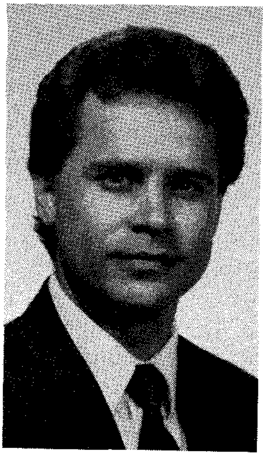
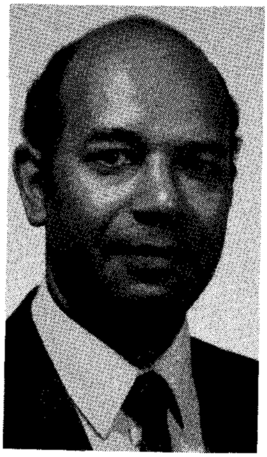
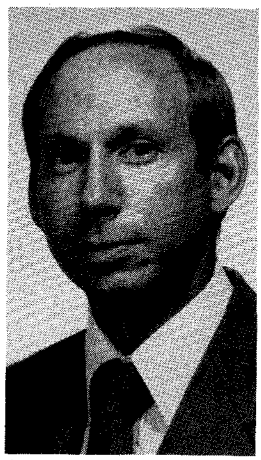
Ductility of Reinforced and Prestressed Concrete Flexural Members







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The current ACI Code[1] specifies separate sets of reinforcement limits for conventionally reinforced and prestressed concrete. The reinforced concrete limit is related to the steel area ratio, ρ, which depends on the definition of member depth and width. This limit is applicable only to rectangular and flanged sections, with or without compression reinforcement.

The limit of reinforcement in prestressed concrete is given in terms of the steel index, ω, which is a function of the ratio, ρ. It accounts for the presence of mild tension and compression reinforcement, thus allowing for partial prestressing. It, too, is applicable only to rectangular and flanged sections. However, this limit does not default to that of a conventionally reinforced section when the area of prestressed steel is set equal to zero. The Code provisions are confusing if the section is flanged and contains both prestressed and nonprestressed steel, or if the section has more than one type of concrete as in composite sections.

The purpose of this paper is to examine the background of both limits and to develop a unified approach that would apply to the entire spectrum of structural concrete members. Several recommendations are discussed. These include the 1986 Supplement to the ACI

Commentary,[2] the Canadian Code,[3] the CEB-FIP Model Code,[4] and the work of Naaman and his associates.[5,6,7] Based on these earlier studies, a very simple yet general formula is proposed herein. It is compatible with the intent of the ACI Code, and is valid regardless of the section shape, reinforcement, and number of concrete types used in composite action. Moreover, it allows the engineer to focus on the factors affecting ductility, which include but are not limited to maximum tension reinforcement.

Numerical examples are provided to illustrate the proposed formula and to compare it with other methods. A proposal for revision of the ACI Code and Commentary, on the basis of the developments given herein, is presented in Appendix B.

Synopsis

A unified ductility formula is proposed for both reinforced and prestressed concrete flexural members. This equation is intended as a substitute for the maximum tensile reinforcement provisions in the ACI Code for both types of construction. Furthermore, the formula allows designers to adjust other factors that affect ductility, such as concrete confinement in the compression zone. The proposed formula is extremely easy to use in comparison with other available recommendations, is applicable to sections of general shape and reinforcement content, and gives reasonable results. Numerical examples and proposed ACI Code and Commentary revisions are presented.

THEORETICAL DEVELOPMENT

Various codes require member ductility by limiting the maximum amount of flexural reinforcement. One of the benefits of ductility is large deflections and thus warning before failure. Ductility is also important for seismic design and moment redistribution in continuous members.

A good indicator of ductility is the section curvature, which is the second derivative of deflection. Referring to Fig. 1, the section curvature, ψ, is defined by the relationship:

$$\psi = \frac{\epsilon_{cu}}{c} \tag{1}$$

where ϵ_{cu} is the maximum usable compressive strain at the extreme concrete fiber and c is the distance from the extreme compression fiber to the neutral axis.

Naaman, Harajli, and Wight[6] have discussed in detail the factors affecting member ductility. An acceptable degree of ductility can be achieved by keeping ψ greater than or equal to a certain minimum value. It is more convenient, however, to work with the nondimensional quantity of member rotation, θ, over a given length.

In plastic design, the length over which plastic hinging develops is essentially an assumed value based on judgment and experience.[8-11] A commonly assumed length is the effective depth (see Ref. 4). Thus, for members with prestressed steel only:

$$\theta_{ps} = \int_0^{d_{ps}} \psi\, dx = \psi\, d_{ps} = \frac{\epsilon_{cu}\, d_{ps}}{c} \geqslant \theta_{min,\, ps} \tag{2a}$$

For conventionally reinforced members:

$$\theta_{ns} = \int_0^{d_{ns}} \psi\, dx = \psi\, d_{ns} = \frac{\epsilon_{cu}\, d_{ns}}{c} \geqslant \theta_{min,\, ns} \tag{2b}$$

Eqs. (2) clearly show that ductility can be improved by reducing the neutral axis depth, c, or by increasing the ulti-

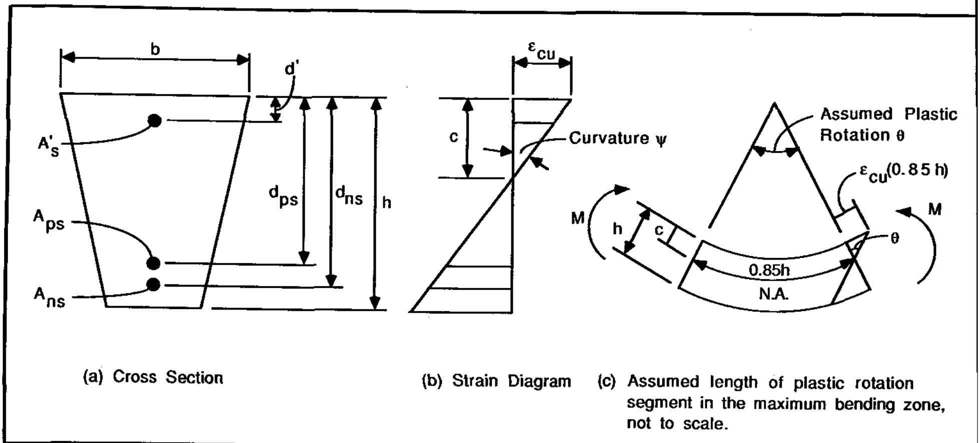

Fig. 1. Flexural ductility relationships.

mate concrete strain, ϵ_{cu}. The latter parameter is recognized by the CEB-FIP Model Code,[4] but not by the ACI or Canadian Codes. Numerous studies have shown that ϵ_{cu} can be increased by confining the concrete with closed ties. Concrete strength and other factors also affect ϵ_{cu}.

The neutral axis depth, c, may be reduced by increasing the concrete compressive strength, adding compression reinforcement, or by decreasing the tensile force. The last factor corresponds to the ACI Code provisions for maximum reinforcement limits.

The ACI Code maximum reinforcement limit for prestressed concrete is based on keeping the strain in the prestressing steel equal to or greater than the yield strain (see Ref. 12). Other assumptions implied in the ACI Code formulas for maximum reinforcement are: decompression steel strain = 0.00585 and ϵ_{cu} = 0.003.

Using these quantities, a corresponding value of d_{ps}/c may be obtained from Fig. 1(b). Substitution into Eq. (2a) yields:

$$\theta_{min,ps} = 0.00715 \qquad (3a)$$

The ACI Code provisions for reinforced concrete are based on limiting the steel area to 0.75 of the theoretical balanced failure area. For Grade 60 steel placed in rectangular sections, the corresponding rotation is:

$$\theta_{min,ns} = \frac{0.003 + \epsilon_y}{0.75} = 0.0068 \qquad (3b)$$

For Grade 40 steel, which is not common anymore, $\theta_{min,ns}$ = 0.0058. For flanged sections, limiting the area of tensile reinforcement to 0.75 of the balanced area does not correspond to a fixed minimum curvature or rotation limit. If the width of the compression zone is nonuniform, 0.75 of the area in the compression zone does not correspond to 0.75 of the neutral axis depth. A more consistent approach is to limit the neutral axis depth to 0.75 of the balanced depth for all section shapes.[13] This will be further clarified by Example 2.

It can be seen from Eqs. (3) that θ_{min} is comparable for both construction types. For the general case of a cross section containing significant quantities of both types of steel, i.e., partially prestressed sections, it would be convenient to have one equation instead of applying Eqs. (2) and (3) separately. A unified and conservative equation may be obtained by setting the ductility limit in terms of the full member depth, h, rather than the effective depths d_{ps} and d_{ns}. Thus, Eqs. (2) become:

$$\frac{c}{h} \leqslant \alpha \, \epsilon_{cu} \tag{4}$$

where α is equal to $d_{ps}/(h\,\theta_{min,ps})$ and $d_{ns}/(h\,\theta_{min,ns})$ for prestressed and conventionally reinforced members, respectively.

Examination of numerous practical applications has revealed that taking $d_{ps}/h = 0.85$ is a reasonable assumption. Therefore, the corresponding value of α is 0.85/0.00715 = 118.9, say 120 for simplicity. For $\alpha = 120$, the corresponding value of d_{ns}/h is 0.0068(120) = 0.816, which is an acceptable ratio for conventionally reinforced sections. Thus:

$$\frac{c}{h} \leqslant 120 \, \epsilon_{cu} \tag{5}$$

Eq. (5) is proposed for use as a minimum ductility requirement in place of the maximum reinforcement limits given in the ACI Code. It contains the two most important parameters that influence ductility, namely, the neutral axis depth and the ultimate concrete strain.

The use of total member depth, h, rather than an effective depth in the ductility criterion can be justified as follows:

1. In members containing both prestressed and nonprestressed steel, the effective depth has no single definition, while the total depth, h, is a well known quantity.

2. The effective depth is a product of the design process. It is possible to complete the strength design of partially prestressed sections, for example, without any need for its calculation.

3. At sections of maximum bending moment, where it is critical to check ductility, the tensile stress resultant is generally farther from the compression face than $0.85h$ for prestressed concrete and $0.816h$ for reinforced concrete. Thus, Eq. (5) is more conservative than the ACI Code limits for the great majority of practical cases.

4. In plastic analysis, the length of segment for calculating rotation is somewhat arbitrary.

5. The Canadian Code uses this concept for its prestressed concrete ductility provisions.

COMPARISON WITH OTHER METHODS

As shown earlier, the proposed formula was developed to give results consistent with the 1983 ACI Code. The 1986 Supplement to the ACI 318-83 Commentary provides an alternative method for calculating the maximum reinforcement index of prestressed members:

$$0.85 \frac{a}{d_{ps}} \leqslant 0.36 \, \beta_1 \tag{6}$$

For noncomposite sections, Eq. (6) is equivalent to the maximum reinforcement limit given in the 1983 Code. Setting $d_{ps} = 0.85h$ and $\epsilon_{cu} = 0.003$ in proposed Eq. (5) yields:

$$\frac{c}{d_{ps}} \leqslant 0.42 \tag{7}$$

which is equivalent to Eq. (6).

In addition to being compatible with the intent of the ACI Code limits, proposed Eq. (5) is easy to use and offers consistent results for the entire spectrum of structural concrete (see the Proposed Commentary Revisions in Appendix B).

Naaman's[7] proposed formula:

$$\frac{c}{d_e} \leqslant 0.425 \tag{8}$$

where

$$d_e = \frac{A_{ps} f_{ps} d_{ps} + A_{ns} f_y d_{ns}}{A_{ps} f_{ps} + A_{ns} f_y} \tag{9}$$

Table 1. Reinforcement limits for conventionally reinforced and prestressed concrete sections by currently available methods.

	REINFORCEMENT LIMITS*		
	Maximum for Under-Reinforced Sections	Maximum for Moment Redistribution	Negative Moment Redistribution in percent
Proposed	All cases: $\frac{c}{h} \le 120\varepsilon_{cu}$	All cases: $\frac{c}{h} \le 80\varepsilon_{cu}$	All cases: $20\left(1-\frac{\left(\frac{c}{h}\right)}{120\varepsilon_{cu}}\right)$
	(1) Reinforced Concrete ACI Section 10.3 (a) Rectangular Sections $\rho_{max} \le 0.75\bar{\rho}_b + \rho'\frac{f'_s}{f_y}$ where $\bar{\rho}_b = 0.85\beta_1\frac{f'_c}{f_y}\left(\frac{87\,ksi}{87\,ksi+f_y}\right)$	ACI Section 8.4 $(\rho-\rho') \le 0.5\bar{\rho}_b$	ACI Section 8.4 $20\left(1-\frac{\rho-\rho'}{\bar{\rho}_b}\right)$
	(b) T-Sections $\rho_{max} \le \frac{b_w}{b}\left[0.75\left(\bar{\rho}_b+\rho_f\right)+\rho'\frac{f'_s}{f_y}\right]$ where $\rho_f = \frac{0.85f'_c(b-b_w)h_f}{f_y b_w d}$ $\rho_{max} = \frac{A_{s,\,max}}{bd}$		
ACI 318-83 Code	**(2) Prestressed Concrete** ACI Section 18.8.1 (a) Rectangular Sections $\omega_p + \frac{d_{ns}}{d_{ps}}(\omega-\omega') \le 0.36\beta_1$ where $\omega_p = \frac{A_{ps}f_{ps}}{bd_{ps}f'_c}$, $\omega = \frac{A_{ns}f_y}{bd_{ns}f'_c}$, $\omega' = \frac{A'_s f_y}{bd_{ns}f'_c}$	ACI Section 18.10.4.3 (a) Rectangular Sections $\omega_p + \frac{d_{ns}}{d_{ps}}(\omega-\omega') \le 0.24\beta_1$	ACI Section 18.10.4.1 (a) Rectangular Sections $20\left[1-\frac{\omega_p + \frac{d_{ns}}{d_{ps}}(\omega-\omega')}{0.36\beta_1}\right]$
	(b) T Sections $\omega_{pw} + \frac{d_{ns}}{d_{ps}}(\omega_w-\omega'_w) \le 0.36\beta_1$ where ω_{pw}, ω_w and ω'_w are computed as for ω_p, ω and ω' except that b shall be the web width, and reinforcement area shall be that required to develop compressive strength of web only.	(b) T Sections $\omega_{pw} + \frac{d_{ns}}{d_{ps}}(\omega_w-\omega'_w) \le 0.24\beta_1$	(b) T Sections $20\left[1-\frac{\omega_{pw} + \frac{d_{ns}}{d_{ps}}(\omega_w-\omega'_w)}{0.36\beta_1}\right]$

* Symbols not defined here are given in the Notation section.

is close to Eq. (7), which is a special form of the proposed limit, Eq. (5). This is a good approach if the designer does not mind the work required to calculate d_e. The authors prefer to use total member depth, h, instead of d_e, for the aforementioned reasons.

The Canadian Code[3] endorses the following expression as a ductility limit for prestressed concrete members:

$$\frac{c}{h} \leqslant 0.36 \tag{12}$$

If $d_{ps} < 0.8h$, the Canadian Commentary[3] recommends that Eq. (10) be replaced by the following limit:

$$\frac{c}{d_{ps}} \leqslant 0.6 \tag{11}$$

If $\epsilon_{cu} = 0.003$, the proposed limit, Eq.

Table 1 (cont.). Reinforcement limits for conventionally reinforced and prestressed concrete sections by currently available methods.

	REINFORCEMENT LIMITS		
	Maximum for Under-Reinforced Sections	Maximum for Moment Redistribution	Negative Moment Redistribution in percent
ACI 318 1986 Supplement to Commentary	(1) **Reinforced Concrete** Same as 1983 Code.	Same as 1983 Code.	Same as 1983 Code.
	(2) **Prestressed Concrete** Section 18.8.1 Commentary Same as 1983 Code, or alternatively: $0.85\frac{a}{d_{ps}} \leq 0.36\beta_1$	Section 18.10.4 Commentary Same as 1983 Code, or alternatively: $0.85\frac{a}{d_{ps}} \leq 0.24\beta_1$	Section 18.10.4 Commentary Same as 1983 Code, or alternatively: $20\left[1 - \frac{\left(0.85\frac{a}{d_{ps}}\right)}{0.36\beta_1}\right]$
Naaman	All cases: $\frac{c}{d_e} \leq 0.42$ where $d_e = \frac{A_p f_{ps} d_p + A_s f_y d_s}{A_p f_{ps} + A_s f_y}$	All cases: $\frac{c}{d_e} \leq 0.28$	All cases: $20\left(1 - 2.36\frac{c}{d_e}\right)$
Canadian Code A23.3 1984	(1) **Reinforced Concrete** Section 10.3.3 $\frac{c}{d} \leq \frac{87\text{ ksi}}{87\text{ ksi} + f_y}$	Not specified	(1) Section 8.4 $\left(30 - 50\frac{c}{d}\right) \leq 20$
	(2) **Prestressed Concrete** Section 18.8.1 $\frac{c}{h} \leq 0.5$	Not specified	(2) Section 18.11.2 $\left(30 - 50\frac{c}{d}\right) \leq 20$

(5), reduces to:

$$\frac{c}{h} \leqslant 0.36 \tag{12}$$

Thus, the Canadian Code has a ductility limit similar in form to the one proposed herein.

The Canadian Code also provides the following equation for calculating the ductility limit of conventionally reinforced concrete members:

$$\frac{c}{d} \leqslant \frac{87\text{ ksi}}{87\text{ ksi} + f_y} \tag{13}$$

where d is the depth to the centroid of the tensile reinforcement. Eq. (13) does, however, have a form that is suitable for cross sections of any shape. Eqs. (10) and (13) permit a greater amount of reinforcement than both the ACI Code and the proposed formula [Eq. (5)].

The CEB-FIP Model Code[4] uses the following formula to determine member ductility:

$$\theta_{adm} = \frac{\epsilon_{cu} d}{c} \tag{14}$$

where θ_{adm} is an admissible plastic rotation. Eqs. (2a) and (2b) have the same form as Eq. (14). The effective depth d is not clearly defined in Ref. 4 and a minimum value for θ_{adm} is not given. Thus, a complete comparison with proposed Eq. (5) is not possible.

A summary of currently available methods for calculating the reinforcement limits for under-reinforced sections, moment redistribution, and percentage of negative moment that may be redistributed is given in Table 1. It is seen that the proposed method requires the least computational work and has the widest scope of application.

On the following pages, two numerical examples are given to illustrate the proposed method for determining ductility requirements in a concrete section.

NUMERICAL EXAMPLES

To illustrate the proposed method for determining ductility requirements in a concrete section, two numerical examples are given. The first example covers a prestressed concrete beam containing both prestressed and nonprestressed reinforcement while the second example considers a conventionally reinforced concrete beam. The results of the proposed design approach are compared with four other methods of calculation.

EXAMPLE 1

A modified version of the beam given in Example 4.2.6 of the PCI Design Handbook,[14] shown in Fig. 2, is considered. The proposed approach and four other methods will be used to investigate if the tension reinforcement meets the ductility requirements.

Given: f_c' (precast) = 5 ksi (34.5 MPa)
f_c' (topping) = 3 ksi (20.7 MPa)
Reinforcement is 10 - ½ in. (12.7 mm) diameter 270 ksi (1862 MPa) low relaxation prestressed strands.
A_{ps} = 1.836 in.^{2} (1185 mm^2)
f_{se} = 162 ksi (1117 MPa)
f_y = 60 ksi (413.7 MPa)
A_{ns} = 2 − #8 = 1.58 in.^{2} (1019 mm^2)
A_s' = 2 − #9 = 2.0 in.^{2} (1290 mm^2)

Solution: From the flexural analysis (see for example Ref. 15):
c = 6.57 in. (166.9 mm)
a = 5.51 in. (140 mm)
β_1 average = 0.839
f_{ps} = 253.61 ksi (1749 MPa)
f_s' = 60 ksi (413.7 MPa)

1. Proposed method

From Eq. (5):

$$\frac{c}{h} = \frac{6.57}{26} = 0.253$$

$< 120\,(0.003) = 0.36$ (ok)
Percent of limit used =
100(0.253)/0.36 = 70.3 percent

2. 1983 ACI Code method

The 1983 Code does not apply to this case unless certain assumptions are made. These include definition of the various steel indexes and of an average concrete strength.

3. 1986 Supplement to the ACI 318-83 Commentary

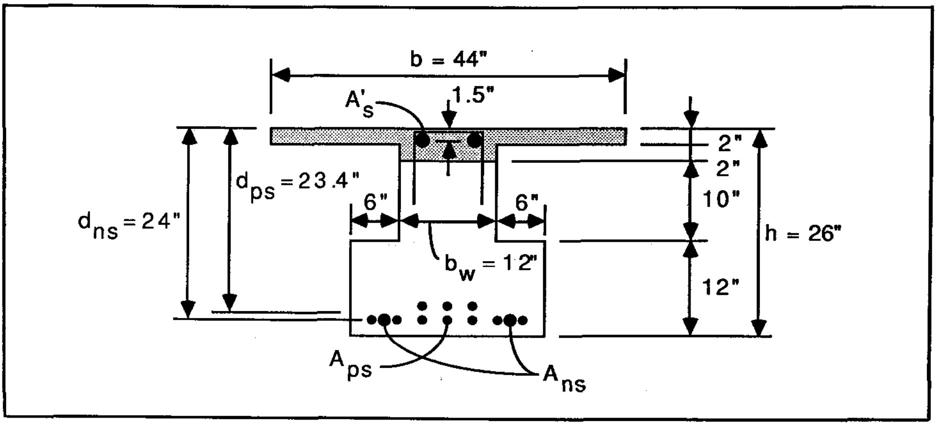

Fig. 2. Partially prestressed beam used in Example 1.

From Eq. (6):

$$0.85\left(\frac{a}{d_{ps}}\right) = 0.85\left(\frac{5.51}{23.4}\right) = 0.200$$

$< 0.36\,(0.839) = 0.302$ (ok)

Percent of limit used = 66.2 percent

4. Naaman's method

From Eq. (9):

$$d_e = \frac{1.836\,(253.61)\,(23.4) + 1.58\,(60)\,(24)}{1.836\,(253.61) + 1.58\,(60)}$$

$= 23.5$ in. (597 mm)

Inserting d_e into Eq. (8) gives:

$$\frac{c}{d_e} = \frac{6.57}{23.5} = 0.279 < 0.425 \text{ (ok)}$$

Percent of limit used = 65.6 percent

5. Canadian Code

From Eq. (10):

$$\frac{c}{h} = \frac{6.57}{26} = 0.253 < 0.5 \text{ (ok)}$$

Percent of limit used = 50.6 percent

This example illustrates the extreme simplicity of the proposed method. Also, the method shows it to be the most conservative. Method 3 overcomes a deficiency and the 1983 ACI Code and is a rational and simple approach. However, it is not applicable to conventionally reinforced sections. Method 4 is identical to Method 3 except that a slightly different depth of steel is used. The Canadian Code is as simple as Method 1 but it is the least conservative of all.

EXAMPLE 2

The ductility requirements for the reinforced concrete beam shown in Fig. 3 will be investigated by five methods. Answers will be expressed in terms of the percent of $A_{ns,max}$ used so that the methods can be accurately compared.

1. Proposed method

From Eq. (5):

$$\left(\frac{c}{h}\right)_{max} = 120\,(0.003) = 0.36$$

$c_{max} = 0.36\,(20) = 7.20$ in. (183 mm)

$a_{max} = \beta_1\, c_{max} = 0.85\,(7.20) = 6.12$ in. (155 mm)

$$A_{ns,max} = \frac{0.85(4)}{60}\,[(76 - 12)4 + 12(6.12)]$$

$= 18.67$ in.^{2} (12,045 mm^2)

Percent of limit used:

100(9.36)/18.67 = 50.1 percent

2. 1983 ACI Code method

$$A_{sf} = \frac{0.85(4)}{60}\,(76 - 12)4$$

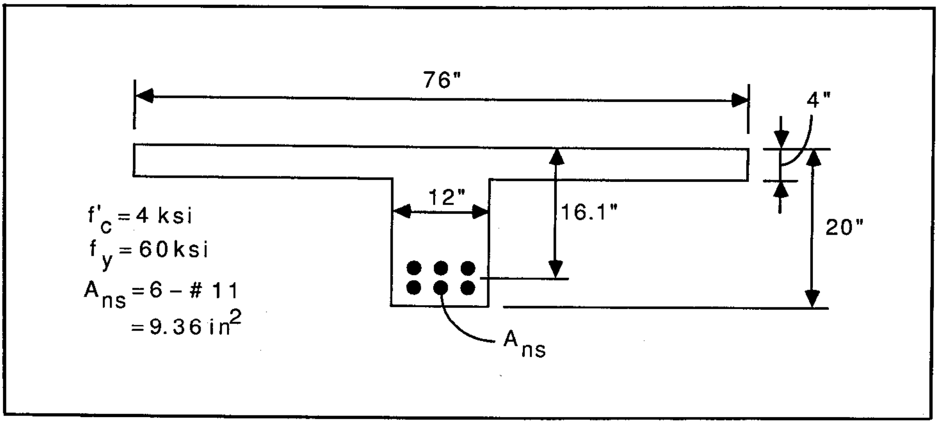

Fig. 3. Conventionally reinforced beam used in Example 2.

$A_{sf} = 14.51$ in.^{2} (9361 mm^2)

The area of steel corresponding to balanced strain conditions for a 12 in. wide rectangular section is:

$$A_{sb} = 0.0285(12)16.1$$
$$= 5.51 \text{ in.}^2 \text{ (3555 mm}^2\text{)}$$

$$A_{ns,max} = 0.75\,(A_{sb} + A_{sf})$$
$$= 15.02 \text{ in.}^2 \text{ (9690 mm}^2\text{)}$$

Percent of limit used = 62.3 percent

3. Modified 1983 ACI Code method

The authors and others[13] recommend that c be limited to $0.75c_b$ for the purpose of maintaining the same minimum section curvature requirements as for rectangular sections.

$$c_{max} = 0.75\,c_b$$
$$= 0.75\left(\frac{\epsilon_{cu}}{\epsilon_{cu} + \epsilon_y}\right) d$$
$$= 0.75\left(\frac{0.003}{0.003 + 0.0021}\right) 16.1$$
$$= 7.10 \text{ in. (180 mm)}$$

$$a_{max} = \beta_1 c_{max} = 0.85\,(7.10)$$
$$= 6.04 \text{ in. (153 mm)}$$

$$A_{ns,max} = \frac{0.85(4)}{60}[(76 - 12)4 + 12(6.04)]$$
$$= 18.61 \text{ in.}^2 \text{ (12,006 mm}^2\text{)}$$

Percent of limit used = 50.3 percent

4. Naaman's method

From Eq. (8):

$$c_{max} = 0.425\,d_e = 0.425\,(16.1)$$
$$= 6.84 \text{ in. (174 mm)}$$

$$a_{max} = \beta_1 c_{max} = 0.85\,(6.84)$$
$$= 5.81 \text{ in. (148 mm)}$$

$$A_{ns,max} = \frac{0.85(4)}{60}[(76 - 12)4 + 12(5.81)]$$
$$= 18.46 \text{ in.}^2 \text{ (11,910 mm}^2\text{)}$$

Percent of limit used = 50.7 percent

5. Canadian Code

From Eq. (13):

$$c_{max} = \left(\frac{87 \text{ ksi}}{87 \text{ ksi} + f_y}\right) d$$
$$= \left(\frac{87}{87 + 60}\right) 16.1$$
$$= 9.53 \text{ in. (242 mm)}$$

$$a_{max} = \beta_1 c_{max} = 0.85\,(9.53)$$
$$= 8.10 \text{ in. (206 mm)}$$

$$A_{ns,max} = \frac{0.85(4)}{60}[(76 - 12)4 + 12(8.10)]$$
$$= 20.01 \text{ in.}^2 \text{ (12,913 mm}^2\text{)}$$

Percent of limit used = 46.8 percent

This example shows that all the methods, except Method 2, give comparable results. Method 2 contains a deficiency in its treatment of nonrectangular sections and should be modified as indicated by Method 3. Methods 1 and 4 are the only two approaches applicable to both reinforced and prestressed concrete. Method 1 is the simplest of all four methods.

* * *

CONCLUSIONS

1. The following ductility limit is recommended for both prestressed and conventionally reinforced concrete flexural members:

$$\frac{c}{h} \leqslant 120\,\epsilon_{cu} \qquad (5)$$

where c is the neutral axis depth at ultimate flexure and h is the total member depth.

2. The proposed limit is simpler and more rational than the current ACI Code limits, and bridges the gap between the conventionally reinforced and prestressed concrete provisions. It is shown to be more conservative than the ACI Code limits for prestressed concrete when $d_{ps} > 0.85h$ and for reinforced concrete when $d_{ns} > 0.816h$, which covers the great majority of practical cases. In addition, it is more reasonable than the ACI Code limit for nonrectangular conventionally reinforced concrete sections, and it allows for adjustments in ϵ_{cu} from the typical value of 0.003 if concrete confinement or other conditions justify it.

3. The proposal by Naaman is an acceptable alternative to the proposed limit. It has the advantage of using an "effective depth" to the center of the tensile stress resultant at ultimate flexure. However, computation of that depth is an extra step that may not be needed for other purposes. Also, no provision is given for a possible increase in the value of ϵ_{cu}.

* * *

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APPENDIX A — NOTATION

The symbols listed below supplement and supercede those given in the ACI 318-83 Code and Commentary (see Refs. 1 and 2).

A_{ns} = area of nonprestressed tension reinforcement

c = distance from extreme compression fiber to neutral axis

c_b = distance from extreme compression fiber to neutral axis under balanced strain conditions

d = depth to centroid of tensile reinforcement, Eq. (13)

d = effective depth of member, Eq. (14)

d_e = equivalent effective depth, Eq. (9)

d_{ns}, d_{ps} = distance from extreme compression fiber to centroid of nonprestressed and prestressed tension reinforcement

h = overall thickness of member

ϵ_{cu} = maximum usable compressive strain at extreme concrete fiber, normally taken equal to 0.003 unless higher values can be justified

ϵ_y = yield strain of mild reinforcement

θ_{min} = minimum required rotation of a member, additional subscripts *ns* and *ps* refer to conventionally reinforced and fully prestressed members, respectively

ρ_{max} = $0.75\ \rho_b$

ψ = section curvature, Eq. (1)

* * *

APPENDIX B — PROPOSED ACI 318-83 CODE AND COMMENTARY REVISIONS

Proposed Code Revisions

It is proposed that the following notation be changed.

Section 8.0: Add:

c = distance from extreme compression fiber to neutral axis

h = overall thickness of member

ϵ_{cu} = maximum usable compressive strain at extreme concrete fiber, normally taken equal to 0.003 unless higher values can be justified

Section 10.0: Add definitions for c, h, and ϵ_{cu}.

Section 18.0: Delete ω_p, ω_w, ω_{pw}, and ω_w', and add definitions for c and ϵ_{cu}.

It is proposed that the following sentences be added at the ends of Sections 8.4.1, 8.4.3, and 10.3.3:

"**8.4.1** — . . . Alternatively, the percent change may be taken as follows:

$$20\left[1-\frac{\frac{c}{h}}{120\,\epsilon_{cu}}\right] \text{ percent}$$

8.4.3 — . . . Alternatively, the section must be designed so that (c/h) is not greater than $80\,\epsilon_{cu}$.

10.3.3 — . . . Alternatively, this ductility requirement may be satisfied if (c/h) is not greater than $120\,\epsilon_{cu}$."

It is proposed that Sections 18.8.1, 18.8.2, 18.10.4.1, and 18.10.4.3 be changed to read as follows:

"**18.8.1** — Amount of prestressed and nonprestressed reinforcement used for computation of moment strength of a member, except as provided in Section 18.8.2, shall meet the following ductility criterion: $(c/h) \leq 120\,\epsilon_{cu}$.

18.8.2 — When the criterion of Section 18.8.1 is not met, the design moment strength shall not exceed the moment strength based on the compression portion of the moment couple.

18.10.4.1 — Where bonded reinforcement is provided at supports in accordance with Section 18.9.2, negative moments calculated by elastic theory for any assumed loading arrangement may each be increased or decreased by not more than:

$$20\left[1-\frac{\frac{c}{h}}{120\,\epsilon_{cu}}\right] \text{ percent}$$

18.10.4.3 — Redistribution of negative moments shall be made only when the section at which moment is reduced is designed such that $(c/h) \leq 80\,\epsilon_{cu}$."

Proposed Commentary Revisions

It is proposed that the following changes be made. Insert the following paragraph at the end of Section 8.4:

"The adoption of an alternative ductility limit in Section 10.3.3 required a corresponding change in Sections 8.4.1 and 8.4.3."

Insert the following paragraph at the end of Section 10.3.3:

"A discussion of the development of the alternative ductility criterion is given in Ref. A.*

The new criterion offers the following advantages:

(a) It is a unified approach for the entire spectrum of structural members, from conventionally reinforced to fully prestressed.

(b) It is relatively easy to use as the quantities c and h are products of the standard design process.

(c) The limit is valid for composite and noncomposite sections of general shape.

(d) It is more conservative than the prior limit when $d < 0.816h$, which covers the most common cases in practice.

(e) It offers designers a clearer picture of the factors influencing ductility, such

*Ref. A is the same as this paper.

as increasing ϵ_{cu} by confining concrete in compression, or decreasing c by increasing concrete compressive strength.

(f) It offers a consistent approach to the calculation of curvature ductilities in all section shapes. The limits in prior editions of the code did not always offer consistent curvature ductilities for flanged sections, as compared with rectangular sections."

Revise Section 18.8.1 to read as follows:

"**18.8.1** — A new ductility limit was adopted for this edition of the code. The new criterion is equivalent to the earlier one for noncomposite rectangular and flanged sections when $\epsilon_{cu} = 0.003$ and $d_p = 0.85h$. It is more conservative when $d_p > 0.85h$, which covers the great majority of flexural members used in practice. The prior steel indexes were confusing for nonrectangular sections and not capable of providing the correct ductility value for composite sections. Additional information and advantages of this approach can be found in Section 10.3.3."

In Section 18.10.4, delete the second and third paragraphs and insert the following paragraph after the first paragraph:

"The adoption of a new ductility limit in Section 18.8.1 of the code required a corresponding change in the allowable percent of moment redistribution and ductility criterion for Sections 18.10.4.1 and 18.10.4.3, respectively. The amount of redistribution allowed depends on the ability of the critical sections to deform inelastically by a sufficient amount. Serviceability under service loads is taken care of by the limiting stresses of Section 18.4. The choice of $80\epsilon_{cu}$ as the limiting ductility value, for which redistribution of moments is allowed, is in agreement with the requirements for conventionally reinforced concrete stated in Section 8.4.3."

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NOTE: Discussion of this article is invited. Please submit your comments to PCI Headquarters by August 1, 1989.