Some partially prestressed concrete members may be designed without following the zero tension requirement under dead load. In these members, cracks caused by dead load or service loads most likely applied at an early stage will remain open permanently under dead load. Since the cracked part of the section cannot resist any tensile stresses, including the tensile stresses caused by time-dependent internal stress redistribution, creep analysis of the section should be performed by taking into account the presence of cracks.

There are many well-known methods for computing time-dependent effects of uncracked sections. However, only a few authors discuss these effects in cracked sections.

This paper deals with a creep analysis for cracked prestressed concrete sections under dead load, based on the following assumptions:

1. The section is designed without consideration of a zero tension requirement under dead load; cracking could be caused by dead load or service loads which are most likely applied before time-dependent effects occur.

2. Plane sections remain plane after bending.

3. The common practice of ignoring tensile stresses in concrete is assumed.

4. The stress-strain relations for bonded prestressed and nonprestressed steel are linear.

5. The concrete strain response to gradually varying stress is given by Bazant's age-adjusted effective modulus method.
A practical method for directly computing the concrete strain under a varying stress was developed by Trost in 1967 and later improved by Bazant who called his method the age-adjusted effective modulus method. He introduced the concept of an aging coefficient which expresses the aging effect on creep of concrete loaded gradually. Neville et al. showed the aging coefficient in the form of graphs, which were derived according to the procedure reported by Bazant by using the 1978 CEB-FIP creep function instead of the ACI creep function.

The response of the concrete strain at time \( t \) is given by the age-adjusted effective modulus formula as follows:

\[
\varepsilon_c(t) = \frac{f_c(t_o)}{E_c} \left[ 1 + \varphi(t, t_o) \right] + \frac{f_c(t) - f_c(t_o)}{E_c} \times \left[ 1 + \chi(t, t_o) \varphi(t, t_o) \right] + \varepsilon_{cs}(t, t_o) \tag{1}
\]

where \( \varepsilon_c(t) \) is total concrete strain at time \( t \), and \( \varepsilon_{cs}(t, t_o) \) is free shrinkage since the time of prestressing \( t_o \), \( \varphi(t, t_o) \) and \( \chi(t, t_o) \) are creep coefficient and aging coefficient at time \( t \) for concrete loaded at age \( t_o \), respectively, and \( f_c(t) \) and \( f_c(t_o) \) are concrete stresses at time \( t \) and \( t_o \), respectively.

From Eq. (1), the concrete stress at time \( t \) is written as:

\[
f_c(t) = \bar{E}_c \left[ \varepsilon_c(t) - \varepsilon_{cs} \right] - \frac{(1 - \chi)}{1 + \chi} \varphi f_c(t_o)
\tag{2}
\]

where \( \bar{E}_c = E_c/(1 + \chi \varphi) \). The concrete stress \( f_c(t) \) is given by subtracting the concrete stress \( f_c(t_o) \) multiplied by a factor \( (1 - \chi)/(1 + \chi \varphi) \) from the concrete stress \( \bar{E}_c \left[ \varepsilon_c(t) - \varepsilon_{cs} \right] \).

Assuming a linear distribution of the total strain \( \varepsilon_c(t) \) in the cracked section, it is necessary to differentiate two cases where the distribution of the concrete stress at time \( t, f_c(t) \), becomes linear and bilinear in the compressed area of the cracked section.

(a) When the value of the second term of \( f_c(t) \) exceeds that of the first term, the concrete stress \( f_c(t) \) becomes negative, which means a tensile stress. Ignoring the tensile stress in the concrete, a linear distribution of \( f_c(t) \) is obtained as shown in Fig. 1 (a).

(b) When the value of the first term of \( f_c(t) \) exceeds that of the second term everywhere in the compressed part of the section, a bilinear distribution of \( f_c(t) \) is obtained as shown in Fig. 1 (b). This occurs because of the nonexistence of \( f_c(t_o) \) below the tip of the crack caused at time \( t_o \).
PROPOSED ANALYTICAL METHOD

The proposed method is based on satisfying equilibrium of forces and moments and compatibility of strains. Typical equilibrium and strain compatibility equations for a general cross section with several layers of prestressed and nonprestressed steel and for the strain diagrams shown in Fig. 2, are summarized below. The symbols are explained in the Appendix. All mathematical derivations and other
Supporting material are not published in the PCI JOURNAL due to space limitations. However, they are kept on file at PCI Headquarters and are available on request.

1. At time $t_o$

**Equilibrium equations:**

\[
\begin{bmatrix} D_1 & D_2 \\ D_2 & D_3 \end{bmatrix} \begin{bmatrix} \epsilon_{ca} (t_o) \\ \Phi (t_o) \end{bmatrix} + \begin{bmatrix} FF1 \\ FF2 \end{bmatrix} = \begin{bmatrix} 0 \\ M_d \end{bmatrix}
\]

where the following notations are introduced for the cracked section:

<table>
<thead>
<tr>
<th>Pretensioning</th>
<th>Post-tensioning*</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D_1$</td>
<td>$D_{1t}(x_o) = E_c A_{e1}(x_o) + E_s A_s + E_p A_p$</td>
</tr>
<tr>
<td>$D_2$</td>
<td>$D_{2t}(x_o) = E_c Q_{e1}(x_o) + E_s Q_s + E_p Q_p$</td>
</tr>
<tr>
<td>$D_3$</td>
<td>$D_{3t}(x_o) = E_c I_{ei}(x_o) + E_s I_s + E_p I_p$</td>
</tr>
<tr>
<td>$FP1$</td>
<td>$\Sigma A_{pm} f_{pm}(t_o)$</td>
</tr>
<tr>
<td>$FP2$</td>
<td>$\Sigma A_{pn} f_{pm}(t_o) y_n$</td>
</tr>
</tbody>
</table>

*Since there is no bond between tendons and concrete at transfer, the terms $A_{pm}$ and $I_n$ should be omitted and $f_{pm}$ can be replaced by $f_{pm}(t_o)$.

Equation for determining neutral axis depth $x_o$:

\[
\epsilon_{ca}(t_o) + \Phi(t_o) (y_1 - x_o) = 0
\]  (7)

Concrete stress at level $k$:

\[
f_{ck}(t_o) = E_c [\epsilon_{ca}(t_o) + \Phi(t_o) y_k]
\]  (8)

Stresses in nonprestressed reinforcement at level $m$ and bonded prestressed steel (pretensioned member only) at level $n$:

\[
\begin{align*}
f_{sm}(t_o) &= E_s [\epsilon_{ca}(t_o) + \Phi(t_o) y_m] \\
f_{pm}(t_o) &= f_{pm} + E_p [\epsilon_{ca}(t_o) + \Phi(t_o) y_n]
\end{align*}
\]  (9)  (10)

Stress in tendon at level $n$ for post-tensioned member:

\[
f_{pn}(t_o)
\]  (11)

Stress in tendon at level $n$ for nonprestressed member:

\[
\begin{align*}
\tilde{D}_{1t}(x_t) &\tilde{D}_{2t}(x_t) \\
\tilde{D}_{3t}(x_t)
\end{align*}
\]

\[
\begin{align*}
\begin{bmatrix} \epsilon_{ca}(t) \\ \Phi(t) \end{bmatrix} + \begin{bmatrix} FF1 \\ FF2 \end{bmatrix} = \begin{bmatrix} 0 \\ M_d \end{bmatrix}
\end{align*}
\]

For post-tensioned members, since the bond between the tendons and ducts is generally established just after transfer, at this state the constants $D_{1t}$s are the same as those for pretensioned members. The steel stresses $f_{pm}$ in tendons are given as:

\[
f_{pm} = f_{pm}(t_o) - E_p [\epsilon_{ca}(t_o) + \Phi(t_o) y_n]
\]  (12)

and $FP1$ and $FP2$ in Eq. (6) are also the same as those for pretensioned members.

By solving Eqs. (3) and (4), $\epsilon_{ca}(t_o)$ and $\Phi(t_o)$ are obtained in terms of $x_o$. Substituting $\epsilon_{ca}(t_o)$ and $\Phi(t_o)$ into Eq. (7), a cubic equation in $x_o$ is found. Thus, $x_o$, $\epsilon_{ca}(t_o)$ and $\Phi(t_o)$ at time $t_o$ are obtained.

2. At time $t$

Equilibrium equations:

\[
\begin{align*}
\begin{bmatrix} FR1 \\ FR2 \end{bmatrix} &= \begin{bmatrix} F1 \\ F2 \end{bmatrix} = \begin{bmatrix} 0 \\ M_d \end{bmatrix}
\end{align*}
\]

where

\[
\begin{align*}
\tilde{D}_{1t}(x_t) &= E_c A_{e1}(x_t) + E_s A_s + E_p A_p \\
\tilde{D}_{2t}(x_t) &= E_c Q_{e1}(x_t) + E_s Q_s + E_p Q_p \\
\tilde{D}_{3t}(x_t) &= E_c I_{ei}(x_t) + E_s I_s + E_p I_p
\end{align*}
\]

\[
\begin{align*}
FR1 &= \Sigma A_{pn} \delta f_{pn}(t) \\
FR2 &= \Sigma A_{pn} \delta f_{pn}(t) y_n
\end{align*}
\]
\[ F_1 = \begin{cases} \bar{E}_c (1 - \chi) \varphi \{ \epsilon_{cg}(t_0) A_{cn}(x_0) + \Phi(t_0) Q_{cn}(x_0) \} + \epsilon_{cs} A_{cn}(x_0) & \text{for } x_t > x_o \\ \bar{E}_c (1 - \chi) \varphi \{ \epsilon_{cg}(t_0) A_{cn}(x_t) + \Phi(t_0) Q_{cn}(x_t) \} + \epsilon_{cs} A_{cn}(x_t) & \text{for } x_t < x_o \end{cases} \]

\[ F_2 = \begin{cases} \bar{E}_c (1 - \chi) \varphi \{ \epsilon_{cg}(t_0) Q_{cn}(x_0) + \Phi(t_0) I_{cn}(x_0) \} + \epsilon_{cs} Q_{cn}(x_0) & \text{for } x_t > x_o \\ \bar{E}_c (1 - \chi) \varphi \{ \epsilon_{cg}(t_0) Q_{cn}(x_t) + \Phi(t_0) I_{cn}(x_t) \} + \epsilon_{cs} Q_{cn}(x_t) & \text{for } x_t < x_o \end{cases} \]

Equation for determining neutral axis depth \( x_t \):

(a) When \( x_t < x_o \):

\[ \epsilon_{cg}(t) + \Phi(t) (y_t - x_t) - \epsilon_{cs} - (1 - \chi) \varphi \{ \epsilon_{cg}(t_0) + \Phi(t_0) (y_1 - x_1) \} = 0 \]  

(b) When \( x_t > x_o \):

\[ \epsilon_{cg}(t) + \Phi(t) (y_t - x_t) - \epsilon_{cs} = 0 \]  

Concrete stress at level \( k \):

(a) Decreasing neutral axis depth as shown in Fig. 1(a):

\[ f_{ck}(t) = \bar{E}_c \{ \epsilon_{cg}(t) + \Phi(t) y_k - \epsilon_{cs} \} - (1 - \chi) \varphi \epsilon_{cg}(t_0) \]  

(b) Increasing neutral axis depth as shown in Fig. 1(b):

\[ f_{ck}(t) = \bar{E}_c \{ \epsilon_{cg}(t) + \Phi(t) y_k - \epsilon_{cs} \} - (1 - \chi) \varphi \epsilon_{cg}(t_0) \]  

Steel stresses in nonprestressed reinforcement at level \( m \) and in bonded tendon at level \( n \):

\[ f_{sm}(t) = E_s \{ \epsilon_{cg}(t) + \Phi(t) y_m \} \]  

\[ f_{pn}(t) = f_{pno} + E_p \{ \epsilon_{cg}(t) + \Phi(t) y_n \} + \delta f_{pn}(t) \]

Solving Eqs. (13) and (14), \( \epsilon_{cg}(t) \) and \( \Phi(t) \) can be obtained in terms of \( x_t \) and substituting them into Eq. (18), a fifth degree equation in \( x_t \) is found. Thus, \( x_t \), \( \epsilon_{cg}(t) \) and \( \Phi(t) \) at time \( t \) can be computed.

For a cracked cross section shown in Fig. 2, the constants in Eqs. (5) and (15) are given as:

\[ A_s = A_{si} + A_{a} \]  

\[ Q_s = A_{si} y_{si} - A_{a} y_{a} \]  

\[ I_s = A_{si} y_{si}^2 + A_{a} y_{a}^2 \]  

\[ A_p = A_p \]  

\[ Q_p = -A_p y_p \]  

\[ I_p = A_p y_p^2 \]  

(a) \( x > h_i \):

\[ A_{cn}(x) = (b - b_{wi}) h_i + b_{wi} x - A_{si} \]  

\[ Q_{cn}(x) = (b - b_{wi}) \frac{h_i}{2} (y_1 - h_i/2) + b_{wi} (y_1 - x/2) - A_{si} y_{si} \]  

\[ I_{cn}(x) = b x - A_{si} \]  

(b) \( y_1 - y_{si} < x < h_i \):

\[ A_{cn}(x) = b x - A_{si} \]  

\[ Q_{cn}(x) = b x (y_1 - x/2) - A_{si} y_{si} \]  

\[ I_{cn}(x) = b x (y_1 - x/2) - A_{si} y_{si}^2 \]

**STRESS ANALYSIS UNDER SERVICE LOADS**

In the analysis of cracked sections, it is advantageous to bring the prestressed section to a condition identical to that of a conventionally reinforced section subject to combined axial force and bending moment. The force required to achieve this state is called the fictitious decompression force by which the reinforcement, prestressed and nonprestressed, must be held externally in order to eliminate stresses in the concrete while no external loads are present. Tadros\(^{10}\) presented a theoretical procedure for determining the fictitious decompression force.

The fictitious stresses in steel reinforcement at the decompression state are given as follows:

\[ f_{sm}^* = E_s (\epsilon_{cg}^* + \Phi^* y_m) \]  

\[ f_{pn}^* = f_{pno} + \delta f_{pn}(t) + E_p (\epsilon_{cg}^* + \Phi^* y_n) \]
\[
\begin{align*}
\epsilon_{cg}^* &= \frac{1}{1 + \chi \varphi} \{ \chi \varphi \epsilon_{cg}(t) + (1 - \chi) \varphi \epsilon_{cg}(t_o) + \epsilon_{cg} \} \\
\Phi^* &= \frac{1}{1 + \chi \varphi} \{ \chi \varphi \Phi(t) + (1 - \chi) \varphi \Phi(t_o) \}
\end{align*}
\]  
(24)

(a) When the neutral axis depth \( x \) under service loads is larger than \( x_o \), but less than \( x_i \), \( x \) can be obtained by solving the following equations:

\[
\begin{bmatrix}
D_{11}(x) & D_{21}(x) \\
D_{21}(x) & D_{31}(x)
\end{bmatrix}
\begin{bmatrix}
\Delta \epsilon_{cg} + (1 - \chi) \varphi \epsilon_{cg}(t_o)/(1 + \chi \varphi) \\
\Delta \Phi + (1 - \chi) \varphi \Phi(t_o)/(1 + \chi \varphi)
\end{bmatrix}
+ \begin{bmatrix}
FP1^* \\
FP2^*
\end{bmatrix}
= \begin{bmatrix}
0 \\
M_d + M_1
\end{bmatrix}
+ \frac{(1 - \chi) \varphi}{1 + \chi \varphi} \begin{bmatrix}
-FP1 \\
M_d - FP2
\end{bmatrix}

\begin{align*}
\Delta \epsilon_{cg} + \frac{(1 - \chi) \varphi}{1 + \chi \varphi} \epsilon_{cg}(t_o) + \left\{ \Delta \Phi + \frac{(1 - \chi) \varphi}{1 + \chi \varphi} \Phi(t_o) \right\}(y_1 - x) &= 0
\end{align*}
\]  
(25)

where \( \Delta \epsilon_{cg} \) and \( \Delta \Phi \) are incremental axial strain and section curvature due to service loads. The following equations may be written:

\[
\begin{align*}
FP1^* &= \Sigma A_{cm} f_{sm}^* + \Sigma A_{pn} f_{pn}^* \\
FP2^* &= \Sigma A_{cm} f_{sm}^* y_m + \Sigma A_{pn} f_{pn}^* y_n \\
D_{11}(x) &= E_c A_{c11}(x) + E_s A_s + E_p A_p \\
D_{21}(x) &= E_c Q_{c11}(x) + E_s Q_s + E_p Q_p \\
D_{31}(x) &= E_c I_{c11}(x) + E_s I_s + E_p I_p
\end{align*}
\]  
(28)

By solving Eqs. (25) and (26), \( \Delta \epsilon_{cg} \) and \( \Delta \Phi \) can be expressed in terms of \( x \) and substituting \( \Delta \epsilon_{cg} \) and \( \Delta \Phi \) into Eq. (27), a cubic equation in \( x \) is obtained.

The stresses in the section can be written as follows:

\[
\begin{align*}
f_{ck} &= E_c \{ \Delta \epsilon_{cg} + \Delta \Phi y_k \}
\end{align*}
\]  
(29)

and the axial strain and section curvature are:

\[
\begin{align*}
\epsilon_{cg} &= \epsilon_{cg}^* + \Delta \epsilon_{cg} \\
\Phi &= \Phi^* + \Delta \Phi
\end{align*}
\]  
(30)

(b) When the neutral axis depth \( x \) is smaller than \( x_o \) and \( x_i \), then by solving the following equations, \( \Delta \epsilon_{cg} \), \( \Delta \Phi \) and \( x \) can be obtained:

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Finally, the stresses in the concrete and steel, and the deformations in the section of the member are given as follows:

\[
\begin{align*}
\Delta \epsilon_{cg} + \Delta \Phi (y_1 - x) &= 0 \\
\epsilon_{cg} = \epsilon_{cg}^* + \Delta \epsilon_{cg}
\end{align*}
\]

\[
\begin{align*}
\phi = \phi^* + \Delta \phi
\end{align*}
\]

**DESIGN EXAMPLE**

The following example illustrates the proposed calculation method and will show readers how to quickly apply the basic computation steps. A prestressed concrete single T section is selected. The dimensions, material properties and moments due to loading are shown in Fig. 3.

The section is analyzed manually under the following conditions:
- **Loading Case 1**: Dead load plus prestressing force at transfer.
- **Loading Case 2**: Application of live load after establishing bond between tendon and concrete.
- **Loading Case 3**: Removal of live load.
- **Loading Case 4**: Dead load plus combined effects of creep and shrinkage of concrete and relaxation of prestressing steel.
- **Loading Case 5**: Application of live load after all losses have occurred.

**Loading Case 1**

First, the section is assumed to be uncracked just after transfer. From the well-known analysis of uncracked prestressed sections, the following stresses in the concrete and steel, and deformation in the section are obtained:

\[
\begin{align*}
f_{c1} &= 35 \times 10^3 (0.06700 + 0.1701 \times 0.326) \times 10^{-3} = 4.29 \text{ N/mm}^2 (622 \text{ psi}) \\
f_{c2} &= 35 \times 10^3 (0.06700 - 0.1701 \times 0.674) \times 10^{-3} = -1.67 \text{ N/mm}^2 (-242 \text{ psi}) \\
f_{s1} &= 200 \times 10^3 (0.06700 + 0.1701 \times 0.276) \times 10^{-3} = 22.79 \text{ N/mm}^2 (3.30 \text{ ksi}) \\
f_{s2} &= 200 \times 10^3 (0.06700 - 0.1701 \times 0.624) \times 10^{-3} = -7.82 \text{ N/mm}^2 (-1.13 \text{ ksi}) \\
f_p &= f_p (t_o) = -1140 \text{ N/mm}^2 (-165.3 \text{ ksi})
\end{align*}
\]

Since the concrete stress \( f_{c2} \) at the extreme tension fiber is in tension, and less than the tensile strength of concrete, the assumption of an uncracked section used here is correct.

**Loading Case 2**

Since full service loads are most likely applied after establishing the bond between the tendon and concrete, it is assumed that any time-dependent deformations have not occurred before this loading. The section can be assumed to be cracked under the service loads.

Assuming the neutral axis of the member is in the web, the section properties of the effective concrete zone are written as:
SECTION PROPERTIES

NET CONCRETE SECTION: $A_c = 0.52947 \text{ m}^2$, $I_c = 0.049158 \text{ m}^4$

NON PRESTRESSED REINFORCEMENT: $A_s = 0.004013 \text{ m}^2$

$Q_s = 0.001986 \times 0.276 - 0.002027 \times 0.624 = -0.00071671 \text{ m}^3$

$I_s = 0.001986 \times 0.276^2 + 0.002027 \times 0.624^2 = 0.00094055 \text{ m}^4$

PRESTRESSED STEEL: $A_p = 0.001115 \text{ m}^2$

$Q_p = 0.001115 \times (-0.574) = -0.00064001 \text{ m}^3$

$I_p = 0.001115 \times 0.574^2 = 0.00036736 \text{ m}^4$

DIMENSIONS cm

BENDING MOMENTS:

DEAD LOAD MOMENT: 1044.5 kN·m

LIVE LOAD MOMENT: 337.5 kN·m

MATERIAL PROPERTIES:

CONCRETE: $E_c = 35 \text{ kN/mm}^2$, TENSILE STRENGTH = 3.3 N/mm$^2$, $\sigma = 2.6$, $\varepsilon_c = 25 \times 10^{-5}$

NON PRESTRESSED STEEL: $E_s = 200 \text{ kN/mm}^2$

PRESTRESSED STEEL: $E_p = 200 \text{ kN/mm}^2$, TENSILE STRESS AT TRANSFER = 1140 N/mm$^2$

LOSS DUE TO STEEL RELAXATION = 5 PERCENT OF INITIAL STRESS

Fig. 3. Example: Dimensions, moments, section and material properties of prestressed concrete T beam.

(Conversion factors: 1 cm = 0.3937 in., 1 N/mm$^2$ = 145 psi, 1 kN/mm$^2$ = 145 ksi.)
\[ A_{ei}(x) = 1.48x0.12 + 0.74x0.05 + 0.32x - 0.001986 \]
\[ + 0.126 + 0.32x \text{ m}^2 \]
\[ Q_{ei}(x) = 0.1776x(0.326 - 0.006) + 0.037x(0.206 - 0.06) + 0.32x(0.326 - x/2) - 0.001986x0.276 \]
\[ = 0.053699 + 0.10432x - 0.16x^2 \text{ m}^2 \]
\[ I_{ei}(x) = 0.1776x(0.12x^2/12 + (0.326 - 0.06)^2) + 0.037x(0.05^2/18 + (0.206 - 0.06/3)^2) + 0.32x(x^2/12 + (0.326 - x/2) - 0.001986x0.276 \]
\[ = 0.013959 + 0.034008x - 0.10432x^2 + 0.10667x^3 \text{ m}^4 \]

From Eq. (12), the initial steel stress \( f_{ps} \) in the bonded tendon can be calculated by referring to the section deformations of Loading Case 1:

\[
\begin{bmatrix} D_{1II} & D_{2II} \\ D_{2II} & D_{3II} \end{bmatrix} \begin{bmatrix} \epsilon_{cs} \\ \Phi \end{bmatrix} + \begin{bmatrix} -1264.3 \\ 725.7 \end{bmatrix} = \begin{bmatrix} 0 \\ 1044.5 + 337.5 \end{bmatrix}
\]

Solving Eq. (38), \( \epsilon_{cs} \) and \( \Phi \) are obtained as follows:

\[
\begin{align*}
\epsilon_{cs} &= (1.2643D_{3II} - 0.65629D_{2II})x10^3/(D_{1II}D_{3II} - D_{2II}^2) \\
\Phi &= (0.65629D_{1II} - 1.2643D_{2II})x10^3/(D_{1II}D_{3II} - D_{2II}^2)
\end{align*}
\]

Substituting Eq. (39) into Eq. (7), an equation for determining the neutral axis depth \( x \) can be written as:

\[
\frac{1.2643D_{3II} - 0.6563D_{2II}}{0.6563D_{1II} - 1.2643D_{2II}} + 0.326 - x = 0
\]

Substituting Eq. (37) into Eq. (40), a cubic equation in \( x \) can be obtained as:

\[
-107.0 - 891.6x - 940.7x^2 + 4720x^3 \\
3524 + 2734x + 7080x^2
\]

and the neutral axis depth \( x \) is calculated to be 0.258 m.

Substituting \( x = 0.258 \) m into Eqs. (37) and (39), \( \epsilon_{cs} \) and \( \Phi \) are obtained as follows:

\[
\epsilon_{cs} = -0.06100x10^3, \Phi = 0.8981x10^4/m
\]

Therefore, the stresses in the section in Loading Case 2 are:

\[
\begin{align*}
f_{c1} &= 8.12 \text{ N/mm}^2 (1177 \text{ psi}) \\
f_{s1} &= 37.40 \text{ N/mm}^2 (5.42 \text{ ksi}) \\
f_{c2} &= -124.26 \text{ N/mm}^2 (-18.02 \text{ ksi}) \\
f_{p} &= -1249.2 \text{ N/mm}^2 (-181.1 \text{ ksi})
\end{align*}
\]

Substituting Eq. (35) into Eq. (5), the constants \( D_{1II}, D_{2II} \) and \( D_{3II} \) for the cracked section with bonded tendon are:

\[
\begin{align*}
D_{1II}(x) &= E_c A_{ei}(x) + E_s A_s + E_p A_p \\
D_{2II}(x) &= E_c Q_{ei}(x) + E_s Q_s + E_p Q_p \\
D_{3II}(x) &= E_c I_{ei}(x) + E_s I_s + E_p I_p
\end{align*}
\]

By replacing \( M_d \) in Eq. (4) with \( (M_d + M_i) \), the equilibrium equations for service loads are written as follows:
Loading Case 3

Since cracking has been caused by the application of live load as shown in the previous calculation (Loading Case 2), and the zero tension requirement is not satisfied under the dead load, the cracks remain open under the dead load after removal of live load. Hence, a cracked section should be assumed in the calculation of the stresses under the dead load. The calculation procedure is similar to that for Loading Case 2, but the right hand term of Eq. (38) should be written in terms of the dead load moment as follows:

\[
0 \quad 1044.5
\]

Solving a cubic equation, \(x_a\) is obtained as 0.572 m and \(\varepsilon_{ca}(t_o)\) and \(\Phi(t_o)\) are:

\[
\varepsilon_{ca}(t_o) = 0.05631 \times 10^{-3} \\
\Phi(t_o) = 0.2286 \times 10^{-3}/m
\]  

(41)

Therefore, the stresses in the cracked section under the dead load before the time-dependent effects occur are found:

\[
f_{c1} = 4.58 \text{ N/mm}^2 (664 \text{ psi}) \\
f_{c2} = 23.88 \text{ N/mm}^2 (3.46 \text{ psi}) \\
f_{c2} = 17.26 \text{ N/mm}^2 (-2.50 \text{ psi}) \\
f_p = -1148.8 \text{ N/mm}^2 (-166.6 \text{ ksi})
\]

Loading Case 4

For this loading case, the creep analysis of cracked sections is performed. By assuming the aging coefficient \(\chi\) is equal to 0.80, the age-adjusted effective modulus \(E_c\) can be obtained from Eq. (2) as follows:

\[
E_c = 351(1 + 0.80x2.6) \\
= 11.36 \text{ kN/mm}^2 (1647 \text{ ksi})
\]

The steel stress loss \(\delta f_p(t)\) due to the relaxation of prestressing steel is:

\[
\delta f_p(t) = 0.05 \times 1140 = 57 \text{ N/mm}^2
\]

Substituting Eq. (35) into Eq. (15), the constants for the cracked section \(D_{11\text{II}}\), \(D_{2\text{II}}\) and \(D_{3\text{III}}\) are obtained as follows:

\[
\begin{align*}
D_{11\text{II}} &= E_c A_{e II} (x_i) + E_s A_s + E_p A_p = (3441 + 3636x_i) \times 10^3 \\
D_{11\text{III}} &= E_c A_{e III} (x_i) + E_s A_s + E_p A_p = (338.9 + 1185x_i - 1818x_i^2) \times 10^9 \\
D_{2\text{III}} &= E_c I_{e e} (x_i) + E_s I_s + E_p I_p = (420.2 + 386.4x_i - 1158x_i^2 + 1212x_i^3) \times 10^9
\end{align*}
\]  

(42)

From Eq. (16), \(FR1\) and \(FR2\) are:

\[
FR1 = A_p \delta f_p(t) = 1115 \times 57 = 63.56 \text{ kN} \\
FR2 = A_p \delta f_p(t) u_p = 63.56x(-0.574) \\
\quad = -36.48 \text{ kN \cdot m}
\]

By assuming a decreasing neutral axis depth with time \((x_i < x_o)\), \(F1\) and \(F2\) are obtained from Eq. (17). By substituting \(\varepsilon_{ca}(t_o) = 0.05631 \times 10^{-3}\) and \(\Phi(t_o) = 0.2286 \times 10^{-3}/m\) given by Eq. (41) into Eq. (17), the following expressions are obtained:

\[
F1 = 11.36 \times 10^3 \times [(1 - 0.80)x_2.6x(0.05631 \times 10^{-3}x(0.2126 + 0.32x_i) \\
+ 0.2286 \times 10^{-3}x(0.053699 + 0.10432x_i - 0.16x_i^2)] \\
+ 0.25 \times 10^{-3}(0.2126 + 0.32x_i)] \\
= (0.7470 + 1.1548x_i - 0.2144x_i^2) \times 10^3
\]

\[
F2 = 11.36 \times 10^3 \times [(1 - 0.80)x_2.6x(0.05631 \times 10^{-3}x(0.053699 + 0.10432x_i - 0.16x_i^2) \\
+ 0.2286 \times 10^{-3}x(0.013959 + 0.02008x_i - 0.10432x_i^2 + 0.10667x_i^3)] \\
+ 0.25 \times 10^{-3}(0.053699 + 0.10432x_i - 0.16x_i^2)] \\
= (0.1892 + 0.3764x_i - 0.6473x_i^2 + 0.1429x_i^3) \times 10^3
\]

From Eqs. (13) and (14), the equilibrium equations are rewritten as:
\[
\begin{bmatrix}
D_{1n} & D_{2n} \\
D_{2n} & D_{3n}
\end{bmatrix}
\begin{bmatrix}
\varepsilon_{ef}(t) \\
\Phi(t)
\end{bmatrix}
= \begin{bmatrix}
-1264.3 & 725.7 \\
-36.48 & 63.56
\end{bmatrix}
+ \begin{bmatrix}
F1 \\
F2
\end{bmatrix}
+ \begin{bmatrix}
0 \\
M_d
\end{bmatrix}
\]

\[
\begin{bmatrix}
1.948 + 1.1548x_r - 0.21616x_r^2 \\
0.5445 + 0.3770x_r - 0.6487x_r^2 + 0.1441x_r^3
\end{bmatrix}
\]

(43)

Solving Eq. (43), \( \varepsilon_{ef}(t) \) and \( \Phi(t) \) are:

\[
\begin{bmatrix}
\varepsilon_{ef}(t) \\
\Phi(t)
\end{bmatrix}
= \begin{bmatrix}
[(1.948 + 1.1548x_r - 0.21616x_r^2)D_{3n} - (0.5445 + 0.3770x_r - 0.6487x_r^2 + 0.1441x_r^3)D_{2n}] / \Delta \\
[(0.5445 + 0.3770x_r - 0.6487x_r^2 + 0.1441x_r^3)D_{1n} - (1.948 + 1.1548x_r - 0.21616x_r^2)D_{2n}] / \Delta
\end{bmatrix}
\]

(44)

where \( \Delta = D_{1n}D_{3n} - D_{2n}^2 \).

Substituting Eqs. (42) and (44) into Eq. (18a), an equation for determining the neutral axis depth can be written as:

\[
\frac{\varepsilon_{ef}(t) - (1 - \chi) \varepsilon_{ef}(t_0) - \varepsilon_{es}}{\Phi(t) - (1 - \chi) \varepsilon_{ef}(t_0) + (y_1 - x_r)}
= \frac{262.2 - 109.9x_r - 3950x_r^2 + 1147x_r^3 - 1007x_r^4}{1055 + 338.5x_r + 1721x_r^2 - 0.02470x_r^3 + 0.03801x_r^4} + 0.326 - x_r = 0
\]

(45)

The above fifth degree equation in \( x_r \) leads to:

\[ x_r = 0.478 \text{ m} \] for \( < x_o = 0.572 \text{ m} \) (ok)

Substituting \( x_r = 0.478 \text{ m} \) into Eqs. (42) and (44), \( \varepsilon_{ef}(t) \) and \( \Phi(t) \) are:

\[
\varepsilon_{ef}(t) = 0.3921 \times 10^{-3},
\Phi(t) = 0.8573 \times 10^{-3}/\text{m}
\]

(46)

and the stresses in the section are obtained by Eqs. (19a), (20) and (21) as follows:

\[
\begin{align*}
f_{c1} &= 11.36 \times 10^5 \times (0.3921 + 0.8573 \times 0.326 - 0.25) \times 10^{-3} \\
&- (1 - 0.80) \times 2.6 \times (0.05631 + 0.2286 \times 0.326) \times 10^{-3} \\
&= 4.02 \text{ N/mm}^2 (583 \text{ psi})
\end{align*}
\]

\[
\begin{align*}
f_{c2} &= 125.77 \text{ N/mm}^2 (18.24 \text{ ksi}) \\
f_{c2} &= -28.55 \text{ N/mm}^2 (-4.14 \text{ ksi}) \\
f_p &= 1097.2 \text{ N/mm}^2 (-159.1 \text{ ksi})
\end{align*}
\]

\textbf{Loading Case 5}

From Eq. (24), the axial strain \( \varepsilon_{ef}(t) \) and the fictitious stresses in the reinforcing steel are obtained by Eqs. (22) and (23) as follows:

\[
\begin{align*}
f_{A} &= 200 \times 10^5 \times (0.3555 + 0.6176 \times 0.276) \times 10^{-3} \\
&= 105.2 \text{ N/mm}^2 (15.25 \text{ ksi})
\end{align*}
\]

\[
\begin{align*}
f_{A} &= 200 \times 10^5 \times (0.3555 - 0.6176 \times 0.624) \times 10^{-3} \\
&= -5.97 \text{ N/mm}^2 (-0.866 \text{ ksi})
\end{align*}
\]

\[
\begin{align*}
f_p &= -1133.9 + 57 + 200 \times 10^5 \times (0.3555 - 0.6176 \times 0.574) \times 10^{-3} \\
&= -1076.7 \text{ N/mm}^2 (-156.12 \text{ ksi})
\end{align*}
\]
Fig. 4. Stresses in concrete and non-prestressed steel. (Creep analysis for cracked section.)
(Conversion factor: $1 \text{ N/mm}^2 = 145 \text{ psi}$.)

Concrete Stresses

- $23.88 \text{ N/mm}^2$
- $8.12 \text{ N/mm}^2$
- $4.02 \text{ N/mm}^2$
- $7.52 \text{ N/mm}^2$

Stresses in non-prestressed steel & strain distributions in section

- $22.79 \text{ N/mm}^2$
- $37.40 \text{ N/mm}^2$
- $23.88 \text{ N/mm}^2$
- $125.77 \text{ N/mm}^2$
- $138.72 \text{ N/mm}^2$

(1) $-1.67 \text{ N/mm}^2$
(2) $22.0 \text{ cm}$
(3) $57.2 \text{ cm}$
(4) $47.8 \text{ cm}$
(5) $22.7 \text{ cm}$

Stresses in non prestressed steel & strain distributions in section

- $4.29 \text{ N/mm}^2$
- $4.58 \text{ N/mm}^2$
- $4.02 \text{ N/mm}^2$
- $7.52 \text{ N/mm}^2$

Loading Case

- $125.77 \text{ N/mm}^2$
- $138.72 \text{ N/mm}^2$

Centroid

$G_c$

$1.5628 \times 10^{-3} \text{/m}$
From Eqs. (31) and (32), the equilibrium equations are given below, provided that $x$ is less than $x_0$.

$$0 = D_{11} \Delta \epsilon_{cg} + D_{21} \Delta \Phi + (1986 \times 105.2 - 2027 \times 5.97 - 1115 \times 1076.7) \times 10^{-3}$$

$$+ (1044.5 + 337.5)$$

$$+ D_{31} \Delta \epsilon_{cg} + D_{32} \Delta \Phi + \{208.9 \times 0.276 - 12.10 \times (-0.624) - 1200.5 \times (-0.574)\}$$

(1044.5 + 337.5)

$$= D_{21} \Delta \epsilon_{cg} + D_{31} \Delta \Phi + \{208.9 \times 0.276 - 12.10 \times (-0.624) - 1200.5 \times (-0.574)\}$$

(47)

Solving Eq. (47), $\Delta \epsilon_{cg}$ and $\Delta \Phi$ are obtained and substituting them into Eq. (33), the following equation for determining the neutral axis depth is obtained:

$$1.0037D_{31} - 0.6277D_{21} + (0.326 - x)$$

$$= \frac{-256.5 - 1097x - 149.4x^2 + 3747x^3}{3701 + 3366x + 5621x^2}$$

$$+ 0.326 - x = 0$$

(48)

Solving Eq. (48), $x$ is $0.227$ m ($< x_0$ and, therefore, the neutral axis is below the bottom of the top flange) and $\Delta \epsilon_{cg}$ and $\Delta \Phi$ are:

$$\Delta \epsilon_{cg} = -0.0934 \times 10^{-3}$$

$$\Delta \Phi = 0.9452 \times 10^{-3}$$

Finally, the stresses and deformations in the section under the service loads are obtained by Eq. (34) as follows:

$$f_{c1} = 7.52 \text{ N/mm}^2 (1089 \text{ psi})$$

$$f_{c1} = 138.7 \text{ N/mm}^2 (20.11 \text{ ksi})$$

$$f_{c2} = -142.6 \text{ N/mm}^2 (-20.67 \text{ ksi})$$

$$f_\Phi = -1203.9 \text{ N/mm}^2 (-174.56 \text{ ksi})$$

$$\epsilon_{cg} = 0.2621 \times 10^{-3}$$

$$\Phi = 1.5628 \times 10^{-3}$$

The distribution of the stresses in the section under various loading conditions is shown in Fig. 4.

In order to reduce the amount of interpolation in solving the equations for determining the neutral axis depth, a computer program has been developed for the creep analysis of cracked prestressed sections, based on the equations presented in this paper. The computer program is available directly through the author.

**DISCUSSION**

1. The effect of free shrinkage on the depth of the neutral axis was investigated on two T sections (Fig. 3), namely, without any compression reinforcement ($A_n$) and with 1986 mm$^2$ of $A_n$. The range of shrinkage considered varies from zero to $25 \times 10^{-6}$.

The results show that the depth of the neutral axis always increases with time for the free shrinkage range smaller than $13 \times 10^{-5}$ (Fig. 5). These results suggest that the neutral axis depth increases with time are not usually expected under normal environmental conditions. This is because the free shrinkage of concrete is usually much larger than this range.

Another result is that the use of compression reinforcement will reduce the axial strain, the section curvature, in addition to the concrete stress at the extreme compression fiber after the occurrence of all time-dependent effects as shown in Table 1.

2. When the effect of free shrinkage larger than $13 \times 10^{-5}$ is taken into account in the creep analysis of cracked sections, the neutral axis will not coincide with the zero strain axis as shown in Fig. 6a. Also, the depth of the zero strain axis increases with time while the depth of the neutral axis decreases with time. Two axes coincide only when the free shrinkage is zero (Fig. 6b).

3. Since the stress-strain relation of steel remains elastic regardless of any time-dependent deformations of concrete, the zero strain axis of the section coincides with the zero stress axis of the steel section. Therefore, the height of the crack in the cracked section can be determined by the zero stress axis of the steel section.
Table 1. Effect of compression reinforcement on $x_t$, $f_{ct}$, $\epsilon_{cg}$ and $\Phi$.

<table>
<thead>
<tr>
<th>$A_{sl}$ (mm$^2$)</th>
<th>1986</th>
<th>0</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>At time $t$</td>
<td>$x_t$ (cm)</td>
<td>47.8</td>
<td>50.6</td>
</tr>
<tr>
<td></td>
<td>$f_{ct}$ (N/mm$^2$)</td>
<td>4.02</td>
<td>4.91</td>
</tr>
<tr>
<td></td>
<td>$\epsilon_{cg} \times 10^3$</td>
<td>0.3921</td>
<td>0.4350</td>
</tr>
<tr>
<td></td>
<td>$\Phi \times 10^5/m$</td>
<td>0.8573</td>
<td>0.9777</td>
</tr>
</tbody>
</table>

Table 2. $\epsilon_{cg}$ and $\Phi$ in cracked section.

<table>
<thead>
<tr>
<th>Creep analysis</th>
<th>Service loads are applied</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Before all losses</td>
</tr>
<tr>
<td></td>
<td>Cracked section</td>
</tr>
<tr>
<td>Dead load</td>
<td>$\epsilon_{cg} \times 10^3$</td>
</tr>
<tr>
<td></td>
<td>$\Phi \times 10^5/m$</td>
</tr>
<tr>
<td>Service loads</td>
<td>$\epsilon_{cg} \times 10^3$</td>
</tr>
<tr>
<td></td>
<td>$\Phi \times 10^5/m$</td>
</tr>
</tbody>
</table>

Fig. 5. Variations of neutral axis depth with free shrinkage $E_{cs}$.
(Conversion factor: 1 cm = 0.3937 in.)
4. The creep analysis may be performed on the basis of an uncracked section if no service loads are applied before occurrence of all time-dependent deformations. However, after occurrence of all losses and application of service loads, cracking will result and any stress calculation afterwards should be performed using a cracked section, even when the live load is removed.

Except for the deformations of section, it can be concluded from Figs. 4 and 7 that different creep analyses give relatively close results in the strain distributions for a cracked section under service and dead loads after all time-dependent deformations have occurred. The axial strain and the curvature in the cracked section are given in Table 2.

Therefore, for a high value of dead
Concrete stresses

<table>
<thead>
<tr>
<th>Stresses in non prestressed steel &amp; strain distributions in section</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.29 N/mm²</td>
</tr>
<tr>
<td>72.0 cm</td>
</tr>
<tr>
<td>-1.67 N/mm²</td>
</tr>
<tr>
<td>0.06700x10⁻⁶</td>
</tr>
<tr>
<td>0.01380</td>
</tr>
<tr>
<td>-7.82 N/mm²</td>
</tr>
<tr>
<td>0.3960x10⁻³</td>
</tr>
<tr>
<td>81.4 cm</td>
</tr>
</tbody>
</table>

Fig. 7. Stresses in concrete and nonprestressed steel and strain distributions under various loading conditions. (Creep analysis is performed on uncracked section.) (Conversion factors: 1 cm = 0.3937 in., 1 N/mm² = 145 psi.)

load degree of prestress $\tilde{\kappa}$ ($=M_{dec}/M_d$), such as 0.8 in this example, the long-term deflections under dead load may be calculated using the results of creep analysis based on uncracked sections regardless of the existence of cracks during time-dependent deformations.

5. In order to compare the long-term curvatures and stresses in nonprestressed steel calculated by two different creep analyses using uncracked and cracked sections, a rectangular section shown in Fig. 8 with various combinations of prestressed and nonprestressed steel is analyzed. All the cross sections have the same ultimate strength computed by ACI 318-83. Variations of long-term section curvature, stress in nonprestressed steel and fictitious decompression force with $\tilde{\kappa}$ are shown in Fig. 9.
BENDING MOMENTS:
DEAD LOAD MOMENT: 975 kNm
LIVE LOAD MOMENT: 625 kNm
REQUIRED FLEXURAL STRENGTH:
$$M_0 = 1.4 M_d + 1.7 M_l = 2427.5 \text{kNm}$$

MATERIAL PROPERTIES:
CONCRETE: \( f_c = 40 \text{ N/mm}^2 \), \( E_c = 35 \text{ kN/mm}^2 \), TENSILE STRENGTH = 3.3 N/mm\(^2\)
CREEP COEFFICIENT, \( \varphi = 26 \), FREE SHINKAGE, \( e_{cs} = 25 \times 10^{-5} \)
NONPRESTRESSED STEEL: \( f_y = 400 \text{ N/mm}^2 \), \( E_s = 200 \text{ kN/mm}^2 \)
PRESTRESSED STEEL: \( f_{pu} = 1900 \text{ N/mm}^2 \) (STRESS-RELIEVED STRAND)
\( E_p = 200 \text{ kN/mm}^2 \), TENSILE STRESS AT TRANSFER = 1140 N/mm\(^2\)
LOSS DUE TO STEEL RELAXATION = 5 PERCENT OF INITIAL STRESS

<table>
<thead>
<tr>
<th>( A_p (\text{mm}^2) )</th>
<th>2400</th>
<th>2057</th>
<th>1714</th>
<th>1371</th>
<th>1029</th>
<th>686</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A_s (\text{mm}^2) )</td>
<td>169</td>
<td>1288</td>
<td>2430</td>
<td>3592</td>
<td>4771</td>
<td>5971</td>
</tr>
<tr>
<td>( A_p + A_s (\text{mm}^2) )</td>
<td>2569</td>
<td>3345</td>
<td>4144</td>
<td>4963</td>
<td>5800</td>
<td>6657</td>
</tr>
</tbody>
</table>

Fig. 8. Example: Dimensions, moments, section and material properties of rectangular beam. (Conversion factors: 1 cm = 0.3937 in., 1 N/mm\(^2\) = 145 psi, 1 kN/mm\(^2\) = 145 ksi, 1 kN·m = 0.7375 kip-ft.)
Cracked section analysis based on decompression force obtained from creep analysis of uncracked section.

Fig. 9. Variations of long-term curvature, stress in nonprestressed steel and decompression force with dead load degree of prestress $K$.

Crack analysis of cracked section.

Decompression force $A_2' - f_p^* + A_4 f_t^*$ (kN)

Stress in non prestressed steel (N/mm$^2$)

Conversion factor: 1 N/mm$^2$ = 145 psi.
It can be observed from Fig. 9 that for a low value of $\kappa$ the result of an uncracked section is always much smaller than that of a cracked section. However, this difference becomes considerably small when the section which was assumed to be uncracked is analyzed by the elastic theory using a fictitious decompression force derived from a creep analysis for an uncracked section and assuming that the section is now cracked under dead load.

For practical use in routine design work, it is suggested that even for lower values of $\kappa$, the creep analysis and computation of fictitious decompression force may be carried out on the basis of uncracked sections. Then the long-term section curvature and steel stresses in a cracked section under dead load can be computed by elastic theory and applying dead load moment to the section at the decompression state.

**CONCLUDING REMARKS**

In this paper, the creep analysis for cracked sections of partially prestressed concrete members, which are designed without following the zero tension requirement in concrete under dead load, has been developed based on Bazant's age-adjusted effective modulus method. The author has also introduced a simplified design method for routine work. The calculation steps of this method are as follows:

1. Creep analysis is performed based on the assumption that the section is not cracked, and by using one of the well-known methods for estimating time-dependent effects in uncracked prestressed concrete sections.

2. Section deformations at the decompression state and the fictitious decompression force are calculated on the basis of the creep analysis for uncracked sections.

3. Section curvature and steel stress in cracked sections under dead load are computed using elastic theory and the fictitious force.

This simple method results in a good approximation of the exact method developed by the author according to the numerical example carried out in this paper.

Although a description of the long-term deflection analysis is not included in this paper, the mean curvature can be computed by taking into account the contribution of concrete in tension from the two calculated curvatures of the section in uncracked and cracked states. The long-term deflection can be obtained by double integration of the mean curvature along the member, while taking into account the support conditions.

The proposed approximate procedure can be used to predict the long-term deflections of cracked prestressed concrete members.
REFERENCES


NOTE: Supplemental information (including mathematical derivations) on this article is available from PCI Headquarters.
APPENDIX — NOTATION

\(A_{el}\) = net area of uncracked concrete section
\(A_{eII}(x)\) = net area of concrete bounded by neutral axis of depth \(x\) and extreme compression fiber
\(A_p\) = area of prestressed reinforcement
\(A_s\) = area of nonprestressed reinforcement

\(D_{1II}(x), D_{2II}(x), D_{3II}(x)\) = various elastic stiffnesses of cracked section. Argument \((x)\) may be omitted. See Eq. (5).

\(\bar{D}_{1II}(x), \bar{D}_{2II}(x), \bar{D}_{3II}(x)\) = various stiffnesses of cracked section when time-dependent effects are considered. See Eq. (15).

\(E_e\) = modulus of elasticity of concrete
\(\bar{E}_e\) = age-adjusted effective modulus of concrete
\(E_p\) = modulus of elasticity of prestressed steel
\(E_s\) = modulus of elasticity of nonprestressed steel

\(f_{ck}\) = concrete stress at level \(k\) (positive for compressive stress)
\(f_c(t)\) = concrete compressive stress at time \(t\)

\(f_{pm}\) = stress in prestressed reinforcement at level \(n\) (negative for tensile stress)
\(f_p(t)\) = stress in prestressed reinforcement at time \(t\) (negative for tensile stress)

\(\Delta f_p(t)\) = loss of tensile stress in prestressing steel at time \(t\) due to relaxation of steel

\(f_{pm}(t_0)\) = stress in prestressed reinforcement at level \(n\) immediately after transfer
\(f_{pm}(t)\) = stress in nonprestressed reinforcement at level \(m\) (negative for tensile stress)

\(f_p(t)\) = stress in nonprestressed reinforcement at time \(t\) (negative for tensile stress)

\(f^*_p\) = fictitious stress in prestressed reinforcement at level \(n\) when stresses in concrete are eliminated

\(f^*_m\) = fictitious stress in nonprestressed reinforcement at level \(m\) when stresses in concrete are eliminated

\(I_{eII}(x)\) = moment of inertia of concrete section bounded by neutral axis of depth \(x\) and extreme compression fiber about centroidal axis of net concrete section

\(I_p\) = moment of inertia of prestressed reinforcement about centroidal axis of net concrete section

\(I_s\) = moment of inertia of nonprestressed reinforcement about centroidal axis of net concrete section

\(M_d\) = moment due to dead load
\(M_{dec}\) = decompression moment, produces zero stress at extreme tension fiber

\(M_l\) = moment due to live load

\(Q_{eII}(x)\) = first static moment of concrete section bounded by neutral axis of depth \(x\) and extreme compression fiber about centroidal axis of net concrete section

\(Q_p\) = first static moment of prestressed reinforcement about centroidal axis of net concrete section

\(Q_s\) = first static moment of nonprestressed reinforcement about

124
centroidal axis of net concrete section

t = time elapsed after transfer
t₀ = time at transfer
x₀ = depth of neutral axis of concrete stress distribution at time t₀
xₜ = depth of neutral axis of concrete stress distribution at time t
yₖ = distance from centroidal axis of net concrete section to a fiber k (positive in upward direction)
y₁ = distance from centroidal axis of net concrete section to extreme compression fiber (positive)
εₑ(t) = concrete strain at time t (positive sign indicates shortening)

εₑ₀(t) = axial strain at centroid of net concrete section at time t
εₑ⁺ = axial strain when concrete stresses are eliminated
Δεₑ₀ = incremental axial strain due to service loads from reference section
εₑₙ = free shrinkage of concrete since time of prestressing t₀
φ = concrete creep coefficient since time of prestressing t₀
Φ(t₀) = section curvature at time t₀
Φ(t) = section curvature at time t
Φ* = section curvature when concrete stresses are eliminated
ΔΦ = incremental section curvature due to service loads from the reference section
χ = aging coefficient at time t for concrete loaded at age t₀
κ = dead load degree of prestress

NOTE: Discussion of this paper is invited. Please submit your comments to PCI Headquarters by September 1, 1987.
Unusually long precast prestressed bulb T beams were used to build a three-span bridge to carry bicycle and pedestrian traffic over a busy interstate freeway and railroad mainline.

I-205 Bikeway Overcrossing Banfield Freeway

The problem confronted by the Bridge Design Section of the Oregon Department of Transportation was to design a three-span bikeway structure to be visually compatible with an existing parallel freeway bridge adjacent to the bikeway while at the same time provide minimum disruption to freeway and rail traffic during construction.

The solution to the problem was to design a structure using six 6 ft 2 in. (1.87 m) deep precast prestressed concrete bulb T beams ranging in length from 110 to 166 ft (33.5 to 50.6 m) with the longest beams weighing 79 tons (72 t) each.

The resulting structure is a 433 ft (132 m) long three-span bridge with a 10 ft 9 in. (3.28 m) wide deck having a total area of 4654 sq ft (433 m²). The detailed plan, elevation and cross section are shown on p. 128.
The three long spans, column orientation and grade line were designed to be aesthetically compatible with the existing parallel freeway bridge adjacent to the bikeway. In addition, the end abutments of the bikeway structure are supported on the wingwall footings of the existing freeway bridge for economy of design as well as visual compatibility with this bridge.

The bikeway structure, which is located in East Portland at the South Banfield Interchange with I-205, carries both bicycle and pedestrian traffic over a railroad mainline and six lanes of interstate freeway. The structure parallel and adjacent to the bikeway is a three-span post-tensioned concrete box girder bridge carrying seven lanes of freeway traffic.

The pile footing supported wing walls of the existing freeway bridge were modified to serve as end abutments for the bikeway structure. This not only reduced construction costs but gives the bridge ends of the bikeway a pleasing continuous appearance in line with the ends of the existing structure. A single column at each interior bent was located to align with the four columns at each interior bent of the existing freeway structure.

Prestressed bulb T beams were chosen for the superstructure because they could be precast, trucked to the job site and erected quickly over a busy freeway and railroad line without the need of falsework support. An unusual feature of this structure is the extremely long [up to 166 ft (50.6 m)] beams. Each span contains two bulb T beams with 5 ft 4¼ in. (1.63 m) wide top flanges. The wide top flanges allowed the entire bridge deck to be poured directly on the bulb T beams without bottom deck forms.

Protective fencing completely encloses the deck surface providing safety for the bikeway traffic as well as for freeway and rail traffic below.

The completed structure provides an attractive, safe crossing for bicycles and pedestrians at an intersection of freeways and a railroad line.

The total cost of the bikeway structure was $266,800 with the precast prestressed concrete portion of the work amounting to $112,000. Based on the above figures, the unit cost was $57.33 per sq ft.
The bikeway structure was completed in January 1986 after a 9 month construction period. During the past year it has become a popular facility for both cyclists and pedestrians.

Credits

Owner: Oregon Department of Transportation — Highway Division, Salem, Oregon.

Architect/Engineer: Bridge Design Section, Oregon Department of Transportation — Highway Division, Salem, Oregon.

Contractor: Auburn Construction Company, Wilsonville, Oregon.

Prestressed Concrete Manufacturer: Concrete Technology Corporation, Tacoma, Washington.