,  $K$ , as follows:

$$K = Af_1^2 / 2(\sqrt{1 + 0.01f_1^2}) \quad (4)$$

where

$A$  = amplitude of oscillation in mm

$f_1$  = fundamental frequency

Allen and Murray<sup>21</sup> recently proposed the following simple criterion for evaluating joist-supported office floors subjected to a loading function similar to that of Ref. 16:

$$\min. \beta W \geq 13e^{0.35f_1} \quad (5)$$

where

$\beta$  = damping ratio (minimum 0.03 recommended for office floors)

$f_1$  = fundamental frequency

The effective mass weight of floor,  $W$ , vibrating in fundamental mode is determined by the following equation:

$$W = w B_j L \quad (6)$$

where

$w$  = unit weight of floor panel including active live load

$B_j$  = effective joist panel width depending on flexural rigidities of structural components

$L$  = joist/beam span

An upper limit of  $B_j$ , equal to two-thirds of the floor's total width perpendicular to the joist/beam, was recommended. Allen and Murray's criterion<sup>21</sup> is based on the ISO's standards.<sup>14</sup>

## FORCING FUNCTIONS AND VIBRATION CRITERIA

After reviewing the state-of-the-practice procedures for evaluating floor vibration perceptibility, the following four criteria are used to predict response and acceptability of double tee floors:

1. Murray's single heel drop as represented by a linear decreasing im-

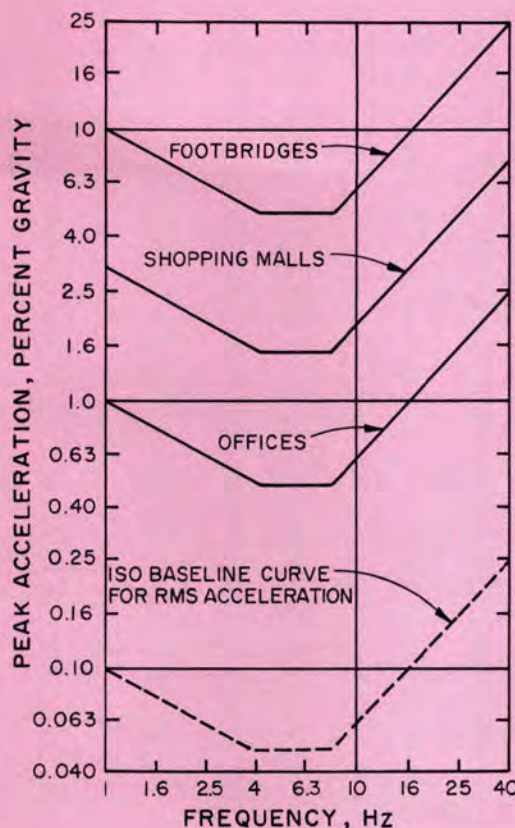


Fig. 4. Recommended acceleration limits for walking vibrations (Refs. 14 and 21).

pulse function (see Fig. 1), with a maximum value of 600 lbs (2670 N) and a duration of 50 milliseconds.

2. Tolaymat's multiple heel drop excitation (see Fig. 2), in which six multilinear heel drops are assumed to be applied at 0.6 second intervals.

3. Allen's continuous multiple harmonic expression [Eq. (3) or Fig. 3] in conjunction with the rms acceleration ( $a_r$ ) and ISO's baseline curve (see Fig. 4), with the baseline multiplier taken equal to 7.

4. The recent joint Allen-Murray cri-

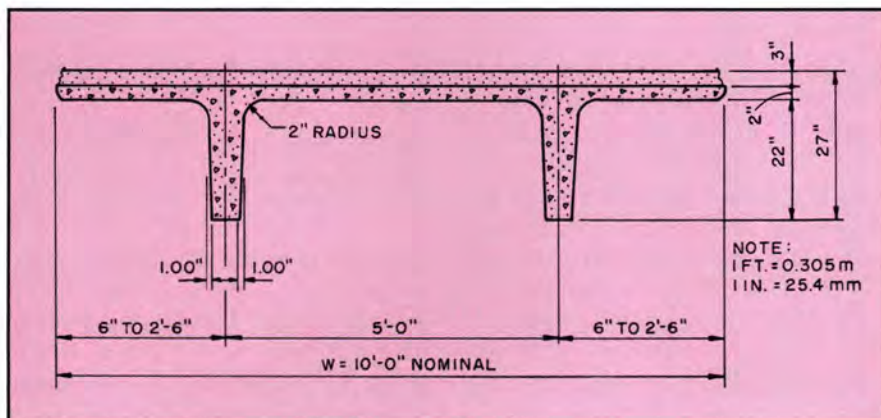


Fig. 5a. Typical composite double tee section.

terion as explained in Eqs. (5) and (6).

Although the first two criteria have been widely used in steel-framed structures, they can also be applied to precast double tee floors because the floor model is structurally similar. As for the third and fourth criteria, they appear to be rational and general with direct application to stemmed floors.

The magnitude of rms acceleration ( $a_r$ ) is calculated from:

$$a_r = \sqrt{\sum a_n^2 / N} \quad (7)$$

where

$N$  = number of acceleration points in time history

$a_n$  = value of acceleration at each point

The crest factor is defined as:

$$\text{Crest factor} = \text{maximum peak acceleration } (a_o/a_r) \quad (8)$$

As Criterion 4 gains more acceptance, it is possible that Criterion 1 may diminish in importance.

## PROBLEM DESCRIPTION

This pilot project is concerned primarily with composite double tee floor systems as shown in Fig. 5a. After consultation with local and regional precasters, the following three precast depths were found to be prevalent: DT18, DT24 and DT32, with common tee widths of 8 to 10 ft (2.4 m to 3.0 m). However, an average width of 9 ft (2.7 m) was used in the present study, since the fundamental frequency changes very little as the width increases from 8 to 10 ft (2.4 m to 3.0 m) as shown in Fig. 6. The deviation of derived dynamic parameters for double tees within this range of widths should be negligible.

Floors were assumed to be simply supported. The number of stems in the main computer runs of the parametric study was set at 17 as shown in Fig. 7. The resulting bay width was 76 ft 6 in. (23 m). The east and west edges of the bay (see Fig. 7) were considered pinned to reflect actual tie conditions.

For a span length,  $L$ , two approximate values were considered for each precast depth ( $h$ ): a short span ( $20h$ ), and a long span ( $28h$ ), which bracket

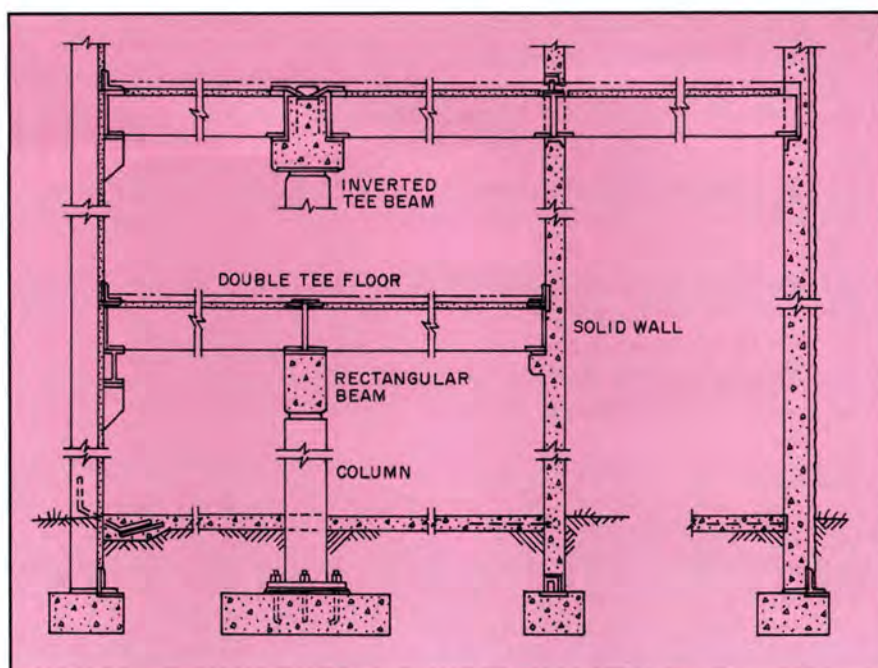


Fig. 5b. Selected framing details for precast concrete office building.

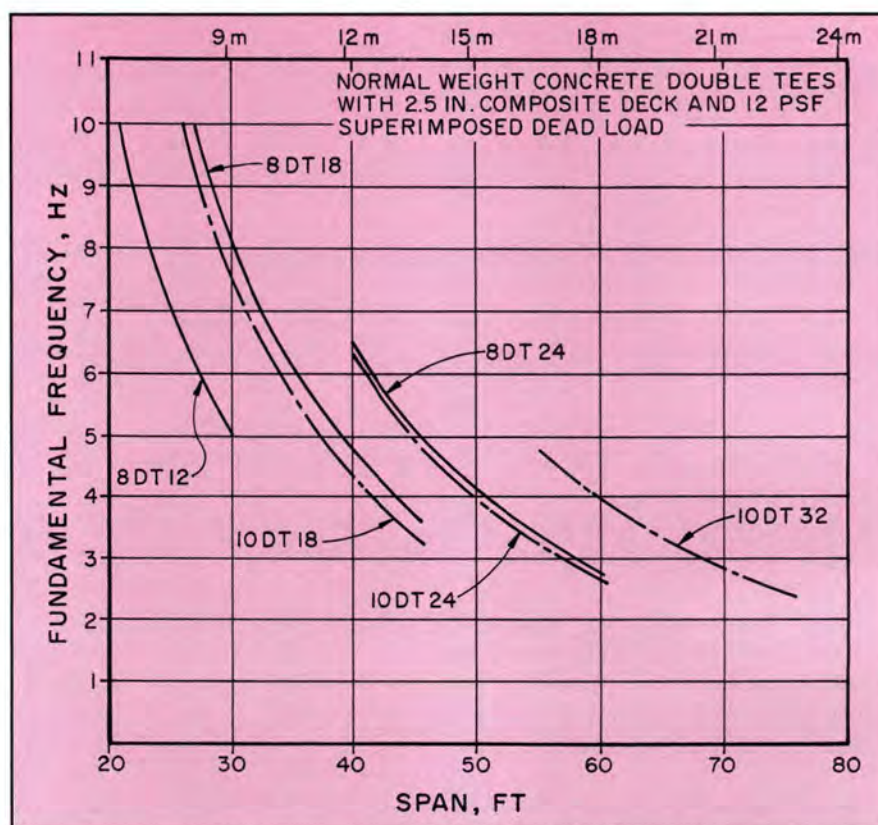


Fig. 6. Fundamental frequencies for a variety of floor tees (Ref. 24).

the majority of real-life structures. Geometries of the double tee sections, summarized and shown in Fig. 8, were borrowed from the PCI Design Handbook, Fourth Edition,<sup>22</sup> and are close to the industry averages.

The assumed cast-in-place (CIP)

topping thickness was a nominal 3 in. (76 mm), equal to the average of currently used values, 2.5 and 3.5 in. (64 and 89 mm). Concrete strengths ( $f'_c$ ) were: 5000 psi (34.5 MPa) for the precast stems and 3000 psi (20.7 MPa) for the flange. It is recognized that this

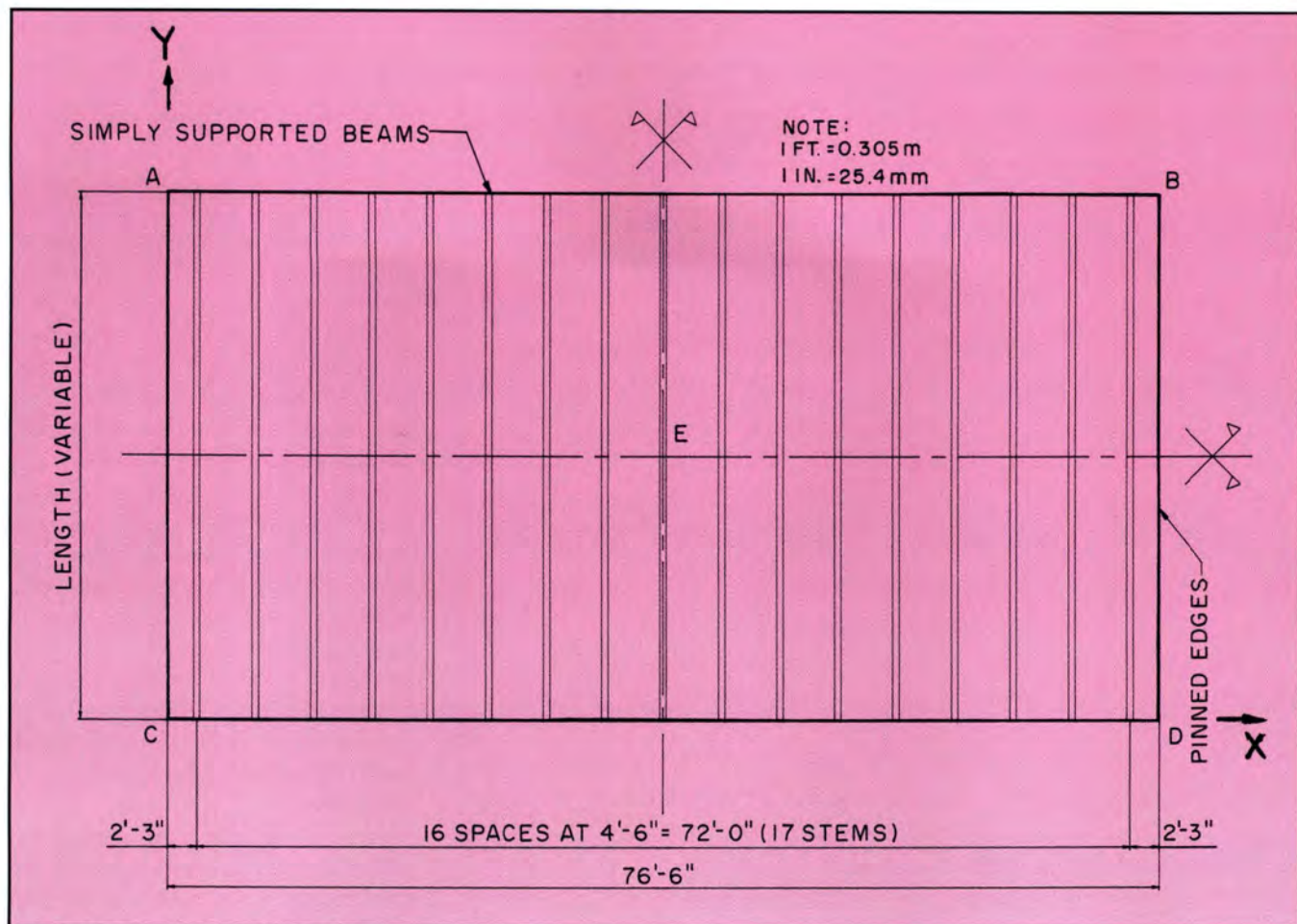


Fig. 7. Plan view of typical floor system.

is a conservative assumption and that actual strengths may be slightly higher. However, one should keep in mind that a higher precision is not warranted in this type of subjective response evaluation.

The corresponding effective Young's moduli used were  $E_1 = 4030$  ksi (27.8 MPa) for the stems and  $E_2 = 3122$  ksi (21.5 MPa) for the horizontal slab elements, which were derived from the familiar ACI Code formula:<sup>1</sup>

$$E_c = 57,000 \sqrt{f'_c} \quad (9)$$

## FINITE ELEMENT MODELING

The physical problem on hand was solved by a linear modal analysis using the general finite element program ADINA.<sup>23</sup> The modal equation is described by:

$$\ddot{x}_i + 2 \xi_i \omega_i \dot{x}_i + \omega_i^2 x_i = r_i \quad (10)$$

where

Table 2. Results based on first criterion with single heel drop (by T. M. Murray). Damping ratio  $D = 0.03$ .

Case No.	Section description and span $L$	First frequency $f_1$ (Hz)	Maximum amplitude $A_1$ (in.)	Maximum peak acceleration $a_o$ (in./sec <sup>2</sup> )	Recommended minimum damping $35 A_1 f_1 + 2.5$ (percent)
1	9DT18 + 3 Span = 30 ft	7.162	0.00380	9.847	3.4
2	Span = 42 ft	3.700	0.00505	6.906	3.2
3	9DT24 + 3 Span = 40 ft	5.642	0.00343	6.552	3.2
4	Span = 54 ft	3.135	0.00419	5.022	3.0
5	9DT32 + 3 Span = 52 ft	4.897	0.00250	4.206	2.9
6	Span = 72 ft	2.606	0.00300	3.139	2.8

Note: 1 ft = 0.3048 m; 1 in. = 25.4 mm.

$x_i$  = generalized displacements  
( $i = r \sim s$ )

$\dot{x}_i$  = generalized velocities

$\ddot{x}_i$  = generalized accelerations

$\xi_i$  = critical damping ratio corresponding to the frequency  $\omega_i$

In Eq. (10),  $r_i$  is defined by:

$$r_i = \{\phi\}_i^T \{F(t)\} \quad (11)$$

where  $\{\phi\}_i$  are the mode shapes and  $\{F(t)\}$  is the dynamic loading vector.

The first five modes were assumed in

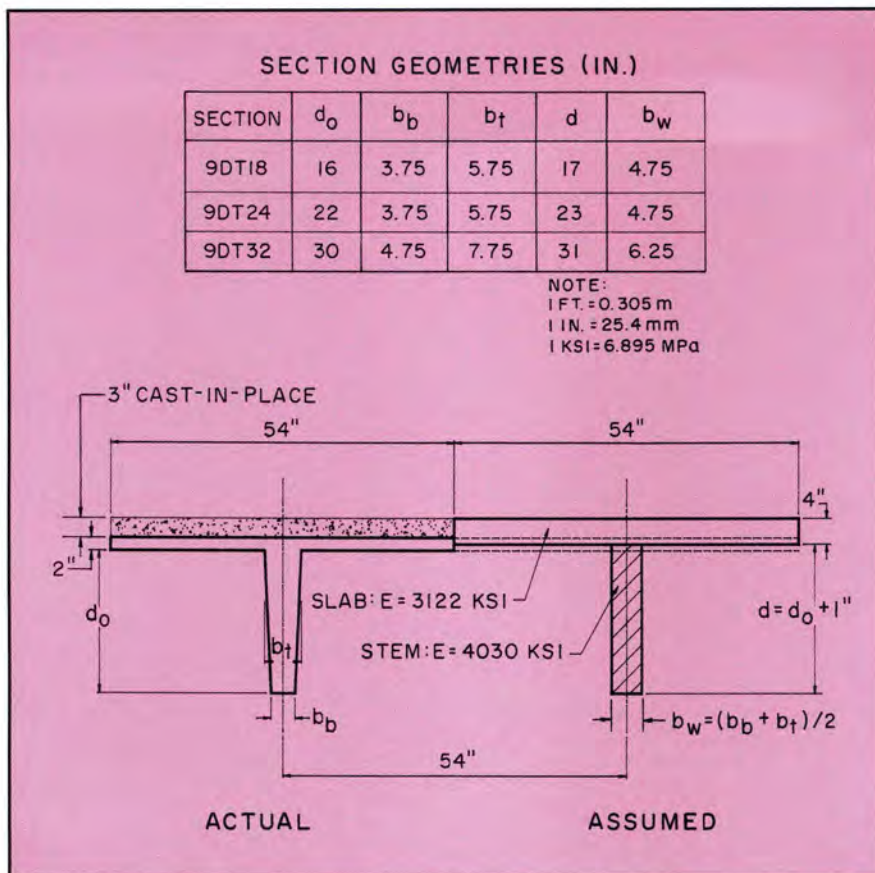


Fig. 8. Actual (left) and assumed tee sections.

the mode superposition analysis. A typical damping ratio of 0.03 (i.e., 3 percent of critical damping) was used, and the time-step size required for numerical solution was typically 0.01 seconds. The double tee structural system was modeled using "shell" elements for the slab and "beam" elements for the stems (see Fig. 8). A standard quadrilateral (four-node) shell element of constant thickness was incorporated in modeling

the horizontal slab.

The stems of the tees were described using a standard rectangular, isoparametric beam element. The composite action of the beam and slab was effected by connecting the centers of the slab and beam with rigid links. This produced the correct constraint relations for displacements of the shell and beam.

As shown in Fig. 7, support for the

structure consisted of a roller at each end of the beams which provided resistance to vertical ( $z$ -direction) movements only. The beams were, therefore, free to rotate at their ends. For structural stability, no  $x$ -displacement was allowed at Points A and B, and hinge support was applied at Points C and D (see Fig. 7). The other edge nodes along Edges AB and CD were modeled as rollers and hinges, respectively. The finite element mesh was proportioned so that the maximum aspect ratio of the quadrilateral elements always remained at 2:1 or less.

In the main model (see Figs. 7 and 8), full continuity and moment-transfer capability were assumed in the 4 in. (100 mm) slab elements. In one case, however, the moment transfer capability at the joints — located 9 ft (2.74 m) on center — was assumed to be negligible or destroyed because of full-depth cracking. In that case, the model would change to a "continuous hinge" assumption.

Since double tees are carried by stocky inverted tee- or L-girders, or by solid precast walls, as shown Fig. 5b, there is generally no need to consider the flexibility of the supports as is sometimes done in composite steel beam framing. Typical discretization of the floor structure is shown in Fig. 9. There were 12 subdivisions in the longitudinal direction. The slab (shell) elements were 2 ft 3 in. (0.69 m) wide in the transverse direction and 4 in. (100 mm) thick (see Figs. 8 and 9). An average thickness for the stem was assumed in the finite element model.

Table 3. Results based on second criterion with six heel drops (by R. Tolaymat). Damping ratio  $D = 0.03$ .

Case No.	Section description and span $L$	First frequency $f_1$ (Hz)	Maximum amplitude $A_{max}$ (in.)	Maximum peak acceleration $a_o$ (in./sec <sup>2</sup> )	Amplitude ratio $A_2/A_1$	$f_1 A_{max}$	Comment
7	9DT18 + 3 Span = 30 ft	7.162	0.00459	12.521	0.877	0.0329	ok
8	Span = 42 ft	3.700	0.00692	10.152	1.138	0.0256	ok
9	9DT24 + 3 Span = 40 ft	5.642	0.00411	10.346	0.940	0.0232	ok
10	Span = 54 ft	3.135	0.00520	7.984	1.030	0.0163	ok
11	9DT32 + 3 Span = 52 ft	4.897	0.00311	6.096	0.978	0.0152	ok
12	Span = 72 ft	2.606	0.00494	4.277	0.860	0.0129	ok

Note: 1 ft = 0.3048 m; 1 in. = 25.4 mm.

## NUMERICAL EXAMPLES AND RESULTS

The main parametric study consisted of analyzing the response of three basic floor sections, as shown in Fig. 8 and here:

- 9DT18+3: minimum span = 30 ft (9.1 m)  
maximum span = 42 ft (12.8 m)
- 9DT24+3: minimum span = 40 ft (12.2 m)  
maximum span = 54 ft (16.5 m)
- 9DT32+3: minimum span = 52 ft (15.8 m)  
maximum span = 72 ft (21.9 m)

The three dynamic forcing (loading) functions described previously (see Figs. 1 to 3) were considered in the numerical study. Dynamic loading was typically applied at the center of the floor system (Point E of Fig. 7). In addition to the self-weight of the tees, dead loads included 37 psf (1773 Pa) for the topping, 15 psf (719 Pa) for partitions and ceiling, and 3.2 psf (153 Pa) for sustained live load. The latter value is based on an assumed ratio (8 percent) of a realistic live load, 40 psf (1917 Pa).

Numerical results and comparisons with the vibration criteria are summarized in Tables 2 to 5. Representative plots for displacements and accelerations are shown in Figs. 10 to 12. Figs. 10 and 11 show the displacements and accelerations for a 9DT24+3 floor spanning 40 ft (12.2 m) and subject to the Tolaymat series of six heel drops. Fig. 12 displays the ratio  $a_n/a_r$  for a 42 ft (12.8 m) long 9DT18+3 floor subjected to the Allen-type loading.

## DISCUSSION

Table 2 indicates a required minimum damping ratio between 2.8 and 3.4 percent. Since the recommended value of damping ratio by Allen<sup>16,21</sup> and other researchers is 3 percent for bare floors, it appears that Cases 4 to 6 will always be satisfactory. Cases 1 to 3 would require either a suspended ceiling or a minimal amount of partitions.

Tables 3 to 5 indicate that all floors are acceptable if a damping ratio of 3

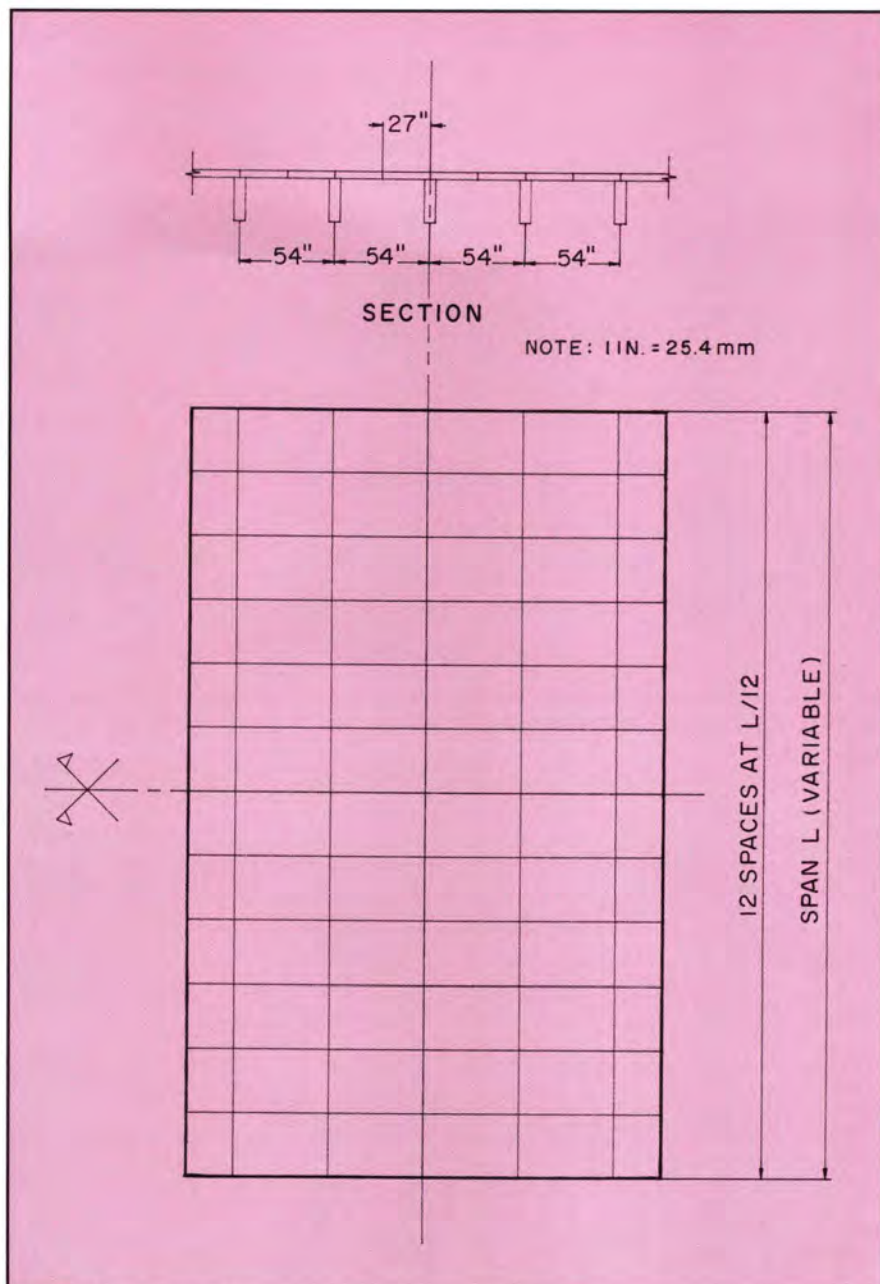


Fig. 9. Partial plan of floor showing discretization of double tees.

Table 4. Results based on third criterion, ISO threshold limits with Allen's forcing function. ISO baseline curve multiplier = 7.0; damping ratio  $\beta = 0.03$ .

Case No.	Section description and span $L$	First frequency $f_1$ (Hz)	rms acceleration $a_r$ (in./sec <sup>2</sup> )	Maximum peak acceleration $a_o$ (in./sec <sup>2</sup> )	Crest factor $a_o/a_r$	Acceptability by ISO (1989) criterion
13	9DT18 + 3 Span = 30 ft	7.162	1.316	5.499	4.18	$a_r < 1.35$ , ok
14	Span = 42 ft	3.700	1.098	4.132	3.76	$a_r < 1.49$ , ok
15	9DT24 + 3 Span = 40 ft	5.642	1.109	3.920	3.53	$a_r < 1.35$ , ok
16	Span = 54 ft	3.135	1.085	3.005	2.77	$a_r < 1.74$ , ok
17	9DT32 + 3 Span = 52 ft	4.897	0.651	2.517	3.87	$a_r < 1.35$ , ok
18	Span = 72 ft	2.606	0.558	1.878	3.37	$a_r < 1.98$ , ok

Note: 1 ft = 0.3048 m; 1 in. = 25.4 mm.

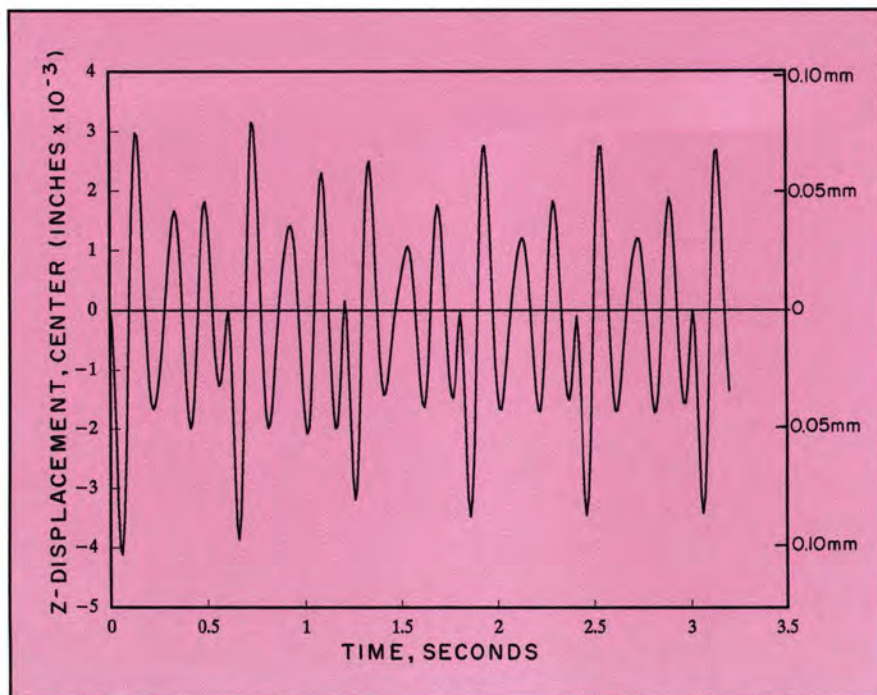


Fig. 10. Center point displacement under multiple heel drops (9DT24 + 3 composite floor).

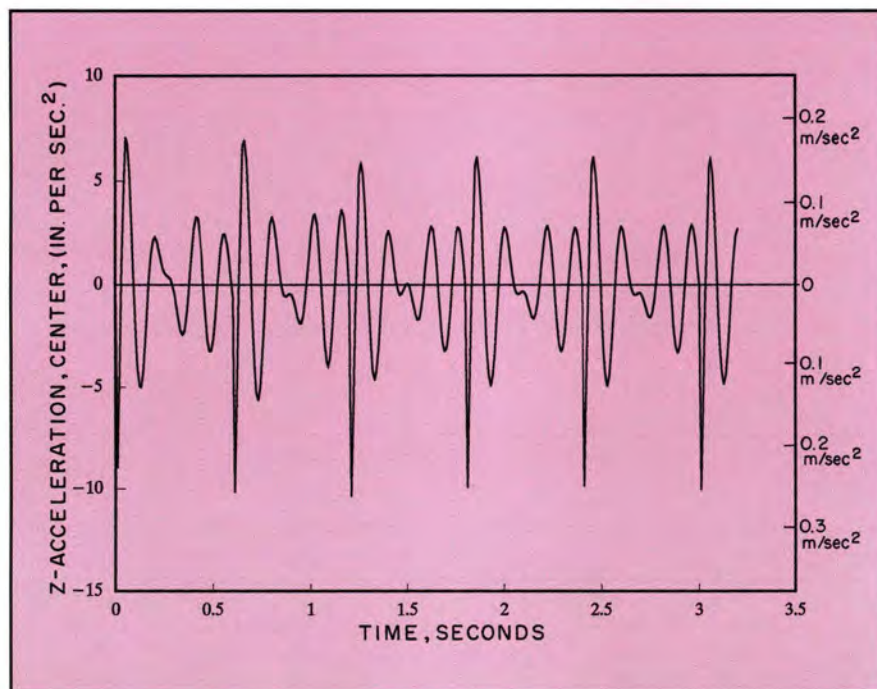


Fig. 11. Center point acceleration under multiple heel drops,  $L = 40$  ft (12.2 m) (9DT24 + 3 composite floor).

percent (for bare floors) is assumed. It is worthy to note that both Cases 8 and 10 in Table 3 and Cases 20 and 22 in Table 5 (representing the same geometry and span) show the closest values to the acceptance limits, despite the fact that tolerance criteria are vastly different.

## SENSITIVITY STUDY

To examine the sensitivity of the dynamic properties variations, a limited study was conducted by changing certain floor parameters moderately. The results are as follows:

### (A) Effect of Damping Ratio

Cases 7, 9, 13 and 15 were re-examined using a damping ratio of 2 percent. The results are shown here:

Case 7 [9DT18 + 3,  $L = 30$  ft (9.1 m)]:  $A_{max} = 0.00466$  in. (0.12 mm);  $a_o = 12.581$  in./sec<sup>2</sup> (320 mm/sec<sup>2</sup>,  $A_2/A_1 = 1.201$ ).

Case 9 [9DT24 + 3,  $L = 40$  ft (12.2 m)]:  $A_{max} = 0.00417$  in. (0.11 mm);  $a_o = 10.871$  in./sec<sup>2</sup> (276 mm/sec<sup>2</sup>),  $A_2/A_1 = 0.935$ .

Both cases are still acceptable by Tolaymat's criterion.

Case 13 [9DT18 + 3,  $L = 30$  ft (9.1 m)]:  $a_r = 1.488$  in./sec<sup>2</sup> (38 mm/sec<sup>2</sup>),  $a_o = 5.525$  in./sec<sup>2</sup> (140 mm/sec<sup>2</sup>);  $a_r > 1.35$  in./sec<sup>2</sup> (34 mm/sec<sup>2</sup>) limit.

Case 15 [9DT24 + 3,  $L = 40$  ft (12.2 m)]:  $a_r = 1.266$  in./sec<sup>2</sup> (32 mm/sec<sup>2</sup>),  $a_o = 3.936$  in./sec<sup>2</sup> (100 mm/sec<sup>2</sup>);  $a_r < 1.35$  in./sec<sup>2</sup> (34 mm/sec<sup>2</sup>) limit.

Using the ISO threshold, Case 13 appears to have a rms acceleration of about 10 percent over the limit. If the baseline multiplier is chosen to be slightly higher (say, 8 instead of 7), then the floor may be considered acceptable.

### (B) Effect of Using Lightweight Precast Concrete Tees

Using a unit weight equal to 115 lbs/ft<sup>3</sup> (1840 kgf/m<sup>3</sup>) instead of the usual 150 lbs/ft<sup>3</sup> (2400 kgf/m<sup>3</sup>) and a uniform Young's modulus  $E_c = 2878$  ksi (19.9 GPa) in Cases 3 and 4 yields the following results by Murray's criterion:

Case 3 [9DT24 + 3,  $L = 40$  ft (12.2 m)]:  $f_1 = 5.24$  Hz, max  $A_1 = 0.00419$  in. (0.11 mm), minimum required  $D = 35 A_1 f_1 + 2.5 = 3.27$  percent.

Case 4 [9DT24 + 3,  $L = 54$  ft (16.5 m)]:  $f_1 = 2.91$  Hz, max  $A_1 = 0.00496$  in. (0.13 mm), minimum required  $D = 35 A_1 f_1 + 2.5 = 3.01$  percent.

By inspection, it is seen that the change in the required damping ratio is negligible.

### (C) Impact of a Refined Mesh

Case 4 was reconfigured using 24 elements per beam instead of the usual 12. The results were:  $f_1 = 3.115$  Hz,  $A_1 = 0.00418$  in. (0.11 mm), and  $a_o = 4.978$  in./sec<sup>2</sup> (126 mm/sec<sup>2</sup>).

The effect on the dynamic parameter is negligible (< 1 percent).

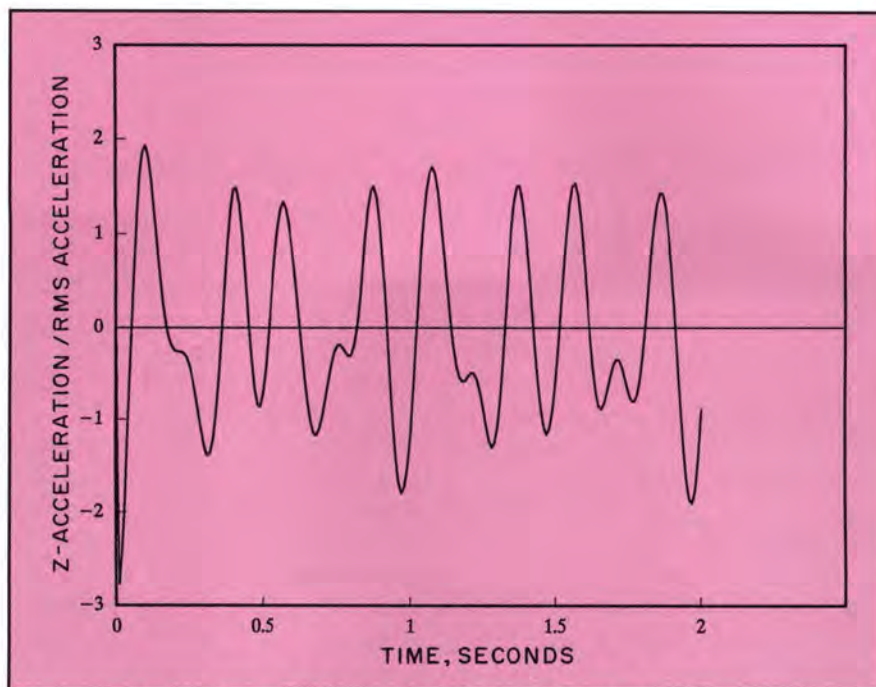


Fig. 12. Center point normalized acceleration ( $a_n/a_r$ ) under Allen type loading.

Table 5. Results based on the joint Allen/Murray approximate criteria for walking vibrations. Damping ratio  $\beta = 0.03$ .

Case No.	Section description and span $L$	First frequency $f_1$ (Hz)	Effective panel width, $B_j$ (ft)	Minimum required $\beta W$ (kips)	Available $\beta W = w B_j L$ (kips)	Comments
19	9DT18 + 3 Span = 30 ft	7.162	0.853L: 25.6	1.06	1.98	ok
20	Span = 42 ft	3.700	35.8	3.56	3.87	ok
21	9DT24 + 3 Span = 40 ft	5.642	0.708L: 28.3	1.80	3.14	ok
22	Span = 54 ft	3.135	36.0*	4.34	5.39	ok
23	9DT32 + 3 Span = 52 ft	4.897	0.559L: 29.1	2.34	5.07	ok
24	Span = 72 ft	2.606	36.0*	5.22	8.68	ok

\* Arbitrary limit of four double tee widths.  
Note: 1 ft = 0.3048 m; 1 in. = 25.4 mm; 1 kip = 4.448 kN.

#### (D) Effect of a Deep Crack in the CIP Deck

Case 16 was re-examined assuming that the CIP deck is incapable of transferring moments at the 9 ft (2.7 m) joint intervals. Only shear transfer is preserved along the joints, which amounts to a virtual continuous hinge along the joint. The computer analysis yielded the following:

$f_1 = 2.792$  Hz,  $a_r = 0.6$  in./sec<sup>2</sup> (15 mm/sec<sup>2</sup>),  $a_o = 1.84$  in./sec<sup>2</sup> (47 mm/sec<sup>2</sup>).

It appears that the frequency has decreased by 11 percent while the rms

and peak accelerations were reduced by 45 and 39 percent, respectively, without affecting the floor acceptability.

#### (E) Impact of a Thinner CIP Deck

Cases 15 and 16 were re-analyzed using a CIP slab that is 2¼ in. (57 mm) thick only. The effective flange thickness becomes equal to 2¼ in. + 1 in. = 3¼ in. (83 mm). Results of the analysis are:

For Case 15: [9DT24 + 3,  $L = 40$  ft (12.2 m)]:

$f_1 = 5.703$  Hz,  $a_r = 1.257$  in./sec<sup>2</sup> (32 mm/sec<sup>2</sup>),  $a_o = 4.28$  in./sec<sup>2</sup> (109 mm/sec<sup>2</sup>).

For Case 16 [9DT24 + 3,  $L = 54$  ft (16.5 m)]:

$f_1 = 3.166$  Hz,  $a_r = 0.928$  in./sec<sup>2</sup> (24 mm/sec<sup>2</sup>),  $a_o = 3.29$  in./sec<sup>2</sup> (84 mm/sec<sup>2</sup>).

It is seen that the frequency change is only about 1 percent. Although the rms acceleration changed by 13 percent, the floors remain within the acceptable range.

#### (F) Effect of Young's Modulus Change

It is common knowledge in the precast concrete industry that the static Young's modulus often varies between 90 and 105 percent of the nominal value defined by ACI 318-89,<sup>1</sup> Eq. (3). On the other hand, many researchers have reported values for the dynamic modulus that are 10 to 15 percent higher than the static value. This implies a possible value of the modulus that may vary between  $0.90 \times 1.10 = 0.99 E_c$  and  $1.05 \times 1.15 = 1.21 E_c$ . Since  $1.21 E_c$  is probably an extreme case, a value of  $1.15 E_c$  was selected for the sensitivity study of Case 14. The following results were obtained based on the increased moduli:

$E_{beam} = 4635$  ksi (32 MPa) and  $E_{slab} = 3590$  ksi (24.8 MPa):

$f_1 = 3.97$  Hz,  $a_r = 1.258$  in./sec<sup>2</sup> (32 mm/sec<sup>2</sup>),  $a_o = 4.114$  in./sec<sup>2</sup> (104 mm/sec<sup>2</sup>).

As expected, the frequency increased by about 7 percent, while the rms acceleration increased by 14 percent. The maximum peak acceleration remained virtually unchanged. The floor is still within the acceptable range.

## CONCLUSIONS AND RECOMMENDATIONS

A computer analytical model using beam and shell elements has been developed to predict the dynamic response of 30 to 72 ft (9.1 to 12.9 m) long double tee floors subject to walking vibrations. Response parameters are used to predict floor acceptability based on four different state-of-the-practice criteria. Based on the results

of this study, the following conclusions for section geometries similar to those in the PCI Design Handbook<sup>22</sup> can be drawn:

1. Using the lengths corresponding to the practical minimum and maximum span-to-depth ratios, all the floors fall within the acceptability range of the stated criteria, provided the damping ratio is 3 to 3.5 percent of critical. Since the accepted minimum damping ratio for a bare floor is usually set at 3 percent, the suspended ceiling and partitions increase the damping ratio substantially and achieving a 3.5 percent damping ratio is usually not a problem.

2. The damping ratio is a major parameter in the floor response and vibrations perceptibility. The presence of partitions and suspended ceilings greatly enhances floor acceptability.

3. The use of lightweight concrete tees or CIP decks that are thinner than 3 in. (76 mm) has a small or negligible effect on the response.

4. The impact of a Young's modulus deviation from the standard ACI value is small.

5. The improved accuracy due to a refined mesh (greater than 12 elements per span) is quite negligible, and, therefore, not warranted.

6. It appears from a single study on a DT24 + 3 floor that the presence of a deep shrinkage crack at the location of tee joints is not detrimental to floor acceptability.

Although this pilot study covered the extreme values of span-to-depth ratios, and the sensitivity study showed negligible or small changes in floor response, future research is needed in the following areas:

a. Sections with large flange widths such as the 12DT sections may require a separate investigation.

b. Pre-topped sections must be investigated in a systematic manner to cover the full range of span-to-depth ratios.

c. Although modeling of hollow-core slabs is significantly more time-consuming, it is feasible, nevertheless, by using powerful computer programs such as ADINA. Because of the significantly different properties, the response parameters for hollow-core

floors will be different from double tee floor systems.

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## APPENDIX — NOTATION

$A$  = initial maximum amplitude of oscillation  
 $A_1$  = maximum amplitude under single heel-drop impact  
 $B_j$  = effective joist panel width  
 $D$  = damping ratio  
 $E_c$  = Young's elastic modulus of concrete  
 $F(t)$  = forcing function  
 $K$  = a sensitivity factor  
 $L$  = span length  
 $N$  = number of accelerations  
 $N_{eff}$  = number of effective beams

$P$  = average weight of person  
 $a_i$  = dynamic load factor  
 $a_n$  =  $n$ th acceleration  
 $a_o$  = maximum peak acceleration  
 $a_r$  = root-mean-square acceleration  
 $f$  = foot stepping frequency  
 $f_1$  = fundamental frequency  
 $f'_c$  = compressive strength of concrete  
 $i$  = harmonic multiple or index number  
 $r_i$  = generalized loads

$t$  = time  
 $W$  = effective weight of floor vibrating in fundamental mode  
 $w$  = unit weight of floor panel including active live load  
 $x_i$  = generalized displacements  
 $\dot{x}_i$  = generalized velocities  
 $\ddot{x}_i$  = generalized accelerations  
 $\beta$  = damping ratio  
 $\xi_i$  = damping ratio  
 $\omega_i$  = vibration frequencies  
 $\{\phi\}_i$  = mode shapes

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