Transfer Control of Prestressing Strands

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Based on a particular bond stress-slip relationship, nonlinear equations are developed for the transfer length and for the draw-in of prestressing strands taking into account the effective or the initial prestress, the concrete strength at transfer and the strand size. These equations can be used as the end block design of a prestressed concrete member or as the control of prestress transfer during production.

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In a pretensioned concrete member, the prestressing force imparted by the strand is transferred to the concrete by bond in the end region of the member. The distance over which the effective prestress is developed is called the transfer length. Its actual value depends on parameters such as prestressing level, strength of concrete at tension release, diameter of strand, type of release (gradual or sudden), surface condition of the strand (clean, oiled, rusted), confining reinforcement around the strand, time elapsed after transfer, position of the strand in the member (top or bottom), type of concrete curing and other factors.

Force transfer analysis was a topic of interest in the early works of Hoyer,[1] Magnel[2] and Guyon.[3] Other investigators have subsequently studied the subject.[4-17]

The ACI 318-89 provisions for development length implicitly include the transfer length (ℓ_t) (based on tests by Hanson and Kaar[5]) as a function of the effective prestress (f_{se}) and the nominal strand diameter (d_b). Substituting SI units:

$$\ell_t = \frac{1}{7}\frac{f_{se}}{3} d_b \qquad (1)$$

Using inch-pound units, its denominator becomes 3 ksi rather than 21 MPa. The same formula is adopted in the *ACI Manual of Concrete Practice 1989*.[18]

The following study is based on a mathematical model of the transfer mechanism without any preliminary assumption for the bond stress distribution. The applied nonlinear bond stress-slip relationship produces nonlinearity in all the derived formulas which are applicable to all types of strands. However, in this paper, the numerical examples and graphs are developed for only ½ in. (12.8 mm) seven-wire strands.

EXISTING EQUATIONS

Guyon[3] expressed the transmission length as a function of the draw-in of prestressing tendon (S) and the initial tendon strain (ε_{si}):

$$\ell_t = \alpha S / \varepsilon_{si} \tag{2}$$

where $\alpha = 2$ or $\alpha = 3$, assuming constant or linear bond stress distribution, respectively.

Olesniewicz[9] developed an empirical formula as a function of the effective prestress, the concrete strength at transfer (f'_{ci}) and the nominal strand diameter:

$$\ell_t = 10\sqrt{\frac{f_{se}}{f'_{ci}}}\, d_b \tag{3}$$

To calculate the lower and upper bound values of the transfer length, he suggested coefficients of 7 and 13, respectively, instead of 10 as in Eq. (3).

Zia and Mostafa[12] developed a linear approach to the transfer length based on several research data for concrete strengths of 2 ksi (14 MPa) < f'_{ci} < 8 ksi (55 MPa). Substituting SI units:

$$\ell_t = 1.5\frac{f_{si}}{f'_{ci}}\, d_b - 117 \tag{4}$$

Using inch-pound units by Eq. (4), the additive term is 4.6 in. rather than 117 mm.

BOND-SLIP RELATION AND TENDON STRESSES OVER TRANSFER LENGTH

The prestress transfer is the same physical phenomenon as the force transfer of a reinforcing bar. In both cases, the steel force is transferred to the concrete by bond stresses which are activated by the slips at the interactional surface. The differences in the two cases are the stress level, the place of maximum slip and the bar geometry.

Governing Equation

Considering equilibrium, compatibility, elastic behavior of steel and concrete, and assuming the same bond-slip behavior over the transfer length[19,29] [see Fig. 1(a) and 1(b)], the governing equation of the phenomenon is given by:

$$\delta''(\xi) - K_p f_b[\delta(\xi)] = 0 \tag{5}$$

This is a second order, ordinary differential equation for the slip distribution, where $\delta(\xi)$ is the slip related to the strand diameter [$\delta(\xi) = s(\xi)/d_b$], $f_b[\delta(\xi)]$ is the bond-slip relationship, ξ is the nondimensional co-ordinate of the section ($\xi = x/d_b$) and

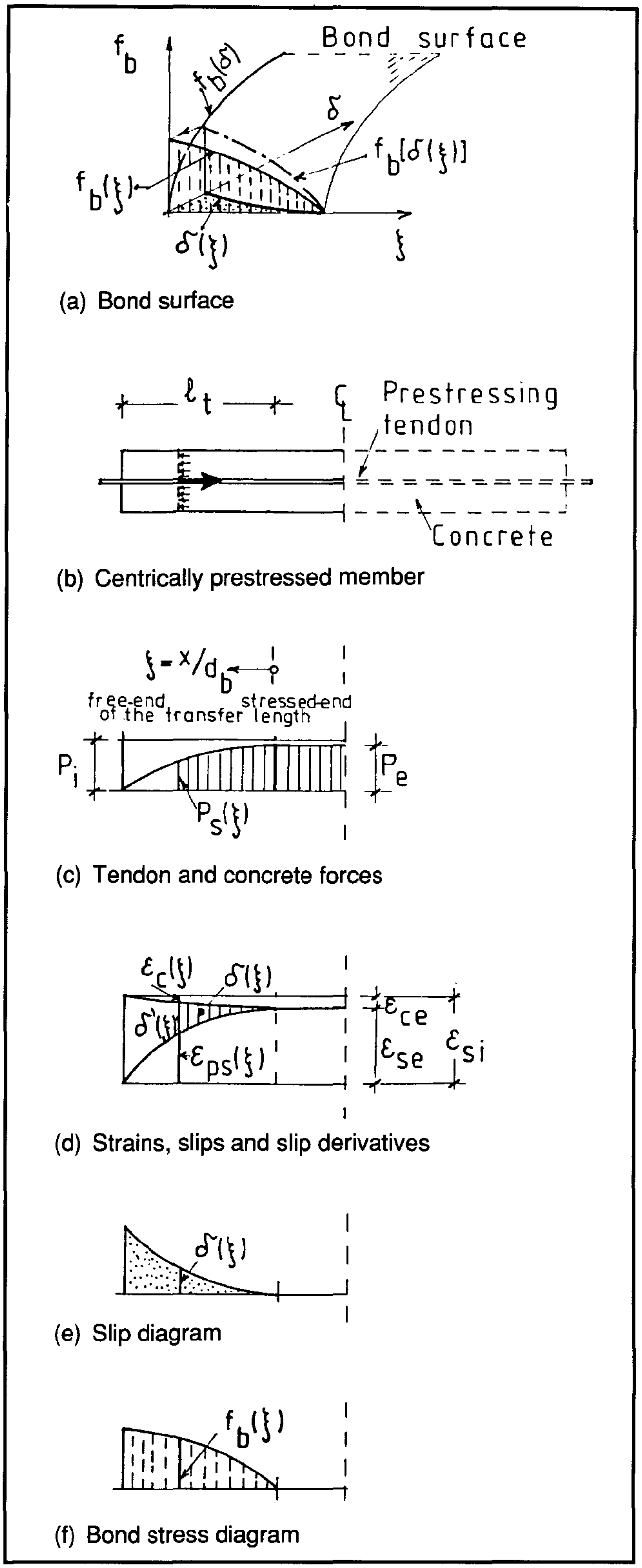


Fig. 1. Schematic diagrams of the transfer analysis.

$$K_p = \frac{4(1 + n\rho_p)}{E_p}\Theta \tag{6}$$

In Eq. (6), $n = E_p/E_c$, $\rho_p = A_{ps}/A_c$ and $\Theta = d_b^2\pi/(4A_{ps})$. Since A_{ps} = 0.155 in.^{2} (100 mm^2) and the definition of the nominal strand diameter is the sum of the diameters of the

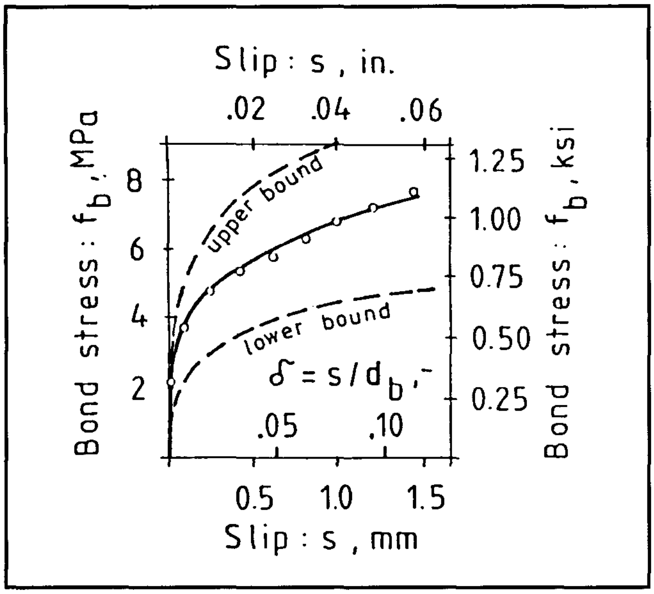


Fig. 2. Bond-slip relationship of a seven-wire strand. Type of Felten and Guilleaume ST 1600/1800-F 100. Note: A_{ps} = 0.155 in.^{2} (100 mm^2), d_b = ½ in. (12.8 mm), f'_{ci} = 5.8 ksi (40 MPa). Bound values are obtained by applying Eq. (9b).

core wire and the two external wires (refer to their value in the next section), Θ = 1.287 for seven-wire strands (rather than 1.0). Hence, $K_p = 5.15\,(1+ n\rho_p)/E_p$ for ½ in. (12.8 mm) seven-wire strands.

The origin of the co-ordinate system is at the stressed end of the transfer length where the tendon stress reaches the effective prestress. The co-ordinate axis parallel to the axis of the strand is directed toward the end face of the member [see Fig. 1(b)].

A similar equation to Eq. (5) was used by several researchers for the anchorage analysis of reinforcing bars assuming different bond-slip relationships. These studies include the works by Rehm,[19] Martin,[20] Tepfers,[21] Giuriani,[22] Krips[23] and other investigators. In the present study, a solution is developed for the transfer analysis of prestressing strands.

The solution of the governing equation depends on the bond-slip relationship and the boundary conditions.

Bond-Slip Relationship

The bond-slip relationship gives the bond stress produced by the slip at the interactional surface. For the anchorage analysis of reinforcing bars, linear,[19,21] bilinear[22] and various nonlinear[19,20,24,25] bond-slip relationships were applied.

For the bond-slip relationship of multi-wire strands, the power function:

$$f_b = C\delta^b \tag{7}$$

is proposed, where C (with the dimension of stress) and $0 < b < 1$ (dimensionless) are experimental constants. This bond-slip relationship has the advantages of a good approximation to the test results and of providing a possibility to solve the governing differential equation.

A bond-slip relationship of seven-wire strands was evaluated from test results carried out at the Department of Reinforced Concrete Structures, Budapest University of Technology.[26,27] The type of strand used was Felten and Guilleaume ST 1600/1800 having a cross-sectional area A_{ps} = 0.155 in.^{2} (100 mm^2).

The diameter of the core wire was 0.171 in. (4.35 mm) and of the external wires was 0.166 in. (4.22 mm), providing a nominal strand diameter d_b = 0.50 in. (12.8 mm). The measured modulus of elasticity of the strand was E_p = 28093 ksi (193700 MPa), and the specified concrete strength at transfer was f'_{ci} = 5.8 ksi (40 MPa).

In these tests, the surface deformations of the pretensioned members and the draw-in of prestressing strands were measured during the gradual release of prestress. The tendon stresses and slips were evaluated from the concrete surface strains.[27] The bond stresses were obtained from the change of prestressing force between the points of measurement.[27]

A curve in the form of Eq.(7) was fit by the least squares method providing (see Fig. 2):

$$f_b = 13\left(\frac{s}{d_b}\right)^{0.25} = 13\sqrt[4]{\delta} \tag{8}$$

C = 13 MPa (1.885 ksi) and b = 0.25 for the given seven-wire strand.

Jokela and Tepfers[14] obtained slightly higher bond stresses from pull-out tests using British Bridon seven-wire strand with a cross-sectional area of 0.146 in.^{2} (94.2 mm^2). The difference is consistent because of the 5.9 in. (150 mm) bond length and because the slips were measured at the loaded side. Edwards and Picard[8] obtained somewhat lower bond stresses by pull-out tests with seven-wire strands of 0.144 in.^{2} (93 mm^2) conducted on specimens with 1.5 in. (38 mm) bond length.

Assuming proportionality between the bond stress and the square root of the concrete strength at transfer,[9,15] Eq. (8) yields:

$$f_b = c\sqrt{f'_{ci}}\sqrt[4]{\delta} \tag{9a}$$

where c = 2.055 MPa$^{1/2}$ (0.783 ksi$^{1/2}$) to the mean values of the bond stress and $c\sqrt{f'_{ci}} = C$ of Eq. (8).

Previous pull-out tests[28] with deformed bars resulted in a 35 percent difference of the mean value and the upper or lower bound values of bond stresses. Considering the same scatter for seven-wire strands, Eq. (9a) can be rewritten as (see Fig. 2):

$$f_b = \psi c\sqrt{f'_{ci}}\sqrt[4]{\delta} \tag{9b}$$

where

$\psi = \psi_{0.95}$ = 1.35 for the upper bound of bond stresses
$\psi = \psi_m$ = 1.00 for the mean value of bond stresses
$\psi = \psi_{0.05}$ = 0.65 for the lower bound of bond stresses

Solution of the Governing Equation

The strain analysis at the stressed end of the transfer length [see Fig. 1(d)] provides homogeneous initial values, i.e., $\delta(\xi = 0) = 0$ and $\delta'(\xi = 0) = 0$. Substituting the nonlinear bond-slip relationship given by Eq. (9) into Eq. (5), the derived differential equation is nonlinear even if linear elastic stress-strain relationships for steel and concrete are assumed.

The solution of the governing differential equation gives the slip distribution over the transfer length:[29]

$$\delta(\xi) = \frac{s(\xi)}{d_b} = \kappa \xi^{\frac{2}{1-b}} \tag{10}$$

where κ is a dimensionless coefficient which includes the parameters of the bond-slip relationship, the factor K_p and the concrete strength:

$$\kappa = \left[\frac{\psi c (1-b)^2}{2(1+b)} K_p \sqrt{f'_{ci}} \right]^{\frac{1}{1-b}} \tag{11}$$

For seven-wire strands:

$$\kappa = 0.357 \sqrt[3]{K_p^4 f'^2_{ci}} \tag{11a}$$

Eq. (10) is shown schematically in Fig. 1(e). The power of Eq. (10) is $2/(1 - b) = 8/3$ for ½ in. (12.8 mm) seven-wire strands. To calculate the bound values of the slip distribution, $\kappa_{0.95} = (\psi_{0.95})^{4/3}\ \kappa = 1.49\kappa$ or $\kappa_{0.05} = (\psi_{0.05})^{4/3}\ \kappa = 0.56\kappa$ can be substituted.

The bond stress distribution is calculated substituting Eq. (10) into the bond stress-slip relationship [see Fig. 1(f)]:

$$f_b(\xi) = \psi c \sqrt{f'_{ci}} \kappa^b \xi^{\frac{2b}{1-b}} \tag{12}$$

The coefficient $c\sqrt{f'_{ci}}\kappa^b = 1.59\left(K_p f'^2_{ci}\right)^{1/3}$ MPa and the power is $2b/(1 - b) = 2/3$ for ½ in. (12.8 mm) seven-wire strands. To calculate the upper or lower bound of the bond stress distribution, its mean value given by Eq. (12) (and using $\psi = 1$) is multiplied by 1.49 or 0.56, respectively.

Due to the slip increase, Eq. (12) gives the maximum bond stress at the end face of the concrete member [see Fig. 1(f)]. Not only is this a mathematical simplification, it is also explained by the physical meaning of the bond-slip relationship and by the slight increase of the wire diameters during tension release.

The tendon stress distribution is obtained by substituting the bond stress distribution into the equilibrium equation, yielding:

$$f_{ps} = f_{se} - B\xi^{\frac{1+b}{1-b}} \tag{13}$$

The power $(1 + b)/(1 - b) = 5/3$ for ½ in. (12.8 mm) seven-wire strands and the coefficient:

$$B = \frac{4(1-b)\Theta}{1+b} \kappa^b \psi c \sqrt{f'_{ci}} = 4.91 \sqrt[3]{K_p f'^2_{ci}} \tag{14}$$

When inch-pound units are substituted, the coefficient is 1.36 ksi$^{2/3}$ (rather than 4.91 MPa$^{2/3}$). The coefficient B, given by Eq. (14), has a dimension of stress and for ½ in. (12.8 mm) seven-wire strands (when $n\rho_p = 0$ is substituted), $B = 8.47(f'^2_{ci} / E_p)^{1/3}$. Its coefficient is 8.74 (instead of 8.47) when $n\rho_p = 0.1$. The tendon stress, f_{se}, following the elastic deformation of the concrete after transfer, is given by:

$$f_{se} = f_{si} / \left(1 + n\rho_p\right) \tag{15}$$

Eq. (13) is shown schematically in Fig. 1(c). To calculate the upper or lower bound of the tendon stress distribution, coefficient B given by Eq. (14) (and using $\psi = 1$) is multiplied by 1.49 or 0.56, respectively.

EQUATIONS FOR TRANSFER LENGTH

The following equations are expressed in the simplest possible terms in order to make their application easy; therefore, the coefficients generally have fractional units. The given values correspond to ½ in. (12.8 mm) seven-wire strands. The parametric forms of the formulas are presented in Appendix B using the same equation numbers with an asterisk.

The transfer length as a function of the effective prestress is obtained from Eq. (13) substituting $f_{ps}(\xi = \ell_t / d_b) = 0$ and expressing $\xi = \ell_t / d_b$:

$$\frac{\ell_t}{d_b} = \left(\frac{f_{se}}{d_b}\right)^{\frac{1-b}{1+b}} = K_1 \sqrt[5]{\frac{f_{se}^3}{f'^2_{ci}}} \tag{16}$$

where

$K_1 = 3.11$ MPa$^{-1/5}$ = 4.58 ksi$^{-1/5}$ when $n\rho_p = 0.1$
$K_1 = 3.17$ MPa$^{-1/5}$ = 4.66 ksi$^{-1/5}$ when $n\rho_p = 0$

Substituting the upper bound or the lower bound of the bond-slip relationship into Eqs. (16) or (16*) of Appendix B, the lower bound $(\ell_{t0.05})$ and upper bound $(\ell_{t0.95})$ of the transfer length are obtained successively:

$$\ell_{t0.05} = \ell_t / 1.35^{0.8} = 0.79\ell_t$$

$$\ell_{t0.95} = \ell_t / 0.65^{0.8} = 1.41\ell_t$$

The ratio of the upper bound and the lower bound values of the transfer length obtained is 1.41/0.79 = 1.8. Based on field measurements, den Uijl[31] reported 1.9 and Eq. (3) provides 1.86 for the same ratio.

Substituting Eq. (15) for f_{se} in Eq. (16), the transfer length can be obtained as a function of the initial prestress [(see Fig. 3(a)]:

$$\frac{\ell_t}{d_b} = \left[\frac{f_{si}}{\left(1 + n\rho_p\right)B}\right]^{\frac{1-b}{1+b}} = 2.93 \sqrt[5]{\frac{f_{si}^3}{f'^2_{ci}}} \tag{17}$$

where the coefficient is accounted for $n\rho_p = 0.1$ and it is equal to 4.32 ksi$^{-1/5}$ when substituting inch-pound units.

The transfer length can be expressed as a function of the draw-in from the inverse of Eq. (10) [see Fig. 3(b)] by:

$$\frac{\ell_t}{d_b} = \left(\frac{S}{\kappa d_b}\right)^{\frac{1-b}{2}} = K_2 \sqrt[4]{\frac{S^3}{f'_{ci}}} \tag{18}$$

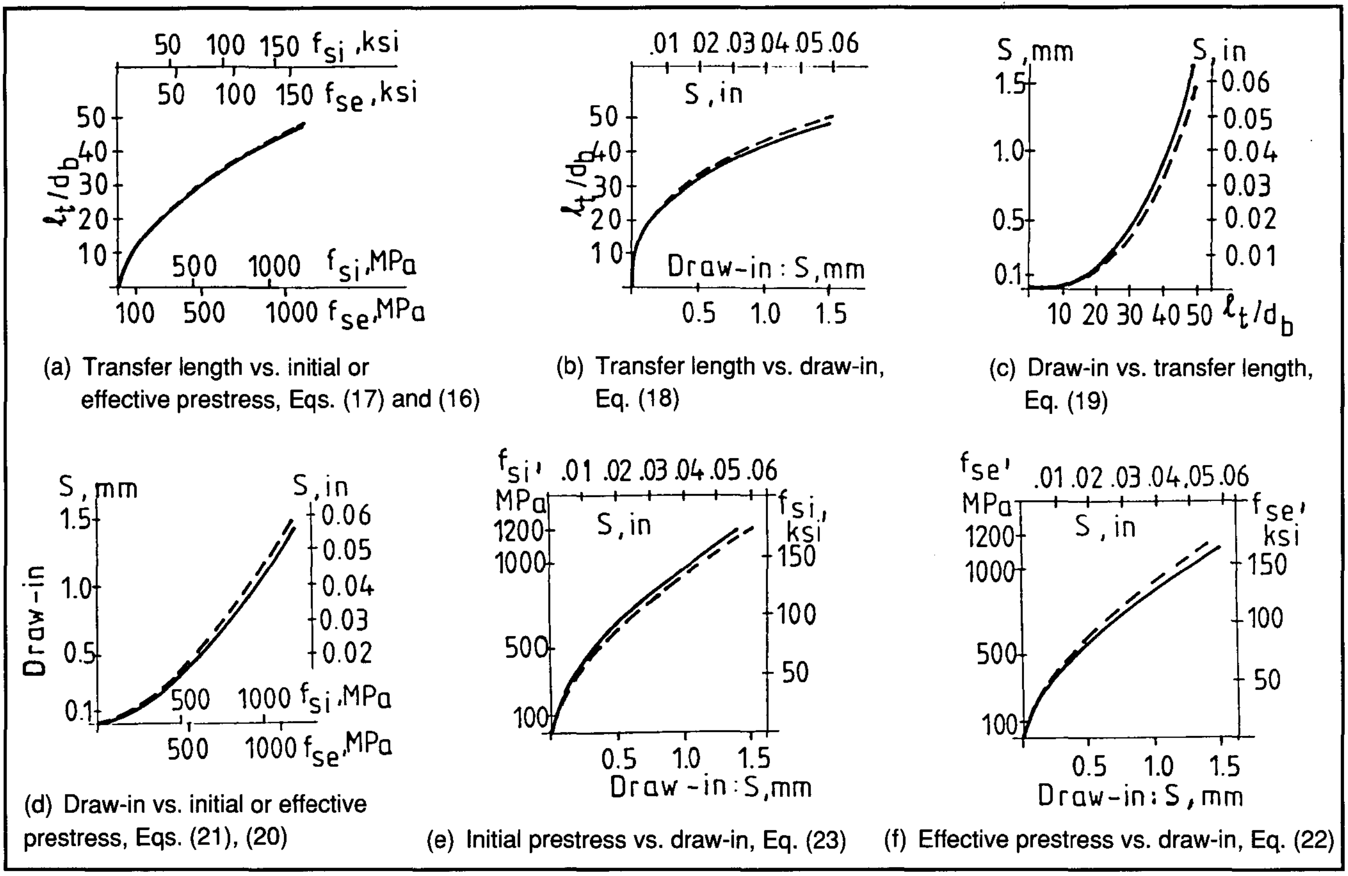


Fig. 3. Application of proposed formulas for seven-wire strands. Note: A_{ps} = 0.155 in.^{2} (100 mm^2), d_b = ½ in. (12.8 mm), E_p = 28280 ksi (195000 MPa), bond-slip relationship: $f_b = c\sqrt{f'_{ci}}\sqrt{\delta}$, where c = 2.055 MPa$^{1/2}$ (0.783 ksi$^{1/2}$).
———— : $n\rho_p = 0.1$ — — — — — : $n\rho_p = 0$.

where

K_2 = 105 MPa$^{1/4}$/mm$^{3/8}$ = 218 ksi$^{1/4}$/in.$^{3/8}$ when $n\rho_p = 0.1$
K_2 = 110 MPa$^{1/4}$/mm$^{3/8}$ = 229 ksi$^{1/4}$/in.$^{3/8}$ when $n\rho_p = 0.0$

Eqs. (16), (17) and (18), together with Eqs. (16*), (17*) and (18*) of Appendix B, show that the transfer length is a nonlinear function of the prestress or the draw-in, the concrete strength, the coefficients of the bond-slip relationship, the moduli of elasticity and the section properties.

EQUATIONS FOR DRAW-IN

The draw-in of the strand is obtained from Eq. (10) substituting $\xi = \ell_t / d_b$ [see Fig. 3(c)]:

$$S = \kappa\left(\frac{\ell_t}{d_b}\right)^{\frac{2}{1-b}} d_b \qquad (19)$$

where κ is given by Eq. (11).

The expected draw-in as a function of the effective prestress is obtained considering $\xi = \ell_t / d_b$ and substituting Eq. (16) into Eq. (10) [see Fig. 3(d)]:

$$S = K_3\left(\frac{f_{se}^{\,2}}{E_p\sqrt{f'_{ci}}}\right)^{0.8} \qquad (20)$$

where

K_3 = 1.44 MPa$^{5/2}$ mm = 0.122 ksi$^{5/2}$ in. when $n\rho_p = 0.1$
K_3 = 1.33 MPa$^{5/2}$ mm = 0.113 ksi$^{5/2}$ in. when $n\rho_p = 0.0$

The draw-in can be expressed as a function of the initial prestress by substituting Eq. (17) into Eq. (10). This yields a formula similar to Eq. (20) including f_{si} rather than f_{se}, but with a slightly different coefficient [see Fig. 3(d)]:

$$S = 1.23\left(\frac{f_{si}^{\,2}}{E_p\sqrt{f'_{ci}}}\right)^{0.8} \qquad (21)$$

where the coefficient is for $n\rho_p = 0.1$ and equal to 104 ksi$^{5/2}$ in. when substituting inch-pound units.

The power of the effective or initial prestress in the previous two equations is $2/(1 + b) = 1.6$. The average field measurements[30] of 58 pretensioned concrete members yielded a value of 1.5.

EQUATIONS FOR PRESTRESS

The effective prestress can be expressed as a function of the measured draw-in from Eq. (20) [see Fig. 3(f)]:

$$f_{se} = 369\sqrt{\frac{\sqrt{f'_{ci}}}{1+n\rho_p}}\,S^{0.625} \qquad (22)$$

Table 1. Powers of the bond stress, tendon stress and slip distributions over the transfer length.

	Guyon[3]	Guyon[3]	Proposed method: Parametric equation	Proposed method: Seven-wire strands
Power of the bond stress distribution	0	1	$\frac{2b}{1-b}$	0.67
Equation number	Eq. (2)	Eq. (2)	Eq. (12)	Eq. (12)
Power of the stress and strain distributions	1	2	$\frac{1+b}{1-b}$	1.67
Equation number			Eq. (13)	Eq. (13)
Power of the slip distribution	2	3	$\frac{2}{1-b}$	2.67
Equation number			Eq. (10)	Eq. (10)

Table 2. Calculated transfer length and draw-in values of a seven-wire strand using Eqs. (16) and (20), respectively. Note: d_b = ½ in. (12.8 mm), f_{si} = 174 ksi (1200 MPa), f_{se} = 158 ksi (1090 MPa), f'_{ci} = 5.8 ksi (40 MPa).

	Substituting f_{se}		Substituting $f_{se} = f_{si}$	
	$n\rho_p = 0.1$	$n\rho_p = 0$	$n\rho_p = 0.1$	$n\rho_p = 0$
$\frac{\ell_t}{d_b}$	47.2	48.1	50.0	51.0
	2%		2%	
S, mm	1.40	1.30	1.64	1.52
in.	0.055	0.051	0.064	0.060
	-7%		-7%	

Similarly, the initial prestress can be expressed as a function of the measured draw-in from Eq. (21) [see Fig. 3(e)]:

$$f_{si} = 369\sqrt{\left(1+n\rho_p\right)\sqrt{f'_{ci}}}\,S^{0.625} \tag{23}$$

The coefficient of Eqs. (22) and (23) is 655 $\text{ksi}^{3/4}$ $\text{in.}^{-5/8}$ when substituting inch-pound units.

If concrete deformations are neglected, i.e., $n\rho_p = 0$, Eqs. (22) and (23) would provide the same result for the effective and initial prestress; but this would not be correct; hence, $n\rho_p$ is not negligible in the last two formulas, but the term can be neglected in the previous equations (see also the next calculation results).

ANALYSIS OF RESULTS

The proposed method, especially Eqs. (16), (17) and (18), clearly shows that the transfer length of a prestressing strand can be expressed separately by either the prestress or the draw-in since both parameters are governed by the slip distribution. Knowing the bond-slip relationship and either the prestress or the draw-in is sufficient to determine the transfer length.

A similar calculation procedure can be used for other tendon types if their bond-slip relationship is available in the form of $f_b = C\delta_b$.

The analysis of the exponents of bond stresses, tendon stresses and slips (see Table 1) show that the computed values lie between the limit values given by the Guyon[3] formula, Eq. (2), since the power of Eq. (12) [$2b/(1-b) = 0.67$] provides a fractional value between 0.0 (indicating a constant) and 1.0 (indicating a linear bond stress distribution).

EXAMPLES AND COMPARISONS

The following procedure can be used in applying the proposed equations:

1. Select d_b, A_{ps}, Θ, E_p [measured or assumed to be 195000 MPa (28280 ksi)] and f'_{ci} at transfer.

2. Assume the parameters of the bond-slip relationship: $b = 0.25$ and $c = 2.055$ $\text{MPa}^{1/2}$ (0.783 $\text{ksi}^{1/2}$) for ½ in. (12.8 mm) seven-wire strands. Note that the coefficients need to be determined for other types of strands.

3. Calculate the transfer length or the draw-in or the prestress using Eqs. (16), (17), (18), (19), (20), (21), (22) or (23) assuming (or measuring) two of these three parameters.

4. Apply Eqs. (10), (12) or (13) to determine the slip, the bond stress or the tendon stress distributions after calculating the coefficients K_p, κ and B.

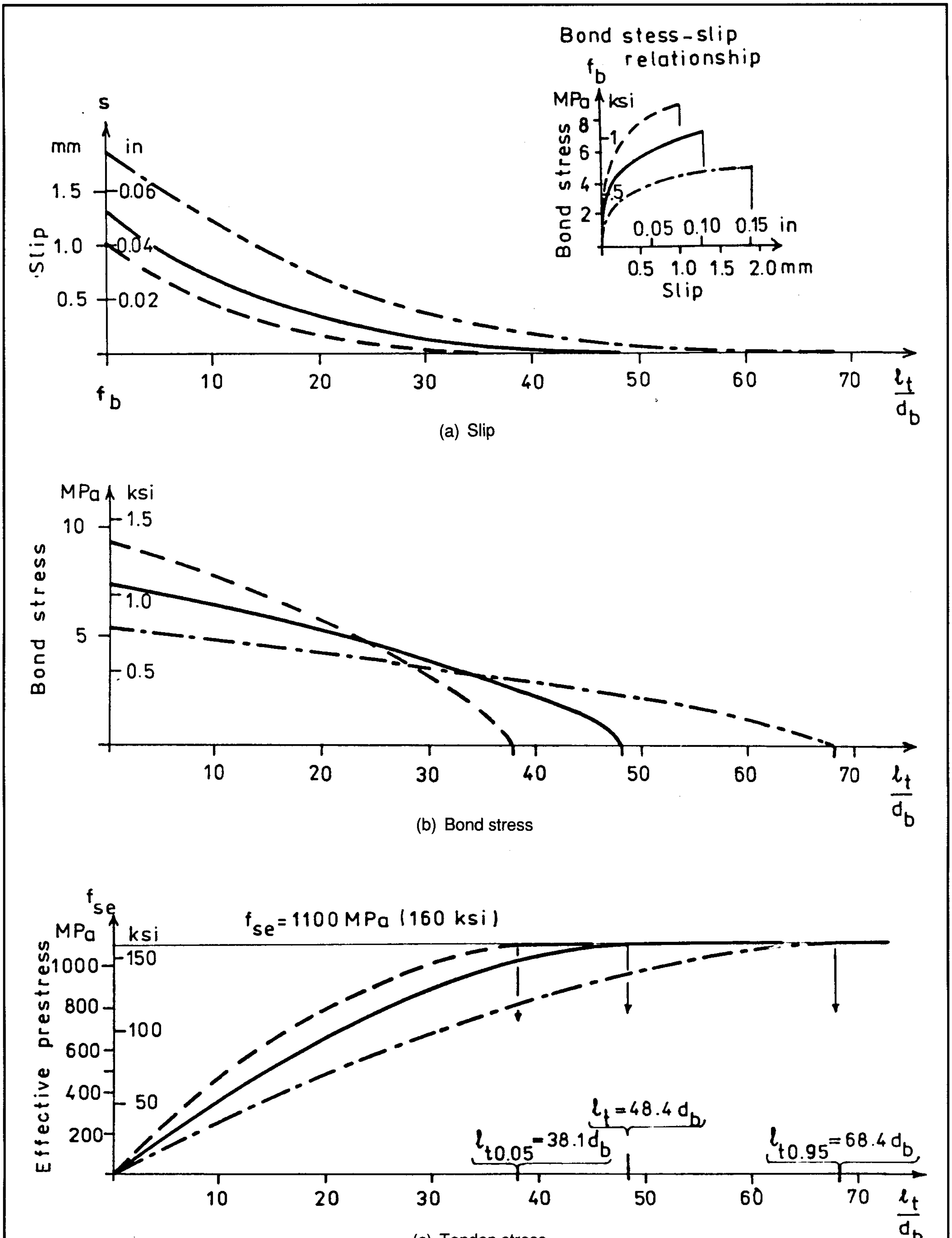
In Fig. 3, calculation results are presented applying Eqs. (16) to (23) for ½ in. (12.8 mm) seven-wire strands. The substituted data are presented in the figure captions. All the curves indicate nonlinear behavior.

In Table 2, calculation results are presented for the transfer length and for the draw-in of seven-wire strands using Eqs. (16) and (20), respectively, and the experimental bond-slip relationship given by Eq. (9). The results of the comparison in Table 2 indicate:

- Neglecting the concrete strains ($n\rho_p = 0$), the transfer length increases by only 2 percent [approximately 0.5 in. (12 mm)] and the draw-in decreases by 7 percent [approximately 0.004 in. (0.1 mm)].
- Substituting the initial prestress instead of the effective prestress in the calculations, the transfer length obtained is longer by 6 to 7 percent [approximately 1.6 in. (40 mm)], and the draw-in is higher by 16 to 17 percent [approximately 0.01 in. (0.25 mm)].

Three series of calculation results are presented in Fig. 4 using Eqs. (10), (12) and (13) for ½ in. (12.8 mm) seven-wire strands assuming the lower bound, the mean and the upper bound of the bond-slip relationship are given by Eq. (9b). This is practically the same problem as the transfer analysis for three different concrete strengths. As indicated by the diagrams, the higher the concrete strength, the smaller the transfer length and the draw-in, and the higher the maximum bond stress.

The proposed transfer length formula, Eq. (16), is graphically compared in Fig. 5 to the ACI 318-89 formula, to Eq. (3) by Olesniewicz and to Eq. (4) by Zia and Mostafa for concrete strengths of 4.4 and 5.8 ksi (30 and 40 MPa), respectively. The diagrams indicate that the proposed Eq. (16) yields to intermediate values of Eqs. (3) and (4). In the usual 145 to 174 ksi (1000 to 1200 MPa) effective prestress region, the coincidence of the ACI 318-83 formula, Eqs. (4) and (16), is good for f'_{ci} = 4.4 ksi (30 MPa) but weaker for higher concrete strengths.



(a) Slip

(b) Bond stress

(c) Tendon stress

Fig. 4. Example for the transfer of a ½ in. (12.8 mm) seven-wire strand considering the lower bound (— - — - —), the mean (————) and the upper bound (— — — — —) of the bond-slip relationship given by Eq. (9b). Note: A_{ps} = 0.155 in.^{2} (100 mm^2), E_p = 28280 ksi (195000 MPa), $n\rho_p = 0$, f'_{ci} = 5.8 ksi (40 MPa).

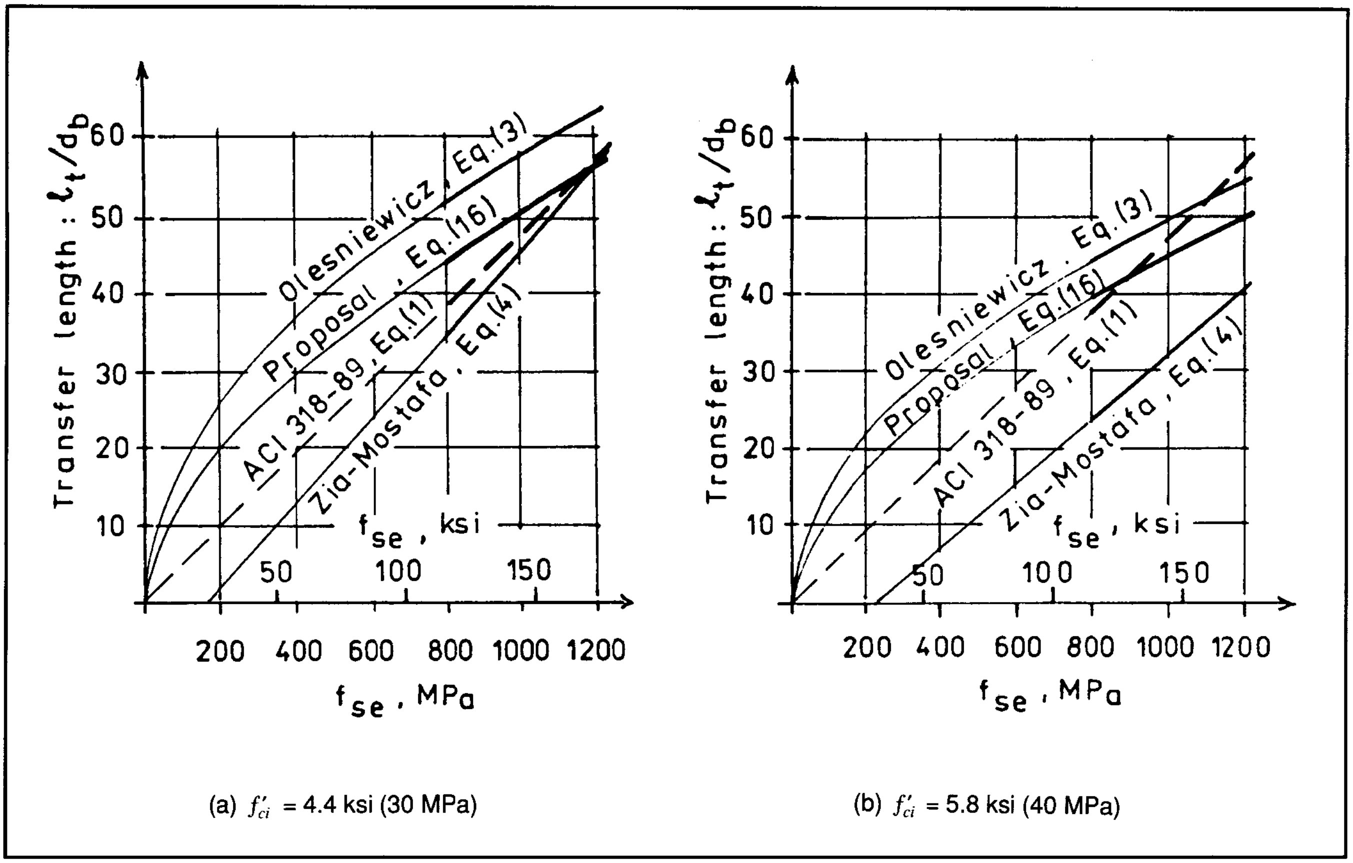


Fig. 5. Comparison of transfer length formulas.

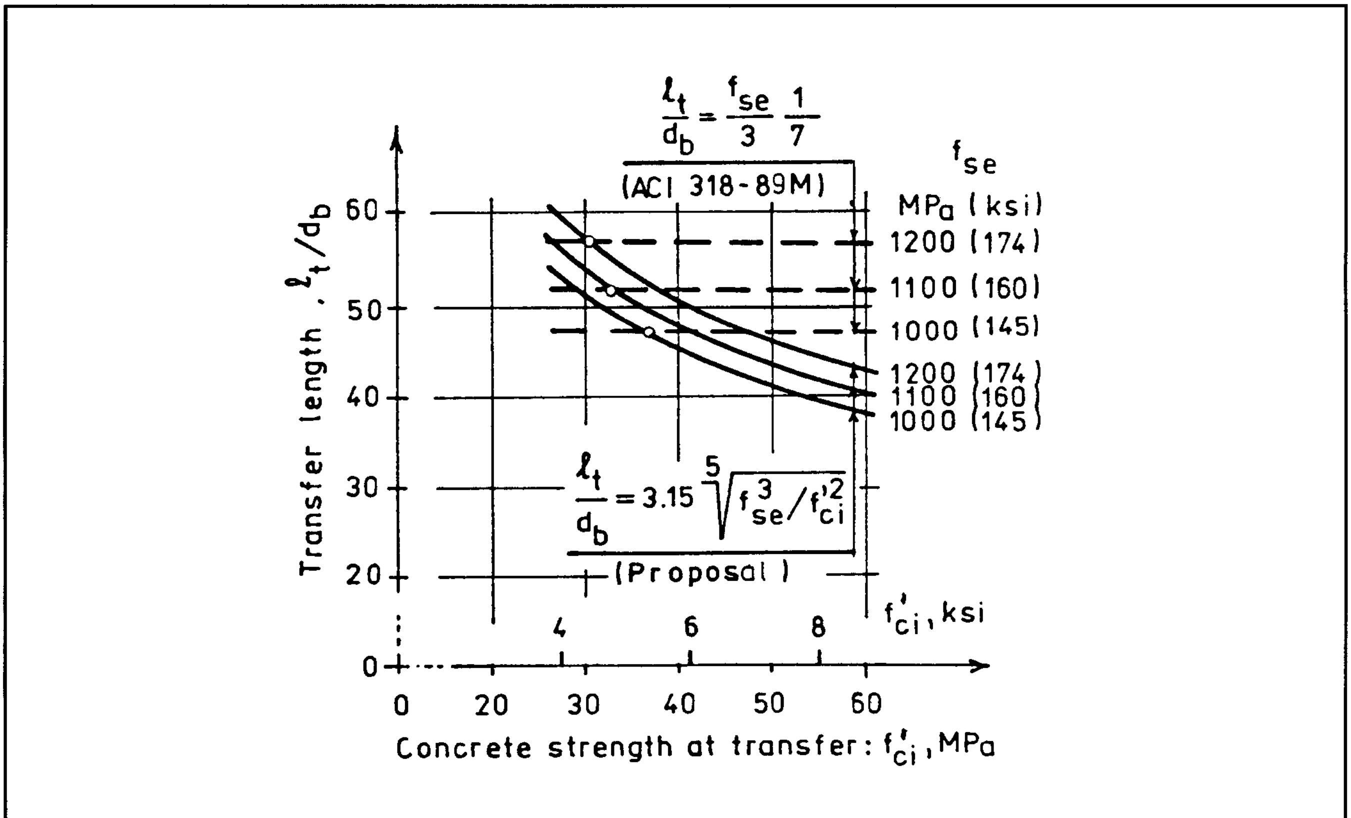


Fig. 6. Transfer length as a function of concrete strength and effective prestress by ACI 318-89 provisions and by proposed formula.

The proposed nonlinear equation and the linear ACI 318-89 formula provide approximately the same transfer length for concrete strength of 4.4 to 5.8 ksi (30 to 40 MPa) (see Fig. 6). For higher concrete strength, the ACI 318-89 formula is more conservative.

RECOMMENDATIONS

Analysis of the transfer length is important in checking the section in which all the effective prestress of the tendon exists and to analyze the prestressing stress distribution close to the support where the moment of applied load is small. The proposed Eqs. (16) or (17) and (13) provide a method to answer these two questions.

During the production of prestressed concrete members, the correct amount of prestress needs to be checked. Without measuring the strains after transfer (which can be time consuming in large production runs), there remains the possibility to measure the prestress, the draw-in (in the case of tension release by hydraulics or saw cutting) and the concrete strength. Eqs. (18), (20), (21), (22) and (23) provide a means to verify that the prestress is transferred to the concrete in a satisfactory way.

CONCLUSIONS

1. The following equations are developed for checking the transfer control of ½ in. (12.8 mm) seven-wire strands based on a nonlinear bond-slip behavior (using SI units, the equations provide mean values):

(a) Average transfer length as a function of the effective prestress:

$$\ell_t / d_b = 3.15 \sqrt[5]{f_{se}^3 / f_{ci}'^2}$$

(b) Tendon stress distribution over the transfer length [measuring the section co-ordinate ($\xi = x/d_b$) from the stressed end of the transfer length]:

$$f_{ps} = f_{se} - \left(8.5 \sqrt[3]{f_{ci}'^2} / E_p\right) \xi^{5/3}$$

(c) Draw-in as a function of the effective prestress:

$$S = 1.4 \left(\frac{f_{se}^2}{E_p \sqrt{f_{ci}'}} \right)^{0.8}$$

(d) Effective prestress as a function of the (measured) draw-in:

$$f_{se} = 369 \sqrt{\sqrt{f_{ci}'} / \left(1 + n\rho_p\right)} S^{0.625}$$

(e) Transfer length as a function of the (measured) draw-in:

$$\ell_t / d_b = 105 \sqrt[4]{\sqrt{S^3} / f_{ci}'}$$

The coefficients, using inch-pound units, are presented in the discussion. The proposed nonlinear transfer length equation given in Conclusion 1(a) and the ACI 318-89 formula provide approximately the same transfer length for concrete strengths of 30 to 40 MPa (4.4 to 5.8 ksi) (see Fig. 6). For higher strength concrete, the ACI 318-89 formula is more conservative.

2. The bond stress-slip relationship of a ½ in. (12.8 mm) seven-wire strand can be assumed to be $f_b = c\sqrt{f_{ci}'}\sqrt{\delta}$, where δ = slip/d_b, f_{ci}' is the specified concrete strength at transfer and c = 2.055 MPa$^{1/2}$ (0.783 ksi$^{1/2}$) is an experimental constant.

3. For seven-wire strands, the bond stress, the tendon stress and the slip distributions have powers of 0.67, 1.67 and 2.67, respectively.

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APPENDIX A — NOTATION

A_c = effective area of concrete section around a prestressing strand (see, for example, CEB-FIP Model Code 1978, Fig. 15.1), mm^2

A_{ps} = area of individual prestressing strand, mm^2

b = power of bond-slip relationship [see Eqs. (7), (8) and (9)]

B = coefficient [see Eq. (14)], MPa

c = $C\sqrt{f'_{ci}}$, MPa$^{1/2}$

C = multiplication factor of bond-slip relationship [see Eqs. (7) and (8)], MPa

d_b = nominal diameter of prestressing strand, mm

E_c = modulus of elasticity of concrete, MPa

E_p = modulus of elasticity of prestressing strand, MPa

f_b = bond stress, MPa

f'_{ci} = specified compressive strength of concrete at transfer, MPa

f_{ps} = stress in prestressing strand, MPa

f_{se} = effective stress in prestressing strand, MPa

f_{si} = initial prestress before losses, MPa

K_p = coefficient [see Eq. (6)], MPa^{-1}

K_1 to K_3 = coefficients for Eqs. (16), (18) and (20)

ℓ_t = transfer length (mean value), mm

$\ell_{t0.95}$; $\ell_{t0.05}$ = upper and lower bound values of transfer length, mm

n = E_p/E_c, modular ratio for prestressing strand

s = slip, mm

S = draw-in (free-end slip) of prestressing strand, mm

x = section co-ordinate measured from stressed end, mm

δ = s/d_b; $\delta' = d\delta/d\xi$; $\delta'' = d^2\delta/d\xi^2$

ε_c = concrete strain

ε_{ce} = concrete strain just after transfer

ε_{ps} = strain of prestressing strand

ε_{se} = tendon strain just after transfer

ε_{si} = initial strain of prestressing strand

ξ = x/d_b

ρ_p = A_{ps}/A_c

κ = coefficient [see Eq. (11)]

ψ = coefficient to take into account the scatter of the bond stresses [see Eq. (9b)]

$\psi_{0.05}$; $\psi_{0.95}$ = coefficient for the lower and upper bound of the bond stresses [see Eq. (9b)]

APPENDIX B — PARAMETRIC EQUATIONS

The parametric form of the equations for transfer length, draw-in and prestress of seven-wire strand are presented here. The equations are denoted by the same number with an asterisk as in the discussion where they are expressed for ½ in. (12.8 mm) seven-wire strands.

$$\frac{\ell_t}{d_b} = \left[\frac{(1+b)E_p^b}{2^{(2-b)}(1-b)^{(1+b)}\left(1+n\rho_p\right)^b \psi c \Theta}\right]^{\frac{1}{1+b}} \left(\frac{f_{se}}{f_{ci}'^{\,1/[2(1-b)]}}\right)^{\frac{1-b}{1+b}} \tag{16*}$$

$$\frac{\ell_t}{d_b} = \left[\frac{(1+b)E_p^b}{2^{(2-b)}(1-b)^{(1+b)}\left(1+n\rho_p\right)^b \psi c \Theta}\right]^{\frac{1}{1+b}} \left(\frac{f_{si}}{f_{ci}'^{\,1/[2(1-b)]}}\right)^{\frac{1-b}{1+b}} \tag{17*}$$

$$\frac{\ell_t}{d_b} = \sqrt{\frac{(1+b)E_p}{2\psi c(1-b)^2\left(1+n\rho_p\right)\Theta\sqrt{f_{ci}'}}\left(\frac{S}{d_b}\right)^{1-b}} \tag{18*}$$

$$S = d_b\left(\frac{(1+b)\left(1+n\rho_p\right)}{8\psi c E_p \Theta\sqrt{f_{ci}'}} f_{se}^2\right)^{\frac{1}{1+b}} \tag{20*}$$

$$S = d_b\left(\frac{1+b}{8\psi c E_p\left(1+n\rho_p\right)\Theta\sqrt{f_{ci}'}} f_{si}^2\right)^{\frac{1}{1+b}} \tag{21*}$$

$$f_{se} = \sqrt{\frac{8\psi c E_p \Theta\sqrt{f_{ci}'}}{(1+b)\left(1+n\rho_p\right)}\left(\frac{S}{d_b}\right)^{1+b}} \tag{22*}$$

$$f_{si} = \sqrt{\frac{8\psi c E_p\left(1+n\rho_p\right)\Theta\sqrt{f_{ci}'}}{1+b}\left(\frac{S}{d_b}\right)^{1+b}} \tag{23*}$$