Summary Paper

Strength of Precast Prestressed Concrete Members With Dapped Ends

by

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CONTENTS

1. Summary and Conclusions ........................................... 60

2. Scope of Study ....................................................... 61

3. Introduction .......................................................... 61

4. Development of Research Program .................................. 62
   — Common Specimen Design and Test Requirements
   — Program of Tests

5. Experimental Program ................................................ 64
   — The Test Specimens
   — Testing Procedures

6. Behavior of Test Specimens ......................................... 68
   — General Behavior
   — Reinforcement Scheme 1
   — Reinforcement Scheme 2
   — Reinforcement Scheme 3
   — Reinforcement Scheme 4
   — Reinforcement Scheme 5
   — Influence of Prestressing Strand Location

References ................................................................. 75

Appendix — Notation ...................................................... 75

Note: This summary paper is a condensed version of PCISFRAD Project No. 6, "Strength of Members With Dapped Ends." The full report is available from PCI Headquarters at $12.00 to firms supporting the sponsored research, $17.00 to PCI Members (non-supporting firms) and $34.00 to non-PCI Members. The summary paper, and the full report, are based on a research project supported by the PCI Specially Funded Research and Development (PCISFRAD) Program. The conduct of the research and the preparation of the final reports for each of the PCISFRAD projects were performed under the general guidance and direction of selected industry Steering Committees. However, it should be recognized that the research conclusions and recommendations are those of the researchers. The results of the research are made available to producers, engineers and others to use with appropriate engineering judgment similar to that applied to any new technical information.
1. SUMMARY AND CONCLUSIONS

Five dapped end reinforcement schemes, suitable for thin stemmed precast prestressed concrete members such as double tees, were investigated. The studies involved subjecting full scale specimens to a combination of shear and outward tension at the bearing plate. Based on these studies, design procedures suitable for use in practice were developed for each of the reinforcement schemes.

The first two reinforcement schemes studied are currently used in precast prestressed concrete structures. The experimental study has resulted in a better understanding of their behavior and refinement of their design.

The three other reinforcement schemes were developed from the first two reinforcement schemes and also from that utilizing vertical hanger reinforcement contained in Section 6.13 of the PCI Design Handbook (Third Edition). These reinforcement schemes simplify fabrication through reduction in reinforcement congestion in the nib of the dapped end and hence facilitate the placing and compaction of the concrete.

Extensive conclusions for design are set out in the project Final Report together with complete details of the proposed design methods and examples of their application. The following is a summary of the principal conclusions:

1. All five of the reinforcement schemes studied are suitable for use in practice.
2. In all cases the horizontal extension of the hanger reinforcement in the bottom of the web should be not less than 1.7 times the specified development length for the reinforcing bar used, in order that the yield strength of the hanger reinforcement can be developed.
3. The design procedures for Reinforcement Schemes 1, 2, 3, and 5 set out in the Final Report and for Reinforcement Scheme 4 set out in Refs. 3 and 1, yield safe estimates of the strength of dapped ends using these reinforcement schemes.
4. The reinforcement schemes using inclined hanger reinforcement provide better control of cracking than Scheme 4 which utilizes vertical hanger reinforcement, particularly if all the prestressing strands are terminated at the end face of the beam web.
5. Draping half the prestressing strands through the nib of the dapped end significantly improved serviceability for all of the reinforcement schemes, by reducing reinforcement stresses and associated cracking at service load. It is therefore strongly recommended that, if at all possible, not less than half the prestressing strands be draped through the nib.
6. It is possible to develop the yield strength of #3 and #4 (9 and 13 mm diameter), 60 grade (414 MPa yield) reinforcing bars by anchoring them with a 180 degree loop transverse to the axis of the member, and having a minimum bend diameter of six bar diameters. (The orientation of the loop is critical, since it can only act as an effective anchorage if compressive stresses in the concrete act across the plane containing the loop and so prevent a splitting failure.)

7. A concentric or near concentric arrangement of hanger reinforcement is preferable. If hanger reinforcement eccentric to the centerline of the web is used, then that eccentricity should be minimized, and special care taken to ensure adequate cover to the lower part of the hanger reinforcement and its horizontal extension.
8. In most of these pretensioned, prestressed concrete beams with dapped ends, it was not possible to develop a full depth beam shear strength greater than the diagonal tension cracking shear, through the provision of web reinforcement. It is therefore proposed...
that for that part of the full depth web adjacent to a dapped end, the nominal shear strength $V_n$ be taken equal to $V_{ci}$, where $V_{ci}$ is equal to the lesser of $V_{ci}$ and $V_{cur}$, calculated for the section distance $h/2$ from the end of the full depth web. In the future some reinforcement detail may be developed which can overcome this problem, but at the present time no test data validating such a detail is available.

9. The effect of the horizontal tension force $N_h$ and of the actual buildup of prestressing force in each strand, must be taken into account when calculating $f_{pe}$ and $f_{pe}$ at $h/2$ from the end of the full depth web, for use in the ACI Code equations for $V_{ci}$ and $V_{cur}$. [The results obtained in this study indicate that it is reasonable to assume a linear increase in prestress force over the transfer length of the strand, taken as 36 in. (915 mm) for clean 0.5 in. (13 mm) seven-wire strand.]

2. SCOPE OF STUDY

The objectives of this research project were to:

1. Attain a better understanding of the behavior of dapped ends in thin stemmed, precast prestressed concrete members such as the double tee.

2. Develop reinforcing schemes and associated methods of design, which combine simplicity of application with economy of fabrication, while providing the margin of safety required by present building codes.

3. INTRODUCTION

The dapped end beam enables the depth of a precast floor or roof structure to be reduced, by recessing the supporting corbel or ledge into the supported beam. However, the "dap" or cutaway section at the end of the beam results in a severe stress concentration at the re-entrant corner.

This can initiate diagonal tension cracking at a lower shear than would otherwise be expected. If suitable reinforcement is not provided close to the re-entrant corner, failure can occur with little or no warning. Dapped ends are also sensitive to horizontal tension forces caused by restraint of shrinkage or creep shortening of members. These effects were discussed by Mast.5

Previous studies6,8 of dapped end beams primarily involved rectangular section members, which were either reinforced concrete beams6,8 or were post-tensioned.7,8 Of the foregoing studies, only that of Mattock and Chan3 included horizontal tension as well as shear.

Martin and Korkosz9 have discussed current design practice. They reported that the most commonly used reinforcement scheme is that utilizing an orthogonal arrangement of reinforcement. Design procedures for this reinforcement scheme appear in the PCI Design Handbook.1 They are essentially similar to the design recommendations contained in Ref. 3. An alternative scheme proposed in the PCI Design Handbook1 is the use of an inclined bar with hooked ends as the hanger reinforcement. A problem with this scheme is the questionable effectiveness of the hooks as anchorage for the hanger reinforcement.

In both the above reinforcement schemes it is also difficult in practice to ensure accurate location of the local
reinforcement in the end of the member, which is essential if this reinforcement is to be able to develop its design strength. To overcome this problem some producers use welded assemblies of reinforcing bars and steel plates, which can be located positively in the end of the member.

4. DEVELOPMENT OF RESEARCH PROGRAM

The research program was developed in consultation with the project steering committee. It was decided to make the test beams represent one-half of an 18 in. (460 mm) deep, 8 ft (2.4 m) wide double tee. A typical test beam is shown in Fig. 1. A dapped end specimen was formed on each end of the test beam, and these specimens were tested separately.

The beams were prestressed by four ½ in. (12.7 mm) diameter 270K strands, typically arranged as shown in Fig. 1, so that two strands passed through the nib and two strands terminated at the front face of the dap. However, in two test beams the strands were carried straight from end to end of the beam, with the strands arranged as shown in Section A-A of Fig. 1. In these beams none of the strand passed through the nib and all the strands terminated at the front face of the dap. The average effective prestress at the time of the test was 156 ksi (1080 MPa).

All beams were provided with ACI Code minimum web reinforcement over the whole full depth length, in the form of a single sheet of welded wire fabric with W2.9 (5 mm diameter) vertical wires at 7.5 in. (190 mm) centers. The flanges of all the beams were reinforced with welded wire fabric.

Complete details of the fabrication of the specimens and of the strengths of the materials used are contained in the project report.\(^2\) The concrete was made from Type 3 portland cement, sand, ¾ in. (20 mm) maximum size glacial outwash gravel and a water reducing admixture. It had a target strength at test of 5000 psi (34.5 MPa). Stress-strain curves were obtained for each reinforcing bar used. These were used in the evaluation of the strain measured in the dapped end tests. The actual yield strengths of the reinforcing bars were used in the design of the test specimens.

Concurrently with the design of the basic test beam, and the design and construction of the formwork, a survey was made of PCI Producer Members to obtain information on currently used reinforcing details and design procedures for dapped ends in double tees. This information was taken into account in the planning of the test program.

Common Specimen Design and Test Requirements

The project steering committee decided that for all specimens the dap should be half the total section depth and that the flange should be cut away for the 6 in. (152 mm) length of the nib. They further decided that the centerline of the reaction should be 4.5 in. (114 mm) from the re-entrant corner of the dap. Also, that the specimens should be subjected to a tension force equal to 20 percent of the end shear.

The target nominal shear strength, \(V_\text{n}\), of the specimens was based on a review of the values of the end shear per web corresponding to the loads and spans tabulated in the PCI Design Handbook,\(^6\) for an untopped 8DT18 section. It was found that for these loads and spans, the maximum required nominal shear strength per web is 23.2 kips (103 kN). It was decided that the specimens should be designed to have a nominal shear strength as close to this value as
possible, taking into account the actual strengths of the available reinforcing bars.

It was proposed that each specimen be subjected to several applications of service load and moderate overload, before being loaded to failure. Using the ACI Code load factors and strength reduction factor for shear, the ratio of service load to nominal strength for the 8DT18 section is close to 0.53 when the end shear is a maximum. The service load shear, $V_s$, was therefore taken equal to $0.53V_n$.

For moderate overload tests it was decided to subject the specimens to a shear corresponding to the load prescribed in the ACI Code, Section 20.4 — Load tests of flexural members, i.e., $0.85 (1.4D + 1.7L)$. The overload shear is therefore $0.85V_n$ or $0.85 (0.85V_n)$, i.e., $0.72V_n$. This corresponds to a 36 percent overload.

Program of Tests

Based on the foregoing discussion, all specimens were subjected to the following series of tests:

Test (a) — Incremental load test to service load shear $V_s = 0.53V_n$.
Test (b) — Nine cycles of loading to service load.
Test (c) — Incremental load test to moderate overload, maximum shear $0.72V_n (1.36V_n)$.
Test (d) — Ten cycles of loading to service load.
Test (e) — Incremental load test to failure.

Reinforcement strains, crack growth and strand draw-in were monitored at each load increment in Tests (a), (c) and (e), and at maximum and minimum loads in alternate loading cycles in Tests (b) and (d). In all tests, the specimens were subjected to a tension force equal to 20
percent of the shear force acting on the dapped end.

Each of the two dapped ends on a test beam was tested separately, using the testing arrangement shown in Fig. 2. The loads were applied to the beam through horizontal bearing plates, and it was also supported on horizontal bearing plates. This resulted in the dapped ends being subjected to a combination of shear perpendicular to the beam axis and a tension force parallel to the beam axis equal to 20 percent of the shear.

The points of application of the loads were chosen so that the distribution of shear and moment in the end of the beam adjacent to the dapped end under test, would approximate that which would be caused by a uniformly distributed load. The load was applied by a testing machine, through a load cell (see Fig. 2).

5. EXPERIMENTAL PROGRAM

The experimental program is summarized in Table 1, which indicates the reinforcement schemes used and the special characteristics of each specimen. The specimens are identified by a number which reflects the reinforcement scheme and a letter which reflects the type of specimen.

The Test Specimens

In all cases the lower ends of the hanger reinforcement were anchored by extending the bars horizontally in the bottom of the beam web. Specimens 1A and 2A were provided with extensions equal to the specified development length for these bars. In both cases the inclined bars did not develop their yield strength. A combined bond and diagonal tension failure occurred in the beam web adjacent to the dapped end. All other specimens were therefore provided with a horizontal extension equal to 1.7 times the specified development length, i.e., the length of a Class C splice.

Details of Reinforcement Scheme 1 are shown in Fig. 3(a). The upper ends of the inclined bars are anchored by
Table 1. Summary of test program.

<table>
<thead>
<tr>
<th>Reinforcement scheme</th>
<th>A Draped strand</th>
<th>B Draped strand</th>
<th>C No strand in nib</th>
<th>D Draped strand</th>
<th>E Draped strand</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1A</td>
<td>1B</td>
<td>1C</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>2</td>
<td>2A</td>
<td>2B*</td>
<td>2C*</td>
<td>2D*</td>
<td>—</td>
</tr>
<tr>
<td>3</td>
<td>—</td>
<td>3B†</td>
<td>3C†</td>
<td>3D†</td>
<td>3E†</td>
</tr>
<tr>
<td>4</td>
<td>—</td>
<td>β = 45°</td>
<td>β = 45°</td>
<td>β = 60°</td>
<td>—</td>
</tr>
<tr>
<td>5</td>
<td>—</td>
<td>4B</td>
<td>4C</td>
<td>—</td>
<td>—</td>
</tr>
</tbody>
</table>

* Compression reinforcement omitted.
† Cover to horizontal extension of hanger reinforcement increased.
‡ β = slope to horizontal of hanger reinforcement and end face of web.
§ Slope of hanger reinforcement = 45°, slope of end face of web = 90°.

Welding to opposite faces of a vertical plate, which is itself welded to a short length of steel angle. The nib flexural reinforcement is welded to the bearing plate, which is in turn attached to the anchorage at the upper end of the inclined bars. The nominal shear strength was initially taken equal to the vertical component of the yield strength of the inclined bars, assuming a truss-like behavior. The assumed behavior model and a design procedure for dapped ends incorporating the first reinforcement scheme are set out in Appendix B of the project report.

Details of Reinforcement Scheme 2 are shown in Fig. 3(b). The upper end of the single inclined bar is anchored by welding to a vertical plate, which is in turn welded to the bearing plate. The nib flexural reinforcement is also welded to the bearing plate. The nominal shear strength of this dapped end is taken equal to the sum of the vertical component of the yield strength of the inclined bar and the calculated shear to cause cracking at the re-entrant corner of the dap. The assumed behavior model and a design procedure for dapped ends incorporating the second reinforcement scheme are set out in Appendix C of the project report.

Both of the first two reinforcement schemes have the advantage of providing positive anchorage in the nib for the inclined reinforcement, and positive location of the reinforcement in the form. The resulting reinforcement assemblies can also be placed in the form as a unit. However, both reinforcement schemes involve a significant amount of welding and congestion of the reinforcement can make placing of the concrete in the dapped end more difficult. Scheme 1 has the further disadvantage that prestressing strands passing through the nib must be threaded between the inclined bars.

The third and fourth reinforcement schemes were devised as simplifications of Reinforcement Scheme 1 and of that using vertical hanger reinforcement described in the PCI Design Handbook. In both cases it was assumed that adequate anchorage could be provided at the upper end of the hanger reinforcement by looping it through 180 degrees, with a bend diameter equal to six times the bar diameter.

The details of Reinforcement Scheme 3 are shown in Fig. 3(c). As for Reinforcement Scheme 1, a truss-like behavior was assumed and the nominal shear strength was initially taken equal to the vertical component of the yield strength of the inclined bars.

The assumed behavior model and a
design procedure for dapped ends using the third reinforcement scheme are set out in Appendix D of the project report.²

The details of Reinforcement Scheme 4 are shown in Fig. 3(d). The closed stirrup used for vertical hanger reinforcement in the PCI Design Handbook¹ is replaced by the looped bar. Shear reinforcement in the nib is provided in the form of a single hairpin bar, tied to one leg of the hanger reinforcement and to a vertical #3 bar welded to the bearing plate. The nib of the dapped end is assumed to act as an inverted corbel and the hanger reinforcement is assumed to carry the entire shear acting on the dapped end. The model for behavior and the design procedure followed were as detailed in Ref. 3. However, because the flange is cut back over the length of the nib, it is necessary to check the flexural strength both at the centerline of the hanger reinforcement and at the vertical face of the dap.

![Fig. 3. Typical details of dapped end reinforcing schemes.](image-url)
The looped bar anchorage used for the hanger reinforcement in the third and fourth reinforcement schemes was satisfactory. In both cases the yield strength of the reinforcement was developed before failure. The loop anchorage is probably effective because compression stresses act in the concrete across the plane containing the loop, preventing a splitting failure of the concrete inside the loop.

The advantages of Reinforcement Schemes 3 and 4 over Schemes 1 and 2 are that the amounts of welding and structural steel used are reduced, as is congestion of reinforcement in the nib. However, the reinforcement is not located as positively in Schemes 3 and 4, although it was found possible to tie it together into fairly rigid reinforcement cages which could be placed in the form as a unit.

Details of the fifth reinforcement scheme are shown in Fig. 3(e). This scheme was developed after experience with the first two reinforcement schemes. It provides positive location of the reinforcement and enables a symmetrical (or near symmetrical) arrangement of hanger reinforcement to be used. It also eliminates the threading of prestressing strands through the reinforcement in the nib and reduces reinforcement congestion in the nib, thus making placing and compaction of concrete in the nib easier. The nominal strength was calculated in the same way as for the second reinforcement scheme.

Testing Procedures

Before testing, the beam was supported temporarily on the crib of wooden blocks seen in Fig. 4 with the dapped end bearing out of contact with its support. Zero load readings were taken at this time on the strain gauges and the strand slip gauges. The beam end was then lifted, the tapered wooden wedge removed from below the beam web and the dapped end bearing brought down into contact with its support. Initial dead load readings of the gauges were then taken. The extension
from the head of the testing machine was then brought into contact with the load cell and the first test commenced. Fig. 4 shows a typical specimen at this point.

The program of Tests (a) through (e) previously outlined was followed. In Tests (a) and (c) load increments of 1.5 kips (6.7 kN) were used; this resulted in increments of shear of about 1 kip (4.5 kN). In the final test, the load increments were reduced to 1 kip (4.5 kN) or 0.5 kip (2.2 kN) at high loads, as distress became evident.

6. BEHAVIOR OF TEST SPECIMENS

Complete information on the strength and behavior of all the test specimens is provided in the project report. This paper summarizes behavior common to all or most of the specimens, and discusses aspects of behavior particular to certain specimens. A summary of the specimen strengths and modes of failure is given in Table 2.

General Behavior

As would be expected, the first crack occurred at the re-entrant corner of the dap, due to the stress concentration at that location. However, the shear at which a crack became visible at this location varied with arrangement of reinforcement, and in particular was affected by the arrangement of the prestressing strands at the end of the beam.

In Fig. 5 are shown the various types of cracks which occurred in most specimens, as they would appear after failure. At service load the re-entrant corner crack was usually 2 to 5 in. (50 to 125 mm) long. This was always the widest crack, and its maximum width was close to the re-entrant corner. In all cases in which half the prestressing strands passed through the nib, cracking behavior under service load was very good. The maximum crack widths under service load were between 0.005 and

![Fig. 5. Typical cracks in a dapped end.](https://via.placeholder.com/150)
Table 2. Summary of specimen strengths and modes of failure.

<table>
<thead>
<tr>
<th>Specimen No.</th>
<th>$V_{n}$ (test) (kips)</th>
<th>$V_{n}$ (calc) (kips)</th>
<th>$V_{n}$(test) / $V_{n}$(calc)</th>
<th>Failure mode</th>
</tr>
</thead>
<tbody>
<tr>
<td>1A</td>
<td>21.82</td>
<td>22.69*</td>
<td>0.96</td>
<td>A &amp; B</td>
</tr>
<tr>
<td>1B</td>
<td>27.93</td>
<td>24.08*</td>
<td>1.16</td>
<td>A</td>
</tr>
<tr>
<td>2A</td>
<td>23.75</td>
<td>22.80*</td>
<td>1.00</td>
<td>A &amp; B</td>
</tr>
<tr>
<td>2B</td>
<td>20.05</td>
<td>23.29*</td>
<td>0.86</td>
<td>C</td>
</tr>
<tr>
<td>2D</td>
<td>23.82</td>
<td>22.41*</td>
<td>1.06</td>
<td>D</td>
</tr>
<tr>
<td>3B</td>
<td>27.93</td>
<td>18.47*</td>
<td>1.51</td>
<td>B &amp; A</td>
</tr>
<tr>
<td>3C</td>
<td>21.16</td>
<td>18.63*</td>
<td>1.42</td>
<td>A &amp; B</td>
</tr>
<tr>
<td>3D</td>
<td>25.24</td>
<td>16.87*</td>
<td>1.50</td>
<td>A &amp; B</td>
</tr>
<tr>
<td>3E</td>
<td>29.51</td>
<td>21.85</td>
<td>1.26</td>
<td>B &amp; A</td>
</tr>
<tr>
<td>4B</td>
<td>27.45</td>
<td>20.00*</td>
<td>1.37</td>
<td>A &amp; B</td>
</tr>
<tr>
<td>4C</td>
<td>19.54</td>
<td>20.00*</td>
<td>0.98</td>
<td>A &amp; B</td>
</tr>
<tr>
<td>5B</td>
<td>24.76</td>
<td>22.91*</td>
<td>1.08</td>
<td>A</td>
</tr>
</tbody>
</table>

Note: 1 kip = 4.45 kN.
* Calculated using original or only method of calculation.
† Calculated using modified method of calculation.
A = Diagonal tension failure in beam web.
B = Flexural bond failure in beam web.
C = Inclined bar burst sideways out of web at bottom corner.
D = Diagonal tension failure in nib.

Note: Beam incorporating Specimens 1C and 2C failed prematurely by flexural bond, due to faulty construction.

0.01 in. (0.13 and 0.25 mm), even after 36 percent overload.

During the moderate overload test (c), the re-entrant corner crack usually extended to within 1 or 2 in. (25 or 50 mm) of the web-flange junction, and a short branch crack would form approximately over the hanger reinforcement. This type of crack is caused by high local bond stresses on the surface of the hanger reinforcement. Also, under the moderate overload a fine crack usually occurred in the nib at the inner edge of the bearing plate, and in some cases a short inclined crack occurred in the nib.

In the final test, secondary inclined cracks propagated from the end face of the web. These cracks formed progressively further from the re-entrant corner, as the tension stress in the hanger reinforcement increased. The secondary inclined cracks on the opposite faces of the web were often connected together by cracks across the end face of the web. In some cases the secondary inclined cracks linked with the re-entrant corner crack, which spread along the web-flange junction as the shear increased.

In some of the specimens fine vertical cracks formed on the end face of the web, approximately over the hanger reinforcement. Also, additional fine inclined cracks usually occurred in the nib at higher loads.

Approaching the failure load, a succession of short flexural cracks usually occurred at the bottom of the web. The first crack formed near the bottom corner of the web, as the tensile stress at this location in the horizontal extension of the hanger reinforcement started to become significant.

In all cases where a secondary failure was avoided, the hanger reinforcement...
yielded before failure, or was very close to yield. However, in almost every case, failure finally occurred as a result of a major diagonal tension crack extending from close to the bottom corner of the web, up to the web-flange junction, 30 to 40 in. (760 to 1015 mm) from the end of the flange. This critical diagonal tension crack was accompanied by slip of the lower prestressing strands as a flexural bond failure occurred. This prevented the development of any truss action by the web reinforcement, because the tension chord of the truss was effectively disabled. Hence, little or no increase in load beyond diagonal tension cracking was possible, despite the provision of minimum shear reinforcement in the beam web.

It was possible to calculate the shear at diagonal tension cracking of the web reasonably closely, providing the actual build up of prestress along the beam, and the effect of the tension force acting on the dapped end, were taken into account when calculating $V_{et}$ and $V_{ew}$. Calculating the shear at $h/2$ from the end of the full depth web, and taking the transfer length of the strand as 36 in. (914 mm), the average ratio of test to calculated diagonal tension cracking shear for all specimens is 1.03, with a standard deviation of 0.07. The 36 in. (914 mm) transfer length for clean $3/8$ in. (13 mm) diameter 270K strand was taken from the study of Kaar, LaFraugh and Mass.  

**Reinforcement Scheme 1**

In Fig. 6 is shown the variation with shear of the stresses in the hanger reinforcement in Specimen 1B. This behavior is typical for specimens in which half of the prestressing strands were
draped through the nib of the dapped end. The stress in the nib flexural reinforcement at the re-entrant corner was similar in magnitude to that in the hanger bars at the same location.

The hanger reinforcement in Specimen 1B developed its yield strength at a shear of 23.8 kips (106 kN). It continued to develop its yield strength or a slightly greater force, until a diagonal tension failure of the beam web occurred at a shear of 27.93 kips (124 kN). The nominal shear stress in the nib at failure of the beam was 810 psi (5.6 MPa).

At the failure shear of 21.82 kips (97.1 kN) in Specimen 1A, the hanger reinforcement was carrying a shear of 17.04 kips (75.8 kN) and the vertical component of the force in the prestressing strands at the nib-beam interface was calculated to be 0.37 kips (1.6 kN). The concrete in the plane of the interface was therefore carrying a shear of 4.41 kips (19.6 kN) at failure of the beam web, but the nib was still intact. This shear corresponds to a nominal shear stress in the nib of 1.8 √f'c psi.

It was therefore decided that the calculated nominal shear strength of Specimen 1B should be taken as the sum of the vertical component of the yield strength of the inclined bars and a shear corresponding to a nominal shear stress of 2 √f'c psi (0.17 √f'c MPa) in the nib. This shear acting in the concrete at the nib-beam interface was taken into account when designing the flexural reinforcement in the nib. This method of shear strength calculation is referred to as the “modified method of calculation” in Table 2. It can be seen that it resulted in a conservative but reasonably close estimate of the strength of Specimen 1B.

Reinforcement Scheme 2

A premature secondary type of failure occurred in Specimen 2B, which was reinforced with a single inclined bar located eccentric to the centerline of the beam. At failure, this bar burst sideways out of the beam near the bottom corner of the web, as splitting occurred in the end face of the web over the bar. Strains measured on the inside and outside faces of the inclined bar near the re-entrant corner, indicated that it was subjected to considerable bending as well as to tension.

Specimen 2D was therefore made, in which the #5 inclined bar was placed closer to the centerline of the beam, and the concrete cover to this bar was increased from 0.75 to 1.0 in. (19 to 25 mm). The strains measured on the inclined bar indicated that a small amount of bending was still occurring, but very much less than in the case of Specimen 2B. The bursting failure was avoided and the strength attained exceeded the calculated nominal strength by 6 percent.

Reinforcement Scheme 3

The primary purpose of testing the specimens using this reinforcement scheme was to check whether a simple 180 degree loop at the upper end of the inclined bar could anchor the bar in the nib well enough to develop its yield strength.

Specimens 3B and 3D were similar, except that the angle of inclination of the hanger bars and of the end face of the web was 45 and 60 degrees to the horizontal in Specimens 3B and 3D, respectively. Both specimens were designed assuming truss-like behavior of the nib at ultimate, with the compression chord provided by the concrete assumed to be horizontal. The calculated nominal shear strength was therefore equal to the vertical component of the yield strength of the inclined bars.

For both Specimens 3B and 3D, the variation with shear of the stresses in the hanger reinforcement was similar to that shown in Fig. 6 for Specimen 1B. In both Specimens 3B and 3D the yield strength of the inclined bars was devel-
Fig. 7. Cracking pattern after failure in Specimen 3B.

oped at ultimate, indicating that the loop anchorage was effective.

It is clear from the strengths obtained in Specimens 3B and 3D that the concrete must have contributed to the shear strength. This contribution probably occurred because the top chord of the analogous truss in the nib was not horizontal, as assumed in calculating the strength, but was in fact inclined. This hypothesis is supported by the orientation of the cracks in the nib above the inclined bars in both specimens.

This can be seen in Fig. 7 for the case of Specimen 3B. A modified calculation of shear strength was therefore made assuming, as in the case of Specimen 1B, that there was a contribution to shear strength from the concrete equal to $2b_d d_d \sqrt{f_e}$ psi (0.17b_d d_d \sqrt{f_e} MPa). It can be seen in Table 2 that this shear contribution resulted in a much better agreement between test and calculated strengths.

Specimen 3E was similar to Specimen 3B, except that the end face of the web was made vertical and the welded wire fabric web reinforcement was carried through to that vertical end face. This specimen was tested to check whether locating the lap between the bottom prestressing strand and the horizontal extension of the inclined bars further from the end of the strand, would prevent a flexural bond failure at ultimate after diagonal tension cracking, and thus allow the web reinforcement to develop additional shear resistance. However, Specimen 3E showed no significant improvement in behavior over that of Specimen 3B.

**Reinforcement Scheme 4**

This reinforcement scheme is an adaptation of the reinforcement scheme using vertical hanger reinforcement contained in the PCI Design Handbook. The purpose of these tests was to determine whether the closed stirrup reinforcement could be replaced by a reinforcing bar looped through 180 degrees near the top face of the beam and bent through 90 degrees at the bottom corner of the beam web, as shown in Fig. 3(d).

Specimen 4B, which had half the prestressing strands draped through the nib of the dapped end behaved very well, developing the yield strength of the
hanger reinforcement before failure. However, cracking at service load was more extensive and wider than in the case of dapped ends with inclined hanger reinforcement, though still within acceptable limits.

The nib flexural reinforcement stress at maximum load was very close to its yield strength of 67.74 ksi (467 MPa), both at the re-entrant corner and also 9 in. (230 mm) in from that corner. This observation verifies the need to provide anchorage for the nib flexural reinforcement for a development length beyond the point at which a 45 degree line from the bottom corner of the web crosses this reinforcement, when using this reinforcement scheme.

**Reinforcement Scheme 5**

This reinforcement scheme was developed to provide positive location and anchorage of the reinforcing bars, while reducing reinforcement congestion in the nib. It also allows symmetrical or near symmetrical arrangement of the hanger reinforcement and eliminates threading of prestressing strand through the dapped end reinforcement. The design method used was the same as for Reinforcement Scheme 2.

The vertical plates have a height approximately one-eighth the height of the web and have a shear strength not less than the yield strength of the hanger bars welded to them. The bend diameter of the hook is a minimum of six bar diameters and its purpose is to control diagonal tension cracking in the nib. Therefore, the proportions of the nib and the hook must be such that a straight line drawn from the center of action of \( V \) on the bearing plate, to the center of action of the resultant concrete compression force at the nib-web interface, crosses the hook not higher than the point of tangency.

The behavior of Specimen 5B was satisfactory both at service load and at ultimate. The maximum shear carried was 8 percent above the calculated nominal shear strength. The variation with shear of the stresses in the hanger reinforcement was similar to that found in Specimen 1B. Anchorage of the hanger reinforcement was satisfactory and cracking within the nib was well controlled by the hook detail, as may be seen in Fig. 8.
Influence of Prestressing Strand Location

In Specimens 3C and 4C, the dapped end reinforcement was the same as in Specimens 3B and 4B. However, while in Specimens 3B and 4B two strands were draped through the nib, in Specimens 3C and 4C all four strands were horizontal from end to end of the beam and terminated at the end face of the beam web, below the re-entrant corner.

For both reinforcement arrangements it was found that in the Type C specimens in which all the strands terminated in the end face of the web, behavior at all stages of loading was inferior to that of the comparable Type B specimens in which half the prestressing strands were draped through the nib.

In the Type C specimens, transfer of prestress produced tensile stresses in the dapped end reinforcement near the re-entrant corner. This resulted in cracks occurring at this location immediately when load was applied. On first application of service load, the cracking of the Type C specimens was as extensive as that which occurred in the Type B specimens after 20 cycles of service load and 36 percent overload. This behavior was due to the much greater stresses which occurred in the reinforcement of the Type C specimens, as may be seen in Fig. 9, which shows the stresses which occurred in the hanger reinforcement of Specimens 4B and 4C.
REFERENCES

4. ACI Committee 318, “Building Code Requirements for Reinforced Concrete, (ACI 318-77),” American Concrete Institute, Detroit, Michigan, 1977.

APPENDIX — NOTATION

\[ a = \text{shear span, defined as distance from support reaction to intersection of hanger and nib flexural reinforcement} \]
\[ A_s = \text{area of nib flexural tension reinforcement} \]
\[ A_{bh} = \text{area of hanger reinforcement} \]
\[ b_d = \text{average width of nib} \]
\[ d_b = \text{reinforcing bar diameter} \]
\[ d_d = \text{effective depth of nib} \]
\[ D = \text{unfactored dead load} \]
\[ f'_c = \text{compressive strength of concrete measured on 6 x 12 in. (150 x 300 mm) cylinders} \]
\[ f_y = \text{yield strength of reinforcing bar} \]
\[ h = \text{overall depth of precast member} \]
\[ h_d = \text{overall depth of nib} \]
\[ l_d = \text{reinforcing bar development length} \]
\[ l_v = \text{distance from re-entrant corner to line of action of vertical reaction acting on nib} \]
\[ L = \text{unfactored live load} \]
\[ N_n = \text{nominal tension force resisted at ultimate (acting at bottom face of nib, parallel to axis of member)} \]
\[ V_{ct} = \text{shear at flexure-shear diagonal tension cracking} \]
\[ V_{cwe} = \text{shear at web-shear diagonal tension cracking} \]
\[ V_d = \text{shear due to unfactored dead load} \]
\[ V_{\text{max}} = \text{maximum shear resisted in test} \]
\[ V_n = \text{nominal shear force resisted at ultimate} \]
\[ V_s = \text{shear at service load} \]
\[ \beta = \text{inclination of end face of beam web to the horizontal} \]
\[ \phi = \text{ACI Code strength reduction factor} \]