

Recommended Practice for the Design of Prestressed Concrete Columns and Walls

prepared by

**PCI Committee on Prestressed
Concrete Columns**

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COMMITTEE STATEMENT

These recommendations for prestressed concrete columns and walls are written in a code format. In the main body of the recommendations, background details and suggestions for satisfying the requirements are not included. Commentaries are included in selected sections for the purpose of amplification of intent. There are 12 chapters in this Recommended Practice plus sections on notation, selected references, a bibliography and sample problems.

References citing research used in developing these recommendations are included for the user desiring to study individual topics in greater detail. A bibliography listing additional references related to prestressed concrete column and wall design has also been included.

An Appendix section contains numerical design examples (together with design aids) to show how the provisions of the report can be applied in practice.

PREFACE

The provisions given in these recommendations are intended to cover minimum requirements for the design of prestressed concrete compression members (columns and walls) in which the use of prestressing steel is of prime importance to ensure their stability and strength.

These recommendations supersede "Recommended Practice for the Design of Prestressed Concrete Columns and Bearing Walls," published in the PCI JOURNAL, V. 21, No. 6, November-December 1976.

COMMENTARY

The development of prestressed concrete has led to the use of prestressed compression elements: columns, walls and piles. Prestressing a structural member designed to carry compression may seem contradictory because some of the capacity of the concrete is "used up" by the application of the prestressing force. The effective prestress levels in columns and walls seldom exceed 10 percent of the concrete compressive strength, and therefore, prestressing has negligible effect on the axial load carrying capacity. Often, prestressed compression members, especially wall panels, support low axial loads and high bending moments. In such cases, pre-

stressing may prove to be beneficial. The use of prestressing strands is much more economical than deformed reinforcing bars in a precasting plant, and furthermore the ACI Building Code waives the minimum reinforcement requirement for prestressed compression members. Under plant controlled conditions, it is less costly to increase member capacity by increasing concrete strength than to increase capacity with added reinforcement.

Columns and walls may have dimensions governed by architectural or fire rating requirements and other conditions, such as thermal insulation or constructability, not dependent on stability or stress. In such cases, manufacturers may elect to prestress the elements merely to avoid cracking during transportation and erection, or for economy in manufacture. In such circumstances, members so prestressed are not properly classed as "prestressed columns." These recommendations (in particular, the minimum prestress of 225 psi) are not intended to apply to those situations. Design should be based, instead, on ACI 318-83,^{1,2} which gives minimum steel requirements.

This recommended practice, prepared by the PCI Committee on Prestressed Concrete Columns, updates the Committee's previous documents.^{3,4}

* * *

NOTATION

C_m	= a factor relating actual moment diagram to an equivalent uniform moment diagram		
D	= dead loads, or related internal moments and forces	M_{2s}	= value of larger factored end moment on compression member due to loads that result in appreciable sidesway, calculated by conventional elastic frame analysis
E_c	= modulus of elasticity of concrete, psi		
EI	= flexural stiffness of compression member		
E_s	= modulus of elasticity of reinforcement, psi	n	= ratio of modulus of elasticity of steel to modulus of elasticity of concrete = E_s/E_c
f'_c	= specified compressive strength of concrete, psi	P_b	= nominal axial load strength at balanced strain conditions
$\sqrt{f'_c}$	= square root of specified compressive strength of concrete, psi	P_c	= critical load (Euler)
f'_{ci}	= compressive strength of concrete at time of initial prestress, psi	P_n	= nominal axial load strength at given eccentricity
f_{ps}	= stress in prestressed reinforcement at nominal strength	P_o	= nominal axial load strength at zero eccentricity
I	= moment of inertia of section resisting externally applied factored loads	P_u	= factored axial load at given eccentricity $\leq \phi P_n$
I_g	= moment of inertia of gross concrete section about centroidal axis, neglecting reinforcement	r	= radius of gyration of cross section
k	= effective length factor for compression members	β_d	= absolute value of ratio of maximum factored dead load moment to maximum factored total load moment, always positive
l_u	= unsupported height of compression member	δ_b	= moment magnification factor for frames braced against sidesway, to reflect effects of member curvature between ends of compression member
L	= live loads, or related internal moments and forces	δ_s	= moment magnification factor for frames not braced against sidesway, to reflect lateral drift resulting from lateral and gravity loads
M_c	= factored moment to be used for design of compression member	η	= correction factor applied for stiffness accounting for P_u/P_o ratio
M_{1b}	= value of smaller factored end moment on compression member due to the loads that result in no appreciable sidesway, calculated by conventional elastic frame analysis, positive if member is bent in single curvature, negative if bent in double curvature	θ	= correction factor for stiffness accounting for flanges of the cross section
M_{2b}	= value of larger factored end moment on compression member due to loads that result in no ap-	λ	= correction factor applied to the gross stiffness of cross section
		ϕ	= strength reduction factor

* * *

CHAPTER 1 — DEFINITIONS

1.1 In addition to the definitions given in Chapter 2 of ACI 318-83, the supplemental definitions in Sections 1.2 to 1.6, herein, are for clarification.

1.2 Beam-Column — Structural element subject to axial compressive loads in combination with flexure.

1.3 Column — A vertical member in which the ratio of the larger overall cross-sectional dimension to the smaller is equal to or less than 3.0, and in which the height is greater than three times the least lateral dimension.

1.4 Flat Wall — A vertical member in which the cross-sectional ratio, defined in Section 1.3, is greater than 3.0, and the section is of constant thickness in the direction of the smaller dimension.

projection, horizontal, level or having no curved surface. It is current industry practice to speak of flat walls and this term will encompass walls that are slightly curved as used for fluid or solid material storage.

1.5 Ribbed Wall — A vertical member in which the load is distributed between all or part of a flat wall section and a monolithic cast rib(s).

1.6 For the purpose of these recommendations, the height of columns and walls is defined as the vertical spanning dimension and the length (width) of walls is defined as the horizontal spanning dimension.

COMMENTARY

The previous committee report⁴ defined the vertical members considered here as "thin" walls. A hesitation to use the word flat existed because of its connotations of expressing smooth, without

COMMENTARY

In developing these recommendations it was realized that the word "length" applied to a wall would normally be associated with the plan dimension; thus, the words "height" and "width" seem more appropriate.

* * *

CHAPTER 2 — SCOPE

2.1 These recommendations apply to columns and walls prestressed with high strength steel meeting the requirements for prestressing steels in Section 3.5.5 of ACI 318-83.

2.2 All provisions of ACI 318-83 not specifically excluded, and not in conflict with the provisions of these recommendations, are to be considered applicable to prestressed concrete columns and walls.

2.3 The following provisions of ACI 318-83 do not apply to prestressed concrete columns and walls, unless specifically noted: Sections 8.4, 8.10.2, 8.10.3, 8.10.4, 8.11, 10.3.2, 10.3.3, 10.5, 10.6, 10.9.1, 10.9.2 and Chapters 13 and 14.

2.4 Design formulas and permissible stresses included in these recommendations are for concrete columns and walls prestressed with bonded tendons.

2.5 Columns and walls containing unbonded tendons should be designed on the basis of a rational analysis, or load test, in which the ultimate flexural strength of the member, including slenderness effects, is properly taken into account. In all cases where the service load is governed by tension, minimum bonded reinforcement in the precompressed tensile zone should be provided according to Section 12.7 of these recommendations.

COMMENTARY

Available research data do not permit development of simplified empirical procedures for the design of columns and walls with unbonded tendons. Despite this shortcoming, the Committee believes that the use of unbonded tendons should not be discouraged. However, design of such members should be based on rational analysis or load test.

The requirement for minimum bonded reinforcement in Chapter 12 is included to prevent the formation, at the service load level, of a single crack at the critical sections.

2.6 The provisions of these recommendations apply to columns and walls which have a minimum average prestress, after all losses, of 225 psi on the gross section. Columns and walls stressed to a lower nominal prestress, used to control cracking and to facilitate handling, shall have minimum reinforcement in accordance with Section 10.9 of ACI 318-83 for columns and Sections 11.10 or 14.3 of ACI 318-83 for walls.

COMMENTARY

The requirement of a minimum average prestress of 225 psi is in conformance with Section 18.11.2 of ACI 318-83. The Commentary on Building Code Requirements for Reinforced Concrete² (ACI 318-83) also deals with this question in detail. Gross section properties may be used in lieu of the uncracked transformed section properties. Structural members in which prestress is added primarily for handling conditions do not fall within the scope of this report.

2.7 When the slenderness ratio, kl_u/r , is below the lower bounds given in Section 8.1.6, the effects of column or wall slenderness may be neglected. For those members with slenderness greater than the upper bound in Section 8.1.6, the provision for approximate slenderness evaluation with these recommendations is not allowed.

* * *

CHAPTER 3 — GENERAL CONSIDERATIONS

3.1 Prestressed concrete columns and walls must be designed for all the forces to which they are subjected and their behavior should be considered with respect to lateral displacement, end conditions, repetitive loading, axial shortening, effects of creep, shrinkage, temperature changes, foundation settlement, cracking, construction or handling loads and required strength.

COMMENTARY

The recommendations for the design of members subject to axial loads and bending are in conformance with the general requirements of Section 18.2 of ACI 318-83.

3.2 Consideration should also be given, where applicable, to requirements for durability, fire resistance and, in the case of walls, for watertightness and insulation. Requirements in Section 4.2 and Chapter 5 of ACI 318-83 should be considered.

3.3 Permissible stresses specified in these recommendations may be exceeded if the design strength of the column or wall member is shown by test or rational analysis to provide the minimum load factors specified in ACI 318-83 taking into account the proper capacity reduction factor, ϕ , and that the performance will not be otherwise impaired.

COMMENTARY

The purpose of the capacity reduction factor is to account in part for: (a) inaccuracies in the methods of calculating design strengths, (b) for variability of the design strength, (c) for the importance of the member in the structure and (d) for the type of potential failure — whether ductile or brittle.

When the design strength has been determined experimentally, all but the

first of the factors listed above are still present. Therefore, the observed mean strength should still be reduced by the capacity reduction factor before comparing the code capacity with factored load effects.

3.4 Members should meet the strength requirements specified in Chapter 9 of ACI 318-83. Special attention should be given to load factors of the local jurisdiction if different from those in ACI 318-83.

3.5 Design should be based on strength and checked for behavior at various service load conditions that may be critical during the life of the structure from the time the prestress is first applied.

3.6 Strength design should be performed by rational analysis, considering equilibrium of forces and compatibility of strains, and based upon accepted values for the mechanical properties of the steel and the concrete.

3.7 All cross sections should be designed for the applied axial compressive load and the accompanying bending moment, with consideration of the slenderness effects.

3.8 Stresses due to prestressing and stress increases due to any change in the cross section should be considered in the design.

3.9 Where grouted tendons are used for prestressed columns or walls, the possibility of member buckling between the points where the concrete and the prestressing steel are in contact should be considered during and after the prestressing operation and until the grouting is complete and has achieved 75 percent of its strength.

3.10 The effect of local buckling of segments of a wall member should be investigated when evaluating the overall stability of the wall.

(see Commentary on next page)

COMMENTARY

Wall panel segments are generally attached at discrete points which are intended to make the entire wall act as an integral unit. Local buckling of the wall, or buckling of the segment before it is integral with the rest of the wall, should be investigated.

3.11 In earthquake resistant structures, a portion of the steel, either prestressed or nonprestressed, should be bonded to the concrete so that under design load conditions, the critical sections will meet the ductility requirements for seismic design as given in Appendix A, ACI 318-83.

CHAPTER 4 — BASIC ASSUMPTIONS

4.1 The strength design of columns and walls for flexural and axial forces should be based upon the applicable assumptions given in Sections 10.2, 10.3, 18.2 and 18.3 of ACI 318-83, and should satisfy conditions of equilibrium of forces and compatibility of strains.

COMMENTARY

The assumptions in Sections 10.2, 10.3, 18.2 and 18.3 of ACI 318-83 are

well accepted and the recommendations in this report endorse these assumptions. Repetition of these assumptions in this chapter is not considered necessary. General requirements and design guidelines for prestressed concrete, outlined in Chapter 18 of ACI 318-83, should also be followed. Other background information on the required strength calculations can be found in the bibliography at the end of this report.

CHAPTER 5 — LIMITING DIMENSIONS

5.1 The limiting dimensions for prestressed concrete columns and walls should be determined by taking into account local and overall stability as well as requirements for concrete placement, the effectiveness of lateral reinforcement (if required), steel protection, fire protection and insulation.

COMMENTARY

The Committee believes that there is no need to recommend a minimum thickness for walls or a minimum dimension for columns. However, when lateral ties are used in a member, the minimum practical column or wall dimension for proper development of a closed tie is about 8 in.

Designers of walls should consider the practical limitations of fabrication, handling, concrete placement and fire protection in selecting the thickness of a wall panel. The Committee suggests a practical overall minimum wall thickness of 3 in.

5.2 Calculation of deflections should be in accordance with Sections 9.5.4.1 and 9.5.4.2 of ACI 318-83.

5.3 The dimensions of columns and walls should be such that, under service load conditions, nonstructural elements attached to columns or walls would not be damaged by lateral deflection and the stresses in the concrete do not exceed the stresses in Chapter 9.

COMMENTARY

In previous committee reports, provisions were given for checking the lateral deflections of the prestressed column or wall in order to establish the minimum section under service load conditions. These provisions for service load deflection calculations were eliminated from this report because the permissible stresses listed in Chapter 9 produce essentially a redundant calculation of minimum section dimensions.

5.4 Calculated lateral deflections of prestressed concrete compression members under factored load should not exceed $l_w/100$.

COMMENTARY

A deflection limitation under factored load was introduced because a *P*-Delta analysis can sometimes indicate large and unreasonable deflections before stability failure.

In calculating deflection, the nonlinear geometric and material properties of

a prestressed compression member should be recognized. The nonlinear effect may be aggravated by an eccentric prestressing force and cracking. Guidance for calculating deflections can be found in the PCI Design Handbook⁸ or references on stability of compression members.⁶⁻¹⁰

The Committee believes that deflections should be checked under factored load particularly in seismic areas where structural elements have greater potential of being loaded to their ultimate capacity and the probability of cracking is higher.

The Committee notes that it is less likely that prestressed columns and/or walls subjected to wind loads would have the section dictated by a deflection criterion.

The deflection limit is based on information in a test report on slender walls.¹¹

5.5 Limits on compression member slenderness are given in Section 8.1.6 of these recommendations.

CHAPTER 6 — EFFECTIVE DIMENSIONS OF WALLS

6.1 For walls, the effective width (the portion of the wall to be considered as effective) for design of members to accommodate each concentrated load or moment should be determined by rational analysis.

In lieu of a rational analysis, the effective width should not exceed:

- The center-to-center distance between loads;
- The length of the loaded portion plus six times the wall thickness on either side;
- The width of the rib (in ribbed wall panels) plus six times the thickness of the wall between ribs on either

side of the rib; or

- 0.4 times the actual height of the wall.

The effect of local stresses in the vicinity of the applied load should be investigated.

COMMENTARY

The provisions in Section 6.1 are not in conformance with Section 14.2 of ACI 318-83, where the width of the wall to be considered as effective for concentrated loads is the width of the bearing plus four times the wall thickness. A theory of elasticity solution to the effec-

tive width is given in Ref. 12. Other more recent analytical studies, based on finite element analysis,¹³ have shown that larger effective widths can be justified. Likewise, recent research at the University of Texas at Arlington indicates that even wider effective widths are appropriate.¹⁴

The use of greater effective widths than suggested here may result in local overstressing (crushing) of the wall below the concentrated load.

This provision first appeared in the previous committee report⁴ and has not produced deficient designs to the Committee's knowledge. This successful industry experience further justifies the liberalization.

6.2 If loading on the wall is uniform, the full width can be considered effective for resisting axial load and moment.

COMMENTARY

Full width section properties are permitted by ACI 318-83 for precast elements subject to flexural loading and are logically extended here to include axial load.

6.3 When the noneffective portion of a ribbed wall, excluded in Section 6.1c, is

carrying compressive load, the strength and slenderness of the noneffective portion should be determined as for a flat wall without considering any support from nearby ribs. This provision may be waived if shown by rational analysis or test that the ribs can provide lateral support.

6.4 The effective width of sandwich panels should be the same as outlined in Sections 6.1, 6.2 and 6.3 of these recommendations. The effective thickness of a sandwich panel may be assumed equal to the thickness of the equivalent solid flat wall, provided the sandwich layers are connected by integrally cast ribs or adequate mechanical shear connectors for full composite action. If the shear connection between wythes of a sandwich panel are less than required for full composite action, the effective thickness of the panel should be taken as the thickness of only one wythe of the panel.

COMMENTARY

In the absence of published literature or test results, the provisions of masonry codes¹¹ pertinent to bonding and tying of masonry wall wythes may be used as a guide in establishing adequate mechanical shear connectors for full composite action.

CHAPTER 7 — SLENDERNESS EFFECTS

7.1 The design of compression members should be based on factored forces and moments determined from an analysis of the structure. Such an analysis should take into account the influence of axial loads and variable moments of inertia on member stiffness and fixed-end moments, the effect of deflection on moments and forces, and the effects of duration of loads. The loads induced by prestressing should be considered with a load factor of 1.0.

7.2 In lieu of the procedure described in Section 7.1, the design of compression members described in this recommendation may be based on the approximate procedure for the evaluation of slenderness presented in Chapter 8.

COMMENTARY

The use of second order (*P*-Delta) analysis to evaluate slenderness is recommended.¹⁵⁻¹⁷ The analysis is usually

done by calculating deflections using elastic analysis methods, but with factored loads. When using the *P*-Delta method, however, often difficult judgments are needed to estimate an appropriate stiffness (EI). The *P*-Delta method for establishing member stability effects usually gives satisfactory results in simple cases. However, when the loading cases produce loads and moments that are close to the critical stiffness value at cracking, the instability point may be missed by using an incorrect stiffness (EI) with the *P*-Delta method unless stiffness is continuously updated. Therefore, the Committee encourages the use of more sophisticated methods of analysis such as iterative computer programs fashioned after the procedures outlined by Nathan.¹⁸ Com-

mercially available computer programs exist for making the more sophisticated analyses.

Likewise, evaluating slenderness effects using the approximate procedures in ACI 318-83 for columns has shown that Section 10.11 of ACI 318-83 renders the design of some prestressed columns and walls unconservative.¹⁹ This is particularly true of sections that are unsymmetrical with respect to the axis of bending, such as double-tee wall members.

Studies at the University of British Columbia by Nathan^{18,20,21-23} with the PCI Committee on Prestressed Concrete Columns serving in an advisory capacity, form the basis of the recommendations for approximate evaluation of slenderness in Chapter 8.

CHAPTER 8 — APPROXIMATE EVALUATION OF SLENDERNESS EFFECTS

8.1 General

8.1.1 The unsupported height l_u of a compression member should be taken as the clear distance between floor slabs, beams, girders, or other elements capable of providing lateral support for the member.

mension in the direction in which stability is being considered for rectangular members, and 0.25 times the diameter for circular members. For other shapes, r may be computed for the gross section considering the effective dimensions as defined in Chapter 6.

COMMENTARY

The general procedure follows that of ACI 318-83. Deviations from those procedures will be noted and discussed. The designer must determine which elements, such as bearing pads between structural members, provide lateral support.

8.1.2 Where column capitals or haunches are present, the unsupported height should be measured to the lower extremity of the capital or haunch in the plane considered.

8.1.3 The radius of gyration r may be taken equal to 0.30 times the overall di-

COMMENTARY

In the design of members for axial load, Section 6.1 defines an effective width for flanged cross sections. When designing flanged members for pure flexure,⁵ however, it has been customary to use the full width of flange. Thus, it is not obvious what width should be used in stability calculations.

The effect of including additional flange area in these computations is to increase the critical buckling load and, therefore, reduce the moment magnification factor. However, use of the effective widths defined in Sections

6.1 and 6.2 may be unduly conservative in certain cases.

If the radius of gyration is based on the full width of the flanged section, r is generally smaller than that based on effective width section properties. Consequently, the slenderness ratio will be more conservative if the full width is used. Use of the full gross cross section for members with small axial load, relative to axial capacity, is usually appropriate.

8.1.4 For members braced against sidesway, the effective length factor k may be taken as 1.0 unless an analysis shows that a lower value may be used.

COMMENTARY

The commentary for ACI 318-83² Section 10.11.2 describes equations and charts that may be considered as analytical justification for k less than 1.0.

8.1.5 For members not braced against sidesway, the effective length factor k should be determined with due consideration of the effects of cracking and reinforcement on relative stiffness, and should be equal to or greater than 1.0.

COMMENTARY

The commentary for ACI 318-83² Section 10.11.2 describes equations and charts that can be used to account for restraint at the ends of the compression member.

8.1.6 For members braced against sidesway, the effects of slenderness may be neglected when kl_u/r is less than $[25-10 (M_{1b}/M_{2b})]$. For members not braced against sidesway, the effects of slenderness may be neglected when kl_u/r is less than 15. For all members with kl_u/r greater than 150, a rational analysis should be performed to evaluate slenderness effects.

COMMENTARY

The lower limits of slenderness stated are less than those in ACI 318-83. The reduction in the unbraced case follows the trend set by the Canadian concrete code²⁴ where all unbraced columns must consider slenderness. The approximate methods in Section 8.2 have been empirically fit to slenderness as low as 25.

The analytical work¹⁹ used for these recommendations showed that strength reduction due to slenderness effects for prestressed columns and walls was significantly more than the ACI recommendations. Analytical studies have clearly shown that the prestressed member has greater strength reduction for slenderness and is more sensitive to slenderness effects than the reinforced concrete columns considered by ACI 318-83.

The upper limit of slenderness has increased from the previous committee report because the slenderness equations have been empirically fit to include a slenderness of 150.²¹

8.1.7 For eccentrically prestressed members, consideration should be given to the effect of lateral deflection due to prestressing in determining the magnified moment.

COMMENTARY

The eccentric prestressing force may cause camber which must be taken into account in computing the magnification factor.

It is conservative to underestimate the prestress losses due to causes other than elastic shortening for such calculations.

8.1.8 The evaluation of the slenderness effects by approximate methods is not recommended for members having unbonded prestressing tendons.

COMMENTARY

The Committee has limited informa-

tion on the behavior of column members having unbonded prestressing.²⁵ Therefore, the applicability of the approximate method for this case is not yet verified.

8.1.9 In addition to Sections 8.1.1 to 8.1.8 of this recommended practice, Sections 10.11.5 to 10.11.7 of ACI Code 318-83 should apply to all designs performed by the approximate method.

8.2 Moment Magnification Factors

8.2.1 Members subject to axial load and flexure should be designed using the factored axial load P_u from a conventional frame analysis and a magnified factored moment M_c defined by:

$$M_c = \delta_b M_{2b} + \delta_s M_{2s} \quad (8.1)$$

$$\delta_b = \frac{C_m}{1 - P_u / \phi P_c} \geq 1.0 \quad (8.2)$$

$$\delta_s = \frac{1}{1 - \Sigma P_u / \phi \Sigma P_c} \geq 1.0 \quad (8.3)$$

and

$$P_c = \frac{\pi^2 EI}{(kl_u)^2} \quad (8.4)$$

COMMENTARY

The general procedure for determining the design bending moment in an axially compressed member follows ACI 318-83. A detailed description of the terms and the commentary pertaining to these provisions are not repeated here but can be found in ACI 318-83 and its commentary.^{1,2} Subscripts "b" and "s" represent braced and sway (unbraced) frames, respectively. For sway frames the effect of story stability must be checked. The δ_s computed is for the entire story based on the use of $\Sigma P_u / \Sigma P_c$.

8.2.2 In lieu of detailed analytical calculations, EI in Eq. (8.4) may be taken

as:

$$EI = \frac{E_c I_g / \lambda}{(1 + \beta_d)} \quad (8.5)$$

$$\lambda = \eta \theta \geq 3.0 \quad (8.6)$$

$$\eta = 2.5 + \frac{1.6}{P_u / P_o} \quad (8.7)$$

$$6 \leq \eta \leq 70$$

For cross sections without a compression flange:

$$\theta = \frac{27}{(kl_u/r)} - 0.05 \quad (8.8)$$

For cross sections with a compression flange:

$$\theta = \frac{35}{(kl_u/r)} - 0.09 \quad (8.9)$$

COMMENTARY

Provisions for calculating EI have changed significantly since the previous publication of this recommended practice.⁴ Development of the EI equation is based on analytical studies conducted by Nathan.¹⁸ The equation for EI is a best fit of analytical results for prestressed concrete columns and wall panels. The equations are intended to apply to members where the average prestress is equal to or greater than 0.50 f_{ps} .

In the commentary to ACI 318-83, it is noted that the EI values were derived for columns with relatively large ratios of P_u / P_o and for column loads above the balance point of the interaction diagram (P_b). In a prestressed member, particularly in a lightly reinforced wall panel with a wide compression flange, it is known that these members have even higher values of P_b / P_o than reinforced concrete columns. Because the normal design range values of P_u / P_o in a prestressed column are considerably lower than those for the reinforced concrete

$$\eta = 2.5 + \frac{1.6}{P_u/P_o}$$

$$6 \leq \eta \leq 70$$

$$\lambda = \eta \theta$$

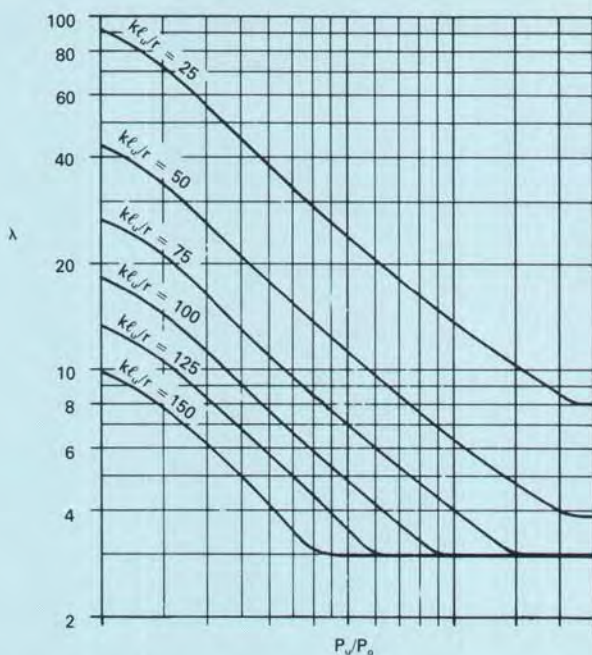
for P_c equation:

$$EI = \frac{E_c I_g \lambda}{1 + \beta_a}$$

(a) Compression flange

$$\theta = \frac{35}{k\ell_e/r} - 0.09$$

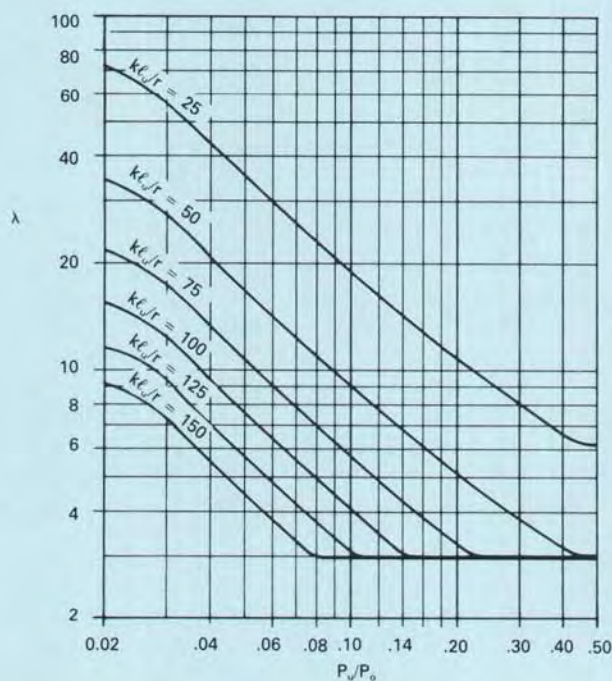
Fig. 8.1



(b) No compression flange

$$\theta = \frac{27}{k\ell_e/r} - 0.05$$

Fig. 8.2



Coefficients, λ , for modified EI

column, the need to modify the ACI 318 provision existed. The difference in EI for the prestressed column and the reinforced column were significant enough to develop unique EI equations for the prestressed column.

In order to retain the form of the equations used in ACI 318-83, it was necessary to include the effect of the strength reduction factor ϕ and the long-term load factor β_d into Eqs. (8.7), (8.8) and (8.9) because Eqs. (8.2) [or Eq. (8.3)] and (8.5) did not properly account for their influence. The ϕ and β_d factors were included in λ because the moment-curvature diagram for a prestressed column or wall is significantly different than the typical reinforced concrete column that was used to develop the ACI stiffness equations.

Nathan's 1985 paper¹⁸ provides additional discussion of this matter.

Design aids for determining λ are shown in Figs. 8.1 and 8.2.

8.2.3 For members braced against side-sway and without transverse loads between supports, C_m in Eq. (8.2) may be taken as:

$$C_m = 0.7 + 0.3 (M_{1b}/M_{2b}) \geq 0.4 \quad (8.10)$$

For all other cases, C_m should be taken as 1.0.

COMMENTARY

Analytical studies by Nathan¹⁹ have indicated that the above expression is a better fit for the bending moment distribution coefficient in Eq. (8.2) than that in ACI 318-83 Eq. (10-12).

CHAPTER 9 — PERMISSIBLE STRESSES IN CONCRETE AND PRESTRESSING STEEL

9.1 Procedures for investigating stresses at transfer of prestress, at service load, at any other loading condition during handling, or under service conditions during the life of the member should be based on the design assumptions given in Section 18.3 of ACI 318-83.

9.2 Computed flexural stresses in a prestressed compression member should not exceed the limits in ACI 318-83 Section 18.4.1 for transfer and Section 18.4.2a and b for service loads. Section 18.4.2c is not applicable. If exposed surfaces are to remain free of discernible cracks, the allowable tensile stresses in normal weight concrete should not exceed $5\sqrt{f'_c}$.

COMMENTARY

Stresses in the prestressed column or wall are not allowed to exceed the limitations specified in Sections 18.4.1 and 18.4.2a and b of ACI 318-83 nor is the

waiver clause of ACI 318-83 Section 18.4.3 permitted. Slender prestressed columns and walls can be sensitive to initial crookedness and cracking and since ACI 318-83 does not specifically address these items, the rigid restrictions on stress limitations in the concrete are recommended by the Committee.

Permissible stress limitations must be compared to stresses caused by load including the effects of secondary bending moment. Service load stress calculations are based on elastic behavior; therefore, the cross section must remain essentially uncracked and linearly elastic. An increase in allowable stresses due to wind or any incidental load is not allowed.

If lightweight aggregate concrete is used, the allowable stress should be modified with the factors in ACI 318-83 Section 11.2.1.2.

9.3 Permissible stresses in Section 9.2 should not be exceeded for handling, transportation or erection. If the member is designed to carry tension without cracking under service conditions, the member should not be allowed to crack under handling, transportation or erection.

COMMENTARY

Handling and storage conditions must not be conducive to causing permanent, nonrecoverable deformations.

9.4 Permissible stresses in the prestressing steel should be in accordance with Section 18.5 of ACI 318-83.

CHAPTER 10 — BEARING STRESSES

10.1 In calculating bearing stresses, the average prestress in the vicinity of the loaded area at the time of loading should be taken into consideration in addition to the design bearing stresses under the loaded area.

COMMENTARY

The contact area between supporting and supported structural elements should be checked to ascertain that local crushing of concrete does not occur. When the contact area occurs at the top of a prestressed column, or especially along the top of a flat prestressed wall panel, the possibility of local crushing in the vertical (supporting) element is higher than in the horizontal (supported) element because of the additional stresses caused by the transfer of the prestressing tendon force.

10.2 The design compressive bearing strength of concrete should be in accordance with Section 10.15.1 of ACI 318-83 and the PCI Design Handbook, Third Edition.

COMMENTARY

Bearing strength calculated using ACI 318-83 deals only with the load applied perpendicular to the bearing support surface, while the PCI Design Handbook⁵ includes loads applied parallel to the support surface.

10.3 When the bearing strength is exceeded in a member, reinforcement should be provided in accordance with recommendations in the PCI Design Handbook, Third Edition.

10.4 Panels or columns should be reinforced for horizontal tensile forces, nominally perpendicular to the direction of the concentrated gravity load, with an additional area of steel in accordance with the PCI Design Handbook, Third Edition.

COMMENTARY

Axial tension caused by restrained shrinkage should be accounted for by placing reinforcement in the direction of the tensile force and/or perpendicular to the potential crack. This additional reinforcement is particularly important to avoid accidental spalling and cracking at the ends of thin stemmed members.

10.5 Post-tensioning anchorages should be reinforced in accordance with ACI 318-83 Section 18.13. Reinforcement required to resist stresses in the concrete due to prestressing strands or tendon anchorages should be additive to the reinforcement required for the design loads in Section 10.3.

COMMENTARY

Reinforcement in the concentrated load areas of a member is intended to

resist bursting, horizontal splitting and spalling forces.²⁶

10.6 Concentrated loads should not be located at or in the vicinity of post-tensioning anchorages, unless it is shown analytically or experimentally that the performance of the anchorage will not

be impaired.

COMMENTARY

This provision emphasizes the necessity of considering external load in addition to the high localized stresses from the effect of the anchorage.

CHAPTER 11 — SHEAR

11.1 The nominal shear strength of prestressed columns and walls should be based on provisions in Sections 11.1, 11.2, 11.4, 11.5, 11.10 and 11.11 of ACI 318-83.

COMMENTARY

Prestressed columns should be designed for shear in the same manner as prestressed beams, i.e., by ignoring the influence of axial load.

Perpendicular to the plane of a flat prestressed wall, shear should be considered as for slabs and footings. Shear forces parallel to the plane of a flat prestressed wall should be considered under the provisions for shear walls.

For the rare case of two-way prestressed walls, the design for shear in the plane of the wall may be based on superimposing the effects of the vertical and horizontal prestressing. An approximate method can be found in Section

11.3.2 of the 1976 committee recommendations.⁴

Specific recommendations for considering the effects of torsion on a prestressed compression member have not been thoroughly researched. Combined shear and torsion of prestressed members²⁷ is not covered by ACI 318-83 but shear and torsion design for flexural members is considered in the Third Edition of the PCI Design Handbook.⁵ The Committee knows of no research relating to the behavior of prestressed columns subject to torsional loads or that specific problems with torsion in a column presents a common design situation.

If combined axial load, shear and torsion exist in a prestressed column, the Committee recommends designing the member as a prestressed concrete beam. Suggested design procedures for shear and torsion can be found in the PCI Design Handbook.

CHAPTER 12 — REINFORCING DETAILS OF PRESTRESSED CONCRETE COLUMNS AND WALLS

12.1 General Reinforcing Details

12.1.1 Structural details of prestressed and nonprestressed reinforcement should conform to Chapter 7 of ACI 318-83 except as modified by these provisions.

12.2 End Regions

12.2.1 Reinforcement in the end anchorage zone of the member should be in accordance with Section 18.13 of ACI 318-83.

12.2.2 End anchorage zone reinforcement

ment is not required in post-tensioned construction if the average bearing stress beneath the anchor is less than $0.5 f_{ci}$.

COMMENTARY

These provisions are generally applicable to post-tensioned concrete construction. Bearing stresses of less than half of the concrete strength directly beneath the anchor plate of post-tensioning tendons do not need supplemental reinforcement to resist bursting or spalling forces induced by the tension. Concentrated bearing stresses of this magnitude have a negligible effect on untied reinforced concrete columns.

12.3 Limits of Longitudinal Reinforcement

12.3.1 All prestressed columns and walls meeting the requirements of Section 18.11 of ACI 318-83 may be designed as prestressed members.

12.3.2 Prestressed flat wall panels and ribbed wall panels may have the minimum reinforcement requirements of ACI 318-83, Section 14.3, waived.

12.4 Lateral Reinforcement

12.4.1 Lateral reinforcement requirements for prestressed columns should conform to ACI 318-83 Section 18.11.2.2 except Section 18.11.2.2a may be waived for prestressed columns.

COMMENTARY

The previous edition of this recommended practice⁴ required that axial load capacity be multiplied by 0.85 when lateral ties were not provided in a prestressed column. Recent research^{14,28} has shown no justification for this reduction. Ties do not lead to a significant increase in column capacity at peak loads, do not necessarily limit column deflections at peak loads, and do not consistently influence the ability to

sustain deformations on the descending branch of the load-deflection curve. These conclusions apply to columns when stability, rather than material behavior, governs load capacity.

Lateral reinforcement should be considered for relatively short, stocky columns where axial load is high, and where member shear force is significant.

12.4.2 Lateral reinforcement in the form of rectangular continuous spiral reinforcement may be substituted for individual lateral ties if the spiral has an area equivalent to that of ties spaced in accordance with ACI 318-83 Section 18.11.2.2b and 18.11.2.2c.

COMMENTARY

Prestressed piles use continuous square spirals as ties and the typical spacing exceeds that of round spirals.

12.4.3 Lateral reinforcement requirements for prestressed ribbed walls meeting the requirements of Section 12.3.1 may have the lateral reinforcement requirement waived if the nominal capacity is multiplied by 0.85.

COMMENTARY

Ribbed wall panels often use the thickened rib as a column. This type of wall panel is frequently designed with a large slenderness and eccentricity. The fact that lateral ties are not typically used in wall panels is cause for making the factor of safety higher than normal. ACI 318-83 Section 18.11.2.3 has already waived the minimum reinforcement requirement in Section 14.3 for a flat wall but does not indicate this waiver applies to ribbed walls.

If the rib in a ribbed wall panel is used as a column, the designer should detail the rib as for a column. However, when the entire panel is considered as a compression member, the lateral reinforcement requirement may be waived

if the nominal axial capacity is multiplied by 0.85. This multiplying factor effectively increases the factor of safety as for an unreinforced concrete wall.

Minimum lateral reinforcement may be needed to improve ductility if shear and torsion are sufficient to cause cracking of the ribbed wall.

12.5 Two-Way Prestressing

12.5.1 Where two-way prestressing is used in walls, no additional horizontal reinforcement is required if the horizontal prestress, after losses, is at least 150 psi.

12.6 Special Reinforcement

12.6.1 Prestressed flat wall panels should have perimeter deformed bar reinforcement near the panel free edges parallel to the direction of the axial prestress. Anchorage for this deformed bar reinforcement should extend a minimum of 24 in. perpendicular to the prestress direction at the longitudinal panel ends. These extensions should be considered for the deformed bar reinforcement only and not as providing reinforcement in the anchorage zone.

12.6.2 The stipulation in Section 12.6.1 may be waived if prestressing steel is placed along the sides within 0.75 of the wall thickness from the edge; except that in such a case, confinement steel, in the form of mesh or cross-reinforcement transverse to the prestressing steel, should be evenly distributed over a 24

in. length at each longitudinal end of the edge tendons.

COMMENTARY

This reinforcement has been found to be necessary to control cracking during handling and transportation, and to help with end block stresses. When the prestressing steel is placed near the longitudinal panel edges, confinement steel is required over a 2 ft length at each end of the tendon to guard against longitudinal splitting.

12.6.3 Reinforcement should be provided in the end anchorage zone perpendicular to the prestressing steel of all flat type wall panels unless experience has shown this reinforcement can be eliminated.

COMMENTARY

In order to control cracking parallel to the prestressing in the end anchorage zone, a minimum area of steel should be uniformly distributed in the transfer length of each layer of prestressing steel.²⁹

12.7 Minimum Bonded Reinforcement for Unbonded Prestressing

12.7.1 Minimum area of bonded reinforcement for members with unbonded tendons should be in accordance with ACI 318-83 Section 18.9.

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REFERENCES

1. ACI Committee 318, "Building Code Requirements for Reinforced Concrete (ACI 318-83)," American Concrete Institute, Detroit, Michigan, 1983.
2. ACI Committee 318, "Commentary on Building Code Requirements for Reinforced Concrete (ACI 318-83)," American Concrete Institute, Detroit, Michigan, 1983.
3. PCI Committee on Prestressed Concrete Columns, "Tentative Recommendations for the Design of Prestressed Concrete Columns," *PCI JOURNAL*, V. 13, No. 5, October 1968, pp. 12-21.
4. PCI Committee on Prestressed Concrete Columns, "Recommended Practice for the Design of Prestressed Concrete Columns and Bearing Walls," *PCI JOURNAL*, V. 21, No. 6, November-December 1976, pp. 16-45. Discussion by L. D. Martin and Committee Closure, *PCI JOURNAL*, V. 23, No. 1, January-February 1978, pp. 92-94.
5. *PCI Design Handbook, Precast and Prestressed Concrete*, Third Edition, Prestressed Concrete Institute, Chicago, Illinois, 1985.
6. Bleich, F., *Buckling Strength of Metal Structures*, Engineering Society Monograph, McGraw-Hill, New York, N.Y., 1952.
7. Galambos, T. V., *Structural Members and Frames*, Prentice-Hall, Inc., Englewood Cliffs, New Jersey, 1968.
8. Gere, J. M., and Weaver, W., *Analysis of Framed Structures*, Van Nostrand Co. Inc., Princeton, New Jersey, 1965, pp. 428-431.
9. Johnston, B. G. (Editor), *Guide to Stability Design Criteria for Metal Structures*, Third Edition, Column Research Council, John Wiley and Sons, Inc., New York, N.Y., 1976.
10. Timoshenko, S. P., and Gere, J. M., *Theory of Elastic Stability*, Second Edition, McGraw-Hill, New York, N.Y., 1961.
11. "Test Report on Slender Walls," ACI-SEASC Task Committee on Slender Walls, February 1980-September 1982, Los Angeles, California.
12. Timoshenko, S. P., and Goodier, J. N., *Theory of Elasticity*, Third Edition, McGraw-Hill Book Company, New York, N.Y., 1970.
13. Chaudwani, R., "Precast Concrete Load Bearing Wall Panels," MASC Thesis, University of British Columbia, 1970.
14. Yuan, Robert L., "Prestressed Concrete Column Behavior," PCI Specially Funded Research and Development Program — Research Project No. 3, Prestressed Concrete Institute, Chicago, Illinois.
15. Wood, B. R., Beaulieu, D., and Adams, P. F., "Column Design by P-Delta Method," *Journal of the Structural Division*, ASCE, V. 102, No. ST2, February 1976, pp. 411-427.
16. Wood, B. R., Beaulieu, D., and Adams, P. F., "Further Aspects of Design by P-Delta Method," *Journal of the Structural Division*, ASCE, V. 102, No. ST3, March 1976, pp. 487-500.
17. Nixon, D., Beaulieu, D., and Adams, P. F., "Simplified Second-Order Frame Analysis," *Canadian Journal of Civil Engineering*, V. 2, No. 4, December 1975, pp. 602-605.
18. Nathan, N. D., "Rational Analysis and Design of Prestressed Concrete Beam Columns and Wall Panels," *PCI JOURNAL*, V. 30, No. 3, May-June 1985, pp. 87-133.
19. Nathan, N. D., "Applicability of ACI Slenderness Computations to Prestressed Concrete Sections," *PCI JOURNAL*, V. 20, No. 3, May-June 1975, pp. 68-85.
20. Nathan, N. D., "Slenderness of Prestressed Concrete Beam-Columns," *PCI JOURNAL*, V. 17, No. 6, November-December 1972, pp. 45-57.
21. Nathan, N. D., "Slenderness of Prestressed Concrete Columns," *PCI JOURNAL*, V. 28, No. 2, March-April 1983, pp. 50-77.
22. Alcock, W. J., and Nathan, N. D., "Moment Magnification Tests of Prestressed Concrete Columns," *PCI JOURNAL*, V. 22, No. 4, July-August 1977, pp. 50-61.
23. Chaudwani, R., and Nathan, N. D., "Precast Prestressed Sections under Axial Load and Bending," *PCI JOURNAL*, V. 16, No. 3, May-June 1971, pp. 10-19.
24. CSA Standard CAN3-A23.3-M84, "De-

- sign of Concrete Structures for Buildings," Canadian Standards Association, Rexdale, Ontario, 1984.
25. Burns, N. H., and Pierce, D. M., "Strength and Behavior of Prestressed Concrete Members With Unbonded Tendons," *PCI JOURNAL*, V. 12, No. 5, October 1967, pp. 15-29.
 26. Marshall, W. T., "A Theory for End Zone Stresses in Pretensioned Concrete Beams," *PCI JOURNAL*, V. 11, No. 2, April 1966, pp. 45-51.
 27. Collins, M. P., and Mitchell, D., "Shear and Torsion Design of Prestressed and Nonprestressed Concrete Beams," *PCI JOURNAL*, V. 25, No. 5, September-October 1980, pp. 32-100. Discussion by K. Cederwall and L. Elfgrén, W. H. Dilger, P. Gergely, K. Kordina and M. Teutsch, P. Mueller, K. S. Rajagopalan, H. T. Solanki, H. Stamenkovic, and Authors, *PCI JOURNAL*, V. 26, No. 6, November-December 1981, pp. 96-118.
 28. Carinci, C. A., and Halvorsen, G. T., "Tie Requirements for Prestressed Concrete Columns," *PCI JOURNAL*, V. 32, No. 4, July-August 1987, pp. 46-79.
 29. Marshall, W. T., and Mattock, A. H., "Control of Horizontal Cracking in the Ends of Pretensioned Prestressed Concrete Girders," *PCI JOURNAL*, V. 7, No. 5, October 1962, pp. 56-74.

* * *

BIBLIOGRAPHY

30. Aburazza, Soliman, "Ultimate Capacity of Prestressed Columns," MS Thesis, University of Denver, August 1974.
31. Anderson, A. R., and Moustafa, S. E., "Ultimate Strength of Prestressed Concrete Piles and Columns," *ACI Journal*, August 1970, pp. 620-635.
32. Aroni, S., "Slender Prestressed Concrete Columns," *Journal of Structural Division*, ASCE, V. 94, No. ST4, April 1968, pp. 875-904.
33. Aroni, S., "Slender Prestressed Concrete Columns," Report No. 67-10, Structures and Material Research, Department of Civil Engineering, University of California, Berkeley, May 1967, 231 pp.
34. Aroni, S., "The Strength of Slender Prestressed Concrete Columns," *PCI JOURNAL*, V. 13, No. 2, April 1968, pp. 19-33.
35. Baha, Z., "A Study of Ultimate Capacity of Prestressed Concrete Load Bearing Wall Panels," PhD Dissertation, Department of Civil Engineering, North Carolina State University, 1973.
36. Bolander, J. Jr., and Naaman, A. E., "Load-Moment Interaction Diagrams as Design Aids for Prestressed Concrete Columns and Poles," Report No. UMCE 85-1, Department of Civil Engineering, University of Michigan, February 1985, 91 pp. See also Bolander, J. Jr., Sowlat, K., and Naaman, A. E., "Design Considerations for Tapered Prestressed Concrete Poles," *PCI JOURNAL*, V. 33, No. 1, January-February, 1988, pp. 44-67.
37. Breckenridge, R. A., "A Study of the Characteristics of Prestressed Concrete Columns," Report No. 18-6, U.S. Civil Engineering Center, University of Southern California, Los Angeles, April 1953.
38. British Standards Institution, "Code of Practice for the Structural Use of Concrete (CP110)," Part 1, 1972.
39. Brown, H. R., and Hall, A. S., "Tests on Slender Prestressed Concrete Columns," Proceedings of the Symposium on Reinforced Concrete Columns, ACI SP-13, American Concrete Institute, Detroit, Michigan, 1965, pp. 179-192.
40. Brown, K. I., "The Ultimate Load Carrying Capacity of Prestressed Concrete Columns Under Direct and Eccentric Loading," *Civil Engineering and Public Works Review* (London), V. 60, No. 705, April 1965, pp. 539-541; No. 706, May 1965, pp. 683-687; No. 707, June 1965, pp. 841-845.
41. Cazaly, L., and Huggins, M. W., *Canadian Prestressed Concrete Institute Handbook*, Canadian Prestressed Concrete Institute, Willowdale, Ontario, Canada, 1964.
42. Chu, K. H., and Chow, H. L., "Effective Column Length in Unsymmetrical Frames," Publication, International Association of Bridge and Structural Engineering (IABSE), V. 29-1, 1969.
43. Cranston, W. B., "Analysis and Design of Reinforced Concrete Columns," Report No. 20, Cement and Concrete Association, London 1972, 54 pp.
44. Efsen, A., "Eccentrically Loaded Prestressed Concrete Columns" (in Danish), FIP Prestressed Concrete Abstracts, Copenhagen, 1963.
45. Ghali, A., and Neville, A. M., *Structural Analysis*, Chapman and Hall, London, 1977, Chapter 15.
46. Guyon, Y., *Prestressed Concrete*, V. 1 and 2, John Wiley & Sons, Inc., New York, N.Y., 1960.
47. Hall, A. S., "Buckling of Prestressed Columns," *Construction Review*, June 1963.
48. Hall, A. S., "Buckling of Prestressed Columns," Cement and Concrete Association, Sydney, Australia, 1961.
49. Hromadik, J. J., "Column Strength of Long Piles," *ACI Journal*, June 1962, Proceedings, V. 59, Part 1, No. 6, pp. 757-778.
50. Itaya, R., "Design and Uses of Prestressed Concrete Columns," *PCI JOURNAL*, V. 10, No. 3, June 1965, pp. 69-76.
51. Jernigan, A. M., "Some Studies on the Behavior of Prestressed Concrete Columns," MS Thesis, Department of Civil Engineering, University of Florida, Gainesville, June 1956.
52. Kabaila, A. P., and Hall, A. S., "Analysis of Instability of Unrestrained Prestressed Concrete Columns With End Eccentricities," Symposium on Reinforced Concrete Columns, ACI Special Publication SP-13, American Concrete Institute, Detroit, Michigan, 1965, pp. 179-192.

- tute, Detroit, Michigan, 1966, pp. 157-178.
53. Kavanagh, T. C., "Effective Length of Framed Columns," *Transactions, ASCE*, V. 127 (1962), Part II, pp. 81-101.
 54. Komendant, E., *Contemporary Concrete Structures*, McGraw-Hill Book Co., New York, 1972.
 55. Knoez, T., *Manual of Precast Concrete Construction*, V. 1, 2 and 3, Bauverlag GMBH, Wiesbaden and Berlin, 1968, 1971, 1970.
 56. Lai, S. M. A., MacGregor, J. G., and Hellesland, J., "Geometric Non-Linearities in Nonsway Frames," *Journal of the Structural Division, ASCE*, V. 109, No. ST12, December 1983, pp. 2770-2785.
 57. Lai, S. M. A., and MacGregor, J. G., "Geometric Non-Linearities in Unbraced Multi-Story Frames," *Journal of the Structural Division, ASCE*, V. 109, No. ST11, November 1983, pp. 2528-2545.
 58. Laszlo, G., "Experimental Research on the Buckling Behaviour of Slender Prestressed Concrete Columns," MASC Thesis, The University of British Columbia, April 1966.
 59. Leonhardt, F., *Prestressed Concrete Design and Construction*, Second Edition, Wilhelm Ernst & Sohn, Berlin, 1964.
 60. Li, S., and Ramakrishnan, V., "Optimum Prestress, Analysis and Ultimate Strength Design of Prestressed Concrete Sheet Piles," *PCI JOURNAL*, V. 16, No. 3, May-June 1971, pp. 60-74.
 61. Lin, T. Y., and Burns, N. H., *Design of Prestressed Concrete Structures*, Third Edition, John Wiley & Sons, New York, N.Y., 1981.
 62. Lin, T. Y., and Itaya, R., "A Prestressed Concrete Column Under Eccentric Loading," *PCI JOURNAL*, V. 2, No. 3, December 1957, pp. 5-17.
 63. Lin, T. Y., and Lakhwara, T. R., "Ultimate Strength of Eccentrically Loaded Partially Prestressed Columns," *PCI JOURNAL*, V. 11, No. 3, June 1966, pp. 37-49.
 64. MacGregor, J. G., "Stability of Multi-Story Concrete Buildings," ASCE-IABSE Conference on Tall Buildings, Bethlehem, Pennsylvania, SoA-Rep 23-3 Planning and Design of Tall Buildings, V. III, 1972, pp. 517-536.
 65. MacGregor, J. G., and Hage, S. E., "Stability Analysis and Design of Concrete Frames," *Journal of the Structural Division, ASCE*, V. 103, No. ST10, Proceedings Paper 13280, October 1977, pp. 1953-1970.
 66. Magnel, Gustave, *Prestressed Concrete*, Third Edition, Concrete Publications Limited, London, 1954.
 67. Mast, Paul E., "Elastic Stability of Flanges of Typical Prestressed Single Tees," *PCI JOURNAL*, V. 11, No. 4, August 1966, pp. 64-76.
 68. Naaman, A. E., *Prestressed Concrete Analysis and Design*, McGraw Hill Book Company, New York, N.Y. 1982.
 69. Nilson, A. H., *Design of Prestressed Concrete*, John Wiley & Sons, Inc., New York, N.Y., 1978, 526 pp.
 70. Osgood, W. R., "The Double Modulus Theory of Column Action," *Civil Engineering*, March 1935, pp. 174-175.
 71. Ozell, A. M., and Jernigan, A. M., "Some Studies of the Behavior of Prestressed Concrete Columns," Technical Progress Report No. 3, Engineering Experiment Station, University of Florida, 1957.
 72. PCI Committee on Prestressed Concrete Poles, "Guide Specifications for Prestressed Concrete Poles," *PCI JOURNAL*, V. 27, No. 3, May-June 1982, pp. 18-29.
 73. PCI Committee on Prestressed Concrete Poles, "Guide for Design of Prestressed Concrete Poles," *PCI JOURNAL*, V. 28, No. 3, May-June 1983, pp. 22-87.
 74. *PCI Design Handbook - Precast and Prestressed Concrete*, Prestressed Concrete Institute, Chicago, Illinois, 1971.
 75. *PCI Design Handbook - Precast Prestressed Concrete*, Second Edition, Prestressed Concrete Institute, Chicago, Illinois, 1978.
 76. Rodgers, T. E., Jr., "Prestressed Concrete Poles: State-of-the-Art," *PCI JOURNAL*, V. 29, No. 5, September-October 1984, pp. 52-103.
 77. Rosenblueth, E., "Slenderness Effects in Buildings," *Journal of the Structural Division, ASCE*, V. 91, No. ST1, February 1965, pp. 229-252.
 78. Rüschi, H., "Researches Towards a General Flexural Theory for Structural Concrete," *ACI Journal*, V. 32, No. 1, July 1960, pp. 1-28.
 79. Salmons, J. R., and McLaughlin, D. G., "Design Charts for Proportioning Rec-

- tangular Prestressed Concrete Columns," *PCI JOURNAL*, V. 27, No. 1, January-February 1982, pp. 120-143.
80. Shanley, F. R., "Inelastic Column Theory," *Journal of Aeronautical Sciences*, V. 4, No. 5, May 1947.
 81. Sheppard, D. A., "Seismic Design of Prestressed Concrete Piling," *PCI JOURNAL*, V. 28, No. 2, March-April 1983, pp. 20-49.
 82. Southwell, R. V., "On the Analysis of Experimental Observations in Problems of Elastic Stability," *Proceedings, Royal Society of London, Series A*, V. 135, April 1, 1932, pp. 601-616.
 83. Sowlat, K., and Naaman, A. E., "Design Aids for Hollow-Cored Prestressed Concrete Poles," Department of Civil Engineering, University of Michigan, Report No. 4MCE No. 84-1, September 1984, 70 pp. See also Bolander, J. Jr., Sowlat, K., and Naaman, A. E., "Design Considerations for Tapered Prestressed Concrete Poles," *PCI JOURNAL*, V. 33, No. 1, January-February 1988, pp. 44-67.
 84. Troxell, G. E., Davis, H. E., and Kelly, J. W., *Composition and Properties of Concrete*, Second Edition, McGraw-Hill Book Company, New York, N.Y., 1956.
 85. Wang, P. T., Shah, S. P., and Naaman, A. E., "Stress-Strain Curves of Normal and Lightweight Concrete in Compression," *ACI Journal*, V. 75, No. 11, November 1978, pp. 603-611.
 86. Wilhelm, W. J., and Zia, P., "Effects of Creep and Shrinkage on Prestressed Concrete Columns," *Journal of the Structural Division, ASCE*, V. 96, No. ST10, October 1970, pp. 2103-2121.
 87. Wilhelm, W. J., "An Analytical Study of the Effects of Creep and Shrinkage on the Behavior and Ultimate Capacity of Long Hinged Prestressed Concrete Columns," PhD Dissertation, Department of Civil Engineering, North Carolina State University, 1968.
 88. Zia, P., and Andrew, J. R., "Ultimate Strength of Rectangular Prestressed Concrete Columns Under Concentric and Eccentric Loadings," North Carolina State University, Raleigh, September 1967.
 89. Zia, P., Andrew, J. R., and Chawla, M. S., "Prestressed Concrete Columns Under Concentric and Eccentric Loadings," North Carolina State University, Raleigh, July 1969.
 90. Zia, P., and Guillermo, E. C., "Combined Bending and Axial Load in Prestressed Concrete Columns," *PCI JOURNAL*, V. 12, No. 3, June 1967, pp. 52-59, Discussion by Laszlo, G., Nathan, N. D., and Authors, *PCI JOURNAL*, V. 12, No. 6, December 1967, pp. 77-88.
 91. Zia, P., and Moreadith, F. L., "Ultimate Load Capacity of Prestressed Concrete Columns," *ACI Journal*, *Proceedings*, V. 63, No. 7, July 1966, pp. 767-788.
 92. Elias, Hani E., and Durrani, A. J., "Confinement of Prestressed Concrete Columns," *PCI JOURNAL*, V. 33, No. 3, May-June 1988, pp. 122-141.

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NOTE: Discussion of this report is invited. Please submit your comments to PCI Headquarters by April 1, 1989.

APPENDIX — DESIGN EXAMPLES

The following two design examples show how some of the provisions contained in the report can be applied. The first example treats slenderness effects for precast prestressed ribbed wall panels and precast prestressed columns.

EXAMPLE 1

This design example considers slenderness effects using moment magnification for precast prestressed ribbed wall panels and precast prestressed columns.

Given: The structure shown is the interior portion of a long building that is isolated from the remaining structure by expansion joints, creating an unbraced (sway) frame.

A structural frame analysis gives the following service load data:

Each wall panel	D	L	W
Axial load (kips)	14.4	7.2	0
Top bending moment (kip-ft)	9.6	4.8	0
Bottom bending moment (kip-ft)	4.2	2.1	17.0*

*Flange of tee wall panel in compression

Each column	D	L	W
Axial load (kips)	115.2	57.6	0
Top bending moment (kip-ft)†	0	0	0
Bottom bending moment (kip-ft)	0	0	3.0

†Pinned

The second example covers the design of slender wall panels.

Reference to the PCI Design Handbook, PCI Column Committee report or ACI Code are given in the right hand margin alongside the calculations.

Wall panel properties (from PCI Design Handbook, Third Edition):

$$A = 401 \text{ in.}^2; I = 20,985 \text{ in.}^4 \quad \text{p. 2-17}$$

$$r = \sqrt{\frac{20,985}{401}} = 7.23 \text{ in.}$$

$$\phi P_o = 1165 \text{ kips}; \phi = 0.7$$

$$P_o = 1165/0.7 = 1664 \text{ kips} \quad \text{p. 2-5}$$

Note that loading is uniform on wall panel. Therefore, full section is effective. 6.2

$$f'_c = 5000 \text{ psi}, E_c = 4,300,000 \text{ psi}$$

Column properties (from PCI Design Handbook, Third Edition):

$$A = 576 \text{ in.}^2; I = 27,648 \text{ in.}^4$$

$$r = \sqrt{\frac{27,648}{576}} = 6.93 \text{ in.} \quad \text{p. 2-55}$$

$$\phi P_o = 1325 \text{ kips}; \phi = 0.7$$

$$P_o = 1325/0.7 = 1893 \text{ kips} \quad \text{p. 2-48}$$

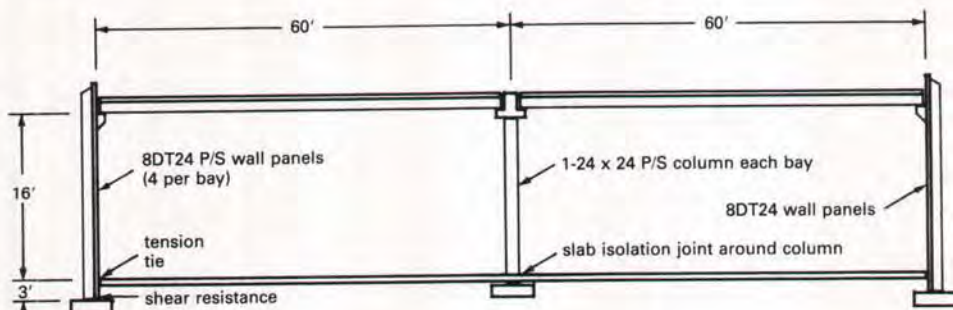
$$f'_c = 5000 \text{ psi}, E_c = 4,300,000 \text{ psi}$$

Problem: Find the magnified moments for the wall panels and columns and check loads against design capacity.

Solution: Use following procedure.

Wall Panel:

- Case 1 (Dead + Live Loads) Gravity
ACI 318-83 9.2.1



$$P_u = 1.4(14.4) + 1.7(7.2) = 32.4 \text{ kips}$$

$$M_2(\text{top}) = 1.4(9.6) + 1.7(4.8) \\ = 21.6 \text{ kip-ft}$$

$$M_1(\text{bottom}) = 1.4(4.2) + 1.7(2.1) \\ = 9.5 \text{ kip-ft}$$

Larger end moment (M_2) occurs at top of wall panel. Therefore, use M_2 to determine β_d .

$$\beta_d = \frac{\text{Factored dead load moment}}{\text{Factored total moment}} \quad 8.2.2 \\ = \frac{1.4(9.6)}{1.4(9.6) + 1.7(4.8)} \\ = 0.62$$

Calculate magnified moment:

Since axial loads on this structural frame are symmetrical and do not contribute to sidesway, the value of k can be taken as 1.0 and $\delta_s M_{2s} = 0$. 8.2.1

$$\frac{kl_u}{r} = \frac{(1.0)(16)(12)}{7.23} \\ = 26.6 > 25 - 10 (M_{1b}/M_{2b}) \quad 8.1.6$$

Therefore, consider slenderness.

$$P_u/P_o = 32.4/1664 = 0.02$$

From Fig. 8.1 or Eqs. (8.6), (8.7) and (8.9): 8.2.2

$$\lambda = \eta \theta \geq 3.0$$

$$\eta = 2.5 + \frac{1.6}{P_u/P_o} \\ = 2.5 + \frac{1.6}{0.02} \\ = 82.5 > 70 \text{ (use } \eta = 70)$$

$$\theta = \frac{35}{kl_u/r} - 0.09 \\ = \frac{35}{26.6} - 0.09 \\ = 1.23$$

$$\lambda = 70(1.23) = 85.8$$

Calculate stiffness [Eq. (8.5)]: 8.2.2

$$EI = \frac{E_c I_g / \lambda}{1 + \beta_d} \\ = \frac{4300(20,985)/85.8}{1 + 0.62} \\ = 649,195 \text{ kip-in.}^2$$

Calculate coefficient for end moment effect [Eq. (8.10)]: 8.2.3

$$C_m = 0.7 + 0.3 (M_{1b}/M_{2b}) \geq 0.4$$

$$C_m = 0.7 + 0.3 (9.5/21.6) \\ = 0.83$$

Calculate Euler buckling load [Eq. (8.4)]: 8.2.1

$$P_c = \frac{\pi^2 EI}{(kl_u)^2} \\ = \frac{\pi^2 (649,195)}{[(1.0)(16)(12)]^2} \\ = 173.8 \text{ kips}$$

Determine ϕ at magnitude of factored axial load: ACI 318-83 (9.3.2)

$$\phi = 0.7 \text{ at } P_u = 0.1 f'_c A_g \\ = 0.1 (5.0)(401) \\ = 200 \text{ kips}$$

$$\phi = 0.9 \text{ at } P_u = 0 \\ \phi = 0.9 - 0.2 (32.4/200) = 0.87$$

Calculate moment magnifying factor [Eq. (8.2)]: 8.2.1

$$\delta_b = \frac{C_m}{1 - P_u/\phi P_c} \geq 1.0 \\ = \frac{0.83}{1 - 32.4/0.87(173.8)} \\ = 1.06$$

Calculate magnified moment [Eq. (8.1)]: 8.2.1

$$M_c = \delta_b M_{2b} + \delta_s M_{2s} \\ = 1.06(21.6) + 0 \\ = 22.8 \text{ kip-ft}$$

• Case 2 (Dead + Live + Wind Loads) ACI 318-83 9.2.2

Wind load contributes to sidesway. Since the bending moment at the top is not affected by wind, the only moments that can be magnified by sidesway are the bottom moments.

$$P_u = 0.75 [1.4(14.4) + 1.7(7.2) + 1.7(0)] \\ = 24.3 \text{ kips}$$

Bottom moments:

$$M_{2b} = 0.75 [1.4(4.2) + 1.7(2.1)] \\ = 7.1 \text{ kip-ft}$$

$$M_{2s} = 0.75 [1.7(17)] \\ = 21.7 \text{ kip-ft}$$

or

$$P_u = 0.9(14.4) + 1.3(0)$$

$$= 13.0 \text{ kips}$$

$$M_{2b} = 0.9(4.2)$$

$$= 3.8 \text{ kip-ft}$$

$$M_{2s} = 1.3(17)$$

$$= 22.1 \text{ kip-ft}$$

To find large end moment at bottom, calculate sustained load factor:

$$\beta_{db} = 0.62 \text{ (see earlier calculation)} \quad 8.2.2$$

$$\beta_{ds} = 0$$

Determine unbraced effective length factor using Jackson-Moreland alignment chart. Fixity at base of wall panel (ψ_A) is approximately equal to the ratio of length of panel above to below floor:

$$8.1.5$$

$$\psi_A = 16/3 = 5.3$$

For top of column that was assumed as pinned:

$$\psi_B = 10 \text{ (max)}$$

$$k = 2.6$$

$$kl_u/r = 2.6(16)(12)/7.23 \quad 8.1.6$$

$$= 69 > 15$$

Therefore, consider slenderness.

$$P_u/P_o = 24.3/1664$$

$$= 0.015$$

From Fig. 8.1 or Eqs. (8.6), (8.7) and (8.9):

$$\lambda = 29 \quad 8.2.2$$

Calculate stiffness for braced loads

$$[\text{Eq. (8.5)}]: \quad 8.2.2$$

$$(EI)_b = \frac{E_c I_g / \lambda}{1 + \beta_{db}}$$

$$= \frac{4300(20,985)/29}{1 + 0.62}$$

$$= 1,920,721 \text{ kip-in.}^2$$

Calculate coefficient for end moment effect (braced):

$$8.2.3$$

$$C_m = 0.83 \text{ (see earlier calculation)}$$

Calculate Euler buckling load (braced case) [Eq. (8.4)]:

$$8.2.1$$

$$(P_c)_b = \frac{\pi^2 EI}{(k_b l_u)^2}$$

$$= \frac{\pi^2 (1,920,721)}{[(1)(16)(12)]^2}$$

$$= 514 \text{ kips}$$

Determine ϕ for axial load (braced case):

$$\phi = 0.9 - 0.2(24.3/200) \quad \text{ACI 318-83}$$

$$= 0.88 \quad 9.3.2$$

Calculate moment magnification factor — braced case moments

$$[\text{Eq. (8.2)}]: \quad 8.2.1$$

$$\delta_b = \frac{C_m}{1 - P_u/\phi P_c}$$

$$= \frac{0.83}{1 - 24.3/0.88(514)}$$

$$= 0.88 \text{ (use 1.0)} \quad 8.2.1$$

Calculate stiffness for sidesway loads

$$[\text{Eq. (8.5)}]: \quad 8.2.1$$

$$(EI)_s = \frac{E_c I_g / \lambda}{1 + \beta_{ds}} \quad 8.2.1$$

$$= \frac{(4300)(20,985)/29}{1 + 0}$$

$$= 3,111,569 \text{ kip-in.}^2$$

Coefficient for end moment effects — sidesway case

$$8.2.3$$

$$C_m = 1.0$$

Calculate Euler buckling load — sidesway case [Eq. (8.4)]:

$$8.2.1$$

$$(P_c)_s = \frac{\pi^2 EI}{(k_s l_u)^2}$$

$$= \frac{\pi^2 (3,111,569)}{[(2.6)(16)(12)]^2}$$

$$= 123.2 \text{ kips}$$

Determine ϕ for sidesway case axial load.

$$\text{Base } \phi \text{ on } \Sigma P_u \text{ and } \Sigma (0.1) A_g f'_c.$$

$$\text{ACI 318-83}$$

$$9.3.2$$

Calculate moment magnification factor — sidesway case [Eq. (8.3)]:

$$8.2.1$$

$$\delta_s = \frac{1}{1 - \Sigma P_u / \phi \Sigma P_c}$$

Note: δ_s for wall panel needs P_u and P_c for column. Hold δ_s calculation for column information.

Column

- Case 1 (Dead + Live Loads) Gravity
 $P_u = 1.4(115.2) + 1.7(57.6)$ ACI 318-83
 $= 259.2$ kips 9.2.1

$$M_2 = 0$$

$$M_1 = 0$$

$$\beta_d = 0 \quad 8.2.2$$

Take $k = 1.0$, i.e., no bending moments to magnify.

$$kl_u/r = (1.0)(16)(12)/6.93$$

$$= 27.7 > 25 - 10 (M_{1b}/M_{2b}) \quad 8.1.6$$

Since kl_u/r is close to the limit to neglect slenderness, ignore slenderness for Case 1 loading.

- Case 2 (Dead + Live + Wind Loads)
ACI 318-83
9.2.2

$$P_u = 0.75 [1.4(115.2) + 1.7(57.6)$$

$$+ 1.7(0)]$$

$$= 194.4 \text{ kips}$$

$$M_{2b} = 0.75 [1.4(0) + 1.7(0) + 1.7(0)]$$

$$= 0$$

$$M_{2s} = 0.75 [1.4(0) + 1.7(0) + 1.7(3.0)]$$

$$= 3.83 \text{ kip-ft}$$

or

$$P_u = 0.9(115.2) + 1.3(0)$$

$$= 103.7 \text{ kips}$$

$$M_{2b} = 0.9(0) + 1.3(0)$$

$$= 0$$

$$M_{2s} = 0.9(0) + 1.3(3.0)$$

$$= 3.9 \text{ kip-ft}$$

Determine effective length using alignment chart. Base fixity taken to be sufficient for: 8.1.5

$$\psi_{base} = 1.0; \psi_{top} = 10.0 \text{ (pinned)}$$

$$k = 1.9 \text{ (see alignment chart)}$$

$$kl_u/r = 1.9(16)(12)/6.93$$

$$= 52.6 > 15 \quad 8.1.6$$

Therefore, consider slenderness.

$$P_u/P_o = 194.4/1893$$

$$= 0.10$$

From Fig. 8.2 or Eqs. (8.6), (8.7) and (8.8):

$$\lambda = 8.6 \quad 8.2.2$$

Calculate stiffness for sidesway loads [Eq. (8.5)]: 8.2.2

$$(EI)_s = \frac{E_c I_g / \lambda}{1 + \beta_d}$$

$$= \frac{4300(27,648)/8.6}{1 + 0}$$

$$= 13,824,500 \text{ kip-in.}^2$$

Calculate coefficient for end moment effect—sidesway case: 8.2.3

$$C_m = 1.0$$

Calculate Euler buckling load — sidesway case [Eq. (8.4)]: 8.2.1

$$(P_c)_s = \frac{\pi^2 EI}{(kl_u)^2}$$

$$= \frac{\pi^2 (13,824,500)}{[(1.9)(16)(12)]^2}$$

$$= 1025 \text{ kips}$$

Calculate moment magnification factors: 8.2.1

$$\delta_b = 1.0$$

$$\delta_s = \frac{1}{1 - \Sigma P_u / \phi \Sigma P_c}$$

Assuming eight wall panels:

$$\Sigma (P_u)_s = 8(24.3) + 194.4$$

$$= 388.9 \text{ kips}$$

$$\Sigma (P_c)_s = 8(123.2) + 1025$$

$$= 2010.6 \text{ kips}$$

Determine ϕ for sidesway case:

ACI 318-83
9.3.2

$$\phi = 0.9 - 0.2 (\Sigma P_u / \Sigma 0.1 A_g f'_c)$$

$$= 0.9 - 0.2 \{388.9 / (0.1)(5)[8(401)$$

$$+ 576]\}$$

$$= 0.86$$

Calculate moment magnification factors: 8.2.1

$$\delta_{s(wall)} = \delta_{s(column)}$$

$$= \frac{1}{1 - \Sigma P_u / \phi \Sigma P_c}$$

$$= \frac{1}{1 - 388.9 / 0.86(2010.6)}$$

$$= 1.29$$

Summary of bending moments and magnification factors.

Building component	Case 1				Case 2			
	δ_b	M_{2b} (kip-ft)	δ_s	M_{2s} (kip-ft)	δ_b	M_{2b} (kip-ft)	δ_s	M_{2s} (kip-ft)
Wall panel	1.06	21.6	0	0	1.0	7.1	1.29	21.7
					1.0	3.8	1.29	22.1
Column	1.0	0	1.0	0	1.0	0	1.29	3.8
					1.0	0	1.29	3.9

Summary of magnified moments ($M_c = \delta_b M_{2b} + \delta_s M_{2s}$) and axial loads.

Building component	Case 1		Case 2	
	P_u (kips)	M_c (kip-ft)	P_u (kip)	M_c (kip-ft)
Wall panel	32.4	22.8	(a) 24.3	35.1
			(b) 13.0	32.3
Column	259.2	0	(a) 194.4	4.9
			(b) 103.7	5.0

A summary of the bending moments and magnification factors of the wall panel/column system for Cases 1 and 2 is shown in the table above. Similarly, a summary of the magnified moments and

axial loads for Cases 1 and 2 is also shown.

See interaction diagrams for comparison of applied loads to design capacity.

The selected members are satisfactory.

★ ★ ★

$$\eta = 2.5 + \frac{1.6}{P_u/P_o}$$

$$6 \leq \eta \leq 70$$

$$\lambda = \eta \theta$$

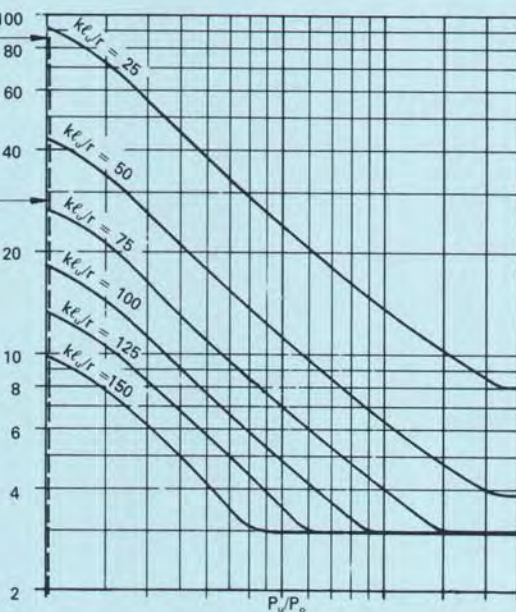
for P_c equation:

$$EI = \frac{E_c I_g / \lambda}{1 + \beta_o}$$

Wall Panel
Case 1

Wall Panel
Case 2

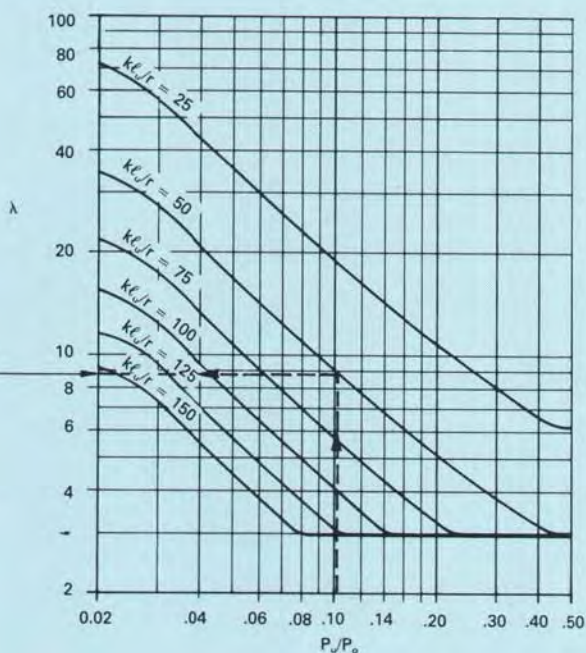
λ



(a) Compression flange

$$\theta = \frac{35}{k\ell_j/r} - 0.09$$

Fig. 8.1

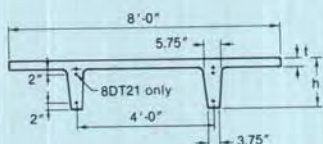


(b) No compression flange

$$\theta = \frac{27}{k\ell_j/r} - 0.05$$

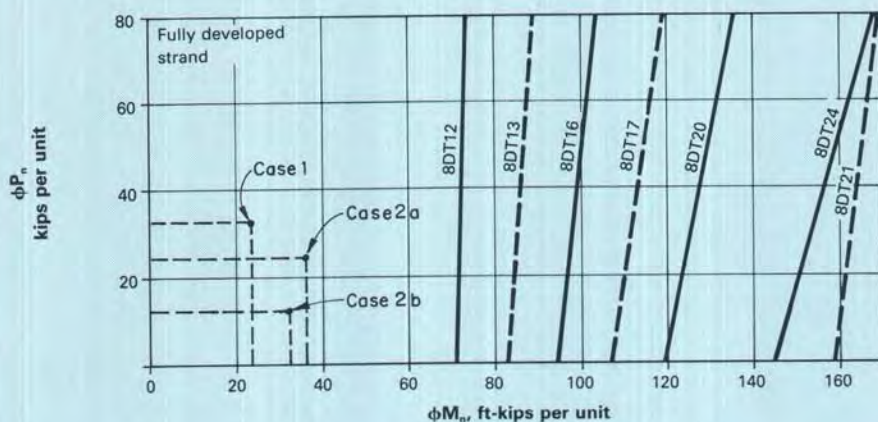
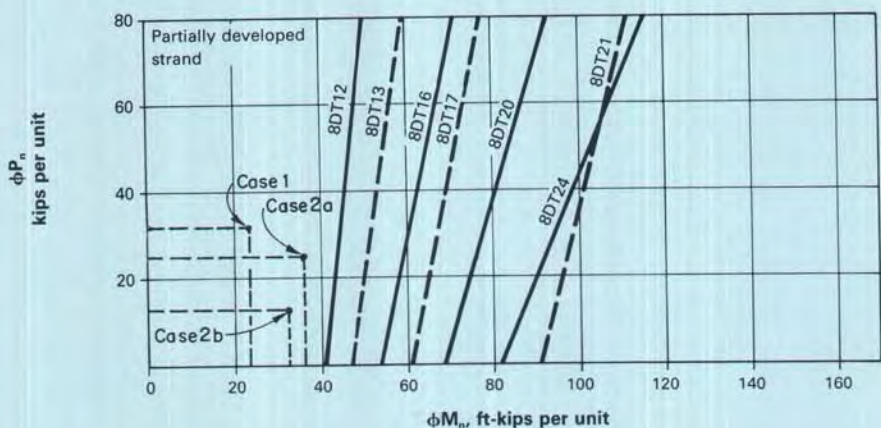
Fig. 8.2

Coefficients, λ , for modified EI



$f'_c = 5000$ psi, normal weight
strand = $\frac{1}{2}$ " dia., $f_{pu} = 270$ ksi

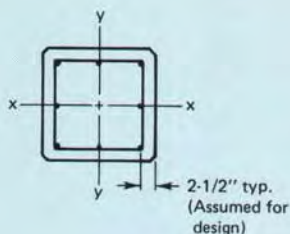
Mark	h in.	t in.	No. strd.	Full Interaction Curve Data					
				Partially Developed Strand Force			Fully Developed Strand Force		
				ϕP_n	ϕP_{nb}	ϕM_{nb}	ϕP_n	ϕM_{nb}	ϕM_o
8DT12	12	2	4	826	549	107	41	509	120
8DT13	13	3	4	1111	799	130	47	767	145
8DT16	16	2	4	939	592	175	55	553	193
8DT17	17	3	4	1224	839	209	61	800	230
8DT20	20	2	4	1052	637	254	68	598	277
8DT21	21	3	6	1324	853	301	91	812	327
8DT24	24	2	4	1165	684	344	82	644	371



Partial interaction curve for prestressed double tee wall panels

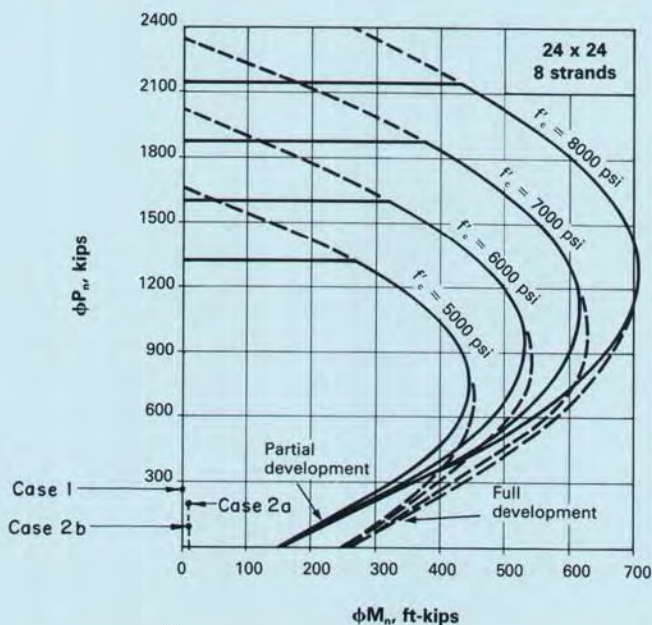
Criteria

1. Minimum prestress = 225 psi
2. All strand assumed 1/2 in. diameter,
 $f_{pu} = 270$ ksi
3. Curves shown for partial development of strand near member end, where $f_{ps} \approx f_{se}$
4. Horizontal portion of curve is the maximum for tied columns = $0.80\phi P_o$
5. $\phi = 0.9$ for $\phi P_n = 0$
= 0.7 for $\phi P_n \geq 0.10f'_c A_g$
Varies from 0.9 to 0.7 for points between

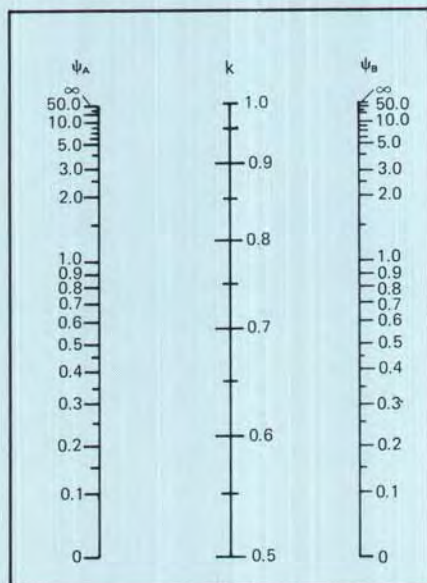


Notation

- ϕP_n = Design axial strength
 ϕM_n = Design flexural strength
 ϕP_o = Design axial strength at zero eccentricity
 A_g = Gross area of the column
 δ = Moment magnifier (Sect. 10.11, ACI 318-83)

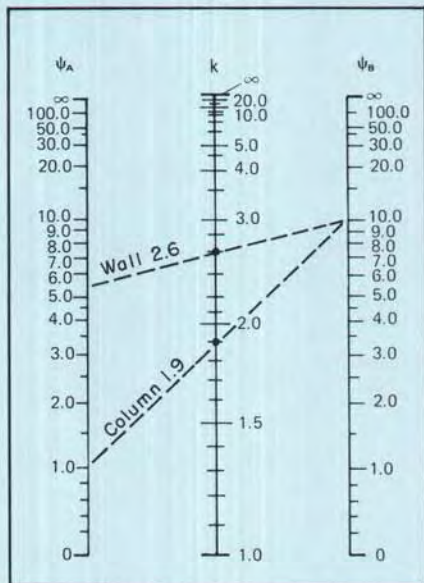


Design strength interaction curves for precast, prestressed concrete columns



(a)

Braced frames



(b)

Unbraced frames

ψ = If connection is rigid, ratio of ΣK of compression members to ΣK of flexural members in a plane. For typical precast connections usually assumed as pinned in analyses, ψ may be taken as 10. Lower values may be used if justified by analysis. For example, ψ at footings may be equal to K_c/K_b as calculated in Sect. 3.8.3.

k = effective length factor

Alignment charts for determining effective length factors

EXAMPLE 2

This example covers the design of slender wall panels.

Given: Precast prestressed wall panels enclose a single story building.

The following design parameters relate to the proportioning of the prestressed wall panel.

- Roof dead load = 0.3 kips per ft of wall
- Roof live load = 0.25 kips per ft of wall
- Eccentricity (e) of load at top of wall = 6 in.
- Structural frame assumed braced:
 $k = 1$
- Concrete strength: $f'_c = 5000$ psi
- Concrete modulus: $E_c = 4.3 \times 10^6$ psi
- Wall prestress: $f_p = 250$ psi ($7/16$ in. diameter 270 kip strand at 12 in. on center)
- Lateral loads:
Seismic: Zone 2
Wind: Basic 80 miles per hr;
Exposure B; Enclosed

Wall panel properties: 6 x 12 in. wide section properties.

$$A_c = 6 \times 12 = 72 \text{ in.}^2$$

$$I_g = 12(6)^3/12 = 216 \text{ in.}^4$$

$$r = \sqrt{216/72} = 1.73 \text{ or } 8.1.3$$

$$[0.3 \times 6 = 1.80 \text{ in.}]$$

$$\text{Panel weight} = (6/12)(150) = 75 \text{ psf} = W_p$$

Panel has an outward initial bow of $1/2$ in. (assumed fabrication tolerance).

Problem: Find the magnified moments in the wall panel using a second order deflection analysis and by the moment magnification procedure. Check the panel for deflection and cracking.

Determine governing loading

Lateral load (use Uniform Building Code)

— Wind suction:

$$q_s = 17 \text{ psf}$$

$$p = C_e C_g q_s I$$

$$= 0.8 \times 1.2 \times 17 \times 1$$

$$= 16.32 \text{ psf} \leq 17 \text{ psf}$$

— Seismic:

$$F_p = Z I C_p W_p$$

$$= 3/8 \times 1 \times 0.3 \times 75$$

$$= 8.44 \text{ psf}$$

Wind load governs: 17 psf > 8.44 psf

Analysis:

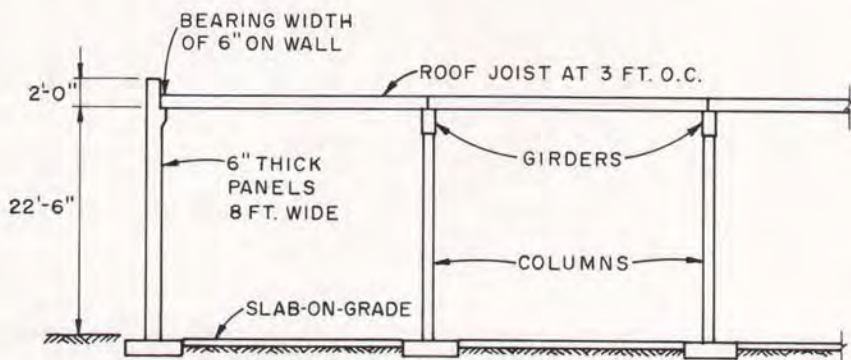
- Check effective width of panel 6.1
 - Center to center spacing of loads = 36 in.
 - Load width plus 6 x thickness each side $6 + 6(6) + 6(6) = 78$ in.
 - $0.4 \times$ wall height
 $0.4(22.5)(12) = 34.9$ in. (controls)

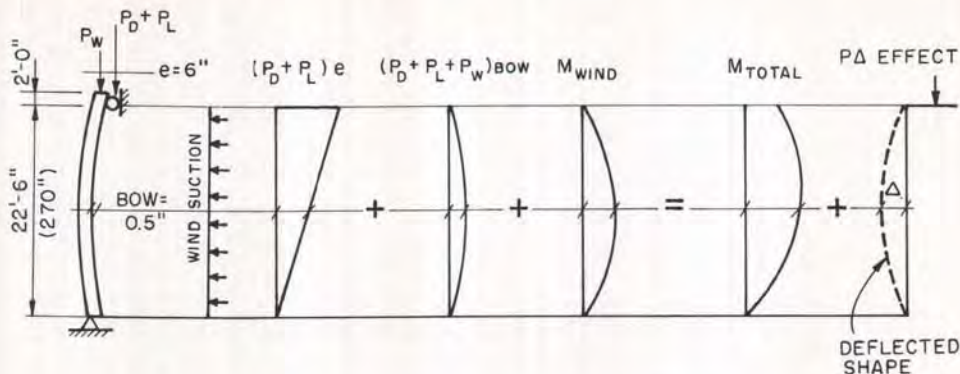
Use 36 in. as effective width. Do all calculations based on 12 in. wide section.

$$\begin{aligned} (P_{wall})_o &= [2 + (22.5/2)]75 \\ &= 994 \text{ psf (use 1.0 kip)} \\ &\text{(half of panel weight causing } P - \Delta \text{ moment)} \end{aligned}$$

— Load factors:

- Case 1 — $U_1 = 1.4D + 1.7L$





- Case 2 — $U_2 = 0.75(1.4D + 1.7L + 1.7W)$
 - Case 3 — $U_3 = 0.9D + 1.3W$
- Note that earthquake load does not control.
- $$M_{u2} = 0.75 \left[3.657 + \left\{ 1.7 \times \frac{0.017(22.5)^2 12}{8} \right\} \right]$$
- $$= 19.202 \text{ kip-in.}$$

Case 1

Top of wall:

$$P_{u1} = 1.4(0.3) + \frac{1.4(2 \times 75)}{1000} + 1.7(0.25)$$

$$= 1.055 \text{ kips}$$

$$M_{u1} = 1.4(0.3)(6) + 1.7(0.25)(6)$$

$$= 5.070 \text{ kip-in.}^2$$

Midheight of wall:

$$P_{u1} = 1.4(0.3) + 1.4(1.0) + 1.7(0.25)$$

$$= 2.245 \text{ kips}$$

$$M_{u1} = [1.4(0.3) + 1.7(0.25)]6/2 + [1.4(0.3) + 1.4(1.0) + 1.7(0.25)]0.5$$

$$= 3.657 \text{ kip-in.}^2$$

Note that the first bracketed term denotes eccentricity moment while the second bracketed term denotes bowing moment.

Case 2

Top of wall:

$$P_{u2} = 0.75(1.055)$$

$$= 0.791 \text{ kips}$$

$$M_{u2} = 0.75(5.070)$$

$$= 3.803 \text{ kip-in.}$$

Midheight of wall:

$$P_{u2} = 0.75(2.245)$$

$$= 1.684 \text{ kips}$$

Note that the last term denotes wind moment.

Case 3

Top of wall:

$$P_{u3} = 0.9 \left(0.3 + \frac{2 \times 75}{1000} \right)$$

$$= 0.405 \text{ kips}$$

$$M_{u3} = 0.9(0.3 \times 6)$$

$$= 1.620 \text{ kip-in.}$$

Midheight of wall:

$$P_{u3} = 0.9(0.3 + 1.0)$$

$$= 1.170 \text{ kips}$$

$$M_{u3} = 0.9 \left[(0.3)(6/2) + (1 + 0.3)0.5 \right] + 1.3 \left[\frac{0.017(22.5)^2 12}{8} \right]$$

$$= 18.177 \text{ kip-in.}$$

in which the first and second terms within the first bracket denote Pe and $P \Delta_{bow}$ moments, respectively.

A summary of the loading at the top and midheight of the wall for Cases 1, 2 and 3 is given on the next page.

Loading summary

Case	Top		Midheight	
	P_u kips	M_u kip-in.	P_u kips	M_u kip-in.
Case 1	1.055	5.070	2.245	3.657
Case 2	0.791	3.803	1.684	19.202
Case 3	0.405	1.620	1.170	18.177

Calculate sustained load factors: 8.2.2

Case 1 at midheight:

$$\beta_{d1} = \frac{1.4(0.3)(6/2) + 1.4(0.3 + 1.0)(0.5)}{3.657}$$

$$= 0.593$$

Case 2 at midheight:

$$\beta_{d2} = \frac{0.75[(1.4)(0.3)(6/2) + (1.4)(0.3 + 1.0)(0.5)]}{19.202}$$

$$= 0.085$$

Case 3 at midheight:

$$\beta_{d3} = \frac{0.9[0.3(6/2) + 1.3(0.5)]}{18.177}$$

$$= 0.077$$

Note: It is conservative to use the largest β_d especially since panel stiffness changes significantly when panel cracks.

Solution using $P - \Delta$ procedure (second order analysis)

Calculate ϕ :

$$\phi = 0.7 \text{ at } 0.1 f'_c A_g$$

$$= 0.1(5)(72) = 36 \text{ kips}$$

$$\phi = 0.9 \text{ at } P_u = 0$$

$$\phi = 0.9 - 0.2 (P_u/P_{0.1})$$

For Case 2:

$$\phi = 0.9 - 0.2 (1.684/36) = 0.89$$

$$EI = \frac{\phi E_c I_g}{1 + \beta_d}$$

$$= \frac{0.89(4300)(216)}{1 + 0.593}$$

$$= 518,915 \text{ kip-in.}^2$$

Deflection at midspan (Case 2)

1. Due to P_e :

$$\Delta = \frac{ML^2}{16EI} = \frac{0.75(2.535)(270)^2}{16(518,915)}$$

$$= 0.0167 \text{ in.}^2$$

2. Due to $P \Delta_{\text{bow}}$:

$$\Delta = \frac{5ML^2}{48EI} = \frac{5(0.75)(1.123)(270)^2}{48(518,915)}$$

$$= 0.123 \text{ in.}^2$$

3. Due to wind:

$$\Delta = \frac{5ML^2}{48EI} = \frac{5(0.75)(21.946)(270)^2}{48(518,915)}$$

$$= 0.2409 \text{ in.}^2$$

Summation of deflections ($\Sigma \Delta$):

$$0.0167 + 0.0123 + 0.2409 = 0.2699 \text{ in.}$$

4. Deflection due to $P - \Delta$ moment at midspan:

$$\Delta = \frac{P \Delta' L^2}{8EI}$$

$$= \frac{0.75[(1.4)(1.3) + 1.7(0.25)](270)^2 \Delta'}{8(518,915)}$$

$$= 0.0296 \Delta'$$

Iteration 1:

$$\Delta = 0.0296(0.27) = 0.0080 \text{ in.}$$

$$\Delta' = 0.2699 + 0.0080 = 0.2779 \text{ in.}$$

Iteration 2:

$$\Delta = 0.0296(0.278) = 0.0082 \text{ in.}$$

$$\Delta' = 0.2699 + 0.0082 = 0.2781 \text{ in.}$$

Iteration 3:

$$\Delta = 0.0296(0.278) = 0.0082 \text{ in.}$$

$$\Delta' = 0.2699 + 0.0082 = 0.2781 \text{ in.}$$

Note: The Δ' of the last two iterations are convergent.

M_u at midspan (Case 2)

$$M_u = 19.202 + 1.684(0.278)$$

$$= 19.670 \text{ kip-in.}$$

Design loads for $P - \Delta$ analysis (at midspan):

$$P_u = 1.684 \text{ kips}$$

$$M_u = 19.670 \text{ kip-in.}$$

Solution using moment magnification procedure

Loading Case 2 controls at midspan:

$$P_{u2} = 1.684 \text{ kips}$$

$$M_{u2} = 19.202 \text{ kip-in.}$$

$$\beta_d = 0.593 \text{ (conservative)}$$

$$kl_u/r = 1.0(270)/1.73$$

$$= 156 > 150 \text{ (ok)}$$

Note that $k = 1$ is conservative and $r = 1.8$ by Section 8.1.3).

$$\phi P_o = 205 \text{ kips}$$

(see PCI Design Handbook, Fig. 2.6.5)

$$P_o = 205/0.7 = 292.86 \text{ kips}$$

$$P_u/P_o = 1.684/292.86 = 0.0058$$

Determine EI : 8.2.2

$$EI = \frac{E_c I_g / \lambda}{1 + \beta_d} \quad \text{Eq. (8.5)}$$

$$\lambda = \eta \theta > 3.0 \quad \text{Eq. (8.6)}$$

$$\eta = 2.5 + \frac{1.6}{P_u/P_o} \quad \text{Eq. (8.7)}$$

$$= 2.5 + \frac{1.6}{1.684/292.8}$$

$$= 280.7$$

$$\eta_{max} = 70 \text{ (use } \eta = 70)$$

$$\theta = \frac{27}{kl_u/r} - 0.05 \quad \text{Eq. (8.8)}$$

$$= \frac{27}{156} - 0.05$$

$$= 0.123$$

$$\lambda = 70(0.123) = 8.62 > 3.0$$

(see Fig. 8.2)

$$EI = \frac{4300(216)/8.62}{1 + 0.593}$$

$$= 67,639 \text{ kip-in.}^2$$

Determine magnified moment: 8.2.1

$$P_c = \frac{\pi^2 EI}{(kl_u)^2} \quad \text{Eq. (8.4)}$$

$$= \frac{\pi^2 (67,639)}{(1.0 \times 270)^2}$$

$$= 9.16 \text{ kips}$$

$$\phi = 0.89 \text{ (earlier calculation)}$$

• Moment magnification factor: Eq. (8.2)

$$\delta_b = \frac{C_m}{1 - P_u/\phi P_c}$$

$$C_m = 1.0 \quad 8.2.3$$

$$\delta_b = \frac{1.0}{1 - 1.684/0.89(9.16)}$$

$$= 1.26$$

$$M_c = \delta_b M_{u2} \quad \text{Eq. (8.1)}$$

$$= 1.26(19.202)$$

$$= 24.201 \text{ kip-in.}$$

• Check axial load and bending moment on interaction diagram.

	Second order analysis	Moment magni- fication
Axial load (kips)	1.684	1.684
Bending moment (kip-in.)	19.670	24.201
(kip-ft)	1.64 (ok)	2.02 (ok)

Factored moment deflection check: 5.4
 $EI = 67,639 \text{ ksi}$

Note: This is the stiffness at ultimate. Panel is most likely cracked or near cracking.

$$M_u = 24.20 \text{ kip-in. per ft width}$$

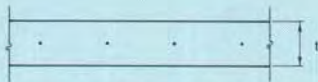
Deflection due to magnified moment:

$$\Delta = \frac{5 M_u L^2}{48 EI}$$

$$= \frac{5(24.20)(270)^2}{48(67,639)}$$

$$= 2.71 \text{ in. } (L_u/99 > L_u/100 \text{ max})$$

Deflection is very close to limit (say ok).

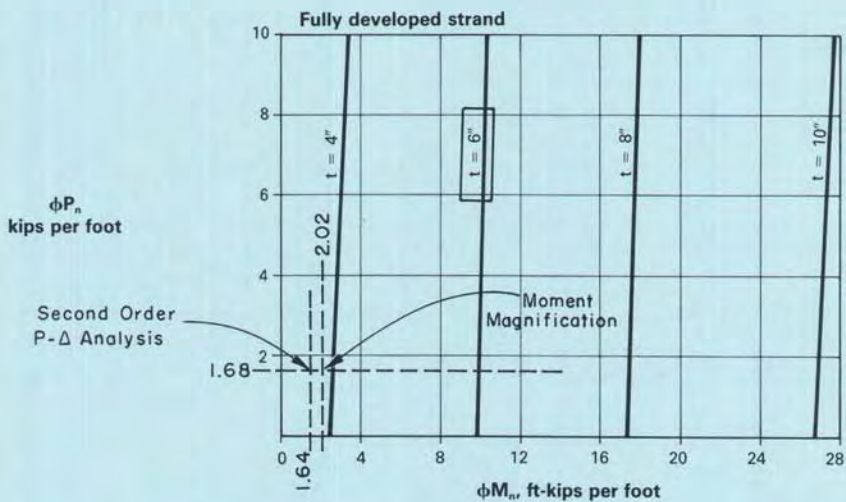
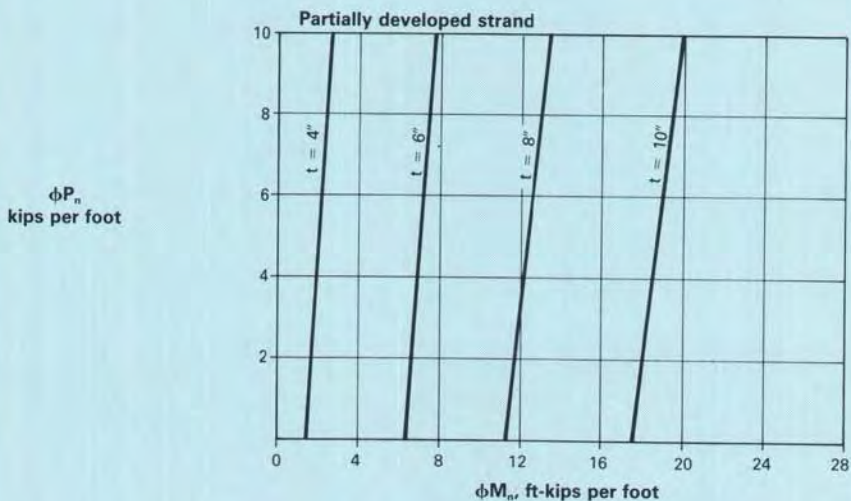


Curves based on minimum prestress of 250 psi

$f'_c = 5000$ psi

$f'_{pu} = 270$ ksi

t in.	Full Interaction Curve Data						
	Partially Developed Strand Force				Fully Developed Strand Force		
	ϕP_o	ϕP_{nb}	ϕM_{nb}	ϕM_o	ϕP_{nb}	ϕM_{nb}	ϕM_o
4	140	64	6.2	1.9	65	6.0	2.5
6	205	90	13.8	6.4	87	13.4	9.7
8	273	121	25.0	11.3	116	23.8	17.3
10	341	152	39.4	17.5	145	37.2	26.7



Partial interaction curve for prestressed solid wall panels

$$\eta = 2.5 + \frac{1.6}{P_u/P_o}$$

$$6 \leq \eta \leq 70$$

$$\lambda = \eta \theta$$

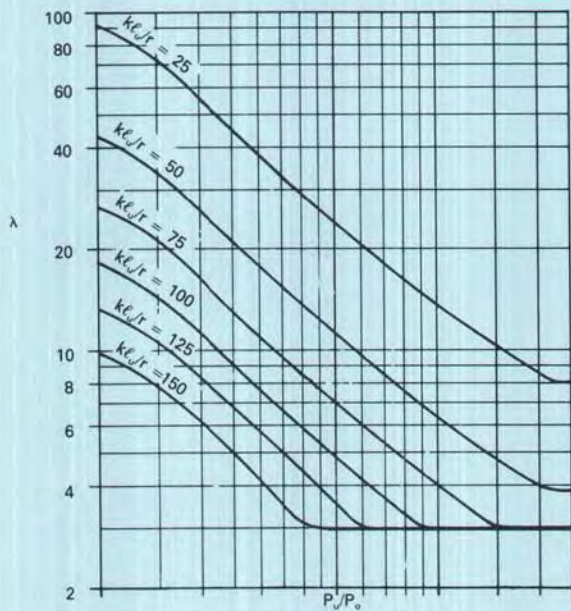
for P_o equation:

$$EI = \frac{E_s I_g / \lambda}{1 + \beta_s}$$

(a) Compression flange

$$\theta = \frac{35}{k\ell_u/r} - 0.09$$

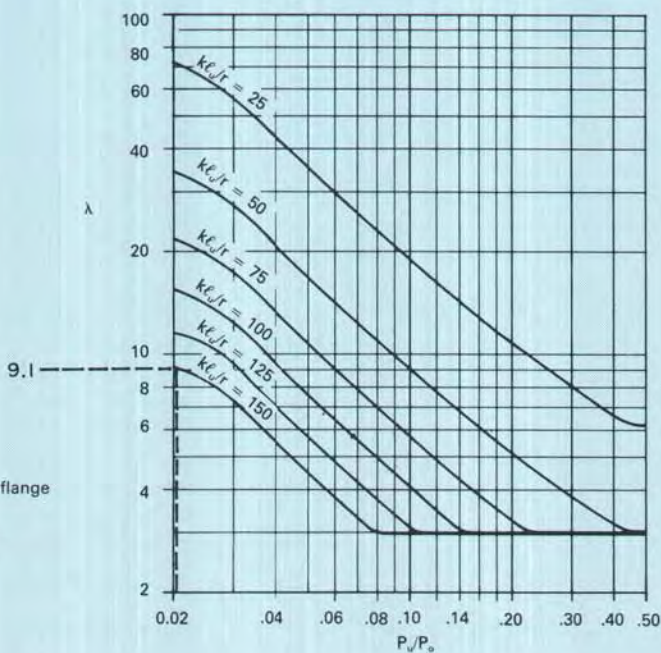
Fig. 8.1



(b) No compression flange

$$\theta = \frac{27}{k\ell_u/r} - 0.05$$

Fig. 8.2



Coefficients, λ , for modified EI