Flexural Ductility of Structural Concrete Sections

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This paper investigates in detail the flexural ductility of structural concrete sections, i.e., reinforced concrete (RC), prestressed concrete (PC) and partially prestressed concrete (PPC) sections. Flexural ductility is defined as the ratio of the section ultimate and yield curvatures. Realistic, rather than conventional, standard estimates are attempted on the basis of actual material behavior, consistent description of all behavior states, and unified analysis of structural concrete sections. The effects of section shape, degree of prestressing, material laws, compression reinforcement, and other parameters are considered. Finally, some ACI Code revisions are proposed.

Ductility of structural concrete sections ensures sufficient deformability and avoids premature failure in either tension or compression zones (breaking of reinforcement or crushing of concrete, respectively). Thus, it may be considered a criterion for determining feasible ranges of reinforcement percentages, between some minimum and maximum (or balanced) values.

However, although reinforcement limitations are related to flexural ductility, they are not its direct expression or quantitative measure, but only reflect minimum ductility requirements implied by standard codes. Flexural ductility is usually defined by the ratio of ultimate to yield curvatures, and, hence, it is an intrinsic property of structural concrete sections.

Flexural ductility of structural concrete has been widely investigated[1-10] because of its implications in inelastic moment redistribution, energy absorbing and earthquake resistant structural design, joint detailing of precast RC/PC structures, and other applications. Some important results from analytical and experimental investigations of the flexural behavior of reinforced concrete (RC), prestressed concrete (PC) and partially prestressed concrete (PPC) sections in general and their ductility in particular have been reported in a number of papers published in the PCI JOURNAL.[5,8,9]

Analytical investigations are based on either code specified or more realistic behavior models. To various degrees, structural codes:

- Neglect the need for consistent and continuous coverage of structural concrete throughout all behavior states.
- Do not offer a unified treatment for RC, PC, and PPC.
- Do not consider all relevant factors.
- Perpetuate some questionable assumptions, definitions, and design trends.

The first three points are self evident. The last one is illustrated by a recent study[8] which proposes a replacement of existing upper limits of reinforcement:

$$(\rho - \rho')/\rho_b \leq 0.5 \text{ or } \omega_p + (d_{ns}/d_{ps})\ (\omega - \omega') \leq 0.24\beta_1$$

for RC and PC in ACI 318-83,[11] and $c/d < 600/(600 + f_y)$ in the CSA Code.[12] The recommended upper limit[8] $c/h \leq 120\varepsilon_{cu}$ replaces these code maximum values of the steel reinforcement, and is not a direct expression of section ductility. Despite its generality and wide acceptance,[12,13] the superiority of the c/d or c/h limitation over the one in ρ or ω is not evident. Indeed, for a given section, c is unknown before the calculation of the resisting moment, whereas ρ or ω are basic design (geometry and material) data. Hence, c is adequate for analysis, but inconvenient for design, when the section geometry and reinforcement amount are not yet known.

For all its simplicity and versatility, the proposed limitation $c/h \leq 120\varepsilon_{cu}$ only reflects some specified ultimate deformation of concrete and no other relevant parameters. Again, flexural ductility is an intrinsic property of structural concrete sections. As such it cannot properly be defined by standard limitations, but rather by the basic governing parameters, i.e., section geometry and mechanical properties of materials.

Structural designers may require some documented information on:

1. Flexural behavior (M-ϕ curves), based on realistic models of material behavior.
2. Behavior throughout States I, II and III (i.e., uncracked, cracked and post-yield, respectively).
3. Ductility of RC, PPC and PC sections.
4. Effects of section shape, degree of prestressing, material σ-ε laws, in addition to material strengths, flange width, and compression to tension steel ratios, which have previously been investigated.
5. Lower and upper bounds of the effective flexural ductility that can be counted upon in design, when current practice is followed (i.e., realistic ρ, ρ', f_y and f_c' values, and other parameters).

An attempt to provide such information has been presented in a comprehensive analytical study published elsewhere.[3] As a followup, this paper explores in detail the flexural ductility of structural concrete sections with a view to removing some limitations of past investigations. Specifically, it is our aim to predict ductility factors, based on consistent assumptions throughout all behavior states, valid equally for RC, PC and PPC sections, and considering all governing parameters. The objective is to offer a coherent theoretical understanding of inelastic section behavior, along with some reliable conclusions for structural engineering practice.

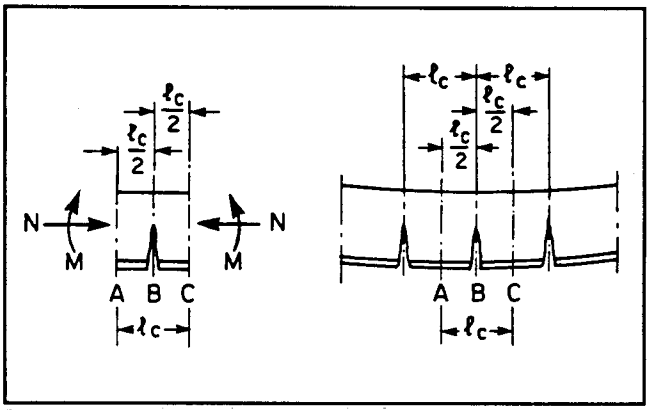


Fig. 1. Typical element for which the local moment-curvature law is studied.

ANALYTICAL MODEL

The moment-curvature (M-ϕ) constitutive law presented in Refs. 3, 6 and 14 has been adopted for the study

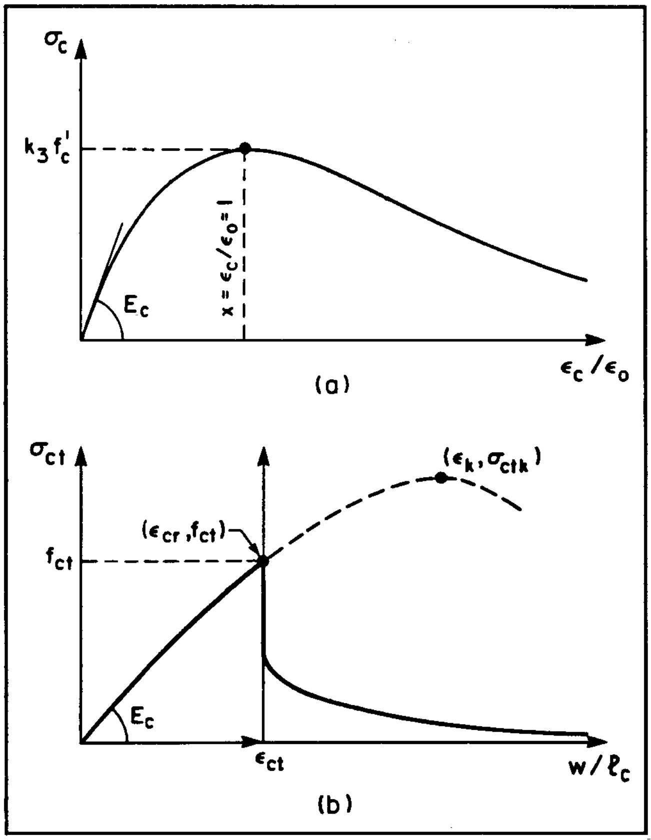


Fig. 2. Constitutive laws for concrete: (a) compression;[7] (b) tension.[18]

of the ductility properties of structural concrete sections. This M-ϕ law is based on realistic material laws and takes into consideration the effects of cracking, steel-concrete interaction and tension stiffening through the analytical model developed in Refs. 15 and 16 for RC elements and later extended to PC or PPC elements.[17] In the following, only the main assumptions inherent to the moment-curvature law are recalled.

Assumptions

1. Quasi-static (monotonic, nonrepeated, nonreversible) loading.
2. Negligible shear effects.
3. Linear strain distribution.
4. Known material stress-strain relationships (analytical, experimental, and other studies).
5. Uni-axial stress-strain laws valid for section analysis.

The moment-curvature law is determined from the study of an element of length equal to the crack spacing l_c, assuming the moment is constant along the element and expressing the compatibility and equilibrium conditions at the cracked section B (Fig. 1). The curvature is defined as the ratio between the relative rotation of two sections (A and C in Fig. 1) and the crack spacing l_c.

Material Laws

The material laws adopted for the section analysis are shown in Figs. 2 to 4, along with the relevant notation.

The stress-strain relationship adopted for concrete in compression was proposed by Sargin,[7] and accounted for the effects of the loading rate and duration, degree of confinement, strain at peak, strain softening effect and other factors.

The law for concrete in tension (Fig. 2b) is based on experimental results.[18] It assumes a parabolic stress-strain relation up to cracking and a hyperbolic stress-crack opening relation after the onset of cracking.

The stress-strain laws adopted for the mild and reinforcing steel and prestressing steel are also those proposed by Sargin.[7] A third linear branch has been added to the original σ-ε law for the prestressing steel. This linear branch starts at the stress f_p', for which the tangent to the nonlinear portion of the analytical model passes through the ultimate strength point (Fig. 3 and Ref. 3).

The models assumed to represent the bond of mild reinforcing steel and prestressing steel are shown in Fig. 4.[17]

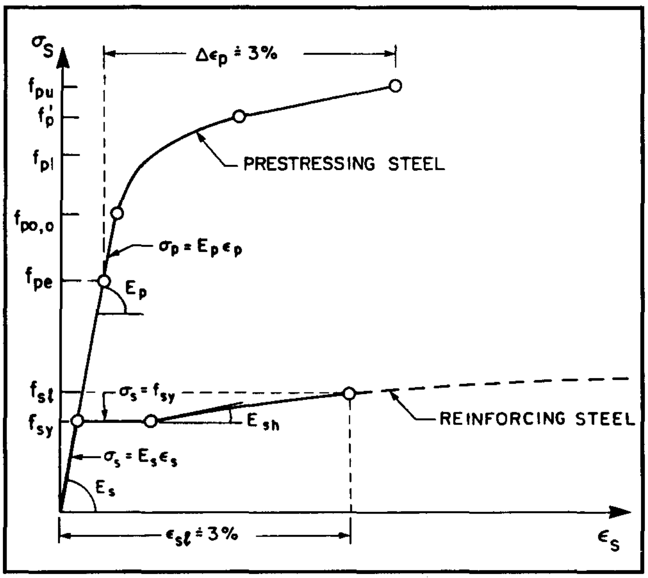


Fig. 3. Constitutive laws for reinforcing steel (nonprestressed) and prestressing steel.[7]

Computational Features

A general computer program, MOCURO (MOment CUrvature ROtation), has been developed to automatically handle the governing conditions of section response at all loading states.[6] Any symmetrical concrete section with multiple layers of mild reinforcing steel and/or prestressing steel, under either negative or positive moment, may be analyzed. The program accepts any experimental, analytical or assumed point by point material constitutive law.

Initially, the program computes the cracking, yielding and ultimate limit states. Successively, any desired number of points of the M-ϕ curve in the first (uncracked), second (cracked) and third (post-yielding) states can be computed by imposing the curvature and solving the equilibrium equations by an iterative procedure.

The output includes a summary of the section data, the

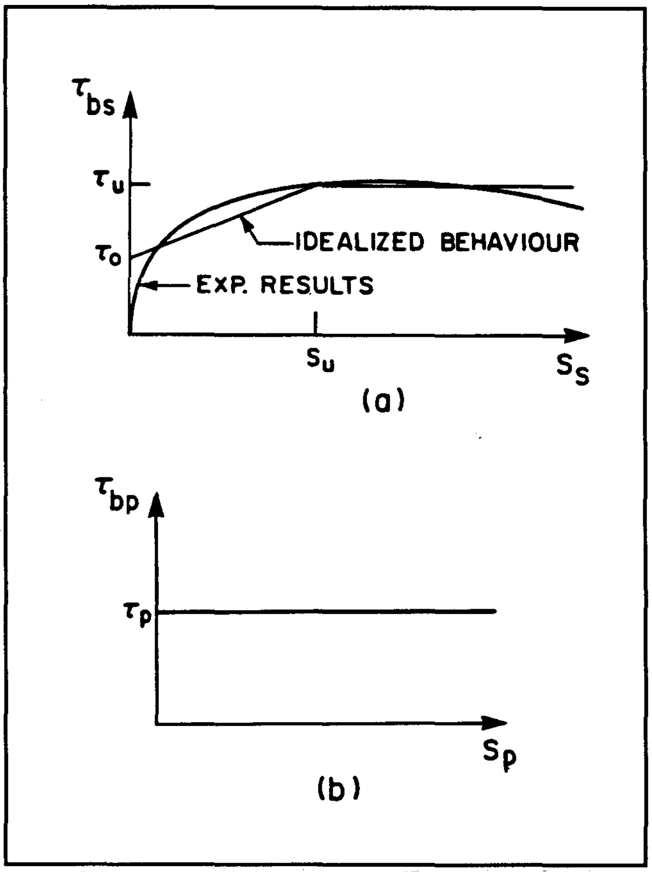


Fig. 4. Bond-slip constitutive relations:[17] (a) reinforcing steel; (b) prestressing steel.

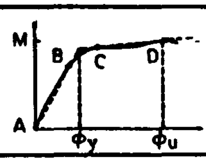
elastic stiffness, the cracking, yielding, and ultimate moments and curvatures, the ductility factor ϕ_u/ϕ_y and M, $M/(f_c'bd^2)$, ϕ, ϕl_c, the tangent stiffness, reinforcing steel and prestressing steel stresses, as well as the crack opening values for each computed point of the M-ϕ law.

A complete description of the program characteristics and of the input-output procedures are given in the MOCURO-User's Manual.[19]

Table 1. Numerical constants for material laws.

CONCRETE COMPRESSION		$f_c' = 40$ MPa ; $E_c = 29930$ MPa	$\epsilon_o = 0.00264$; $A \cong 2.5$	$D \simeq 0.362$; $k_3 = 0.8$
CONCRETE TENSION		$f_{ct} = 4.5$ MPa ;	$c_2 = 2$; $c_1/l_c = 12000$	
MILD STEEL		$f_{sy} = 400$ MPa ; $E_s = 200000$ MPa	$f_{su} = 600$ MPa ; $\epsilon_{su} = 7\%$	$\epsilon_{sh} = 1\%$; $E_{sh} = 6500$ MPa
MILD STEEL BOND		$\tau_o = 3$ MPa ;	$\tau_{su} = 10$ MPa ; $s_u = 0.5$ mm	
PRESTRESSING STEEL		$f_{po,p} = 1300$ MPa ; $E_p = 190000$ MPa	$f_{pl} = 1580$ MPa, $\epsilon_p = 1\%$; $f_{pu} = 1860$ MPa, $\epsilon_{pu} = 3.5\%$	$f_p' = 1740$ MPa ; $\epsilon_p' = 1.9\%$
PRESTRESSING STEEL BOND		$\tau_p = 4$ MPa		

Table 2. Definitions of reinforcement index, yielding and ultimate conditions.

AUTHOR	ω	ϕ_y	ϕ_u
CEB - MC 78	$\frac{A_p f_{py} + A_s f_y}{bdf_c'}$	$\left(\begin{array}{l}\varepsilon_s = \varepsilon_y \\ \varepsilon_p = \varepsilon_{py}\end{array}\right)$	$\varepsilon_c = 0.35\%$ or $\varepsilon_s = 1.00\%$ or $\Delta\varepsilon_p = \varepsilon_p - \varepsilon_{pe} = 1.00\%$
ACI (318 - 83)	$\frac{A_p f_{ps} + A_s f_y}{bdf_c'}$	$\left(\begin{array}{l}\varepsilon_s = \varepsilon_y \\ \varepsilon_p = \varepsilon_p(f_{py})\end{array}\right)$	$\varepsilon_c = 0.3\%$
COHN, BARTLETT	$\frac{A_p f_{ps} + A_s f_s}{b\,df_c'}$	$\varepsilon_s = \varepsilon_y$, $\gamma \neq 1$; $\varepsilon_p = \varepsilon(f_{p,0.05})$, $\gamma = 1$	$\frac{\partial M}{\partial \phi} = 0$
NAAMAN, HARAJLI, WIGHT	$\frac{A_p f_{ps} + A_s f_s}{b\,df_c'}$		$\frac{\partial M}{\partial \phi} = 0$
COHN, RIVA	$q = \frac{A_p f_{pu} + A_s f_{st}}{b_w df_c'}$; $f_{st} = \sigma(\varepsilon_{st} \doteq 3\%)$	$\varepsilon_s = \varepsilon_y = 0.2\%$, $\gamma \neq 1$; $\Delta\varepsilon_p = \varepsilon_p - \varepsilon_{pe} = 0.2\%$, $\gamma = 1$	$\frac{\partial M}{\partial \phi} = 0$ or $\varepsilon_{st} = \Delta\varepsilon_p = \varepsilon_{pu} - \varepsilon_{pe} \doteq 3\%$ or $\varepsilon_p = \varepsilon_{pu} = 3.5\%$

( ) = implied definition

PARAMETERS GOVERNING FLEXURAL DUCTILITY

A comprehensive study was conducted to ascertain the influence of the basic material and geometric parameters that govern the behavior of flexural elements on both the $\dot{M}$-ϕ constitutive law and the ductility factor ϕ_u/ϕ_y.

The material parameters are: σ-ε law for concrete in compression; σ-ε law for concrete in tension; σ-ε law for mild steel; σ-ε law for prestressing steel; bond-slip constitutive law for mild reinforcing steel and prestressing steel.

The geometric parameters are: shape of the section; tension reinforcement index; compression reinforcement index; prestressing to total reinforcing steel ratio (mixed reinforcement index); degree of prestressing; and stirrup percentage.

The loading parameters (axial loading, loading repetition, duration and reversal) have not been investigated in this study.

Unless otherwise specified, the material parameters have been kept constant. The values assumed for the parameters involved in the σ-ε laws (Figs. 2 through 4) are summarized in Table 1.

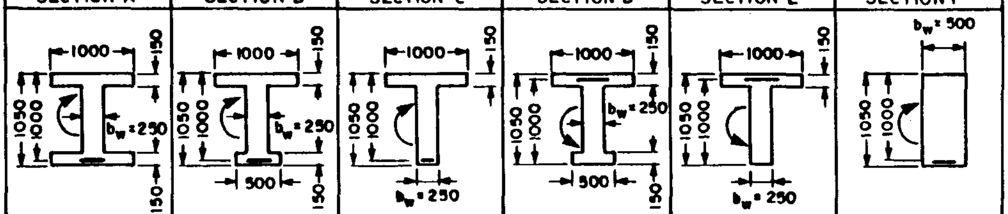
The most important geometric parameters appear to be the section shape, reinforcement index (ω), and mixed reinforcement index (γ). Various definitions of ω and of the yielding and ultimate curvatures (ϕ_y and ϕ_u), can be found in the literature. A summary of some available definitions is given in Table 2.

The yielding curvature ϕ_y is defined as the curvature at which the transition from the second (cracked) to the third (post-yielding) state of the moment-curvature law takes place. In reinforced concrete sections, this is a well defined point that coincides with yielding of the reinforcing steel. In partially and fully prestressed sections, it is not possible to identify a unique transition point: in these cases the curvature ϕ_y has only a conventional meaning, which is especially useful for the definition of the ductility factor ϕ_u/ϕ_y. Various definitions of ϕ_y have been proposed in the literature (Table 2); in this study the yielding curvature ϕ_y of reinforced and partially prestressed sections ($\gamma < 1$) is the curvature at which reinforcing steel reaches its yield strain $\varepsilon_y = 0.2$ percent.

The yield curvature of fully prestressed concrete sections ($\gamma = 1$) is the curvature at which the strain increment of prestressing steel from its effective value equals the yield strain of reinforcing steel (i.e., $\Delta\varepsilon_p = \varepsilon_p - \varepsilon_{pe} = \varepsilon_y = 0.2$ percent, where ε_{pe} is the effective prestressing steel strain,

Table 3. Specimen data for parametric study, effects of section shape, q, γ, f'_c.

$\bar{K} = 1$
STIRRUPS #10 @ 300
f_s, f_p SEE TABLE 1
s_u = 0.5 mm ALL CASES



						SECTION A						SECTION B						SECTION C						SECTION D						SECTION E						SECTION F					
ℓ_c (mm)	f'_c (MPa)	τ_s (MPa) τ_o	τ_s (MPa) τ_u	τ_p (MPa)	ω / γ	0.05	0.10	0.15	0.20	0.25	0.30	0.05	0.10	0.15	0.20	0.25	0.30	0.05	0.10	0.15	0.20	0.25	0.30	0.05	0.10	0.15	0.20	0.25	0.30	0.05	0.10	0.15	0.20	0.25	0.30	0.05	0.10	0.15	0.20	0.25	0.30
100	40	3	10	4	0.0		•		•		•																										•		•		•
					1.0		•		•		•																										•		•		•
200	30	3	10	4	0.0	•	•	•	•	•	•																									•	•	•	•	•	•
					1.0	•	•	•	•	•	•																									•	•	•	•	•	•
	40	2	7	4	0.0								•		•		•																								
					0.6								•		•		•																								
		3	10	3	0.5								•		•		•																								
					1.0								•		•		•																								
				4	0.0	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•
					0.2	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•
					0.4	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•
					0.6	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•
					0.8	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•
					1.0	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•
				5	0.5								•		•		•																								
					1.0								•		•		•																								
		4	13	4	0.0								•		•		•																								
					0.5								•		•		•																								
	50	3	10	4	0 0		•		•		•																										•		•		•
					1.0		•		•		•																										•		•		•
	60	3	10	4	0.0		•		•		•																										•		•		•
					1.0		•		•		•																										•		•		•
300	40	3	10	4	0.0		•		•		•																										•		•		•
					1.0		•		•		•																										•		•		•

and ε_y is the reinforcing steel yield strain). This definition has the advantage of simplicity and ensures a consistent yielding behavior description for RC, PPC and PC sections.

Various definitions of the ultimate curvature ϕ_u are available in the literature (Table 2). Typically, the definition of the ultimate limit state in building codes is based on the assumption of a limit strain for concrete (ACI 318-83[11]) or for both concrete and steel (CEB-MC 78[13]). On the other hand, a frequently used definition is that the ultimate limit state corresponds to the maximum moment capacity.[2,5]

In this study the latter definition has been adopted for sections in which the ultimate condition is characterized by concrete crushing. For all other sections, the ultimate limit state is identified by the failure of prestressing steel for PC or PPC sections ($\gamma \neq 0$) or by the reinforcing steel reaching a given limiting strain for RC sections ($\gamma = 0$). This limiting strain is assumed approximately equal to the strain increment of prestressing steel from its effective to its ultimate value in an equivalent PC or PPC section, i.e., $\varepsilon_{sl} = \Delta\varepsilon_p = \varepsilon_{pu} - \varepsilon_{pe} \cong 3$ percent.

Although in some cases the actual ultimate limit state of RC sections could occur for reinforcing steel strains higher than the selected limiting strain (since the ultimate strain for mild steel is much higher than 3 percent, and can be as high as 15 ÷ 20 percent), the adopted definition has the advantage of ensuring a consistent description of the behavior of RC, PPC and PC sections at the ultimate limit state.

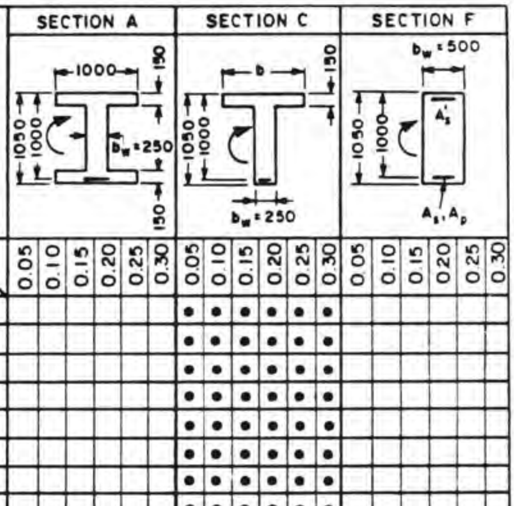
The reinforcement index essentially reflects the ultimate flexural behavior of concrete sections,[2] and its definition should depend on that assumed for the ultimate limit state of the section. As one of the main parameters that characterizes the section behavior, ω is especially useful in the design process. Because the final design and actual response are not known, the definition of ω should not be dependent on the steel stresses at the ultimate limit state and should only reflect the material characteristics and section geometry.

To avoid confusion between the definitions of the reinforcement index commonly adopted in the literature and the definition adopted in this study, the former (listed in Table 2) are indicated by the symbol ω, and the latter will be referred to as q, where:

$$q = \frac{A_p f_{pu} + A_s f_{sl}}{b_w d f'_c} \quad (1)$$

Table 4. Specimen data for parametric study, effects of analytical model, compression flange width and compression reinforcement.

f'_c = 40 MPa
f_{sy} = 400 MPa
f_{pu} = 1860 MPa
f_{pe} = 1116 MPa ($\bar{K}$ = 1)



MODEL	ω'_s/ω_s	b/b_w	γ \ ω	A 0.05	A 0.10	A 0.15	A 0.20	A 0.25	A 0.30	C 0.05	C 0.10	C 0.15	C 0.20	C 0.25	C 0.30	F 0.05	F 0.10	F 0.15	F 0.20	F 0.25	F 0.30
COHN RIVA	0.0	1.0	0.0							•	•	•	•	•	•						
		1.5	0.0							•	•	•	•	•	•						
		2.0	0.0							•	•	•	•	•	•						
		2.5	0.0							•	•	•	•	•	•						
		3.0	0.0							•	•	•	•	•	•						
		3.5	0.0							•	•	•	•	•	•						
		4.0	0.0							•	•	•	•	•	•						
		4.5	0.0							•	•	•	•	•	•						
		5.0	0.0							•	•	•	•	•	•						
	0.00	—	0.0													•	•	•	•	•	•
	0.25	—	0.0													•	•	•	•	•	•
	0.50	—	0.0													•	•	•	•	•	•
	0.75	—	0.0													•	•	•	•	•	•
	1.00	—	0.0													•	•	•	•	•	•
COHN RIVA	0.0	—	0.0	•	•	•	•	•	•							•	•	•	•	•	•
			1.0	•	•	•	•	•	•							•	•	•	•	•	•
ACI	0.0	—	0.0	•	•	•	•	•	•							•	•	•	•	•	•
			1.0	•	•	•	•	•	•							•	•	•	•	•	•
CEB	0.0	—	0.0	•	•	•	•	•	•							•	•	•	•	•	•
			1.0	•	•	•	•	•	•							•	•	•	•	•	•

Consistent with the proposed definition of the ultimate limit state, the reinforcing steel stress f_s at this limit state is assumed equal to the steel stress at the proposed limit strain, $f_{sl} = \sigma_s(\varepsilon_{sl})$, and, therefore, it is lower than the ultimate stress of the material, $f_{su} = \sigma_s(\varepsilon_{su})$. In the above definition, q is based on the specified ultimate prestressing steel and mild steel stresses, f_{pu} and f_{sl}, respectively.

The other ω definitions in Table 2 are based on either the prestressing steel and reinforcing steel stresses at the ultimate limit state of the section, f_{ps} and f_s, respectively,[2,5] or the steel yielding stresses, f_{py} and f_y.[11,13] The proposed q is referred to the web width of the concrete section, b_w, rather than to the width of the compression zone, b. This more explicitly illustrates the effect of the flange width on the ductility of structural concrete sections.

The mixed reinforcement index[2] γ expresses the proportion of prestressing steel to the total steel in a section:

$$\gamma = \frac{A_p f_{pu}}{A_p f_{pu} + A_s f_{sl}} \quad (2)$$

This definition implies that, for sections with a given q value and whose ultimate limit state corresponds to the steel reaching its limiting strain, the ultimate moment value is independent of γ.

Finally, the degree of prestressing $\bar{\kappa}$ is defined[2] as:

$$\bar{\kappa} = \frac{f_{pe}}{f_{pa}} \quad (3)$$

where f_{pe} and f_{pa} are the effective and admissible (i.e., service stress, $f_{pa} = 0.6 f_{pu}$) prestressing steel stresses, respectively.

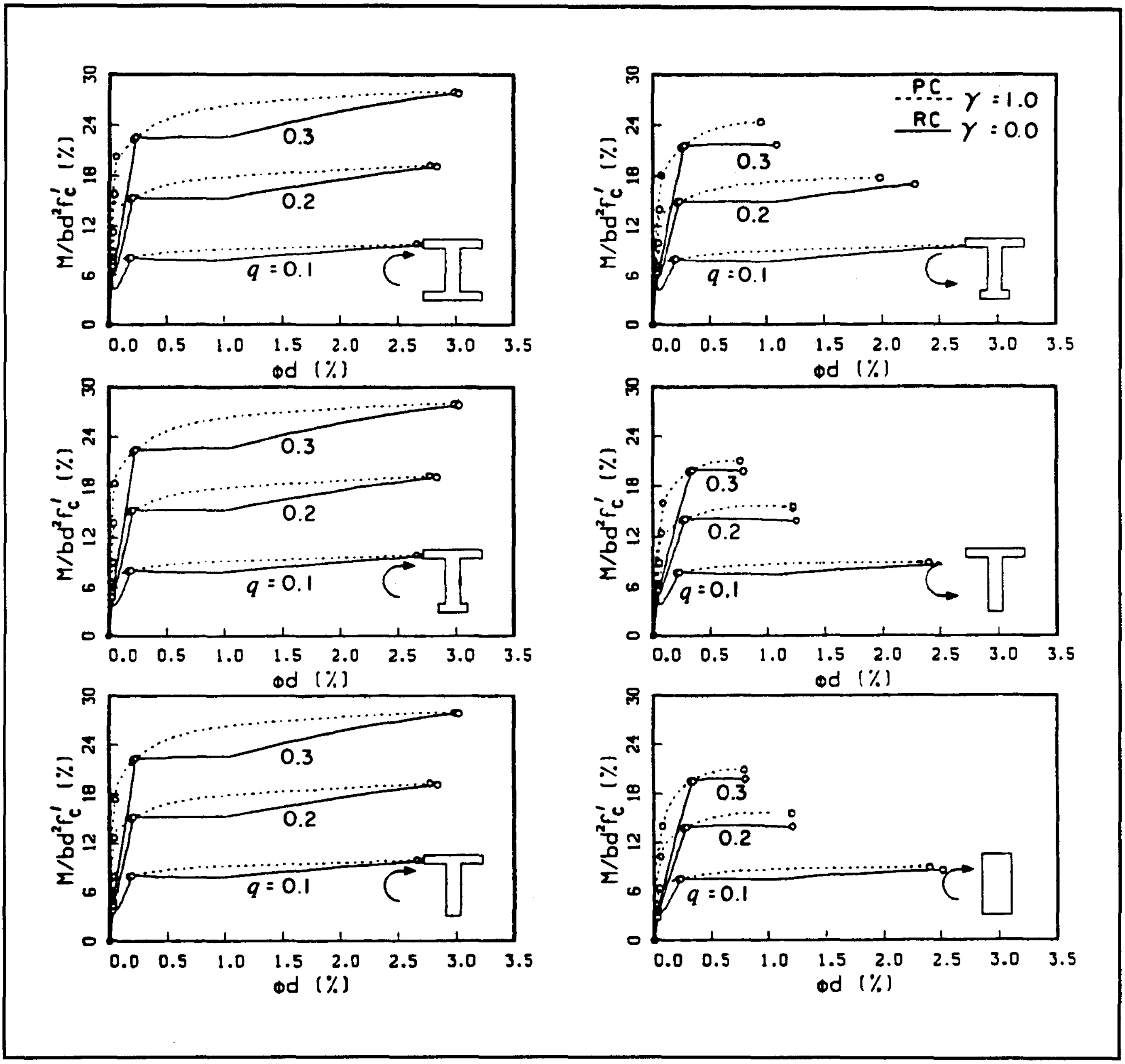


Fig. 5. Effect of section shape on the moment-curvature relationship.

PARAMETRIC STUDY OF FLEXURAL DUCTILITY

A set of 468 computer simulated tests was performed to study the influence of geometric parameters, material characteristics and various analytical models on the moment-curvature law and ductility factor. The program of the parametric study is summarized in Tables 3 and 4, where black dots indicate the combinations of parameters investigated.

Table 3 assembles the data of specimens selected mainly for studying the effect of section shape (A . . . F), degree of prestressing γ (0 . . . 1), reinforcement index q (0.05 . . . 0.30) and concrete grade, f_c' (30 . . . 60 MPa or 4.35 . . . 8.70 ksi). In addition, the influence of crack spacing, l_c [100, 200, 300 mm (4, 8, 12 in.)] and bond stresses τ_s, τ_p were examined.[3]

Table 4 identifies the specimens investigated for comparing the proposed analytical model with those implied by the ACI Standard[11] and the CEB Model Code,[13] as well as for emphasizing the major effect of compression flange width and compression reinforcement on the ductility factor.

For all tests, the stirrup area and spacing, and degree of prestressing $\overline{\kappa}$ ($\overline{\kappa} = 1$, with $f_{pe} = f_{pa} = 0.6 f_{pu}$), are constant. For all cases studied the yielding point is assumed to correspond to $\varepsilon_s = \varepsilon_y = 0.2$ percent if $\gamma < 1$, or $\Delta\varepsilon_p = \varepsilon_p - \varepsilon_{pe} = 0.2$ percent if $\gamma = 1$. The ultimate limit strains for mild reinforcing steel and prestressing steel are assumed to correspond to $\varepsilon_{sl} = 3.0$ percent and $\varepsilon_{pu} = 3.5$ percent, respectively (Table 2).

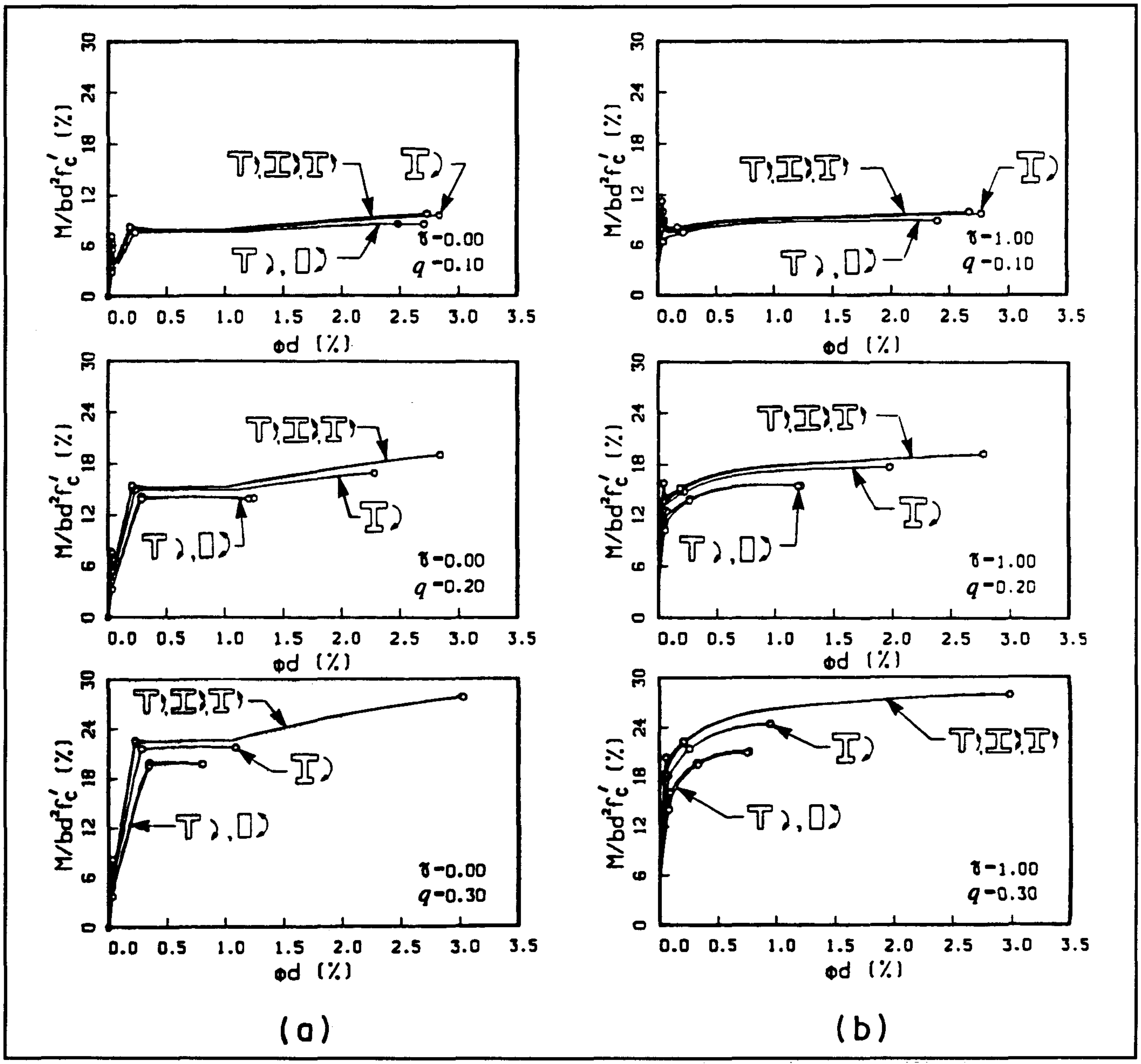


Fig. 6. Effect of section shape on the moment-curvature relationship: (a) reinforced concrete sections; (b) fully prestressed concrete sections.

Influence of Section Shape, q and γ

A thorough discussion on the influence of these parameters on the M- ϕ law and a complete set of plots is given elsewhere.[3,6] The present discussion is focused on the effects of these parameters on the ductility factor ϕ_u/ϕ_y only.

Figs. 5 and 6 show the influence of the section shape, reinforcement index q, and mixed reinforcement index γ on the moment-curvature law. They demonstrate that PC sections have a flatter inelastic response than RC sections, which display a more pronounced strain hardening.

The influence of the section shape on ϕ_u/ϕ_y is illustrated by Fig. 7. The combined effects of the section shape, q, and γ, shown in Fig. 8, may be summarized as follows:

- The ductility is generally improved by the introduction of prestressing, as already shown.[2,5] For design practice, it is conservative to consider the ductility factor of an RC section for any PPC or PC section with the same q.
- For Sections A, B and C, ϕ_u/ϕ_y is scarcely affected by q. For these sections, assuming a constant value of the ductility factor approximately equal to 14 appears to be conservative for any amount of reinforcement.
- For rectangular and T-sections under negative moment (Sections F and E), the ductility factor shows a hyperbolic variation with q. In these cases ϕ_u/ϕ_y varies between approximately 14 (for q = 0.05) and 2.5 (for $q \geq$ 0.25).
- The different trends of the ϕ_u/ϕ_y curves may be explained by the varying types of failure that characterize the ultimate limit state of various sections. For wide flanged sections (A, B and C), an almost constant ultimate curvature is obtained because failure is always governed by the steel reaching its limit strain. For the T-section under negative moment (E) and rectangular section (F), where failure is governed by concrete crushing, curvatures are re-

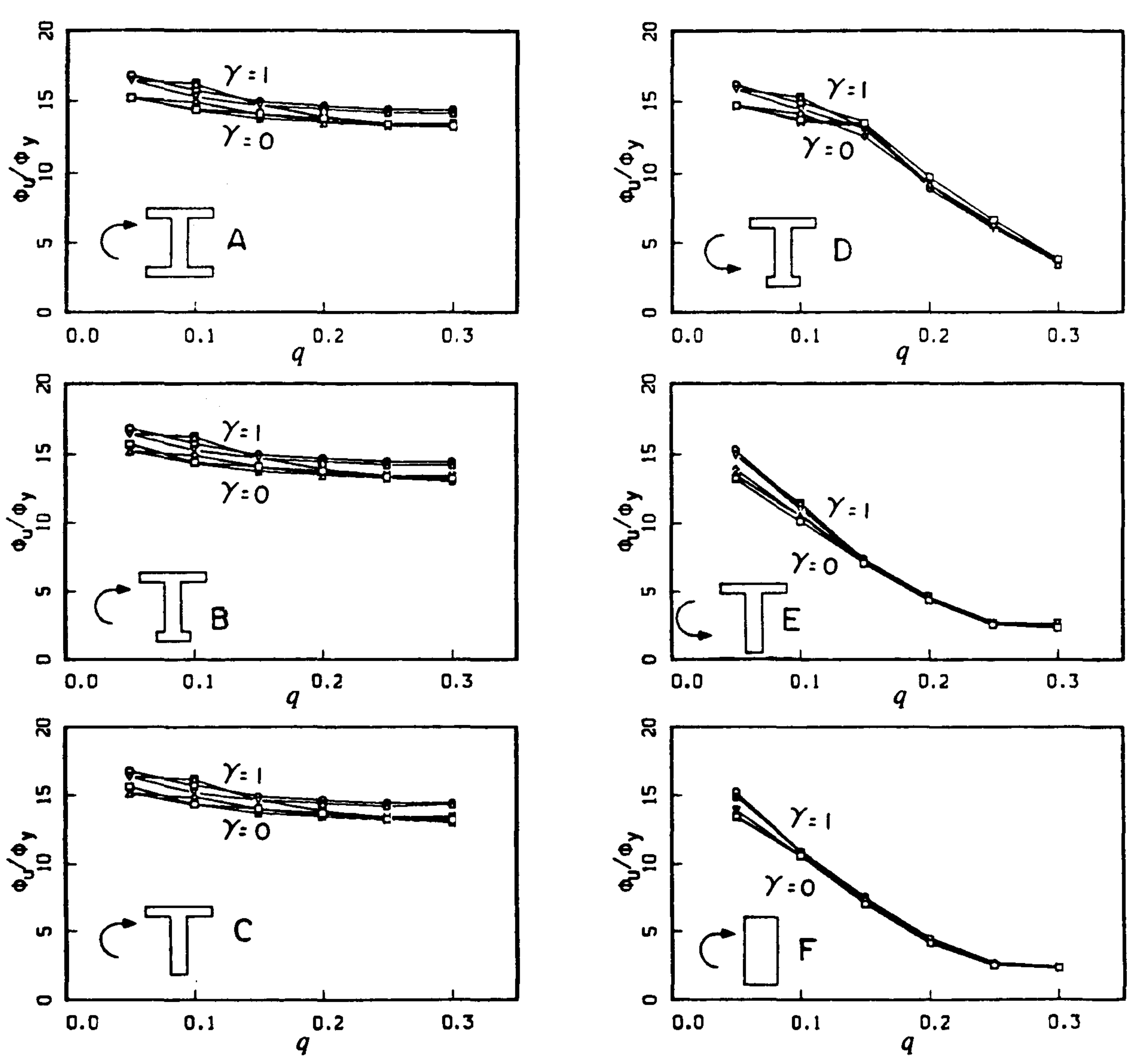


Fig. 7. Effect of mixed reinforcement index on the ductility factor vs. mechanical steel percentage relationship.

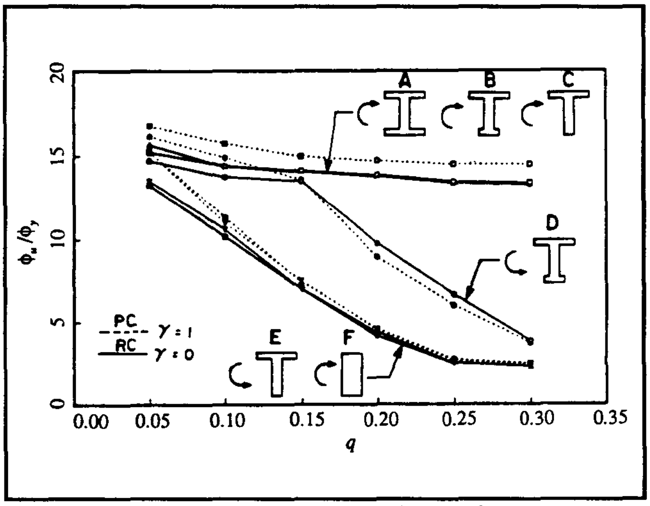


Fig. 8. Effect of section shape on the ductility factor.

duced by increasing q values. Section D shows a behavior intermediate between the two preceding cases. More precisely, for $q \leq 0.15$ the ultimate limit state is characterized by steel failure and the ductility factor has a trend similar to Sections A, B and C. For $q > 0.15$ failure is governed by concrete crushing, resulting in a behavior analogous to Sections E and F.

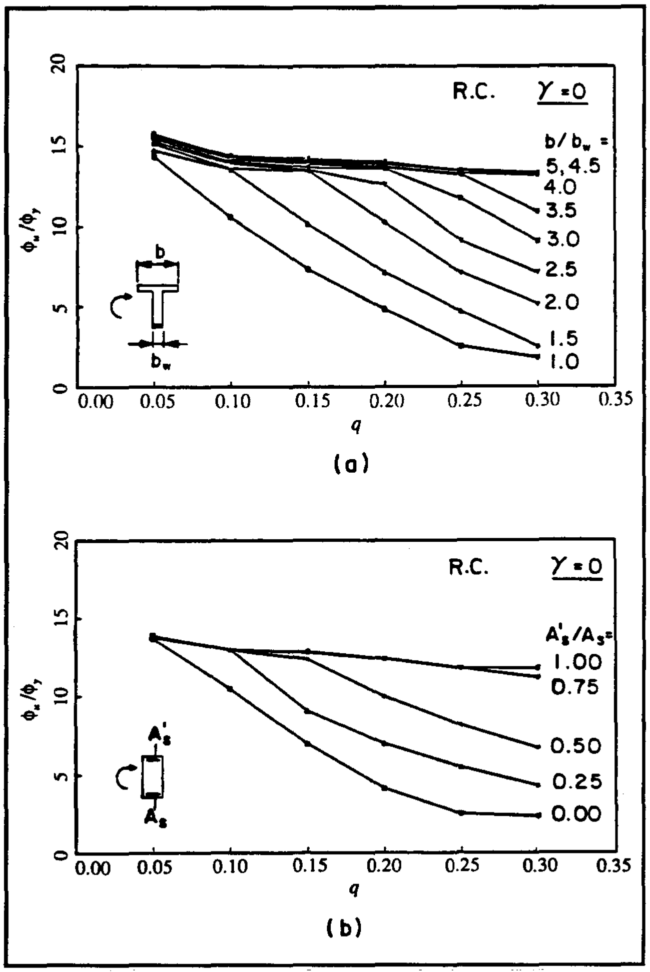


Fig. 9. Ductility factor vs. reinforcement index variation for RC and PC sections: (a) effect of flange to web width ratio; (b) effect of compression reinforcement.

Influence of Flange Width and Compression Reinforcement

A set of numerical tests was performed on a T-section and on a doubly reinforced rectangular section (Sections C and F, Table 4) to assess and compare the influence of A_s'/A_s on the ductility factor ϕ_u/ϕ_y. Only the case $\gamma = 0$ has been studied, as previous results (Fig. 7) have indicated that γ has little influence on the ductility and that the ductility factor of RC sections gives a conservative estimate of ϕ_u/ϕ_y for any value of γ. Figs. 9a and 9b summarize the effects of b/b_w and A_s'/A_s on ϕ_u/ϕ_y.

Fig. 9a shows that increasing the size of the compression flange increases the ductility of the section. For $b/b_w = 1.0$ the ductility factor varies hyperbolically with q. Increasing b/b_w to 4 results in the ductility factor becoming almost constant and independent of q ($\phi_u/\phi_y \approx 14$). Values of b/b_w higher than 4 do not affect the ϕ_u/ϕ_y ratios.

The progressive change of behavior with the increase of b/b_w is related to the type of failure that characterizes the ultimate limit state. For any considered q value, the ultimate limit state corresponds to the concrete crushing for $b/b_w = 1$ and to the steel reaching its limit strain for $b/b_w \geq 4$. For b/b_w values between 1 and 4, the ultimate limit state corresponds to the steel reaching its limit strain for low q values and to the concrete crushing for high q values.

Comparison of Figs. 9a and 9b demonstrates that the effect of A_s'/A_s on the ductility is similar to that of b/b_w. As expected, the ductility factor increases A_s'/A_s values, and becomes almost constant for $A_s'/A_s \geq 0.75$. From Fig. 9a we note that for $A_s'/A_s = 0.25$ the ductility factor ϕ_u/ϕ_y is always higher than 5, even at rather high q (ρ) values ($q = 0.25$ or $\rho = 2$ percent).

Influence of the Analytical Model

A set of tests with different analytical models was performed to compare results based on the ACI 318-83[11] and CEB-MC 78[13] assumptions and definitions with those in this investigation.

The ACI Code allows the use of any realistic material law for the nonlinear analysis of concrete sections. The code only prescribes a maximum allowable concrete strain $\varepsilon_{cu} = 0.3$ percent and suggests an unlimited elastic perfectly plastic behavior for reinforcing steel. Accordingly, a comparison with the ACI standard model was performed using for concrete under compression and prestressing steel the same constitutive laws adopted throughout this study, neglecting reinforcing steel strain hardening, and limiting the concrete ultimate strain to 0.3 percent.

According to CEB-MC 78, a parabola-rectangle and an elastic-plastic stress-strain relationship for concrete and mild steel, respectively, were adopted. For the prestressing steel the material law given in the CEB-MC 78 was adopted. In both the CEB and ACI models the tension stiffening effect is neglected.

Only the I and rectangular sections (A and F in Table 4) were analyzed for the cases in which prestressing is either

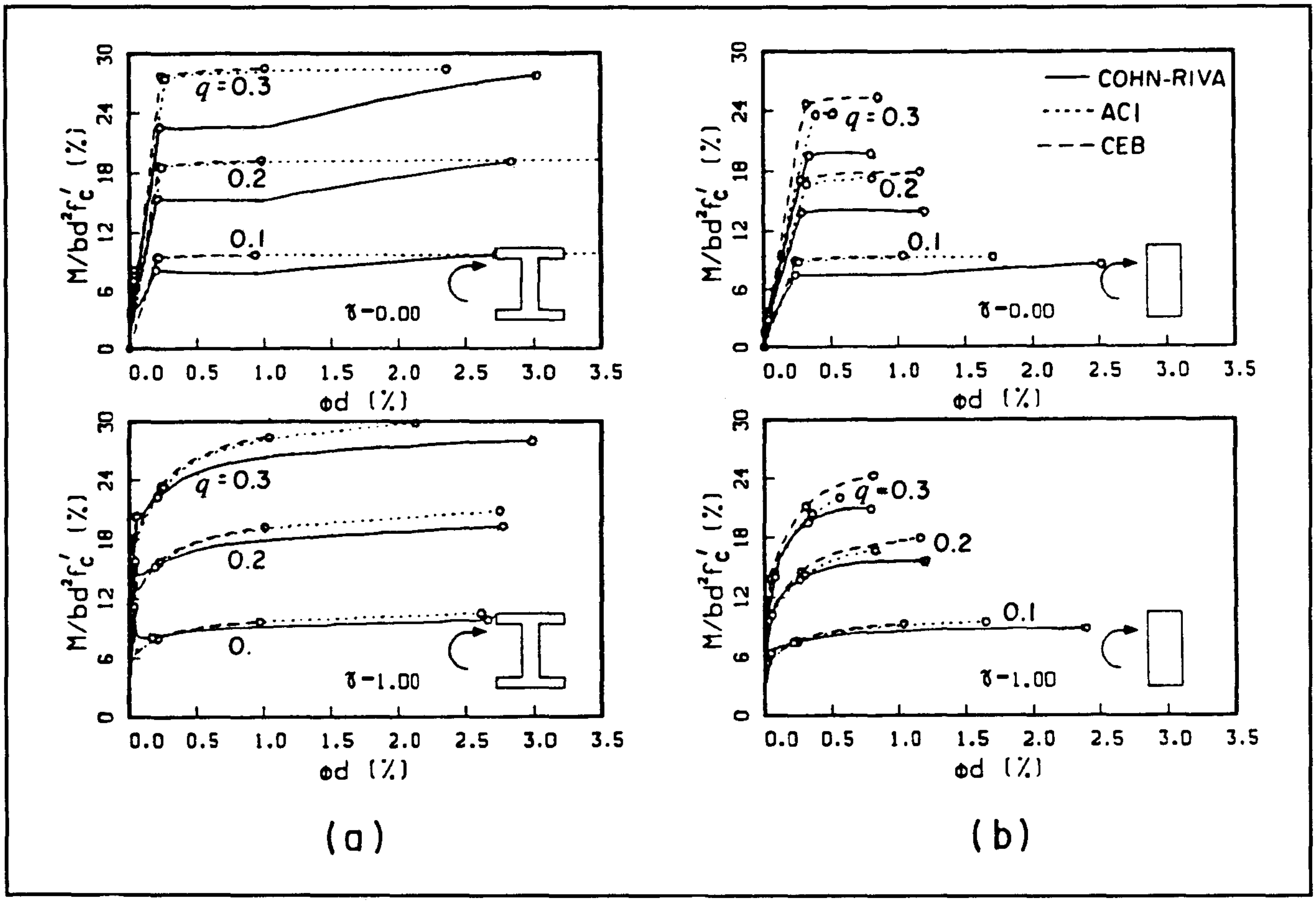


Fig. 10. Moment-curvature relationships according to models by Cohn-Riva, ACI and CEB codes:
(a) I-section; (b) rectangular section.

full or absent ($\gamma = 1$ or $\gamma = 0$, respectively). To obtain comparable moment and curvature results as functions of q (or ω) and γ, the reinforcement area for each section was determined by defining ω for the ACI and CEB models with respect to the web width b_w. The moment-curvature plots in Fig. 10 and the ductility factor plots in Fig. 11 suggest the following remarks:

- The ultimate moments according to ACI 318-83 and CEB-MC 78 models are generally in good agreement. Some differences are noted for prestressed sections where the ultimate moment according to ACI is slightly larger because no limitations are imposed on the steel strain.
- Comparing the ultimate moments according to the ACI and CEB codes and those resulting from the proposed analytical model, two different cases can be identified. For sections whose ultimate limit state corresponds to the steel reaching its limit strain, as defined in this study, the ultimate moments are in good agreement with the code values

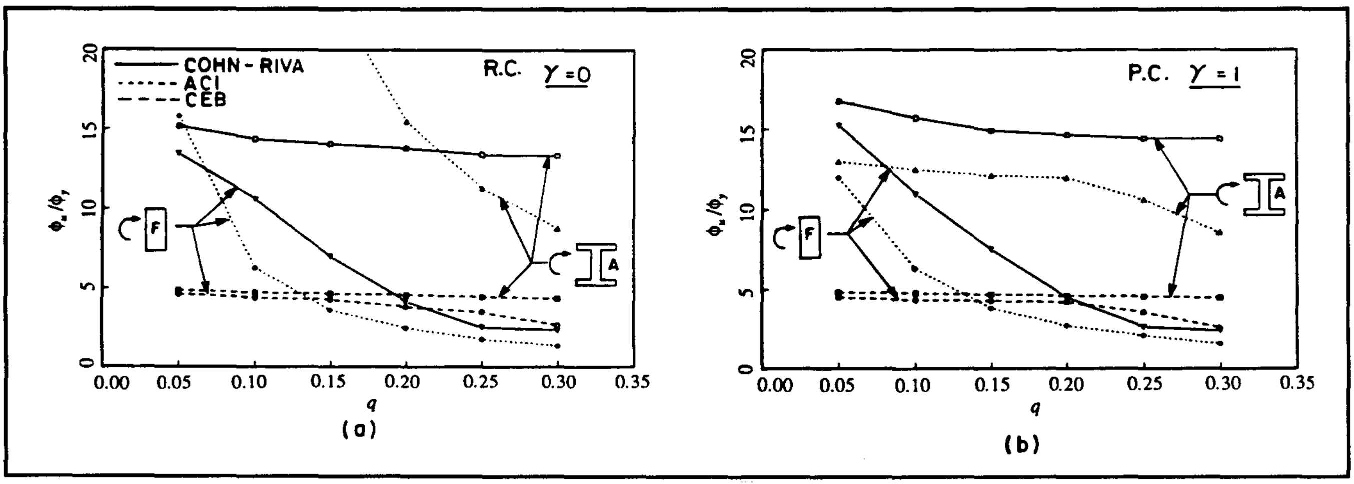


Fig. 11. Ductility factor vs. reinforcement index relationships according to models by Cohn-Riva, ACI and CEB codes:
(a) RC sections; (b) PC sections.

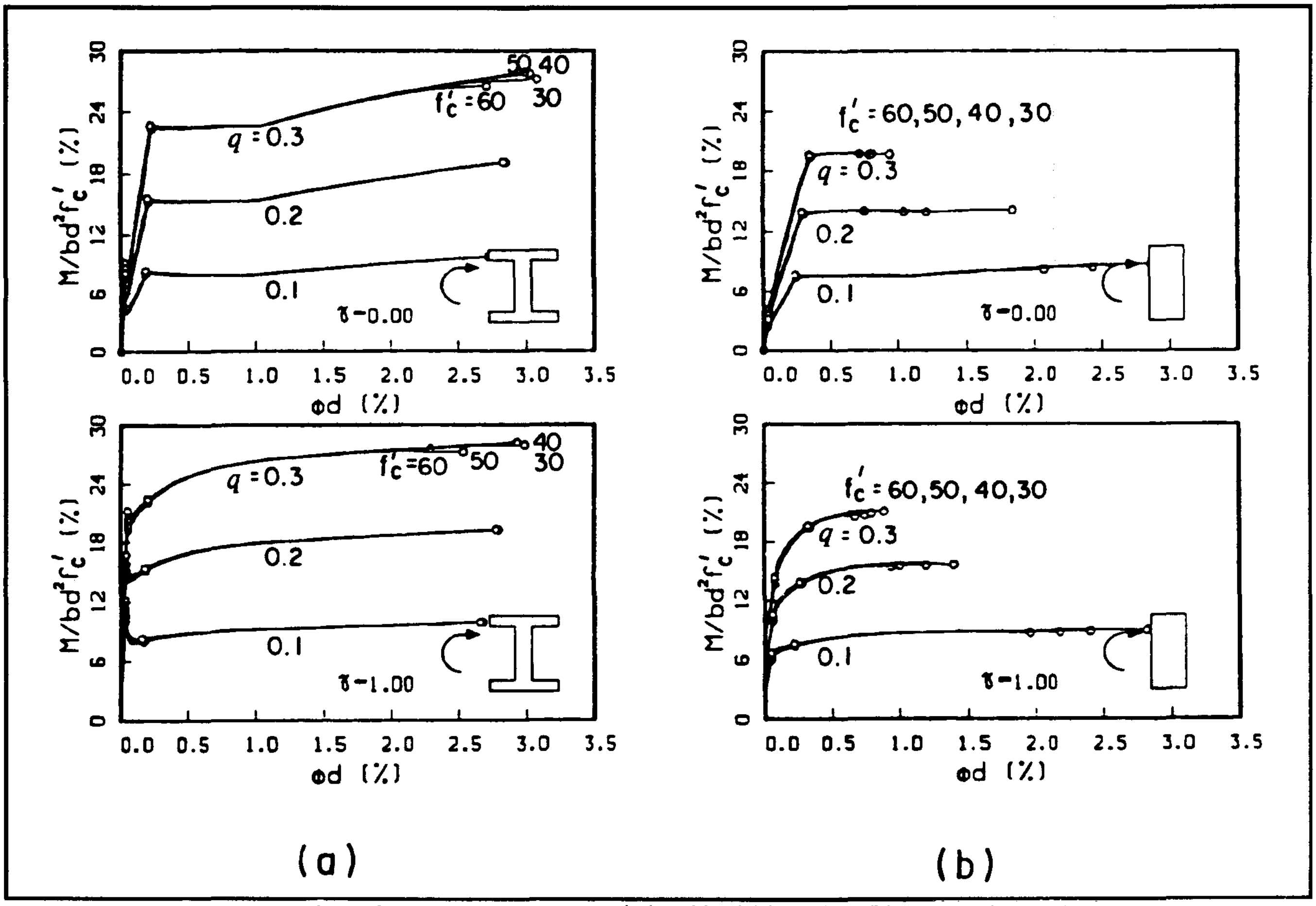


Fig. 12. Effect of concrete grade on the moment-curvature relationship: (a) I-section; (b) rectangular section.

(Sections A and F with q or $\omega = 0.10$). However, for sections whose ultimate limit state is governed by concrete crushing (Section F with q or $\omega > 0.10$), the ultimate moments resulting from analysis with the proposed model are lower than those given by analyses according to the ACI and CEB models.

These differences are mainly due to the adopted material laws and the definition of q. The main difference between the definitions of ω given by the ACI and CEB codes and the definition of q adopted herein (Table 2) is that, while both codes define ω with respect to the yield stress of the steel, we define q with respect to a higher stress value, equal to the stress at the proposed limiting strain. Hence, for an RC section with a given steel area we can write:

$$\omega = \frac{A_s f_y}{b_w d f_c'} = \frac{f_y}{f_{sl}} q \approx 0.8\, q \qquad (4)$$

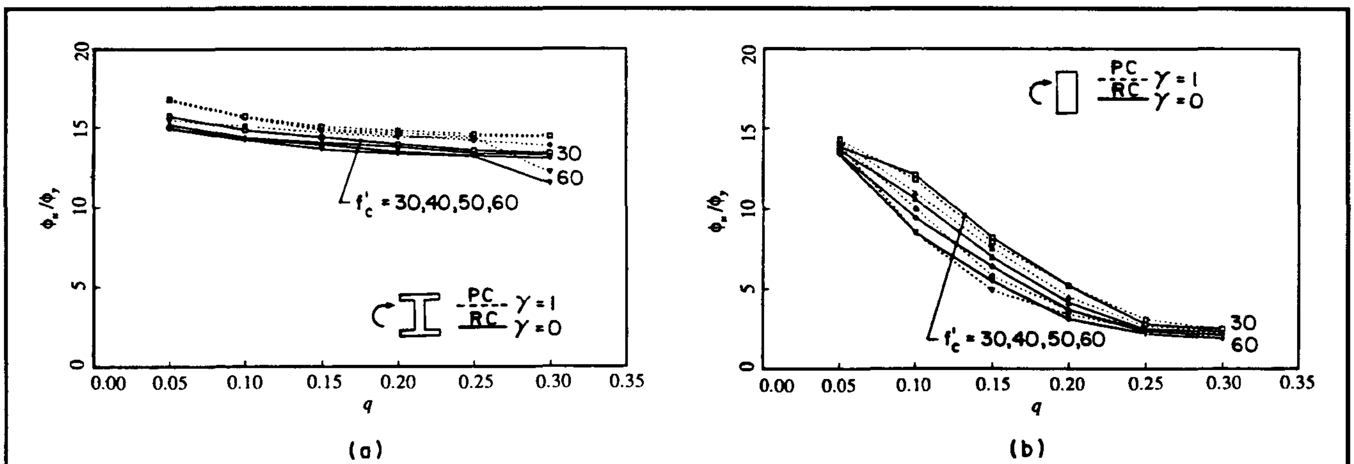


Fig. 13. Effect of concrete grade on concrete and reinforcing steel strains at the ultimate limit state: (a) $f_c' = 30$ MPa (4.35 ksi); (b) $f_c' = 60$ MPa (8.70 ksi).

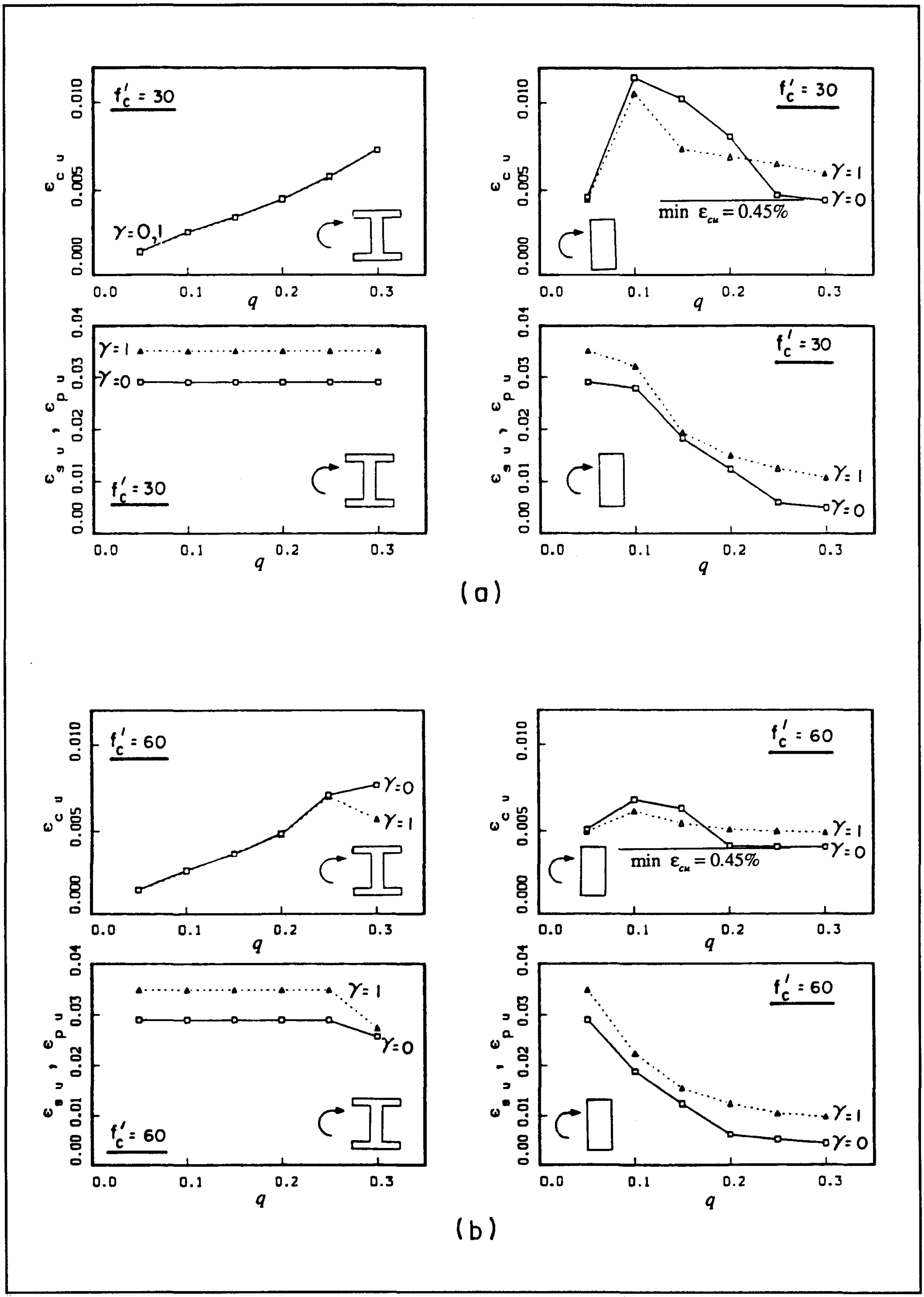


Fig. 14. Effect of concrete grade on the ductility factor: (a) I-section; (b) rectangular section.

Thus, if two sections are such that q for the former is equal to ω of the latter, the former will have less reinforcing steel, and, if $\varepsilon_{su} < \varepsilon_{sl}$, its ultimate moment will be lower.

- M_y values according to the CEB and ACI formulations are in good agreement. The M_y value determined with the proposed analytical model is always lower because of the lower amount of steel provided to the section if $q = \omega$.
- The yielding curvature is not much affected by the adoption of different analytical models or definitions of the reinforcement index.
- Some major differences between the ultimate curvatures (Fig. 10) and ductility factors (Fig. 11), obtained by adopting the ACI or CEB recommendations or the proposed analytical models, are noted.

The material models and limiting strain values (Figs. 10 and 11) of the ACI 318-83 Code[11] result in ductility factors inferior to the theoretical ϕ_u/ϕ_y values for all cases, except for RC I-sections and $q < 0.22$. In this case, the assumed infinite plastic behavior of the reinforcing steel results in considerable overestimates of ϕ_u/ϕ_y (Fig. 11).

The CEB-MC 78 model[13] leads to gross underestimates of ductility factors for rectangular and I-sections of both RC and PC sections with (practical) reinforcement indices $q \leq 0.20$ (Fig. 11) because of the limiting concrete strain $\varepsilon_{cu} = 0.35$ percent and steel strain $\varepsilon_{sl} = 1.0$ percent. Code recommended limiting strains are realistic for strength calculations, but are excessively conservative for deformation analyses. This is particularly true for I-sections or lightly reinforced rectangular sections, when the actual failure takes place for steel strains well beyond the CEB limit of $\varepsilon_s = 1.0$ percent.

The greater ductility present in highly reinforced sections is due to the adopted concrete law, which considers a plastic (rather than a strain softening) behavior of concrete in compression. This assumption may be considered acceptable, since it influences only highly reinforced sections, which have a limited application in engineering practice.

Influence of the Concrete Compression Strength

The influence of f_c' on the M-ϕ law and the section ductility ϕ_u/ϕ_y has been studied on the I and rectangular sections (A and F in Table 3). The results are plotted in Figs. 12-14 and suggest the following remarks:

- The effect of concrete grade on the M-ϕ curves is relatively minor, although higher grades tend to reduce the ultimate concrete strains, particularly at low steel percentages (Figs. 12, 13a and 13b). The concrete grade does not significantly affect the ductility of I-sections, but has some impact on that of rectangular sections (Fig. 14). It is interesting to note that for increasing f_c' values, the ductility factor ϕ_u/ϕ_y decreases under a constant q value, but increases under a constant ρ value, as independent analytical

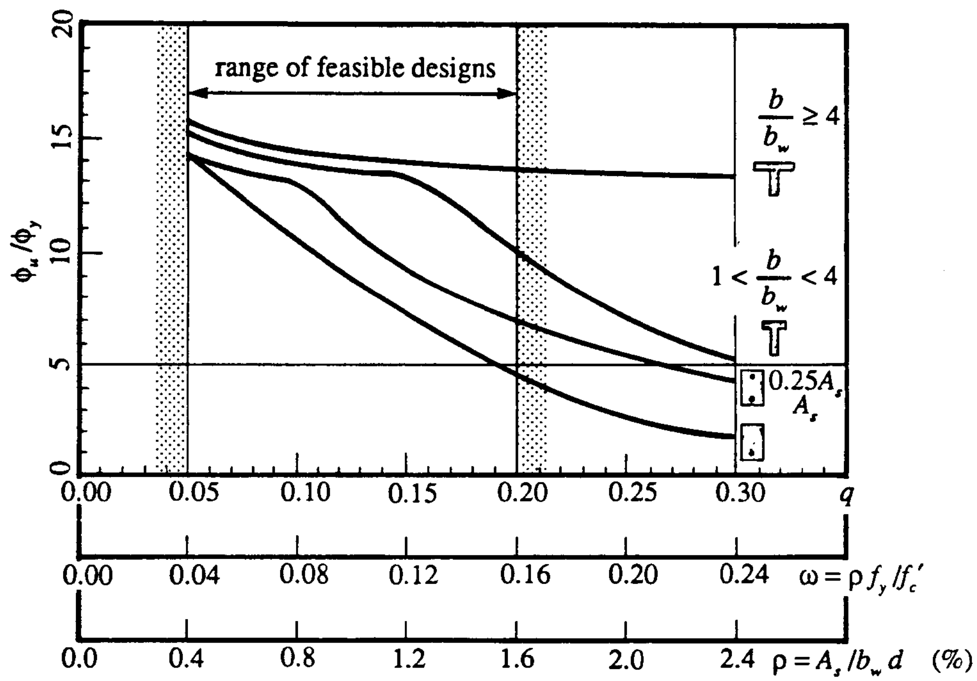


Fig. 15. Combined effects of section shape and compression reinforcement on the ductility factor [for $f_c' = 40$ MPa (5.8 ksi) and $f_y = 400$ MPa (58 ksi)].

studies have shown.[3,4] This apparently conflicting result is explained by observing that, for an RC section, $q = (f_{sl}/f_c')\,\rho$. Hence, to keep a constant q value for increasing f_c' values, it is necessary to increase the steel percentage ρ of the section, with a resulting overall decrease of flexural ductility.

- The concrete strength has a negligible effect on M_y and M_u, while ϕ_y and ϕ_u decrease for increasing values of f_c'. For a given q value, the reduction in yielding curvatures is negligible, but the reduction in ultimate curvatures is relevant for sections failing by concrete crushing [for RC rectangular sections and $f_c' = 60$ MPa (8.70 ksi)], ϕ_u can be up to 30 percent smaller than for $f_c' = 30$ MPa (4.35 ksi). Since the ultimate curvature decreases with increasing f_c', the ductility factor is also proportionally reduced [up to 40 percent of the difference between ϕ_u/ϕ_y for $f_c' = 30$ MPa (4.35 ksi) and $f_c' = 60$ MPa (8.70 ksi)], as shown in Fig. 14b.
- For the rectangular section, the ultimate concrete strain ε_{cu} is always larger than 0.45 percent and can be as high as 1 percent ($q = 0.10$ and $f_c' = 30$ MPa, Fig. 13a). This is due to the existence of an extended strain softening branch in the σ_c-ε_c law.

Ductility Factors and Moment Redistribution

Fig. 15 summarizes the effects of q, compression flange to web width ratio b/b_w, and compression to tension steel ratio A_s'/A_s on the ductility factor ϕ_u/ϕ_y for $f_c' = 40$ MPa (5.8 ksi) and $f_y = 400$ MPa (58 ksi), and determines the range of feasible design solutions from the viewpoint of section ductility. Although the curves refer to RC sections, results may be considered safe lower bounds for any structural concrete section.

Fig. 15 shows that for sections with a reinforcement index $q \le 0.20$ ($\rho \le 1.6$ percent) and $A_s'/A_s \ge 0.25$ (which are common practical solutions), the ductility factor ϕ_u/ϕ_y is always larger than 5. Ductility factors of at least 14 are ensured for $0.05 \le q \le 0.20$ (or $0.4 \le \rho \le 1.6$ percent). For RC sections still higher values may be obtained removing the limitation imposed on the maximum allowable steel strain ($\varepsilon_{sl} = 3.0$ percent).

This is an important finding with regard to the amounts of moment redistribution permissible in design. As known, Sections 8.4 and 18.10 of ACI-318[11] link allowable moment redistribution to ρ, ρ', ρ_b for RC and ω, ω', ω_b for PC.

Theoretically, the maximum amount of permissible moment redistribution y_θ may be determined from earlier investigations[21] as:

$$y_\theta = (\phi_u/\phi_y - 1)/(15 + \phi_u/\phi_y - 1)$$

i.e., as a direct function of the ductility ratio.

It can be seen that for values ϕ_u/ϕ_y of at least 5 (i.e., $q \le 0.20$), $y_\theta \ge 0.21$ follows.[21] Accordingly, the ACI 20 percent maximum redistribution is always safe for any RC or PC section shape with $q \le 0.20$ ($\rho \le 1.6$ percent).

A similar conclusion is obtained from an independent extensive computer investigation on the effective moment redistribution of structural concrete frameworks.[22]

CONCLUSIONS

A comprehensive analytical investigation of RC, PPC and PC structural concrete members[3] has attempted to eliminate some inconsistencies and limitations of code based behavior models by: (1) adopting realistic material laws, (2) considering a continuous description of all behavior states, (3) using a unified formulation for RC, PPC and PC, and (4) considering the effects of all governing factors. Application of the MOCURO computer program[19] has enabled the automatic calculation of ductility factors and a parametric study on a large set of concrete sections.

Analysis of related results presented in this study supports the following conclusions:

1. Prestressing induces a relatively flat response in the inelastic range and, therefore, plastic analysis methods are applicable to continuous PC structures.

2. Prestressing has a positive effect on the section ductility: ϕ_u/ϕ_y increases with the amount of prestressing (γ). Hence, ductility factors of RC members may be used as conservative estimates for the corresponding PC and PPC members.

3. Typical prestressed beams I, boxed, and flanged sections (with $b/b_w \ge 4$) have a ductility factor of at least 14.

4. Moment redistribution of at least 20 percent is ensured for sections of any shape and material combinations, provided the reinforcement percentage, ρ, for RC, or reinforcement index, ω, for PC and PPC, do not exceed 1.6 percent and 0.16, respectively.

5. Sections 8.4 and 18.10.4 of ACI 318-83 Code[11] could be modified as follows:

- **8.4.1** and **18.10.4.1** . . . "negative moments" . . . increased or decreased by not more than 20 percent.
- **8.4.2** and **18.10.4.2** unchanged.
- **8.4.3** . . . ρ or $\rho - \rho'$ is not greater than 1.6 percent or b/b_w is at least 2 for flanged sections.
- **18.10.4.3** . . . $\omega + \omega_p - \omega'$ is not greater than 0.16 or b/b_w is at least 2 for flanged sections.

6. The above conclusions ensure the possibility of moment redistribution, even for fully prestressed concrete members ($\gamma = 1$). However, design solutions based on redistributed moments are only possible if cracking under service loads is permissible,[23] i.e., moment redistribution should be restricted to reinforced or partially prestressed concrete structures only.

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APPENDIX A — NOTATION

The following symbols are used in this paper in addition to standard ACI 318-89 notation:

f_{sl} = reinforcing steel stress at assumed limiting strain
l_c = crack spacing
M = bending moment referred to centroid of gross cross section
M_u = ultimate moment
M_y = yielding moment
$q = \dfrac{A_p f_{pu} - A_{pl} f_{sl}}{b_w d f'_c}$ = reinforcement index
s_p = prestressing steel slip
s_s = reinforcing steel slip
$\gamma = \dfrac{A_p f_{pu}}{A_p f_{pu} + A_s f_{sl}}$ = mixed reinforcement index
$\Delta \varepsilon_p$ = prestressing strain increment
$\Delta \varepsilon_s$ = reinforcing steel increment
ε_{pu} = prestressing steel ultimate strain
ε_{sl} = 3 percent = assumed reinforcing steel limiting strain at ultimate limit state
ε_{su} = reinforcing steel ultimate strain
$\overline{\kappa}$ = degree of prestressing
σ_c = concrete stress
σ_p = prestressing steel stress
σ_s = reinforcing steel stress
τ_p = prestressing steel bond stress
τ_s = reinforcing steel bond stress
ϕ = curvature
ϕ_u/ϕ_y = ductility factor