Optimization of Precast Prestressed Concrete Bridge Girder Systems

A practical approach to the optimal design of precast, prestressed concrete highway bridge girder systems is presented. The approach aims at standardizing the optimal design of bridge systems, as opposed to standardizing girder sections. Structural system optimization is shown to be more relevant than conventional girder optimization for an arbitrarily chosen structural system. Bridge system optimization is defined as the optimization of both longitudinal and transverse bridge configurations (number of spans, number of girders, girder type, reinforcements and tendon layout). As a result, the preliminary design process is much simplified by using some developed design charts from which selection of the optimum bridge system, number and type of girders, and amounts of prestressed and non-prestressed reinforcements are easily obtained for a given bridge length, width and loading type.

A review of American bridge construction practice enables the identification of some broad trends in the selection of bridge superstructure systems. Although this selection reflects some regional and historical variations, it appears to be primarily affected by the span length, as shown in Fig. 1. The figure suggests that reinforced and prestressed concrete solid or voided slab and light girders are suitable for short spans not exceeding 65 ft (20 m). However, for medium spans up to 130 ft (40 m), prestressed concrete stringers or box girders are preferred. For spans longer than 130 ft (40 m), prestressed concrete box girders or other forms of construction become desirable.

Fig. 1 shows that several alternative solutions exist for any given span range. For example, for spans of 65 to 100 ft (20 to 30 m), no less than eight candidate solutions are available. How does a designer decide on the best system selection for a given project? Bridge systems may be selected either by relying on existing tradition and accumulated experience, or by minimizing the costs under the specific conditions of the project. In the first case, the solutions may not be the most
economic, while in the second case they may be very time consuming, particularly if the number of bridges to be designed is large.

If existing practice is to be taken as a starting point in bridge selection, the exhaustive survey of the performance of highway bridges in the United States, published in the May-June 1992 PCI JOURNAL, can be used with great benefit. The survey indicates that since their introduction in 1950, prestressed concrete bridges have captured almost a quarter of the total bridge market, and since 1985, about 65 percent of the bridges with spans between 60 and 120 ft (18.3 and 36.6 m). The survey also indicates that, during the last 40 years, about 35,000 bridges built (or close to half of all prestressed concrete bridges) are of the stringer (I-girder) type (42.1 percent simple and 6 percent continuous stringers).

Thus, the stringer system is the most common prestressed concrete bridge type in the United States, a feature it retains throughout 25 states for spans of 60 to 80 ft (18.3 to 24.4 m). It is also noted that from 1985 to 1989, prestressed concrete stringer and box girder types have each covered about one-third of the prestressed concrete bridge market.

On the other hand, if “custom-tailored” optimal designs are desired for individual bridges, designers may seek optimization at any of three possible levels: (1) bridge members, (2) bridge (longitudinal and transverse) configurations, or (3) overall bridge system.

Level 1, member optimization of a predetermined structure, is the simplest and most widely reported optimization procedure. It involves the optimization of girder cross sections of specified structure types such as simple span prestressed concrete slabs, single span steel girders, single span prestressed concrete girders, single span box girders, multi-span simple girders, continuous prestressed concrete box girders, continuous reinforced concrete girder and frame bridges.

Level 2, configuration or layout optimization, is concerned with finding the best longitudinal and transverse member arrangement within a given bridge system, i.e., the number of spans, the position of intermediate supports, simply supported or continuous members, etc. Much less work is reported on this type of optimization than on member optimization.

Level 3, system optimization, attempts to identify the overall features of the structural system, including material(s), structural type and configuration, as well as member sizing. This is the most complex design problem and only few attempts to solve it are known. Among these are the studies by McDermott, Abrams and Cohn on tall building systems, Gellatly and Dupree on surface effect vehicles, Kulka and Lin on medium two-span continuous prestressed concrete bridge systems, and Mafi and West on short simple span bridge systems.

It is apparent that the overall economic impact increases with higher optimization levels; i.e., optimizing the overall bridge system is considerably more effective in reducing total costs than optimizing its components.

Whether bridge systems are selected by following current practice or through formal optimization techniques, the development of some standards for selecting optimal bridge systems for various ranges of geometry would have considerable practical significance. These standard optimal systems could serve both as guidelines for preliminary designs and also as yardsticks against which final bridge designs could be evaluated.

A comprehensive investigation on this topic, in progress at the University of Waterloo, aims at developing optimization procedures and solutions for short and medium span length highway bridge systems. The approach consists of first optimizing configurations of major bridge types (prestressed concrete stringers and box girders, steel stringers,
etc.) and then identifying optimal systems from the various
pre-optimized configurations using sieve-search tech­
niques. Results on optimization of simple span bridge
systems have been reported elsewhere.

The object of this paper is to present further results on the
optimization of continuous I-girder systems and to compare
them with the earlier single span solutions. More precisely,
the following problem is addressed: What is the optimal pre­
stressed concrete stringer system for given bridge length,
width, traffic loading and standard provisions? Specifically,
what are the longitudinal and transverse configurations that
result in minimum superstructure cost and what are the corre­
ponding prestressed and non-prestressed reinforcements?

It is emphasized that “an optimal bridge superstructure”
is understood as one of minimum total cost, using standard­
ized girder sections and traffic loading. In this study, CPCI
I-sections and Ontario Highway Bridge Design Code
(OHBDC) loading, safety and design specifications, as
well as Ontario costs, are used. The sensitivity of optimal
solutions to the changes that occurred in the new edition of
the OHB Code is investigated. Alternative use of stan­
dard AASHTO-PCC sections and AASHTO specifications
is also considered. Worth noting is the recent interest in
standardizing, across the United States, the use of pre­
stressed concrete I-sections for bridge design which could
be achieved through the formal optimization (Level 1) pro­
cedures described here and elsewhere.

This paper presents a general approach to optimization at
Levels 1, 2 and 3, and readily usable results of optimal
stringer configurations for precast, prestressed concrete
girders. The paper is organized in three main sections: the
first part details the formulation of the optimization pro­
cedure; the second part presents its practical implementa­
tion to bridges having design parameters within the investiga­
tion range; the third part offers some practical recom­
mendations, extension to AASHTO specifications and design examples.

**OPTIMAL BRIDGE DESIGN PROCEDURE**

**Design Variables**

Given a bridge of total length $L$, width $W$ and some speci­
fied truck and lane loads, the aim of the design is to deter­
mine the minimum cost of the bridge system. The optimum
bridge is defined by the best structural system type (longitu­
dinal and transverse bridge configurations) and optimum de­
sign of its individual members (slab, girders, diaphragms,
etc.). In this paper, the selected structural system consists of
a reinforced concrete slab on precast, prestressed concrete
I-girders, as it represents about 40 percent of the short and
medium span bridges built in the United States and Canada.

The optimum longitudinal configuration is defined by the
number of spans, restraint type (simply supported or contin­
uous) and span length ratios (Fig. 2). The transverse config­
uration is defined by the number of girders (or girder spacing
and slab overhang) (Fig. 3). Simply supported bridges
(with one, two and three spans) and continuous bridges with
two and three spans are considered, although the results are
conservative and may be used for four or more equal spans.

Finally, the optimum designs of the girder and slab are
defined by the girder cross section dimensions (Figs. 4 and
5a), slab thickness, amounts of prestressed and non­
prestressed flexural reinforcements in the girder and slab,
as well as the tendon eccentricities at midspan and supports
(Figs. 5a and 5b).

**Objective Function**

The assumed merit function is the unit superstructure cost
and is defined as:

$$Z = \left[ n \left( C_s L + C_{cs} V_{cs} + C_{w} W_{w} + n' C_{pc} W_{pc} + C_{f} W_{f} + C_{m} W_{m} \right) \right] / WL$$

where

- $n$ = number of girders
- $n'$ = number of positive moment connections (at piers)
- $C_s$ = cost of precast girder per length (including cost of
  fabrication, prestressing, delivery and erection)
- $C_{cs}$ = concrete cost in slab and diaphragms
- $C_{f}$ = non-prestressed steel unit cost
- $C_{pc}$ = unit cost of positive moment connection at piers
  including cost of steel and bending of bars (assumed
  $C_{pc} = 1.7 C_p C_{vp}$ where $C_{vp} =$ volume of positive
  moment reinforcement)
- $V_{cs}$ = volume of concrete in slab and diaphragms and
  $W_{w}, W_{pc}, W_{f}, W_{m}, W_{pc}$ are the weights of non-prestressed
  steel in slab and diaphragms, negative moment
  steel in slab at piers and positive moment steel in
girders, respectively.

**Design Constraints**

In the design of precast concrete girders, all serviceability
limit states (SLS) and ultimate limit states (ULS) require­
ments of the Ontario Highway Bridge Design Code
(OHBD) are to be satisfied. However, only the flex­
ural constraints at SLS, transfer and ULS are considered, in
addition to the side constraints that reflect specific OHBD
requirements. Other constraints (camber, deflection, vertical
and interface shears, etc.) could easily be added but, in gen­
eral, they affect only marginally the flexural design and are
left to be checked at the final design stage. Special attention
must be given to the connection design and detailing for the
positive moments at piers induced by the creep and shrink­
age effects.

(a) Transfer Stresses Constraints — Normal stresses in
concrete $\sigma_0$ due to girder self-weight should be within the
allowable limits at midspan and support critical sections:

$$f_E \leq \sigma_0 \leq f_c$$

where $f_E$ and $f_c$ are the allowable tensile and compressive
stresses at transfer, respectively. For the girder with a con­
crete having $f_c^1 = 40$ MPa (5800 psi), $f_c = -0.24 \sqrt{f_c^1} = -1.3$
MPa (-190 psi) and $f_E = 0.6f_c^1 = 18$ MPa (2610 psi).

(b) Service Limit State Constraints — In the design of
prestressing steel for girders and non-prestressed reinforce­
Fig. 2. Longitudinal bridge configurations: (a) one simple span, (b) two simple spans, (c) three simple spans, (d) two continuous spans, (e) three continuous spans.

Fig. 3. Transverse bridge configuration.

ment for positive moments at piers, the normal stresses in concrete ($\sigma_c$), mild steel ($\sigma_s$) and jacking prestress ($\sigma_j$) should be:

For concrete:

$$f_s \leq \sigma_s \leq f_{sc}$$  \hspace{1cm} (3a)$$

where $f_s = -0.48\sqrt{f_c'} = -3$ MPa (-435 psi) and $f_{sc} = 0.45f_c' = 18$ MPa (2610 psi) are the allowable tensile and compressive stresses at service, respectively.

For non-prestressed steel:

$$\sigma_s \leq 0.6f_s$$  \hspace{1cm} (3b)$$

For prestressing steel at jacking:

$$\sigma_j \leq 0.8f_{ps}$$  \hspace{1cm} (3c)$$

The SLS moment is defined as:

$$M = M_g + M_s + M_a + 0.75 M_L + 0.85 M_{cr}$$  \hspace{1cm} (4)$$

where $M_g$, $M_s$, and $M_a$ are the girder, slab and asphalt pavement self-weight moments, respectively. Also, $M_L$ and $M_{cr}$ are the live load moment and creep and shrinkage moment, respectively.

(c) Ultimate Limit State Constraints — The ultimate load moment $M_u$ should be less than or equal to the resisting moment of the section:

$$M_u = 1.1 M_g + 1.2 M_s + 1.5 M_a + 1.4 M_L \leq \phi M_n$$  \hspace{1cm} (5)$$

where $M_n$ is the nominal resisting moment of the section and $\phi = 0.85$ is the flexural strength reduction factor. In the new OHBD Code, the single understrength factor ($\phi = 0.85$) has
been replaced by strength reduction factors for concrete \( \phi_c = 0.75 \), steel \( \phi_s = 0.90 \) and prestressing steel \( \phi_p = 0.95 \).

(d) Side Constraints — These reflect some specific OHBDC requirements:

\[ S \leq 15 \ t \] \hspace{1cm} (6a)
\[ t \geq 225 \text{ mm} \ (8.9 \text{ in.}) \] \hspace{1cm} (6b)
\[ S' \leq \text{Min} \ [0.6 \ S, 1.8 \text{ m} \ (5.9 \text{ ft})] \] \hspace{1cm} (6c)
\[ \omega \leq 0.30 \] \hspace{1cm} (6d)
\[ c_{\text{hot}} \geq 100 \text{ mm} \ (3.9 \text{ in.}) \] \hspace{1cm} (6e)
\[ c_{\text{top}} \geq 80 \text{ mm} \ (3.1 \text{ in.}) \] \hspace{1cm} (6f)

where \( S, S', t, \omega, c_{\text{hot}} \) and \( c_{\text{top}} \) are the girder spacing, slab overhang, slab thickness, total tension reinforcement index of the section, and bottom and top minimum concrete covers, respectively. The total reinforcement index is defined as:

\[ \omega = (A_{ps} f_{ps} + A_i f_i) / b d f'c \] \hspace{1cm} (7)

Dead and Live Load Analysis

In order to properly account for the construction sequence of this type of bridge, two analytical models are used to compute the moments at different critical sections.

(a) Model 1: Simply Supported Beam — Initially, the precast girders are placed on the substructure and the analysis under the girder and slab self-weights is carried out as for simple beams. At this stage, only the girder section is effective in computing the stresses.

(b) Model 2: Continuous Beam — Subsequently, continuity is achieved by adding non-prestressed reinforcement over the piers, and the analysis for the live load and pavement weight is carried out as for a continuous beam. Composite (slab and girder) action develops and must be considered in evaluating the stresses. However, the live load moment obtained should be multiplied by the transverse distribution factor \( S/D_d \) and augmented by the impact factor \( I \) (dynamic load effect), conservatively estimated as 0.4 for truck load and 0.1 for uniform lane load. In OHBDC,\textsuperscript{24,30} \( D_d \) values (given for two-, three- and four-lane bridges) depend on the torsional and flexural parameters \( \alpha \) and \( \theta \), respectively, in addition to the design lane width:

\[ \alpha = D_{xy} + D_{xt} + D_1 + D_2 / 2(D_x D_y)^{0.5} \] \hspace{1cm} (8a)
\[ \theta = W / 2 \ell_c \left( D_e / D_y \right)^{0.25} \] \hspace{1cm} (8b)

where \( D_x, D_y, D_{xy}, D_{xt}, D_1 \), and \( D_2 \) are the flexural, torsional and coupling rigidities in both transverse and longitudinal directions, respectively. The \( D_d \) factors have been derived by using the orthotropic plate analogy for simple span bridges.\textsuperscript{24} In order to use the same factors for continuous bridges, an effective length \( \ell^* \) is determined for all critical sections to compute the flexural parameter in Eq. (8b) which may be taken as shown\textsuperscript{24,30} in Fig. 6. In the new edition of the OHBDC Code,\textsuperscript{25} the determination of the \( D_d \) factors has been simplified to only depend on the bridge type and span length.

Creep and Shrinkage Moments Analysis

(a) Creep Moments due to Prestressing and Dead Loads — Continuity of this bridge type requires that positive moments developed over piers due to creep of the prestressed girders and the effects of live load on remote spans be considered. Positive moments due to creep under sustained (prestressing and dead) loads are partially counteracted by the negative moments due to differential shrinkage between the cast-in-place deck slab and precast girders. Of the several methods of creep analysis proposed in the literature,\textsuperscript{31,32,33} the Tröst-Bazant age-adjusted elastic modulus method\textsuperscript{34} is used to compute the prestress and dead load restraint moments.

For illustration purposes, consider a continuous girder with \( n \) spans, initially assumed freely supported (Fig. 7a), and subsequently connected for continuity at an age \( t_1 \) (Fig. 7b). Using the age-adjusted elastic modulus method, the moment \( M^*_n(t) \) induced by creep at a support \( n \) is given by:

\[ M^*_n(t) = M_n \left[ V(t, t_n) - V(t_1, t_n) / [1 + \chi V(t_1, t_n)] \right] \] \hspace{1cm} (9)
where $M_n$ is the moment at support $n$ if the structure is continuous and monolithically cast in a single stage.

The restraint moments at the supports are determined by using any elastic analysis method (e.g., the three-moment equation); $v(t, t_o)$ is the creep coefficient (ratio of creep strain to initial elastic strain); $t_o$ is the age at loading; $t_1$ is the age at which continuity is achieved and $\chi$ is the aging coefficient introduced because the creep moment develops gradually with time (Tröst proposes $\chi = 0.8$). Assuming $t_1 = 28$ days and $t = \infty$, the following values are obtained from ACI-209:33 $v(t_1, t_o) = 0.96 v_n$ and $v(t, t_1) = 0.54 v_n$ where $v_n$ is the ultimate creep factor which may be conservatively taken as 4.5.

Since $v(t_1, t_o) = 0.42 v_n$ (i.e., 42 percent of the ultimate creep will have occurred at the time the continuity is achieved), the connections should develop the moments from the remaining creep of 0.54 $v_n$. Substitution of parameters in Eq. (9) yields:

$$M_n(t = \infty) = 0.80 M_n$$

(10)

(b) Shrinkage Moments — Differential shrinkage between the reinforced concrete deck slab and the prestressed concrete girders induces the moment $M_{sh}$ in the composite section:

$$M_{sh} = A_s E_s \varepsilon_{sh} y_s$$

(11)

where

$A_s$ = area of effective slab deck

$E_s$ = Young’s modulus of slab concrete

$y_s$ = distance from centroid of composite section to centroid of slab

$\varepsilon_{sh}$ = differential shrinkage strain$^2 = 10^{-4}$

The (shrinkage) restraint moments induced by differential shrinkage can also be obtained using the three-moment equation; however, the presence of creep reduces shrinkage restraint moments. In the absence of accurate data, the OHBDC recommends the use of a 60 percent creep reduction factor. Therefore, the final moment $M_{cm}$ at support $n$ due to the combined effects of creep and shrinkage is:

$$M_{cm} = 0.8 \left( M_{pu} + M_{pu} \right) - 0.4 M_{sh}$$

(12)

where $M_{pu}$, $M_{pu}$, and $M_{sh}$ are the restraint moments at support $n$ due to prestressing, dead load and shrinkage, respectively.

**Solution of the Optimization Problem**

The optimization problem, defined by Eqs. (13a) to (13l), includes both discrete and continuous design variables; the constraints are nonlinear functions of the design variables. Solution of such a problem is very difficult and requires the use of mixed integer programming algorithms which are not efficient and may not converge to an optimum. To overcome this difficulty, a two-stage optimization with continuous variables only is performed by using nonlinear programming methods. This approach has been used for the optimization of simply supported bridge girders.22

The two-stage optimization operates as follows:

(a) **Maximum Feasible Girder Spacing** — For each simple span and continuous bridge and precast girder, the maximum feasible girder spacing is determined for span lengths varying from 10 to 30 m (33 to 98 ft) by solving the optimization problem defined by Eqs. (13a) to (13l), but maximizing the girder spacing instead of minimizing the cost as objective function. Once the maximum feasible girder spacing is determined, for every given bridge width and span length, the minimum number of girders resulting in the minimum superstructure cost can be obtained.

(b) **Optimum Reinforcements** — After the optimum number of girders has been determined, the total superstructure cost can be minimized by solving the problem defined by Eqs. (13a) to (13l) with the minimization of the prestressed and non-prestressed reinforcements as objective function.

These two optimization problems can be solved by any nonlinear programming technique. In this paper, the
GAMS/MINOS program, based on the projected Lagrangian algorithm, has been used. The description of the algorithm is given elsewhere.335

OPTIMAL DESIGN OF BRIDGE SYSTEMS — PRACTICAL APPLICATION

Investigation Outline

To achieve the stated objectives of the investigation, the above-mentioned optimization procedure and the GAMS/MINOS program have been used to produce optimal bridge designs for a large number of precast, prestressed concrete girder bridges. Some 165 optimal solutions have been generated for the following variations of the main design data indicated by the dots (Table 1):

(a) Five longitudinal bridge configurations: one, two and three simply supported spans, and two and three continuous equal spans
(b) Three girder types: CPCI 900, 1200 and 1400
(c) Three bridge widths: W = 8, 12 and 16 m (26, 39 and 52 ft) (i.e., two-, three- and four-lane bridges)
(d) Five span lengths: 10, 15, 20, 25 and 30 m (33, 49, 66, 82 and 98 ft) (whenever feasible)

Table 1. Specimen data for parametric investigation for bridge widths W = 8, 12 and 16 m (26, 39 and 52 ft).

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Metric (SI) conversion factor: 1 m = 3.28 ft.

For all designs, the following values are adopted as pre-assigned parameters:
- Class A highway
- Slab thickness: t = 225 mm (8.9 in.) (OHBDC minimum)
- Minimum OHBDC slab reinforcement (i.e., p = 0.3 percent top and bottom reinforcements in both directions)
- Concrete grades: f'c = 40 MPa (5800 psi); f'cl = 30 MPa (4350 psi) for girder and f'c = 30 MPa (4350 psi) for slab
- Steel grades: fpu = 1860 MPa (270 ksi) for prestressing steel and fpu = 400 MPa (60 ksi) for mild steel
- Unit costs based on average Ontario costs always expressed in Canadian dollars: Cq = $514, 486, 580 per m for CPCI 900, 1200 and 1400, respectively; Cqs = $617 per m and Cq = $1410 per t

For simple span bridges, Eq. (1) with Cpc = W/m = 0 applies, but the additional cost of deck joints, Cal = $1410 per m, must be added.

Solutions of the 165 bridge designs include:
- Optimal girder spacing and the corresponding minimum number of girders
- Optimal prestressing (force and tendon layout)
- Non-prestressed steel reinforcements for girder and slab
- Minimum cost of superstructure per unit deck area
- Identification of active constraints

Optimal Design Solutions for Simple and Two- and Three-Span Continuous CPCI Girders

Optimal solution features for the simple and two- and three-span continuous girders are presented in Table 2 and Figs. 8 and 9. With the completed optimal designs for the simple and two- and three-span continuous girders, the optimal superstructure costs for discrete bridge lengths up to 100 m (330 ft), three bridge widths and three CPCI girder types in five longitudinal bridge configurations are summarized in Table 3.

Table 2. Maximum feasible girder spacing, optimum girder spacing and optimum number of girders for single-, two- and three-span continuous CPCI girder bridges.
Finally, the optimization results are synthesized in Fig. 10, which identifies the optimum precast girder bridge system (i.e., the best longitudinal configuration, as well as the optimum number and type of girders) for a specified bridge length and width.

(a) Optimum Girder Spacing (Minimum Number of Girders) — Optimization results show that the optimum girder spacing is the same for two- and three-span continuous girders. The maximum feasible and optimum girder spacing and corresponding number of girders are shown in Table 2 for the three bridge widths [8, 12 and 16 m (26, 39 and 52 ft)] investigated. From Table 2, it is seen that the maximum feasible girder spacing depends on the span length, girder type and slab thickness. However, the optimum girder spacing depends on the bridge width in addition to the above-mentioned parameters.

It is also noted that, for a given precast girder section, bridge span and width, the optimal girder spacing is less than the maximum feasible value and always corresponds to the maximum feasible slab overhang. For a given precast girder section and bridge width, the optimum girder spacing (or number of girders) is a constant that minimizes superstructure cost up to a certain span length. Beyond this span length, an extra girder is required, at an increased superstructure cost. For CPCI 1200 and 1400 girders, these span lengths are \( \ell = 20 \) m (65.6 ft) and \( \ell = 25 \) m (82.0 ft), respectively.

(b) Optimum Prestressing Force and Tendon Layout — Optimization results show that the optimum prestressing force and total reinforcement index for the three-span continuous girders are up to 2 percent higher than for those corresponding to the two-span continuous girders. This minor difference is due to the fact that a large part of the load (girder and slab self-weights) is carried in the same way (simple beam) by both configurations.

Further, since both resist the same dead load moments and only marginally different live load moments, the variations of the optimum prestressing force with span length for various CPCI girders and bridge widths are illustrated in Fig. 8b for the three-span continuous girders only. However, this plot can be conservatively used for continuous beams with any number of equal spans. The optimum prestressing

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Metric (SI) conversion factor: 1 m² = 10.76 sq ft.
Fig. 8. Optimum prestressing force for CPCI girder bridges: (a) simple span bridges, (b) continuous span bridges.

Note: 1 kN = 0.225 kips; 1 m = 3.28 ft.
forces for simply supported girder sections shown in Fig. 8 are seen to be 10 to 40 percent higher than for the corresponding continuous girder sections. As expected, it is seen that for a given span length, the amount of pre-stressing decreases for larger girder depths. The lowest values always correspond to the CPCI 1400 section, which requires very limited prestressing for spans smaller than 15 m (49 ft). This suggests that for short spans, a reinforced concrete girder with the same depth will be able to sustain the same loads.

The optimum prestressing force always corresponds to the maximum midspan tendon eccentricity (i.e., minimum concrete cover), with the tensile service stress constraint being active. However, for the CPCI 900 section and \( \ell = 15 \) m (49 ft), the tendon eccentricity is not maximal because the transfer tensile stress constraint also becomes active and thus limits the feasible eccentricity. For the end support eccentricity, any feasible solution can be used since its effect on the optimum design is negligible. However, taking the smallest feasible value may reduce the amount of positive moment reinforcement at the piers.

(c) Optimum Total Reinforcement Index — The total reinforcement index is found to be rather low (\( \omega_{\text{max}} = 0.06 \)) for the three investigated girder sections. This suggests that the ductility constraint [Eq. (6d)] is redundant for this type of girder. Furthermore, the identical values obtained for all bridge widths indicate that the number of lanes does not influence the total reinforcement index.

Some additional non-prestressed reinforcement is required in compliance with the ULS flexural strength criterion. This reinforcement percentage varies between 0.85 and 0 percent, with higher percentages needed for shorter spans. Variations of \( \omega \) with span length for the three precast girder types are shown in Fig. 9. Since the amount of prestressing is known from Fig. 8, and the total reinforcement index is known from Fig. 9, the amount of non-prestressed reinforcement is easily determined from Eq. (7).

(d) Optimum Unit Cost of Superstructure — Table 3 shows unit superstructure costs (i.e., cost per deck area) for bridge lengths varying from 10 to 90 m (33 to 295 ft), three bridge widths [8, 12 and 16 m (26, 39 and 52 ft)], three CPCI girder types and the five longitudinal bridge configurations investigated in this paper. From Table 3 it is noted that:

- Configurations 2-S and 3-S are never competitive because of the added cost of the deck joints and are reported here for the sake of completeness only.
- For the competitive configurations (1-S, 2-C and 3-C), costs are almost constant up to a certain span length below which they increase sharply, due to an increase in the required number of girders. As an example, the cost of CPCI 1200 at \( \ell = 25 \) m (82 ft) increases on average by 17 percent vs. the cost at \( \ell = 20 \) m (66 ft).
- In general, unit cost decreases for increasing number of lanes. Thus, the superstructure cost for CPCI 1400 and...
\( \ell = 25 \text{ m (82 ft)} \) of a four-lane bridge is 10 percent cheaper than that of a two-lane bridge.

- For short span bridges \( \ell \leq 13 \text{ m (43 ft)} \), both CPCI 1200 and 900 may be considered equally competitive for all configurations, because CPCI 1200 is a little cheaper than CPCI 900. However, if the unit costs of the two girders are equal, the CPCI 900 girder will be optimal for this span range. For \( 13 < \ell \leq 22 \text{ m and } \ell > 22 \text{ m (43 < } \ell \leq 72 \text{ ft and } \ell > 72 \text{ ft)} \), the CPCI 1200 and 1400 are optimal, respectively.

- For total bridge lengths \( L \leq 66 \text{ m (217 ft)} \), the optimal system always consists of CPCI 1200 sections, with three, four or five girders for \( W = 8, 12 \text{ or } 16 \text{ m (26, 39 or 52 ft)} \), respectively.

- For \( L > 66 \text{ m (217 ft)} \), the optimal system always consists of CPCI 1400 sections, with four to seven girders depending on \( W \) and \( L \) (Fig. 10 and Table 4).

(e) **Governing Criteria (Active Constraints)** — In all cases investigated, the service tensile stress \( \sigma_s = f_s \) (Eq. (13c)) is the only active SLS constraint. Therefore, all other SLS constraints may be disregarded, except for some special cases where transfer stress constraints may also be active and need to be considered [for \( \ell = 25 \text{ and } 30 \text{ m (82 and 98 ft)} \), \( \sigma_s = f_s \) (Eq. (13b)) governs for CPCI 1200 and 1400, respectively, while \( \sigma_s = f_s \) (Eq. (13b)) governs for CPCI 900 and \( \ell = 15 \text{ m (49 ft)} \)].

**Optimum Bridge Superstructure System**

At this point, the problem to be solved is: What is the optimum system for some given bridge length and width; i.e., which of the five longitudinal configurations and transverse configurations result in the most economical superstructure?

The answer is provided by Fig. 10, which is a synthesis of optimal design solutions in Tables 2 and 3. Fig. 10 illustrates the variations of optimal bridge costs per unit deck area \( Z \) with the total bridge length \( L \), for various longitudinal configurations and girder sections. Its study justifies the following remarks:

- Solutions 2-S and 3-S (two and three simple spans) are not plotted because they are not competitive (high additional costs of the required deck joints).

- The optimum longitudinal configuration depends only on the total bridge length and is independent of its width.

- Bridge configurations 1-S, 2-C and 3-C are optimal for short \([L \leq 27 \text{ m (89 ft)}]\), short to medium \([27 < L \leq 44 \text{ m (89 < } L \leq 144 \text{ ft)}]\) and medium \([55 < L \leq 100 \text{ m (180 < } L \leq 328 \text{ ft)}]\) length bridges, respectively.

- For \( 44 < L \leq 55 \text{ m (144 < } L \leq 180 \text{ ft)} \), the three-span con-
continuous (Solution 3-C) bridge is not necessarily optimal although its superstructure cost is about 7.5 percent cheaper than that of the two-span continuous (Solution 2-C) bridge. Determination of the optimum configuration should be based on the minimum total bridge cost (superstructure plus substructure), as discussed in the next section.

• For $27 < L \leq 44$ m (89 < $L \leq 144$ ft), the two-span continuous (Solution 2-C) bridge is optimal with CPCI 900 and 1200 as girder sections. The optimum number of girders is three, four and five for $W = 8$, 12 and 16 m (26, 39 and 52 ft), respectively, as shown in Fig. 10.

• For $55 < L \leq 100$ m (180 < $L \leq 328$ ft), Solution 3-C is the only configuration that remains feasible. The CPCI 1200 and 1400 are the optimal girder sections for bridge lengths smaller or larger than 65 m (213 ft), respectively.

• For $L > 65$ m (213 ft), the three-span continuous bridge with CPCI 1400 girders should be compared to the four-span continuous bridge with CPCI 1200 or 1400 girders and other systems — such as voided slab or box girder bridges — which may be competitive for medium to long span bridges.

• For short span bridges with $\ell < 13$ m (43 ft) [i.e., $L < 13$ m (43 ft) for Solution 1-S, $L \leq 26$ m (85 ft) for Solution 2-C and $L < 39$ m (128 ft) for Solution 3-C], CPCI 900 and 1200 sections are competitive because their unit costs are almost equal. In such cases, a final selection may be based on some alternative objective function (minimum weight or minimum reinforcement) or response criterion (minimum deflection).

Solution Sensitivity Considerations

The optimal solutions presented in this study are based on a set of parameters consistent with current design practice. The parameters to be considered in concrete bridge design are longitudinal configuration, bridge length and width, traffic loading, slab thickness, girder (standard or non-standard) section type, concrete grade, superstructure and substructure cost parameters.

In this section, the validity and possible application of the optimal solutions obtained outside the range of adopted parameters are examined.

(a) Longitudinal Configuration — Optimal bridge systems of 1-S, 2-S, 3-S, 2-C and 3-C type (Fig. 2) have been identified. The optimization procedure can be applied to any number of simple or continuous spans. For any span length, results of the present study of 2-C and 3-C configurations are still valid and conservative for 4-C, 5-C, etc., configurations.

(b) Bridge Length ($L$) — With the three standard girder sections and five longitudinal configurations considered, optimal solutions for bridges not exceeding 100 m (328 ft) have been determined. Longer bridges may be designed with other than adopted standard girders (e.g., CPCI 1900, 2300 or AASHTO-PCI Type V, VI, etc.) or longitudinal configurations (4-C, 5-C, etc.). It also must be noted that for bridges with spans in excess of 40 m (131 ft), other structural systems than reinforced concrete slab on precast, pre-stressed concrete I-girders may represent competitive alternatives (prestressed concrete box girders or other forms of construction).

(c) Bridge Width ($W$) — Although optimization has been conducted for bridge widths $W = 8, 12$ and 16 m (26, 39 and 52 ft), additional data are provided in Table 4 which lists the optimal number of girders for bridge widths varying between 8 and 16 m (26 and 52 ft). Determination of the prestressing force is possible by interpolating in Fig. 8 or by using Eq. (14).

(d) Traffic Loading — The study is based on the OHBD Code loading specifications for Class A highways (average daily truck traffic higher than 1000 or traffic higher than 4000).

In the new edition of the OHBD Code, the two 140 kN (31.5 kips) axles have been increased to 160 kN (36 kips), which will induce an increase in the live load moment of smaller than 14 percent. However, the corresponding impact factor (function of the number of axles governing the design) is equal to 0.25 for the span lengths investigated in this paper. Since an impact factor of 0.4 has been assumed in this study, which is 60 percent higher than the new 0.25 value, these optimal solutions remain feasible within the new OHBD Code requirements.

It will also be shown in the next section that the design results are comparable to those based on AASHTO HS20 loading, and may also be used for lighter loadings (e.g., AASHTO HS15 and H20 loadings).

(e) Slab Thickness ($t$) — Optimal solutions assume a slab thickness $t = 225$ mm (8.9 in.), the minimum required by the OHBD. Optimal designs will remain valid for slab thicknesses of at least 200 mm (8 in.).

(f) Girder Section Type — The optimization program can readily include section variables in order to determine the optimal girder sections (Fig. 5a) in addition to system optimization variables. Consideration of general economy (formwork, fabrication and transportation costs) suggests that using standardized girder sections is more economical, although not structurally optimal. This is why the CPCI 900, 1200 and 1400 girders have been used in the present investigation. The CPCI 1900 and 2300 girders are not included because related costs are unavailable.

(g) Concrete Grade ($f'_{c}$) — Girder optimization has assumed $f'_{c} = 40$ MPa (5800 psi). Results remain valid on the conservative side for higher concrete grades.

(h) Superstructure and Substructure Costs — Optimization of precast, prestressed concrete girder bridges has been investigated for the minimum cost of superstructure, assuming relatively moderate or low substructure costs. The effect of substructure costs are briefly discussed in the next section.

Although absolute costs of bridge systems will vary with the specific unit costs, the optimal design solutions will be insensitive to variations of these unit costs as long as their relative magnitudes remain unchanged.

* Private communication with Kris Basi, Ministry of Transportation, Ontario, Canada, September 1990.
### PRACTICAL RECOMMENDATIONS

#### Optimality Criterion for Prestressing Force

The service tensile stress constraint, which is always active for this type of bridge, may be adopted as an optimality criterion for the determination of the prestressing force:

\[
P = \left[ f_t + (M_g + M_o) / S_b + (M_a + 0.75M_d) / S_{bc} \right] / (1/A + e/S_b)
\]

where \( S_b \) and \( S_{bc} \) are the section moduli with regard to bottom fibers of the girder and composite sections, respectively.

#### Substructure Cost Consideration

Fig. 10 shows that the optimization of the superstructure yields unique optimum bridge designs, except at two specific length ranges where two configurations are competitive and the final choice is to be made by comparing their total costs.

- For \( 27 < L \leq 32 \text{ m} \) (89 < \( L \leq 105 \text{ ft} \)), the superstructure cost for the 1-S configuration (feasible for the CPCI 1400 girder only) is on average 24 percent (rather high) more expensive than the two-span continuous (2-C) configuration, which may be considered as the optimum (Fig. 10). However, if the cost of the intermediate pier in the 2-C configuration is very high (e.g., very difficult access site), the 1-S configuration may be preferable.
- For \( 44 < L \leq 55 \text{ m} \) (144 < \( L \leq 180 \text{ ft} \)), the 2-C and 3-C configurations may be competitive, and the final choice will depend on their substructure cost ratio and the superstructure to substructure costs ratio, as is shown in the following. Let \( \delta_1 \) be the ratio of the total bridge costs for the 2-C and 3-C configurations:

\[
\delta_1 = \frac{(\text{sup}+\text{sub})_{2-C}}{(\text{sup}+\text{sub})_{3-C}}
\]

where "\( \text{sup} \)" and "\( \text{sub} \)" are the costs of the superstructure and substructure, respectively. Division of both numerator and denominator by \( \text{sup}_{2-C} \) yields:

\[
\frac{\delta_1}{\text{sup}_{2-C}} = \frac{1 + \frac{\text{sub}_{2-C}}{\text{sup}_{2-C}}}{\frac{\text{sup}_{3-C}}{\text{sup}_{2-C}}} = \frac{1 + \delta_2}{\delta_3}
\]

It is noted from Fig. 10 that the superstructure cost ratio \( \text{sup}_{3-C}/\text{sup}_{2-C} = 0.93 \) (constant) for all three bridge widths considered. Hence, the ratio \( \delta_1 \) depends on two parameters:

- \( \delta_2 = \frac{\text{sub}_{2-C}}{\text{sup}_{2-C}} \)
- \( \delta_3 = \frac{\text{sub}_{3-C}}{\text{sub}_{2-C}} \)

From Eq. (15e), it can be concluded that:

- 3-C is optimal for \( \delta_3 = 1 \).
- 2-C is optimal for \( \delta_3 > 1.15 \).
- 3-C and 2-C are competitive for \( \delta_3 < 1.15 \) and \( \delta_3 \leq 0.7 \).

#### AASHTO Implementation

(a) AASHTO-PCI Girders vs. CPCI Girders —

From Figs. 4 and 11, it is seen that the precast CPCI girders 900, 1200 and 1400 have very similar geometries with the AASHTO-PCI girders Types II, III and IV, respectively.
July-August 1993

Table 6. Comparison of OHBDC* and AASHTO* HS20 live load moments.

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<th>Code moments and ratios</th>
<th>Simple span (1-S)</th>
<th>Two-span continuous (2-C)</th>
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Metric (SI) conversion factor: 1 kN-m = 8.850 kip-in.

Table 5, which compares the geometrical and mechanical characteristics of the two sets of girders, indicates that they have virtually identical section moduli $S_L$ and $S_D$. Differences between CPCI and AASHTO-PCI section moduli are inferior to 8 percent for Types II and IV vs. CPCI 900 and 1400, and less than 4 percent for Type III vs. CPCI 1200. This suggests that the AASHTO-PCI Types II, III and IV are structurally equivalent to CPCI 900, 1200 and 1400, respectively.

(b) AASHTO* vs. OHBDC* Design Requirements

**SLS Constraints**

**Allowable Stresses: OHBDC** and AASHTO adopt the same allowable tensile and compressive stresses at SLS, i.e., $-0.5 \sqrt{f_t}$ and $0.4 \sqrt{f_c}$, respectively.

**Effective Service Moments:**

**OHBDC:**

$$M = M_D + 0.75 M_L$$

(16a)

**AASHTO:**

$$M = M_D + M_L$$

(16b)

where $M_D$ and $M_L$ are the moments due to dead and live loads, respectively.

Although the OHBDC truck (lane) load moments are higher than the AASHTO (HS20) truck (lane) load moments (see Fig. 12), the OHBD Code considers only 75 percent of these moments at SLS. Moreover, transverse distribution factors in the OHBDC are larger than the AASHTO values ($D_d = 1.8$ to 2.15 vs. 1.67). Hence, OHBDC and AASHTO live load moments should be compared on the basis of their effective truck (or lane) load design moments, which include the transverse distribution factor $S/D_D$, SLS load combination factor and impact factor $I$. These effective design live load moments $M^*_L$ may be expressed as:

**OHBDC:**

$$M^*_L = 0.75 \left(1 + I \right) \left(S/D_D \right) M_{dl}/2$$

(16c)

**AASHTO:**

$$M^*_L = (1 + I) \left(S/1.67 \right) M_{dl}/2$$

(16d)

where $M_{dl}$ is the moment due to the governing truck or lane load. In the OHBD Code, $D_d$ is not a constant as in AASHTO, but a function of the number of lanes, torsional and flexural parameters of the bridge. In the new OHBDC Code, the determination of $D_d$ has been simplified to depend on the span length only (for girders of the same bridge type). For comparison purposes, one may conservatively adopt $D_d = 1.9$. The ratio of OHBDC to AASHTO (HS20) maximum truck (lane) load moments, $R_1$, and the ratio of their corresponding effective design moments, $R_2$, are related by:

$$R_2 = 0.628 R_1$$

(16e)

For the simple and two-span continuous girders, Table 6 shows the maximum truck (lane) load moments by the two codes, their ratios $R_1$ and ratios $R_2$ of the effective design moments. The real differences between the AASHTO and OHBDC service live load moments, illustrated by ratio $R_2$, remain inferior to 6 percent, and, therefore, for all practical purposes, identical designs will satisfy SLS requirements of both codes.

**ULS Constraints**

The ultimate limit state (ULS) flexural requirements in OHBDC and AASHTO codes are:

**OHBDC:**

$$M_u = 1.1 M_g + 1.2 M_s + 1.5 M_d + 1.4 M_L \leq 0.85 M_n$$

(5)

**AASHTO:**

$$M_u = 1.3 M_D + 2.17 M_L \leq 0.90 M_n$$

(17a)

In AASHTO, a constant overload factor is applied on the total dead load, while in OHBDC this factor varies from 1.1 (for girder weight moment) to 1.5 (for asphalt pavement weight moment). From Table 6, it can be assumed on average that the OHBDC live load moment is 60 percent higher than the AASHTO moment $R_1 = 1.6$. Taking the AASHTO live load moment as a reference, and including the transverse distribution factors, the OHBDC live load moment is 40 percent higher than the AASHTO moment (1.6 x 1.67 = 1.4). In addition, since the main dead load moment is due to the girder and slab weight moments, the various dead load factors in Eq. (5) are substituted by a single factor, approximately 1.2, and the OHBDC ULS requirement becomes:

$$M_u = 1.2 M_D + 1.96 M_L \leq 0.85 M_n$$

(17b)

In the new OHBD Code, the single understrength factor (0.85) has been substituted by strength reduction factors for concrete (0.75), steel (0.90) and prestressing steel (0.95).
Fig. 12. OHBDC and AASHTO live loadings: (a) OHBDC truck and lane loadings [Axles 2 and 3 have been changed to 160 kN (36 kips) in the new OHBDC], (b) AASHTO HS20 truck and lane loadings.
However, since the optimal girder designs obtained are always under-reinforced with low reinforcement indices, the corresponding equivalent understrength factor will be higher than 0.85. Hence, the original assumption \((\phi = 0.85)\) is more conservative than the new code safety requirements.

Let \(\lambda_0\) be an overall safety factor defined as the ratio of the ultimate resisting moment to the service moment. The overall safety factors \(\lambda_0\) for the two codes are:

- **OHBDC:**
  \[
  \lambda_0 = M_s/\phi M_{D+L} = 1.41 + 0.953 M_L/M_{D+L} \quad (17c)
  \]

- **AASHTO:**
  \[
  \lambda_0 = M_s/\phi M_{D+L} = 1.44 + 0.967 M_L/M_{D+L} \quad (17d)
  \]

where \(M_{D+L}\) is the service dead and live load moment. The difference between the two safety factors does not exceed 2 percent for all practical values of \(M_L/M_{D+L}\), and it is concluded that the same design will be feasible for both the AASHTO and OHBDC codes.

**Optimal Design Procedure**

With the design aids resulting from the optimization study of precast, prestressed I-girder systems, the preliminary optimal design of a bridge system consists of four simple steps to determine: (a) system selection, (b) optimum prestressing force and tendon layout, (c) non-prestressed steel for ULS flexural strength (if needed), and (d) bottom and top reinforcements at piers.

- **(a) System Selection** — Given the total bridge length \(L\), width \(W\) and OHBDC or AASHTO (HS20) loadings, use Fig. 10, Table 2 and Table 4 to determine the optimal configuration, number and type of girders.

- **(b) Optimum Prestressing and Tendon Layout** — Given the span length \(L\), the optimum prestressing is obtained from Figs. 8(a) and 8(b) for simple and continuous span bridges, respectively. For bridge widths between the values investigated in this paper, interpolate linearly from Fig. 8 or use Eq. (14).

The tendon layout adopted for the type of girders investigated has two depressed points at the third span from each support (Fig. 5b). The eccentricity at the middle third span \((e_c)\) should be taken as the maximum feasible value (i.e., for minimum concrete cover), while the eccentricity at the support \((e_s)\) could have any value that satisfies the transfer stress constraints. Let \(P_{lim}\) be a certain limit value for the effective prestressing force:

\[
P_{lim} = (f_{pc} S_b + f_{ps} S_p) A_c / \beta (S_b + S_p) \quad (18a)
\]

where \(\beta\) is the ratio of the initial to effective prestressing forces. Thus:

If \(P \leq P_{lim}\) \(e_s \leq S_b / A_c - f_{ps} S_p / \beta P\) (18b)

and if \(P > P_{lim}\) \(e_s \leq -S_b / A_c + f_{pc} S_b / \beta P\) (18c)

- **(c) Non-Prestressed Steel for ULS Flexural Strength** — Determine \(\phi\) from Fig. 9 as a function of the span length. Find the non-prestressed reinforcement from Eq. (7).

**Design Example 1:**

**Member vs. System Optimization**

The relevance and benefits of system optimization vs. member optimization are illustrated by a cost comparison of optimal designs for a reinforced concrete slab on a precast, prestressed I-girder bridge with a total length \(L = 60\) m (197 ft) and width \(W = 16\) m (52 ft) (four lanes) for two cases: (a) specified longitudinal configuration and (b) variable longitudinal configuration.

- **(a) Design 1: Specified Configuration**
  - Consider a two-span simply supported girder bridge (2-S) with \(\ell = 30\) m (98 ft).
  - Optimum girder section is CPCI 1400, the only feasible alternative for \(\ell = 30\) m (98 ft).
  - Design requires seven CPCI 1400 girders with \(P = 3700\) kN (832 kips) and \(e_c = 535\) mm (21 in.) (maximum feasible eccentricity).
  - From Table 3, unit superstructure cost per deck area \(Z = $451\) per \(m^2\) \((Z = $42\) per sq ft\), yielding a total superstructure cost of about $433,000.

- **(b) Design 2: System Optimization**
  - From Fig. 10, the optimum solution for \(L = 60\) m (197 ft) and \(W = 16\) m (52 ft) is the three-span continuous (3-C) bridge system.
  - optimum solution requires five CPCI 1200 girders.
  - From Fig. 8b, the optimum prestressing force is \(P = 2300\) kN (517 kips) and \(e_c = 427\) mm (17 in.) (maximum feasible eccentricity).
  - From Fig. 10 or Table 3, unit superstructure cost per deck area \(Z = $335\) per \(m^2\) \((Z = $31\) per sq ft\), yielding a total superstructure cost of about $322,000.

The 25.7 percent cost reduction of Design 2 vs. Design 1 demonstrates the economic importance of system optimization vs. member optimization.

**Design Example 2:**

**AASHTO-PCI Application**

A reinforced concrete slab on precast, prestressed AASHTO-PCI girders bridge of total length \(L = 130\) ft (39.7 m) and width \(W = 40\) ft (12.2 m) is to be designed according to AASHTO specifications for the HS20 loading. The slab thickness is 8.5 in. (216 mm) with an asphalt pavement of 3.5 in. (89 mm). For the girders, \(f_{pc} = 6000\) psi (41 MPa) and \(f_{ps} \approx 4500\) psi (31 MPa), and for the slab, \(f_{ps} = 4500\) psi (31 MPa). Also, \(f_{pc} = 270\) ksi (1860 MPa) and \(f_{ps} = 60\) ksi (414 MPa).

- **(a) Optimal Bridge System Selection**

From Fig. 10, for \(L = 130\) ft (= 40 m) and \(W = 40\) ft (=12 m), the best structural system consists of four CPCI 1200 two-span continuous [span length \(\ell = 65\) ft (20 m)] girders,
equivalent to four AASHTO-PCI Type III girders and \( S = 9.5 \text{ ft} (2.9 \text{ m}) \) girder spacing.

(b) Optimum Prestressing and Tendon Layout
From Fig. 8b and for a span length of 65 ft (=20 m), the optimum prestressing for the AASHTO-PCI Type III is 483 kips (2150 kN) with eccentricity at midspan of 17 in. (427 mm). The tendon layout is depressed at the third span points [21.6 ft (6.58 m)] from each support, as shown in Fig. 5b. The end eccentricity is easily determined from the transfer stress constraints.

(c) Non-Prestressed Steel for ULS Flexural Strength
From Fig. 9b, for \( f_0 = 65 \text{ ft} (20 \text{ m}) \) (and AASHTO-PCI Type III), the total reinforcement index is \( \omega = 0.03 \). The amount of non-prestressed reinforcement is obtained from Eq. (7):
\[
A_s = (\omega bd f'_c - A_{ps} f_{ps})/f_s = 0.37 \text{ in.}^2 (239 \text{ mm}^2)
\]

(d) Bottom and Top Reinforcements at Piers
Eqs. (17d) and (17e) yield the positive (bottom) and negative (top) moment reinforcements at the piers as 3.2 in.\(^2\) (2065 mm\(^2\)) and 6.9 in.\(^2\) (4452 mm\(^2\)), respectively.

CONCLUSIONS
1. The general optimization approach presented in the paper is ideally suited for determining the optimal designs of precast, prestressed concrete sections, members or bridge systems.
2. Standardization of optimal bridge systems through optimization of longitudinal and transverse bridge configurations is the main objective of the paper. The overall design economy may rationally be improved by optimizing bridge systems rather than their (standardized) components.
3. Optimal longitudinal configurations are simply supported girders (1-S) for bridge lengths not exceeding 27 m (89 ft); two-span continuous girders (2-C) for bridge lengths varying between 28 and 44 m (92 and 144 ft); and three-span continuous girders (3-C) for bridge lengths between 55 and 100 m (180 and 328 ft). For bridge lengths of 44 to 55 m (144 to 180 ft), both optimal superstructure systems 3-C and 2-C are competitive as long as the substructure cost of the former remains moderate; otherwise, the two-span continuous system is more economic.
4. Optimal girder sections: For total bridge lengths \( L \leq 66 \text{ m} \) (217 m), the optimal system always consists of CPCI 1200 sections (with three, four or five girders for \( W = 8, 12 \) or 16 m (26, 39 or 52 ft), respectively). For \( L > 66 \text{ m} \) (217 ft), the optimal system always consists of CPCI 1400 sections (with four to seven girders depending on \( W \) and \( L \)) (Fig. 10 and Table 4).
5. Optimal system selection may be obtained directly from Fig. 10 by specifying the bridge length \( (L) \) and width \( (W) \). Although Fig. 10 identifies optimal bridge systems under OHBDC traffic loading, it may conservatively be used for lighter loading schemes.
6. Maximum feasible girder spacing is insensitive to bridge width and depends only on the bridge configuration (simple or continuous), span length, girder type and slab thickness. However, the optimal girder spacing depends on the bridge width in addition to all other parameters.
7. Prestressing reinforcement of I-girders may be found directly from Fig. 8, which corresponds to maximum feasible eccentricity (i.e., minimum concrete cover) at midspan. The support eccentricity may be determined from Eqs. (18b) and (18c). Additional non-prestressed reinforcement required at midspan may be determined from Fig. 9 and Eq. (7). The non-prestressed continuity reinforcement at the piers is determined from Eqs. (13d) and (13e).
8. The total reinforcement index of prestressed girders is rather low and is marginally affected by the bridge width. It increases with span length and decreases with girder depth.
9. Although the superstructure costs of various bridge systems (Table 3) depend on the adopted unit costs, optimal bridge systems are unaffected by absolute values of the latter as long as their ratios remain comparable. Hence, the system effectiveness will conserve the trends found in this investigation, while real costs may vary with the specified economic conditions of a particular project.
10. Design aids provided in the paper (Figs. 8, 9 and Tables 2 and 4) may be used to generate preliminary designs, to be detailed in compliance with any additional economic or structural requirements (shear strength, deflection and other parameters). Alternatively, basic design parameters may be determined by using the prestressing force optimality criterion.
11. AASHTO implementation of bridge system optimization is direct and only requires substitution of standard sections AASHTO-PCI Type II, III and IV for CPCI 900, 1200 and 1400 sections, respectively.
12. Extensions of bridge system optimization procedures are necessary and will be considered separately. In particular, the following topics deserve further investigation: optimization of precast, prestressed concrete I-girder sections to be standardized, other competitive transverse configurations (prestressed concrete voided slab or box girder, composite steel girder-concrete deck, etc.) and comprehensive sensitivity studies.

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APPENDIX A — NOTATION

The following symbols are used in this paper in addition to the standard ACI and PCI notation:

- **A** = girder cross-sectional area
- **C<sub>cs</sub>** = concrete cost in slab and diaphragms
- **C<sub>dj</sub>** = cost of deck joints
- **C<sub>g</sub>** = cost of precast girder per length
- **C<sub>pc</sub>** = unit cost of positive moment connection at piers
- **C<sub>s</sub>** = non-prestressed steel unit cost
- **c<sub>bot</sub>** = bottom concrete cover
- **c<sub>top</sub>** = top concrete cover
- **D<sub>d</sub>** = design load distribution factor
- **D<sub>v</sub>, D<sub>y</sub>** = bridge flexural rigidity in transverse and longitudinal directions, respectively
- **D<sub>xy</sub>, D<sub>yx</sub>** = bridge torsional rigidity in transverse and longitudinal directions, respectively
- **D<sub>1</sub>, D<sub>2</sub>** = bridge coupling rigidity in transverse and longitudinal directions, respectively
- **f<sub>sp</sub>, f<sub>sc</sub>** = allowable tensile and compressive stresses at service, respectively
- **f<sub>mp</sub>, f<sub>mc</sub>** = allowable tensile and compressive stresses at transfer, respectively
- **e<sub>e</sub>, e<sub>c</sub>** = tendon eccentricities at end and midspan sections, respectively
- **I** = impact factor
- **I<sub>g</sub>, I<sub>gc</sub>** = moments of inertia of girder and composite sections, respectively
- **L** = total bridge length
- **l** = span length
- **l<sup>*</sup>** = equivalent span length for live load moment computation in continuous girders
- **M, M<sub>D+L</sub>** = total service moment
- **M<sub>a</sub>** = asphalt pavement weight moment
- **M<sub>cm</sub>** = moment at support **n** due to combined effects of creep and shrinkage
- **M<sub>Dn</sub>** = support **n** restraint moment due to dead load
- **M<sub>g</sub>** = girder weight moment
- **M<sub>L</sub>** = live load moment
- **M<sub>n</sub>** = nominal resisting moment; moment at support **n** if structure is continuous and monolithically cast in a single stage
- **M<sub>pn</sub>** = support **n** restraint moment due to prestressing
- **M<sub>s</sub>** = slab weight moment
- **M<sub>sm</sub>** = support **n** restraint moment due to shrinkage
- **M<sub>u</sub>** = ultimate load moment
- **M<sup>*</sup><sub>n</sub>** = moment induced by creep at support **n**
- **n** = number of girders
- **n<sub>p</sub>** = number of piers

- **n<sub>ss</sub>** = number of critical span sections
- **R<sub>1</sub>** = ratio of OH BDC and AASHTO HS20 truck (lane) load moments
- **R<sub>2</sub>** = ratio of OH BDC and AASHTO HS20 effective design moments
- **S** = girder spacing
- **S<sup>’</sup>** = slab overhang
- **S<sub>sp</sub>, S<sub>bc</sub>** = section moduli with regard to bottom fibers of girder and composite sections
- **sup** = superstructure cost
- **sub** = substructure cost
- **t** = slab thickness
- **t<sub>0</sub>** = concrete age at loading
- **t<sub>1</sub>** = concrete age at which continuity is achieved
- **V<sub>cs</sub>** = volume of concrete in slab and diaphragms
- **V<sub>pc</sub>** = volume of positive moment reinforcement
- **W** = bridge width
- **W<sub>s</sub>** = weight of non-prestressed steel in slab and diaphragms
- **W<sub>sn</sub>** = weight of negative moment steel in slab at piers
- **W<sub>sp</sub>** = weight of positive moment steel in girders
- **y** = distance between centroids of girder and composite sections
- **y<sub>bc</sub>** = distance from centroid of composite section to bottom fiber
- **y<sub>l</sub>** = distance from centroid of girder section to top fiber
- **Z** = superstructure cost per deck area
- **α** = bridge torsional parameter
- **β** = ratio of initial and effective prestressing forces
- **δ<sub>l</sub>** = ratio of total bridge costs of 2-C to 3-C configurations
- **δ<sub>2</sub>** = ratio of substructure to superstructure costs for 2-C configuration
- **δ<sub>3</sub>** = ratio of substructure costs of 3-C to 2-C configurations
- **Φ** = flexural strength reduction factor
- **χ** = aging coefficient
- **λ<sub>0</sub>** = ratio of ultimate resisting moment to service moment
- **v(t, t<sub>0</sub>)** = creep coefficient (ratio of creep strain to initial elastic strain)
- **v<sub>u</sub>** = ultimate creep factor
- **ω** = total reinforcement index
- **σ<sub>sp</sub>, σ<sub>sc</sub>** = concrete stresses at transfer and service, respectively
- **θ** = bridge flexural parameter