Reliability Based Parametric Study of Pretensioned AASHTO Bridge Girders

Based on reliability analysis, a parametric study is conducted on typical prestressed concrete I-girders and spread box beams to investigate their structural safety. The effects of changes in initial prestress, section size and allowable concrete stresses on the required number of strands and on the reliability index are considered. It is shown that current bridge design specifications result in non-uniform safety for different spans and girder spacings. Therefore, code calibration at ultimate must take into account the allowable stresses criterion.

Compared to other types of bridges, the percentage of prestressed concrete bridges built annually in North America over the past 40 years has increased dramatically. For example, out of the total 8000 bridges built per year during the 1950s in the United States, only a few were made of prestressed concrete. In contrast, today, based on data furnished by the National Bridge Inventory,1 prestressed concrete bridges constitute about 50 percent of the 5000 new bridge projects constructed each year.

Current methods for designing prestressed concrete bridges are governed by the AASHTO Specifications,2 which are based on the ACI Code,3 engineering experience and research. During the last few years, the design criteria by which bridges are designed have come under increasing scrutiny and re-evaluation.

In engineering practice, prestressed concrete structures are typically designed to satisfy allowable initial and final stresses at service load conditions. The ultimate flexural capacity of the section is also checked. However, in the design of pretensioned bridge girders, ultimate conditions usually are not critical.

In this study, a parametric study is carried out on typical prestressed concrete I-girders and box beams designed by AASHTO. Since absolute safety in a structure is unattainable due to economic restraints and inherent uncertainties in applied loads and internal resistance, the parametric study uses reliability methods to investigate structural safety.4 The effects of changes in initial prestress,
section size and allowable concrete stresses on the required number of prestressing strands and on the reliability index are considered.

Conventional methods of code calibration for an ultimate limit state are usually based on considering the mean girder resistance, $\mu_R$, to be equal to:

$$\mu_R = \lambda \left[ \alpha_D (D) + \alpha_{L+I} (L+I) \right] \left( \frac{1}{\phi} \right) \quad (1)$$

where

- $\alpha_D, \alpha_{L+I} =$ dead and live load (including impact) factors, respectively
- $D, L, I =$ nominal dead, live and dynamic load effects, respectively
- $\phi =$ resistance reduction factor
- $\lambda =$ mean-to-nominal ratio

Eq. (1) is convenient to use because it does not require any design computations. The formula works equally well for steel and reinforced concrete members in flexure but not always for prestressed concrete elements. The reason for this inconsistency is that current (and proposed) bridge design specifications for prestressed concrete sections are based on satisfying allowable concrete stresses.

The ultimate flexural capacity is checked later on in the design process and, in most practical situations, it does not govern the design. Therefore, actual designs for prestressed concrete girders should be worked out independently because the reliability at ultimate depends on the actual number, location and final stress of the prestressing strands.

Fig. 1 shows the required number of strands vs. simple span length for a typical PENN-DOT 28/63 I-girder (the first and second numbers represent, respectively, the bottom flange width and the total beam depth, in inches). The girders are designed according to the AASHTO Specifications (1989) and have a spacing of 8 ft (2.44 m).

In the useful range of application, the curves representing the allowable stress condition are consistently above the curve representing the ultimate strength requirement. Fig. 2 shows an equivalent plot for a spread 48/54 box

<table>
<thead>
<tr>
<th>Types of stress</th>
<th>Stress</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial stress at transfer of prestressed concrete member</td>
<td>Tension</td>
</tr>
<tr>
<td></td>
<td>Compression</td>
</tr>
<tr>
<td>Final stress under design loads</td>
<td>Tension</td>
</tr>
<tr>
<td></td>
<td>Compression</td>
</tr>
</tbody>
</table>

* For non-corrosive environments, multiply this value by 2.
section. The results indicate that similar trends governing I-girders are also applicable to box beams.

The objective of this paper is to provide a realistic estimate for the reliability of newly designed prestressed concrete bridge girders — hence, justifying why such bridges are more reliable than others. The effects of changing certain design parameters, such as initial prestress, section size and allowable concrete stresses, on the reliability are also investigated.

**METHOD OF DESIGN**

The AASHTO Specifications (1989) are used for the design of typical simple span I-girder and spread box prestressed concrete bridges. AASHTO's Specifications for highway bridges require interior girders to be designed to resist a factored moment, $M_u$, equal to:

$$M_u = 1.3(M_D) + 2.17(M_L)(GDF)(1 + i)$$

(2)

where

- $M_D$ = dead load moment
- $M_L$ = live load moment per lane
- $GDF$ = live load girder distribution factor (applied to a wheel load)
- $i$ = an impact coefficient which is a function of span length and has a value smaller or equal to 0.30

Note that the dead load moment, $M_D$, is estimated by preselecting deck dimensions, girder section, concrete barriers and wearing surface. The live load moment, $M_L$, is based on influence lines and the HS20-44 truck or lane loading, whichever governs.

The live load distribution factor, GDF, is computed from the following equations:

- For interior I-girders

$$GDF = \frac{S}{5.5}$$

(3)

- For interior spread box beams

$$GDF = \frac{2N_t}{N_B} + [0.07W - N_t(0.10N_t - 0.26) - 0.20N_B - 0.12] \frac{S}{\ell}$$

(4)
edge about structural behavior, material properties and loading becomes available.

Recently, limit state philosophy has been introduced in newly developed design codes in Europe and Japan (for example, BS5400 in the United Kingdom and Specification of the Japan Roads Association). The Ontario Ministry of Transportation introduced the first edition of the “Ontario Highway Bridge Design Code” in 1979. The third edition of this reliability-based code is expected to be published soon.

In the United States, the change of philosophy from an “allowable stress” to “limit state” format started first with the concrete industry and is continuing with the various steel design standards. In 1986, AISC published the LRFD Code for steel structures. A similar work covering the AASHTO Specifications is currently underway (NCHRP Project 12-33). All of these developments strengthen the importance of achieving (or striving to achieve) uniform reliability in engineering structures.

RELIABILITY MODELS

The structural design codes have evolved over the years from a set of rules in use by individual design engineers or contractors because it was deemed necessary to establish uniformity of cost and risk. Such codes continue to develop as construction methods change and as more knowl-
viceability and damage accumulation.

Ultimate limit states result in partial or total collapse of the structure and they occur under an extreme loading condition, such as flexure, shear or buckling.

Serviceability limit states, on the other hand, involve functionality problems under service loads and do not result in failure. Examples of serviceability limit states include excessive deflection, vibration and crack opening.

Fatigue is a limit state involving damage accumulation. For such limit states, failure occurs under service loads after a number of load applications.

In general, the reliability of a structural member for the ultimate flexural capacity limit state can be expressed by the use of a failure function, $G$, as:

$$G(R,Q) = R - Q$$  \hspace{1cm} (5)

where $R$ is the resistance and $Q$ is the total load effect.

Failure occurs if $G$ is less than or equal to zero. Load components and resistance are random by nature because of the inherent variability in material and load, lack of statistical data, mathematical idealization, approximate design procedures and human error. Therefore, $G$ is a random variable since it is a combination of random variables, as indicated by Eq. (5). Structural safety can be measured in terms of a reliability index, $\beta$, as:

$$\beta = \frac{\mu_G}{\sigma_G}$$  \hspace{1cm} (6)

in which $\mu_G$ and $\sigma_G$ denote the mean and standard deviation of $G$, respectively. The relationship between probability of failure, $P_f$, and reliability index is expressed as:

$$P_f = 1 - \Phi(\beta)$$  \hspace{1cm} (7)

where $\Phi$ is the cumulative standard normal distribution function ($\mu = 0$ and $\sigma = 1$). Fig. 3 shows a typical probability distribution of $G$ and a graphical definition of the reliability index.

If both $R$ and $Q$ are independent normal (Gaussian) random variables, then $\beta$ can be evaluated from:

$$\beta = \frac{\mu_R - \mu_Q}{\sqrt{\sigma_R^2 + \sigma_Q^2}}$$  \hspace{1cm} (8)

where

$\mu_R, \mu_Q =$ mean values of $R$ and $Q$, respectively

$\sigma_R, \sigma_Q =$ standard deviations of $R$ and $Q$, respectively

In the event that $R$ and $Q$ are log-normally distributed random variables, then $\beta$ can be approximated by the following equation:

$$\beta = \frac{\ln(\mu_R) - \ln(\mu_Q)}{\sqrt{V_R^2 + V_Q^2}}$$  \hspace{1cm} (9)

where $V_R$ and $V_Q$ represent, respectively, the coefficients of variation of $R$ and $Q$.

In this study, the reliability index is computed using the Rackwitz-Fiessler method because $R$ and $Q$ do not have the same shape of probability distributions. This procedure is based on approximating the true probability density functions of the random vari-

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Fig. 7. Strand ratio vs. simple span length for I-girders based on AASHTO design.

Fig. 8. Strand ratio vs. simple span length for 48 in. (1.22 m) wide spread box beams based on AASHTO design.
LOAD AND RESISTANCE VARIABLES

The statistical parameters of load and resistance are needed to evaluate the reliability index. The maximum load effects on a highway bridge are due to dead load, live load, dynamic load, environmental loads and accidental loads (braking forces, vehicle collision and other factors). Environmental loads do not govern for short and medium span superstructures and, hence, they are not treated in this study. Therefore, the total load effect, \( Q \), for a bridge girder can be represented by:

\[
Q = [D + LP + I]E
\]

where
\( D = \) dead load
\( L = \) live load
\( I = \) dynamic load
\( P = \) live load influence factor
\( E = \) total load analysis factor

Note that \( E \) and \( P \) are random variables which account for uncertainties in modeling the live load and the total load combination, respectively. The probability distribution of \( Q \) is approximated by a normal distribution, based on the central limit theorem. Statistical parameters of \( Q \) are computed using Turkstra’s rule.

For dead load, it is convenient to consider four components according to quality control procedures. These components are weight of precast concrete members, \( D_1 \), cast-in-place concrete elements, \( D_2 \), asphalt, \( D_3 \), and miscellaneous items, \( D_4 \) (such as luminaires and signs). The bias (mean-to-nominal) ratios and coefficients of variation (COV) of dead load are shown in Table 2.

Nowak’s live load model is used in this study. The live load model is based on truck surveys in North America. Ref. 10 shows that the governing combination for multiple-lane short and medium span bridges is two trucks traveling side-by-side. Actual or more accurate girder distribution factors are needed to determine the live load means per girder.

The following equation is used to evaluate GDF for 1-girders:

\[
GDF = 0.15 + \left( \frac{S}{3} \right)^{0.6} \left( \frac{S}{\ell} \right)^{0.2}
\]

The equivalent GDF equation for a spread box beam is:

\[
GDF = \left( \frac{S}{2} \right)^{0.6} \left( \frac{S}{\ell} \right)^{0.125}
\]

where \( h \) is the depth of the precast box.

The dynamic load on bridges is generally due to the dynamic properties of the structure, the suspension system of the vehicle, and surface roughness and bumps. The mean value of impact is considered equal to 15 percent of live load with a rather high coefficient of variation of 0.80.
Monte Carlo simulation was used to evaluate the statistical parameters of the ultimate moment capacity of pretensioned I-girders of different sizes. A typical probabilistic moment-curvature relationship for an AASHTO Type III I-section at the mean and one standard deviation above and below the mean is shown in Fig. 4. The analysis shows that the mean-to-nominal ratio and coefficient of variation of the moment capacity at ultimate are governed by the statistics of the pre-stressing strands and are equal to 1.04 and 0.08, respectively. The probability distribution of the flexural capacity at ultimate is considered log-normal. The same statistical parameters are used for box beams since the behavior of such sections in flexure is similar to I-girders.

**GENERAL ANALYSIS**

The AASHTO Specifications are used for the design of typical simple span I-girder and spread box beam bridges. The girder spacing considers ranges between 6 to 12 ft (1.82 to 3.65 m) for the I-girders and 6 to 10 ft (1.82 to 3.05 m) for the box beams. The span length for the I-girders varies between 60 to 120 ft (18.2 to 36.5 m) and for box beams 40 to 100 ft (12.2 to 30.5 m). All bridges are assumed to carry at least two traffic lanes, girders act compositely with the deck, and strands are draped at the third points.

In addition to the girder weight, all bridges have two normal size parapets, 1 in. thick (25.4 mm) average haunch, 1 ft wide (0.305 m) diaphragms, 30 psf (1.44 kN/m²) wearing surface and 15 psf (0.72 kN/m²) stay-in-place formwork. The thickness of the cast-in-place concrete deck varies with the girder spacing. The shapes of the pre-stressed concrete I-girders and box beams used in this study are presented, respectively, in Figs. 5 and 6.

Nominal final concrete strengths of 6500 psi (44.8 MPa) in the pretensioned I-girder and 4500 psi (31.0 MPa) in the deck are used. Concrete strength at transfer is considered equal to 5500 psi (37.9 MPa). The pre stressing steel is composed of 0.5 in. (12.7 mm) low relaxation strands with 270 ksi (1862 MPa) ultimate strength.

It is convenient to define the strand ratio, \( \eta \), as the ratio of the number of strands required to satisfy the allowable stress requirement, \( N_{ser} \), to the number of strands needed for the ultimate strength condition, \( N_{ult} \), that is:

\[
\frac{N_{ser}}{N_{ult}} = \eta
\]

(13)

If \( \eta \) is greater than 1, then it may represent the additional factor of safety prestressed concrete bridge girders possess beyond the minimum value required by AASHTO.

The deterministic analysis shows that allowable stresses govern the design of prestressed concrete girders for the spans being studied. Figs. 7 and 8 show the strand ratio vs. span length for I-girders and 48 in. wide (1.22 m) box beams, respectively. The shaded areas represent the variation in \( \eta \) due to designs based on different girder spacings. The plots are generated for corrosive (\( f_c = 3 \sqrt{f_{ct}} \))

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**Fig. 11. Strand ratio vs. girder depth based on AASHTO design.**

**Fig. 12. Reliability index vs. girder depth based on AASHTO design.**
PARAMETRIC STUDY

A 100 ft (30.5 m) long prestressed concrete bridge with center-to-center girder spacing equal to 8 ft (2.44 m) is considered in the parametric study. The objective of such an analysis is to find the effects of changes in the prestressed concrete section, initial prestressing force, and allowable concrete stresses on the strand ratio as well as on reliability. The analysis is carried out on I-girders and 36 in. (0.915 m) and 48 in. (1.22 m) wide box beams. The reference designs, which are based on AASHTO in a corrosive environment \( f_u = 3 f_u' \), are presented in Table 3.

The effect of variation in section size of the girder on the strand ratio and on the reliability index is shown in Figs. 11 and 12, respectively. As expected, as the depth of the section increases, the required number of strands decreases due to an increase in eccentricity and section modulus. However, Fig. 11 indicates that the difference between the applied moment due to factored loads and actual strength gets smaller as the size of the girder increases.

The effect of changes in the initial prestress on the strand ratio is demonstrated in Fig. 13. The reference girders for I-sections and box beams are modified to include an initial prestress at transfer in the range of 65 to 85 percent of ultimate stress, \( f_u' \). The results of the analysis indicate that as the initial stress increases, the strand ratio and number of strands decreases. This decrease in the number of strands is accompanied by a decrease in the ultimate flexural capacity.

For example, the I-girders prestressed with 0.65\( f_u' \) and 0.85\( f_u' \) required 56 and 37 strands, respectively. However, the capacity of the section with the 56 strands was 11,636 kip-ft (15,778 kN-m), and the other, 8099 kip-ft (10,982 kN-m). Both values are much higher than the applied 6993 kip-ft (9483 kN-m) midspan.

and non-corrosive \( \left( f_u = 6 \sqrt{f_u'} \right) \) environments. The ratios vary between 1.06 and 1.55 depending on span, girder spacing, allowable concrete tensile stress and size of girder.

Fig. 9 shows the range of reliability indices for I-girders based on ultimate (factored) moment and allowable stresses for corrosive and non-corrosive environments. A similar plot is generated in Fig. 10 for 48 in. (1.22 m) wide boxes. The lower values of the indices which are based on \( (1.3M_D + 2.17M_{L,+}) \), without regard to service load stresses, represent girder designs at smaller girder spacings.

Fig. 13. Strand ratio vs. allowable final concrete tensile stress.

**Table 3. Summary of girder designs used in the parametric study.**

<table>
<thead>
<tr>
<th>Section</th>
<th>No. of strands</th>
<th>( P_{act} ) (kips)</th>
<th>( e ) at midspan (in.)</th>
<th>( 0M_a ) (kip-ft)</th>
<th>( 1.3M_D + 2.17M_{L,+} ) (kip-ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>28/63 (I)</td>
<td>44</td>
<td>1363</td>
<td>26.9</td>
<td>9456</td>
<td>6993</td>
</tr>
<tr>
<td>36/60 (B)</td>
<td>42</td>
<td>1301</td>
<td>22.9</td>
<td>8469</td>
<td>5575</td>
</tr>
<tr>
<td>48/54 (B)</td>
<td>44</td>
<td>1363</td>
<td>21.5</td>
<td>8171</td>
<td>5613</td>
</tr>
</tbody>
</table>

Metric (SI) conversion factors: 1 in. = 25.4 mm; 1 kip = 4.448 kN; 1 kip-ft = 1356 N-m.

Fig. 14. Reliability index vs. allowable final concrete tensile stress.
moment due to factored loads. The probabilistic analysis indicates a similar trend to the deterministic analysis, as presented in Fig. 14.

The sensitivity of the strand ratio and reliability index to changes in the allowable final tensile concrete stress, $f_t$, is studied. Several girders were designed using a range of $f_t$ which varied between zero and $12f_t'$. The results are shown in Figs. 15 and 16 and indicate a strong relationship between the strand ratio (and consequently the reliability index) and $f_t$.

2. Girder design is usually governed by allowable service load requirements. In most practical cases, the actual ultimate flexural stress is 5 to 50 percent higher than the required strength due to the applied factored loads. Higher values are for designs which limit allowable concrete tensile stresses to $3\sqrt{f_t'}$, instead of $6\sqrt{f_t'}$.

3. Actual I-girder reliability indices are at least 30 percent higher than that required by design codes to satisfy ultimate load effects.

4. The difference between the reliability of box beams for designs based on factored load effects and allowable stresses is much larger than the corresponding difference for I-girders.

5. A decrease in section size and initial prestress can result in designs which have higher reliability indices due to an increase in number of strands. Further, shallower sections may be more economical if they satisfy service load conditions, since the extra cost of strands is more than compensated for by the lesser girder cost per foot.

CONCLUSIONS

Simply supported pretensioned I-girders designed by AASHTO are investigated using reliability methods. Reliability indices are computed based on actual designs for a range of span lengths and girder spacings. The results of this study regarding pretressed concrete bridge girders in flexure at ultimate lead to the following conclusions:

1. Current bridge design specifications result in non-uniform safety for different spans and girder spacings.

RECOMMENDATIONS

Based on the above-mentioned findings, the following recommendations regarding AASHTO are suggested:

1. Code calibration at ultimate must take into account the allowable stresses criterion.

2. A reduction in live load is needed when checking service stress levels, similar to the reduction factor currently being used in the Ontario Highway Bridge Design Code.

3. Either an increase in the allowable final tensile concrete stress or the use of crack width as a serviceability check instead of allowable stresses is recommended.

4. More sophisticated girder distribution factors are needed to ensure uniform reliability.

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REFERENCES


3. ACI Committee 318, “Building Code Requirements for Reinforced Concrete,” ACI 318-89, American Concrete Institute, Detroit, MI, 1989.


This Appendix provides a flow chart for computing the reliability index using the Rackwitz-Fiessler method (1978) for the limit state function: \( G = R - Q \).

**Guess the design point \((R', Q')\)**

(For example, \( R' = Q' = \{\mu_R + \mu_Q\}/2\))

**Approximate the actual distributions of \( R \) and \( Q \) by Normal ones \((R' \text{ and } Q')\) at the design point \((R', Q')\). That is,**

\[
\begin{align*}
  f_{R'}(R') &= f_{R}(R') \\
  F_{R'}(R') &= F_{R}(R') \\
  f_{Q'}(Q') &= f_{Q}(Q') \\
  F_{Q'}(Q') &= F_{Q}(Q')
\end{align*}
\]

The statistical parameters of \( R' \) and \( Q' \) are computed from:

\[
\begin{align*}
  \sigma_{R'} &= \varphi \{\Phi^{-1}[F_{R}(R')]/f_{R}(R')\} \\
  \mu_{R'} &= R' - \sigma_{R'} \Phi^{-1}[F_{R}(R')] \\
  \sigma_{Q'} &= \varphi \{\Phi^{-1}[F_{Q}(Q')]/f_{Q}(Q')\} \\
  \mu_{Q'} &= Q' - \sigma_{Q'} \Phi^{-1}[F_{Q}(Q')]
\end{align*}
\]

**The reliability index, \( \beta \), is approximated by the expression:**

\[
\beta = (\mu_{R'} - \mu_{Q'})/\sqrt{\sigma_{R'}^2 + \sigma_{Q'}^2}
\]

**Calculate the new design point from:**

\[
\begin{align*}
  R^*_n &= \mu_{R'} - \beta \frac{\sigma_{R'}}{\sigma_{R'}^2 + \sigma_{Q'}^2} \\
  Q^*_n &= \mu_{Q'} + \beta \frac{\sigma_{Q'}}{\sigma_{R'}^2 + \sigma_{Q'}^2}
\end{align*}
\]

**Is \( |(R^*_n - R'_{n})/R^*_n| < \text{tolerance?} \)**

**END**
APPENDIX B — NOTATION

The following mathematical symbols, which are used throughout the text, are defined in alphabetical order.

\( D \) = dead load effect
\( D_1 \) = cast-in-place dead load effect
\( D_2 \) = precast dead load effect
\( D_3 \) = asphalt load effect
\( e \) = eccentricity
\( E \) = total load analysis factor
\( f \) = probability density function
\( F \) = cumulative distribution function
\( f_c' \) = concrete strength
\( f_{ct} \) = concrete strength at transfer
\( f_{pu} \) = ultimate prestress
\( g \) = failure function
\( \text{GDF} \) = girder distribution factor
\( h \) = depth of box section
\( i \) = impact coefficient
\( l \) = dynamic load effect
\( L \) = live load effect
\( \ell \) = span length
\( M_n \) = nominal flexural capacity
\( M_u \) = ultimate moment due to applied factored loads
\( M_D \) = dead load moment
\( M_L \) = live load moment per lane
\( N_B \) = number of beams
\( N_L \) = number of lanes
\( N_{ser} \) = number of strands required to satisfy allowable stresses
\( N_{str} \) = number of strands required to satisfy ultimate strength
\( P \) = live load influence factor
\( P_f \) = probability of failure
\( P_{init} \) = initial prestressing force
\( P_{ef} \) = effective prestressing force
\( R \) = resistance
\( S \) = girder spacing
\( \nu \) = coefficient of variation
\( W \) = roadway width
\( \alpha_D \) = dead load factor
\( \alpha_{L+I} \) = live load and impact factor
\( \beta \) = reliability index
\( \lambda \) = bias or mean-to-nominal ratio
\( \eta \) = strand ratio
\( \mu \) = mean value
\( \sigma \) = standard deviation
\( \phi \) = resistance reduction factor
\( \Phi \) = cumulative standard normal distribution function
\( \Phi^{-1} \) = inverse of cumulative standard normal distribution function
\( \Phi \) = standard normal probability density function