READER COMMENTS

Influence of Concrete Strength and Load History on the Shear Friction Capacity of Concrete Members*

by Joost Walraven, Jerome Frénay and Arjan Pruijssers

Comments by Alan H. Mattock, S. T. Mau and Thomas T. C. Hsu, and Authors

ALAN H. MATTOCK†

The authors are to be congratulated on an interesting and carefully executed study.

The writer is fully in agreement with the authors' contention that concrete strength is a factor in shear friction strength. It was omitted from the Modified Shear Friction Eq. (5) because, at the time that the equation was proposed, most of the available test data related to specimens with a concrete strength of about 4000 psi (28 MPa).

Subsequently, tests were carried out on specimens made from 6000 psi (41 MPa) concrete, and in 1976 the following equation was proposed:[23]

$$\nu_u = 4.5 f_c'^{0.545} + 0.8 (\rho_v f_y + \sigma_n) \text{ psi}$$
$$\not> 0.3 f_c' \text{ psi} \tag{10}$$

$$\nu_u = 0.467 f_c'^{0.545} + 0.8 (\rho_v f_y + \sigma_n) \text{ MPa}$$
$$\not> 0.3 f_c' \text{ MPa}$$

This equation is compared in Fig. 15 with the data reported in Ref. 23 for initially cracked monolithic specimens made of normal weight concrete having an average concrete strength f_c' of 5963 psi (41.1 MPa). It can be seen that the equation represents the data reasonably well. It also represents equally well the earlier data[3] from tests of lower strength concrete specimens.

Also shown in Fig. 15 is a line representing the authors' Eq. (6) for specimens made of this particular strength of concrete. It can be seen that for this series of specimens, Eq. (6) is a little on the unconservative side. For these eight specimens, the average value of ν_u (test)/ν_u (calc.) is 0.93 using Eq. (6) and 1.05 using Eq. (10). The coefficients of variation are 0.08 and 0.06, respectively.

The authors' equation is substantiated by careful and extensive statistical analysis. However, the writer prefers the form of Eq. (10) since this reflects the two most significant contributors to shear transfer strength. The first term reflects the resis-
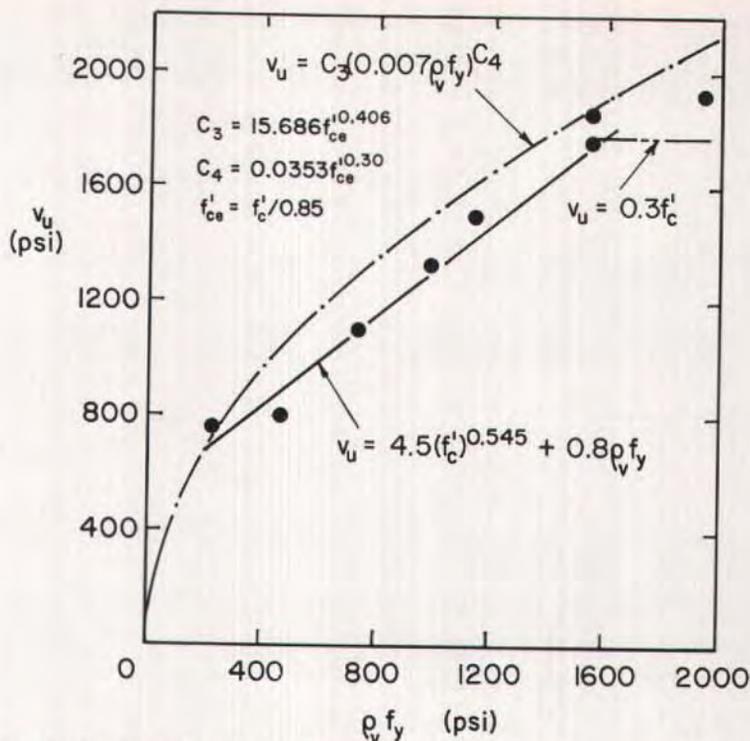
Fig. 15. Comparison with alternative equations of data from tests (Ref. 23) of specimens made from 6000 psi (41 MPa) normal weight concrete.

tance to shearing off of asperities on the crack faces, and the second term reflects the frictional resistance to sliding. It is proper that the resistance to shearing off of the asperities should be represented by a term containing the concrete strength. It is also proper that the term reflecting the frictional resistance should be independent of the concrete strength.

The writer also believes that an upper limit to shear transfer strength should be stated, to reflect the change in behavior of specimens with very high $\rho_v f_y$, in which the initial crack "locks up" and the mode of failure changes to that of an initially uncracked shear plane.

The writer does not propose Eq. (10) for use in practice, because in 1976 the equation appeared to be too complex for design office use. However now that electronic calculators are so widely available, this equation could readily be used in practice.

Reference

23. Mattock, A. H., "Shear Transfer Under Monotonic Loading, Across an Interface Between Concretes Cast at Different Times," Report SM 76-3, Department of Civil Engineering, University of Washington. (Part 1 of Final Report to National Science Foundation, Grant No. Eng74-21131.)

S. T. MAU* and THOMAS T. C. HSU†

The authors have made an important contribution in expanding the applicability of the shear friction design formula to the range of high strength concrete. The proposed formula, Eq. (6), includes two

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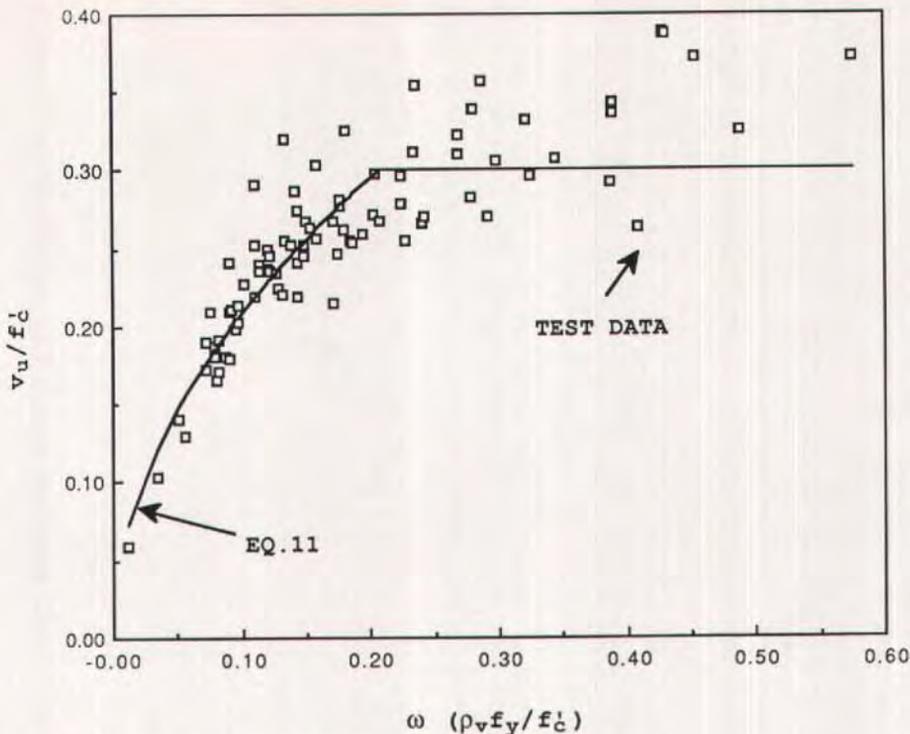
Fig. 16. Shear strength as a function of the reinforcing index.

parameters, $(\rho_v f_y)$ and f_c'. The form of the formula is determined by curve fitting the test data. As a result, it is rather inconvenient to use and a design chart must be provided.

It is our opinion that a more judicious choice of the parameters should lead to a simpler formula for design. In studying the behavior of shear transfer across an initially uncracked shear plane[24] and the shear strength of low-rise shearwalls,[25] it was found that the nondimensional reinforcing index ω, defined as $\rho_v f_y / f_c'$, is the dominant factor in the determination of the normalized shear strength, defined as v_u / f_c'. This is valid when the yielding of reinforcing steel precipitates the failure.

Even though the shear friction phenomenon across a cracked plane may be different from that of the shear failure across an initially uncracked plane, the interaction between the steel and concrete is likely to involve the same factor. Thus, a good correlation could be found between the normalized shear strength and a single parameter, the reinforcing index ω.

As shown in Fig. 16, the following formula fits the data well:

$$\frac{v_u}{f_c'} = 0.66 \sqrt{\omega} < 0.3 \qquad (11)$$

The constant cutoff at high ω value is in the region where the steel does not yield at failure and is meant to neglect the slight increase of shear strength with excessive reinforcement. Note that Eq. (11) is based on standard cylinder strength, f_c'. The cube strengths of concrete, f_{cc}', reported in the paper were converted to f_c' by multiplying a factor of 0.85, recommended by the authors.

Eq. (11) has been compared to the 88 data points reported in the paper. In the region where the steel should yield at failure ($v_u / f_c' < 0.3$), the mean value of the
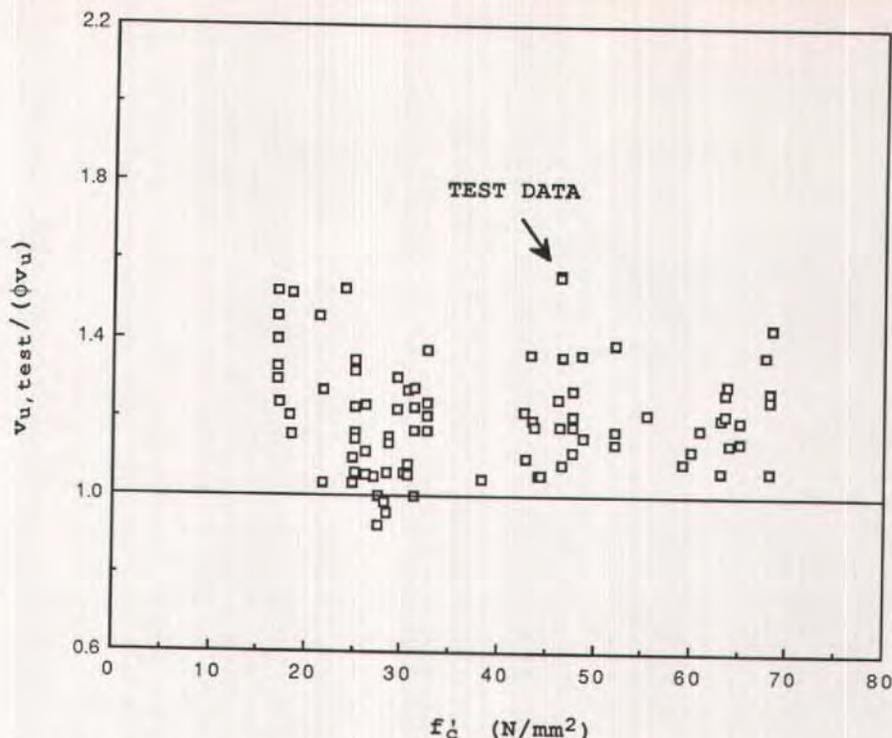
Fig. 17. Comparison between experimental shear stresses and the recommended values based on Eq. (11).

test-to-calculated shear strength ratio for 60 specimens is 1.003, and its coefficient of variation is 0.111. In the region of excessive reinforcement ($\nu_u/f_c' > 0.3$), the mean value of 28 specimens is 1.058, and its coefficient of variation is 0.130.

It is seen that the present proposal, Eq. (11), has the same accuracy as the authors' formula, Eq. (6), in the practical region useful for designers. By adopting the same strength reduction factor $\phi = 0.85$ as originally proposed by Mattock,[10] the test data normalized by the design shear strength are plotted in Fig. 16 as a function of f_c'.

A comparison of Fig. 17 with Fig. 12 reveals that the present proposal [Eq. (11)] is as good as the authors' formula [Eq. (6)] in predicting the effect of the concrete strength. However, the present

proposal has the following advantages: (1) It identifies a physically meaningful parameter, the reinforcing index ω, as the single most important factor; (2) The design formula is dimensionless and easy to use; and (3) The design formula is in a form that deviates the least from Mattock's modified shear friction formula, i.e., Eq. (9).

References

24. Hsu, T. T. C., Mau, S. T. and Chen, B., "Theory on Shear Transfer Strength in Reinforced Concrete," *Structural Journal*, American Concrete Institute, V. 84, No. 2, March-April 1987, pp. 149-160.

25. Mau, S. T., and Hsu, T. T. C., "Shear Design and Analysis of Low-Rise Structural Walls," *ACI Journal*, V. 83, No. 2, March-April 1986, pp. 306-315.
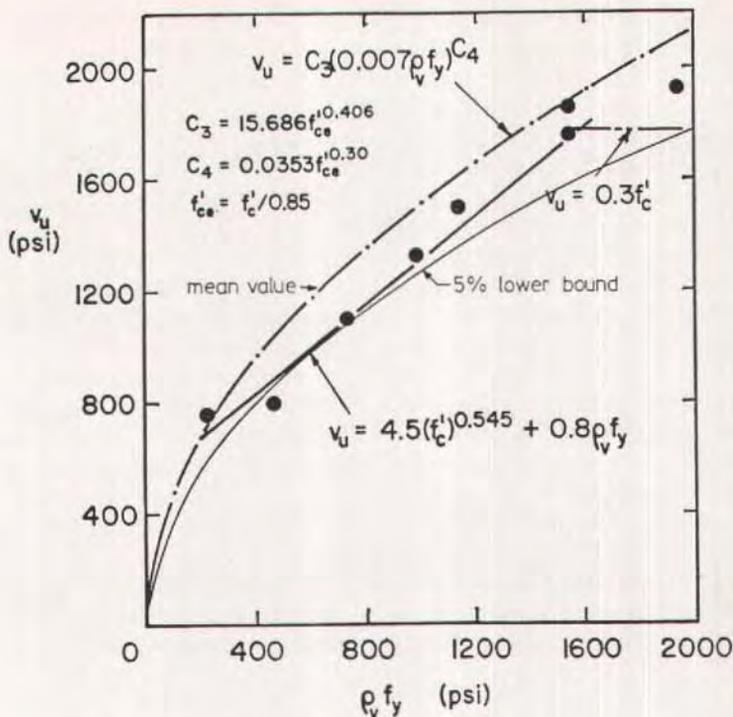
Fig. 18. Comparison of authors' and Mattock's equations.

AUTHORS' CLOSURE by JOOST WALRAVEN,* JEROME FRÉNAY† and ARJAN PRUIJSSERS‡

The authors would like to thank Professors Mattock, Mau and Hsu for their comments. In both contributions it is confirmed that the concrete strength should be considered in the formulation of the shear friction capacity.

With regard to Professor Mattock's comments, it is quite well understood that in former formulations of the shear friction capacity the role of the concrete strength

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was neglected. As shown in Fig. 11 of the article, the modified shear friction equation was already reasonably accurate for concrete strengths up to $f_c' = 5000$ psi (35 N/mm²), which covered most of the applications in former years.

Only in the last ten years has there been an increased interest in higher strength concretes. This has resulted in shear friction tests on high strength concrete specimens, enabling a new evaluation of existing shear friction formulas. The authors endeavored to derive a statistical expression with optimum accuracy for the widest possible range of parameter variation and found this in the power function represented by Eq. (6) in the article.

Fig. 15 of Professor Mattock's discussion could suggest that this function is slightly unconservative because the experimental values represented in this diagram lay below the curve. However, in de-

veloping the design chart (Fig. 9 in the article), the 5 percent lower bound was used. Fig. 18 shows that this approach yields safe results. (The fact that the experimental values are only slightly above the lower bound curve could be explained by the fact that the points refer to tests on interfaces between concretes cast at different times: it is possible that the penetration of the particles in such a case is less than in a crack through monolithic concrete.)

The proposal by Professor Mattock to replace the constant term in the modified shear friction equation, Eq. (9), by a term depending on the concrete strength would indeed improve the accuracy of this expression, but would still be too conservative if compared with the new data for $6000 < f'_c < 10000$ psi $(40 < f'_c < 70 \text{ N/mm}^2)$. It is, however, apparent that a simplification of the statistical expression proposed by the authors, Eqs. (6) and (7) in the arti-

cle, would be welcome if the same accuracy could be maintained.

Professors Mau and Hsu are to be congratulated for finding such an equation. Their expression:

$$\frac{\nu_u}{f'_c} = 0.66 \sqrt{\omega} < 0.3$$

with $\omega = f_{sy}/f'_c$ is really simple and has an accuracy which is almost as good as the more complicated statistical function proposed by the authors (general $x = 1.019$ and $s = 0.127$ for Mau/Hsu's equation and $x = 1.001$ and $s = 0.109$ for the authors' equation in comparison with 88 results). The expression is therefore certainly a good alternative for the use of the design chart.

The constructive comments of the discussers, pushing the frontier of knowledge, are highly appreciated.

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