READER COMMENTS

Prestress Loss and Deflection of Precast Concrete Members*

by Maher K. Tadros, Amin Ghali, and Arthur W. Meyer

Comments by Lucian Nedelcu, A. Scanlon, B. Fitzpatrick and J. Warwaruk, Don R. Weiss, and Authors

LUCIAN NEDELCU†

I read the authors' paper with much interest and wish to express my deep respect for their accomplishment. In a climate where engineers test and find individual formulas for the ultimate design of prestressed concrete members, a step back to the elastic theory represents an act of courage. In this respect, the paper is excellent in its recognition and consideration of tension stiffening. Unfortunately, the manner in which the subject is treated is not complete.

To consider the tension stiffening of the prestressed concrete member, the authors' propose two empirical formulas: one for calculating the effective moment of inertia (Eq. 4) and the other for calculating the effective centroid distance (Eq. 7). To compute the cracked moment of inertia, the authors suggest using the empirical formula proposed by Naaman (Eq. 2) and the exact approach proposed by Jittawait (Figs. 1 and 2 from the paper).

Fig. A plots the three moments of inertia (I'_e, I_{cr1}, and I_{cr2}) as functions of the bending moment applied to the member presented in Example 1, where I_e' is the effective moment of inertia proposed by the authors (Eq. 4 and Figs. 1 and 2); I_{cr1} is the cracked moment of inertia proposed by Jittawait (Figs. 1 and 2); and I_{cr2} is the cracked moment of inertia proposed by Naaman (Eq. 2).

Looking at Fig. A, the difference between the moment of inertia values when the applied bending moment is M = 4924 kip-in.) is practically insignificant: I_e' = 7670 in.^{4}; I_{cr1} = 7400 in.^{4}; and I_{cr2} = 7675 in.^{4} As such, I ask the authors why it was necessary to propose a method where the tension stiffness is less than 4 percent. Furthermore, other methods presented ignore tension stiffening: the cracked moment of inertia is either constant (Naaman) or slightly variable (Jittawait). Theoretically, the cracked moment of inertia at rupture has to be equal to the moment of inertia of the gross section.

To get a smaller time deflection of the member, the authors propose another empirical approach which considers a loss in the compressive force in concrete, ΔP_c, located at the centroid of the combined

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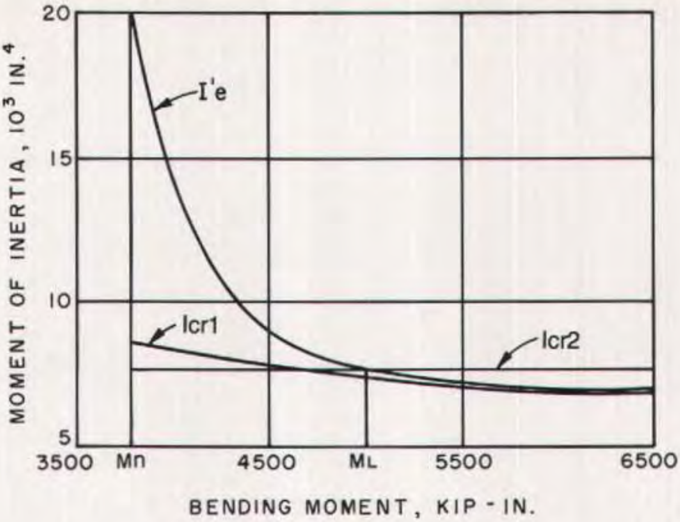

Fig. A. Moment of inertia versus bending moment for a cracked prestressed concrete section.

steel area. As defined, ΔP_c seems to be a loss in both the prestressed and nonprestressed steel. This approach needs some explanation as I do not understand how nonprestressed steel can exhibit a loss of nonexistent stress.

Regarding the location of ΔP_c, the authors' proposal is not clear. Assuming the authors are correct in their approach, the next step could lead to a prestressed concrete member having nonprestressed reinforcement placed at the top. In doing so, the centroid of the combined steel area would move above the centroid of the cross section, achieving a gain rather than a loss in time-dependent deflection.

To compute the loss in prestressing steel, the authors use a method presented in Appendix A. This method, like many others used today, is based on empirical data rather than the actual behavior of prestressed members.

The main shortcomings of the proposed methods are:

1. The loss due to thermal treatment of the concrete is not considered.

2. The loss due to elastic shortening of the member is computed incorrectly: elastic shortening for members using the pretensioning method is independent of vertical loads (including the self weight of the member) and of tendon profile (only the eccentricity at the end of the member is of interest). In other words, only the loads that cause strain at the end of the beam generate elastic shortening of the member and a loss in prestressing force. For members using the post-tensioning system, computation of elastic shortening is more complicated since both the vertical loads and the tendon profile influence elastic shortening of the member.

DON R. WEISS*

The authors present a rational approach to calculating deflections of prestressed concrete members, discussing three basic points: time-dependent effects, nonprestressed steel effects and cracking effects. A simplified approach that accounts for

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Table A. Deflection comparison of plant produced products.*

		BEAM A				BEAM B				BEAM C				BEAM D (with topping)				BEAM E (with topping)			
		Measured	PCI Handbook	Proposed Method		Measured	PCI Handbook	Proposed Method		Measured	PCI Handbook	Proposed Method		Measured	PCI Handbook	Proposed Method		Measured	PCI Handbook	Proposed Method	
				Design	Actual			Design	Actual			Design	Actual			Design	Actual			Design	Actual
Release	P_{oo} (kips)	—	141	142	142	—	327	328	329	—	274	271	274	—	374	371	373	—	370	371	373
	f'_{ci} (psi)	3620	3620	3000	3620	3650	3650	3000	3650	4540	4540	3300	4540	4640	4640	4100	4640	4640	4640	4200	4640
	δ (in.)	0.125	0.09	0.11	0.10	1.00	0.56	0.64	0.60	0.94	0.84	1.07	0.93	1.75	1.19	1.32	1.25	1.75	1.39	1.51	1.45
Erection	P_{ce} (kips)	—	128	120	121	—	294	256	260	—	249	227	230	—	336	304	304	—	348	314	312
	f'_c (psi)	6350	6350	5000	6350	6300	6300	5000	6300	7880	7880	6000	7880	7900	7900	6000	7900	8000	8000	6000	8000
	δ (in.)	0.125	0.15	0.19	0.17	1.30	0.99	1.15	1.06	1.00	1.43	1.66	1.47	2.40	2.09	2.41	2.28	3.12	2.46	2.82	2.69
Final Long Term	δ (in.)	—	−0.13	−0.12	−0.11	—	+0.51	+0.55	+0.55	—	−0.21	−0.40	−0.29	—	—	—	—	—	—	—	—

*Description of products in Table 1.

Beam A — 8 in. span deck; L = 23 ft; SDL = 20 psf; SLL = 40 psf; 5½ in. ϕ 270 K low relax.

Beam B — 8 in. span deck; L = 27 ft; SDL = 5 psf; SLL = 125 psf; 12½ in. ϕ 270 K low relax.

Beam C — 8 DT 24; L = 68 ft; SDL = 15 psf; SLL = 16 psf; 10½ in. ϕ 270 K low relax with single point depress.

Beam D — 10 ft 8 in. DT 28 with 2 in. topping; L = 61 ft; SDL = 5 psf; SLL = 30 psf; 14½ in. ϕ 270 K low relax with single point depress.

Beam E — 10 ft 8 in. DT 28 with 3 in. topping and extra cricket topping at one end; L = 61 ft; SDL = 5 psf; SLL = 30 psf; 14½ in. ϕ 270 K low relax with double depress.

environmental conditions and the presence of nonprestressed steel is a welcome contribution to assist the precast concrete industry in predicting camber and deflection of a serviceable product. It is encouraging to have a method available with adjustable multipliers to account for the above effects in any combination.

Being a producer in Arizona with non-average environmental conditions, the proposed method allows for adjusted time-dependent multipliers by selecting representative creep coefficients and free shrinkage values. I agree with the authors that camber and deflection calculations should be viewed as an estimate. I also believe the proposed method enhances the estimate for release and erection cambers.

Table A is included to provide a deflection comparison of plant produced products with the PCI method and the proposed method. In the table, I have used the sign convention of downward deflection as negative.

Recognizing that many variables influence camber and deflection, the compared examples are based on quality control concrete compression reports for both release strength and 28-day strength. To eliminate the variance of dunnage support locations, some erection camber values were obtained with the product erected at the jobsite.

The proposed method reflects higher prestress forces at release and lower, final prestress forces as compared to the PCI method. Especially with low relaxation strand, this correlation tends to agree with a trend of actual release cambers being higher than PCI calculated cambers. Less growth from release to erection camber is also experienced with actual cambers.

The proposed method to date is applicable only to untopped sections and simple spans. I would like to see an expanded report encompassing variable multipliers for members with composite topping and for members with cantilevers.

COMMENTS by A. SCANLON,* B. FITZPATRICK,** and J. WARWARUK†

The authors have presented a valid procedure for estimating the flexural stiffness of partially prestressed members that may crack under service load. However, requiring engineers to consider the effects of prestressing force when calculating section properties, and the variation in eccentricity of prestress with cracking, introduces a degree of complexity in the calculations that is unwarranted by the expected accuracy of the results. Based on preliminary results of a study currently underway at the University of Alberta, it is believed that a simple extension of the I-effective concept for conventionally reinforced members provides an acceptable level of accuracy for partially prestressed members.

Using the notation of the ACI Building Code (ACI 318-83) the effective moment of inertia is given by:

$$I_e = \left(\frac{M_{cr}}{M_a}\right)^3 I_g + \left[1 - \left(\frac{M_{cr}}{M_a}\right)^3\right] I_{cr}$$

where $M_{cr} = \dfrac{f_r I_g}{y_t}$

M_a = applied moment in member at stage deflection is computed.

Using the expression with the 4th power modifies slightly the rate of transition between I_g and I_{cr}. However, for purposes of this discussion, the third power equation is used. The same expression can be used for partially prestressed members except that the term M_a must be modified to ensure that the ratio (M_{cr}/M_a) equals unity at the moment level where cracking occurs and decreases with increasing applied mo-

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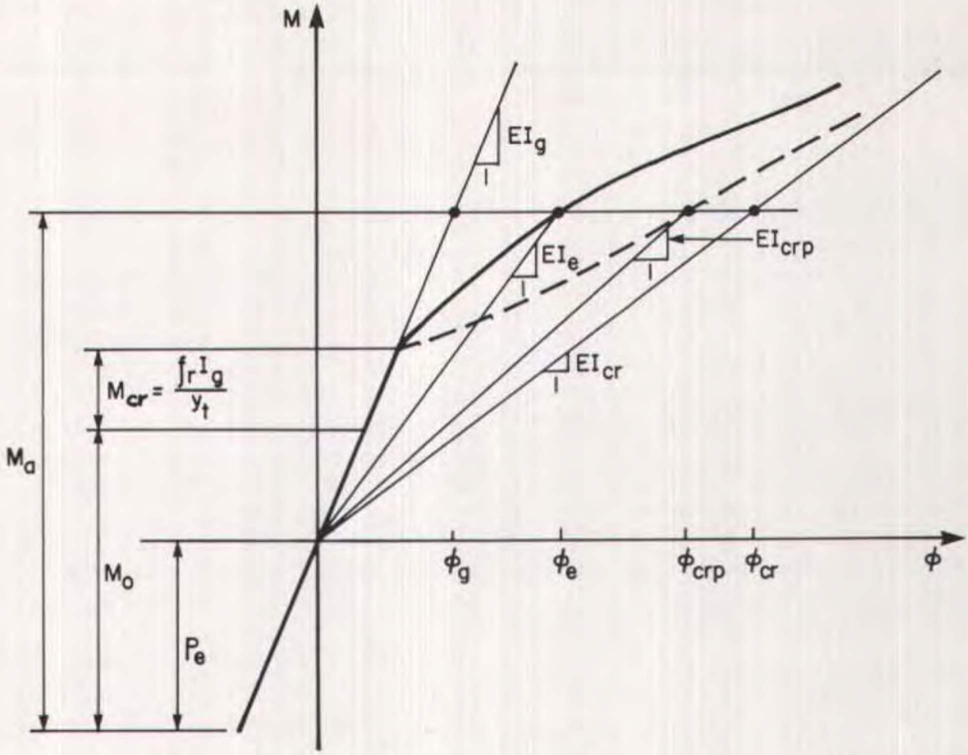

Fig. B. Moment-curvature relationship for typical section.

ment. This is done by replacing M_a by:

$$M_{ap} = M_a - M_o$$

where M_o = decompression moment

$$= Pe + \frac{PI_g}{Ay_t}$$

This is essentially the method used by the authors except they include the effect of prestressing in calculating I_{cr}. It would be preferable to use different notation to differentiate between the two interpretations of a cracked transformed section. We endorse the suggestion by Shaikh[31] that the symbol I_{crp} be used if the effect of prestressing is included and I_{cr} if prestress is ignored. This terminology will be used in the following.

The question remains as to whether using I_{crp} instead of I_{cr} in the I_e equation produces an increase in accuracy. The $M-\phi$ relationship for a typical section is shown in Fig. B. In the figure, M represents the net moment acting on the section as given by applied moment minus Pe, the moment due to prestress. At a particular load level represented by M_a, four separate secant stiffness quantities relating M and ϕ can be identified: EI_g, EI_e, EI_{crp}, and EI_{cr}. Of these, only EI_g and EI_{cr} are independent of moment level. (They are also independent of prestress level.) EI_{crp} equals EI_g at the cracking moment level, and approaches EI_{cr} at higher moment levels. The I_e equation represents a weighting between I_g and I_{cr} (or I_{crp}).

At moment levels slightly above the cracking moment level, I_e is heavily weighted towards I_g while at higher moment levels, the weighting is heavy towards the cracked transformed section when I_{crp} approaches I_{cr}. The result is that

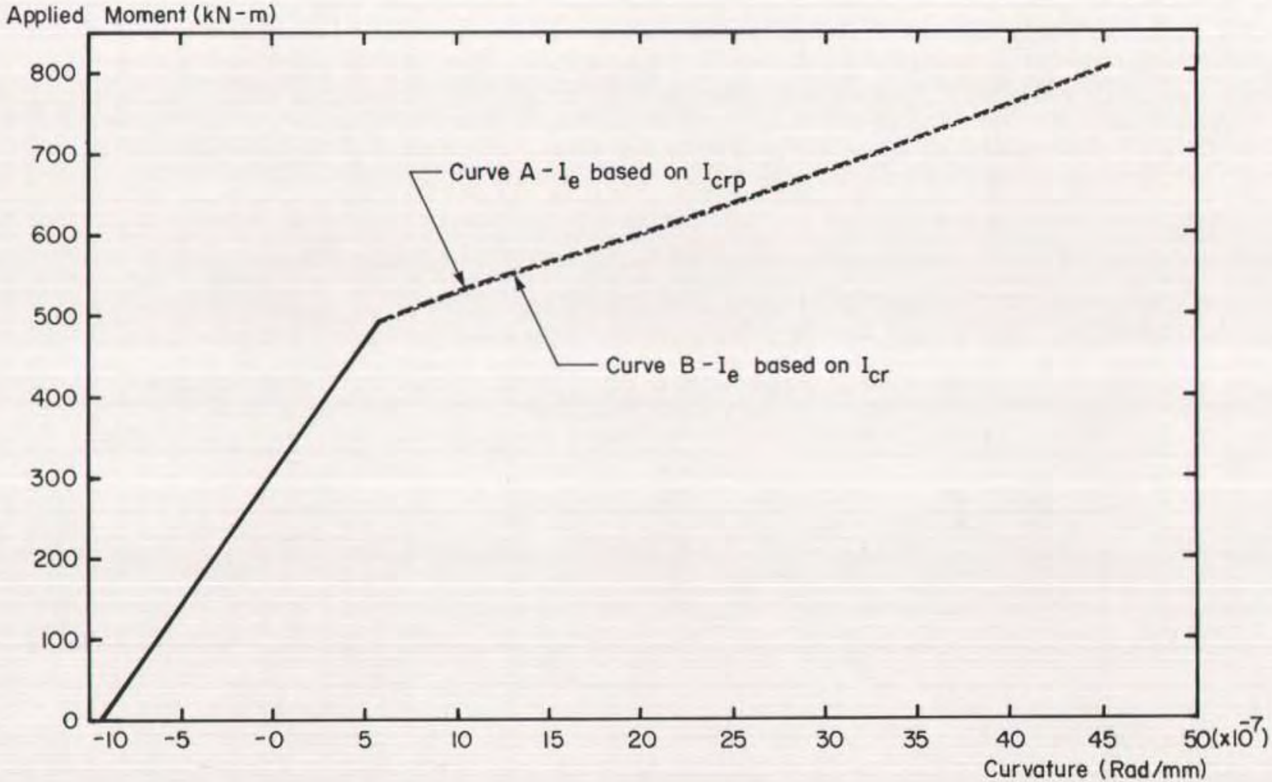

Fig. C. Moment-curvature relationships for cross section of Example 1.

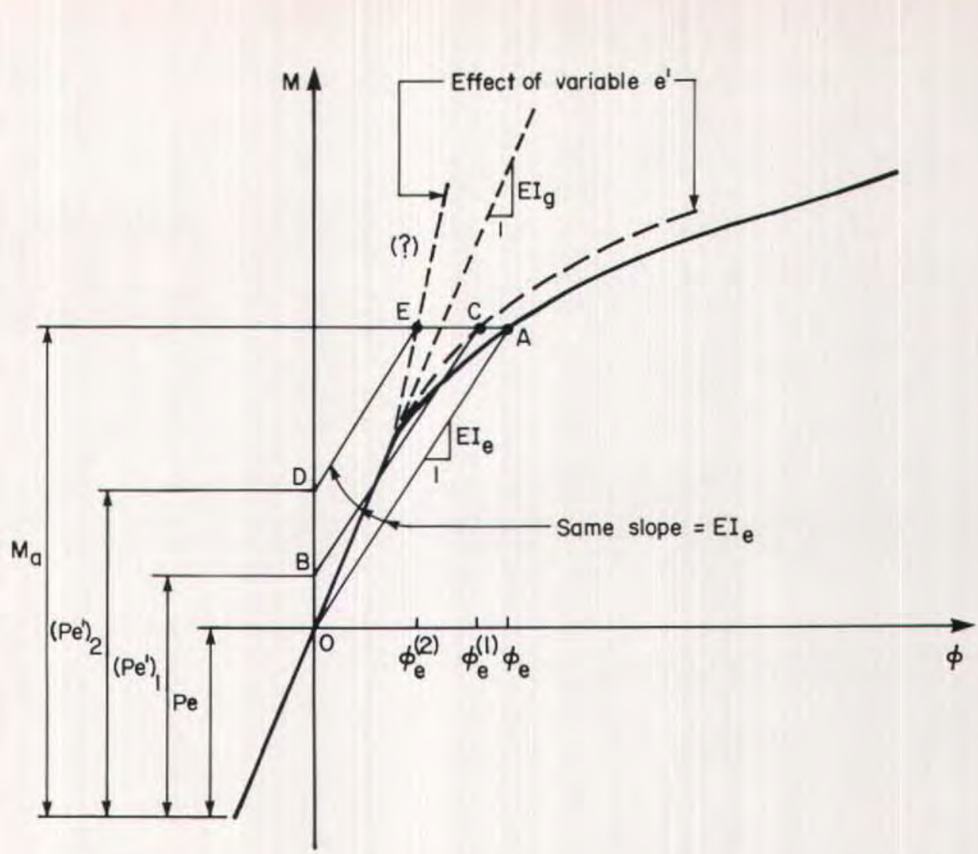

Fig. D. Effect of variable e' on calculated moment-curvature relationship.

I_e calculated by the two methods produces similar results as shown in Fig. C where moment-curvature relationships have been developed for the section given in the authors' Example 1, based on I_e calculated using I_{crp} (Curve A) and also using I_{cr} (Curve B). Similar results have been obtained for a wide variety of cross sections and prestress levels.

The difference between the two curves (A and B) is insignificant from a practical point of view. In fact, a more likely source of inaccuracy in the moment-curvature relationship is in calculating the moment required to cause cracking, due to the high variability in the effective tensile strength of concrete.[32]

In Eq. (3) the authors introduce the term Pe_{cr} in calculating ϕ_{cr} and in Fig. 3 the term Pe' is introduced to calculate ϕ_e. By doing this, the authors have effectively introduced an additional variable into the interpolation formula providing the transition from stiffness of gross section to stiffness of cracked (no tension) section. This seems unnecessary and could possibly lead to a situation where the $M-\phi$ relationship showed stiffening rather than softening after cracking.

Fig. D shows the secant stiffness EI_e passing through the origin of coordinates. Using the authors' approach, the secant stiffness passes through point B when the "effective" prestress moment, including cracking, equals $(Pe')_1$. If the term Pe' becomes large enough, conceivably the situation represented by line *DE* could be reached where the "effective" prestress moment equals $(Pe')_2$ and the $M - \phi$ plot lies to the left of the EI_g stiffness line. Presumably, cracking cannot produce an increase in stiffness even if prestressing is present.

In summary, while we believe the authors' approach to calculating I_e based on I_{crp} is valid, a simpler and equally valid estimate can be obtained using I_{cr} as originally suggested by Branson and Trost,[15] [although Branson and Trost calculate (M_{cr}/M_a) in a somewhat different form]. Also, there appears to be no significant benefit to be obtained from including variable eccentricity in the curvature calculation. In fact, this approach could, in principle, produce an increase rather than a decrease in stiffness after cracking which is obviously not possible.

References

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32. Ramsay, R. J., Mirza, S. A., and MacGregor, J. G., "Monte Carlo Study of Short Time Deflections of Reinforced Concrete Beams," *ACI Journal*, Proceedings V. 76, No. 8, August 1979, pp. 897-918.

CLOSURE BY MAHER K. TADROS,* AMIN GHALI,** and ARTHUR W. MEYER†

The authors appreciate the discussion by Mr. Weiss, Dr. Scanlon et al, and Mr. Nedelcu discussing prestress loss, time-dependent dead load deflection, and instantaneous live load deflection. These points will be considered separately as follows.

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Prestress Loss

Mr. Nedelcu has raised questions concerning computation of prestress loss. The term prestress loss, ΔP_c, is used in the paper to refer to the loss of compression force on the concrete component of the cross section. This is the quantity to be used in computing stresses and deformation of concrete. It is a more important quantity than the loss of tension in prestressed steel. Equilibrium dictates that the loss of compression in concrete is equal to the loss of tension in prestressed steel, only when nonprestressed steel is not present.

Nonprestressed steel undergoes a stress change with time, even though it is not initially stressed. A simple example is offered to Mr. Nedelcu in order to convince him of this fact. Consider a concrete prism reinforced with a steel bar along its axis. As time elapses, shrinkage of concrete develops, forcing the bar to be in compression. An equal and opposite force (i.e., tension) is generated in the concrete in order to maintain equilibrium. The method proposed by the authors rationally accounts not only for the effects of shrinkage but also for creep and relaxation. It fully considers the interaction between the three component materials.

It is unclear to the authors what Mr. Nedelcu means by the "loss due to thermal treatment of concrete." The proposed procedure is valid regardless of the method of concrete curing, provided that representative properties of concrete are used in its application.

Mr. Nedelcu is incorrect in stating that the elastic shortening loss is independent of gravity loads or tendon profile. Basic compatibility and equilibrium conditions must be satisfied for any given time with consideration of all forces applied at that time. Elastic shortening losses have already been covered in detail in the literature (Refs. 3, 4, 5, 9, and 25). See particularly the discussion of Ref. 3 in the March-April 1976 PCI Journal.

Time-Dependent Deflection

The three main features of the authors' proposed procedure, as opposed to other available methods, are that (a) it allows for use of variable material properties, such as modulus of elasticity, creep and shrinkage of concrete; (b) it rationally accounts for the presence of nonprestressed steel; and (c) it accurately considers the effects of cracking under live load. The field measurement study provided by Mr. Weiss is very valuable, yet it covers only feature (a) above. The authors would like to encourage other engineers and prestressed concrete producers to collect and document additional data in order to enable correlation between measured and predicted deflections for the full range of practical applications.

Analysis of the data supplied by Mr. Weiss leads to several observations. First, there is a close agreement between the proposed method and the PCI method for the beams considered. Second, the improved results obtained by the proposed methods, over those obtained by PCI, is primarily due to the use of larger creep and shrinkage coefficients to more accurately represent the dry environmental conditions prevailing in Arizona. The authors expect that more significant differences between the two methods would exist if the beams contained a combination of prestressed and nonprestressed steel.

The third observation is that significant differences exist between the results of the prediction procedures and the measured deflections. It should be noted that deflection of concrete members is a random variable. Existing prediction methods attempt to establish an estimate of the mean, or average, value of that variable. Unfortunately, no guidelines are available to designers at present for estimating the degree of variability. It is important however to have a correct estimate of the average, as the authors have attempted, by using only widely accepted theory and avoiding unnecessary assumptions.

The authors have studied the deflection sensitivity of the beams considered to variations of the contributing parameters. It is possible to observe a 20 percent variation between calculated and measured concrete properties such as modulus of elasticity, creep and shrinkage. Additional variation can result from erroneous application of the formulas used to predict these properties. Examples of such errors are use of wrong concrete age at time of prestress release in calculating the modulus of elasticity, E_{ci}, or use of inaccurate relative humidity in calculating creep and shrinkage. These errors should be avoided in order to keep deflection variation to a minimum.

The results of separately varying the modulus of elasticity and the creep coefficients by ±20 percent are shown in Table B. It can be seen that a 20 percent decrease in modulus of elasticity results in about an equal increase in the corresponding deflection. An increase in E_{ci} of 20 percent, however, results in an average decrease in deflection of about 14 percent. This can be explained by noting that increased E_{ci} results in reduced prestress loss and associated downward deflection. Deflection is not as sensitive to variation in creep as it is to variation in E_{ci}, as shown in the table. Examination of the formulas given in the paper, for example Table 2, will confirm this observation.

It is interesting to note that, in most of the cases considered, the measured deflections are within the bounds obtained by varying E_{ci} and creep by 20 percent.

Live Load Deflection

Mr. Nedelcu and Dr. Scanlon et al have presented discussion on computation of the instantaneous deflection due to live load. There are two specific issues related to this topic: (1) theoretical calculation of deflection, assuming that concrete in the tension zone of a cracked members is incapable of resisting any tension, and (2) empirical consideration of the stiffening effects of uncracked concrete in the ten-

Table B. Parametric Study of Weiss' Beams.

		Beam 1		Beam 2		Beam 3		Beam 4		Beam 5	
		δ	% Change	δ	% Change	δ	% Change	δ	% Change	δ	% Change
	Measured	0.125	—	1.00	—	0.94	—	1.75	—	1.75	—
Computed at release	Base Parameters	0.12	—	0.67	—	1.11	—	1.38	—	1.58	—
	0.80 E_{ci}	0.14	+16.7	0.82	+22.4	1.33	+19.8	1.67	+21.0	1.91	+20.9
	1.20 E_{ci}	0.10	−16.7	0.57	−14.9	0.96	−13.5	1.18	−14.5	1.35	−14.6
Computed at erection	Measured	0.125	—	1.30	—	1.00	—	2.40	—	3.12	—
	Base Parameters	0.19	—	1.19	—	1.70	—	2.50	—	2.94	—
	0.8 E_{ci}	0.23	+21.0	1.42	+19.3	1.94	+14.1	2.94	+17.6	3.45	+17.3
	1.20 E_{ci}	0.17	−10.5	1.03	−13.4	1.51	−11.2	2.17	−13.2	2.56	−12.9
	1.20 (Creep)	0.21	+10.5	1.30	+ 9.2	1.85	+ 8.8	2.74	+ 9.6	3.22	+ 9.5
	0.80 (Creep)	0.17	−10.5	1.08	− 9.2	1.54	− 9.4	2.26	− 9.6	2.65	− 9.9

Table C. Instantaneous total load curvature of the 0.4ℓ section of Example 1.

	Section assumed uncracked			Tension stiffening ignored			Tension stiffening considered		
	I_g*	e_g†	$\phi_g \times 10^5$‡	I_{cr}*	e_{cr}†	$\phi_{cr} \times 10^5$‡	I_e*	e'†	$\phi_c \times 10^5$‡
Accurate values	20985	11.48	2.71	7400	15.15	5.17	7670	15.08	5.04
Approximate values	I_{cr} computed with $P = 0$			7254	15.15	5.28	7529	15.08	5.13
	Percentage error			2	—	2	2	—	2
	I_{cr} computed with $P = 0$, and $e_{cr} = e' = e_g$			7254	11.48	7.83	7529	11.48	7.55
	Percentage error			2	24	52	2	24	50

*Units in in.^{4}
†Units in in.
‡Units in in.$^{-1}$

sion zone, the so called "tension stiffening."

In regard to the theoretical calculation of deflection (ignoring tension stiffening), Dr. Scanlon et al have raised two points related to this issue. The first point is whether or not the axial force effect on the cracked section moment of inertia should be ignored. The second point questions the need for considering the shift in cross section centroid due to cracking.

The proposed method for calculation of stresses, strains and curvatures is based on well established principles of engineering mechanics, namely stresses being linearly proportional to strains, and plane sections remaining plane after deformation. No additional assumptions or approximations were introduced by the authors. The method is consistent with the theory of analysis of cracked prestressed members established by Naaman,[7] Tadros,[10] Nilson,[11] Inomata,[12] and others.

It is commonly agreed that the properties of a cross section change with cracking. These properties include both the moment of inertia and the centroid location. These cracked section properties, are further known to be related to the applied forces. To study the influence of varying these two properties on the cross section deformation, refer to Fig. 3 of the paper, and Table C. The authors agree with the discussers that, for the 0.4 ℓ section of Example 1, ignoring the axial force in determining I_{cr}, results in a practically negligible error of 2 percent. However, this approximation may produce considerable error in other cases, especially if the reinforcement content is relatively low, or if the eccentricity of the applied compressive prestress force is close to the cracking eccentricity.

Referring to Fig. 2, for example, shows that if $n\rho$ is about 0.01, the value of k_{pr} can be as high as 1.2. Ignoring P corresponds to $k_{pr}=1$, i.e., 20 percent error. Fig. E gives another example of the very significant error that can result from ignoring the axial force in determining cracked section properties. (A detailed discussion of Fig. E is given later.) Furthermore, the authors do not believe that there is a significant advantage, if any, gained from this approximation, for the following reasons: (a) Other properties, such as the centroidal depth, are more sensitive to the loading level and, therefore, it would be less con-

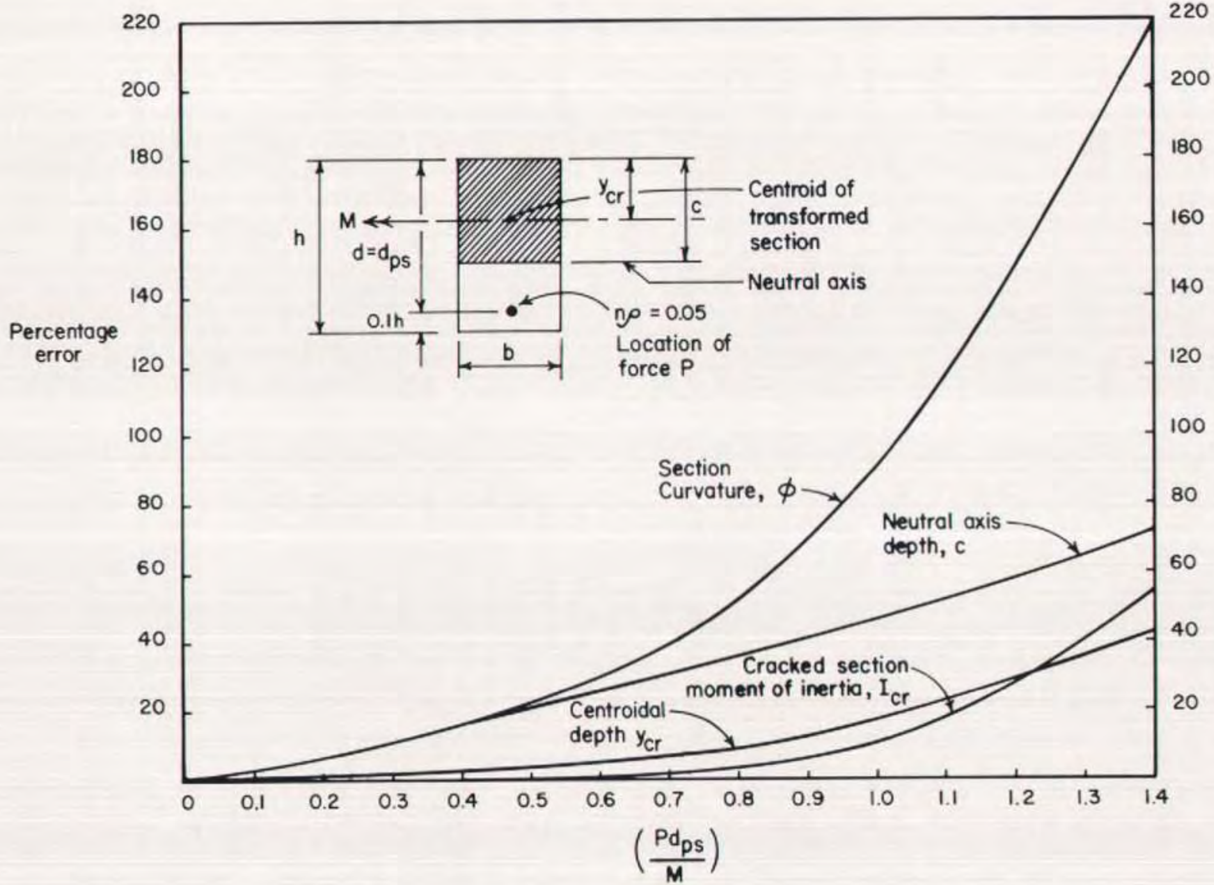

Fig. E. Error resulting from using Scanlon et al recommended approximations.

fusing to consistently consider the axial force in determining all properties. (b) Design aids, such as the graphs in Figs. 1 and 2, and microcomputer programs, are already available for calculating section properties. These aids are much quicker than classic, long-hand calculations. In fact, it would require an additional step in a computer program to set $P = 0$ in the routine used in computing I_{cr}.

The second point raised in the discussion was the need for considering the change in the prestress force eccentricity upon cracking. It should be emphasized here that theoretical analysis of a given cracked section for a given set of applied forces yields a unique solution for the section centroid. Thus, the prestress force eccentricity cannot be considered "variable" as Dr. Scanlon et al and Fig. D indicate. That loading level, when used with the correct section properties, gives the correct curvature value in the moment-curvature diagram. Table C, shows that neglecting the shift in cross section centroid can result in a 52 percent error in section curvature of the beam in Example 1. Note that the shift in centroid upon cracking is especially significant in lightly reinforced flanged sections which are common in practice, yet hardly used in experimental research. Ref. 16 provides further illustration of the significance of this error in deflection calculation. Also note that the two approximations combined, i.e., ignoring P in computing I_{cr} and ignoring the shift in centroid due to cracking, aggrevate the error.

Another example of the significance of the approximations proposed by Scanlon et al is presented in Fig. E. A rectangular section with $n\rho = 0.05$ and with varying degrees of cracking, represented by Pd_{ps}/M, is studied, using the equations derived in Ref. 10. As shown, the error in section properties, if $P = 0$, can be very significant. Approximating I_{cr}, and the corresponding curvature, can result in an error as high as 55 percent, for $Pd_{ps}/M = 1.40$. Compounded error resulting from using approximate I_{cr} and from using gross concrete section centroid rather than cracked section centroid, can amount to 220 percent. Obviously, these errors cannot be tolerated. Please note the correlation between Fig. E and the top portions of Figs. 2 and 3 of the paper.

Considering the second issue, tension stiffening, since concrete has some capacity in tension, the actual curvature of a cracked section, ϕ, is less than the theoretical value ϕ_{cr}. The use of "effective" cross section properties is one way of empirically accounting for tension stiffening. Two other approaches are also discussed in the paper. Regardless of the approach adopted by the designer, the following facts should be recognized: (1) Tension stiffening is not founded on theoretical development. As more test results become available, more refinement of this empirical proposal is likely to occur. (2) Curvatures resulting from tension stiffening should lie between the theoretically correct values ϕ_g and ϕ_{cr}, calculated for uncracked and for cracked sections. (3) Consideration of tension stiffening may be negligible in some applications. For example, as pointed out by Mr. Nedelcu, the tension stiffening effect on the 0.4 ℓ section of Example 1 is small (Table C). (4) It is conservative to ignore tension stiffening, since its consideration reduces the theoretical cracked member deflection.

Editor's Note

A computer program written for the IBM PC microcomputer is available from the PCI at a nominal charge. The program provides computation of the deflection as proposed by the method presented in this paper, as well as by more detailed calculations.

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READER COMMENTS

Design of Reinforced and Prestressed Concrete Inverted T Beams for Bridge Structures*

by S. A. Mirza and R. W. Furlong

Comments by Basile G. Rabbat and Authors

BASILE G. RABBAT†

Professors Mirza and Furlong are to be commended for their timely paper clearly describing potential local and overall failure modes of inverted T beams for bridge structures. The following comments are intended to complement the paper and point out additional design and detailing considerations that deserve attention.

Transverse Horizontal Tensile Force

As indicated on page 122 of the paper, transverse flange reinforcement should be provided to resist a transverse horizontal tensile force at the bearing pad locations. This force is due to indeterminate causes such as restrained creep and shrinkage, seasonal and diurnal temperature effects, and acceleration and braking forces. Consideration of this force is specified for the design of brackets and corbels in Section 11.9.3.4 of ACI 318-83,[2] and in the newly adopted Section 8.16.6.8.3(d) of the 1984 Interim AASHTO Specifications.[11] The same force should be considered in the case of inverted T beams.

The horizontal tensile force is denoted N_{uc}. Refs. 2 and 11 specify that tensile force N_{uc} should not be taken less than $0.2\,P_u$ unless special provisions are made to avoid tensile forces at the bearing pads (P_u is the factored stringer reaction). Reinforcement A_n to resist tensile force N_{uc} should satisfy the following:

$$N_{uc} \leq \phi A_n f_y$$

or

$$A_n \geq \frac{N_{uc}}{\phi f_y}$$

where f_y = specified yield strength of reinforcement; $\phi = 0.85$.

The strength reduction factor ϕ is taken equal to 0.85 for consistency with the provision for brackets and corbels. This is specified in the 1984 AASHTO Interim Specifications,[11] Section 8.16.6.8.3(a), and in the ACI 318-83,[2] Section 11.9.3.1. The justification for this criterion is given in the Commentary to ACI 318-83.[9] It states:

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"Corbel and bracket behavior is predominantly controlled by shear; therefore, a single value of $\phi = 0.85$ is specified for all design conditions." The same justification applies to local failure modes in inverted T beam ledges.

The design example of Appendix B of the paper (referred to below as design example) did not account for tensile force N_{uc}. Area of reinforcement to resist this force at interior and exterior stringers is 0.9 and 0.6 in.^{2}, respectively. The area of this reinforcement is equal to 28 percent of the area of flange transverse flexural reinforcement A_{sf}, computed in Appendix B. Note that reinforcement A_n should be added to flexural reinforcement A_{sf}. Tensile force N_{uc} also causes transverse flange flexure as discussed below.

Flange Transverse Flexure

To compute the flange transverse flexure, the authors suggest that the shear span "a" be taken from the center of the bearing pad to the surface of the web. The writer believes that this distance should be increased to account for two factors.

The first factor concerns the critical section for moment. Flexural cracks from flange bending start at the web-flange junction and extend diagonally to the hanger reinforcement. They then vertically follow the hanger reinforcement. It has been suggested that the critical section for flexure be taken at the center of web stirrups, and not at the web face.[12] This is depicted in Fig. A.

The second factor would account for the effect of superstructure creep, shrinkage, and temperature movements. These effects result in longitudinal and rotational movements of the stringer ends. The longitudinal superstructure movement, depicted δ in Fig. A, can be easily calculated. This movement would dictate the critical location for the stringer vertical reaction.

For a 70 ft span restrained at one end, a coefficient of thermal expansion of 6 x 10^{-6} in./in./°F, and a 40°F drop in temperature, the thermal movement is 0.2 in. For the same span condition and a combined creep and shrinkage coefficient of 0.0006 in./in., the corresponding shortening is 0.5 in. Therefore, the total longitudinal stringer movement δ is 0.7 in. in this example.

A third factor that will increase the flange transverse flexure is the eccentricity of the horizontal tensile force N_{uc} discussed above. This force produces a bending moment equal to $N_{uc} (h_f - d_f)$, where h_f = overall flange depth; d_f = effective flange depth.

If the above factors are considered for the flange flexure of the design example, the flange transverse flexural moment would increase by 36 percent over the value computed in the design example. As discussed earlier, a strength reduction factor ϕ equal to 0.85 should be used to compute the flange flexural reinforcement, A_{sf}, instead of $\phi = 0.90$ used in the paper.

The authors recommend that the flange transverse flexural reinforcement be (1) calculated based on a lever arm equal to 0.8 d_f, and (2) distributed over an effective flange length $(B + 5a)$. Could the authors clarify the following:

1. Is derivation of the effective flange length $(B + 5a)$ directly related to the lever arm 0.8 d_f?

2. Can the reinforcement needed to resist the flange transverse flexure be calculated based on conventional flexural theory, as specified for brackets and corbels?[2,11] In this case the lever arm would be closer to 0.9 d_f.

3. If the larger shear span "a_f" discussed above is used to compute the flange transverse flexural moment and corresponding reinforcement, A_{sf}, should the effective flange length be modified accordingly?

4. What should the effective flange length be to distribute reinforcement A_n?

Serviceability

The authors emphasize the importance of the serviceability consideration when designing the hanger reinforcement. The

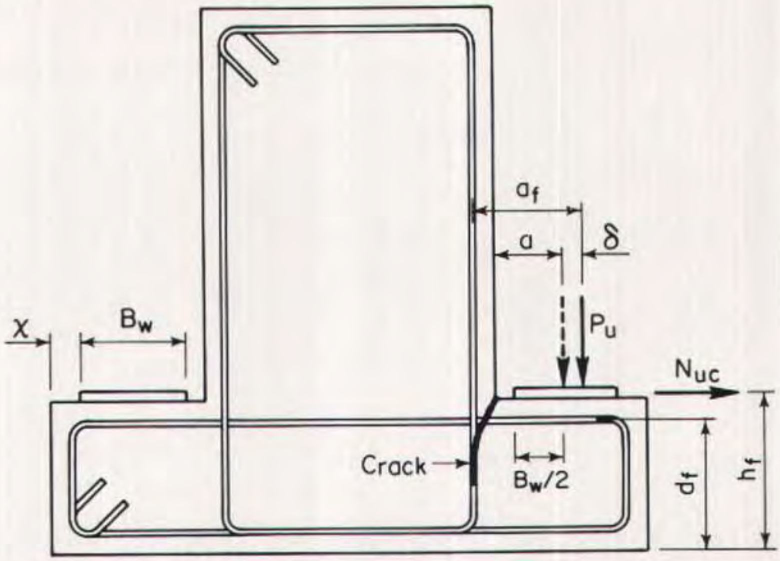

Fig. A. Parameters for flange transverse flexure.

tests referred to in the paper indicate that cracks occurred at the web-flange junction under service loads. These tests are described in detail in Ref. 7. On page 41 of Ref. 7, the authors report that "some yielding of hangers was observed under nominal service loads." Further, "as a serviceability control, it seems desirable to limit the hanger stress to values less than $\frac{2}{3}f_y$." What was the basis for limiting the hanger service stress at $\frac{2}{3}f_y$? This limit seems too high.

A limiting value may be computed from the expression:

$$f_s \leq \frac{D + L}{1.3\,[D + 5/3\,L]/\phi} \cdot f_y$$

where f_s = allowable hanger stress under service loads; f_y = specified yield strength of reinforcement; D = service dead load; L = service live load including impact; ϕ = 0.85 (as discussed earlier).

In the design example, for an interior stringer, $D = 70$ kips and $L = 60$ kips. The allowable hanger stress should be limited to $0.50\,f_y$.

The authors recommend resisting the stringer service reaction by hanger reinforcement distributed over an effective flange length $(B + 3a)$. This effective length is derived from test loads at measured hanger yielding.[7] Based on his experimental experience, the writer believes that yielding of reinforcement often occurs before it is detected by electrical resistance strain gages. Onset of reinforcement yielding will only be detected if the cracks occur at the exact location of the strain gages. Is some conservatism in computing the effective flange length to distribute the hanger reinforcement at service loads warranted? Most important, if cracking can occur at the web-flange junction under service loads, shouldn't the hanger reinforcement be checked for fatigue?

For the design example, a comparison of the hanger stress range with the allowable stress range given in Section 8.16.8.3 of Ref. 13 indicates that the hanger reinforcement is inadequate to resist fatigue. The interior stringer dead load reaction is

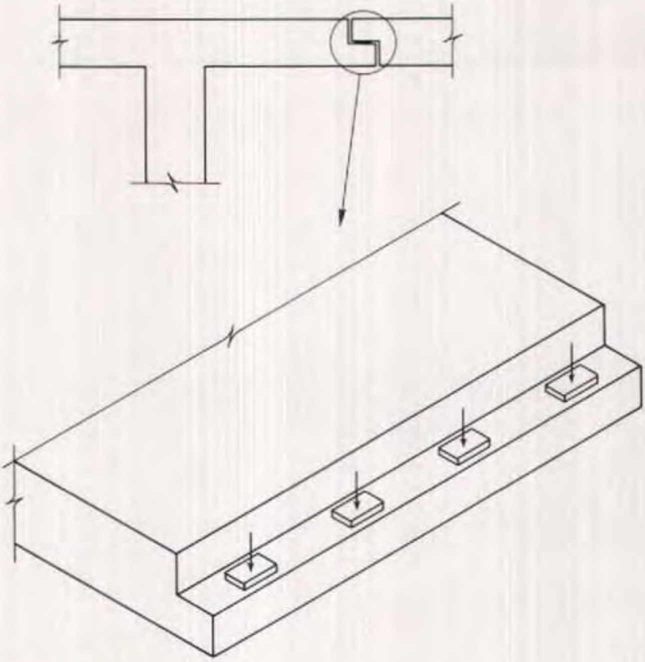

Fig. B. Hinge ledge.

70 kips and the live load plus impact reaction 60 kips. Therefore, total service load is 130 kips. If the limiting hanger stress is $\frac{2}{3} f_y$, the minimum hanger stress f_{min} is $\frac{2}{3}$ (60) (70/130) = 21.5 ksi.

The stress range, f_r, due to live load and impact is $\frac{2}{3}$(60) (60/130) = 18.5 ksi.

The allowable stress range, f_f, which is computed from Eq. (8-60) of Ref. 13, is as follows:

$$f_f = 21 - 0.33 f_{min} + 8\,(0.3)$$
$$= 21 - 21.5/3 + 2.4 = 16.23 < 18.5 \text{ ksi}$$

Therefore, the fatigue provisions of the AASHTO Specifications are violated. This further confirms that the limiting hanger stress at service loads should be less than recommended in the paper.

Web Reinforcement

The authors state on page 127 of the paper that "the stirrups designed as hangers can then be used as part of the web reinforcement resisting flexural shear." The justification given in the paper is that "hanger failure cracks occur at the junction of the web and the flange, whereas the flexural shear failure cracks occur in the web above the flange. Consequently, the yielding of stirrups acting as hanger reinforcement and as shear reinforcement takes place at different locations in the stirrups." What happens if many parallel and well distributed diagonal cracks intersect the web-flange junction? Would hanger and shear stresses become additive in the web reinforcement? In independent studies, Professors Collins and Mitchell,[14] and Ramirez and Breen[15] have recommended that hanger reinforcement be added to the shear and torsional web reinforcement.

Shear Strength

The shear strength provided by concrete as given by Equation (11-3) of ACI 318-83[2] is $V_c = 2\sqrt{f'_c}\, b_w d$. To compute the flexural shear strength of inverted T beams, the authors have modified Eq. (11-3). They propose using area A_e, shown in Fig. 10 of the paper, instead of $b_w d$. This results in an increase of the concrete contribution to shear resistance. This approach violates the ACI Code[2] and the AASHTO Specifications.[13] Eqs. 9 and 10 of the paper are based on a limited number of tests. Considering the importance of bent caps for the structural integrity of a bridge, such liberalization of the shear resistance of concrete is not warranted.

Torsional Strength

In Eq. 12 of the paper, the authors limit the torsional strength using a nominal concrete stress of $18\sqrt{f'_c}$. This high torsional shear stress violates the ACI Code.[2] Based on Sections 11.6.6.1 and 11.6.9.4 of ACI 318-83, the torsional strength should be limited to a nominal concrete stress of $12\sqrt{f'_c}$.

Detailing

A detailing provision specified for brackets and corbels should also be considered for inverted T beam ledges. To avoid localized bearing or shear failure under the bearing pad, the code requires that "Bearing area of load on bracket or corbel shall not project beyond straight portion of primary tension bars A_s, nor project beyond interior face of transverse anchor bar (if one is provided)". (Section 8.15.5.8.7 of Ref. 11 and Section 11.9.7 of Ref. 2). As a result of this provision, the width of each flange should be increased in the design example. The distance to be increased is denoted "x" in Fig. A. Distance "x" accounts for the concrete cover, and size and diameter of flange transverse reinforcement. For the given example, $x = 2 + 5/8 + \frac{2.5}{2} = 3.88$ in., $x =$ 4 in. should be used instead of the provided 1 in. Accordingly, the given flange width of 70 in. should be increased to 76 in. If the reinforcement detail of Fig. 8 of the paper is used, this dimension may be decreased slightly.

Related Application

As indicated in the paper, current codes[2,3] and specifications[11,13] do not include provisions for design of inverted T beams subjected to intermittent concentrated forces applied to the flange. Another structural element where a gap exists in the code is the design of ledges used in hinges, as depicted in Fig. B. Do the authors feel that the design provisions recommended for inverted T beams can be extended to the design of ledges as shown in Fig. B? In particular, is the effective flange length specified in the paper applicable also to hinge ledges when the following effects are considered:

1. Bracket-type friction in the flange
2. Transverse flange flexure
3. Hanger reinforcement

The authors' insight into this problem would be appreciated.

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11. "Interim Specifications, Bridges", American Association of State Highway and Transportation Officials, Washington, D.C., 1984.
12. "Notes on ACI 318-83, Building Code Requirements for Reinforced Concrete with Design Applications," edited by Gerald B. Neville, Portland Cement Association, Fourth Edition, 1984, page 16-2.
13. "Standard Specifications for Highway Bridges", 13th Edition, American Association of State Highway and Transportation Officials, Washington, D.C., 1983, 394 pp.
14. Collins, M. P., and Mitchell, D., "Shear and Torsion Design of Prestressed and

*Note: References 1, 2, 3, 7, and 9 appear in the references section of the original article (PCI JOURNAL, V. 30, No. 4, July-August 1985, p. 132).

Non-Prestressed Concrete Beams," PCI JOURNAL, V. 25, No. 5, September-October 1980, pp. 32-100.

15. Ramirez, J. A., and Breen, J. E., "Proposed Design Procedures for Shear and Torsion in Reinforced and Prestressed Concrete," Research Report No. 248-4F, Center for Transportation Research, The University of Texas at Austin, Austin, Texas, November 1983, 270 pp.

AUTHORS' CLOSURE by S. A. MIRZA* and R. W. FURLONG†

The authors are indebted to Dr. Rabbat for the relevance and importance with which his comments succeed as a complement to the paper. The horizontal tension force directed away from the face of the T beam stem should have been acknowledged by the use of such a force in the design example. An area of reinforcement adequate to sustain the horizontal force must be added to the flexural tension area as indicated by Dr. Rabbat. Several initial studies of bracket behavior (Ref. 8) included the application of horizontal tension force as the shelf load was applied. The empirical derivation of the width within which bars were effective took into account the fact that the distance from the center of flexural compression to the tension force N was greater than the distance from the center of flexural compression to the tension reinforcement.

The empirical source of the design recommendations must be cited also for the definition of the parameter "a". Dr. Rabbat is correct in his observation that the actual effective distance a_f should be larger than "a" as in Fig. A. The authors sought to avoid confusion in the definition "a" as it was specified. That definition served as the parameter against which observed responses were normalized in the interpretation of the strength measurements. The zone ($B + 5\,a$) within which flexural steel could be considered effective included some shelf flexural bars that were not yielded; the contribution or participation of bars outside that zone was observed, but excluded for design. The width to be used for design could have been expressed in terms of some larger 'effective' cantilever arm as suggested (from the hanger rod to a bearing zone outside the centroid of the bearing plate), but such a definition was not used for data reduction.

The effective width for flexural response is related also to the recommended use of 0.8 d_f as the internal lever arm for the bracket. Instead of seeking a refined estimate for d_f dependent upon an effective width of the compression zone, it was deemed more practical and less ambiguous simply to use a conservative estimate of d_f. The bracket design procedures of Ref. 2 employed the same estimate of d_f.

The serviceability criterion that was suggested was derived only to limit the size of cracks in the concrete under service load and fatigue of the reinforcement was not considered. The actual stress in any point of a hanger bar or in a shelf flexural bar is dependent on the location of cracks. The distribution of forces among bars is influenced by cracking as well, and the nominal stress range from dead load to maximum service load may be greater than the actual change in stress. Nevertheless, it is helpful to be aware that fatigue might be significant. In lieu of estimating the hanger bar stress as suggested by Dr. Rabbat, service load cracking and stresses could be reduced simply by an increase in the numerator 3 P_s in Eq. (8b). For example, if the limit service stress were ½ f_y instead of 2/3 f_y, the numerator would become 4 P_s.

The test observations which state that prior to failure, hanger forces need not be superimposed on flexural shear and torsional forces were related again to the empirical derivation for the zone in which hangers can be considered effective at concentrated loads. Reinforced concrete, heterogeneous and cracked before fail-

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ure, does not respond as an elastic continuum. For strength considerations, even if concentrated loads are located at a spacing equal to or less than the effective hanger distance ($B + 2\,d_f$), it is unlikely that the hanger crack and flexural or torsional cracks would occur simultaneously at the back of the shelf where hanger forces are greatest. However, if stringer spacing is less than ($B + 2\,d_f$), recommendations of Refs. 14 and 15 may be appropriate. If stringer spacing exceeds the effective hanger distance, the extension of cracking brings into action additional stirrups. For strength design, flexural and torsional forces did not appear to be additive to hanger forces within the space ($B + 2\,d_f$) before failure.

The use of the effective shear area A_e again reflected the observed response of test beams. Whether or not liberalization of the formulation of shear resistance is warranted, it was reported as it was observed. The transverse reinforcement around the periphery of the flange provided helpful resistance to torsion, but that reinforcement was not included in torsional strength analysis. Perhaps a more sophisticated estimate of V_c including the effect of shear span and longitudinal reinforcement would have indicated that the entire area A_e was not effective. Applying the less sophisticated limit stress level of $2\sqrt{f'_c}$ to the entire area A_e correlated with test results better than did the use of stem area $b_w\,d$.

Sketches of the beam cross sections should have shown chamfered edges along the shelf. Dr. Rabbat is correct in directing attention to that shelf detail. The bearing pad should not project beyond the exterior anchor bar for flexural steel in the shelf.

Finally, the authors considered the flange of the inverted T beam as a ledge, and all of the recommendations of the paper would apply to the slab ledge indicated in Fig. B. The "web" for the slab ledge simply has an infinite thickness, but flange shear and flexure as well as hanger mechanisms should form in the same way as those observed from the research specimens with webs of finite thickness.

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