Transfer and Development Length of Epoxy Coated and Uncoated Prestressing Strand

Thomas E. Cousins
Assistant Professor
Department of Civil Engineering
Auburn University
Auburn, Alabama

David W. Johnston
Professor
Department of Civil Engineering
North Carolina State University
Raleigh, North Carolina

Paul Zia
Distinguished University Professor
Department of Civil Engineering
North Carolina State University
Raleigh, North Carolina

This paper presents the development of analytical equations for transfer length, flexural bond length, and development length of pretensioned strand in prestressed concrete, and compares them to experimental results in the literature. The transfer length equation assumes a plastic zone and an elastic zone within the transfer length. Similarly, the flexural bond length equation assumes plastic and elastic zones, and the development length is the sum of the transfer length and flexural bond length. The analysis considers both uncoated and epoxy coated strand. A numerical example is included to show a typical calculation for the development length of epoxy coated prestressing strand.

In pretensioned concrete, the force in the prestressing strand is transferred to the concrete by bond in the end region of the member. The distance from the end of the member over which the effective prestressing force is developed is called the transfer length. The flexural bond length is the additional bond length required to develop the strand stress from the effective prestress to ultimate stress at the ultimate flexural strength of the member. The development length is the sum of the transfer length and the flexural bond length.

Several experimental and analytical investigations of the transfer length and flexural bond length of prestressing strand have been conducted over the years. Based on these investigations, various empirical equations have been proposed for estimating the transfer and development lengths. This paper presents the development of new analytical equations for transfer length, flexural bond length, and development length, and compares them to experimental results found in the literature. The analysis considers both uncoated and epoxy coated strand.[1-3]

RESEARCH SIGNIFICANCE

Several equations for transfer and development lengths of uncoated prestressing strand have been presented in the literature. These are generally empirical equations and are primarily based on interpretations of experimental research conducted prior to 1965. This paper presents the derivation of new equations based upon an elastic-plastic model and compares the predictions to both recent and earlier experimental research findings.

In addition, this paper proposes equations to predict the transfer and development lengths of epoxy coated prestressing strand. The earlier equations did not consider the effect of the epoxy coating on transfer length and development length. The proposed equations are applicable to both uncoated and epoxy coated prestressing strand.

PREVIOUS METHODS

Based on test results by Kaar, LaFraugh, and Mass[4] for transfer length, and Hanson and Kaar[5] for flexural bond length, the American Concrete Institute[6] recommends an empirical equation for development length, L_d, as follows:

$$L_d = \left[\frac{f_{se}}{3} + (f_{ps} - f_{se}) \right] d_b \qquad (1)$$

where

f_{se} = effective stress in prestressing steel after losses, ksi

d_b = nominal diameter of prestressing strand, in.

f_{ps} = stress in prestressed reinforcement at nominal strength, ksi

The first term in the equation is the transfer length and the second term is the flexural bond length. This equation first appeared in the ACI Code in 1963 and remains as the basis for design today.

Zia and Mostafa[7] developed empirical equations for transfer length and flexural bond length of prestressing strand based on a linear regression analysis of available research data published before 1977. The proposed equation for transfer length, L_t, was:

$$L_t = [1.5\,(f_{si}/f_{ci})\,d_b] - 4.6 \qquad (2a)$$

and for flexural bond length, L_{fb}, was:

$$L_{fb} = 1.25\,(f_{su} - f_{se})\,d_b \qquad (2b)$$

where

f_{si} = initial stress in strand before losses, ksi

f_{ci} = compressive strength of concrete at transfer, ksi

d_b = nominal diameter of prestressing strand, in.

f_{su} = ultimate strength of prestressing strand, ksi

f_{se} = effective stress in prestressing strand after losses, ksi

Zia and Mostafa's equation for transfer length allows for adjustment for different concrete strengths at release, different strand sizes, and different initial prestressing. This equation is more conservative than the ACI equation for larger strand sizes, but gives similar results for smaller size strands. Their equation for flexural bond length is based on a re-evaluation of test results from Hanson and Kaar.[5] Zia and Mostafa concluded that the flexural bond length given in the ACI Code needed to be increased 25 percent.

Martin and Scott[8] re-evaluated the available test results and proposed a transfer length of 80 strand diameters for all sizes of strand, which is considerably more conservative than the ACI Code equation.

PROPOSED ANALYTICAL MODEL FOR TRANSFER LENGTH

In a pretensioned concrete beam, the force in the strand is transferred to the concrete after the concrete has gained sufficient compressive strength to withstand the stresses from the prestressing force. The stress in the concrete at the free end is zero, and it increases over the transfer length as shown in Fig. 1. The strand is held in tension by the concrete, and the force in the steel must be equal in magnitude and opposite in direction to the force in the concrete at any particular location.

Within the transfer length, the steel stress varies from zero to the effective prestress (f_{se}). Also, the transfer of stress from the steel to the concrete indicates the existence of bond stresses between the strand and concrete within the transfer length. The bond stress, $u(l)$, is related to the steel stress, f, by the following equation:

$$u(l) = \left(\frac{df}{dl} \right) \left(\frac{A_s}{\pi d} \right) \qquad (3)$$

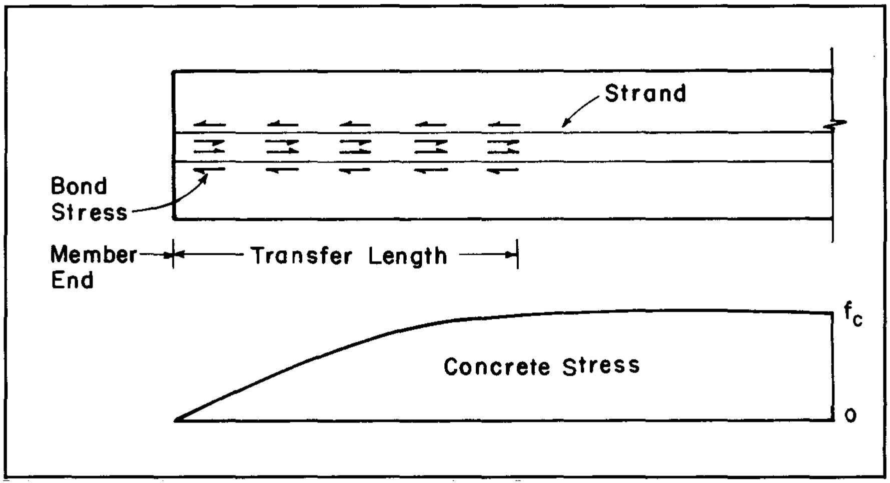

Fig. 1. Stress transfer from strand to concrete.

where

A_s = cross-sectional area of strand
d = nominal diameter of strand
l = length measured from free end
df/dl = slope of steel stress vs. length relationship at any distance l from the end of a specimen

Therefore, the bond stress at any point along the beam is simply the slope of the steel stress vs. length curve at that point, multiplied by a constant.

Guyon[9] suggested a model of the bond region based on the following assumptions:

(1) For small displacements of the strand relative to the concrete, bond stress is proportional to slip.
(2) At the end of the proportional section, the bond stress maintains a maximum or yield value.

For the transfer length model developed in this paper, assumptions similar to Guyon's will be made. For small displacements of the strand relative to the concrete, bond stress will be considered proportional to slip and this region will be called the elastic zone. From the elastic zone to the end of the member, the bond stress maintains a maximum or yield value and this region will be called the plastic zone. Fig. 2 shows the end of a prestressed beam with the elastic and plastic zones labeled and a plot of the idealized bond stress [$u(l)$].

Eq. (3) implies that the plastic transfer bond stress (U_t) is proportional to the slope of the linear portion of the curve for steel stress vs. distance. A characteristic U_t for each strand type can be obtained from the experimental results as well as the slope, B, of the bond stress curve [$u(l)$] in the elastic zone. Hence, the length of the elastic zone is:

$$L_{te} = \frac{U_t}{B} \qquad (4)$$

where

B = bond modulus (slope of bond stress curve in the elastic zone), psi/in.
L_{te} = length of elastic zone of transfer length, in.

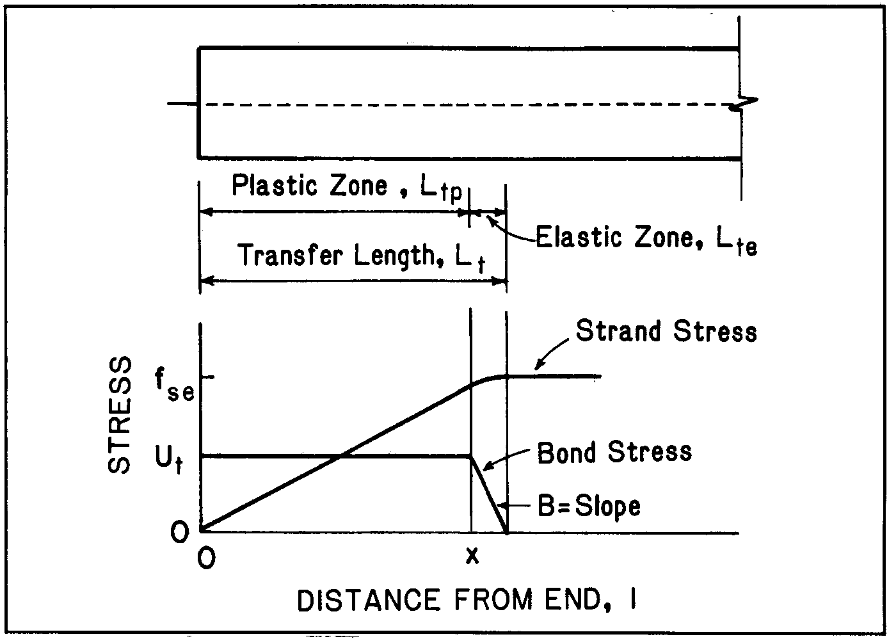

Fig. 2. Assumptions of transfer length model.

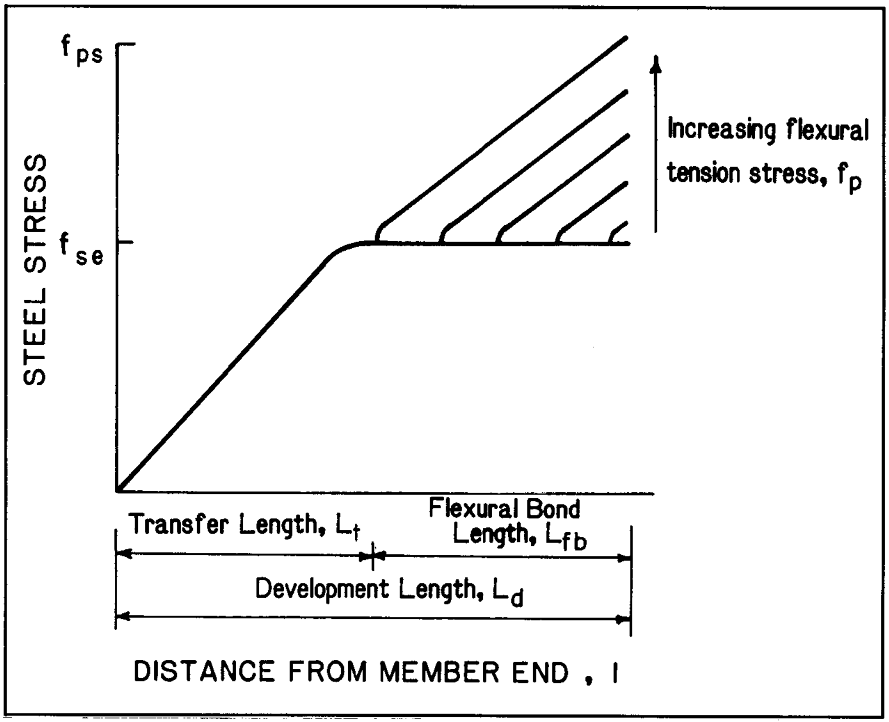

Fig. 3. Assumptions of development length model.

In Fig. 2, the steel stress at point x, f_{px}, is f_{se} minus the area under $u(l)$ in the elastic zone, or:

$$f_{px} = f_{se} - 0.5\left(\frac{U_t}{B}\right)\left(\frac{\pi d}{A_s}\right) \qquad (5)$$

The area under the $u(l)$ curve in the plastic zone should equal the prestress force at x, F_{px}, as shown below:

$$F_{px} = L_{tp}\,\pi d\,U_t \qquad (6)$$

where L_{tp} is the length of the plastic zone. Therefore, the equation for the length of the plastic zone is:

$$L_{tp} = \frac{f_{se}A_s}{\pi d\,U_t} - 0.5\left(\frac{U_t}{B}\right) \qquad (7)$$

The complete equation for the entire transfer length is:

$$L_t = 0.5\left(\frac{U_t}{B}\right) + \frac{f_{se}A_s}{\pi d\,U_t} \qquad (8)$$

This equation for transfer length

appears to be independent of the concrete compressive strength (f_c'). However, research on bond has suggested that bond strength is proportional to $\sqrt{f_c'}$. Thus, U_t can be redefined as $U_t'\sqrt{f_{ci}'}$, where f_{ci}' is the concrete compressive strength at transfer. Then, the equation for transfer length becomes:

$$L_t = 0.5\left(\frac{U_t'\sqrt{f_{ci}'}}{B}\right) + \frac{f_{se}A_s}{\pi d\, U_t'\sqrt{f_{ci}'}} \quad (9)$$

The above equations will be applied to the experimental results, and compared to other equations for transfer length found in the literature.

ANALYTICAL MODEL FOR DEVELOPMENT LENGTH

The development length is the sum of the transfer length and flexural bond length, the latter being the length beyond the transfer length required to develop the ultimate flexural strength of the member. The variation of the steel stress over the development length at ultimate moment conditions of a pretensioned member is idealized in Fig. 3.

The flexural bond length is assumed to have elastic and plastic zones of bond stress similar to the elastic and plastic zones in the transfer length model. As the applied moment increases, the stress in the strand increases and proceeds towards the end of the transfer length as a wave (Fig. 3).

When the wave reaches the transfer length, a general bond slip failure occurs. However, a flexural failure would occur if the ultimate moment capacity of a section is reached before the bond stress wave reaches the transfer length. The leading edge of the wave is the region of elastic bond stress. The plastic zone extends from the end of the elastic zone to the point of maximum moment.

From observations of transfer length data, the elastic region was determined to be relatively short. Thus, the plastic zone will be assumed to cover the entire flexural bond length. At failure, the increase in the strand force would be resisted by the plastic bond stress over the flexural bond length. From equilibrium of force:

$$(f_{ps}-f_{se})A_s = (U_d)\pi d(L_{fb}) \quad (10)$$

Then the flexural bond length is given by:

$$L_{fb} = (f_{ps} - f_{se})\left(\frac{A_s/\pi d}{U_d}\right) \quad (11)$$

or restating the equation as a function of f_c':

$$L_{fb} = (f_{ps} - f_{se})\left(\frac{A_s/\pi d}{U_d'\sqrt{f_c'}}\right) \quad (12)$$

where

L_{fb} = flexural bond length, in.

f_{ps} = stress in strand at flexural failure, ksi

f_{se} = effective prestress, psi

U_d = plastic bond stress for development, psi

U_d' = $U_d\sqrt{f_c'}$, psi

d = nominal strand diameter, in.

A_s = area of prestressing strand, sq in.

Combining Eqs. (9) and (12), the development length is $L_t + L_{fb}$, or:

$$L_d = 0.5\left(\frac{U_t'\sqrt{f_{ci}'}}{B}\right) + \left(\frac{f_{se}A_s}{\pi d\, U_t'\sqrt{f_{ci}'}}\right) + (f_{ps} - f_{se})\left(\frac{A_s/\pi d}{U_d'\sqrt{f_c'}}\right) \quad (13)$$

In the above equations, the plastic bond stresses U_t and U_d are assumed to be inherently different. In the transfer length, the strand stress is reduced at release and the strand diameter is enlarging, creating more friction. While in the flexural bond length, the strand stress is increased due to applied load and the Poisson effect reduces the strand diameter causing a reduction in friction.

APPLICATION OF TRANSFER LENGTH MODEL TO EXPERIMENTAL RESULTS

The transfer length model will first be applied to the results of the authors' research program which have been reported previously.[1-3] This research program included an experimental investigation of transfer length and flexural bond length of epoxy coated and uncoated prestressing strand in three diameters [3/8, 1/2, and 0.6 in. (9.5, 12.7, and 16.2 mm)]. The epoxy coated strand has a crushed glass (or grit) embedded in the epoxy coating to improve bonding characteristics.

Fig. 4 shows a comparison of uncoated and epoxy coated strand. By varying the quantity of grit, low grit density (CL), medium grit density (CM), and high grit density (CH) strand were produced in the 1/2 in. (12.7 mm) diameter (Fig. 5) and were included in this study. In the 3/8 and

Fig. 4. Uncoated and coated prestressing strand.

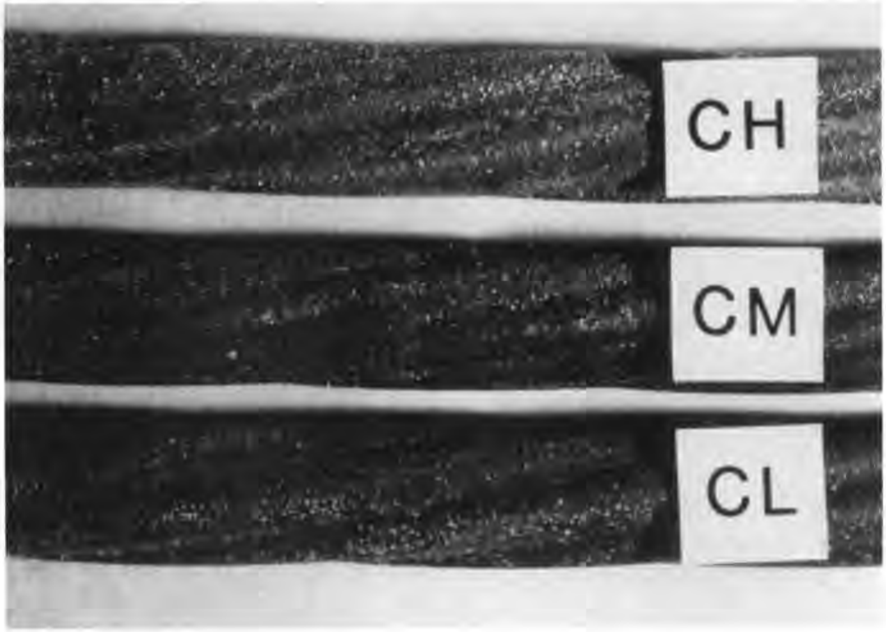

Fig. 5. Grit variation in coated strand.

0.6 in. (9.5 and 16.2 mm) diameter, medium grit density epoxy coated strand was used.

Although the strand was intended to have these three grit densities, a visual inspection of the coated strand suggested that some of the different densities were actually similar. If grouped by visual impressions, these two groups would be:

(1) ½ and ⅜ in. (12.7 and 9.5 mm) diameter medium coated and ½ in. (12.7 mm) diameter high coated strand.

(2) ½ in. (12.7 mm) diameter low coated and 0.6 in. (16.2 mm) diameter medium coated strand.

The latter generally had less grit than the former.

Two types of specimens were fabricated and tested: transfer length specimens and development length specimens. The transfer length specimens were square in cross section with one concentric strand, and the development length specimens were rectangular in cross section with one eccentric strand.

Transfer length results were obtained from both types of specimens by measuring surface strains with a Whittemore type gage and are listed in Tables 1 and 2, respectively. The development length specimens were subsequently tested using an iterative testing scheme to determine development length either in static tests or bond fatigue tests.

The first letter for the specimen identification in Tables 1 and 2 designates the type of specimen as transfer length (T), static development length (S), or bond fatigue development length (F). The number in the specimen identification indicates strand diameter as 3 for ⅜ in. (9.5 mm), 5 for ½ in. (12.7 mm), and 6 for 0.6 in. (16.2 mm). The letter designation at the end of the specimen identification accounts for the duplicates of specimens. Thus, T3UNA is transfer specimen end A with ⅜ in. (9.5 mm) diameter uncoated strand. Transfer length results from both transfer length and development length specimens are used in the following discussion.

The equation developed for transfer length is:

$$L_t = 0.5\left(\frac{U_t}{B}\right) + \frac{f_{se}A_s}{\pi\, d\, U_t} \qquad (8)$$

To calculate the transfer length, U_t, B, and f_{se} must be determined from the experimental data. These were derived using the concrete strain data from the transfer length and development length specimens for each type of strand. The concrete strain for each strand type was plotted from the free end towards the center of each beam so that one plot was obtained for each end of each specimen.

To obtain the effective prestress, the prestress losses were subtracted from the measured stress in the strand immediately before the concrete was placed. Due to equilibrium of forces at any section through the specimen, the shape of the strand stress plot could be approximated from the shape of the concrete strain plot, since the two plots are similar.

Fig. 6 shows an example of the average concrete strain vs. distance for one end of a specimen (T3CMA). Also shown in the figure is the de-

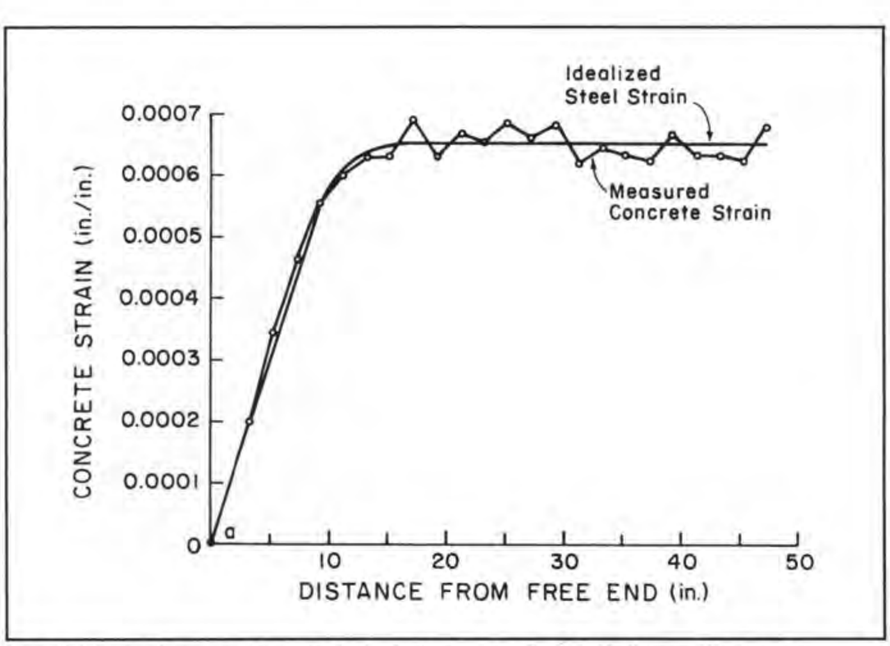

Fig. 6. Example concrete and steel strain vs. length. (Note: 1 in. = 25.4 mm).

rived steel strain superimposed over the concrete strain. In this case for Specimen T3CMA, the effective prestress of 188.3 ksi (1298 MPa) is based on the average concrete strain beyond the transfer length.

The plastic bond stress, U_t, is the slope of the steel stress plot from a to b multiplied by $A_s/\pi d$, and is 1.150 ksi (7.927 MPa). The slope of the bond stress plot within the elastic zone, B, is U_t divided by the length of the elastic zone or 177.0 psi/in. (48.19 MPa/mm).

Table 1 contains values for U_t, B and f_{se} derived in this same manner for each of the transfer length specimens. Similar results from the development length specimens are contained in Tables 2a, b and c. In both tables, the values for f'_{ci} and measured transfer length are for one day after transfer.

Based on the results from the transfer length specimens, the length of the elastic zone of the transfer length was approximately 13 percent of the total transfer length, and the bond stress slope, or modulus, was highly variable.

Therefore, in the calculations of transfer length for Table 2, B is assumed to be the average value from all the transfer length specimens which was 300 psi/in. (81.3 MPa/mm). Since there were different numbers of specimens in each group corresponding to a particular strand diameter and coating type, this is the average of the average B values from each group.

Considering specimens from Tables 1, 2a, 2b and 2c, the values of U_t fall into three ranges. The first range includes 0.6, ½, and ⅜ in. (16.2, 12.7, and 9.5 mm) diameter uncoated strand with average plastic bond stress values of 412, 362, and 419 psi (2841, 2496, and 2889 kPa), respectively. This leads to an average plastic bond stress value of 390 psi (2689 kPa) for uncoated strand.

Table 1. U_t, B, and calculated transfer lengths for transfer length specimens.

Beam end	f'_{ci} (psi)	U_t (psi)	U'_t	B (psi/in.)	f_{se} (ksi)	L_t Measured (in.)	L_t* Calculated (in.)	Measured / Calculated
T3UNA	4190	595	9.2	108.0	187.0	26.0	31.8	0.82
B	4190	368	5.7	0.0	182.8	35.0	31.1	1.13
C	4190	354	5.5	80.0	183.7	36.0	31.3	1.15
D	4190	313	4.8	52.0	184.2	42.0	31.3	1.34
T5UNE	4110	252	3.9	0.0	180.9	74.0	41.7	1.77
F	4110	333	5.2	0.0	180.9	54.0	41.7	1.29
G	4110	272	4.2	0.0	178.6	64.0	41.2	1.55
H	4110	288	4.5	288.0	178.3	58.0	41.1	1.41
T3CMA	4190	1150	17.8	177.0	188.3	13.0	14.5	0.90
B	4190	1240	19.2	349.0	188.9	11.0	14.6	0.75
C	4190	1033	16.0	344.0	185.4	15.0	14.3	1.05
D	4190	1026	15.9	513.0	186.1	13.0	14.4	0.90
T5CMA	4110	1154	18.0	330.0	188.0	17.0	19.1	0.89
B	4110	1178	18.4	0.0	187.5	15.0	19.1	0.79
T5CHA	4110	1276	19.9	0.0	176.9	13.0	18.1	0.72
B	4110	1276	19.9	0.0	176.9	13.0	18.1	0.72
C	4110	1154	18.0	210.0	176.9	19.0	18.1	1.05
D	4110	1059	16.5	177.0	174.3	19.5	17.8	1.10
T5CLA	4110	730	11.4	0.0	176.2	23.0	26.4	0.87
B	4110	563	8.8	282.0	176.9	32.5	26.5	1.23
C	4110	582	9.1	167.0	176.7	27.0	26.4	1.02
D	4110	821	12.8	205.0	176.9	21.0	26.5	0.79
T6CMA	4190	551	8.5	0.0	179.4	37.5	31.2	1.20
B	4190	809	12.5	231.0	179.4	27.0	31.2	0.87
C	4190	713	11.0	57.0	179.0	38.5	31.1	1.24
D	4190	571	8.8	95.0	178.7	37.5	31.1	1.21

Average = 1.07
Standard Deviation = 0.26

*Based on average values of B and U'_t.

Metric (SI) conversion factors: 1 in. = 25.4 mm; 1 ksi = 6.895 MPa; 1 psi = 6.895 kPa; 1 psi/in. = 0.271 kPa/mm.

The second range is for coated strand with a light grit density. A visual inspection of the 0.6 in. (16.2 mm) diameter medium coated strand revealed that its grit density was very similar to the grit density for the ½ in. (12.7 mm) diameter light coated strand. The plastic bond stress values are also similar and thus consistent with the visual evaluation. Therefore, the average value for the plastic bond stress for strand with a light density grit was 725 psi (4999 kPa).

The third range of values includes ½ and ⅜ in. (12.7 and 9.5 mm) diameter medium coated and ½ in. (12.7 mm) diameter high coated strand. Again, a visual inspection of these three strands showed approximately the same amount of grit embedded in each of their coatings. Their plastic bond stress values were also fairly consistent. Thus, the average plastic bond stress for strand with a heavier grit density was determined to be 1086 psi (7488 kPa).

The effect of different concrete compressive strengths can be determined by redefining U_t as $U'_t\sqrt{f'_{ci}}$ as shown in Eq. (9). The values of U'_t for each strand type as related to U_t and f'_{ci} are also shown in Tables 1 and 2. Again, the strand types are divided into the following three groups:

(1) ⅜, ½, and 0.6 in. (9.5, 12.7, and 16.2 mm) diameter uncoated strand.

(2) ½ in. (12.7 mm) coated light and 0.6 in. (16.2 mm) diameter coated medium strand.

(3) ½ in. (12.7 mm) coated medium and heavy, and ⅜ in. (9.5 mm) diameter coated medium strand.

Within these three groups, U'_t is again consistent.

Results of transfer length tests of uncoated strand from several previous studies are presented in Table 3. The transfer length model was applied to these results. Because the elastic portion of the transfer length is small in comparison to the total transfer length, the slope (B) of the elastic portion of the bond stress curve was assumed as found previously to be 300 psi/in. (81.5 kPa/mm).

Applying the transfer length model to these results, values for U_t' were determined from the experimental data presented in the literature and are listed in Table 3. Each transfer length from the literature represents the results from one end of a specimen. Also included in the table is the concrete compressive strength at the time of transfer, the effective prestress level, and the measured transfer length one day after transfer. Two of the papers, by Kaar[4] and by Over,[10] are concerned with the transfer length of uncoated strand with 250 ksi (1724 MPa) ultimate strength, while the rest of the results in the table are for uncoated or coated strands with 270 ksi (1862 MPa) ultimate strength.

Based on the results from Tables 1, 2 and 3, the values for U_t' for uncoated strand range from 3.8 to 11.2, with an average of 6.7 and a median of 6.85. It is recommended that U_t' be taken as 6.7 and U_t as $6.7\sqrt{f_c'}$ for uncoated 7-wire strand. The next to last column of Tables 1, 2 and 3 contains the transfer length calculated from Eq. (9) with U_t' as 6.7, and the last column is the ratio of measured to calculated transfer length. The average for these ratios is reported in Table 4.

Typically, the proposed model overestimates the transfer length of the specimens in Table 3, and underestimates the transfer length of the specimens from Tables 1 and 2. Nevertheless, the overall average ratio of measured to calculated transfer length for uncoated strand (from Tables 1 and 2 and the literature) is about 1.0 with a standard deviation of 0.24. Thus, with the appropriate values for U_t' and B, the transfer length from all available research can be reasonably approximated.

One previous study, by Dorsten, Hunt, and Preston,[12] considered the transfer length of epoxy coated strand, and it discussed an experimental investigation of the transfer length of ½ in. (12.7 mm) diameter coated and uncoated strand. Again, the slope for the elastic portion of the bond stress curve will be assumed as 300 psi/in. (81.5 kPa/mm). From the experimental results of Dorsten, Hunt, and Preston, U_t' was determined and is shown in Table 3. Also included in the table are the concrete compressive strengths at the time of transfer, the effective prestress level, and the measured transfer length.

As stated previously, the values for U_t' for coated strand from Tables 1 and 2 fall into two groups of coated strands. The first is for ½ in. (12.7 mm) diameter light coated and 0.6 in. (16.2 mm) diameter medium coated strand with an average U_t' of 11.0. The other is for ½ in. (12.7 mm) diameter medium coated, ½ in. (12.7 mm) diameter high coated, and ⅜ in. (9.5 mm) diameter medium coated strand with an average U_t' of 16.5.

Application of the theory to the test results from Dorsten, Hunt, and Preston yields an average U_t' of 9.5 which

Table 2a. U_t and calculated transfer lengths for development length specimens.

Beam end	f_{ci}' (psi)	U_t (psi)	U_t'	f_{se} (ksi)	L_t Measured (in.)	L_t* Calculated (in.)	Measured / Calculated
S3UNA	4120	481	7.5	199.9	34.0	34.2	0.99
B	4120	391	6.1	194.9	34.0	33.4	1.02
C	4120	393	6.1	195.9	38.0	34.0	1.12
D	4120	380	5.9	200.1	38.0	34.3	1.11
E	4120	400	6.2	199.4	36.0	34.2	1.05
F	4120	372	5.8	195.9	38.0	33.6	1.13
G	4120	379	5.9	194.5	38.0	33.3	1.14
H	4120	365	5.7	194.8	38.0	33.4	1.14
F3UNA	4810	466	6.7	193.9	30.0	30.9	0.97
B	4810	361	5.2	194.8	30.0	31.0	0.97
C	4810	539	7.8	194.3	26.0	31.0	0.84
D	4810	543	7.8	195.5	26.0	31.2	0.83
S5UNA	4060	396	6.2	200.7	49.0	46.5	1.05
B	4060	451	7.1	200.7	47.0	46.5	1.01
C	4410	317	4.8	199.6	57.0	44.4	1.28
D	4410	328	4.9	200.1	59.0	44.5	1.33
E	4410	391	5.9	198.2	49.0	44.1	1.11
F	4410	315	4.7	198.2	63.0	44.1	1.43
G	4410	512	7.7	199.9	49.0	44.5	1.10
H	4410	484	7.3	198.7	45.0	44.2	1.02
I	4810	393	5.7	193.7	46.0	43.1	1.07
J	4810	414	6.0	195.6	44.0	43.5	1.01
F5UNA	6720	380	4.6	195.2	38.0	35.5	1.07
B	6720	388	4.7	195.2	38.0	35.5	1.07
C	6720	312	3.8	195.2	32.0	35.5	0.90
D	6720	322	3.9	195.2	44.0	35.5	1.24
E	6720	385	4.7	193.8	51.0	35.3	1.44
F	6720	309	3.8	193.8	33.0	35.3	0.93
S6UNA	4750	490	7.1	195.8	44.0	49.6	0.89
B	4750	490	7.1	195.8	50.0	49.6	1.01
C	4750	381	5.5	198.6	56.0	50.3	1.11
D	4750	518	7.5	198.1	44.0	50.2	0.88
E	4750	345	5.0	197.6	62.0	50.1	1.24
F	4750	371	5.4	199.9	68.0	50.6	1.34
F6UNA	4740	351	5.1	195.1	61.0	49.4	1.23
B	4740	445	6.5	193.2	60.0	49.0	1.22
C	4740	367	5.3	191.3	60.0	48.5	1.24
D	4740	363	5.3	195.6	60.0	49.6	1.21

Average = 1.08
Standard Deviation = 0.15

*Based on average values of B and U_t'.

Metric (SI) conversion factors: 1 in. = 25.4 mm; 1 psi = 6.895 kPa; 1 ksi = 6.895 MPa.

is closely aligned with the group of ½ in. (12.7 mm) diameter light coated and 0.6 in. (16.2 mm) diameter medium coated strand. The values of U'_t are greatly affected by grit density and values of U'_t ranging from 9.5 to 16.5 may be applicable to coated strand depending on the grit density in the coating.

Table 2b. U_t and calculated transfer lengths for development length specimens.

Beam end	f'_{ci} (psi)	U_t (psi)	U'_t	f_{se} (ksi)	L_t Measured (in.)	L_t* Calculated (in.)	Measured / Calculated
S3CMA	4120	942	14.7	195.9	16.0	15.1	1.06
B	4120	993	15.5	192.6	16.0	14.9	1.07
C	4120	991	15.4	192.4	14.0	14.9	0.94
D	4120	875	13.6	194.0	16.0	15.0	1.07
F3CMA	4810	1011	14.6	196.2	12.0	14.3	0.98
B	4810	881	12.7	195.3	16.0	14.9	1.07
C	4810	1395	20.1	193.4	10.0	14.1	0.71
D	4810	777	11.2	193.9	18.0	14.1	1.28
S5CMA	4060	858	13.5	202.5	23.0	20.5	1.12
B	4060	858	13.5	202.5	19.0	20.5	0.93
C	4410	757	11.4	194.2	19.0	19.1	0.99
D	4410	758	11.4	198.5	23.0	19.4	1.19
E	4410	1127	17.0	196.6	17.0	19.3	0.88
F	4410	1466	22.1	195.7	13.0	19.2	0.68
G	6720	1621	24.4	199.7	14.0	16.6	0.84
H	6720	880	13.3	198.7	22.0	16.5	1.33
I	6720	663	10.0	197.3	27.0	16.4	1.65
J	6720	805	12.1	198.3	25.0	16.5	1.52
F5CMA	4410	1127	17.0	196.6	19.0	19.7	0.96
B	4410	1280	19.3	197.1	17.0	19.7	0.86
C	6720	1122	13.7	195.9	14.0	16.3	0.86
D	6720	1485	18.1	198.3	18.0	16.4	1.10
S5CHA	3890	1363	21.9	196.0	19.0	20.3	0.94
B	3890	1285	20.6	197.9	19.0	20.4	0.93
C	3890	1117	17.9	195.0	17.0	20.2	0.84
D	3890	1123	18.0	196.0	19.0	20.3	0.94
							Average = 1.03 Standard Deviation = 0.22

*Based on average values of B and U'_t.

Metric (SI) conversion factors: 1 in. = 25.4 mm; 1 psi = 6.895 kPa; 1 ksi = 6.895 MPa.

Table 2c. U_t and calculated transfer lengths for development length specimens.

Beam end	f'_{ci} (psi)	U_t (psi)	U'_t	f_{se} (ksi)	L_t Measured (in.)	L_t* Calculated (in.)	Measured / Calculated
S5CLA	3890	915	14.7	197.2	30.0	30.2	0.99
B	3890	664	10.6	197.7	31.0	30.2	1.03
C	3890	664	10.6	197.7	39.0	30.2	1.29
D	3890	661	10.6	196.7	21.0	30.1	0.70
S6CMA	4750	952	13.8	198.5	22.0	32.5	0.66
B	4750	814	11.8	198.0	26.0	32.4	0.80
C	4740	1036	15.0	195.0	30.0	32.0	0.94
D	4740	836	12.1	193.6	32.0	31.7	1.01
F6CMA	4740	739	10.7	192.7	26.0	31.6	0.82
B	4740	660	9.6	194.8	32.0	31.9	1.00
C	4740	656	9.5	193.9	38.0	31.8	1.19
D	4740	557	8.1	193.6	38.0	31.7	1.20
							Average = 1.06 Standard Deviation = 0.21

*Based on average values of B and U'_t.

Metric (SI) conversion factors: 1 in. = 25.4 mm; 1 ksi = 6.895 MPa; 1 psi = 6.895 kPa; 1 psi/in. = 0.271 kPa/mm.

Since the average values of U'_t from Dorsten, Hunt, and Preston's work and from ½ in. (12.7 mm) diameter light coated and 0.6 in. (16.2 mm) diameter medium coated strand from the authors' research investigation (as reported in Tables 1 and 2) are close, an average value for U'_t of 10.6 was used in calculating the transfer length for both groups of strand. Table 3 lists the transfer length calculated based on Eq. (9) using these recommended values as well as the ratio of measured to calculated transfer length for Dorsten, Hunt, and Preston's data. Tables 1 and 2 have the calculated transfer lengths for the two groups of coated strands along with the ratio of measured to calculated transfer lengths. A value of 16.5 was used for U'_t for the group of ½ in. (12.7 mm) diameter medium and heavy coated and ⅜ in. (9.5 mm) diameter medium coated strand in Tables 1 and 2.

The average ratios of measured to calculated transfer lengths are presented in Table 4. As shown in Table 4, Eq. (9), with the recommended values for U'_t and B, predicts the average transfer lengths of the coated strand from the authors' research within standard deviations of 0.19 and 0.21 for the two groups. Eq. (9) and the recommended values of U'_t and B have approximately the same consistency in predicting the transfer lengths from the literature. Therefore, values from 9.5 to 16.5 for U'_t, and B as 300 psi/in. (81.5 kPa/mm) when used in Eq. (9), give acceptable results for the transfer length when compared to experimental results of strands with varying grit densities.

COMPARISON OF DIFFERENT EQUATIONS FOR TRANSFER LENGTH

Several empirical equations for determining transfer lengths of prestressing strand have been presented. They are:

(1) First term of Eq. (1), which is

Table 3. Analysis of transfer length results.

Authors	Strand type (in.)	f_c' (psi)	f_{se} (ksi)	U_t'	L_t Measured (in.)	L_t* Calculated (in.)	Measured / Calculated
Kaar (Ref. 4)	3/8	3400	168.6	7.2	25.5	33.7	0.76
				7.4	28.5	33.7	0.85
		5015	158.8	6.0	21.5	26.5	0.81
				7.1	25.5	26.5	0.96
	1/2	3525	155.3	5.8	36.0	41.2	0.87
				6.7	43.5	41.2	1.06
		4930	146.6	4.9	33.5	33.0	1.02
				5.5	41.0	33.0	1.24
	0.6	3180	159.3	7.4	42.5	59.6	0.71
				8.1	49.0	59.6	0.82
		5465	153.4	6.6	27.5	44.7	0.62
				8.6	39.5	44.7	0.88
Janney (Ref. 11)	1/2-SC	4115	175.8	8.1	33.0	43.2	0.76
				8.1	33.0	43.2	0.76
	1/2-R	4090	175.8	11.2	24.0	43.3	0.55
				10.6	25.0	43.3	0.58
Over (Ref. 10)	3/8	4180	133.0	4.9	30.0	24.3	1.23
					30.0	24.3	1.23
	1/2	5500	150.0	5.6	35.0	32.0	1.09
					35.0	32.0	1.09
Dorsten (Ref. 12)	1/2	4000	188.9	10.8	27.0	46.5	0.58
				10.8	27.0	46.5	0.58
			187.1	13.7	21.0	46.3	0.45
				13.7	21.0	46.3	0.45
			188.1	10.7	27.0	46.4	0.58
				10.3	28.0	46.4	0.60
			186.7	9.6	30.0	46.2	0.65
				8.5	34.0	46.2	0.74
	Coated 1/2	4000	186.4	7.2	40.0	32.2	1.24
				12.5	23.0	32.2	0.71
			186.5	9.3	31.0	32.2	0.96
				9.6	30.0	32.2	0.93
			185.9	9.9	29.0	32.1	0.90
				8.2	35.0	32.1	1.09
							Average = 0.83

Metric (SI) conversion factors: 1 psi = 6.895 kPa;; 1 ksi = 6.895 MPa; 1 in. = 25.4 mm.

the transfer length component of the ACI equation for development length.[6]

(2) Martin and Scott's Eq. (8).

(3) Zia and Mostafa's Eq. (7).

(4) Proposed equation [Eq. (9)].

Table 5 shows a comparison of transfer length calculated from each of these equations to the measured transfer lengths for each strand diameter and coating type from the research reported in Tables 1 and 2 and Table 3. The transfer lengths for each type of strand calculated using Eq. (9) and the recommended parameters are included in the table. The transfer lengths calculated in the literature fall between the measured transfer lengths for coated and uncoated strands of each size.

While Martin and Scott's results are less than the measured transfer lengths for uncoated strand, their equation is the most conservative of the equations in the literature. The transfer lengths from both the ACI equation and Zia and Mostafa's equation are closest to the transfer lengths for coated strand but are nonconservative for uncoated strand.

The calculated transfer lengths based on Eq. (9) most closely approximate the results from this experimental program and that of Dorsten, Hunt, and Preston. The experimental results for uncoated strand from the rest of the literature are most closely predicted by the equations for transfer length from the literature. Overall, as shown in Table 5, Eq. (9) provides the best estimate.

APPLICATION OF DEVELOPMENT LENGTH MODEL TO EXPERIMENTAL RESULTS

An analytical equation for flexural bond length as developed previously is:

$$L_{fb} = (f_{ps} - f_{se})\left(\frac{A_s/\pi d}{U_d}\right) \qquad (11)$$

To use this equation to predict the flexural bond length of prestressing strand, values for U_d must be derived from the experimental results. Also, U_d was redefined as $U_d'\sqrt{f_c'}$ in Eq. (12) in an effort to include the effect of f_c' on the flexural bond length. The results of the development length tests from Cousins, Johnston, and Zia[1-3] are compiled in Table 6.

Two failure moments are shown for each type of strand to give an upper and lower limit on U_d'. The first mo-

Table 4. Average measured to calculated transfer length ratios.

Strand group	Others		Cousins et al.		Overall	
	Average	Standard Deviation	Average	Standard Deviation	Average	Standard Deviation
½ in. CL 0.6 in. CM	0.97	0.16	1.00	0.19	1.00	0.18
⅜ in. CM ½ in. CM ½ in. CH	NA	NA	0.99	0.21	0.99	0.21
⅜ in. UN ½ in. UN 0.6 in. UN	0.80	0.23	1.13	0.19	1.01	0.26

NA: No data available.

Table 5. Comparison of equations for transfer length.

Strand diameter (in.)	Coating type	Transfer length (in.)								
		Measured	Kaar	Janney	Over	Dorsten	Calculated	ACI	M&S	Z&M
3/8	UN	34.1	25.3	NA	30.0	NA	33.3	22.9	30.0	24.0
	CM	14.2	NA	NA	NA	NA	13.3	23.2	30.0	24.0
1/2	UN	49.7	38.5	28.5	35.0	26.9	44.2	30.2	40.0	33.8
	CL	28.3	NA	NA	NA	NA	30.1	29.5	40.0	32.3
	CM	18.9	NA	NA	NA	31.3	17.3	30.8	40.0	34.5
	CH	17.3	NA	NA	NA	NA	16.5	29.4	40.0	32.3
0.6	UN	55.5*	39.6	NA	NA	NA	51.6	39.4	48.0	41.8
	CM	30.4	NA	NA	NA	NA	35.5	35.8	48.0	41.1

*From flexural bond length specimen. M&S: Martin and Scott. Z&M: Zia and Mostafa. NA: No data available. Metric (SI) conversion factor: 1 in. = 25.4 mm.

ment represents the test average for the flexural failures and the second is the test moment of a bond slip failure closest to the test moment of a flexural failure. The stress in the strand at failure (f_{ps}) was calculated using Eq. (18-3) of ACI 318-83.[6]

The concrete compressive strength, f'_c, corresponds to that at the time of the development length test. The flexural bond length was found by subtracting the calculated transfer length (Table 5) from the embedment lengths corresponding to the moment listed in Table 6. Then U'_d can be found using Eq. (12) as shown in the last column of Table 6.

To develop recommended values for U'_d for particular groups of strand types, the U'_d values are divided into groups corresponding to those of the transfer length model. The groups are shown in Table 7 with the average U'_d for each group in the last column. Except in the group of ½ in. (12.7 mm) diameter medium coated, ½ in. (12.7 mm) diameter heavy coated, and ⅜ in. (9.5 mm) diameter medium coated strand, the values of U'_d are less consistent than desirable. Therefore, more investigation into U'_d values is recommended.

Table 6. Calculation of U'_d values for different types of strand.

Strand diameter (in.)	Coating type	M_{fail} (in.-kips)	f_{se} (ksi)	f_{ps} (ksi)	f'_c (psi)	L_{fb} (in)	U_d (psi)	U'_d
3/8	UN	90	197.1	252.7	5340	20.7	193	2.6
		88	197.1	252.7	5340	23.7	169	2.3
	CM	95	193.6	252.7	5340	7.7	554	7.6
		92	193.6	252.7	5340	10.7	399	5.5
1/2	UN	215	199.5	250.9	5160	74.8	67	0.9
		185	199.5	250.9	5160	60.8	82	1.1
	CL	215	195.8	251.7	5540	33.9	161	2.2
		187	195.8	251.7	5540	11.9	458	6.2
	CM	204	197.3	250.9	5160	12.7	411	5.7
		184	197.3	250.9	5160	9.7	538	7.5
	CH	213	196.9	251.7	5540	13.5	395	5.3
		193	196.9	251.7	5540	10.5	508	6.8
0.6	UN	376	198.1	254.5	6640	80.4	81	1.0
		358	198.1	254.5	6640	74.4	87	1.1
	CM	374	194.6	254.5	6640	28.5	242	3.0
		343	194.6	254.5	6640	12.5	552	6.8

Metric (SI) conversion factors: 1 in. = 25.4 mm; 1 in-kip = 0.113 kN-m; 1 ksi = 6.895 MPa; 1 psi = 6.895 kPa.

The development lengths based on the proposed model using the average U'_d values of Table 7 are compared to the experimental results of the development length tests in Table 8. For each type of strand, the table shows the calculated flexural bond length and calculated development length. The calculated development length is the sum of the calculated flexural bond length and the calculated transfer length of Table 5.

Also shown in Table 8 is the measured development length which is the shortest embedment length of a flexural failure. The last column shows the ratio measured to calculated development lengths. However, due to the inconsistent values of U'_d, the calculated values of development length are not as close as desirable to the measured values, except for ⅜ in. (9.5 mm) diameter medium coated, ½ in. (12.7 mm) diameter medium coated, and ½ in. (12.7 mm) diameter high coated strand.

COMPARISONS OF DIFFERENT EQUATIONS AND RESULTS FOR DEVELOPMENT LENGTH

Two empirical equations for development length have been discussed previously. These two equations, the ACI equation and Zia and Mostafa's equation, are based on the work of Hanson and Kaar.[5] The results of the research reported herein as compared to Hanson and Kaar's work, the ACI equation, and Zia and Mostafa's equation are shown in Table 9. Table 9 shows the measured and calculated development length followed by:

(1) Measured development length from Hanson and Kaar.

(2) ACI equation for development length.

(3) Zia and Mostafa recommended development length.

In general, the calculated development length using Eq. (12) is shorter than the measured length as reported herein, and longer than that found by Hanson and Kaar. The ACI equation and Zia and Mostafa's equation underestimate the development lengths of uncoated strand reported by Hanson and Kaar. The development lengths of coated strand reported herein are overestimated by the ACI equation and Zia and Mostafa's equation except for ½ in. (12.7 mm) diameter low coated and 0.6 in. (16.2 mm) diameter medium coated strand.

Table 7. Average U_d' values for each group.

Strand diameter (in.)	Coating type	U_d' Low	U_d' High	Average for group
3/8	UN	2.3	2.6	1.32
1/2	UN	0.9	1.1	
0.6	UN	1.0	1.1	
1/2	CL	2.2	6.2	4.55
0.6	CM	3.0	6.8	
1/2	CM	5.7	7.5	6.40
1/2	CH	5.3	6.8	
3/8	CM	5.5	7.6	

Metric (SI) conversion factor: 1 in. = 25.4 mm.

Table 8. Calculation of development length for different types of strand.

Strand diameter (in.)	Coating type	U_d'	L_{fb} Calculated (in.)	L_d Calculated (in.)	L_d Measured (in.)	Measured / Calculated
3/8	UN	1.32	41.5	74.8	57.0	0.76
	CM	6.40	9.1	22.8	24.0	1.05
1/2	UN	1.32	52.8	97.0	119.0	1.23
	CL	4.54	16.1	46.2	64.0	1.37
	CM	6.40	11.4	29.1	30.0	1.03
	CH	6.40	11.2	27.7	30.0	1.08
0.6	UN	1.32	60.3	111.9	132.0	1.18
	CM	4.55	18.6	54.1	64.0	1.18

Metric (SI) conversion factor: 1 in. = 25.4 mm.

Table 9. Comparison of equations for development length.

Strand diameter (in.)	Coating type	Development length (in.) Measured	Calculated	H&K	ACI	Z&M
3/8	UN	57.0	68.7	60.0	43.8	50.1
	CM	24.0	22.8	NA	45.4	50.1
1/2	UN	119.0	88.5	80.0	55.9	65.9
	CL	64.0	47.4	NA	57.5	67.2
	CM	30.0	28.9	NA	57.6	68.0
	CH	30.0	27.9	NA	56.8	66.6
0.6	UN	132.0	109.0	NA	73.2	84.1
	CM	64.0	57.4	NA	71.7	86.0

H&K: Hanson and Kaar. Z&M: Zia and Mostafa. NA: No data available.

SUMMARY OF SUGGESTED EQUATIONS AND PARAMETERS FOR TRANSFER LENGTH AND DEVELOPMENT LENGTH

The suggested equation for transfer length is:

$$L_t = 0.5\left(\frac{U_t'\sqrt{f_{ci}'}}{B}\right) + \frac{f_{se}A_s}{\pi\, d\, U_t'\sqrt{f_{ci}'}} \qquad (9)$$

where the f_{ci}' in psi at transfer is used and $\sqrt{f_{ci}'}$ has units in psi. Recommended values of U_t' for uncoated strand and for two gorups of coated strand are:

(1) For uncoated strand, $U_t' = 6.7$.

(2) For coated strand with low grit density, $U_t' = 10.6$.

(3) For coated strand with medium to high grit density, $U_t' = 16.5$.

Based on the results for transfer length specimens, B was highly variable, and an average value of 300 psi/in. (81.5 kPa/mm) was used in the equations for transfer length.

The equation for development length is the sum of the equations for transfer length and flexural bond length. The equation for flexural bond length is:

$$L_{fb} = (f_{ps} - f_{se})\left(\frac{A_s/\pi d}{U_d'\sqrt{f_c'}}\right) \qquad (12)$$

where the f_c' is psi at 28 days is used and $\sqrt{f_c'}$ has units of psi. The development length is:

$$L_d = L_t + L_{fb}$$

Again, the recommended values for U_d' for the three groups of strand are:

(1) For uncoated strand, $U_d' = 1.32$.

(2) For coated strand with a medium to high density of grit, $U_d' = 6.40$.

(3) For coated strand with a low density of grit, $U_d' = 4.55$.

CONCLUSIONS

Analytical models were developed to predict the transfer and development lengths of epoxy coated and uncoated prestressing strand. Based on analysis of the available experimental results and the proposed analytical models, the following conclusions can be drawn:

1. The transfer length model developed herein predicts transfer lengths from available research within an acceptable degree of accuracy.

2. The development length model developed herein predicts development lengths, within an acceptable degree of accuracy. However due to the limited amount of research in this area, more experimental verification of the development length model parameters would be desirable.

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REFERENCES

1. Cousins, T. E., Johnston, D. W., and Zia, P., "Bond of Epoxy Coated Prestressing Strand," FHWA/NC/87-005, Center for Transportation Engineering Studies, Department of Civil Engineering, North Carolina State University, Raleigh, NC, December, 1986, 191 pp.
2. Cousins, T. E., Johnston, D. W., and Zia, P., "Transfer Length of Epoxy Coated Prestressing Strand," *ACI Materials Journal*, V. 87, No. 3, May-June 1990, pp. 193-203.
3. Cousins, T. E., Johnston, D. W., and Zia, P., "Development Length of Epoxy Coated Prestressing Strand," *ACI Materials Journal*, V. 87, No. 4, July-August 1990, pp. 309-318.
4. Kaar, P., LaFraugh, R., and Mass, M., "Influence of Concrete Strength on Strand Transfer Length," PCI JOURNAL, V. 5, No. 8, October 1963, pp. 47-67.
5. Hanson, N., and Kaar, P., "Flexural Bond Tests," *ACI Journal*, V. 56, No. 1, January, 1959, pp. 783-802.
6. ACI Committee 318, "Building Code Requirements for Reinforced Concrete (ACI-318-83)," American Concrete Institute, Detroit, MI, 1983, 111 pp.
7. Zia, P., and Mostafa, P., "Development Length of Prestressing Strand," PCI JOURNAL, V. 22, No. 5, September-October 1977, pp. 54-66.
8. Martin, L., and Scott, N., "Development of Prestressing Strand in Pretensioned Members," *ACI Journal*, V. 73, No. 8, August 1976, pp. 453-456.
9. Guyon, Y., *Prestressed Concrete*, Volume 1, John Wiley and Sons, Inc., New York, N.Y., 1960.
10. Over, R. S., and Au, T., "Prestress Transfer Bond of Pretensioned Strands in Concrete," *ACI Journal*, Proceedings V. 62, No. 11, November 1965, pp. 1451-1460.
11. Janney, J. R., "Report on Stress Transfer Length Studies on 270 k Prestressing Strand," PCI JOURNAL, V. 8, No. 1, February 1963, pp. 41-45.
12. Dorsten, V., Hunt, F., and Preston, H., "Epoxy Coated Seven-Wire Strand for Prestressed Concrete," PCI JOURNAL, V. 29, No. 4, July-August 1984, pp. 120-129.

APPENDIX A — EXAMPLE OF DEVELOPMENT LENGTH CALCULATION

The transfer length, flexural bond length, and development length will be calculated for a beam with ½ in. (12.7 mm) diameter coated medium prestressing strand and the following typical properties:

(1) $f_{ci}' = 4500$ psi (31 MPa) (concrete compressive strength at transfer)

(2) $f_c' = 6000$ psi (41 MPa) (concrete compressive strength at 28 days after transfer)

(3) $f_{sei} = 188{,}000$ psi (1296 MPa) (effective prestress after transfer)

(4) $f_{se} = 160{,}000$ psi (1103 MPa)

(5) $f_{ps} = 255{,}000$ psi (1758 MPa) (stress in prestressing strand at nominal strength)

(6) $f_{pu} = 270{,}000$ psi (1862 MPa) (guaranteed ultimate tensile strength)

(7) $A_s = 0.153$ in.^{2} (99 mm^2)

(8) $d = 0.5$ in. (12.7 mm)

The plastic transfer bond stress coefficient (U_t'), plastic bond stress coefficient for development (U_d'), and the bond modulus *(B)* are 16.5, 6.4 and 300 psi/in. (81.5 kPa/mm), respectively, for ½ in. (12.7 mm) diameter coated medium prestressing strand. The transfer length equation (L_t) is as follows:

$$L_t = 0.5\left(\frac{U_t' \sqrt{f_{ci}'}}{B}\right) + \frac{f_{se} A_s}{\pi d U_t' \sqrt{f_{ci}'}} \tag{9}$$

Substituting yields:

$$L_t = 0.5\left(\frac{16.5\sqrt{4500}}{300}\right) + \frac{188{,}000\,(0.153)}{\pi\,(0.5)\,(16.4)\sqrt{4500}}$$

and

$$L_t = (1.85 + 16.54)$$
$$= 18.4 \text{ in. } (467 \text{ mm})$$

The equation for flexural bond length (L_{fb}) is:

$$L_{fb} = (f_{ps} - f_{se})\left(\frac{A_s/\pi d}{U_d' \sqrt{f_c'}}\right) \tag{12}$$

and substituting yields:

$$l_{fb} = (25{,}000 - 160{,}000) \times \left[\frac{(0.153)/\pi\,(0.5)}{6.4\sqrt{6000}}\right]$$
$$= 18.7 \text{ in. } (475 \text{ mm})$$

The resulting development length *(L_d)* is as follows:

$$L_d = L_t + L_{tb}$$
$$= 18.4 + 18.7$$
$$= 37.1 \text{ in. } (942 \text{ mm})$$