Effect of Initial Strand Slip on the Strength of Hollow-Core Slabs

Under certain conditions, some of the typical machine processes for making hollow-core slabs may sometimes produce concrete with insufficient compaction or paste formation in the region surrounding the pretensioned strand. This can result in a deficiency in the bond capacity which is evidenced by excess initial strand slip upon transfer of the prestressing force through transverse saw-cutting of the product. The producer is then faced with the problem of evaluating the effect of such slip on the structural capacity of the product.

Several tests and studies carried out in past years on hollow-core slabs\(^1\)\(^2\) have established that excess initial strand slip does have adverse effects on the capacity of such slabs. Anderson and Anderson\(^1\) suggested that the magnitude of "free end slip" is directly related to transfer bond quality and to flexural bond quality, and can be used as a direct measure of transfer length if the transfer of prestress is assumed to take place linearly. An allowable free end slip was thus computed directly from the transfer length equation derived from the ACI Commentary\(^3\) (see Fig. 12.9, \(l_t = f_{sed}b/3\)).

In 1980, Mast\(^\dagger\) suggested that the flexural bond length could also be ex-

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*Formerly, Graduate Assistant, Civil, Environmental, and Architectural Engineering, University of Colorado, Boulder, Colorado.

pressed linearly (in proportion to free end slip), that bilinear strand development diagrams could be created for individual strands, and that a similar procedure, utilizing the weighted average of the computed bilinear development diagrams for all strands, would enable the computation of a bilinear moment capacity diagram for each end of any specific member. These concepts are collectively referred to as the “slip theory” in this paper.

In order for prestress transfer length to be limited to the ACI Code value \( f_{cds/3} \), the Anderson-Anderson equation limits the allowable initial strand slip to the range of approximately \( \frac{1}{32} \) to \( \frac{3}{32} \) in. (1 to 3 mm), depending on strand diameter and the initial and final levels of prestress in the strand.

Preliminary tests on slabs with initial strand slips exceeding the allowable values were conducted in Colorado Springs and the results appeared to verify the applicability of the slip theory. Additional tests were conducted at the University of Wisconsin, Milwaukee, to evaluate the relationship of excess initial strand slip to observed capacity and to compare the results of these tests with the slip theory. A close correlation resulted, but there were insufficient data to evaluate that comparison in the flexural bond region of the strand development curve.

The availability of several rejected slabs from the Colorado Springs production facility provided an opportunity to obtain additional data for verification.

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**Synopsis**

The object of this experimental study was to examine the effects of excess initial strand slip on strand development in hollow-core slabs. Results were obtained from eleven concentrated load tests conducted on seven hollow-core slab test specimens. The observed capacities in the tests were compared to predicted capacities computed according to two methods:

1. **ACI 318-83 Section 12.9.1, Development of Prestressing Strand** — This section of the ACI Code is intended for conditions where consolidation of the freshly placed concrete around pretensioned strands is sufficient to create normal bond quality. It is not applicable to cases where substandard bond quality, as evidenced by excess initial strand slip, may result in increased prestress transfer length and flexural bond length of the strands.

2. **A strand slip theory**, suggested by Mast in 1980 and based on portions of the work of Anderson and Anderson, which extends the ACI formulas to account for the effects of excess initial strand slip.

Results of the test program appear to confirm the applicability of the ACI Code equations where initial strand slip is limited to allowable values, but indicate that these equations do not safely predict the capacity of slabs having excess initial strand slip. The strand slip theory, on the other hand, appears to adequately predict the reduced capacity and failure mode of slabs having such excess slip.

The test program was conducted in the laboratories of the Civil, Environmental, and Architectural Engineering Department of the University of Colorado at Boulder.
of the slip theory over a broader range of conditions. In the following, this theory is first outlined and then applied. The current tests are described and their results are compared to predictions by the existing ACI Code equations and by the slip theory.

**THE STRAND SLIP THEORY**

Section 12.9 of the ACI Code provides that development length \( l_d \) shall not be less than:

\[
l_d = \left( f_{ps} - 2 \frac{f_m}{3} \right) d_b
\]

in which the symbols are defined in the Notation section (see Appendix).

Eq. (1a) can be rewritten as:

\[
l_d = l_t + l_b = \frac{f_m}{3} d_b + (f_{ps} - f_m) d_b
\]

in which the first term represents the prestress transfer length \( l_t \), and the second term represents the flexural bond length \( l_b \). When excess initial end slip is evident upon transfer of the prestress, a greater development length may result. In fact, Section 12.9 of the ACI Commentary to the Code states that this provision “... may not represent the behavior of strand in low water-cement ratio, no-slump concrete.”

The slip theory assumes that the initial slip of the strand from the saw-cut face of the concrete is a direct indication of the bond quality of the concrete. This slip is therefore directly related to the transfer length \( l_t \) and the flexural bond length \( l_b \).

The measured initial strand slip \( \delta \) is related to the strand force by assuming a linear increase of this force, as shown in Fig. 1. The average strand force in the transfer zone \( F_{t(ave)} = F/2 \), is then related to the end slip through the steel strains:

\[
\delta = \frac{F_{t(ave)}}{A_p E} \frac{l_t}{2A_p E}
\]

Rearranging this relation and introducing the steel stress \( f_{st} = F/2A_p \), the transfer length can be written as:

\[
l_t = \frac{2 \delta A_p E}{f_{st}} = \frac{2 \delta E}{f_{st}}
\]

When \( l_t \), as obtained from Eq. (3), is larger than \( l_t \) derived from Eq. (1b), it can be deduced that the strand slip is greater than the allowable value, \( \delta_{all} \). This value, \( \delta_{all} \), can be found by equating \( l_t \) from Eq. (1b) to \( l_t \) from Eq. (3):

\[
2 \delta E = \frac{f_m d_b}{3}
\]

or

\[
\delta_{all} = \left( \frac{1}{6} \right) \frac{f_m f_{st} d_b}{E}
\]

The allowable slip, \( \delta_{all} \), is defined as the initial strand slip at the saw-cut end of a slab which results in a transfer

<table>
<thead>
<tr>
<th>Prestress levels in strand</th>
<th>Allowable initial strand slip</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Strand diameter</td>
</tr>
<tr>
<td></td>
<td>3/16 in.</td>
</tr>
<tr>
<td>( f_{st} = 150 \text{ ksi} ) / ( f_m = 120 \text{ ksi} )</td>
<td>0.040 in. (1/32 in.)</td>
</tr>
<tr>
<td>( f_{st} = 165 \text{ ksi} ) / ( f_m = 135 \text{ ksi} )</td>
<td>0.050 in. (2/32 in.)</td>
</tr>
<tr>
<td>( f_{st} = 180 \text{ ksi} ) / ( f_m = 150 \text{ ksi} )</td>
<td>0.060 in. (2/32 in.)</td>
</tr>
</tbody>
</table>

Note: 1 in. = 25.4 mm; 1 ksi = 6.895 MPa.
length equal to that computed by the ACI equation, \( l_t = f_{se} d_b / 3 \), for specific values of \( f_{se}, f_{oe}, d_b, \) and \( E \).

Table 1 shows allowable initial strand slip for various prestress levels and strand diameters of \%\ and \% in. (9.5 and 13 mm). The tabulation is based on \( E = 28,000 \) ksi (193,000 MPa).

As the transfer length \( l_t \) increases to a value of \( l_{t1} \) according to Eq. (3), the flexural bond length \( l_b \) is also assumed to increase to a value of \( l_{b1} \) in the ratio derived from Eq. (1b):

\[
l_{b1} = 3 \left( \frac{f_{ps} - f_{se}}{f_{se}} \right) l_b
\]

Fig. 2a shows the developable strand stress \( f_{dev} \) over the transfer length and the flexural bond length, both according to ACI Section 12.9 (disregarding excess initial strand slip) and also as affected by excess initial slip according to the slip theory, i.e., Eqs. (3) and (5).

For this latter case, assuming a linear variation of steel stress, the developable steel stress will be:

\[
f_{dev} = \frac{x}{l_{t1}} f_{se} \quad \text{where} \quad x \leq l_{t1}
\]

\[
f_{dev} = f_{se} + \frac{(x - l_{t1})}{l_{b1}} (f_{ps} - f_{se})
\]

where \( l_{t1} \leq x \leq l_{d1} \)

\[
f_{dev} = f_{ps} \quad \text{where} \quad l_{d1} \leq x
\]

in which \( x \) is the distance from the slab end.

The average developable strand stress, \( f_{dev(ave)} \), for all strands, at each slab end, can be computed using the weighted average of the bilinear development diagrams for all strands (see Appendix A).

The nominal moment strength \( M_n \) for any section can be calculated as a function of the average developable strand stress, \( f_{dev(ave)} \), according to the strength design method in Chapter 10 of the ACI Code, as follows:
SLIP THEORY

\[ Q_n \]

\[ p \]

\[ r \]

\[ Q_r \]

\[ A \]

\[ C \]

\[ f_{ps} \]

\[ r_f \]

\[ f_{dev}(SLIP \ THEORY) \]

\[ f_{dev}(ACI) \]

\[ t_S \]

THEORY

LOW CORE SLAB

H=DESIGN SPAN

END OF

BRG.
	BRG.

NORTH	SOUTH

a. DEVELOPABLE STRESS IN STRAND \( f_{dev} \) ACCORDING TO

ACI EQUATIONS AND STRAND SLIP THEORY

\[ f_{se} \]

\[ M \]

\[ M_{n(SLIP \ THEORY)} \]

\[ M_{n(ACI)} \]

\[ M_{FAIL} \]

\[ M_n \]

\( \rho \)

\[ f_{dev(amp)} \]

\[ f_{slip} \]

\[ f_{se} \]

\[ M_{n(ACI)} \]

\[ M_{FAIL} \]

\[ M_n(SLIP \ THEORY) \]

\[ M_n \]

END OF

SPECIMEN

\[ \ell \]

NORTH	SOUTH

END OF

SPECIMEN

\[ \ell \]

NORTH	SOUTH

b. COMPARISON OF OBSERVED ULTIMATE MOMENT CAPACITY

WITH ACI AND STRAND SLIP THEORY PREDICTIONS

Fig. 2. Comparison of developable stress and flexural capacity.

\[ M_n = A_{ps}\cdot f_{dev(amp)} \cdot d \cdot (1 - 0.59\rho \cdot f_{dev(amp)} / f_{se}) \]

(7)

A plot of \( M_n \) is shown in Fig. 2b. It provides a moment strength envelope, upon which a moment diagram due to the factored applied loads can be superimposed to check the flexural strength. In Fig. 2b, the dash-dotted moment diagram, representing any load distribution (here a point load), permits prediction of the flexural failure by matching the peak moment with the moment strength envelope. Where loads are applied sufficiently close to the end of the specimen, the strength of the slab
Fig. 3. Cross section and strand information (test slabs).

is reduced because of decreasing capacity of the strand where it is not yet fully developed. The strength of the specimen in this region is reduced further where excess initial strand slip occurs.

The shear strength of pretensioned slabs can also be affected by an increase in the prestress transfer length due to excess initial strand slip. In ACI Eq. (11.13):

\[
V_{cef} = (3.5 \sqrt{f_c'} + 0.3 f_{pe}) b_w d + V_p
\]

where \( f_{pe} \) is the effective concrete stress due to prestress at the centroid of the critical concrete shear section. If, due to excess initial strand slip, the transfer
Table 2. Summary of load test data.

<table>
<thead>
<tr>
<th>Test No.</th>
<th>Design span L (in.)</th>
<th>Shear span A (in.)</th>
<th>Avg. initial slip δ at near end (in.)</th>
<th>Slip theory transfer length $l_t^\prime$ (in.)</th>
<th>Slip theory flexural bond length $l_b^\prime$ (in.)</th>
<th>Applied load at failure $P_f$ (kips)</th>
<th>Mode of failure</th>
<th>Predicted failure load $P$ (kips)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Per ACI Code, governed by</td>
</tr>
<tr>
<td>1</td>
<td>331.5</td>
<td>9.75</td>
<td>0.208</td>
<td>68</td>
<td>124</td>
<td>22.78</td>
<td>Bond</td>
<td>24.37x</td>
</tr>
<tr>
<td>2</td>
<td>311.5</td>
<td>20.50</td>
<td>0.135</td>
<td>47</td>
<td>83</td>
<td>17.88a</td>
<td>Bond</td>
<td>25.53x</td>
</tr>
<tr>
<td>3</td>
<td>246.0</td>
<td>20.25</td>
<td>0.135</td>
<td>50</td>
<td>94</td>
<td>20.23</td>
<td>Bond</td>
<td>26.28x</td>
</tr>
<tr>
<td>4</td>
<td>228.0</td>
<td>44.75</td>
<td>0.141</td>
<td>47</td>
<td>86</td>
<td>16.78</td>
<td>Bond</td>
<td>30.73x</td>
</tr>
<tr>
<td>5</td>
<td>331.5</td>
<td>45.75</td>
<td>0.182</td>
<td>63</td>
<td>115</td>
<td>11.83</td>
<td>Bond</td>
<td>27.44x</td>
</tr>
<tr>
<td>6</td>
<td>276.0</td>
<td>69.75</td>
<td>0.177</td>
<td>61</td>
<td>112</td>
<td>12.63</td>
<td>Bond</td>
<td>32.40x</td>
</tr>
<tr>
<td>7</td>
<td>331.5</td>
<td>69.75</td>
<td>0.063b</td>
<td>23</td>
<td>43</td>
<td>15.00</td>
<td>Bond-Flexure (c)</td>
<td>13.25x</td>
</tr>
<tr>
<td>8</td>
<td>331.5</td>
<td>69.75</td>
<td>0.156</td>
<td>57</td>
<td>107</td>
<td>11.60</td>
<td>Bond</td>
<td>13.25x</td>
</tr>
<tr>
<td>9</td>
<td>331.5</td>
<td>92.75</td>
<td>0.177</td>
<td>60</td>
<td>110</td>
<td>10.39</td>
<td>Bond</td>
<td>13.55x</td>
</tr>
<tr>
<td>10</td>
<td>331.5</td>
<td>117.75</td>
<td>0.177</td>
<td>61</td>
<td>112</td>
<td>10.92</td>
<td>Flexure-Shear (d)</td>
<td>13.51</td>
</tr>
<tr>
<td>11</td>
<td>331.5</td>
<td>139.50</td>
<td>0.203</td>
<td>70</td>
<td>128</td>
<td>10.75</td>
<td>Bond</td>
<td>14.24</td>
</tr>
</tbody>
</table>

Note: 1 in. = 25.4 mm; 1 kip = 4.448 kN.

- **a** — Test No. 2 reached a maximum load of 17.88 kips after cracking at 19.53 kips.
- **b** — Test No. 7 was the only test specimen that had initial strand slip (<3/32 in.) conforming to ACI allowable limits.
- **c** — Three strands exhibited slippage during the test, then the remaining three strands broke.
- **d** — Flexure-shear crack caused a catastrophic failure with no loss of bond.
- **x** — Controlling prediction according to the ACI Code.
- **y** — Controlling prediction according to the slip theory.

**ACI** $l_t = 22$ in., $l_b = 41$ in. for Test Nos. 3, 7, and 8.

**ACI** $l_t = 25$ in., $l_b = 44$ in. for Test Nos. 1, 2, 4, 5, 6, 9, 10, and 11.
length is increased in accordance with Eq. (3), then the effective prestress $f_{pe}$ is correspondingly reduced, as is the shear strength which is given by Eq. (8).

Flexure-shear failure is predicted by ACI Eq. (11.11):

$$V_{ei} = 0.6 \sqrt{f'c} b_w d + V_d + \frac{V_i M_{cr}}{M_{max}} \quad (9)$$

in which

$$M_{cr} = (I/y_1)(6 \sqrt{f'c} + f_{pe} - f_d)$$

Any reduction of the effective prestress $f_{pe}$ in accordance with the strand slip theory will therefore reduce the cracking moment $M_{cr}$ and with it the shear strength $V_{ei}$.

**TEST PROGRAM**

Seven hollow-core slabs, of cross sections, strand patterns, and material properties shown in Fig. 3, were provided by Stresscon Corporation, Colorado Springs, Colorado. Six of these seven slabs had excess initial strand slip of average value greater than 0.08 in. (2 mm) at each end, as shown in Column 4 of Table 2. In no case was the average slip greater than ¼ in. (6 mm).

Concentrated loads were applied at various points for which the slip theory and the ACI Code predicted widely differing strengths. Figs. 4, 5 and 6 show the test setup, and Table 2 shows span lengths and load points for the eleven tests.

The slabs were supported with a bearing length of 3 in. (76 mm), similar to prevailing construction practice. A neoprene strip [11/16 in. (17 mm) thick] was provided at the bearing over the full width of the slab to alleviate stress concentrations at the support. A full width roller at one end and a pin support at the other end were used to permit end rotation and prevent axial constraints. The point load was applied by hydraulic jack and distributed through a W6x15 transverse beam over the full width of the slab.

Instrumentation consisted of a load cell and deflection gage at the load point. Additional slip of the strands during the test was measured by LVDT gages.
Fig. 5. Testing arrangement.

bearing on four of the strand ends and attached to the end of the slab. Concrete strains were measured in some of the specimens at the level of the steel and on the compression side of the slab over the transfer zone, but these strains are not reported.

TEST RESULTS

The results of the test program are summarized in Table 2. The actual applied failure load for each test is shown in Column 7 and the mode of failure is described in Column 8.

Note that the initial strand slip did not exceed \( \delta_{\text{all}} \) in Test No. 7 and that the test load was applied (Fig. 12) at a distance from the end of the slab approximately equal to the strand development length calculated in accordance with the ACI equation. This permitted a check on the applicability of the ACI equation to a case where the initial strand slip was within acceptable limits, and the results confirm its applicability in a situation where even a small increase in development length would have resulted in a premature failure by loss of bond capacity in the strands.

It should be noted that both the ACI equations and the slip theory predicted a shear/moment failure (\( V_{\text{sl}} \)) at a value slightly less than its flexural capacity but that the actual mode of failure was in flexure accompanied by tensile failure of half of the strands in the slab. The failure was ductile, with significant warning deflection prior to failure, and the actual failure load exceeded the predicted flexure failure load by 6 percent.

In Test No. 10, the slip theory predicted a premature failure by loss of bond at a test load of 9.02 kips (40.12 kN). In fact, the test load at failure occurred at 10.92 kips (48.57 kN), but the mode of failure was in shear/flexure (\( V_{\text{sl}} \)) and there was no loss of bond prior to the sudden failure of this specimen.

In all of the remaining nine cases in
which excess initial strand slip occurred, the mode of failure was loss of bond capacity in the strands (strand development). In none of these cases was bond failure predicted by the ACI equations, while the slip theory predicted bond failure in all of these cases.

In most of the tests, some of the strands experienced additional slip during application of increasing test load. Despite this partial bond loss, these slabs were able to carry additional load, suggesting that the strands which exhibited additional slip were able to sustain tensile capacity while the remaining strands were able to continue to carry increasing tensile load until they, too, lost bond and general bond loss failure resulted.

This phenomenon, typical for most tests, is shown for Test No. 8 in Figs. 7 and 8, showing the deflection and additional strand slip during loading, respectively. The apparent drops in load indicated in Fig. 7 are the result of a temporary reduction in the hydraulically applied load upon the sudden increase in slab deflection which accompanies flexural cracking.

COMPARISON BETWEEN PREDICTED AND OBSERVED STRENGTHS

Columns 9, 10 and 11 of Table 2 indicate, for each test, the predicted failure load computed by applicable ACI Code equations for $V_{ccu}$, $V_{cb}$ and development capacity or tensile capacity (designated "f") of the strand, respectively. Note that the ACI equations, which are not intended to account for excess initial strand slip, predict shear failure in Tests No. 1 through No. 9 and tensile failure of the strand in Tests No. 10 and No. 11, whereas bond failure was the actual mode of failure in all cases except Tests No. 7 and No. 10 as described above. In all cases where initial strand slip exceeded $\delta_a$, the ACI equations predicted
higher capacities than the failure loads observed in the tests.

Columns 12, 13 and 14 indicate, for each test, the predicted failure load computed by the slip theory, which correctly predicted the failure mode in all cases except Test No. 10, described above. It also predicted conservative failure loads in all cases except Test No. 4, where it overpredicted failure by 5 percent.

In Fig. 9, the ratios of observed failure moment to predicted failure moment are plotted against the location of the criti-

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**Fig. 7. Slab deflection versus applied load (Test No. 8).**

---

**Fig. 8. Additional strand slip during test versus slab deflection (Test No. 8).**
cal moment section along the span of the test specimen. A value of unity of these ratios indicates exact agreement between prediction and observation. A value larger than unity indicates a safe prediction, while a value less than unity indicates an unsafe prediction.

Fig. 9 shows that, in general, the ACI formulas cannot be used to safely predict failure for products having excess initial strand slip, while predictions by the strand slip theory are generally conservative. The slip theory appears to underestimate the actual capacity by a wide margin for very short shear spans, possibly because of the simplified assumption that strand development within the prestress transfer length is linear rather than convexly curved.

Figs. 10, 11 and 12 examine failure moments in detail for three cases: Test No. 5, in which the critical section falls within the transfer length; Test No. 9, in which the critical section is within the flexural bond length; and Test No. 7, in which the initial strand slip does not exceed $\delta_{\text{ult}}$, and the load is applied at a distance slightly greater than the development length computed by the ACI equation (which is nearly identical to that computed by the slip theory).

These diagrams illustrate the differences in failure mode wherein Test No. 7 is a flexural failure with significant ductility; Test No. 9, while eventually failing in loss of bond, also showed significant ductility; and Test No. 5, which failed in loss of bond and which would have resulted in a sudden failure under a gravity load test with little or no warning deflection.

Fig. 13 shows the unpublished results of the series of tests performed at the University of Wisconsin at Milwaukee.*† These tests were conducted on 8 in. (203 mm) machine cast slabs in which the manufacturing process was deliberately altered in order to produce

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Fig. 10. Load Test No. 5 (load applied within transfer length).
Fig. 11. Load Test No. 9 (load applied within flexural bond length).

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Fig. 12. Load Test No. 7 (load applied at the computed ACI development length). Note that initial strand slip did not exceed ACI allowable.
slabs having excess initial strand slip. The test specimens were 10 ft (3.05 m) and 13 ft 4 in. (4.06 m) long and pretensioned with 3/8 in. (9.5 mm) and 1/2 in. (12.7 mm) diameter strand, respectively.

The ratios of observed to predicted failure moments are plotted against location of critical moment within the span. In general, the ACI Code predictions are not conservative, whereas the slip theory safely predicts failure in 16 out of 17 cases, and is generally more conservative when the critical section occurs within the transfer length.

**CONCLUSIONS**

1. For prestress transfer length to be limited to the ACI Code value ($f_{pc} l_y/3$), the maximum allowable initial strand slip, $\delta_{all}$, varies from approximately $\frac{1}{8}$ to $\frac{1}{2}$ in. (1 to 3 mm), depending upon the strand diameter and levels of initial and final prestress.

2. In Test No. 7, in which initial strand slip did not exceed the allowable limit, the equations of Section 12.9.1 of the ACI Code appeared to correctly predict strand development length.

3. On test specimens where the initial strand slip exceeded the allowable limits, the ACI Code equations did not appear to safely predict the capacity or failure mode of such specimens and the equations appeared to be unconservative by a significant margin.

4. The strand slip theory, which extends the ACI Code equations to account for excess initial strand slip, appeared to give reasonable, conservative predictions for the capacity of test specimens having such excess slip, and correctly predicted the mode of failure.

5. The specimens having excess initial strand slip generally failed through loss of bond on the strand, with little warning deflection and diminished ductility.

These conclusions are based on results of the testing of 8 in. (200 mm) thick hollow-core slabs, machine cast in an extrusion processing using relatively dry, tamped concrete, and in some cases...
deliberate alteration to the machine process was required in order to generate excess initial strand slip. The strength of slabs of other proportions, and those cast by other processes, need further study.

It should also be noted that application of test loads through hydraulic pressure tends to obscure and perhaps modify the potential for the sudden brittle failure, which was experienced under more realistic gravity load applications in the plant test series in cases where premature bond loss was the mode of failure.

It is recommended that future testing include gravity load applications to better evaluate this behavior.

ACKNOWLEDGMENTS

Original concepts, and subsequent review and comments were provided by Robert F. Mast. Initial experimental testing, technical guidance, and financial support were provided by Donald R. Logan (coauthor) through Stresscon Corporation, Colorado Springs, Colorado.

Additional testing, review and comments were also provided by Roger Becker, Computerized Structural Design, Inc., Milwaukee, Wisconsin.

The test series, reported herein, were carried out in the laboratories of the Civil, Environmental, and Architectural Engineering Department of the University of Colorado, Boulder, under the direction of Professor Kurt H. Gerstle (coauthor).

This paper is based upon a Master of Science thesis submitted to the Department by Mark D. Brooks (coauthor).

REFERENCES

3. ACI Committee 318, “Building Code Requirements for Reinforced Concrete (ACI 318-83),” and “Commentary on Building Code Requirements (ACI 318-83),” American Concrete Institute, Detroit, Michigan, 1983.
APPENDIX A — SAMPLE CALCULATIONS

The following sample calculations are based on data obtained from Test No. 2.

1. Parameters given (refer to Fig. 3).
   a. Section properties
      \[ A = 246.5 \text{ in.}^2 \]
      \[ Y_{top} = 3.928 \text{ in.} \]
      \[ Y_{bot} = 4.172 \text{ in.} \]
      \[ b_w = 14.75 \text{ in.} \]
      \[ l = 1730.2 \text{ in.}^4 \]
      \[ S_{top} = 451.9 \text{ in.}^3 \]
      \[ S_{bot} = 414.8 \text{ in.}^3 \]
      \[ w = 61.6 \text{ psf (144 pcf)} \]

   b. Material strengths and modulus of elasticity (assumed)
      \[ f'_c = 5000 \text{ psi} \]
      \[ f_{pu} = 270 \text{ ksi} \]
      \[ E = 28000 \text{ ksi} \]

   c. Prestress losses as a percentage of jacking stress (assumed)
      5 percent at time of initial strand slip measurements immediately after transfer of prestress.
      15 percent at time of load test.

   d. Cross section area of strands and strand location measured from bottom of member to centerline of strand
      \[
      \begin{array}{|c|c|c|}
      \hline
      \text{Diameter of strand} & \text{Area} & \text{Location} \\
      \hline
      \frac{3}{8} \text{ in.} & 0.085 \text{ in.}^2 & 1.6875 \text{ in.} \\
      \frac{1}{2} \text{ in.} & 0.153 \text{ in.}^2 & 1.75 \text{ in.} \\
      \hline
      \end{array}
      \]

   e. Measured initial strand slips (see Table A1) north end of test specimen (as cast in production facility). Measurements made to center wire of 7-wire strand using tire tread gage calibrated in 32nds of an inch.

2. The testing arrangement for Test No. 2 is indicated in Fig. A1. The test specimen was supported by 3 in. neoprene bearing pads resting on pin and roller supports. Test No. 2 was the second test on the specimen and the damaged end from the first test was removed prior to the start of the second test.

3. Strand stresses and development length per ACI 318.
   \[ A_p = 2x0.085 + 4x0.153 = 0.782 \text{ in.}^2 \]
   \[ d = (2x0.085(8 - 1.6875) + 4x0.153(8 - 1.75))/0.782 = 6.26 \text{ in.} \]
   \[ b = 48 \text{ in.} \]
   \[ \rho = 0.782/(48x6.26) = 0.00260 \]
   \[ f_{pu} = f_m(1 - 0.5 \rho f_{pu}/f'_c) \]
   \[ = 270(1 - 0.5x0.0026x270/5) = 251 \text{ ksi} \]
   \[ f_{uc} = 0.85 F_j/A_p \]
   \[ = 0.85[(2x13.8 + 4x28.9)/0.782] = 156 \text{ ksi (avg)} \]
   Refer to Eq. (1b):
   \[ l_d = f_w d_2/3 + (f_{pu} - f_w) d_b \]
   \[ l_t = f_w d_3/3 - l_b = (f_{pu} - f_w) d_b \]
   Individual strands
   \[
   \begin{array}{|c|c|c|c|}
   \hline
   \text{Strand diameter, } d_b, \text{ in.} & 32nds of an inch & \% & \% \\
   \hline
   \text{Jacking force, } F_j, \text{ kips} & 13.8 & 28.9 & 28.9 \text{ ksi} \\
   \text{Jacking stress, } f_j, \text{ ksi} & 162.4 & 188.9 & 188.9 \text{ ksi} \\
   \hline
   \end{array}
   \]

Table A1. Measured initial strand slips.

<table>
<thead>
<tr>
<th>Strand designation</th>
<th>S_1</th>
<th>S_2</th>
<th>S_3</th>
<th>S_4</th>
<th>S_5</th>
<th>S_6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Strand diameter, d_b, in.</td>
<td>%</td>
<td>1/2</td>
<td>1/2</td>
<td>1/2</td>
<td>1/2</td>
<td>%</td>
</tr>
<tr>
<td>Jacking force, F_j, kips</td>
<td>13.8</td>
<td>28.9</td>
<td>28.9</td>
<td>28.9</td>
<td>28.9</td>
<td>13.8</td>
</tr>
<tr>
<td>Jacking stress, f_j, ksi</td>
<td>162.4</td>
<td>188.9</td>
<td>188.9</td>
<td>188.9</td>
<td>188.9</td>
<td>162.4</td>
</tr>
<tr>
<td>Measured initial slip, 32nds of an inch</td>
<td>2</td>
<td>6</td>
<td>5</td>
<td>6</td>
<td>5</td>
<td>2</td>
</tr>
</tbody>
</table>
Fig. A1. Test arrangement (Load Test No. 2).

\[ f_{se} = 0.85 \times 188.9 = 161 \text{ ksi} \]

\[ l_t = 161 \times 0.50/3 = 26.8 \text{ in.} \]

\[ l_b = (251 - 161) \times 0.5 = 45.0 \text{ in.} \]

Weighted average of all strands

\[ l_t = \frac{(2 \times 0.085 \times 17.3 + 4 \times 0.153 \times 26.8)/0.782}{25} = 25 \text{ in.} \]

\[ l_b = \frac{(2 \times 0.085 \times 42.4 + 4 \times 0.153 \times 45.0)/0.782}{44} = 44 \text{ in.} \]

\[ l_d = 25 + 44 = 69 \text{ in.} \]

4. Development length based on slip theory, using measured initial strand slips.
   a. Each strand individually
      [Refer to Eq. (3)]
      (1) Transfer length, \( l_t = 2 \delta E / f_{si} \)
      where \( E = 28,000 \text{ ksi} \)
      \( f_{si} = 95 \text{ percent} \)
      \( \delta = \text{measured initial strand slip} \)

   Examples:
   Strand \( S_1 \):
   \[ \delta = 2/32 \text{ in.} = 0.0625 \text{ in.} \]
   \[ l_t = \frac{2 \times 0.0625 \times 28,000}{0.95 \times 162.4} = 23 \text{ in.} \]

   Strand \( S_2 \):
   \[ \delta = 6/32 \text{ in.} = 0.1875 \text{ in.} \]
   \[ l_t = \frac{2 \times 0.1875 \times 28,000}{0.95 \times 188.9} = 59 \text{ in.} \]

   b. Weighted average, Strands \( S_1 \) through \( S_8 \), based on individual \( l_t \), \( l_b \), and \( A_p \) for each strand (see Table A2).
   (2) Flexural bond length
      [Refer to Eq. (5)]:
      \[ l_b^* = \frac{3(f_{ps} - f_{se})l_t}{f_{se}} \]

   Examples:
   Strand \( S_1 \):
   \[ l_t^* = 23 \text{ in.} \]
   \[ l_b^* = \frac{3(251 - 0.85 \times 162.4)}{0.85 \times 162.4} \times 23 \]
   \[ = 56 \text{ in.} \]

   Strand \( S_2 \):
   \[ l_t^* = 59 \text{ in.} \]
   \[ l_b^* = \frac{3(251 - 0.85 \times 188.9)}{0.85 \times 188.9} \times 59 \]
   \[ = 99 \text{ in.} \]

(3) Development length, \( l_d^* = l_t^* + l_b^* \)

Examples:
   Strand \( S_1 \):
   \[ l_d^* = 23 + 56 = 79 \text{ in.} \]
   Strand \( S_2 \):
   \[ l_d^* = 59 + 99 = 158 \text{ in.} \]
5. Fig. A2 shows the developable stresses in the prestressing strand as they are limited by development length. The computation of the values of $f_{dev}$ at the point of application of the test load are as follows:

a. Per ACI formulas:
$$f_{dev} = 156 \times \frac{22}{24} = 143 \text{ ksi}$$

b. Per slip theory:
$$f_{dev} = 156 \times \frac{22}{47} = 73 \text{ ksi}$$

6. The stresses from Fig. A2 are used as follows:

a. Ultimate moment capacity. The developable stresses are utilized to compute ultimate moment capacity at the point of application of load and predicted test loads are computed after deducting the moment due to specimen weight at the same location.

b. Shear capacity. Since shear capacity depends on the level of prestress at any section considered, Fig. A2 is also used to calculate the value of the prestressing force at each section investigated, up to a maximum average stress level in the strands of 156 ksi for the example shown in Fig. A2.

Table A2. Weighted averages of strands.

<table>
<thead>
<tr>
<th>Strand</th>
<th>$A_{ps}$</th>
<th>$l_i$</th>
<th>$l_i^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_1$</td>
<td>0.085 x 22.7 = 1.93</td>
<td>x 55.7 = 4.73</td>
<td></td>
</tr>
<tr>
<td>$S_2$</td>
<td>0.153 x 58.5 = 8.95</td>
<td>x 98.8 = 15.12</td>
<td></td>
</tr>
<tr>
<td>$S_3$</td>
<td>0.153 x 48.8 = 7.47</td>
<td>x 82.3 = 12.59</td>
<td></td>
</tr>
<tr>
<td>$S_4$</td>
<td>0.153 x 58.5 = 8.95</td>
<td>x 98.8 = 15.12</td>
<td></td>
</tr>
<tr>
<td>$S_5$</td>
<td>0.153 x 48.8 = 7.47</td>
<td>x 82.3 = 12.59</td>
<td></td>
</tr>
<tr>
<td>$S_6$</td>
<td>0.085 x 22.7 = 1.93</td>
<td>x 55.7 = 4.73</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.782</td>
<td>36.70</td>
<td>64.88</td>
</tr>
</tbody>
</table>

$l_i^{(avg)} = 36.70/0.782 = 47 \text{ in.}$

$l_i^{(avg)} = 64.88/0.782 = 83 \text{ in.}$

$l_i^{(avg)} = 47 + 83 = 130 \text{ in.}$
APPENDIX B — NOTATION

\( A_{ps} \) = area of prestressed reinforcement in tension zone, sq in.
\( b_w \) = web width, in.
\( d \) = distance from extreme compression fiber to centroid of longitudinal prestressed tension reinforcement, in.
\( d_b \) = nominal diameter of prestressing strand, in.
\( E \) = modulus of elasticity of prestressing strand, psi
\( f'c \) = compressive strength of concrete, psi
\( f_d \) = stress due to unfactored dead load, at extreme fiber of section where tensile stress is caused by externally applied loads, psi
\( f_{de} \) = developable stress in prestressing strand at section, psi
\( f_i \) = jacking stress level in prestressing strand, psi
\( f_{pc} \) = compressive stress in concrete (after allowance for all prestress losses) at centroid of cross section resisting externally applied loads, psi
\( f_{pe} \) = compressive stress in concrete due to effective prestress forces only (after allowance for all prestress losses) at extreme fiber of section where tensile stress is caused by externally applied loads, psi
\( f_{ps} \) = stress in prestressed reinforcement at nominal strength of test specimen, psi
\( f_{pu} \) = specified tensile strength of prestressing strand, psi
\( f_{se} \) = effective stress in prestressed reinforcement (after allowance for all prestress losses), psi
\( f_{si} \) = stress in prestressed reinforcement following transfer, psi
\( F_{i\text{ave}} \) = average initial prestressing force in the transfer zone following transfer, lbs
\( F_j \) = jacking force in prestressed reinforcement, lbs
\( F_t \) = maximum force in prestressed reinforcement following transfer, psi
\( I \) = moment of inertia of section resisting externally applied loads, in.\(^4\)
\( l_b \) = flexural bond length according to the ACI Code, in.
\( l_{bd} \) = flexural bond length according to the slip theory, in.
\( l_d \) = development length according to the ACI Code, in.
\( l_{d'} \) = development length according to the slip theory, in.
\( l_t \) = transfer length according to the ACI Code, in.
\( l_{t'} \) = transfer length according to the slip theory, in.
\( L \) = design span, in.
\( M_{cr} \) = moment causing flexural cracking at section due to externally applied loads = \( S_b(6 \sqrt{f'c} + f_{pe}) \), lb-in.
\( M_{faa} \) = observed ultimate moment, at failure, lb-in.
\( M_{max} \) = maximum factored moment at section due to externally applied loads, lb-in.
\( M_n \) = nominal moment strength at section, lb-in.
\( M_u \) = predicted ultimate moment, lb-in.
\( P \) = predicted failure load (applied), kips
\( P_u \) = applied failure load, kips
\( V_{ei} \) = nominal shear strength provided by concrete when diagonal cracking results from combined shear and moment, lbs
\( V_{ce} \) = nominal shear strength provided by concrete when diagonal cracking results from excessive principal tensile stress in web, lbs
\( V_d \) = shear force at section due to unfactored dead load, lbs
\( V_t \) = factored shear force at section due to externally applied loads

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occurring simultaneously with $M_{\text{max}}$, lbs

$V_p = \text{vertical component of effective prestress force at section, lbs}$

$\delta = \text{measured initial strand slip at end of specimen, in.}$

$\rho = \text{ratio of prestressed tension reinforcement, } A_{ps}/bd$

$\phi = \text{strength reduction factor}$

---

**METRIC (SI) EQUIVALENTS**

<table>
<thead>
<tr>
<th>Metric (SI)</th>
<th>Equivalent (English)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 ft = 0.3048 m</td>
<td>1 psi = 0.006895 MPa</td>
</tr>
<tr>
<td>1 in. = 25.4 mm</td>
<td>1 ksi = 6.895 MPa</td>
</tr>
<tr>
<td>1 in.$^2$ = 0.000654 m$^2$</td>
<td>1 kip = 4.448 kN</td>
</tr>
</tbody>
</table>

**NOTE:** Discussion of this article is invited. Please submit your comments to PCI Headquarters by October 1, 1988.