

# Strengthening of Concrete Beams by External Prestressing



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*The use of external prestressing tendons as a means of strengthening or upgrading concrete flexural members was experimentally investigated. Sixteen beam specimens were first subjected to cyclic fatigue loading at a constant load range to induce fatigue deformations. Then, they were externally prestressed and subjected to monotonically increasing load to failure. As a result of external prestressing, the nominal flexural strengths of the beams were increased by up to 146 percent and the induced fatigue deflections were reduced by up to 75 percent. External tendons using a draped profile were relatively more effective in increasing the flexural strength than tendons with a straight profile. The stress ranges and mean stress levels in the internal tension reinforcement decreased considerably, which imply a significant improvement in the fatigue life of the strengthened beams.*

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**E**xternal prestressing is a prestressing system in which the concrete structural members are prestressed longitudinally using tendons located completely outside the concrete section. Today, external prestressing is considered one of the most powerful techniques for strengthening or rehabilitating existing concrete bridge structures.<sup>1</sup> Also, because of the construction speed and economy associated with the use of external tendons, it is becoming popular in the construction of new concrete bridges.<sup>2-4</sup>

Several evaluations of existing bridges which were either constructed or strengthened using external tendons, together with the results of limited experimental studies on external prestressing, have been reviewed by

Trinh.<sup>5</sup> All these studies show that there are distinct benefits in using external prestressing.

The advantages of an external tendon system have been best described by Virlogeux<sup>1</sup> and, more recently, have been emphasized by Rabbat and Solat.<sup>4</sup> Some of these advantages include:

- More economical construction
- Easier tendon layout, placement and consolidation of concrete
- Better corrosion protection as compared to a conventional tendon system

The use of an external tendon system to strengthen existing concrete bridges is conceptually different from that used in the traditional design and construction of new bridges. In the design of new concrete bridges, external



tendons constitute the primary reinforcement; hence, the analysis and design could be achieved using methods similar to unbonded post-tensioned construction.<sup>4</sup>

When used for strengthening, however, external tendons represent only a part of the total flexural reinforcement. The remaining reinforcement could be mild reinforcing steel, internal prestressing steel, or a combination of both, depending on the type of concrete structural system to be rehabilitated. Also, more importantly, the level of externally applied load and the extent of deformation (crack widths and deflections) at which external prestressing is utilized to strengthen existing concrete structural members differs from those when used in the design of new concrete members. The corresponding differences makes the assessment of the advantages of external prestressing with regard to the structural behavior different in both cases.

A limited number of experimental studies have dealt with external prestressing as a means of strengthening concrete flexural members.<sup>5,6</sup> Also, a simple methodology for the analysis of beams prestressed or partially prestressed with external or internal unbonded tendons in the linear elastic cracked and uncracked range of behavior was described by Naaman.<sup>7</sup>

Unfortunately, most of the experimental studies dealing with external prestressing were concerned with increasing the flexural strength by providing additional external tendons along with the internal reinforcement before subjecting the beams to applied load. Except for some field applications and evaluations, no experimental study has yet been undertaken to examine the advantages of external prestressing in improving the service load behavior and simultaneously increasing the flexural resistance of concrete members.

## OBJECTIVE

The objective of this investigation was to examine experimentally the benefits of external prestressing as used in the strengthening of concrete flexural members and to evaluate its

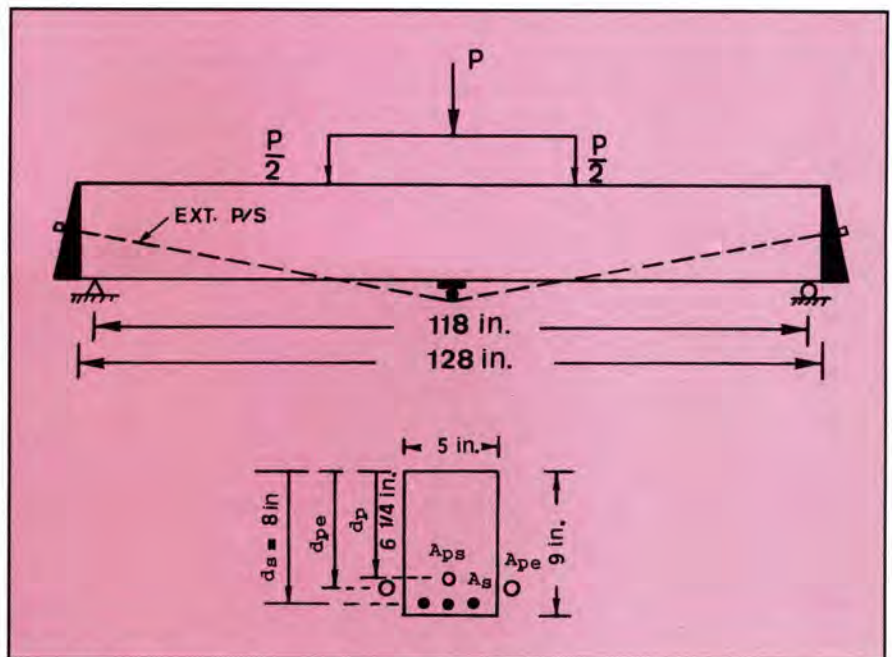


Fig. 1. Beam dimensions and steel layout.

effect on both the service load behavior and nominal flexural resistance.

## EXPERIMENTAL PROGRAM

Sixteen beam specimens with a 5 x 9 in. (127 x 229 mm) rectangular cross section and simply supported on a 9.83 ft (3.0 m) span were tested. All specimens were loaded in four-point bending using two symmetrical concentrated loads applied at one-third the span length.

To simulate actual conditions of concrete flexural members that require strengthening or rehabilitation, large fatigue deformations were induced in the specimens prior to external prestressing by subjecting them to about 5 to 10 thousand cycles of large amplitude fatigue loading at a constant load range. The load range varied between a minimum load  $P_{min}$  and a maximum load  $P_{max}$  equal to about 30 and 80 percent, respectively, of the calculated ultimate flexural load resistance of the specimens.

The load levels  $P_{min}$  and  $P_{max}$  were selected to simulate self-weight and self-weight plus superimposed dead load and full service overload, respectively. The choice of a large amplitude fatigue loading was intended to induce large fatigue deformations at relatively small number of cycles without encoun-

tering fatigue failure of the specimens.

At the end of cyclic fatigue loading, the specimens were externally prestressed while loaded with  $P_{min}$  and then subjected to a monotonically increasing load to failure.

## Test Variables

The test variables included three different concrete structural systems:

- Reinforced concrete (R/C)
- Prestressed concrete (PC)
- Partially prestressed concrete (PPC)

For each case, three contents of tension reinforcement or reinforcing index (defined in accordance with ACI 318-89<sup>8</sup>) were selected (except two for the PPC system).

For each type of specimen and level of reinforcing index, two profiles of the external prestressing steel were examined: straight horizontal tendon profile and a single-point draped tendon profile with a deviator (saddle) at midspan. The depth to the bottom of the deviator for the deviated tendons was 10.75 in. (273 mm) and their eccentricity at the anchorage ends was zero. The eccentricity of the straight tendons at the anchorage ends was 3.12 in. (80 mm).

Two magnitudes of external prestressing force were selected depending on the level of reinforcing index. Specimens with a relatively low reinforcing index were externally pre-



Table 1. Summary of prestressing steel and mild reinforcing steel parameters.

Beam specimen	External prestressing steel (wires)			Internal prestressing steel (seven-wire strands)		Reinforcing steel $A_s$ (in. <sup>2</sup> )	Yield stress of reinforcing steel $f_y$ (ksi)	Concrete strength $f'_c$ (ksi)	
	Profile	$A_{ps}$ (in. <sup>2</sup> )	$f_{pe}$ (ksi)	$f_{pu}$ (ksi)	$A_{ps}$ (in. <sup>2</sup> )				$f_{pe}$ (ksi)
B1D	Draped	2 (5 mm) 0.06	135.30	233	1 (5/16) 0.058	147	2 (6 mm) plain bars	60.00	5.25
B1S	Straight		127.50		162	0.088	40.00	5.00	
B2D	Draped	2 (7 mm) 0.12	110.70	207	2 (5/16) 0.116	161	2 (6 mm) plain bars	60.00	5.25
B2S	Straight		124.00		147	0.088	40.00	4.80	
B3D	Draped	2 (7 mm) 0.12	114.00	207	2 (3/8) 0.17	153	2 (6 mm) plain bars	60.00	5.10
B3S	Straight		132.50		156	0.088	40.00	5.00	
B4D	Draped	2 (5 mm) 0.06	127.50	233	0	–	2 (10 mm) 0.24	45.00	4.40
B4S	Straight		141.00		–	–	68.60	4.00	
B5D	Draped	2 (7 mm) 0.12	122.00	207	0	–	3 (12 mm) 0.53	80.00	4.70
B5S	Straight		114.50		–	–	62.00	5.48	
B6D	Draped	2 (7 mm) 0.12	111.00	207	0	–	3 (14 mm) 0.72	81.00	4.90
B6S	Straight		117.00		–	–	81.00	3.80	
B7D	Draped	2 (7 mm) 0.12	119.00	207	1 (5/16) 0.058	158	2 (10 mm) 0.24	46.00	4.50
B7S	Straight		125.00		158	–	68.60	5.53	
B8D	Draped	2 (7 mm) 0.12	121.00	207	2 (3/8) 0.17	145	2 (12 mm) 0.35	80.00	5.00
B8S	Straight		119.50		151	–	80.00	5.60	

(1) The area of compression steel was zero for all specimens.

(2) Span length = 118.5 in.; total beam length = 128 in.; beam width = 5 in.;  $d_p$  (interior prestressing steel) = 6.25 in.;  $d_s$  (reinforcing steel) = 8.0 in.

(3) For draped external prestressing steel, the depth to the bottom of deviator at midspan = 10.75 in., and eccentricity at end support = 0.0 in.

(4) For straight external prestressing steel, eccentricity at end supports = 3.12 in.

Metric conversion factors: 1 in. = 25.4 mm; 1 in.<sup>2</sup> = 645 mm<sup>2</sup>; 1 ksi = 4.46 kN.

stressed with a force equal to about 8.5 kips (38 kN), while specimens with a moderate and high reinforcing index were externally prestressed with a force equal to approximately 15.0 kips (67 kN).

Specimen geometry and reinforcement layout are shown in Fig. 1. Beam parameters and input variables are listed in Table 1.

### Materials and Beam Fabrication

Seven-wire strands, with  $\frac{3}{8}$  in. (9.5 mm) diameter and Grade 270 (1862 MPa) and with  $\frac{5}{16}$  in. (8.0 mm) diameter and Grade 250 (1725 MPa), were used as the pretensioned internal prestressing steel. The external prestressing steel consisted of either 5 or 7 mm (0.197 or 0.276 in.) diameter wires with ultimate strength equal to 233

Table 2. Stress and strain properties of prestressing steel.

Properties	5/16 in. strand Grade 250	3/8 in. strand Grade 270	7 mm plain wire Grade 207	5 mm plain wire Grade 233
Ultimate breaking stress $f_{pu}$ (ksi)	284.0	287.0	207.0	233.0
Stress at 1 percent extension (kips)	242.0	245.0	192.0	203.0
Strain at $0.6f_{pu}$ (in./in.)	0.0061	0.0061	0.0041	0.0047
Modulus of elasticity (ksi)	28,300.0	28,200.0	30,000.0	30,000.0
Cross-sectional area (in. <sup>2</sup> )	0.058	0.085	0.06	0.03

Metric conversion factors: 1 ksi = 6.895 MPa; 1 kip = 4.46 kN; 1 in. = 25.4 mm.

and 207 ksi (1607 and 1427 MPa), respectively. The stress-strain properties of the internal and external prestressing steel are given in Table 2.

Grade 60 (occasionally Grade 40) deformed reinforcing bars were used in the reinforced concrete specimens and as bonded non-prestressed reinforcement in the partially prestressed

concrete specimens. Shear stirrups, made out of No. 2 (6 mm) plain bars, were provided in the shear span of all specimens at a constant spacing equal to 4 in. (102 mm). Two longitudinal No. 2 (6 mm) plain bars were provided in the tension zone of the fully prestressed concrete specimens to support the shear stirrups.



The concrete mix was prepared using Type I cement, crushed stone of  $\frac{3}{8}$  in. (10 mm) maximum aggregate size and fine masonry sand. The aggregate:sand:cement proportions by weight were 1:0.75:0.4 with a water-cement ratio equal to 0.42. The concrete strength was determined from three 6 x 12 in. (152 x 305 mm) cylinders taken from each specimen. The concrete compressive strengths at the time of testing for all beam specimens are listed in Table 1.

Prestressed and partially prestressed concrete specimens were precast/pre-tensioned inside a 23 ft (7 m) long bed using a 40 kips (178 kN) capacity and 3 in. (76 mm) stroke center hole hydraulic ram. The anchorage stress was monitored accurately using strain gauges attached to the surface of the prestressing strands. The strains measured were converted to stresses using a predetermined apparent modulus of elasticity (because of the spiral orientation of the strands) equal to 33,200 ksi (229 GPa) for the  $\frac{3}{8}$  in. (9.5 mm) strands and 32,600 ksi (225 GPa) for the  $\frac{5}{16}$  in. (8.0 mm) strands, respectively.

The prestressing force was released three days after concrete casting using flame cutting. The stress in the internal prestressing steel (effective prestress) at the time of testing was calculated using the time-step method described in Ref. 9. The effective prestress of the internal prestressing steel at the time of testing is listed in Table 1.

### Instrumentation

The strains in the tensile reinforcement during testing were measured using electrical resistance strain gauges (ERSGs) attached to the surface of the reinforcement. Two strain gauges were used for each prestressing strand and one strain gauge was attached on each outward reinforcing bar. Two strain gauges were used on each exterior prestressing wire at one-fourth the span length from either side. The corresponding layout of the strain gauges was used to check any difference in the stress between the anchorage and prestressing ends due to possible friction at the deviator for the draped external tendon profile. The difference was negligible.



Fig. 2. Closeup view of deviator.

Deflection was measured at mid-span using two linear voltage differential transducers (LVDTs) attached on either side of the specimens.

Strain gauge readings during tensioning of the internal and external prestressing steel, and strain gauge and LVDT readings during testing were monitored automatically using a computerized data acquisition system.

### External Prestressing

Each specimen was externally prestressed using two wires located at symmetrical distances of 1 in. (25 mm) from either side relative to the longitudinal axis of the beam. The two wires were tensioned simultaneously from one end using the same ram used in tensioning the internal prestressing steel. Special care was exercised to balance the prestressing force in the wires to avoid biaxial bending of the specimens.

The wedge anchored prestressing wires were supported directly on an 8 in. (203 mm) wide x 9 in. (228 mm) deep x  $\frac{3}{4}$  in. (19 mm) thick bearing plate attached to the ends of the beam. A tapered bearing plate was used for the deviated tendons to comply with their inclined profile. The deviator at midspan (see Fig. 2) consisted of a 1 $\frac{1}{2}$  in. (38 mm) diameter semi-cylindrical rod welded to a 3 in. (75 mm) wide x 2 in. (50 mm) thick x 8 $\frac{1}{2}$  in. (215 mm) long rectangular plate attached at the bottom surface of the beam at midspan.

The target effective prestress  $f_{pe}$  in the external prestressing steel was  $0.6f_{pu}$ . The corresponding stress was monitored accurately using the readings of the strain gauges attached to the surface of the prestressing wires along with the readings of the pressure gauge of the hydraulic ram used in the prestressing operation. Values of the stress in the external prestressing steel (effective prestress) for the various beam specimens are listed in Table 1.

### Testing Procedure

The test was conducted using an MTS closed loop servo-hydraulic machine with a 100 metric ton capacity dynamic actuator and with stroke and load control capabilities. The supports allowed both horizontal and angular movement of the beams and, hence, simulated roller supports.

The specimens were first loaded to  $P_{max}$  and then subjected to two statically applied load cycles between  $P_{min}$  and  $P_{max}$ . While the level of  $P_{max}$  in the cyclic load range was selected in advance (about 80 percent of the calculated nominal flexural load), it was increased or decreased in some specimens depending on the level of damage (crack widths and deflections) encountered in the first cycle and during subsequent cycles.

The specimens were then subjected to high cycle fatigue loading at a frequency of 1 to 1.5 Hz, depending on the compliance of the beam specimen. The fatigue loading was interrupted at



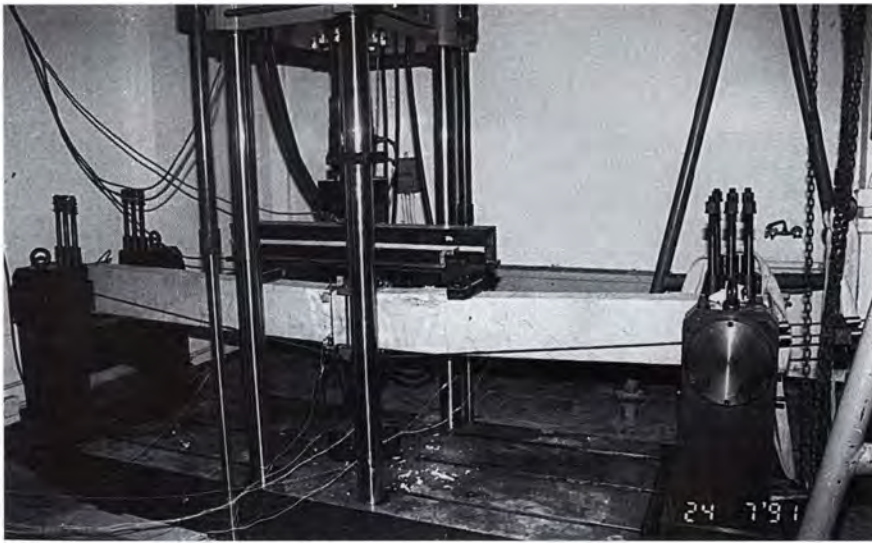


Fig. 3. Typical view of the test setup.

Table 3. Fatigue loading history of the beam specimens.

Beam specimen	$P_{min}$ (kips)	$P_{max}$ (kips)	Number of fatigue cycles
B1D	2.24	5.38	15,000
B1S	2.24	4.48	10,000
B2D	3.36	7.85	10,000
B2S	3.36	7.85	5000
B3D	4.04	9.64	7000
B3S	4.04	9.64	7000
B4D	1.57	4.48	10,000
B4S	1.57	4.48	5000
B5D	4.48	13.45	10,000
B5S	4.04	10.09	7000
B6D	6.28	15.25	5000
B6S	6.28	15.25	Failed at 500
B7D	2.91	6.73	5000
B7S	2.91	6.73	5000
B8D	5.61	13.45	5000
B8S	5.61	13.45	5000

Metric conversion factor: 1 kip = 4.46 kN.

a prescribed number of cycles and a statical cycle between  $P_{min}$  and  $P_{max}$  was conducted to collect measurements from various measuring devices.

At the end of fatigue loading, each specimen was externally prestressed while loaded with a constant load equal to  $P_{min}$ . By selecting a load control test, it was possible for the actuator head of the testing machine to move automatically upward during external prestressing and thus maintain a constant preset actuator load (equal to  $P_{min}$ ).

All measurements, such as beam deflections and strains in the internal prestressing and reinforcing steel, were recorded immediately before and immediately after external prestressing. Following external prestressing, the specimens were subjected to a monotonically increasing load to failure using a stroke control test. The total testing time took an average of 3 to 5 hours, depending on the frequency of the cyclic load and the ultimate load capacity and ductility of the beam specimen.

A typical view of the beam specimens during testing is shown in Fig. 3.

## TEST RESULTS

A summary of the relevant test results is given here. Further details of the experimental program and test results are reported in Ref. 10.

The fatigue loading histories for the various beam specimens are given in Table 3. Typical plots of the load-deflection response of the beam specimens during cyclic fatigue loading (before external prestressing) and their responses after external prestressing are shown in Fig. 4. A summary of the ultimate flexural strengths and deflection characteristics before and after external prestressing is provided in Table 4.

All beam specimens cracked during the first load cycle to  $P_{max}$ . The average crack spacings were 4.7, 3.4 and 3.8 in. (118, 86 and 97 mm) for the fully prestressed, partially prestressed and reinforced concrete specimens, respectively. Specimens B6, B7 and B8, which were heavily reinforced, showed significant inclined shear cracks in the vicinity of the supports.

The level of fatigue deformations induced during cyclic fatigue loading depended mainly on the type of the concrete structural system. In general, the progressive increase in deformations (deflections) with increasing number of loading cycles tended to be smaller for the reinforced and partially pre-

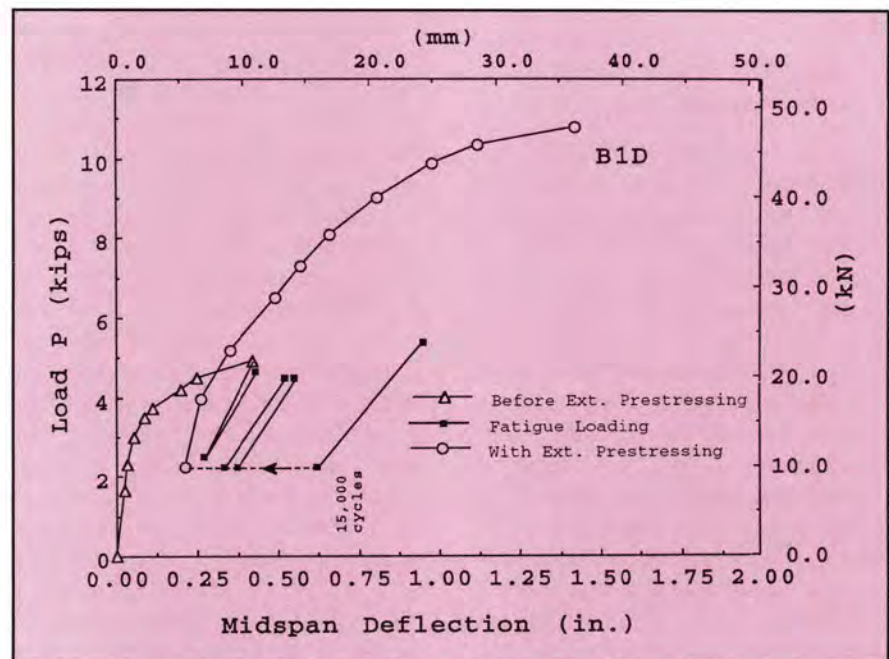


Fig. 4. Typical load-deflection response of beam specimens.



stressed concrete specimens than for the prestressed concrete specimens.

This observation is consistent with the experimental and analytical findings of Refs. 11 to 13, in which reinforcing bars in partially prestressed concrete beams were reported to play a major role in enhancing their fatigue life and improving their deformation performance as compared to precracked fully prestressed beams. Because the beam was heavily reinforced and the concrete strength was relatively lower than anticipated, only one specimen (B6S) failed in fatigue of concrete in compression, at about 500 cycles.

### Load-Deflection Response

It can be seen in Fig. 4 that the induced fatigue service load deflections between  $P_{min}$  and  $P_{max}$  were reduced considerably upon external prestressing. As shown in Table 4, the percentage reduction of deflection at minimum load level  $P_{min}$  for the various beam specimens varied between 34 percent (Beam B5S) and 75 percent (Beam B7D) and was most significant for the fully prestressed and partially prestressed concrete specimens. It should be pointed out that while the cracks in the fully prestressed and partially prestressed concrete specimens closed completely upon external prestressing, they remained slightly open (visible) in the reinforced concrete specimens.

As indicated in Fig. 4, the load-deflection response after external prestressing exhibited three stages of behavior. In the first stage, and for specimens in which the cracks closed completely after external prestressing (prestressed and partially prestressed specimens), the flexural stiffness was very close to the stiffness during the initial loading before external prestressing. As the cracks tended to reopen with increasing applied load, a second stage of behavior was seen to initiate with a reduced flexural stiffness, followed by a third stage which corresponded to the "yield" of the internal tension reinforcement.

Because of controlled crack growth due to external prestressing, the flexural stiffness of the specimens in the second stage of behavior was significantly

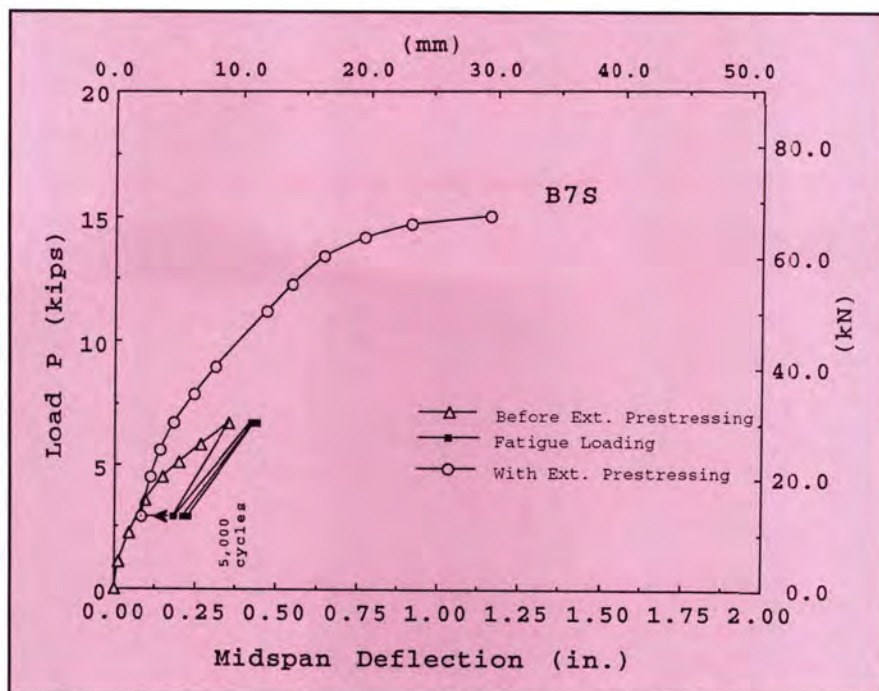


Fig. 4 (cont'd). Typical load-deflection response of beam specimens.

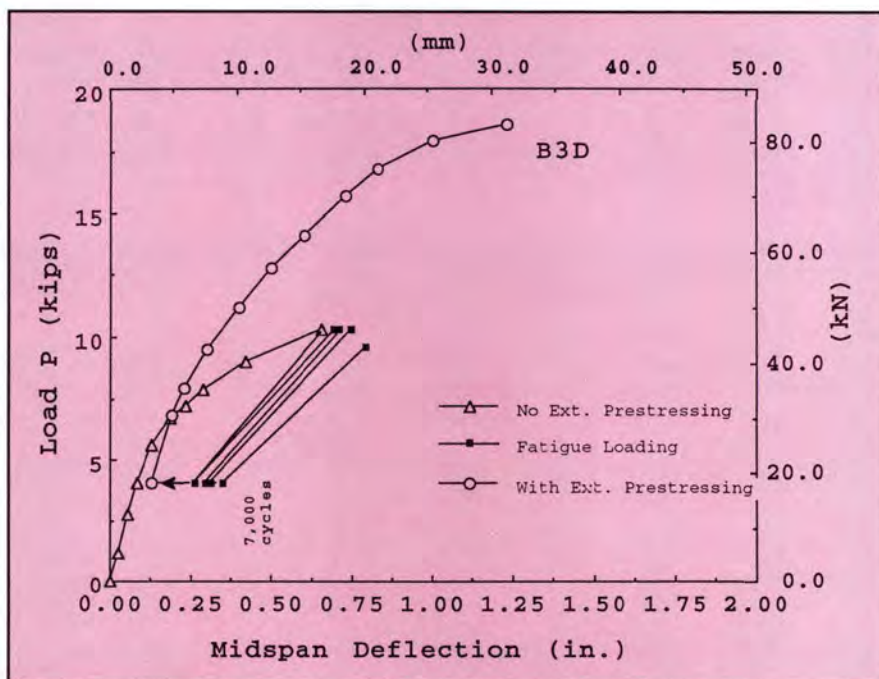


Fig. 4 (cont'd). Typical load-deflection response of beam specimens.

larger than their stiffness in the post-cracking range during initial loading but approximately equal to their stiffness in the last fatigue loading cycle before external prestressing. For specimens in which the cracks did not close completely after external prestressing (reinforced concrete specimens), only two stages of load-deflection response were predominant as shown for Specimen B5D in Fig. 3.

In general, because of a stiffer response at service load, the live load deflection (deflection mobilized between  $P_{min}$  and  $P_{max}$ ) was generally lower after external prestressing as compared to the deflection observed in the last fatigue cycle before external prestressing.

The reduction in deflection at minimum load level  $P_{min}$  (upward deflection caused by external prestressing)



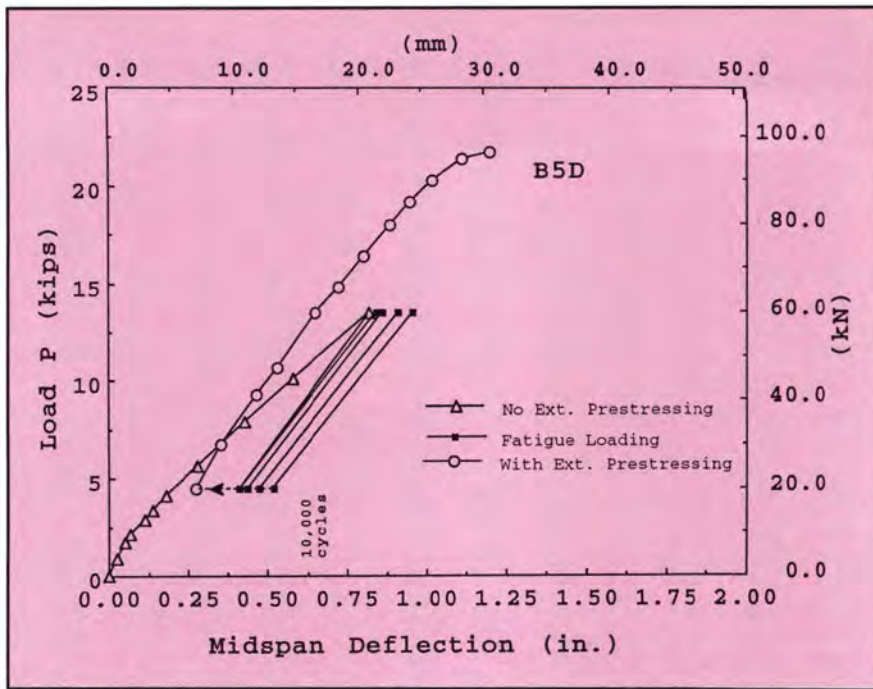


Fig. 4 (cont'd). Typical load-deflection response of beam specimens.

for specimens externally prestressed using the draped profile was, on the average, 35 percent larger than those externally prestressed using the straight profile (see Table 4). In the

ory, assuming linear elastic behavior and that everything else is the same (including external prestressing force and flexural stiffness of the members), the reduction in deflection as a result

of external prestressing is expected to be about 36 percent larger for the deviated profile than for the straight profile used in this investigation.

The ultimate deflections of the beam specimens externally prestressed using the deviated tendon profile were of the same order of magnitude as those externally prestressed using the straight horizontal profile (Table 4). However, the ultimate deflection tended to decrease, as expected, with increasing content of tension reinforcement.

### Ultimate Flexural Resistance

Since the depth of the deviated external prestressing steel decreased toward the end supports, flexural failure of all beam specimens externally prestressed using the deviated tendon profile occurred at either one of the critical sections under the point of application of the loads (one-third the span length). On the other hand, failure of all beam specimens externally prestressed using the straight profile occurred inside the constant moment region at or very close to midspan.

Table 4. Summary of ultimate flexural strength and deflection characteristics of beams before and after external prestressing.

Beam specimen	Ultimate flexural strength $M_u$ (kip-in.)			Measured deflection (in.) at $P_{min}$			Measured ultimate deflection (in.)
	Calculated* before external prestressing	Measured after external prestressing	Percent increase	Before external prestressing	After external prestressing	Percent reduction	
B1D	117.7	212.6	81	0.62	0.21	66	1.42
B1S	106.6	177.6	67	0.28	0.12	57	1.64
B2D	182.5	310.4	70	0.48	0.20	58	1.23
B2S	168.6	256.4	52	0.51	0.19	63	1.24
B3D	238.6	368.4	54	0.35	0.13	63	1.23
B3S	227.8	288.3	27	0.41	0.21	49	1.15
B4D	83.3	205.0	146	0.62	0.39	37	2.10
B4S	123.7	209.0	69	0.29	0.11	62	1.36
B5D	294.2	426.9	45	0.52	0.28	46	1.20
B5S	239.7	326.9	36	0.50	0.33	34	1.13
B6D	384.9	465.0	21	0.51	0.33	35	1.03
B6S	—	—	—	—	—	—	—
B7D	155.6	332.1	113	0.27	0.07	75	1.26
B7S	195.3	298.5	53	0.23	0.08	63	1.16
B8D	349.5	477.0	36	0.35	0.17	51	1.19
B8S	362.4	393.7	9	0.35	0.22	37	0.98

\* Calculated using ACI 318-89.

Metric conversion factors: 1 kip-in. = 0.1133 kN-m; 1 in. = 25.4 mm.



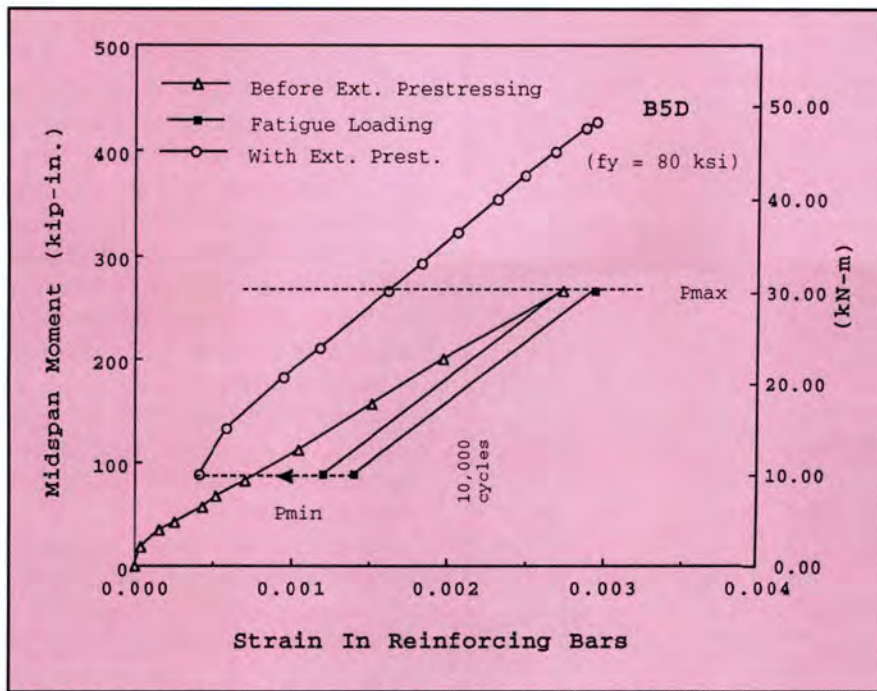


Fig. 5. Variation of the strain in reinforcing steel with applied load.

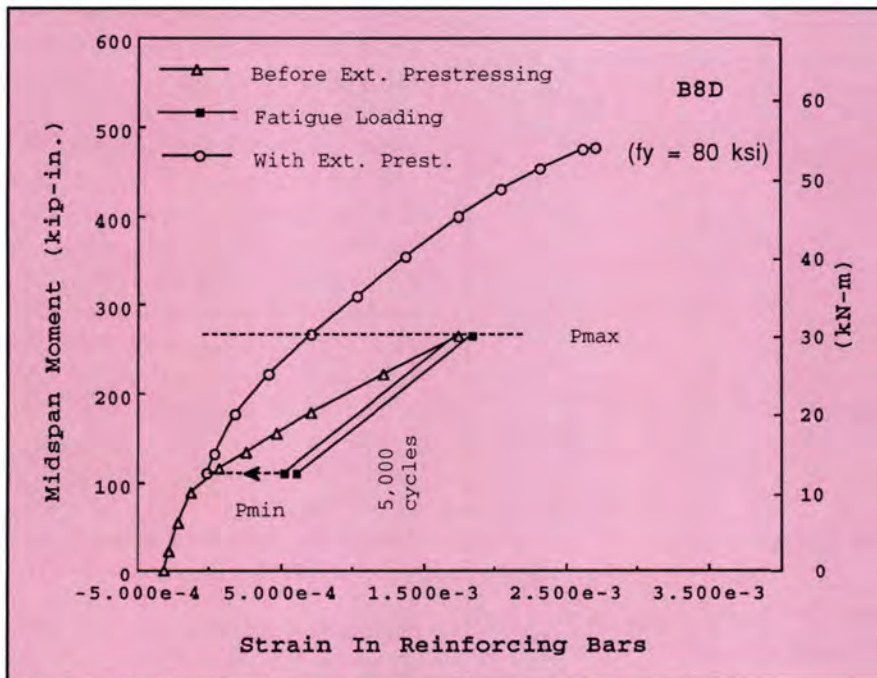


Fig. 5 (cont'd). Variation of the strain in reinforcing steel with applied load.

Because of their free vertical movement relative to the longitudinal axis of the beam, external tendons with a straight horizontal profile encountered a progressive reduction in their depth with increasing beam deflection to failure. The corresponding reduction, measured at midspan in a few specimens, was almost equal to the increase in central beam deflection throughout the load-deflection re-

sponse after external prestressing.

Columns 2 and 3 of Table 4 show a comparison between the calculated ultimate flexural resistance of the beam specimens before external prestressing and the observed flexural strength after external prestressing. The ultimate flexural strength before external prestressing was calculated using the ACI 318-89<sup>8</sup> approach, in which the actual yield stresses  $f_y$  of the mild re-

inforcing steel as given in Table 1 were used.

As shown in Table 4 (Column 4), the increase in the nominal flexural resistance of the various beam specimens due to external prestressing varied between a minimum of 9 percent (Beam B8S) and a maximum of 146 percent (Beam B4D). The corresponding increase depended mainly on the content of internal tension reinforcement in the specimens and the level of external prestressing force.

The increase in ultimate flexural strength for specimens externally prestressed using a straight tendon profile were consistently lower than for those externally prestressed using a deviated profile. This is attributed partly to the larger depth of the deviated external tendons in the vicinity of the critical bending region, and partly to the progressive reduction of depth of the straight external tendons with increasing member deflection to failure.

It should be pointed out that, despite the formation of significant shear cracks in the shear span of heavily reinforced specimens (B6, B7, B8) and with the increase in the ultimate load capacity due to external prestressing, none of the specimens failed in a shear mode. On the contrary, the width of the shear cracks (using visual observations) was less at nominal flexural strength after external prestressing than their width at service load level  $P_{max}$  before external prestressing.

### Stresses in the Tension Reinforcement

**Internal reinforcement** — The stresses in the internal prestressing steel were calculated from the strain gauge readings using the stress-strain curves of the prestressing steel with properties as given in Table 1. For specimens in which the stress in the prestressing steel during the first cycle to  $P_{max}$  was beyond its proportional limit, the corresponding stresses during subsequent cycles were calculated taking into account the hysteresis characteristics of the stress-strain response assuming linear unloading and reloading paths with a slope equal to the modulus of elasticity of the prestressing steel.



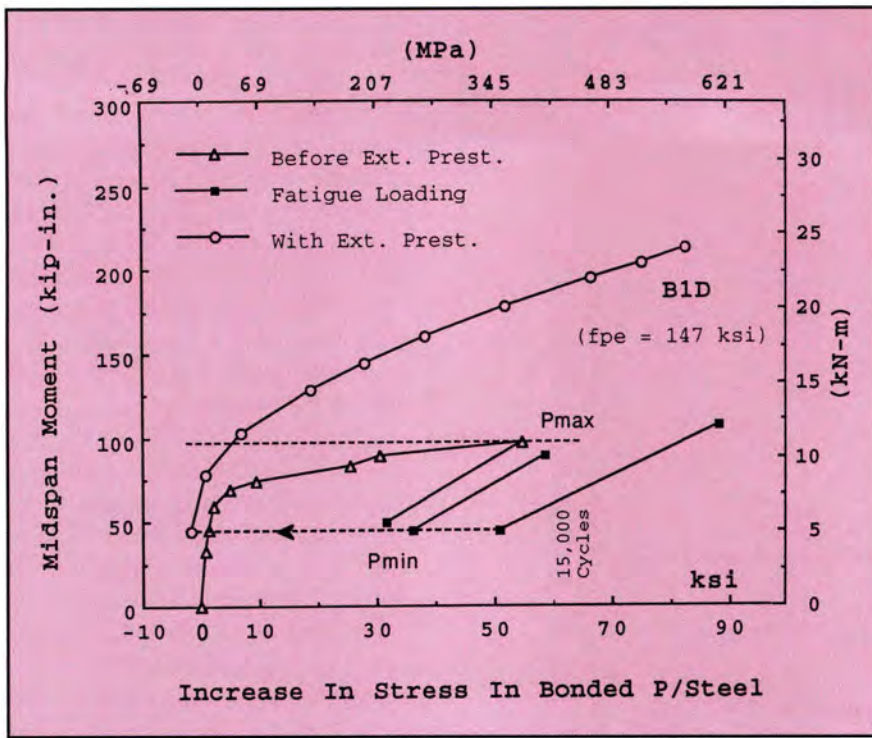


Fig. 6. Change in stress in internal prestressing steel with applied load.

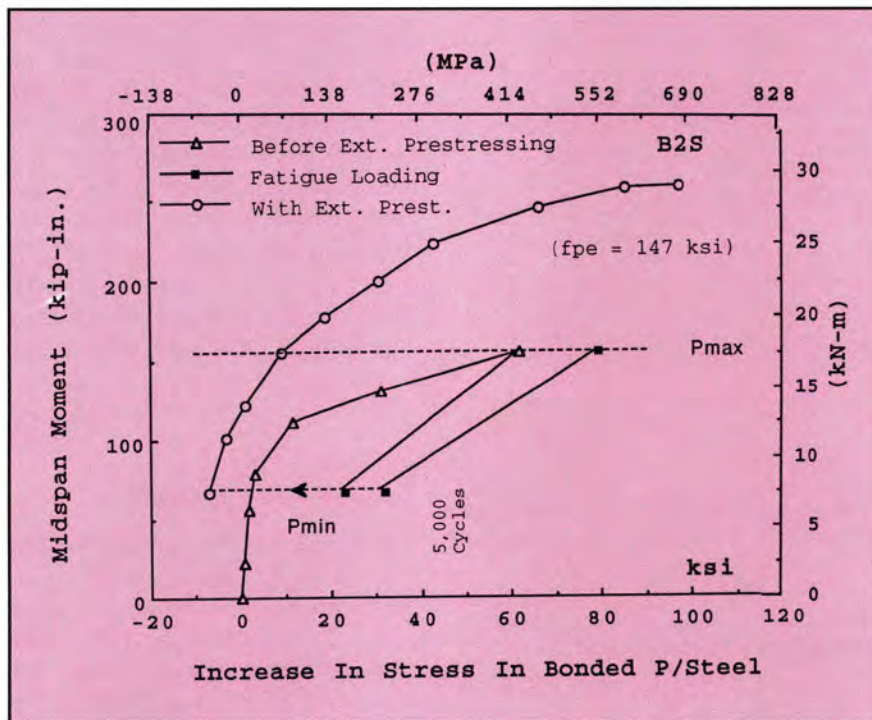


Fig. 6 (cont'd). Change in stress in internal prestressing steel with applied load.

Typical curves showing the variation of reinforcing steel strain and increase in stress in the prestressing steel (above effective prestress) with applied midspan moment during the entire loading history before and after external prestressing are shown in Figs. 5 and 6, respectively. A sum-

mary of stress ranges and mean stress levels in the internal prestressing and reinforcing steel measured during the last fatigue loading cycle before external prestressing and during the monotonic cycle to failure after external prestressing is given in Tables 5 and 6.

It can be seen in Figs. 5 and 6 that

the increase in strain in the reinforcing steel and internal prestressing steel with applied load before and after external prestressing is similar to the load-deflection response of the specimens. The increase of strain in the tensile reinforcement with cyclic fatigue loading is attributed to the stress redistribution across the depth of the member due to the cyclic creep process of concrete in compression.<sup>11,12</sup> Because of their small area, the progressive increase in strain/stress due to the cyclic creep was more pronounced in the prestressing strands than in the mild reinforcing steel.

Many tests have shown that the fatigue life of a concrete flexural member is controlled by the fatigue life of its tensile reinforcement.<sup>11-15</sup> The two primary factors that control the fatigue life of tensile reinforcement are the stress range and the mean stress level induced during cyclic loading. High stress ranges and/or high mean stress levels have long been known to reduce the fatigue life of steel reinforcement whether tested freely in the air<sup>16,17</sup> or as part of concrete flexural members.<sup>11-15</sup>

It can be observed in Figs. 5 and 6 that the strains and stresses in the internal tension reinforcement, and consequently the mean strain/stress levels, were reduced considerably after external prestressing. For example, the strain in the reinforcing steel in Beam B5D (Fig. 5) decreased from about 0.0014 to about 0.0004 at  $P_{min}$ , and from 0.003 to about 0.0016 at  $P_{max}$ , before and after external prestressing, respectively. Also, the increase in stress in the prestressing steel above effective prestress in Beam BID (see Fig. 6) decreased from 50 ksi (345 MPa) to about -3 ksi (21 MPa) at  $P_{min}$ , and from about 90 ksi (620 MPa) to about 6 ksi (41 MPa) at  $P_{max}$ .

Furthermore, because of a stiffer response under applied service load as mentioned earlier, external prestressing resulted in a significant reduction in the stress ranges of the internal tension reinforcement (see Tables 5 and 6). This is particularly true for the stress ranges in the prestressing steel of the prestressed and partially prestressed concrete specimens.

Therefore, external prestressing is expected to prolong the fatigue life of



Table 5. Fatigue stresses of the internal prestressing steel before and after external prestressing.

Beam specimen	Stress range in internal prestressing steel between $P_{min}$ and $P_{max}$ (ksi)		Mean stress level in internal prestressing steel (ksi)	
	Before external prestressing	After external prestressing	Before external prestressing	After external prestressing
B1D	37.2	10.4	216.6	150.5
B1S	22.2	9.3	196.1	170.4
B2D	40.6	4.8	201.4	154.5
B2S	47.5	16.1	202.7	148.1
B3D	43.0	6.8	190.2	154.0
B3S	37.5	14.7	196.5	168.0
B7D	16.9	8.5	191.8	158.0
B7S	18.0	5.8	176.7	157.2
B8D	31.7	12.8	171.3	155.8
B8S	24.9	18.0	177.9	165.1

Metric conversion factor: 1 ksi = 6.895 MPa.

Table 6. Fatigue stresses of the reinforcing steel before and after external prestressing.

Beam specimen	Stress range in reinforcing steel between $P_{min}$ and $P_{max}$ (ksi)		Mean stress level in reinforcing prestressing steel (ksi)	
	Before external prestressing	After external prestressing	Before external prestressing	After external prestressing
B4D	21.1	24.8	34.5	-8.5
B4S	30.9	15.0	39.0	20.8
B5D	45.5	35.3	57.3	23.8
B5S	29.2	30.5	32.6	2.4
B6D	31.7	26.6	48.0	28.0
B6S	—	—	—	—
B7D	23.6	8.0	29.0	-7.9
B7S	24.9	9.9	39.0	9.7
B8D	35.6	21.0	35.7	10.0
B8S	35.0	28.4	35.8	19.2

Metric conversion factor: 1 ksi = 6.895 MPa.

concrete flexural members subjected to repetitive type loading by delaying fatigue failure of the internal tension reinforcement.

**External prestressing steel** — Typical figures showing the increase in stress in the prestressing steel with applied midspan moment are shown in Fig. 7.

Since the external prestressing steel is unbonded to the concrete, the stress ranges in the external tendons mea-

sured between  $P_{min}$  and  $P_{max}$  during the monotonic cycle to failure were generally small. Except for Specimens B5D and B6D, where the stress ranges were 19.9 and 17.5 ksi (137 and 121 MPa), respectively, the stress ranges for the remaining specimens varied between a minimum of 4.1 ksi (28.3 MPa) for Specimen B1S and a maximum of 12.9 ksi (89 MPa) for Specimen B8D, with an average of about 8.5 ksi (58.6 MPa). Therefore, the

aforementioned improvements in the fatigue characteristics of the internal tension reinforcement are achieved at no risk of fatigue failure of the external tendons.

The stresses in the external prestressing steel at ultimate flexural strength were below yield for all beam specimens. The deviated tendons mobilized a larger increase in stress as compared to the straight tendons (see Fig. 7). In general, the stress levels at ultimate flexural strength decreased as the content of tension reinforcement in the specimens increased.

Being unbonded, the increase in strain in the external prestressing steel under applied load depends on the increase in tendon elongation between the anchorage ends. For a deviated external tendon profile of the type used in this investigation, the increase in tendon elongation, and, therefore, the increase in strain in the external prestressing steel, is linearly proportional to the increase in beam deflection at midspan (location of saddle or deviator).

Considering the geometry of deformation for beams with deviated external tendon profiles, the following general equation can be derived to describe the increase in strain  $\Delta\epsilon_{ps}$  and, correspondingly, the stress  $f_{ps}$  in the external tendons as a function of central beam deflection:

$$\Delta\epsilon_{ps} = \frac{4e_m}{L^2} \delta_o \quad (1)$$

and

$$\epsilon_{ps} = \epsilon_{pe} + \Delta\epsilon_{ps} \quad (2)$$

In the linear elastic range:

$$f_{ps} = f_{pe} + \left( \frac{4E_{ps}e_m}{L^2} \right) \delta_o \quad (3)$$

In the nonlinear range:

$$f_{ps} = F(\epsilon_{ps}) \quad (4)$$

where

$L$  = span length

$e_m, e_s$  = eccentricity of external tendons at midspan and anchorage ends, respectively

$\delta_o$  = increase in central beam deflection with applied load after external prestressing



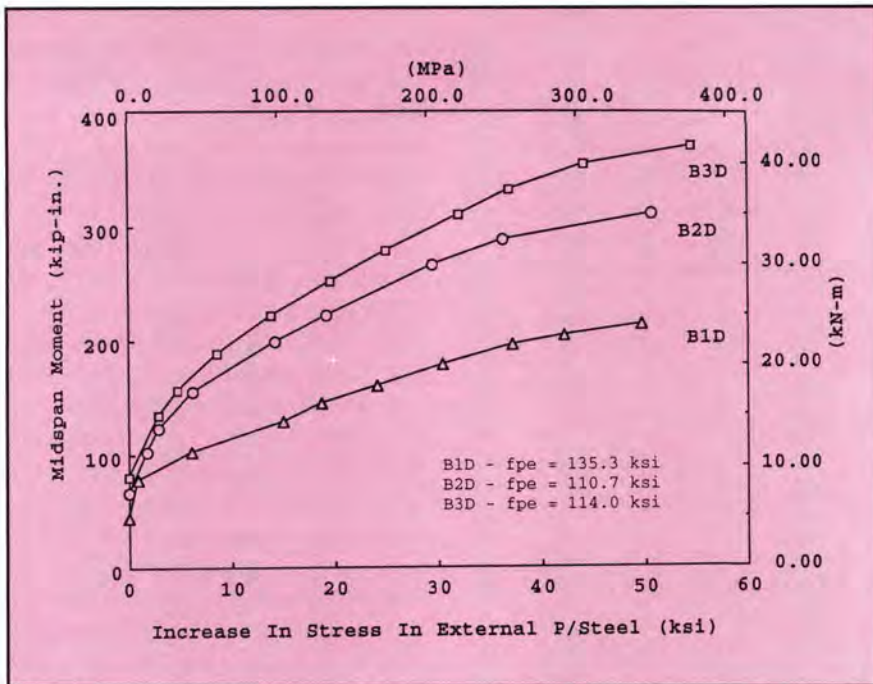


Fig. 7. Increase in stress in external prestressing steel with applied load.

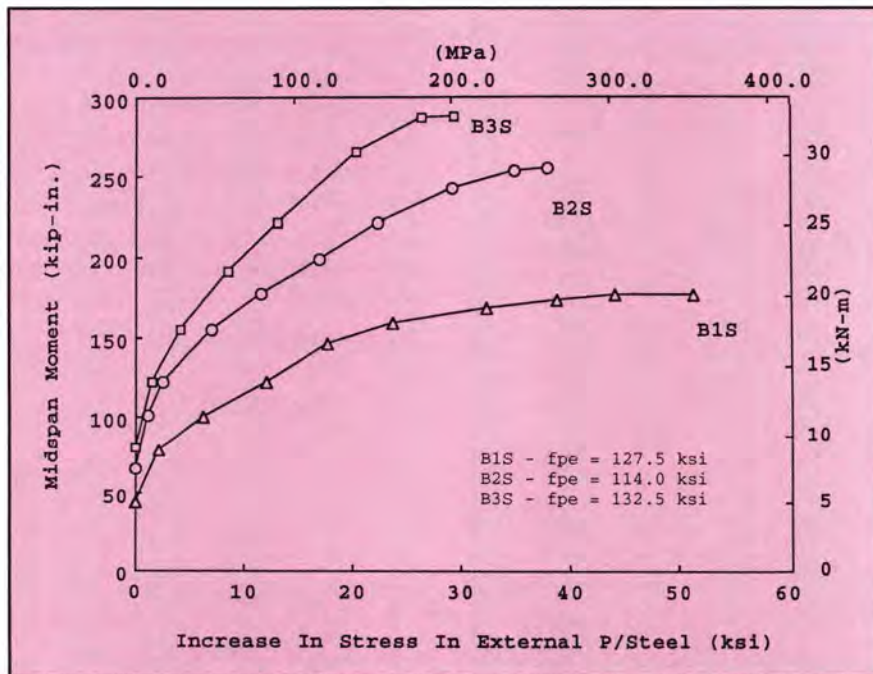


Fig. 7 (cont'd). Increase in stress in external prestressing steel with applied load.

$f_{pe}$ ,  $\epsilon_{pe}$  = effective prestress and effective prestrain, respectively, in external prestressing steel

Note that Eq. (1) is independent of the eccentricity  $e_s$  at the anchorage ends. Eq. (1) can also be applied for a straight horizontal tendon profile ( $e_m = e_s$ ) in an approximate but conservative manner assuming a single crack concept, i.e., the parts of the member to the

right and to the left of midspan remain straight during member deflection.

The variation of calculated  $\Delta\epsilon_{ps}$  with measured midspan deflection is shown for the straight and deviated tendons in Fig. 8. Four different levels of strain corresponding to four different levels of applied load, including ultimate, were selected for every specimen. Fig. 8 clearly shows that the agreement between the strains predicted using Eq. (1)

and the experimental results is excellent for the deviated tendon profile and is reasonably good for the straight tendon profile.

## CONCLUSIONS

The following conclusions are drawn based on the results of this experimental investigation:

1. External prestressing is a very powerful technique for the strengthening or rehabilitation of concrete flexural members. Providing external prestressing steel by a relatively moderate amount resulted in an increase in the nominal flexural resistance by up to 146 percent. This was achieved without significant reduction in ductility or ultimate flexural deformation of the member.

2. External prestressing can be used very effectively to control cracking and re-establish the service load deflections of concrete flexural members subjected to severe loading conditions. For the types of external prestressing steel profile and for the levels of external prestressing force used in this investigation, external prestressing was shown to: (1) reduce the crack widths or close the cracks completely, (2) reduce significantly the service load deflections induced under cyclic fatigue loading, and (3) lead to a stiffer service load-deflection response and, hence, reduce the live load deflections.

3. Because of the progressive reduction of the depth of the straight external tendons with increasing member deformation to failure, external prestressing using a straight horizontal profile was relatively less effective in increasing the flexural resistance as compared to a deviated profile. On the other hand, the service load-deflection response, and the ultimate deflections of the specimens, were similar for the two types of profiles used in this investigation.

4. In addition to the primary advantage of increasing the flexural strength and reducing the service load deflections, external prestressing can also prolong the fatigue life of concrete flexural members subjected to repetitive type loading. External prestressing was shown to reduce considerably the mean stress levels and stress ranges in the internal tension rein-



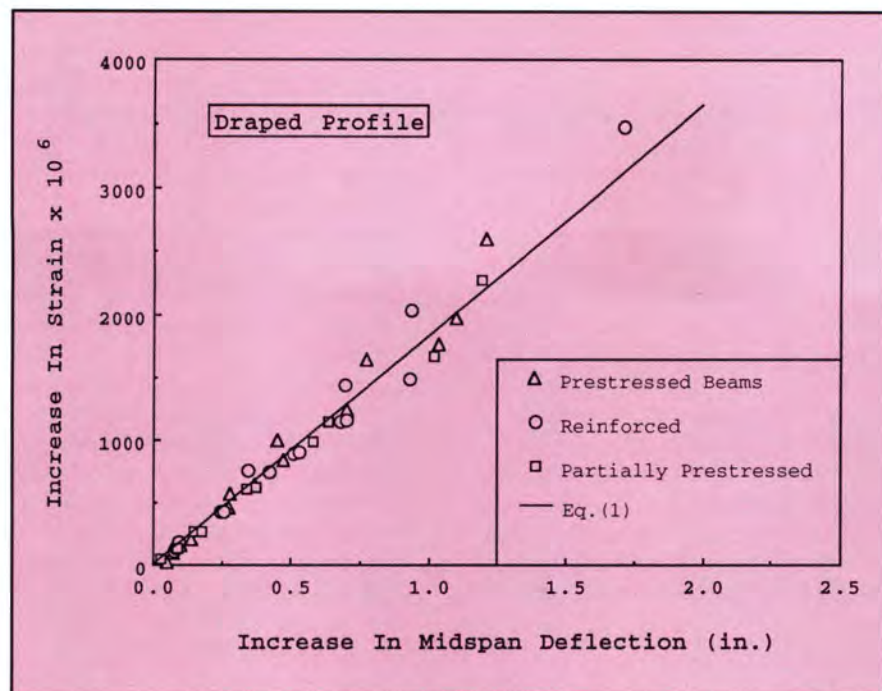


Fig. 8. Variation of strain in external prestressing steel with increase in beam central deflection.

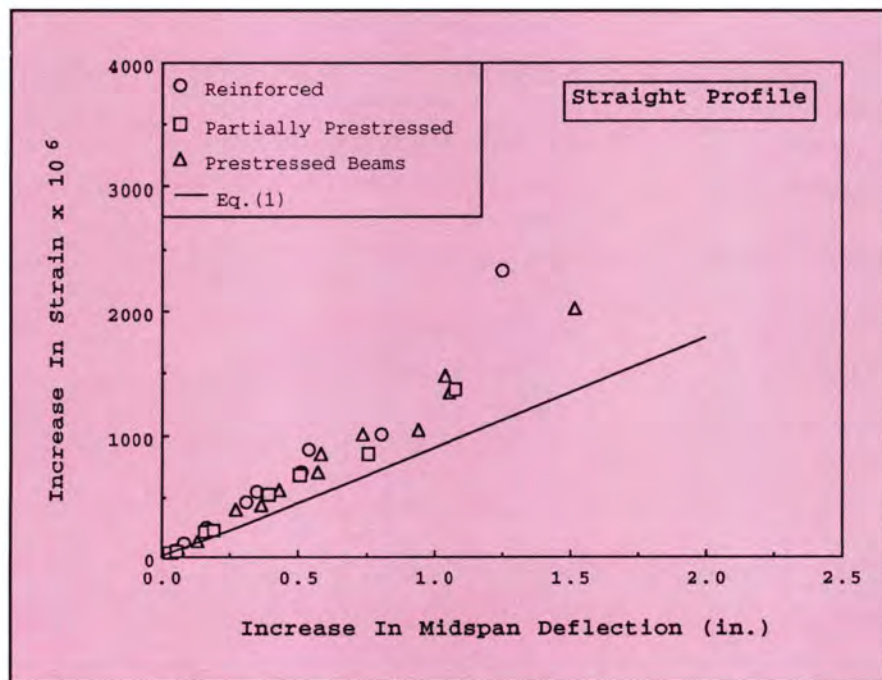


Fig. 8 (cont'd). Variation of strain in external prestressing steel with increase in beam central deflection.

forcement, which is of primary importance to the fatigue life of concrete flexural members. Because the externally prestressed tendons are unbonded to the concrete, and in effect their stress ranges are relatively small (as observed in this investigation), the corresponding improvement in the fatigue characteristics of the internal

tension reinforcement is achieved at no risk of fatigue failure of the external tendons.

5. Within the range of external prestressing force used in this investigation and for the amount of shear reinforcement provided, all beam specimens strengthened by external prestressing failed in a flexural mode

without any indication of shear deterioration. Apparently, the shear cracks in beam specimens with a large content of tension reinforcement were more adverse during the cyclic fatigue loading before external prestressing than at ultimate load resistance after external prestressing.

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## APPENDIX — NOTATION

$A_{pe}$ = area of external prestressing steel	$f'_c$ = concrete compressive strength
$A_{ps}$ = area of internal prestressing steel	$f_{pe}$ = effective prestress in prestressing steel
$A_s$ = area of mild tension steel reinforcement	$f_{ps}$ = stress in prestressing steel
$d_p$ = depth to center of internal prestressing steel	$f_{pu}$ = ultimate strength of prestressing steel
$d_{pe}$ = depth to center of external prestressing steel	$f_y$ = yield stress of mild reinforcing steel
$d_s$ = depth to center of mild tension steel	$L$ = span length
$e_m$ = eccentricity of external prestressing steel at midspan	$M_u$ = ultimate moment
$e_s$ = eccentricity of external prestressing steel at support	$\epsilon_{pe} = f_{pe}/E_{ps}$
$E_{ps}$ = modulus of elasticity of prestressing steel	$\epsilon_{ps}$ = strain in the prestressing steel
	$\delta_a$ = increase in midspan deflection