Lateral Stability of Long Prestressed Concrete Beams — Part 2

Robert F. Mast, P.E.
Chairman
ABAM Engineers
A Member of the Berger Group
Federal Way, Washington

Long-time PCI Professional Member **Robert F. Mast** is one of the four co-founders of ABAM Engineers, headquartered in Federal Way, Washington. The firm, which recently celebrated its 40th anniversary, is today a member of the Berger Group, a worldwide consulting organization. During his professional career, Mr. Mast has been responsible for the design of many important buildings, bridges and special structures. As a partner in the firm, he was involved in the innovative use of prestressed concrete in the first bulb tee bridge, the Walt Disney World and Seattle monorails, Disney World's Space Mountain, the Indonesian LPG floating vessel and many other noteworthy structures. Among his many engineering contributions, he was one of the original developers of the shear-friction principle, widely used in the design of precast concrete connections. For his paper on this topic, Mr. Mast won ASCE's T.Y. Lin Award. He is also an authority on stability problems associated with the handling and transportation of long prestressed concrete members, which is the subject of this paper.

A theory for evaluating the lateral stability of long prestressed concrete I-beams supported from below is developed. The same principles are also applied to hanging beams, extending the work presented earlier in Part 1 of this paper. The theory includes consideration of the post-cracking behavior of prestressed concrete beams under lateral loads. Data from the testing of a full-sized prestressed concrete beam are presented to verify the theory. A method of computing factors of safety related to lateral stability is presented. A numerical example is included, giving computations for the factors of safety for both hanging and bottom-supported beams. A computer program in BASIC is furnished to facilitate these computations.

art 1 of this paper[1] dealt with prestressed concrete beams hanging from lifting loops. The stability of beams hanging from loops is primarily a function of the elastic stiffness properties of the beam and may readily be determined using the methods given in Part 1.

This Part 2 paper deals with the stability of beams when supported on elastic supports below the beam. Such supports may be bearing pads or transportation equipment. The understanding of the behavior of a beam supported on elastic supports was found to be different from and far more complex than that for a beam hanging from lifting loops.

It was found that rollover of beams supported from below is determined primarily by the properties of the support

rather than the beam. Long prestressed concrete I-beams of ordinary proportions (such as the PCI BT-72), when supported from below, were usually found to have sufficient lateral bending strength to withstand greater angles of inclination than can be resisted by the supports.

Part 1 stated that Part 2 would give methods for determining the lateral stability of beams supported from below, and that this requires evaluation of the rotational stiffness of supports and the post-cracking behavior of prestressed concrete I-beams subjected to lateral loads. The solution to these questions proved more difficult than the author had anticipated because both the properties of the support and the post-cracking properties

Fig. 1. Equilibrium of beam on elastic support.

of the beam may be nonlinear. Analytical solutions to the problems were developed, and the results were verified by a full-scale test.

This Part 2 paper supplements Part 1. Eqs. (1) to (15) were given in Part 1. Additional equations given in this Part 2 paper begin with Eq. (16). Part 1 is still valid, with two exceptions: the author recommends using Eq. (22) instead of Eqs. (1) and (2), and using Eq. (30) instead of Eqs. (14) and (15).

BACKGROUND

Classic studies of lateral buckling of beams are based on the assumption that the beams are rigidly restrained from rotation at the supports. Buckling is caused by the middle part of the span twisting relative to the support, creating a sideways deflection. This type of buckling is important in steel I-beams, which have low torsional stiffness.

The torsional stiffness of an I-beam varies as the cube of the thickness of the web and flanges. Concrete I-beams, with relatively thick webs and flanges, are 100 to 1000 times stiffer in torsion than steel I-beams. As a result, lateral buckling of the classic type is seldom critical in a concrete beam. But, when the supports have roll flexibility, the beams may roll sideways, producing lateral bending of the beam. This is the cause of most lateral stability problems involving long concrete I-beams.

The approach may be greatly simplified by assuming the beam to be rigid in torsion. For concrete I-beams with webs and flanges 6 in. (150 mm) or more in thickness, the torsional stiffness of the beams will normally be much greater than the roll stiffness of the supports. The assumption of torsional rigidity for the beam transforms the problem from a buckling problem to a bending and equilibrium problem.

GENERAL SOLUTION

The equilibrium of a slender beam on elastic supports is shown in Fig. 1. This figure is similar to Fig. C1 of Part 1, but with a positive quantity y being used for the height of
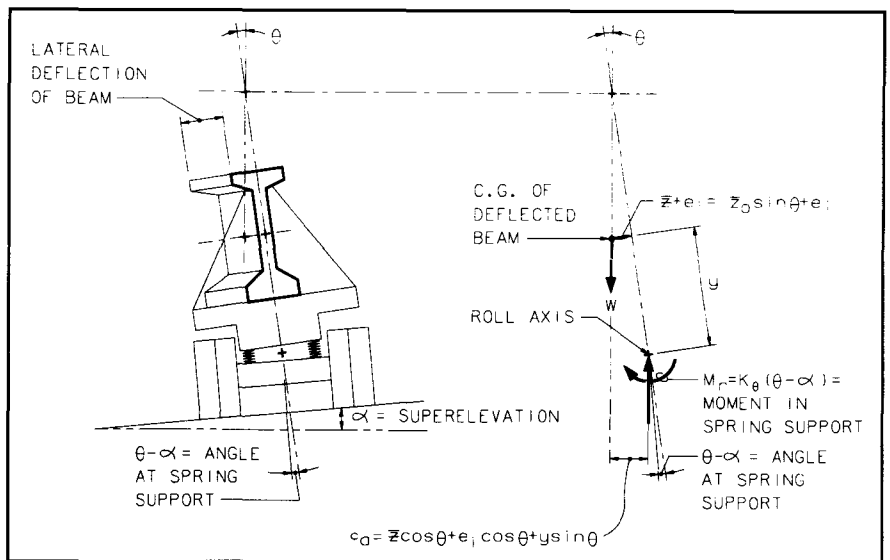
the center of gravity of the beam above the roll axis. The applied overturning moment arm c_a about the roll axis due to the weight of the beam is:

$$c_a = \bar{z} \cos \theta + e_i \cos \theta + y \sin \theta \qquad (16)$$

where

θ = roll angle of major axis of beam with respect to vertical

$\bar{z}$ = lateral deflection of center of gravity of curved arc of deflected beam

e_i = initial eccentricity of center of gravity of beam

y = height of center of gravity of beam above roll axis

The resisting moment arm c_r is equal to the resisting moment of the spring supports divided by the weight W of the beam:

$$c_r = \frac{K_\theta}{W} (\theta - \alpha) \qquad (17)$$

where

K_θ = sum of rotational spring constants of supports

α = superelevation angle or tilt angle of supports

The rotational spring constant K_θ has units of moment divided by rotation angle in radians. The rotational spring constant of an elastic support is found by applying a moment and measuring the rotation. The quantity K_θ is equal to the moment divided by the rotation angle.

It is convenient to let $r = K_\theta / W$. The quantity r has a physical interpretation. It is the height at which the total beam weight W could be placed to cause neutral equilibrium with the spring for a given small angle θ (see Fig. 2). For neutral equilibrium, the overturning moment will just equal the resisting moment when the member supporting W is displaced by a small angle. The quantity r may be called the radius of stability.

The equilibrium angle θ may be found by equating c_a to c_r. Since both c_a and c_r may be nonlinear functions of θ, the solution may be done graphically or by numerical iteration.

When a beam is hanging from lifting loops, the support spring constant K_θ is normally zero, and the resisting moment and resisting lever arm are provided by the weight of the beam itself. In this case, it is convenient to substitute the positive quantity y_r for the negative quantity $-y$, and move it from the applied moment arm to the resisting moment arm side of the equation.

Also, $\bar{z}_o \sin \theta$ may be substituted for $\bar{z}$, where $\bar{z}_o$ is the theoretical lateral deflection of the center of mass of the deflected shape of the beam, with the full dead weight applied laterally. Thus, for hanging beams:

$$c_a = \bar{z}_o \sin \theta \cos \theta + e_i \cos \theta \quad (18)$$

$$c_r = y_r \sin \theta \quad (19)$$

Fig. 2. Definition of radius of stability r.

When c_a and c_r are equated and the small angle approximations $\sin \theta = \theta$ and $\cos \theta = 1$ are made, the equations of Part 1 result. The equations for c_a and c_r represent a more general solution to the stability of hanging beams, as will be demonstrated later in this paper.

In Part 1, linear elastic behavior of hanging beams was assumed, and thus the quantity $\bar{z}$ could be replaced by $\bar{z}_o \sin \theta$, where $\bar{z}_o$ is the theoretical lateral deflection of the center of gravity of the beam with the full dead weight applied laterally, using the gross lateral moment of inertia I_g. The general solution requires that $\bar{z}$ be computed using cracked section stiffness, which varies with the roll angle θ when θ exceeds the tilt angle θ_{max}, at which cracking begins.

Once cracking occurs, the neutral axis acquires a large inclination with respect to the axes of the beam, and the top flange cracks across about one-half of its width. This causes the centroid of the compressive force to shift laterally within the beam. For long I-beams, the prestress force (and the balancing compressive force) is typically 1000 kips (4500 kN) or more.

The primary lateral load resistance is derived from this

Fig. 3. Midspan compressive stress block in a prestressed concrete I-beam, without tilt.

BIAXIAL BENDING OF CRACKED PRESTRESSED I-BEAMS

A common method of assessing the lateral bending strength of a long prestressed concrete beam is to limit the tensile stresses in the corner of the top flange to the modulus of rupture of the concrete, and to provide reinforcement in the top flange. The author has found that this procedure grossly underestimates the lateral bending strength of commonly used prestressed concrete I-beams, such as the PCI BT-72. These I-beams have the ability to resist lateral bending by a lateral shift in the centroid of the compressive force within the beam.

Fig. 3 illustrates the compressive stress block at midspan of a long prestressed concrete beam without tilt, where there is no lateral shift of the compressive forces. In long (say over 120 ft, or 36 m) beams, it is ordinarily not possible to place the prestressing force low enough to fully compensate for the dead weight of the beam. Even when supported a few feet from each end, there is usually some residual compression in the top flange.

In very round numbers, the stresses shown in Fig. 3 might typically be 500 psi (3.5 MPa) in the top and 2500 psi (17.2 MPa) in the bottom. Fig. 4 shows the compressive stress block in the same beam when tilted 15 degrees from the vertical.
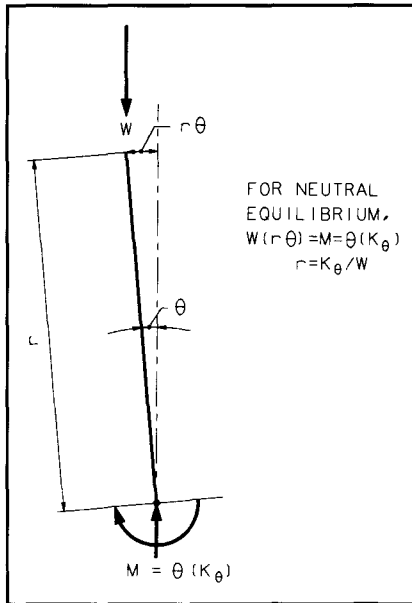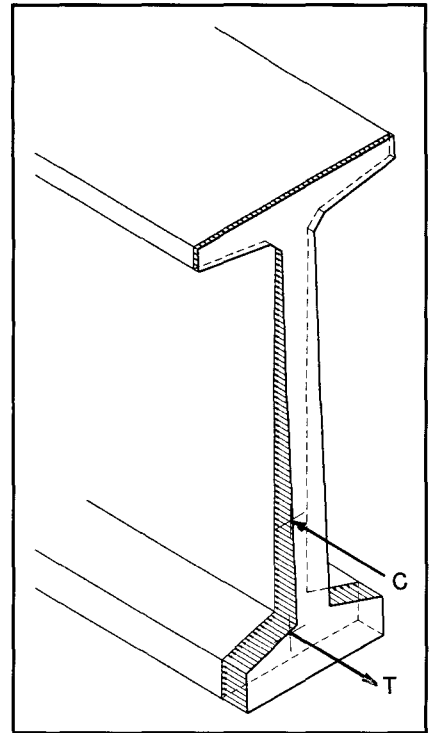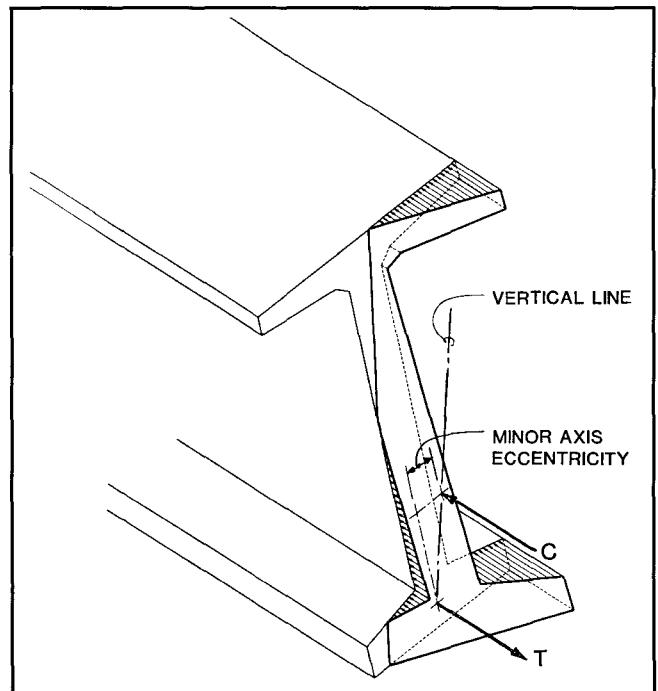

Fig. 4. Midspan compressive stress block in a prestressed concrete I-beam, with 15 degree tilt.

Fig. 5. Overall view of test setup for 149 ft (45.4 m) girder.

Fig. 6. Test beam near maximum tilt of 32 degrees.

large force acting on a lateral (minor axis) eccentricity of several inches (or, say, 200 mm), as illustrated in Fig. 4. Of course, reinforcement in the flanges also provides lateral moment resistance, but its contribution is relatively small compared to the resisting moment provided by the prestress force of 1000 kips (4500 kN) or more and the lateral eccentricity of the balancing compressive force.

ANALYTICAL STUDIES

The analysis of a cracked concrete section subjected to biaxial bending is quite complex. The slope and depth of the neutral axis are both unknown, and both must be found by successive approximations. Once the neutral axis slope and location are found, the net section in compression must be analyzed as a section subjected to unsymmetrical bending, with principal axes inclined to the neutral axis and also inclined to the gross section major and minor axes. Furthermore, a complete solution to the problem involves the analysis of many sections along the length of the beam, so that the deflection and center of gravity of the deflected shape may be found by numerical integration.

The author wrote a computer program to solve this analytical problem. The program is an elastic analysis program. Tensile stresses are neglected in computing stresses and the neutral axis location, but a crude approximation of ten-

sion stiffening was attempted in the deflection calculations. The author also has a program that computes nominal strength in biaxial bending using a rectangular stress block. This program does not compute deflections.

The results of the analysis of several actual large I-beams in the 125 to 150 ft (38 to 46 m)

Fig. 7. End view near maximum tilt of 32 degrees.

span range indicated that the beams can tolerate tilt angles of 25 to 30 degrees prior to failure. This result appeared to be at odds with experience. The program was checked as thoroughly as the author was able, but it is difficult to provide a completely independent check to this complex program. It was decided, therefore, that a test would be necessary to verify the accuracy of the analytical program.

TEST PROGRAM

Because the lateral bending of a tilted beam is a self-weight effect, it was decided that a full-scale test would be necessary. The Precast/Prestressed Concrete Institute, the Portland Cement Association, Concrete Technology Corporation, the University of Washington, and BERGER/ABAM Engineers jointly sponsored the test of a 149 ft (45.4 m) beam. The beam was available from a previous job and was tested at the Concrete Technology Corporation yard. Fig. 5 shows an overall view of the test setup.

The beam was supported on steel cradles and slowly tipped under controlled conditions. The supports were located 11 ft (3.3 m) in from each end, which is typical for shipping a beam of this length. Tilt and deflection readings were taken at intervals during the test. Data were also collected from strain gauges located at two sections near midspan. A detailed description of the test results will be published as a separate paper.

Three tests were made. In the first test, the beam was tipped to an angle of approximately 15 degrees and then tipped back to vertical. Although calculations indicated that the top flange would be cracked at the 15 degree angle (see Fig. 4), the beam showed no sign of any damage or permanent set after being brought back to vertical.

The second test was performed with the beam braced with a king post bracing system commonly used in Washington State when transporting long beams. Little difference from the first test was observed at a tilt of approximately 15 degrees. The beam was then tipped back to vertical, and the

bracing removed. Again, the beam showed no evidence of having been tipped 15 degrees once it was brought back to vertical.

The third test was to destruction, without bracing. Figs. 6 and 7 show the beam as it neared its maximum tilt angle of 32 degrees. Fig. 8 shows the comparison of predicted and actual test results. The failure tilt angle was almost exactly that predicted, and the tilt angle vs. minor axis deflection curve has the same shape as predicted, but with actual deflections being about 10 to 15 percent less than predicted. This is believed to be due to tension stiffening effects within the cracked sections, which were not fully accounted for.

The test results also verified a fundamental assumption used in both Parts 1 and 2 of this paper, that is, the torsional flexibility of the beams may be neglected. At the failure roll angle of 32 degrees, the twist at midspan relative to the supports amounted to about 4 percent of the roll angle. At a 15 degree roll angle, the twist was about 2 percent of the roll angle.

The test results and the computer predictions agree remarkably well. This gives the author confidence that the computer program can be used to predict the behavior of other beams with properties different from the test beam.

EFFECTIVE STIFFNESS OF CRACKED BEAMS

The author's computer program was used to analyze the effective lateral (minor axis) stiffness of nine different long I-beams, as the stiffness decreases due to cracking at various tilt angles. The beams included modified AASHTO beams, PCI bulb tees, and Washington and Oregon beam sections. All beams had a minor axis moment of inertia of at least 4 percent of the major axis moment of inertia, and an average prestress level of at least 1200 psi (8.3 MPa).

Fig. 8. Predicted tilt and deflection behavior, and test results.
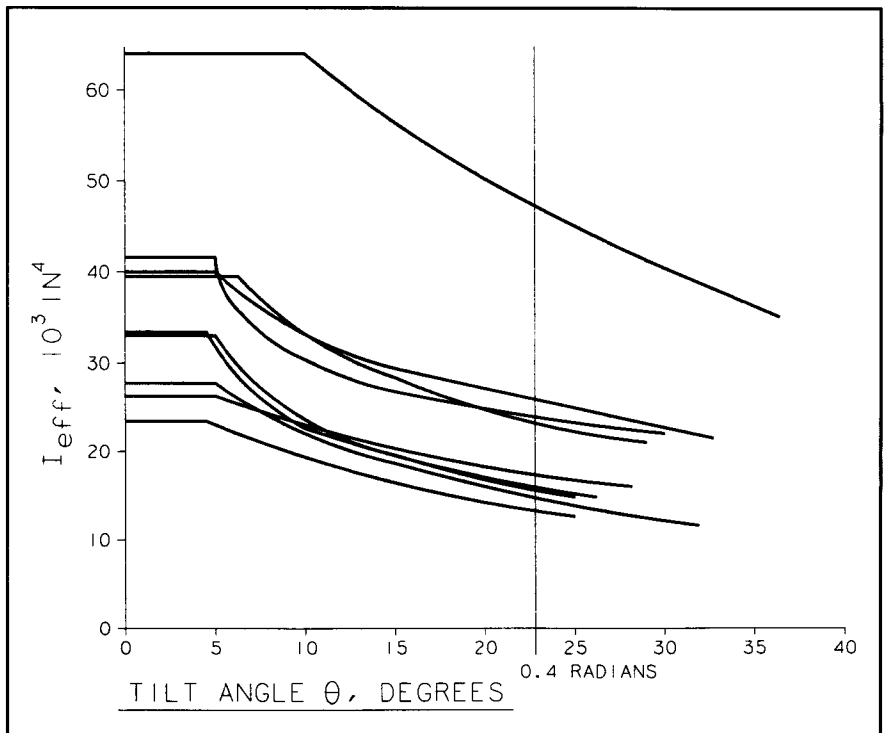
Fig. 9. Variation of effective lateral stiffness I_{eff} with tilt angle.

Fig. 9 shows the results. All beams showed similar relationships between stiffness and tilt angle, and all beams had a predicted tilt angle at nominal strength of at least 25 degrees. The end points of the curves indicate the predicted failure point.

A simplified relationship for the effective stiffness is proposed for long prestressed concrete I-beams of ordinary proportions, such as the PCI BT-72.
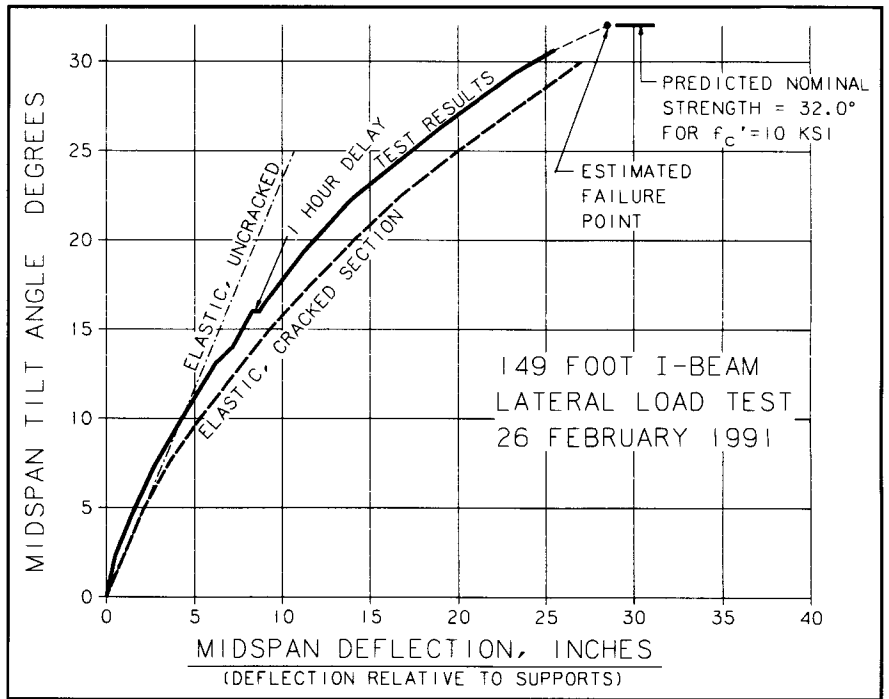
- For tilt angles that produce top flange tensile stresses less than $7.5\sqrt{f_c'}$, use the gross I_g.
- For tilt angles that produce top flange tensile stresses in excess of $7.5\sqrt{f_c'}$, use an effective stiffness:

$$I_{eff} = I_g/(1 + 2.5\ \theta) \tag{20}$$

- Assume the maximum θ at failure θ'_{max} to be 0.4 radians (or 23 degrees).

Fig. 10 compares Eq. (20) to the nine beams analyzed. Although the tests show no sudden change in stiffness at first cracking, it is certainly true that deflections are less predictable after cracking. Therefore, the assumption of a sudden loss of stiffness after cracking is conservative. The length of the plateau prior to cracking will vary depending on the top flange stresses.

PART 1 REVISITED

The equations in Part 1 of "Lateral Stability of Long Prestressed Concrete Beams" were derived using the assumption that the beam remained uncracked, and the maximum tilt angle θ_{max} was defined as that at which the tensile stress in a corner of the top flange reached the modulus of rupture. Subsequent analyses, confirmed by tests, show that beams can sustain much larger tilt angles, but with reduced stiffness due to cracking.

The tilt angle at failure may be taken as 0.4 radians, and the stiffness at angles between θ_{max} and 0.4 radians may be taken using Eq. (20). Because $\bar{z}_o$ is a function of stiffness, it is also a function of θ for cracked beams. This dependency of $\bar{z}_o$ on θ makes it more difficult to solve for θ, compared to the solution for uncracked beams in Part 1.

This difficulty may be overcome by plotting curves for applied overturning moment arm and resisting moment arm, and solving for the intersection which represents equilibrium. Moment arms are used instead of applied and resisting moments. Moment arms are obtained by dividing moments by the weight W. This eliminates W from all the equations, and produces a moment arm measured in inches (or millimeters), which is easier to visualize than a bending moment measured in kip-in. (or kN-m).

Fig. 11 shows how the applied and resisting moment arms are defined. Taking moments about the center of gravity of the cross section at the lifting point, the applied moment arm c_a is that due to the lateral deflection of the center of mass of the deflected shape of the beam, and is given by Eq. (18). The resisting moment arm c_r is given by Eq. (19).

Fig. 12 shows a plot of c_a and c_r. Using the small angle approximations $\sin \theta = \theta$ and $\cos \theta = 1$, the curves for c_a and c_r are straight lines. The equilibrium angle θ is defined by the intersection of the curves for c_a and c_r, and Eq. (5) may be derived geometrically from Fig. 12.

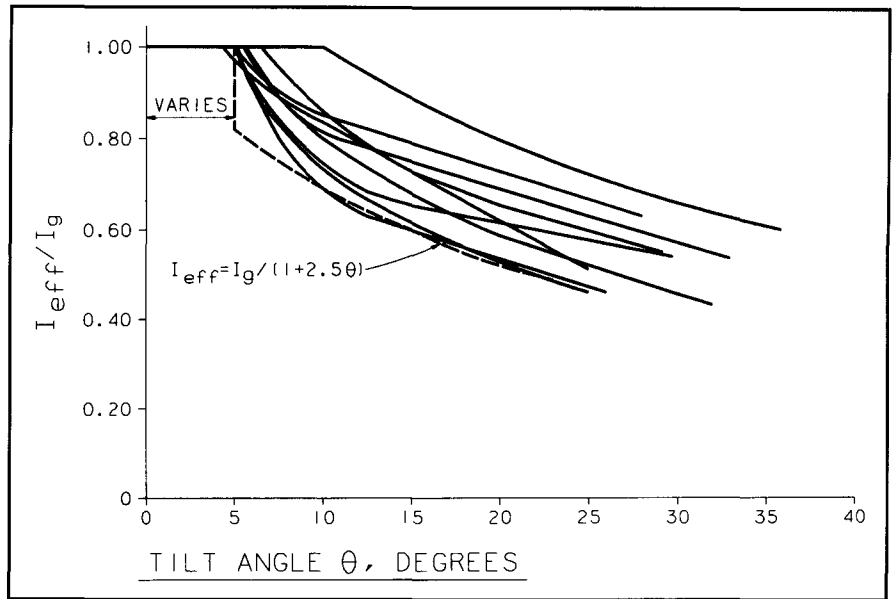The factor of safety may be defined as the ratio of resist-

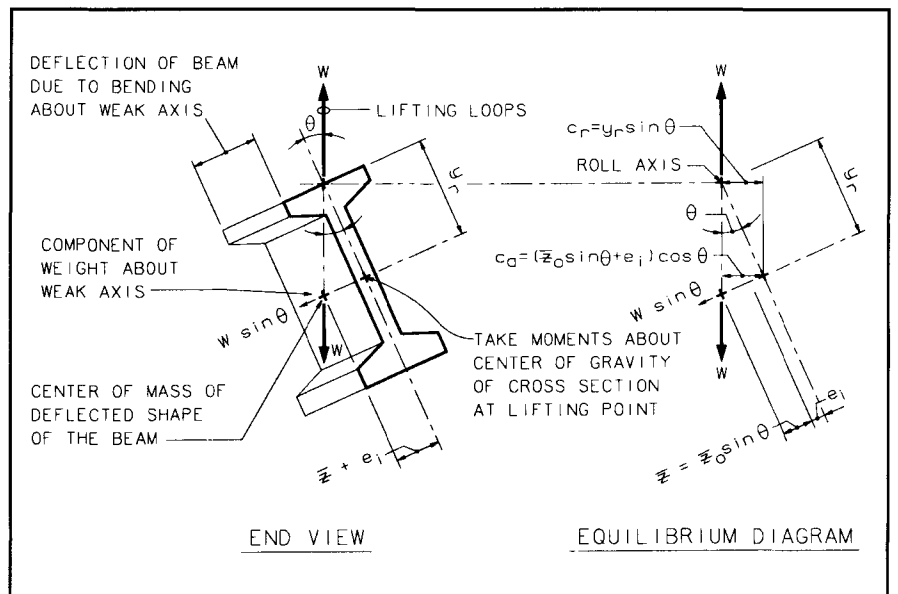Fig. 10. I_{eff} vs. θ relationship, and proposed equation.

Fig. 11. Applied moment arm c_a and resisting moment arm c_r for hanging beam.

ing moment arm c_r to applied moment arm c_a:

$$FS = \frac{c_r}{c_a} = \frac{y_r \theta}{\bar{z}_o \theta + e_i} \qquad (21)$$

The initial roll angle θ_i of a rigid beam is e_i /y_r. When θ is equal to θ_{max}, the tilt angle θ at which cracking is expected, substituting $\theta_i y_r = e_i$ and simplifying, the factor of safety against cracking FS is found:

$$FS = \frac{1}{\bar{z}_o /y_r + \theta_i /\theta_{max}} \qquad (22)$$

Fig. 13 shows Eq. (22) graphically. Eq. (22) should be used to replace both Eqs. (1) and (2) given in Part 1. The results are identical when either $\bar{z}_o /y_r$ or θ_i/θ_{max} is zero. Eq. (22) gives lower factors of safety when these two ratios are positive, for it considers the combined effect of the two
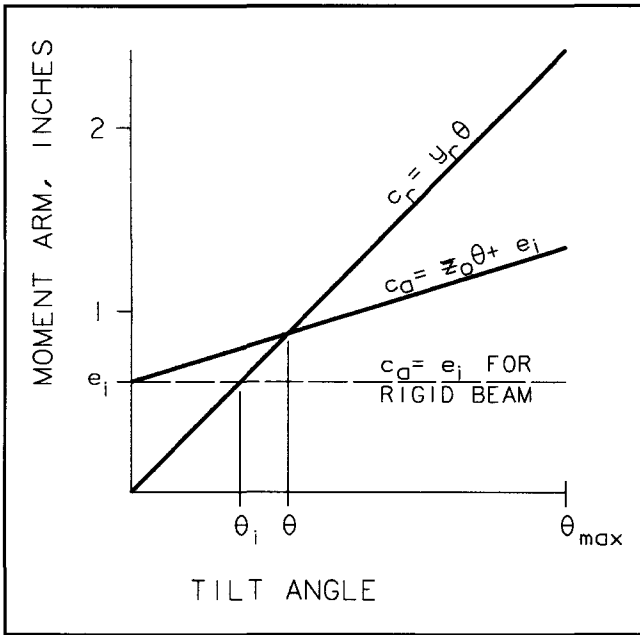
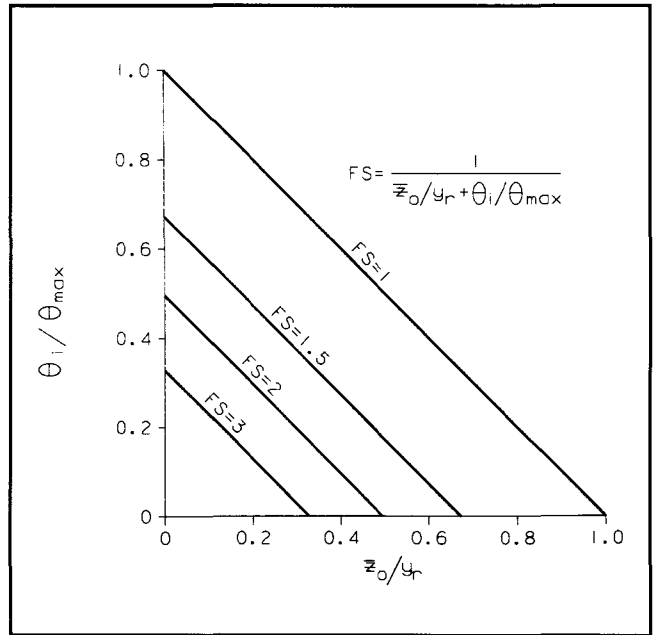Fig. 12. Moment arms vs. tilt angle for uncracked hanging beam.

Fig. 13. Factors of safety against cracking for a hanging beam.

ratios varying simultaneously, while Eqs. (1) and (2) consider the ratios varying one at a time.

The factor of safety against failure FS' may be analyzed by carrying the curves of Fig. 12 beyond the cracking point. A calculation for c_a and c_r is carried out by Eqs. (18) and (19) for increments of the tilt angle θ beyond the cracking point. For each angle θ, the quantity $\bar{z}'_o$ is computed using I_{eff} from Eq. (20) for that angle. The equivalent calculation is:

$$\bar{z}'_o = \bar{z}_o (1 + 2.5\,\theta) \qquad (23)$$

The results are shown in Fig. 14. Note that the scales have changed when comparing this figure to Fig. 12. The pre-cracking behavior shown in Fig. 12 is shown in the lower left corner of Fig. 14.

The curve for the resisting moment arm c_r is almost a straight line, since at 0.4 radians, sin θ = 0.97 θ. The curve for the applied moment arm c_a curves upward at higher θ angles as the stiffness decreases and $\bar{z}_o$ increases. The ratio of c_r/c_a is also plotted, using the scale to the right. The maximum ratio, about 1.8, occurs at about 0.16 radians, and this is the factor of safety FS' against failure.

One could also think of the factor of safety in terms of the difference in inches (or millimeters) between c_r and c_a. This difference continues to increase beyond 0.16 radians, reaching a maximum of about 4 in. (100 mm) as θ nears 0.4 radians. This 4 in. (100 mm) margin may be thought of as a tolerance against failure on the applied and resisting moment arms.

The maximum factor of safety against failure FS' and the tilt angle θ'_{max} at which this occurs may be found as follows. Eq. (21) may be modified by substituting the cracked section $\bar{z}'_o$ as given in Eq. (23):

$$FS = \frac{c_r}{c_a} = \frac{y_r\theta}{\bar{z}_o\theta(1 + 2.5\theta) + e_i} \qquad (24)$$
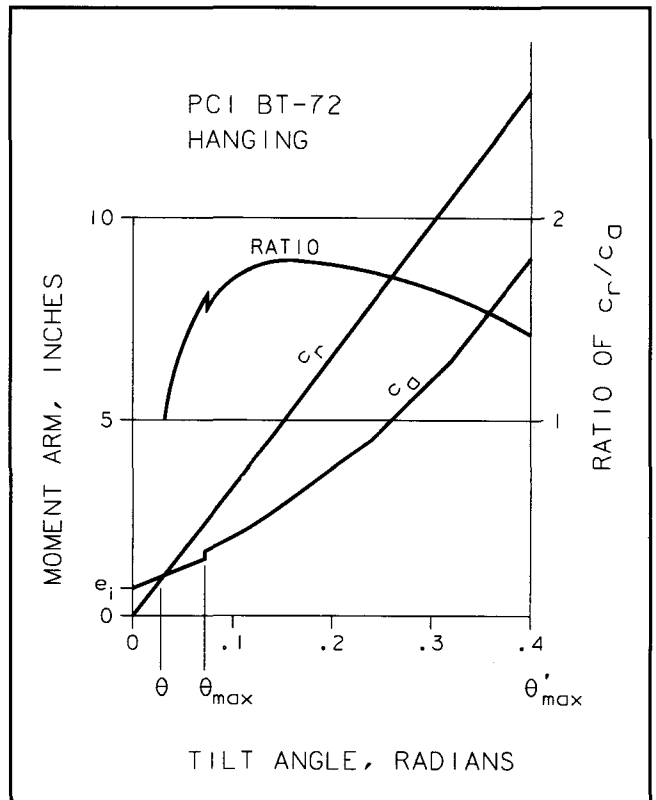

Fig. 14. Moment arms vs. tilt angle for a hanging beam, after cracking.

Eq. (24) may be differentiated to find the tilt angle θ'_{max} at which the maximum ratio of c_r/c_a occurs. The result is:

$$\theta'_{max} = \sqrt{e_i / (2.5\,\bar{z}_o)} \qquad (25)$$

The quantities θ'_{max} and $\bar{z}'_o$ may be substituted back in Eq. (21) to find the maximum factor of safety against failure FS':

$$FS' = \frac{y_r\,\theta'_{max}}{\bar{z}'_o\,\theta'_{max} + e_i} \qquad (26)$$

If the factor of safety against failure FS' is less than the factor of safety against cracking FS, this indicates that the maximum factor of safety occurs just before cracking. In this case, FS' should be taken equal to FS.

BEAMS SUPPORTED FROM BELOW

When a beam is supported on flexible supports such as bearing pads or truck and trailer, a similar situation occurs in which there is a tendency for the beam to roll about a roll axis. In this case, the roll center is below the beam, and y_r would be negative. As discussed earlier, it is convenient to replace it with a positive quantity y, as shown in Fig. 1.

Because the roll axis is beneath the center of gravity, the support must be capable of providing resistance to rotation. This resistance is expressed as an elastic rotational spring constant K_θ. Taking moments about the roll axis and substituting $\bar{z}_o \sin \theta$ for $\bar{z}$ (see Fig. 1).

$$W[\bar{z}_o \sin \theta \cos \theta + e_i \cos \theta + y \sin \theta] = K_\theta (\theta - \alpha) \qquad (10)$$

where α is the superelevation angle or tilt angle of the support.

Using the small angle approximations $\sin \theta = \theta$ and $\cos \theta = 1$:

$$W(\bar{z}_o \theta + e_i + y \theta) = K_\theta (\theta - \alpha) \qquad (11)$$

Solving Eq. (11) for θ and substituting r for K_θ/W:

$$\theta = \frac{\alpha r + e_i}{r - y - \bar{z}_o} \qquad (12)$$

When r is very large, i.e., the support is very stiff, θ approaches α, the tilt angle of the support. When $r = 0$, Eq. (12) is identical to Eq. (5) of Part 1, since $y_r = -y$. Eq. (12) is an expanded version of Eq. (5).

Taking moments about the roll axis, the equations for applied moment arm c_a (see Fig. 15) and resisting moment arm c_r are:

$$c_a = (\bar{z}_o \sin \theta + e_i) \cos \theta + y \sin \theta \qquad (27)$$

$$c_r = r (\theta - \alpha) \qquad (28)$$

Eq. (28) is derived from the fact that the resisting moment is equal to $K_\theta (\theta - \alpha)$ and r is defined as K_θ/W.

Again, the factor of safety is defined as the ratio of the resisting moment arm c_r to applied moment arm c_a. Also, the small angle approximations $\sin \theta = \theta$ and $\cos \theta = 1$ are used:

$$FS = \frac{c_r}{c_a} = \frac{r(\theta - \alpha)}{\bar{z}_o \theta + e_i + y\theta} \qquad (29)$$

Substituting the tilt angle θ_{max} at which cracking is expected, the factor of safety against cracking FS is found:

$$FS = \frac{r(\theta_{max} - \alpha)}{\bar{z}_o \theta_{max} + e_i + y \theta_{max}} \qquad (30)$$

When Eq. (30) was applied to actual beams which have been successfully shipped, it was found that the computed
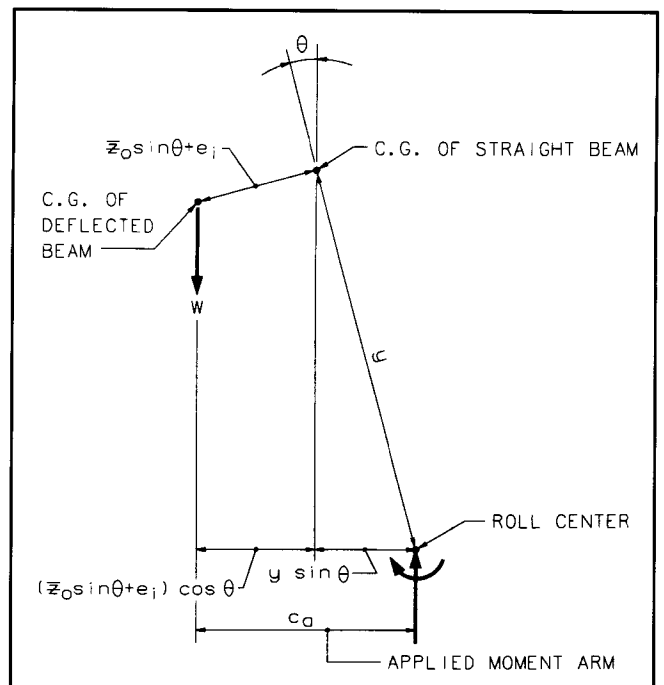

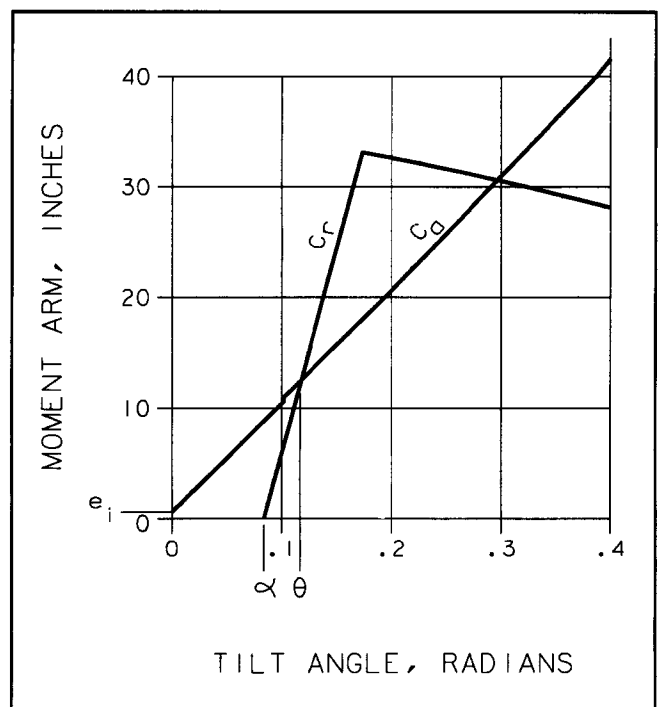Fig. 15. Applied moment arm for beam supported from below.

Fig. 16. Moment arms vs. tilt angle for a beam on truck and trailer.

factors of safety against cracking were often close to or less than unity. To understand the factor of safety against failure, one must look at the post-cracking behavior. This is done as before, by plotting curves of applied moment arm and resisting moment arm.

Fig. 16 shows plots of c_a and c_r for the beam used as a design example by Imper and Laszlo.[2] Note that the moment arms are much larger than for the same beam while hanging (Fig. 14). The applied moment is dominated by the large y

that causes over-turning about the roll axis. The quantity $\bar{z}_o$ is of less importance than when the beam is hanging. The resisting arm c_r was computed using a K_θ of 40,500 kip-in. per radian (4600 kN-m per radian). Actual rigs used to transport a beam of this size could vary considerably from this stiffness.
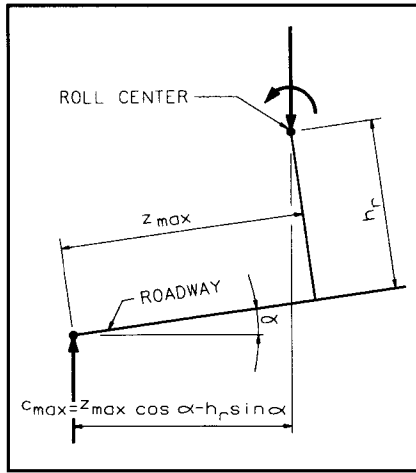

Fig. 17. Maximum resisting moment arm for a beam on truck and trailer.

Fig. 16 shows that the equilibrium tilt angle when on a superelevation of 0.08 radians is about 0.113 radians, whereas the tilt angle causing cracking is 0.10 radians. The rotational flexibility of the trailer causes the extra tilt. One might suggest that stiffer hauling rigs are needed. But, a little flexibility is necessary — not only to absorb road shock, but also to prevent excessive torsional stresses when the truck and trailer are on unequal roadway slopes.

Calculations and experience both show that tilt angles in excess of that which theoretically causes cracking can be tolerated. The test beam showed no observable change in behavior until tilt angles of approximately twice the theoretical tilt angle at cracking were reached.

In most cases in which a beam is supported from below, the maximum resisting moment arm c_r is governed by rollover. The maximum factor of safety against rollover may be found as follows. This derivation neglects the weight of the hauling rig; hence, it contains an additional factor of safety. This is thought to be prudent, since spring rates may become highly nonlinear near rollover.

The resisting moment arm c_r is limited by the geometry of the hauling rig. Assuming a height of roll center h_r (normally about 24 in., or 600 mm) and a distance from center-line to the center of dual tires z_{max} (normally about 36 in., or 915 mm), the maximum resisting moment arm c_{rmax} may be found (see Fig. 17):

$$c_{rmax} = z_{max} \cos \alpha - h_r \sin \alpha \qquad (31)$$

Substituting Eq. (28) into Eq. (31) and solving for the tilt angle θ'_{max}, at which the resisting moment arm is maximum:

$$\theta'_{max} = \frac{z_{max}\cos \alpha - h_r \sin \alpha}{r} + \alpha$$

Using the usual small angle approximations:

$$\theta'_{max} = \frac{z_{max} - h_r \alpha}{r} + \alpha \qquad (32)$$

To find the factor of safety FS' against rollover, Eq. (30) may be modified by substituting the cracked section $\bar{z}'_o$ for $\bar{z}_o$ and θ'_{max} for θ_{max}.
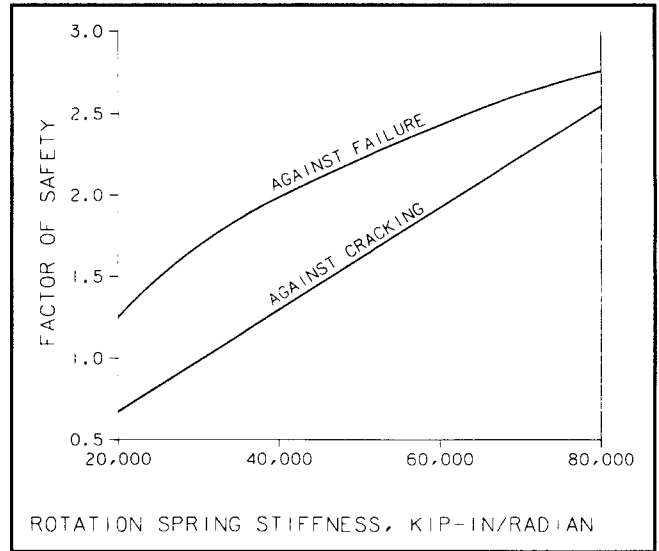

Fig. 18. Importance of rotational stiffness of truck and trailer.

$$FS' = \frac{r(\theta'_{max} - \alpha)}{\bar{z}'_o \theta'_{max} + e_i + y \theta'_{max}} \qquad (33)$$

IMPORTANCE OF ROTATIONAL SPRING STIFFNESS

The effect of rotational spring stiffness K_θ on factors of safety is shown in Fig. 18. The 136 ft (41.5 m) PCI BT-72 bulb tee described in Appendix B was analyzed, with supports 9 ft (2.7 m) from each end. A 6 percent superelevation was used in the analysis.

For K_θ = 30,000 kip-in. per radian (3390 kN-m per radian), the factor of safety against cracking is unity, whereas for K_θ = 60,000 kip-in. per radian (6780 kN-m per radian), the factor of safety against cracking is almost 2. This points up the need for determination of the roll stiffness of transport vehicles being used.

MEASURING ROLL STIFFNESS OF VEHICLES

The roll stiffness (rotational spring stiffness) of transport vehicles is an important parameter in evaluating the safety of slender beams during transportation. Information on the roll stiffness of transport vehicles is not normally available. A method for determining roll stiffness is given below.

The roll stiffness of a vehicle may be determined by placing a weight on the vehicle at various eccentricities to the vehicle centerline. The weight should be of the same order of magnitude as the beam reaction. A handy weight is the beam itself. One end of the beam may be secured, and the other end placed on the vehicle at eccentricities of, say, 10 and 20 in. (250 and 500 mm) either side of the centerline.

Because the cross member tilts under eccentric loads, it is necessary to use a narrow bearing strip of hard material between the beam and the cross member, in order to know the eccentricity of the load on the trailer. Rotations may be determined by measuring the vertical movement at either end

of the cross member used for chaining the beam. Fig. 19 shows such measurements being taken. The roll stiffness is the average of the values obtained by dividing the eccentric moment by the rotation in radians.

It is assumed that the cross member is rigid, and that the beam is rigidly attached to it during transportation. Horizontal measurements could also be taken and used to locate the height of the roll center. In most cases this is unnecessary, and it may be assumed that the roll center is at the height of the axles, about 24 in. (600 mm) above the ground.

A very limited number of measurements indicates that K_θ may be expected to be in the range of 3000 to 6000 kip-in. per radian (340 to 680 kN-m per radian) per dual-tire axle. The higher values apply to rigs without leaf springs, in which the spring is primarily in the tires. For instance, a steer trailer with four dual and one single axle might be expected to have a roll stiffness of 4.5 x (3000 to 6000) = 13,500 to 27,000 kip-in. per radian (1530 to 3060 kN-m per radian). The total K_θ for the rig is the sum of the stiffnesses of the tractor and trailer. The total K_θ may be conservatively taken as twice the value of the unit with the lower stiffness.

The above figures are based on very limited data and must be used with caution until more data are available from the field. Also, these values apply to axles for which the load is balanced mechanically. Axles with air suspension may contribute little, if any, to roll stiffness. For critical shipments, it is strongly recommended that measurements be made of the roll stiffness of the actual vehicles to be used.

An alternative method for measuring roll stiffness is as follows. With the rig on a level surface, measure the tilt angle of the beam (presumably near zero) at the two supports and find the average. Then, with the vehicle parked on pavement with a substantial cross slope, measure the tilt angle of the beam at the two supports and find the average. Take θ as the difference of the initial readings and those on the cross slope. Then estimate r ($= K_\theta/W$) is obtained as follows:

$$r = \frac{(y + \bar{z}_o)\theta}{\theta - \alpha} \quad (34)$$

Eq. (34) corrects for the flexibility of the beam. The cross slope angle α should be about as large as that anticipated during delivery, in order to get accurate results. Both truck and trailer should be parked on the same cross slope, which may be created by the use of blocking beneath one side.

BEARING PADS

Elastomeric bearing pads also provide a "springy" support for prestressed concrete beams. The rotational spring constant K_θ is determined by the dimensions and properties of the pad. Information on the rotational properties of bearing pads is given in Ref. 3. When the load is outside the kern of the bearing pad, the rotational spring constant becomes highly nonlinear. Test results in this area are lacking, for it is generally the object of design to keep the load within the kern.

Based on the data contained in Ref. 3, the author has devised an empirical formula for the effective modulus of elasticity in rotation E_r for bearing pads:

Fig. 19. Measuring roll stiffness of vehicle by placing eccentric loads on vehicle and measuring tilt.
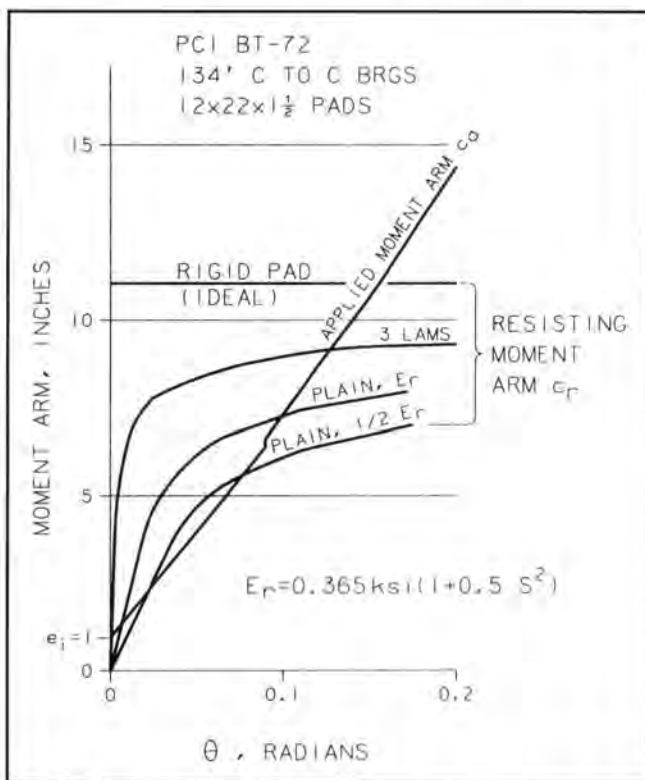

Fig. 20. Moment arms for a beam on bearing pads.

$$\text{In U.S. units: } E_r = 0.365 \ (1 + 0.5S^2) \text{ ksi}$$
$$\text{In SI units: } E_r = 2.52 \ (1 + 0.5S^2) \text{ MPa} \quad (35)$$

where S is the shape factor. When the load is outside the kern, S is computed using the perimeter of the area that is in compression. Eq. (35) gives the modulus of elasticity in rotation E_r for laminated pads. It has been suggested that E_r for plain pads should be taken as half that for laminated pads.

A typical bearing pad for a long span PCI BT-72 bulb tee might be 12 x 22 x 1½ in. (300 x 560 x 38 mm), with the thickness being determined by considerations of thermal expansion and contraction. For such a thickness, a laminated pad, say of three thicknesses of elastomer, would normally be used, although plain pads have also been used. Fig. 20 shows the applied and resisting moment arm curves for a

134 ft (41 m) span PCI BT-72 supported on plain and laminated pads.

It is apparent that a serious stability problem exists with plain pads of this thickness. The laminated pad appears to provide more than adequate stability. But, note that even with the laminated pad, the rollover angle is about 0.13 radians or 7½ degrees, far less than the tilt capability of the beam.

The applied moment curve in Fig. 20 was computed assuming an initial eccentricity of 1 in. (25 mm). The actual initial eccentricity is the sum of that due to sweep and that due to setting the beam off-center of the pads. This value could well exceed 1 in. (25 mm), in which case the applied moment arm curve would be shifted upward so that it intersects the moment arm axis at a value equal to the initial eccentricity. It is apparent that setting a beam a few inches (several centimeters) off center of the pad, even temporarily, can have serious consequences.

The resisting moment arm curves in Fig. 20 are based on the estimate of E_r given by Eq. (35). At present, test data are not available to verify this equation. PCI is currently sponsoring research on the rotational stiffness of bearing pads.

WIND ON BEAMS SUPPORTED ON BEARING PADS

Wind forces on beams produce applied moments that must be added to other applied moments. For instance, a wind pressure of 10 psf (480 Pa) produces a wind load of 8.16 kips (36.3 kN) on the beam, which is 0.0727 of the beam weight. For the beam supports at 134 ft (41 m) center-to-center, $\bar{z}_o$ is 21.24 in. (539.5 mm) and the lateral shift of the center of mass $\bar{z}$ is 1.54 in. (39.2 mm).

This represents an additional initial eccentricity due to the deflection caused by the wind load. Additionally, the wind load itself causes an overturning moment about the bottom of the bearing pads, which divided by the beam weight produces a moment arm of 2.73 in. (69.2 mm). The total moment arm at zero tilt angle θ is the sum of assumed initial eccentricity e_i plus these two added quantities. Thus, the applied moment arm c_a at zero tilt is 6.35 in. (161 mm).

As shown in Fig. 21, this will cause the beam to roll over, even with laminated pads. For a beam erected in an elevated structure, a 10 psf (480 Pa) wind load is a distinct possibility. This emphasizes the importance of bracing the ends of bridge beams against rollover as soon as they are erected.

EFFECT OF OVERHANGS

The beneficial effect of supporting a long beam at some distance in from the ends is well known.[1,2] However, moving the supports inward increases the bottom fiber compressive stress and may increase the required concrete strength. Figs. 22 and 23 show the factors of safety for a 136 ft (41.5 m) PCI BT-72, with various overhang distances a. Factors of safety against cracking and failure are read on the left scale. The factors of safety are dependent on the prestressing, which was assumed to be that of the example problem in Ref. 2 and Appendix B. And, for the supported beam, the
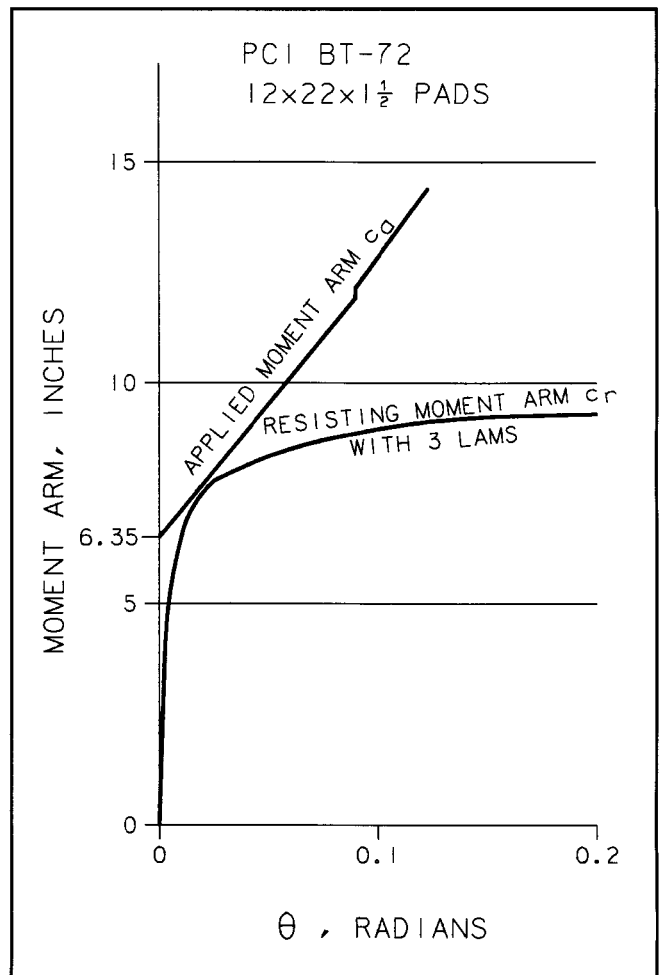

Fig. 21. Moment arms for a beam on bearing pads, in wind.

rotational spring stiffness was assumed to be as in Appendix B of this paper. Required concrete strengths are read on the right scale.

Fig. 22 shows a rapid increase in factors of safety for a hanging beam as the overhang is increased. But, increasing the overhang increases the bottom fiber stress and the required concrete strength to satisfy the $0.6f'_{ci}$ criterion. The best compromise is an a distance in the range of 7 to 9 ft (2.1 to 2.7 m) for this beam.

Fig. 23 shows the factors of safety for a supported beam on a 6 percent slope, with supports having a K_θ of 40,500 kip-in. per radian (4600 kN-m per radian). The factors of safety for supported beams are much less sensitive to overhang distance. For supported beams, the stability of the beam is much more a function of the stiffness properties of the support rather than the beam. But, the factor of safety against cracking is determined by the top fiber stresses in the beam. Past practice has been to support the beam on the truck at the lifting points. Fig. 23 would indicate that some deviation in location of support points on the truck and trailer is permissible.

Eq. (8) from Part 1 may be used to determine the effect of overhangs on the quantity $\bar{z}_o$:

$$\bar{z}_o = \frac{w}{12EIl}\left(\frac{1}{10}l_1^5 - a^2 l_1^3 + 3a^4 l_1 + \frac{6}{5}a^5\right) \qquad (8)$$
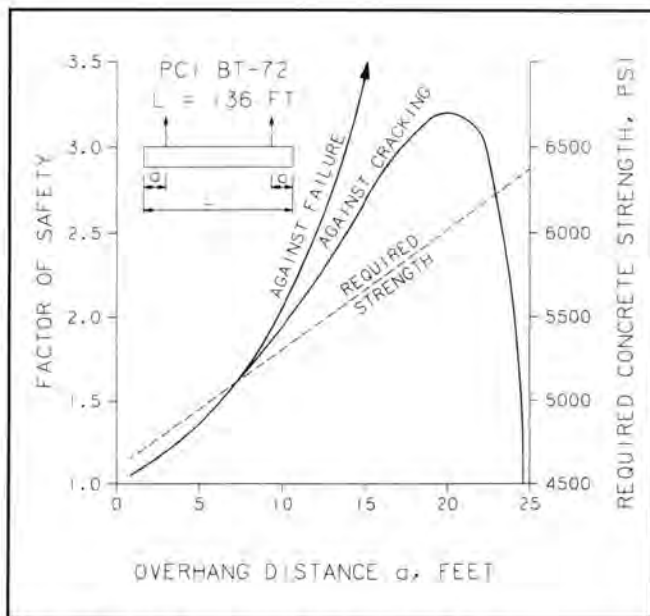
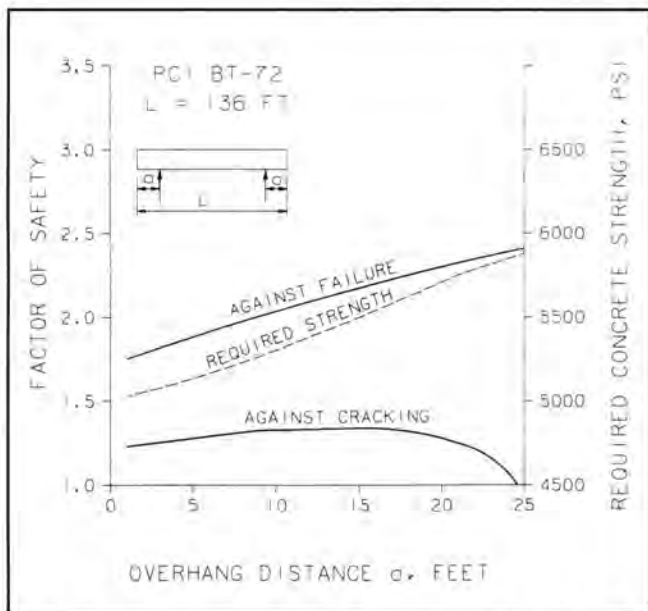Fig. 22. Effect of overhangs for a hanging beam.

Fig. 23. Effect of overhangs for beam on truck and trailer.

Fig. 24. King post bracing of a long prestressed concrete I-beam.

The other parameters needed to construct Figs. 22 and 23 are dependent on the prestressing and on the properties of the supports. Although Figs. 22 and 23 may be used to evaluate the effects of overhang distance in a general way, calculations must be made (or use program ROLLEM) to evaluate specific cases.

Unequal overhangs are sometimes necessary during shipping. It is sufficiently accurate to use the average overhang in stability calculations, but the stress at the support should be checked using the actual overhang.

WHAT FACTOR OF SAFETY IS NECESSARY?

The necessary factor of safety cannot be determined from scientific laws; it must be determined from experience. The author tentatively recommends using a factor of safety of 1.0 against cracking and 1.5 against failure. This applies to both hanging and supported beams.

For hanging beams, the weight and stiffness of the beam is reasonably well known. The major unknown is the initial eccentricity e_i caused by sweep and lifting loop tolerance. By measuring the tilt angle θ of beams while hanging, a fabricator may use Eq. (5) to determine the actual values of e_i being encountered during production, and adjust the calculations accordingly:

$$\theta = \frac{e_i}{y_r - \bar{z}_o} \qquad (5)$$

For supported beams, the major unknowns are the roll stiffness of the supporting vehicles and the transverse slope of the roadway. It should be noted that transverse slopes occur for reasons other than superelevation. On soft ground — on a shoulder or at the construction site — one side may sink into the ground, creating a large transverse slope.

These unknowns primarily affect the factor of safety against cracking. The author believes that many beams have been successfully shipped with a theoretical factor of safety against cracking of less than unity.

As shown in the girder test, exceeding somewhat the theoretical cracking load does not produce any noticeable effect. But, until actual factors of safety used in the past can be better documented, a minimum factor of safety against cracking of 1.0 is tentatively recommended. The factor of safety against failure is basically the factor of safety against overturning of the rig. A factor of safety against overturning of 1.5 is believed to be adequate.

The recommended minimum factors of safety apply to calculations that do not account for creep and impact. Impact is normally of significance during hauling, but stability is primarily a problem when traversing areas of high superelevation at low speed. It is recommended that the effects of impact and superelevation be accounted for separately. This was also recommended by Imper and Laszlo.[2] Lifting and traversing superelevated roads are short-term activities, for which creep effects need not be considered. The author recommends against leaving long beams in a tilted position for a time sufficient for creep effects to be significant.

TEMPORARY KING POST BRACING

Long prestressed concrete I-beams are often braced during transportation using a king post truss system similar to that shown in Fig. 24. Prestressing strands are partially tensioned against bearing plates at each end of the beam and against steel king posts at the middle of the beam. Such bracing has been in use for many years, and it is widely believed that the bracing is of considerable benefit in stiffening the beams during transportation.

The author believes that such bracing is of very little benefit. The steel area of the prestressing strands is too small to make a significant contribution to the lateral stiffness of the beam. For instance, consider a PCI BT-72, braced with two strands on each side, with the strands 36 in. (0.9 m) from the axis of the beam.

The *EI* of the strands, about the weak axis of the beams, is only 13.3 percent of the gross *EI* of the beam. Furthermore, the strands are not bonded; rather, they act more like a truss. An indeterminate truss analysis shows that the bracing reduces the lateral deflection by 5 percent. For comparison, moving the supports 1 ft (0.305 m) [from 9 to 10 ft (2.75 to 3.05 m) from the end] also reduces lateral deflection by 5 percent.

It could be argued that the bracing adds more to lateral strength than to stiffness. This is true; however, as previously shown, prestressed concrete I-beams have sufficient lateral bending strength without bracing. The full-scale test showed very little difference in behavior at a 15 degree tilt (well beyond the cracking tilt angle) with and without bracing, and the presence of bracing reduced the lateral deflection by only 7 percent.

To improve the factor of safety against cracking, temporary prestressing of the top flange, as recommended by Imper and Laszlo,[2] is more effective than lateral bracing. Also, horizontal stiffening trusses with mild steel chords are effective.

CONCLUSIONS AND RECOMMENDATIONS

1. A theoretical analysis of the post-cracking behavior of prestressed concrete I-beams subjected to tilting has been developed. This theoretical analysis has been verified by a full-scale test.

2. Long prestressed concrete I-beams of ordinary proportions (such as the PCI BT-72) normally have sufficient strength to withstand a minimum tilt of 0.4 radians (23 degrees). This is a greater tilt angle than can be resisted by ordinary supports such as bearing pads or transportation equipment.

3. Conclusion 2 leads to the following thought: In past accidents which occurred during the transportation of pre-stressed concrete I-beams, it is likely that the I-beam did not fail until something else triggered the failure. This could be breakage of chains or pins, or rollover of the vehicle.

4. The results of the full-scale test verify that king post bracing using strand does little to improve the lateral bending stiffness and strength of prestressed concrete I-beams during handling and transportation. For prevention of cracking, the use of temporary top flange tensioning proposed by Imper and Laszlo[2] is more effective.

5. A common fundamental approach, using moment arm vs. tilt angle diagrams, can be used to solve both the problem of beams hanging from loops and that of beams supported on elastic supports.

6. The common fundamental approach shows that there are important differences between the case of hanging beams and that of beams on elastic supports. For hanging beams, the most important parameter is the lateral bending stiffness. For beams on elastic supports, the roll stiffness of the support is the most important parameter.

7. The roll stiffness of transportation vehicles is not well known. Because of the critical role this property plays in the lateral stability of beams, tests of the roll stiffness of vehicles are needed. Procedures for such tests are given.

8. When beams are set on elastomeric pads, it is important that the weight of the beam be concentric on the pad. Eccentricity — even temporary eccentricity — may cause rollover in extreme circumstances. Also, beams set on elastomeric pads should be braced against rollover at each end, to prevent the possibility of rollover in high winds.

REFERENCES

1. Mast, R. F., "Lateral Stability of Long Prestressed Concrete Beams — Part 1," PCI JOURNAL, V. 34, No. 1, January-February 1989, pp. 34-53.
2. Imper, Richard R., and Laszlo, George, "Handling and Shipping of Long Span Bridge Beams," PCI JOURNAL, V. 32, No. 6, November-December 1987, pp. 86-101.
3. Roeder, C. W., Stanton, J. F., and Taylor, A. W., "Performance of Elastomeric Bearings," NCHRP Report 298, Washington, D.C., October 1987, 100 pp.

APPENDIX A — NOTATION

a = length of overhang

A_c = concrete area of beam

b = top flange width

b_b = bottom flange width

c_a = applied overturning moment arm

c_r = resisting moment arm

c_{rmax} = maximum resisting moment arm, governed by rollover

e = eccentricity of prestress force

e_i = initial eccentricity of center of gravity of beam

E_c = modulus of elasticity of concrete

EI = flexural stiffness; product of modulus of elasticity and moment of inertia

E_r = effective modulus of elasticity in rotation of a bearing pad

f_b = bottom fiber stress

f_c' = concrete cylinder strength at time of handling or shipping

f_{ci}' = initial concrete cylinder strength at time of transfer of prestress

f_r = modulus of rupture of concrete

f_t = top fiber stress

FS = factor of safety against cracking

FS$'$ = factor of safety against failure

h = overall depth of beam

h_{cg} = height of center of gravity of beam above road

h_r = height of roll center above road

I = major axis moment of inertia of beam

I_{eff} = effective cracked section lateral (minor axis) moment of inertia

I_g = gross lateral (minor axis) moment of inertia

K_θ = sum of rotational spring constants of supports

l = overall length

l_l = length between supports

M = moment

M_g = self-weight bending moment of beam at harp point

M_{gmsp} = self-weight bending moment at midspan

M_{lat} = lateral bending moment at cracking

M_r = resisting moment

P = prestress force at time of shipping

P_i = initial prestressing force after transfer of prestress

r = radius of stability K_θ/W

R = radius of curvature

S = shape factor of bearing pad

S_b = bottom section modulus

S_t = top section modulus

w = weight per unit length of beam

w_c = unit weight of concrete

W = total weight of beam

y = height of center of gravity of beam above roll axis (beam supported from below)

y_b = height above soffit of centroid of concrete section

y_r = height of roll axis above center of gravity of beam (hanging beam)

y_s = height above soffit of centroid of prestressing force

y_t = $h - y_b$

$\bar{z}$ = lateral deflection of center of gravity of beam

z_{max} = distance from centerline of vehicle to center of dual tires

$\bar{z}_o$ = theoretical lateral deflection of center of gravity of beam with full dead weight applied laterally

$\bar{z}_o'$ = theoretical lateral deflection of center of gravity of beam with full dead weight applied laterally, computed using I_{eff} for tilt angle θ under consideration

α = superelevation angle, or tilt angle of support

Δ = camber

θ = roll angle of major axis of beam with respect to vertical

θ_i = initial roll angle of a rigid beam = e_i/y_r

θ_{max} = tilt angle at which cracking begins

θ'_{max} = tilt angle at maximum factor of safety against failure

APPENDIX B — SAMPLE CALCULATIONS

These sample calculations are based on the example given in Ref. 2. Please refer to Ref. 2 for additional details.

GIVEN

PCI BT-72 bridge beam

Depth h = 72 in. (1829 mm)

Top flange width b = 42 in. (1067 mm)

Bottom flange width b_b = 26 in. (660 mm)

Unit weight of concrete w_c = 155 lb/ft³ (2483 kg/m³)

Initial concrete strength f'_{ci} = 4500 psi (31 MPa)

Modulus of elasticity $E_c = 33w_c^{1.5} f_{ci}'^{0.5}$

In SI: $E_c = 0.043\ w_c^{1.5} \sqrt{f_c'}$

Area A_c = 767 in.² (495,000 mm⁴)

Strong axis I = 545,894 in.⁴ (227,218,000,000 mm⁴)

y_b = 36.6 in. (930 mm)

Weak axis I_g = 37,634 in.⁴ (15,664,000,000 mm⁴)

Overall length l = 136 ft (41.5 m)

Pickup points at distance a = 9 ft (2.74 m) from each end

Harp points at $0.4l$

Initial prestress force P_i = 1232 kips (5480 kN) (after initial losses)

Location of P_i, y_s = 5 in. (127 mm) above soffit at harp points

REQUIRED

Find the factors of safety against cracking and against failure when the beam hangs from lifting loops.

1. Check stresses at harp points and required concrete strength:

$w = A_c w_c / 144 = 767\,(0.155)/144$
$\qquad = 0.8256$ kip/ft (12.05 kN/m)

$W = wl = 0.8256\,(136) = 112.28$ kips (500 kN)

M_g at harp point at $0.4l$

$M_g = (W/2)\,(0.96)\,(l/4 - a)\,(12\text{ in./ft})$
$\qquad = (112.28/2)\,(0.96)\,(136/4 - 9)\,(12)$
$\qquad = 15926$ kip-in. (1800 kN-m)

$f_t = P_i/A_c - P_i\,e/S_t + M_g/S_t$
$\qquad = 1232/767 - 1232\,(31.6)/15421 + 15926/15421$
$\qquad = 0.114$ ksi (0.786 MPa)

$f_b = P_i/A_c + P_i\,e/S_b - M_g/S_b$
$\qquad = 3.149$ ksi (21.7 MPa)

f'_{ci} required $= f_b/0.6 = 5248$ psi > 4500 psi
$\qquad\qquad\qquad\qquad$ (36.2 MPa $>$ 31 MPa)

Use f'_{ci} = 5.25 ksi (36.2 MPa)

2. Find modulus of elasticity:

$E_c = 33w_c^{1.5} f_c'^{0.5}$
$\qquad = 33\,(155)^{1.5}\,(5248)^{0.5}/1000$
$\qquad = 4613$ ksi (31807 MPa)

3. Compute initial eccentricity e_i:

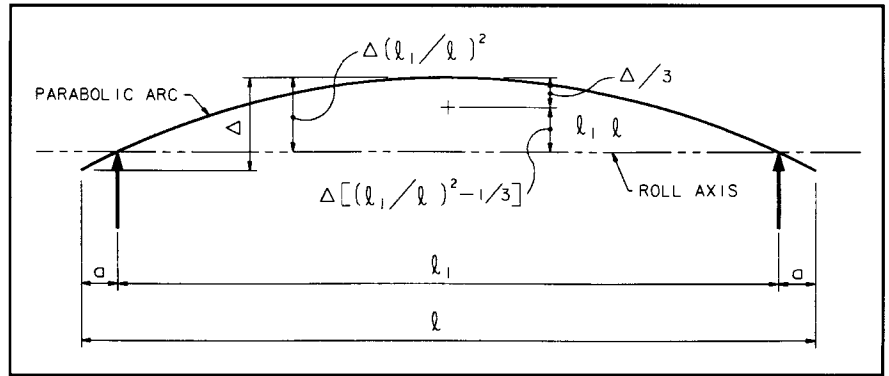For beams hanging from lifting loops, the author recommends using a sweep of one-half of the PCI sweep tolerance and a lifting loop placement tolerance of ¼ in (6 mm). Others may desire to use a different formula for e_i, based on their observation of the actual tilt angles observed for hanging beams. To evaluate e_i due to sweep, the distance must be found between the roll axis and the center of gravity of the arc of the curved beam (see Fig. B1). The curved shape is assumed to be a parabola, and the formulas are derived from the properties of a parabola. Half the PCI sweep tolerance is ⅟₁₆ in. (1.6 mm) per 10 ft (3.05 m), or 0.85 in. (22 mm) for the 136 ft (41.5 m) length. The offset factor (see Fig. B1) is:

$(l_1/l)^2 - \tfrac{1}{3} = (118/136)^2 - \tfrac{1}{3} = 0.419$

$e_i = 0.85\,(0.419) + 0.25 = 0.607$ in. (15 mm)

4. Estimate camber and correct y_r for camber. Camber may be estimated from the midspan curvature R:

$R = EI/M$, where $M = P_i e - M_{gmsp}$

$M_{gmsp} = W/2\,(l/4 - a)$
$\qquad\quad = (112.28/2)\,(136/4 - 9)\,(12\text{ in./ft})$
$\qquad\quad = 16,842$ kip-in. (1903 kN-m)

$R = 4613\,(545,894)/(38931 - 16842)$
$\qquad = 114,000$ in. (2896 m)

The camber Δ (measured with respect to the beam ends) is computed as follows:

$\Delta = l^2/8R$
$\qquad = [136(12)]^2/[8(114,000)]$
$\qquad = 2.92$ in. (74 mm)

The height of the center of gravity of the cambered arc above the roll axis is computed using the same formula as was used for sweep:

$y_r = y_t - \Delta\,(0.419)$
$\qquad = 72 - 36.6 - 2.92\,(0.419)$
$\qquad = 34.18$ in. (868 mm)

Note: Camber has only a small effect on y_r. Thus, instead of the above calculations, one may simply subtract a guess [say between 1 and 2 in. (25 and 50 mm)] from y_t.

5. Compute $\bar{z}_o$ from Eq. (8):

$$\bar{z}_o = \frac{w}{12EIl}\left(\frac{1}{10}l_1^5 - a^2 l_1^3 + 3a^4 l_1 + \frac{6}{5}a^5\right)$$

$\qquad = 10.86$ in. (276 mm)

Fig. B1. Offset of centroid of a curved arc.

6. Compute θ_i:

$$\theta_i = e_i/y_r = 0.607/34.18$$
$$= 0.01775$$

7. Compute the tilt angle θ_{max} at cracking:

$$f_r = 7.5 \sqrt{f'_{ci}}$$
$$= \left(7.5 \sqrt{5248}\right)/1000$$
$$= 0.543 \text{ ksi (3.74 MPa) (tension)}$$

$$f_t = 0.114 \text{ ksi (0.786 MPa) (compression) from Step 1}$$

$$M_{lat} = (f_r + f_t)(I_g)/(b/2)$$
$$= (0.543 + 0.114)(37634)/21$$
$$= 1179 \text{ kip-in. (133 kN-m)}$$

$$\theta_{max} = M_{lat}/M_g = 1179/15926$$
$$= 0.0740$$

8. Compute factor of safety against cracking FS from Eq. (22):

$$FS = \cfrac{1}{\bar{z}_o/y_r + \theta_i/\theta_{max}}$$

$$= \cfrac{1}{10.86/34.18 + 0.01775/0.0740}$$

$$= 1.79$$

This compares to a factor of safety of 1.67 calculated in Ref. 2.

9. Find factor of safety against failure FS'. Use Eq. (25) to find tilt angle at failure:

$$\theta'_{max} = \sqrt{e_i/(2.5 z_o)}$$
$$= \sqrt{0.6066/[2.5(10.86)]}$$
$$= 0.1495$$

Use Eq. (26) to find FS'.

$$FS' = \cfrac{y_r \theta'_{max}}{\bar{z}_o \theta'_{max} + e_i}$$

where

$$\bar{z}'_o = \bar{z}_o(1 + 2.5\,\theta'_{max})$$
$$= 10.86[1 + 2.5(0.1495)]$$
$$= 14.92 \text{ in. (379 mm)}$$

$$FS' = \cfrac{34.18(0.1495)}{14.92(0.1495) + 0.6066}$$

$$= 1.80$$

Note: If FS' is less than FS, FS' should be taken equal to FS.

Check the same beam during transportation.

GIVEN

Concrete strength $f'_c = 5500$ psi (38 MPa)
Add two strands in top flange, per Ref. 2

Prestress force, $P = 1251.5$ kips (5567 kN)
Location of P, $y_s = 7.91$ in. (201 mm) above soffit
Superelevation angle $\alpha = 0.06$ radians (different from the 0.08 used in Ref. 2)
Steer trailer with four dual and one single axle, stiff suspension
Height of roll center $h_r = 24$ in. (610 mm) above road
Height of center of gravity of beam $h_{cg} = 108$ in. (2743 mm) above road

REQUIRED

Find factors of safety against cracking and against rollover during transportation.

10. Estimate K_θ and find r:

Assume $K_\theta = 4500$ kip-in. (509 kN-m) per radian per dual axle
$K_\theta = 4.5(4500) = 20,250$ for trailer
Use two times this for total K_θ for rig
$K_\theta = 40,500$ kip-in. (4577 kN-m) per radian
$r = K_\theta/W = 40,500/112.28$
$= 360.7$ in. (9162 mm)

11. Find tilt angle θ from Eq. (12):

$$\theta = \cfrac{\alpha r + e_i}{r - y - z_o}$$

$$y = h_{cg} - h_r = 108 - 24 = 84 \text{ in. (2134 mm)}$$

Increase y by 2 percent, to allow for camber. Then, $y = 85.68$ in. (2176 mm)

For shipping, assume PCI sweep tolerances plus 1 in. (25.4 mm) off center of truck/trailer. Use offset factor of 0.419 as computed in Step 3.

$$e_i = 1.70(0.419) + 1$$
$$= 1.71 \text{ in. (43 mm)}$$

Ratio $\bar{z}_o$ from Step 5, by square root of ratio of concrete strengths.

$$\bar{z}_o = 10.86 \sqrt{(5248/5500)}$$

$$= 10.61 \text{ in. (269 mm)}$$

$$\theta = \cfrac{0.06(360.7) + 1.71}{360.7 - 85.68 - 10.61}$$

$$= 0.0883$$

12. Check stresses at harp points:

$$f_t = P/A - Pe/S_t + M_g/S_t$$
$$= 1251.5/767 - 1251.5(28.69)/15421 + 15926/15421$$
$$= 0.336 \text{ ksi (2.32 MPa)}$$

$$f_b = P/A + Pe/S_b - M_g/S_b$$
$$= 2.971 \text{ ksi (20.5 MPa)}$$

13. Add lateral bending to f_b, and find required concrete strength:

$$M_{lat} = \theta(M_g) = 0.0883(15926)$$
$$= 1406 \text{ kip-in. (159 kN-m)}$$

$$f_b = 2.971 + 1406(13)/37634$$
$$= 3.457 \text{ ksi (23.84 MPa)}$$

$f'_c = f_b/0.6 = 5759$ psi (39.7 MPa)

$E_c = 33(155)^{1.5}(5759)^{0.5}$

$\quad = 4833$ ksi (33324 MPa)

Adjust $\bar{z}_o$ from Step 5 by ratio of E_c's

$\bar{z}_o = 10.86\ (4613/4833)$

$\quad = 10.37$ in. (263 mm)

14. Estimate camber and correct y for camber:

The effect of camber on stability during hauling is small. Estimate that camber increases y by 2 to 86 in. (51 to 2184 mm).

15. Find the tilt angle θ_{max} at cracking:

$f_r \quad = 7.5\sqrt{5759} = 0.569$ ksi (3.9 MPa) (tension)

$f_t \quad = 0.336$ ksi (2.3 MPa) (compression) from Step 12

$M_{lat} = (f_r + f_t)(I_g)/(b/2)$

$\quad = (0.569 + 0.336)(37634)/21$

$\quad = 1622$ kip-in. (183 kN-m)

$\theta_{max} = M_{lat}/M_g = 1622/15926$

$\quad = 0.1018$

16. Compute factor of safety against cracking FS from Eq. (30):

$$FS = \frac{c_r}{c_a} = \frac{r(\theta_{max} - \alpha)}{\bar{z}_o \theta_{max} + e_i + y\theta_{max}}$$

$$= \frac{360.7(0.1018 - 0.06)}{10.37(0.1018) + 1.71 + 86(0.1018)}$$

$$= 1.31$$

17. Find tilt angle θ'_{max} at maximum resisting moment arm from Eq. (32):

$$\theta'_{max} = \frac{z_{max} - h_r\alpha}{r} + \alpha$$

$$= \frac{36 - 24(0.06)}{360.7} + 0.06$$

$$= 0.1558$$

18. Compute $\bar{z}'_o$ at θ'_{max} from Eq. (20):

$\bar{z}'_o = \bar{z}_o(1 + 2.5\,\theta'_{max})$

$\quad = 10.37\,[1 + 2.5\,(0.1558)]$

$\quad = 14.41$ in. (366 mm)

19. Compute factor of safety against rollover FS' from Eq. (33):

$$FS' = \frac{c_r}{c_a} = \frac{r(\theta'_{max} - \alpha)}{(\bar{z}_o \theta'_{max} + e_i + y\,\theta'_{max})}$$

$$= \frac{360.7(0.1558 - 0.06)}{14.41(0.1558) + 1.71 + 86(0.1558)}$$

$$= 1.99$$

These computations were done with a 0.06 superelevation angle, whereas 0.08 was used in Ref. 2. If these calculations were done with a superelevation angle of 0.08, the factor of safety against cracking would be less than unity. The reason for the difference is that the calculations in Ref. 2 do not account for the additional tilt caused by the roll flexibility of the vehicle.

APPENDIX C — SIMPLE BASIC PROGRAM

A simple BASIC program is shown (see Fig. C1) that allows a design engineer to quickly evaluate the factors of safety against cracking and failure for hanging and supported beams. The determination of the optimum overhang distance a is a matter of judgment. The program enables the trial of various a distances without re-entering other data.

The logic of the program is straightforward. The steps in the computation closely follow the steps in the example problem given in Appendix B. The user may readily alter the program to suit his/her needs.

ERRATA TO PART 1

1. Appendix C. In Eq. (10), delete the first pair of parentheses.
2. Appendix E. In the summation $\Sigma\,bt^3/3$, in.[3] should be in.[4]

```basic
1       REM     PROGRAM ROLLEM                  8/12/92
10      REM   REFER TO "LATERAL STABILITY OF LONG PRESTRESSED CONCRETE BEAMS,
15      REM   PART 2," BY ROBERT F. MAST, PCI JOURNAL
20      XLHARP=0.4
25      INPUT "HEIGHT OF ROLL AXIS ABOVE CG OF SECTION, IN";YRSPT
30      INPUT "AREA, SQ IN";AC
35      INPUT "DEPTH, IN";D
40      INPUT "YB, IN";YB
45      INPUT "VERTICAL MOMENT OF INERTIA, IN^4";I
50      INPUT "TOP FLANGE WIDTH, IN";B
55      INPUT "LATERAL MOMENT OF INERTIA, IN^4";IG
60      INPUT "OVERALL LENGTH, FT";L
65      INPUT "CONCRETE STRENGTH, PSI"; FCPMIN
70      INPUT "CONCRETE DENSITY, PCF";GAMMA
75      INPUT "PRESTRESS FORCE, KIPS";P
80      INPUT "HEIGHT OF PRESTRESS FORCE ABOVE SOFFIT AT MIDSPAN, IN";YS
85      UNITW=AC*GAMMA/144/1000
87      W=UNITW*L
90      YT=D-YB
92      ST=I/YT
93      SB=I/YB
95      ECC=YB-YS
100     INPUT "A DISTANCE, FT";A
110     MG=W/2*((1-(2*(0.5-XLHARP))^2)*L/4-A)*12
130     FTOP=P/AC+(-P*ECC+MG)/ST
135     FBOT=P/AC+(P*ECC-MG)/SB
140     FCP=FCPMIN
145     IF FBOT/0.6*1000>FCPMIN THEN FCP=FBOT/0.6*1000
150     E=33*GAMMA^1.5*SQR(FCP)/1000
155     L1=L-2*A
160     FACTOR=(L1/L)^2-1/3
165     EIN=L/10*.125/2*FACTOR+1/4
167     MGMSP=W/2*(L/4-A)*12
170     RADIUS=E*I/(P*ECC-MGMSP)
175     CAMBER=L^2*144/8/RADIUS
177     YR=YRSPT-CAMBER*FACTOR
180     ZZEROBAR=UNITW/(12*E*IG*L)*(L1^5/10-A^2*L1^3+3*A^4*L1+6*A^5/5)*1728
185     THETAIN=EIN/YR
190     MLAT=(7.5*SQR(FCP)/1000+FTOP)*IG/(B/2)
195     THETAMAX=MLAT/MG
200     THETAEQ=EIN/(YR-ZZEROBAR)
205     FS=1/(ZZEROBAR/YR+THETAIN/THETAMAX)
210     THETAPMAX=SQR(EIN/2.5/ZZEROBAR)
215     ZPZEROBAR=ZZEROBAR*(1+2.5*THETAPMAX)
220     FSP=YR*THETAPMAX/(ZPZEROBAR*THETAPMAX+EIN)
225     IF FSP<FS THEN THETAPMAX=THETAMAX:FSP=FS
250     PRINT
252     PRINT "FCP = ";FCP
254     PRINT "CAMBER = ";CAMBER
256     PRINT "YR = ";YR
258     PRINT "INITIAL ECCENTRICITY = ";EIN
260     PRINT "TOTAL WEIGHT = ";W
262     PRINT "MG      = ";MG
264     PRINT "FTOP AND FBOT FOR PLUMB BEAM = ";FTOP;FBOT
266     PRINT "MAX M LAT = ";MLAT
268     PRINT "ZZEROBAR = ";ZZEROBAR
270     PRINT "ZPZEROBAR = ";ZPZEROBAR
272     PRINT "THETA INITIAL = ";THETAIN
274     PRINT "EQUILIBRIUM THETA = ";THETAEQ
276     PRINT "CRACKING THETA = ";THETAMAX
278     PRINT "MAXIMUM THETA AT FAILURE OR ROLLOVER = ";THETAPMAX
280     PRINT "FACTOR OF SAFETY AGAINST CRACKING = ";FS
282     PRINT "FACTOR OF SAFETY AGAINST FAILURE OR ROLLOVER = ";FSP
300     INPUT "NEW  'A'  DISTANCE?  Y OR N";A$
310     IF A$<>"N" THEN PRINT: GOTO 100
320     INPUT "ROTATIONAL SPRING CONSTANT, K-IN/RADIAN";KTHETA
```

Fig. C1. BASIC program ROLLEM.

```
330      INPUT "HEIGHT OF CG OF BEAM ABOVE ROAD, IN";HCG
340      INPUT "ZMAX (NORMALLY 36), IN";ZMAX
350      INPUT "HEIGHT OF ROLL AXIS ABOVE ROAD (NORMALLY 24), IN";HR
360      INPUT "SUPERELEVATION ANGLE, RADIANS";ALPHA
370      INPUT "BOTTOM FLANGE WIDTH, IN";BB
380      INPUT "CONCRETE STRENGTH, PSI"; FCPMIN
390      INPUT "PRESTRESS FORCE, KIPS";P
400      INPUT "HEIGHT OF PRESTRESS FORCE ABOVE SOFFIT AT MIDSPAN, IN";YS
490      R=KTHETA/W
495      ECC=YB-YS
497      YSPT=HCG-HR
500      INPUT "A DISTANCE, FT";A
502      L1=L-2*A
505      FACTOR=(L1/L)^2-1/3
510      E=33*GAMMA^1.5*SQR(FCPMIN)/1000
515      EIN=L/10*.125*FACTOR+1
520      ZZEROBAR=UNITW/(12*E*IG*L)*(L1^5/10-A^2*L1^3+3*A^4*L1+6*A^5/5)*1728
522      THETAEL=(ALPHA*R+EIN)/(R-1.02*YSPT-ZZEROBAR)
525      MG=W/2*((1-(2*(0.5-XLHARP))^2)*L/4-A)*12
527      MGMSP=W/2*(L/4-A)*12
530      FTOP=P/AC+(-P*ECC+MG)/ST
532      FBOT=P/AC+(P*ECC-MG)/SB
535      FBOT2=FBOT+MG*THETAEL/IG*BB/2
540      FCP=FCPMIN
545      IF FBOT2/0.6*1000>FCPMIN THEN FCP=FBOT2/0.6*1000
550      E=33*GAMMA^1.5*SQR(FCP)/1000
555      ZZEROBAR=UNITW/(12*E*IG*L)*(L1^5/10-A^2*L1^3+3*A^4*L1+6*A^5/5)*1728
557      RADIUS=E*I/(P*ECC-MGMSP)
560      CAMBER=L^2*144/8/RADIUS
562      CAMBER=CAMBER*1.85
565      Y=YSPT+CAMBER*FACTOR
590      MLAT=(7.5*SQR(FCP)/1000+FTOP)*IG/(B/2)
595      THETAMAX=MLAT/MG
600      THETAEQ=(ALPHA*R+EIN)/(R-Y-ZZEROBAR)
605      FS=R*(THETAMAX-ALPHA)/(ZZEROBAR*THETAMAX+EIN+Y*THETAMAX)
610      THETAPMAX=(ZMAX-HR*ALPHA)/R+ALPHA
615      ZPZEROBAR=ZZEROBAR*(1+2.5*THETAPMAX)
620      FSP=R*(THETAPMAX-ALPHA)/(ZPZEROBAR*THETAPMAX+EIN+Y*THETAPMAX)
625      IF FSP<FS THEN THETAPMAX=THETAMAX: FSP=FS
650      PRINT
652      PRINT "FCP = ";FCP
654      PRINT "RADIUS OF STABILITY = ";R
656      PRINT "CAMBER = ";CAMBER
658      PRINT "Y = ";Y
660      PRINT "INITIAL ECCENTRICITY = ";EIN
662      PRINT "TOTAL WEIGHT = ";W
664      PRINT "MG     = ";MG
666      PRINT "FTOP AND FBOT FOR PLUMB BEAM = ";FTOP;FBOT
668      PRINT "MAX M LAT = ";MLAT
670      PRINT "ZZEROBAR = ";ZZEROBAR
672      PRINT "ZPZEROBAR = ";ZPZEROBAR
676      PRINT "EQUILIBRIUM THETA = ";THETAEQ
678      PRINT "CRACKING THETA = ";THETAMAX
680      PRINT "MAXIMUM THETA AT FAILURE OR ROLLOVER = ";THETAPMAX
682      PRINT "FACTOR OF SAFETY AGAINST CRACKING = ";FS
684      PRINT "FACTOR OF SAFETY AGAINST FAILURE OR ROLLOVER = ";FSP
700      INPUT "NEW  'A'  DISTANCE?  Y OR N";A$
710      IF A$<>"N" THEN PRINT: GOTO 500
9999     STOP
32767    END
```

Fig. C1. (cont'd.) BASIC program ROLLEM.