Stress and Strain Analysis in Prestressed Concrete: A Critical Review

by

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Reinforced concrete structures without prestressing generally crack and in some cases have excessive deflection. If some of the reinforcement is prestressed steel, cracking and deflection can be controlled. The amount of prestressing may be sufficient to avoid cracking. This condition is referred to as full prestressing, as opposed to partial prestressing where controlled cracking is allowed. Partial prestressing is widely used and is often considered preferable to the two extremes of no prestressing and full prestressing. When full prestressing is adopted, nonprestressed steel is commonly provided to control cracking due to temperature and shrinkage.

Thus, in general, cross sections of reinforced concrete members have two types of reinforcements: prestressed and nonprestressed.

In current practice, the stress and strain in a section due to prestressing are calculated by considering the effect of a compressive force on a plain concrete section. The loss in tension in the prestressed steel due to creep, shrinkage and relaxation is estimated; and the time-dependent changes in stress and strain are determined accordingly. Equations which give the loss in tension in a prestressing tendon can be found in many codes and design recommendations (e.g., Refs. 1, 2 and 3).

This concept, which ignores the nonprestressed steel, results in errors in prediction of stresses and in larger errors in strains and deflections. It will be shown that the magnitudes of the errors are large with the amount of nonprestressed steel common in fully prestressed or in partially prestressed sections.

A much more accurate approach is adopted in the CPCI Design Manual.[4]

An equation is given to calculate ΔP_c representing the change in the resultant compression on the concrete taking into account (approximately) the effect of the nonprestressed steel. The force, ΔP_c, is considered to act on a plain concrete section and the time-dependent changes in stress and strain are calculated accordingly; the advantages and limitations of this concept will be discussed.

This paper suggests that empirical equations which predict the loss in tension in prestressed tendons are not needed. Accuracy in stress and strain analysis can be achieved by consideration of equilibrium and compatibility and recognition that, in general, prestressed sections have three components: concrete, prestressed steel and nonprestressed steel. The approach is relatively simple and is general, i.e., it applies to reinforced and composite sections with or without prestressing.

The paper will also show that when partial prestressing is used, the amount of nonprestressed steel in the cross sections significantly influences the load level at which cracking occurs, the curvatures or deflections after cracking, the changes in stresses in the reinforcement at cracking and the crack widths.

Where appropriate, numerical examples show the various discrepancies in current design methods.

Synopsis

Prestressed concrete members are generally provided with nonprestressed steel in addition to the prestressed reinforcement. It is shown by an example that the extreme fiber stresses and the strains after creep, shrinkage and relaxation cannot be accurately determined by treating the prestressing as a force on a plain concrete section, ignoring the nonprestressed steel.

A more accurate and more general approach is suggested, which applies to reinforced concrete sections with or without prestressing and to composite sections. Only basic equilibrium and compatibility equations are used. Thus, use of empirical equations for estimating the loss of prestressing is avoided. The PCI Design Handbook equations for prestress losses are critically reviewed.

SIGN CONVENTION

Tensile stress and the corresponding strain are positive (see Fig. A1, Appendix A). A positive bending moment and the corresponding curvature are positive. The symbol y denotes a coordinate defining position of any fiber with respect to a reference point O; y is positive for fibers below O. The symbol Δ indicates a change in value; a positive and a negative Δ mean respectively an increase and a decrease in value. The relaxation of prestressed steel, $\Delta\sigma_{pr}$, is commonly a reduction in tension; thus a negative value.

CONVENTIONAL AND ACCURATE ANALYSES

In a conventional analysis of prestressed concrete sections, empirical equations are used to calculate $P_{effective}$, the tensile force in the prestressing steel after occurrence of creep, shrinkage and relaxation; different empirical equations give different results. But the correct value of $P_{effective}$ should satisfy the equilibrium of forces and strains in concrete and steel.

In this paper the term "accurate" analysis refers to an analysis in which the nonprestressed steel is considered and compatibility of strains in concrete and in the prestressed and the nonprestressed steel is ensured. Only basic mechanics equations are used, without recourse to empirical equations.

The results of accurate and conventional analyses will be compared. The term "conventional" analysis will mean an analysis in which the nonprestressed steel is ignored. For a meaningful comparison, the use of an empirical equation for $P_{effective}$ is also avoided in the conventional analysis by the use of basic mechanics ensuring equilibrium and compatibility. Thus, in the special case of a section without nonprestressed steel the conventional and the accurate analyses are the same.

EQUILIBRIUM AND COMPATIBILITY REQUIREMENTS

The accurate analysis can be executed in four simple steps which are presented briefly in a separate section; more details and derivation of the method are given in Refs. 5 and 6. The present paper compares results of several analyses to show the errors in the conventional analysis in which the nonprestressed steel is ignored.

A correct analysis must satisfy the following compatibility and equilibrium conditions:

1. The strains in steel and concrete at any fiber must be equal before and after the time-dependent changes. An exception where this compatibility requirement does not apply is at a post-tensioned tendon at the time of prestressing; the requirement applies only to the time-dependent strains occurring after anchorage of the tendons.

The time-dependent strain in a nonbonded tendon can differ slightly from the strain in the adjacent concrete, but this difference is ignored here.

2. In a statically determinate structure, the prestressing produces no reactions. At the time of prestressing, the tensile force in the tendon and the compressive forces in the concrete and in the nonprestressed steel are three forces in equilibrium. They thus have zero resultant components. Also, the changes ΔP_c, ΔP_{ps} and ΔP_{ns} in the forces in the three components due to creep, shrinkage and relaxation must not disturb the equilibrium. In other words, the sum of the three Δ values must be zero and their moments about any reference point O must vanish.

Prestressing of statically indeterminate structures produces reactions and internal forces, which require structural analysis of the static indeterminacy, before the stresses and strains at any section can be calculated. The forces in the concrete, in the prestressed and in the nonprestressed steel in any section of a statically indeterminate structure must have resultants equal to the statically indeterminate internal forces calculated a priori.

The results of stress and strain analyses given in this paper are for sections of statically determinate simple beams. For these cases it can be verified that the forces in the concrete, the prestressed and the nonprestressed steel have zero resultants, before and after the time-dependent changes.

EXAMPLE

A simplified structure will be analyzed by the conventional and by the accurate methods and the results compared. The structure shown in Fig. 1 represents a 1 ft (305 mm) wide strip of a post-tensioned, simply supported solid slab.

At time t_o, the structure is subjected to dead load q = 0.40 kip/ft (5.8 kN/m) and an initial prestressing force P = 290 kips (1300 kN), which is assumed constant over the length. At time t, after occurrence of creep, shrinkage and relaxation, a uniform live load p = 1.00 kip/ft (14.6 kN/m) is applied. The intensity, p, is sufficient to produce cracking at midspan. The tensile strength of concrete at time t is f_{ct} = 360 psi (2.5 MPa). The modulus of elasticity of concrete, $E_c(t_c)$,

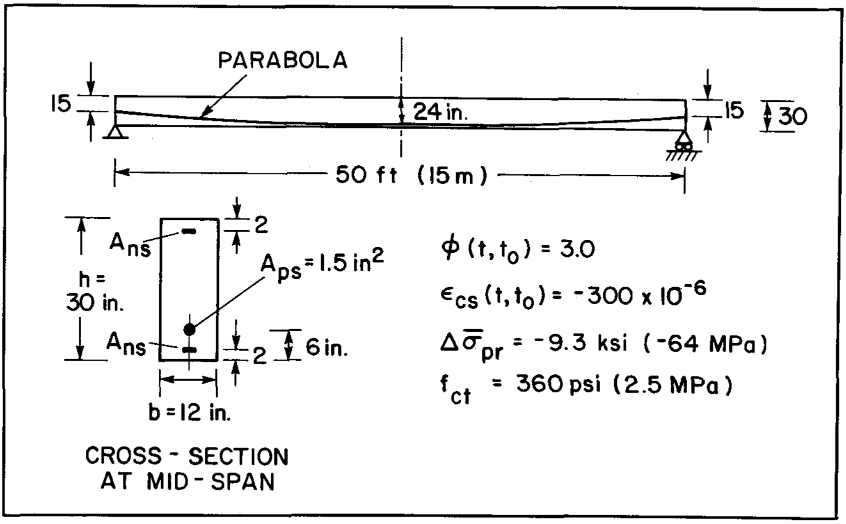

Fig. 1. Example of a post-tensioned slab. Note: 1 in. = 25.4 mm.

is 4350 ksi (30.0 GPa) and is assumed constant with time. The modulus of elasticity of the prestressed and the nonprestressed steel, E_s, is 2900 ksi (200 GPa).

Nonprestressed steel of equal cross section area, A_{ns}, is provided at top and bottom. The steel ratio, $\rho_{ns} = A_{ns}/bh$, is considered variable between zero and 1 percent. Other data are given in Fig. 1.

The objective of the analyses is to determine stresses, strains and deflections at time t_o and at time t just before and after application of the live load. The same values of the creep coefficient, $\phi(t,t_o)$, the free shrinkage, $\epsilon_{cs}(t,t_o)$, and the reduced relaxation, $\Delta\bar{\sigma}_{pr}$, are used in the accurate and in the conventional analyses. The two analyses differ only in that the nonprestressed steel is ignored in the conventional analysis. The parameters ϕ, ϵ_{cs} and $\Delta\bar{\sigma}_{pr}$ are defined in Appendix B.

For clarity of presentation and for ease in verification of the results presented herein, the simplest cross section is chosen. Also, the variation of the initial prestressing force P because of friction is ignored and the difference in the cross section area of the tendon and that of the prestressing duct is neglected. In addition, the modulus of elasticity of concrete is assumed constant with time. None of these simplifications is essential for the method of analysis used.

The bending moment at midspan due to q and p are, respectively, 1500 and 3750 kip-in. (169 and 424 kN-m).

Stresses and Deflections Before Cracking

Table 1 gives the concrete stresses at midspan at time t_o immediately after prestressing and at time t after occurrence of creep, shrinkage and relaxation, but before application of the live load. It can be seen that the stress at the bottom fiber varies between -1026 and -502 psi (-7.08 and -3.46 MPa) as the nonprestressed steel ratio, ρ_{ns}, is increased from zero to 1 percent. The stress difference is greater than f_{ct}.

In other words, ignoring the nonprestressed steel substantially overestimates the compressive stress provided by prestressing to prevent or to control cracking by subsequent live load; the

Table 1. Stresses and deflections at midspan before cracking in Example (Fig. 1).

Nonprestressed steel ratio ρ_{ns} (percent)			0	0.2	0.4	0.6	0.8	1.0	0.4 with reduced ϕ & ϵ_{cs}
Concrete stresses at time t (psi)		σ_{top}	−302	−276	−246	−215	−184	−155	−250
		σ_{bot}	−1026	−879	−759	−659	−574	−502	−969
Change of force in the three materials between t_0 and t (kips)	Concrete	ΔP_c	52	76	97	114	128	140	59
	Prestressed steel	ΔP_{ns}	0	−28	−52	−72	−88	−102	−28
	Nonprestressed steel	ΔP_{ps}	−52	−48	−45	−42	−40	−38	−30
Deflection at time t before application of the live load (10^{-3} in.)			−923	−794	−696	−621	−560	−510	−553
Ratio of deflection at time t, before application of live load to the instantaneous deflection			2.56	2.32	2.13	2.00	1.88	1.78	1.69
Steel stresses at time t, before live load application (ksi)		σ_{ns} (bot)	−36	−33	−30	−28	−26	−24	−20
		σ_{ps}	159	161	163	165	167	168	173

Remarks: Negative deflection means camber.
Conversion factors: 1 psi = 6.9×10^{-3} MPa; 1 kip = 4.45 kN.

overestimation is of the same order of magnitude or greater than the tensile strength of concrete. The compressive stress reserve, intended to counteract the tension due to live load, is substantially eroded as a result of the presence of nonprestressed steel.

Table 1 also gives the force changes ΔP_c, ΔP_{ps} and ΔP_{ns} in the concrete, the prestressed steel and the nonprestressed steel due to creep, shrinkage and relaxation. It can be seen that the change in forces in the concrete and in the prestressed steel are equal in absolute value only when $\rho_{ns} = 0$. In other cases, ΔP_c is substantially larger than $|\Delta P_{ps}|$; the difference between the two quantities is equal to $|\Delta P_{ns}|$, which represents the compressive force picked up by the nonprestressed steel.

The sign of ΔP_c is positive indicating a decrease of the initial compressive force in concrete. The negative ΔP_{ns} indicates an increase in compression. Also, the negative ΔP_{ps} indicates loss of tension in the prestressing tendon.

The nondimensional graphs in Fig. 2 represent the variation of ρ_{ns} versus $(|\Delta P_{ps}| / \Delta_{ref})$ or $(\Delta P_c / \Delta_{ref})$; where Δ_{ref} is a reference force equal to $\Delta P_c / |\Delta P_{ps}|$ when $\rho_{ns} = 0$. The difference between the ordinates of the two curves in Fig. 2 is equal to $|\Delta P_{ns}| / \Delta_{ref}$; it represents the relative increase in compressive force in the nonprestressed steel.

In a conventional analysis, the compressive stress in concrete after creep, shrinkage and relaxation will be greatly overestimated when considered equal to $P_{effective} = P + \Delta P_{ps}$; where P is the absolute value of the initial prestressing force. Accordingly, the absolute value of

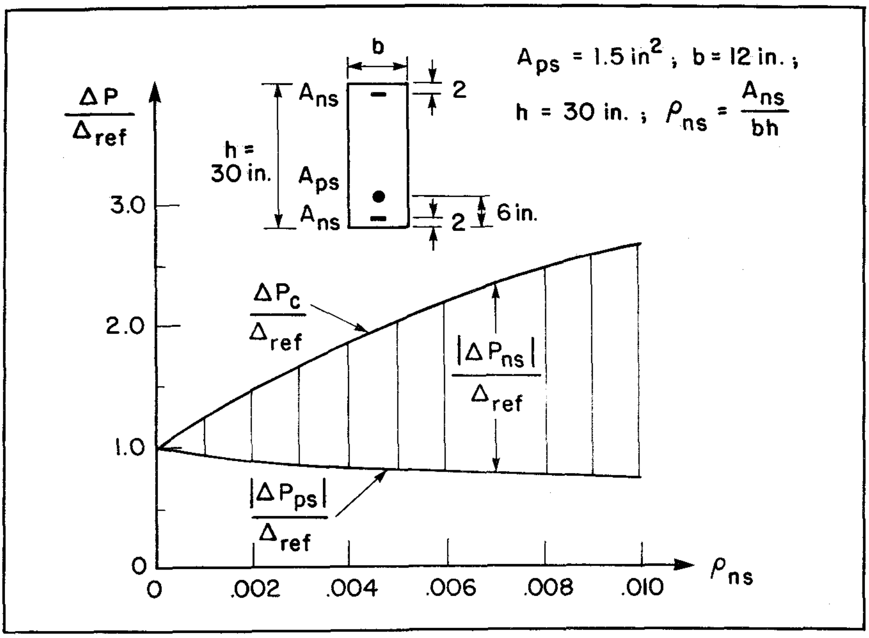

Fig. 2. Relative time-dependent change in forces in concrete, prestressed steel and nonprestressed steel.

the compressive stresses in concrete after losses will be greatly overestimated.

The loss in tension in prestressing steel, ΔP_{ps}, should not be used to calculate the loss in compression in the concrete. The empirical equations given in various codes and design recommendations for estimating ΔP_{ps} are of limited use, if any. It would be better to delete such equations because they are not accurate and the answer they give is frequently used incorrectly as equal in absolute value to ΔP_c.

Table 1 also gives the deflection at the center of span with varying ρ_{ns}. The negative sign indicates camber. It is clear that camber is overestimated if nonprestressed steel is ignored. Also, it can be seen that the deflection after creep, shrinkage and relaxation cannot be accurately predicted by multiplying the instantaneous deflection by a specified number, because such a number must vary with ρ_{ns} and with the creep, shrinkage and relaxation parameters (varies between 1.69 and 2.56 in this particular example).

Stresses, Deflections After Cracking and Crack Width

Table 2 gives the stresses and central deflection after application of live load p = 1 kip/ft, which is more than sufficient to produce cracking. The corresponding moment at midspan is 3750 kip-in. The load p_{cr} and the corresponding moment M_{cr} at midspan at which cracking first occurs are also given in Table 2.

Values which are frequently of concern in design for serviceability when cracking is allowed is the crack width and the change in steel stresses due to live load. The former is of concern for corrosion protection and the latter for safety against fatigue. Table 2 gives the crack width w divided by the crack spacing s; it also gives the stress changes $\Delta\sigma_{ps}$ and $\Delta\sigma_{ns}$ in the prestressed

Table 2. Stresses, deflection and crack width at midspan after live load application in Example (Fig. 1).

Nonprestressed steel ratio ρ_{ns} (percent)		0	0.2	0.4	0.6	0.8	1.0	0.4 with reduced ϕ & ϵ_{cs}
Live load bending moment at which cracking occurs (kip-in.)		2600	2400	2300	2200	2100	2000	2700
Ratio of uniform load intensity P_{cr} at which cracking occurs to p (p = 1 kip/ft)		0.69	0.65	0.61	0.58	0.56	0.54	0.73
Deflection after application of p (10^{-3} in.)		1250	1229	1182	1128	1074	1022	976
Steel stresses after application of p (ksi)	σ_{ns} (bot)	−7	−6	−5	−5	−4	−4	−2
	σ_{ps}	180	181	182	183	183	183	187
Stress changes in steel caused by p (ksi)	$\Delta\sigma_{ns}$ (bot)	29	27	25	23	21	20	18
	$\Delta\sigma_{ps}$	22	20	19	17	16	15	14
Ratio of crack width to crack spacing (10^{-3})		0.72	0.68	0.65	0.62	0.58	0.55	0.39

Conversion factors: 1 kip/ft = 14.6 kN/m; 1 ksi = 6.9 MPa.

and in the nonprestressed steel due to p = 1 kip/ft.

Based on the results in Table 2, the following remarks can be made:

(a) Ignoring the nonprestressed steel will cause the deflection to be overestimated.

(b) The level of loading at which cracking occurs will be overestimated when cracking is ignored.

(c) The steel stress increments, $\Delta\sigma_{ps}$ and $\Delta\sigma_{ns}$, decrease with the increase in ρ_{ns}. Thus, ρ_{ns} can be selected to ensure safety against fatigue.

(d) The crack width can be controlled by appropriate choice of ρ_{ns}. The ratio w/s, crack width to crack spacing, is included in Table 2 rather than w, because s depends not only on ρ_{ns}, but also on how the nonprestressed steel is arranged in the section. The crack spacing s decreases with the increase of ρ_{ns}. Thus, w decreases faster than the ratio w/s as ρ_{ns} is increased.

(e) The stress in the bottom nonprestressed steel is in compression despite cracking.

STRESS AND STRAIN IN NONCRACKED SECTIONS

The instantaneous and time-dependent changes in stress and strain in noncracked sections can be determined in four steps (Fig. 3):

Step 1 — Apply the initial prestressing force and the dead load bending moment which becomes effective at the time of prestressing on a transformed section composed of A_c plus ($\alpha_{ps}A_{ps}$ + $\alpha_{ns}A_{ns}$); where the subscripts *ps* and *ns* refer to the prestressed and the nonprestressed steel, respectively; α is equal to E_{ps} or E_{ns} divided by E_c (t_o).

Here, the transformed section includes only the prestressed and the nonprestressed steel bonded to the concrete at prestress transfer. Thus, A_{ps} should be included when pretensioning is used, but when all the prestressing is post-tensioned in one stage, A_{ps} and the area of the duct should be excluded.

When the structure is statically indeterminate, the indeterminate bending moment due to prestressing should be

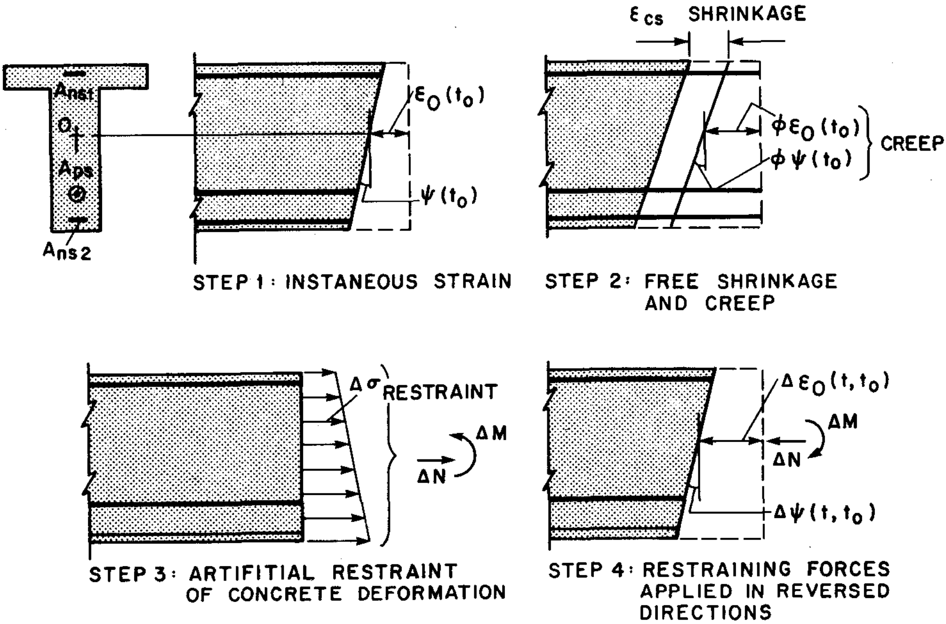

Fig. 3. Steps of analysis of time-dependent strain and stress.

included in the value of the moment.

The values $\epsilon_O(t_o)$ and $\psi(t_o)$ defining the distribution of the instantaneous strain can be determined by Eqs. (A1) [Appendix A]; where $\epsilon_O(t_o)$ is the value of the instantaneous strain at an arbitrary reference point O and $\psi(t_o)$ is the curvature which is equal to the slope $d\epsilon/dy$ of the strain diagram (Fig. A1). Multiplication by $E_c(t_o)$ gives $\sigma_O(t_o)$ and $\gamma(t_o)$, which represent respectively the stress at Point O and the slope $d\sigma/dy$ of the stress diagram (Fig. A1).

Step 2 — Determine the hypothetical change in strain distribution due to creep and shrinkage of concrete if they were free to occur. The strain change at O is equal to $[\phi\epsilon_O(t_o) + \epsilon_{cs}]$ and the change in curvature is $\phi\,\psi(t_o)$; where ϕ is the creep coefficient and ϵ_{cs} is the free shrinkage (see Appendix B).

Step 3 — Calculate artificial stress which when gradually introduced on the concrete during the period t_o to t will prevent occurrence of the strain determined in Step 2. The restraining stress at any fiber y is:

$$\Delta\sigma_{restraint} = \bar{E}_c\{\phi[\epsilon_O(t_o) + \psi(t_o)y] + \epsilon_{cs}\} \quad (1)$$

where $\bar{E}_c$ is the age-adjusted modulus of elasticity of concrete [Eq. (B6)].

Step 4 — Determine by Eqs. (A3) a force at O and a moment which are the resultants of $\Delta\sigma_{restraint}$. The strain in concrete due to relaxation of the prestressed steel can be artificially prevented by the application, at the level of the prestressed steel, of a force equal to $(\Delta\sigma_{ps}A_{ps})$.

This force is substituted by a force of the same magnitude at O plus a couple. Summing up gives ΔN and ΔM, the restraining normal force and the couple required to prevent artificially the strain due to creep, shrinkage and relaxation.

To eliminate the artificial restraint, apply ΔN and ΔM in reversed directions on an age-adjusted transformed section composed of A_c plus $(\bar{\alpha}_{ps} A_{ps} + \bar{\alpha}_{ns} A_{ns})$ where $\bar{\alpha}$ is equal to E_{ps} or E_{ns} divided by $\bar{E}_c(t,t_o)$.

The strain distribution at time t is the sum of the strains determined in Steps 1 and 4, while the corresponding stress is the sum of stresses calculated in Steps 1, 3 and 4.

Commentary:

1. Superposition of strains or stresses can be done simply by summing up the increments $\Delta\epsilon_O$ and $\Delta\psi$ in various steps or $\Delta\sigma_O$ and $\Delta\gamma$, using the same reference point O in all steps. For this reason, Eqs. (A3) and (A4) are used here rather than the more familiar Eqs. (A5) and (A6) which require that O be the section centroid.
2. The four steps give the stress and strain at time t, without preceding the analysis by an estimate of the loss in tension in the prestressed steel. No empirical equation is involved for loss calculation.
3. The compatibility of strain in concrete and steel is maintained at all the reinforcement layers.
4. The procedure of analysis is similar to the well-known displacement method of structural analysis, where the displacements are prevented artificially by restraining forces which are subsequently eliminated by application of the same forces in reversed directions.
5. The same four steps apply to reinforced concrete sections without prestressing simply by setting $A_{ps} = 0$; the effect of cracking will be discussed in the following section.
6. Composite sections made of more than one type of concrete, cast or prestressed in stages or sections made of concrete and structural steel, can be analyzed by the same procedure.[6]
7. The deflection can be determined from the curvature by numerical integration. In simple or continuous beams of constant or variable cross sections, the following equation can be used to give approximately the deflection at midspan:

$$D = \frac{l^2}{96}(\psi_1 + 10\psi_2 + \psi_3) \qquad (2)$$

where ψ_1 and ψ_3 are the curvatures at the ends and ψ_2 is the midspan curvature. This geometric relation is exact when the variation of ψ between the three values is parabolic.

STRESS AND STRAIN IN CRACKED SECTIONS

Consider a prestressed section for which the distribution of stress at any time t due to prestressing and dead load are defined by two known quantities: $\sigma_O(t)$ and $\gamma(t)$. Assume that due to live load the section is subjected to additional bending moment M and axial force N at centroid O. Consider the case when the magnitudes of N and M are sufficient to produce cracking. To determine the strain and stress changes due to live load, partition M and N:

$$N = N_1 + N_2 \quad ; \quad M = M_1 + M_2 \qquad (3)$$

Note that N_1 and M_1, referred to as the decompression forces,[7] represent the forces which are just sufficient to bring the stresses in the concrete to zero. Eqs. (A3) can be used to calculate N_1 and M_1 as the stress resultants of a stress distribution defined by $-\sigma_O(t)$ and $-\gamma(t)$.

Cracking is produced by N_2 and M_2, which can be considered to act on reinforced concrete sections, without distinction between the prestressed and nonprestressed steels (except for their respective E_s values). After decompression, N_2 and M_2 act on a section with zero initial stress in the concrete.

The forces N_2 and M_2 are considered to act on a transformed section composed of the area of concrete in compression plus $(\alpha_{ps} A_{ps} + \alpha_{ns} A_{ns})$ where α is E_{ps} or E_{ns} divided by $E_c(t)$. The depth c of the compression zone is determined by the equilibrium equation and by the assumption that plane cross sections remain plane. For rectangular or T-sections, the determination of c involves the solution of a cubic equation given in various references (e.g., Ref. 6).

Once c is determined, and the area properties of the transformed cracked section are calculated, the equations of Appendix A can be used to determine the stress and strain changes due to N_2 and M_2.

The total change in strain due to N and M is the sum of the change in the decompression stage due to N_1 and M_1, plus the change due to N_2 and M_2. But the change in stress due to N_2 and M_2 gives the total stress in concrete after cracking. The changes in stresses in the prestressed and the nonprestressed steels due to N_2 and M_2 are the values to be used in calculating the crack width (see Ref. 8 or 6) or to assess the fatigue life of the structure. It should be noted that the stresses due to N_2 and M_2 represent the total stress in the concrete after cracking but not the total stresses in the reinforcements.

The above procedure gives the strain and stress in a section due to transient live load producing cracking. If the load producing cracking is sustained and it is required to determine the changes in strain and stress between time t and a later time $\bar{t}$ due to creep, shrinkage and relaxation, Steps 2 to 4 of the preceding section can be applied for a cross section of depth c, which is the depth of the compression zone at time t. This ignores the time-dependent change in c, and results in negligible error.[5]

COMPUTER PROGRAMS

The steps discussed above for analysis of the instantaneous and time-dependent stresses and strains and the changes in these values after cracking involve repetitious calculations of section properties, which can be easily performed by the use of programmable calculators. The computer program CRACK[9] can be used to give the stresses, strains and displacements in reinforced concrete and composite members with or without prestressing. The program CRACK, which is usable on IBM microcomputers, does not perform an analysis of statically indeterminate forces.

A computer program CPF, Cracked Plane Frames,[10] performs the same analyses as CRACK and in addition determines the statically determinate internal forces and changes in these forces due to creep, shrinkage and relaxation and due to cracking. The effect of cracking involves nonlinear iterative analysis discussed in Ref. 11. Program CPF is suitable for composite structures built and prestressed in stages (e.g., segmental bridges and multistory building frames) and is usable on microcomputers.

EFFECTS OF VARYING CREEP AND SHRINKAGE PARAMETERS

It is sometimes argued that the effort required for an accurate prediction of stresses and strains in service conditions is not justified because accurate values of the creep coefficient ϕ and the free shrinkage ϵ_{cs} are not commonly available. A more rational approach, for important structures, is to perform accurate analyses using upper and lower bounds of the parameters ϕ and ϵ_{cs}.

The analysis for the midspan section in the Example is repeated for the case $\rho_{ns} = 0.4$ with $\phi = 1.5$ and $\epsilon_{cs} = -150 \times 10^{-6}$. The results are given in the last column in Tables 1 and 2. It can be seen that, for the case considered, varying ϕ and ϵ_{cs} by a factor of two has some effect, but the effect is not as important as the effect of ignoring nonprestressed steel.

LOSS OF COMPRESSION IN CONCRETE

As mentioned above, the time-dependent loss in tension in a prestressed tendon is not generally equal in absolute value to the change in compression in the concrete. The following equation gives the time-dependent increment of the resultant force on concrete:

$$\Delta P_c = \beta \left\{ \left[\phi\, \sigma_{cst}(t_o)\, \frac{E_{st}}{E_c(t_o)} + \epsilon_{cs} E_{st} \right] \times A_{st} + \Delta\bar{\sigma}_{pr} A_{ps} \right\} \tag{4}$$

where

$$\beta = \left[1 + \frac{A_{st}}{A_c} \frac{E_{st}}{\bar{E}_c} \left(1 + \frac{y_{st}^2}{r_c^2} \right) \right]^{-1} \tag{5}$$

with $A_{st} = A_{ns} + A_{ps}$ = total steel area; E_{st} = modulus of elasticity of steel assumed the same for the two steel types; y_{st} is the y coordinate of the centroid of A_{st}, measured downward from the centroid of A_c; $\sigma_{cst}(t_o)$ is the concrete stress at time t_o at y equals y_{st}; $r_c^2 = I_c/A_c$, where I_c is the second moment of A_c about an axis through its centroid.

The quantity ΔP_c, commonly positive, represents a loss in compression; it is equal in absolute value to the loss in tension in A_{ps} only in the absence of A_{ns}.

Eq. (4) is based on the assumption that all the reinforcements, A_{ns} and A_{ps}, are concentrated at one layer at distance y_{st} below the centroid of A_c. The compatibility of strains at this layer at time t in the nonprestressed steel, the prestressed steel and the concrete requires that:

$$\frac{\Delta\sigma_{ns}}{E_{st}} = \frac{\Delta\sigma_{ps} - \Delta\bar{\sigma}_{pr}}{E_{st}} = \frac{\phi\, \sigma_{cst}(t_o)}{E_c(t_o)} + \epsilon_{cs} + \frac{\Delta P_c}{\bar{E}_c} \left(\frac{1}{A_c} + \frac{y_{st}^2}{I_c} \right) \tag{6}$$

The first and second terms on the last side of this equation are the creep and shrinkage, while the third term is the strain recovery due to a gradually introduced force, ΔP_c.

For equilibrium, the sum of the change in forces in the three materials must be zero:

$$\Delta P_c + A_{ns} \Delta\sigma_{ns} + A_{ps} \Delta\sigma_{ps} = 0 \tag{7}$$

The solution of Eqs. (6) and (7) for P_c gives Eq. (4).

When the value P_c is determined, it can be used to calculate the time-dependent change in strain in concrete at $y = y_{st}$ as the sum of the three terms on the last side of Eq. (6). The corresponding change in stress is simply equal to the last of the three terms multiplied by $\bar{E}_c$. The time-dependent changes in steel stresses, $\Delta\sigma_{ns}$ and $\Delta\sigma_{ps}$, also follow from Eq. (6). The change in curvature between t_o and t is given by:

$$\Delta\psi(t, t_o) = \phi\psi(t_o) + \frac{\Delta P_c\, y_{st}}{\bar{E}_c I_c} \tag{8}$$

where $\psi(t_o)$ is the instantaneous curvature.

Eq. (4), which is adopted in the CPCI Design Manual[4] is strictly valid only when A_{ns} and A_{ps} are situated in one layer. When this condition is satisfied, use of Eqs. (4) or the four steps in Fig. 3 gives identical results. Eq. (4) also applies when the centroids of A_c, A_{ps} and A_{ns} coincide and the section is subjected to concentric force without bending moment.

When Eqs. (4) and (6) are applied in other cases, an approximate value of ΔP_c will be obtained; the force P_c should be applied on A_c at the centroid of A_{st}. This approximate analysis will not ensure compatibility of strains in concrete and steel at all reinforcement layers.

Prestressed concrete designers accustomed to using an equation for the prestress loss may prefer Eq. (4) to the four steps represented in Fig. 3. However, it should be noted that Eq. (4) gives the change in compressive force on the concrete and not the loss in tension in the prestressed tendon. Also, the inaccuracies in prediction of curvature and the extreme fiber stresses should be recognized.

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For a section without nonprestressed steel, the time-dependent changes in the forces in the concrete and in the prestressed steel are of equal and opposite signs; $\Delta P_c = -\Delta P_{ps}$. Thus, Eq. (4), reversed in sign, can be used to give the loss in tension in the prestressed steel due to creep shrinkage and relaxation. An example will be analyzed below using Eq. (4) and the results compared with equations given by ACI-ASCE Committee 423. The equations and the example are taken from Ref. 3.

Consider a double T pretensioned section of gross area 615 in.^{2} (Fig. 4); distance from centroid to soffit = 21.98 in.; moment of inertia of gross area about centroidal axis = 59720 in.^{4}; cross section area of prestressed steel A_{ps} = 1.836 in.^{2} and its centroid is at 4.43 in. about the soffit.

At time t_o, a prestressing force P = 372 kips and a dead load bending = 5232 kip-in. are introduced. It is required to determine the instantaneous change in stress in the prestressed steel and the changes in the same parameter due to creep, shrinkage and relaxation between t_o and a later time t.

The modulus of elasticity of concrete at t_o is $E_c(t_o)$ = 2500 ksi. Consider creep coefficient, ϕ = 1.6; free shrinkage, $\epsilon_{cs} = -184 \times 10^{-6}$; intrinsic relaxation of prestressed steel = -5 ksi. Note that the last two quantities are not explicitly given in Ref. 3; the values used here had to be derived from the given data. Also, for simplicity, the self weight and the weight of concrete topping are assumed to have been introduced at t_o, producing the moment given above.

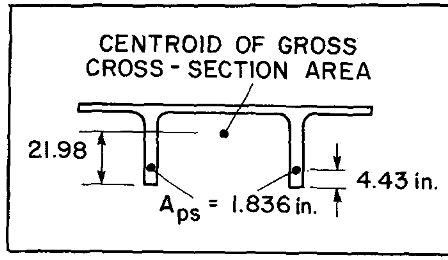

Fig. 4. Pretensioned section treated by accurate analysis and by equations from Ref. 3.

At the time of prestressing, the transformed section composed of A_c plus $\alpha_{ps} A_{ps}$ has the following properties: A = 634 in.^{2}; distance between centroid and soffit = 21.46 in.; moment of inertia about centroidal axis = 65320 in.^{4} Distance from centroid of transformed section to prestressed steel = 21.46 − 4.43 = 17.03 in.

At the level of prestressed steel, the concrete stress at t_o, immediately after transfer [Eq. (A6)] is:

$$\sigma_{cst}(t_o) = -\frac{372}{634} + \frac{5232 - 372(17.03)}{65320} \times (17.03) = -0.874 \text{ ksi}$$

The instantaneous change in stress in the prestressed steel is:

$$\Delta\sigma_{ps}(t_o) = \frac{E_{ps}}{E_c(t_o)} \sigma_{cst}(t_o) = \frac{2800}{2.5}(-0.874) = -9.789 \text{ ksi}$$

The symbols used here for the two instantaneous stress changes have the following equivalents in Ref. 3: $f_{cir} \equiv -\sigma_{cst}(t_o)$ and ES $\equiv -\Delta\sigma_{ps}(t_o)$. The two equations recommended in Ref. 3 for the two parameters [Eqs. (4.5.3) and (4.5.2)] give: f_{cir} = 0.734 psi and ES = 8.221 ksi. The answers differ because of the approximation resulting from the use of an empirical coefficient, K_{cir}, included in Eq. (4.5.3) of Ref. 3.

Because there is no prestressed steel, $E_{st} \equiv E_{ps}$ and $A_{st} \equiv A_{ps}$. Division of Eq. (4) by A_{ps} and reversal of sign of the right-hand side, gives the following expressions for the changes in the stress in the prestressed steel due to the three causes:

Creep:

$$(\Delta\sigma_{ps})_{creep} = \beta\,\phi\sigma_{cst}(t_o)\,\frac{E_{ps}}{E_c(t_o)}$$

$$= -11.888 \text{ ksi}$$

Shrinkage:

$$(\Delta\sigma_{ps})_{shrinkage} = \beta\,\epsilon_{cs}E_{ps}$$

$$= -3.910 \text{ ksi}$$

Relaxation:

$$(\Delta\sigma_{ps})_{relaxation} = \beta\,\Delta\bar{\sigma}_{pr}$$

$$= -3.036 \text{ ksi}$$

The symbols used here for the three time-dependent stress changes have the following equivalents in Ref. 13: CR = $-(\Delta\sigma_{ps})_{creep}$; SH $\equiv$ $-(\Delta\sigma_{ps})_{shrinkage}$; RE $-(\Delta\sigma_{ps})_{relaxation}$. Eqs. (4.5.4), (4.5.6) and (4.5.7) of Ref. 3 give: CR = 13.153; SH = 5.157 and RE = 3.937. The answers differ mainly because of the absence of the dimensionless multiplier β = 0.759 from the equations of Ref. 3. The value of β, calculated by Eq. (5), is always smaller than 1.

It should be noted that the equations of Ref. 3 cannot be applied in the more general case when nonprestressed steel is provided. The answers obtained by the equations of the same reference do not satisfy exactly the compatibility conditions, i.e., Eq. (6). It can be verified that the answers evaluated by Eq. (4) satisfy the compatibility requirements; also, the same answers can be checked following the four steps presented in Fig. 3.

CONCLUSIONS

Reinforced concrete members are generally composed of three components: concrete, nonprestressed and prestressed steels. Ignoring the nonprestressed steel in prestressed concrete members can result in large errors in prediction of stresses and deformations. A relatively simple procedure, applicable to noncracked and cracked sections, can be used for a more accurate analysis based on equilibrium of forces and compatibility of strains in the three components. Two available computer programs which can be used for the analysis are described.

The current practice of analyzing the effect of prestressing by considering a compressive force on a plain concrete section can produce significant errors. It is suggested that such an analysis should be treated as preliminary, particularly when the deflections are of concern or when cracking is not allowed. The example presented shows that the error in prediction of stresses in concrete after creep, shrinkage and relaxation is of the same order of magnitude as the tensile strength of concrete.

The loss in tension in the prestressed tendon due to creep, shrinkage and relaxation should not be used to represent the loss of compressive force on the concrete. Empirical equations included in many codes of practice for the loss in tension in the prestressed steel are not really useful. It is more useful to include in the codes an equation for ΔP_c, representing the change in the resultant force on the concrete [e.g., Eq. (4)].

The limitations of the equation should be mentioned. The basic requirements of equilibrium and compatibility which lead to more accurate and more general analysis should be stated. Code commentaries and recommended practices should give the steps of analysis which satisfy the requirements. Unnecessary approximations should be avoided and the equations which include empirical values not accurate in all cases should be eliminated.

REFERENCES

1. AASHTO, *Standard Specifications for Highway Bridges,* American Association of State Highway and Transportation Officials, 13th Edition, 1983, Washington, D.C.
2. *Ontario Highway Bridge Design Code,* Ministry of Transportation and Communications, Ontario, 2nd Edition, 1983.
3. *PCI Design Handbook – Precast and Prestressed Concrete,* 3rd Edition, Precast/Prestressed Concrete Institute, Chicago, Illinois, 1985, pp. 4-40 to 4-42.
4. Canadian Prestressed Concrete Institute, *Metric Design Manual,* 2nd Edition, 1987, CPCA publication, Ottawa (see p. 3-26).
5. Ghali, A., "A Unified Approach for Prestressed and Nonprestressed Concrete Structures," PCI JOURNAL, V. 31, No. 2, March-April 1986, pp. 118-137. See also discussion, PCI JOURNAL, V. 32, No. 1, 1987, pp. 133-140.
6. Ghali, A., and Favre, R., *Concrete Structures: Stresses and Deformation,* Chapman and Hall, London and New York, 1986, 352 pp.
7. Tadros, M. K., "Expedient Serviceability Analysis of Cracked Prestressed Concrete Beams," PCI JOURNAL, V. 27, No. 6, November-December 1982, pp. 67-86.
8. Favre, R., Beeby, A. W., Falkner, H., Kopma, M. and Schiessl, P., *Cracking and Deformations CEB Manual,* published and distributed by the Swiss Federal Institute of Technology, Lausanne, Switzerland, 1985.
9. Ghali, A., and Elbadry, M. M., *Manual of Computer Program Crack* — Research Report No. CE85-1, Department of Civil Engineering, The University of Calgary. Program is available on diskettes for IBM microcomputers, 1985.
10. Elbadry, M. M., and Ghali, A., *Manual of Computer Program CPF:* Cracked Plane Frames in Prestressed Concrete. Research Report No. CE85-2, Department of Civil Engineering, The University of Calgary, 1985.
11. Elbadry, M. M., and Ghali, A., "Serviceability Design of Continuous Prestressed Concrete Structures," PCI JOURNAL, V. 34, No. 1, January-February 1989, pp. 54-91.
12. Ghali, A., and Trevino, J., "Relaxation of Steel in Prestressed Concrete," PCI JOURNAL, V. 30, No. 5, 1957, pp. 82-94.
13. Trost, H., "Auswirkungen des Superpositionspringzips auf Kriechen — und Relaxations — Problems bei Beton and Spannbeton," *Beton-und Stahlbetonbau,* V. 62, No. 10, pp. 230-238 and No. 11, 1967, pp. 261-269.

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APPENDIX A — RELATIONSHIPS BETWEEN FORCES, STRESSES AND STRAINS IN A SECTION

The equations presented below give the strain and stress distributions in a cross section subjected to a bending moment M and a normal force N at an arbitrary reference point O (Fig. A1). The cross section properties A, B and I are given, representing respectively the area, its first and second moment about an axis through the reference point. Use of Hooke's law and the assumption that cross sections remain plane and equilibrium consideration give:

$$\epsilon = \epsilon_O + \psi y \quad ; \quad \sigma = E(\epsilon_O + \psi y) \qquad \text{(A1)}$$

$$N = \int \sigma \, dA \quad ; \quad M = \int \sigma y \, dA \qquad \text{(A2)}$$

Substitution of Eq. (A1) into Eq. (A2) gives:

$$N = A\sigma_O + B\gamma \quad ; \quad M = B\sigma_O + I\gamma \qquad \text{(A3)}$$

Substitution of $\sigma_O = E\,\epsilon_O$ and $\gamma = E\,\psi$ into Eqs. (A3) and solution gives:

$$\epsilon_O = \frac{IN - BM}{E(AI - B^2)} \quad ; \quad \psi = \frac{-BN + AM}{E(AI - B^2)} \qquad \text{(A4)}$$

When O is at the centroid, $B = 0$ and Eqs. (A3) and (A4) take the more familiar forms:

$$N = A\sigma_O \quad ; \quad M = I\gamma \qquad \text{(A5)}$$

$$\epsilon_O = \frac{N}{EA} \quad ; \quad \psi = \frac{M}{EI} \qquad \text{(A6)}$$

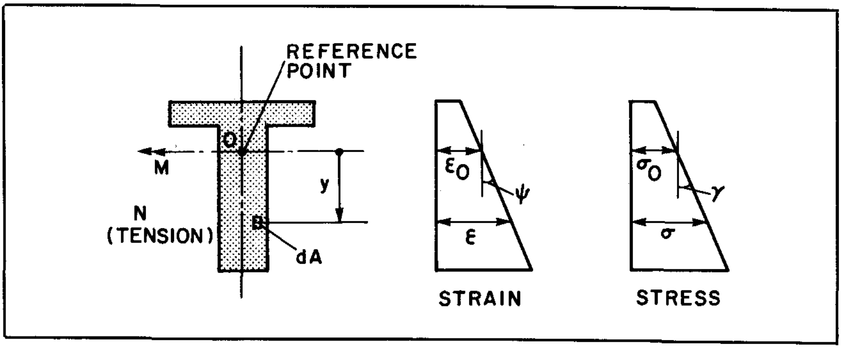

Fig. A1. Definition of symbols and convention for positive, y, N, M, ϵ, ψ, σ and γ.

* * *

APPENDIX B — RELAXATION, CREEP AND SHRINKAGE

The term *intrinsic relaxation* of prestressed steel, $\Delta\sigma_{pr}$, will be used to mean the time-dependent drop in stress in a tendon stretched between two fixed points. The value $\Delta\sigma_{pr}$ is almost zero when the initial stress σ_{po} is equal to 40 percent of the characteristic tensile strength f_{ptk}; but the value $\Delta\sigma_{pr}$ increases with $(\lambda - 0.4)^2$, where $\lambda = \sigma_{po} / f_{ptk}$.

In a concrete member, the distance between the anchored ends of a tendon shortens due to creep and shrinkage of concrete, causing an additional drop of tension. The reduction in tension caused by creep and shrinkage has an effect on the amount of relaxation similar to the effect of reduction in λ. For this reason, in prestressed concrete analysis, a reduced relaxation value should be used:

$$\Delta\bar{\sigma}_{pr} = \chi_r \Delta\sigma_{pr} \qquad \text{(B1)}$$

where χ_r is a relaxation reduction factor given by (see Ref. 12):

$$\chi_r = e^{-(6.7 + 5.3\lambda)\Omega} \qquad \text{(B2)}$$

with

$$\Omega = - \frac{\Delta\sigma_{ps} - \Delta\sigma_{pr}}{\sigma_{pr}} \qquad \text{(B3)}$$

Note that $\Delta\sigma_{ps}$ is the change in stress in the prestressed steel due to the combined effect of creep, shrinkage and relaxation. This value is not known a priori. A value $\chi_r \approx 0.8$ can be assumed and used to determine $\Delta\sigma_{ps}$ from which an improved value of χ_r is determined by Eq. (B2). The assumed and calculated value commonly agree closely after one iteration.

The continuous line ABC in Fig. B1 represents the time variation of strain in concrete due to a stress increment $\Delta\sigma_c(t_o)$ introduced at time t_o and sustained, without change in magnitude. The total strain at time t, instantaneous plus creep, is given by:

$$\Delta\epsilon(t) = \frac{\Delta\sigma_c(t_o)}{E_c(t_o)}(1 + \phi) \qquad \text{(B4)}$$

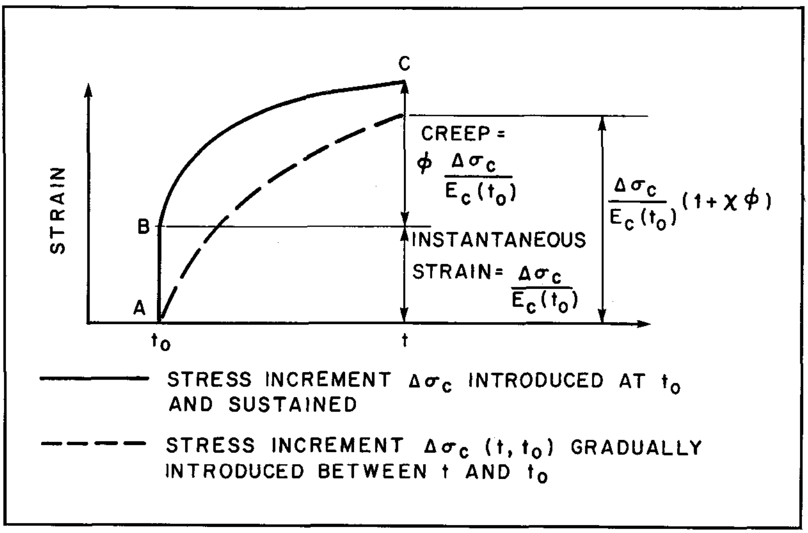

Fig. B1. Time variation of strain caused by a stress increment $\Delta\sigma_c$.

where $\phi = \phi\ (t,t_o)$ is the creep coefficient, representing the ratio of creep occurring between t_o and t to the instantaneous strain.

When a stress increment $\Delta\sigma_c\ (t,t_o)$ is gradually introduced between t_o and t, the strain variation with time can be represented by the dashed line in Fig. B1. The total strain produced during the period t_o to t is:

$$\Delta\epsilon\ (t,t_o) = \frac{\Delta\sigma_c\ (t,t_o)}{\bar{E}_c\ (t,t_o)} \qquad \text{(B5)}$$

where $\bar{E}_c\ (t,t_o)$ is the age-adjusted modulus of elasticity of concrete:[13]

$$\bar{E}_c\ (t,t_o) = \frac{E_c\ (t_o)}{1 + \chi\phi} \qquad \text{(B6)}$$

where χ is the concrete aging coefficient.

The symbol $\epsilon_{cs} = \epsilon_{cs}\ (t,t_o)$ will be used to represent the shrinkage which occurs between t_o and t, when the shortening is free to occur. The values of ϕ and ϵ_{cs} depend upon the shape of the concrete member, the relative humidity of the air, the ages of the concrete at t_o and t. Suggested values of ϕ, ϵ_{cs} and χ are given in Refs. 6 and 8. An approximate value of 0.8 is frequently used for both χ and χ_r.

* * *

APPENDIX C — NOTATION

A = area
B = first moment of area
E = modulus of elasticity
$\bar{E}_c$ = age-adjusted modulus of elasticity of concrete
I = second moment of area
M = bending moment
N = normal force
P = absolute value of prestressing force
t = time or age of concrete
y = coordinate
σ = stress
α = ratio of modulus of elasticity of steel to that of concrete
$\bar{\alpha}$ = ratio of modulus of elasticity of steel to $\bar{E}_c$
λ = σ_{po}/f_{ptk}
χ = aging coefficient for concrete
χ_r = relaxation reduction coefficient
Δ = increment
ϵ = normal strain
ϕ = creep coefficient
ψ = curvature (slope of strain diagram $= d\epsilon/dy$)
γ = slope of stress diagram ($= d\sigma/dy$)

Subscripts

c = concrete
cs = shrinkage
ns = nonprestressed steel
O = reference point
o = time of prestressing
ps = prestressed steel
pr = prestressed steel relaxation

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NOTE: Discussion of this paper is invited. Please submit your comments to PCI Headquarters by August 1, 1990.