The preliminary design of prestressed concrete beams is generally based on working stress limitations. In order to simplify the task of the designer, many time-saving techniques for optimizing beam cross section, prestressing force, and tendon eccentricity have been introduced.

One of the first and most popular design aids is the Magnel diagram, shown in Fig. 1a, which was introduced in 1948.1 The Magnel diagram defines, for a known cross section, the acceptable combinations of eccentricity and prestressing force at a particular location along the span. Although limited in scope, it has been sufficiently useful in its application to warrant continued use and revision. Current textbooks on prestressed concrete design continue to refer to the Magnel diagram as a useful design tool.2,3 The primary limitation of the original Magnel diagram is that the designer must generate a separate diagram for each critical point along the span.

Krishnamurthy has proposed a modification of the Magnel diagram,6,7 as shown in Fig. 1b. This modification graphically represents the safe zone of eccentricity and prestressing force as it relates to the actual position on the cross section. In particular, the method cited in Ref. 7 can be used to obtain non-dimensionalized design charts. The charts apply to a large number of cross sections subjected to arbitrary loading and are easily generated by hand. However, in order to arrive at the tendon profile, the user determines the allowable eccentricity at a number of points along the span. These eccentricities can then be plotted to give the acceptable zone for the entire length of the beam.

The charts presented herein graphi-
cally represent the zone of acceptable eccentricity for the entire span. In addition, a design procedure is proposed which enables the user to select the beam cross section and the tendon profile. The method is applicable to prismatic, symmetrical or unsymmetrical cross sections. Only uniformly loaded, simply supported beams are considered, and the effect of the presence of mild reinforcing steel is ignored.

The current ACI Code⁸ and AASHTO Specifications⁹ stipulate stress limits which form the primary basis for preliminary member selection. These working stress designs often lead to members which satisfy ultimate strength and serviceability requirements. The move toward ultimate strength design⁸,¹⁰ combined with computer-aided optimization techniques has resulted in fewer advancements in working stress design methods. However, the need for a continuation in the refinement of working stress methods should not be overlooked. The recent work by Oribson is a step in this direction.¹¹

**OBJECTIVES**

As mentioned above, the limitations of the Magnel diagram and its modifications are as follows:

1. Each diagram can be used only for known cross-sectional properties.
2. Each use of the diagram is limited to a specific moment; therefore, new moment calculations are required at each critical location along the span.

Given these limitations, Magnel’s definition of the safe zone cannot directly represent the acceptable range of eccentricity for the entire length of the beam in a single application.

To overcome these limitations, a method is proposed for developing design charts which extend the idea of Magnel’s safe zone into a relationship for the entire length of a simply supported beam. The primary objectives in developing these design charts can be summarized as follows:

(a) The charts will not be limited to a specific cross section.
(b) The charts may be used to define the zone of acceptable eccentricity along the entire span.
(c) Generalized charts may be developed in advance, with each chart being applicable to a large number of possible design situations.

In order to accomplish these tasks, the following development is restricted to simply supported, uniformly loaded beams. The assumption of uniform loading is required since the ratio of the self weight of the beam to uniform dead plus live load is used to generalize the basic design equations. The reason for

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**Synopsis**

A proposed design procedure and accompanying design charts for the working stress design of uniformly loaded, simply supported, prestressed concrete beams is presented. The proposed method is formulated through an extension of Magnel's safe zone into a relationship for the entire length of the beam. A brief review of the Magnel diagram is followed by a detailed mathematical development of the proposed design curves.

The curves define the safe zone of eccentricity of the prestressing steel as a function of the horizontal position along the span. A number of different curves, each representing combinations of span length, beam height, and applied loads, are included in each design chart. The design parameters applicable to a given chart include the concrete strength, the prestress loss ratio, and the proportions of the cross-sectional dimensions. Examples are included to demonstrate the use of the proposed design method.
assuming simple supports is due to the necessity of establishing a moment relationship for the span (i.e., moment, $M$, as a function of the distance $x$ from the support). It is, therefore, noted that different support conditions could be accounted for if the appropriate moment relationships were substituted in the following equations.

**BASIC DESIGN EQUATIONS**

The development of the proposed method is based on the four stress constraints which limit the allowable stresses in the top and bottom fibers of a given section during initial and service load conditions. A possible form of these

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Fig. 1. (a) Original Magnel diagram; (b) Krishnamurthy’s modification.
requirements which defines the acceptable eccentricity of the prestressing tendons as a function of the position $x$ along the span may be written as follows:

$$e(x) \leq \frac{f_{ci}S_b}{P_i} - \frac{S_b}{A} + \frac{M_i(x)}{P_i}$$  \hspace{1cm} (1a)$$

$$e(x) \leq - \frac{f_{ci}S_t}{P_i} + \frac{S_t}{A} + \frac{M_t(x)}{P_i}$$  \hspace{1cm} (1b)$$

$$e(x) \succeq \frac{f_{ce}S_t}{R P_i} + \frac{S_t}{A} + \frac{M_s(x)}{R P_i}$$  \hspace{1cm} (1c)$$

$$e(x) \succeq \frac{f_{ce}S_b}{R P_i} - \frac{S_b}{A} + \frac{M_s(x)}{R P_i}$$  \hspace{1cm} (1d)$$

In the above equations, the subscripts $i$ and $s$ refer to initial and service load conditions; subscripts $t$ and $b$ refer to top and bottom fibers; and $R$ is the effectiveness or prestress loss ratio. Note that compressive stresses are taken as positive and tensile stresses as negative. Additionally, the eccentricity, $e$, is taken as positive when below the centroidal axis.

In order to generalize the above inequalities so that they are not limited to a particular cross section, the following definitions are introduced. Given the proportions of any section shown in Fig. 2 (i.e., $b/b_w, h/h_r, b_2/b$), it is easily shown that the properties of the cross section can be expressed in the following form:

$$c = c'h$$  \hspace{1cm} (2a)$$

$$A = A'bh$$  \hspace{1cm} (2b)$$

$$S = S'h^2$$  \hspace{1cm} (2c)$$

$$I = I'h^3$$  \hspace{1cm} (2d)$$

where $c', A', S'$, and $I'$ represent coefficients for a specifically proportioned section.

The initial prestressing force, $P_i$, as defined by Nilson, may be taken as:

$$P_i = A f_{cei}$$  \hspace{1cm} (3)$$

where

$$f_{cei} = f_{ci} - \frac{c_b}{h} (f_{ci} - f_{ti})$$  \hspace{1cm} (4)$$

Combining Eqs. (2a) and (4) gives:

$$f_{cei} = f_{ci} - c_b (f_{ci} - f_{ti})$$  \hspace{1cm} (4a)$$

It is noted that $f_{cei}$ is only a function of the section proportions and concrete strength. Combining Eqs. (1a), (2), and (3) gives:

$$e(x) \leq \frac{f_{ci}S_b (b h^2)}{A' (b h) f_{cei}} - \frac{S_b (b h^2)}{A' (b h)} + \frac{M_i(x)}{A' (b h) f_{cei}}$$  \hspace{1cm} (5)$$

which reduces to:

$$e(x) \leq \frac{S_b h}{A'} \left(1 - \frac{f_{ci}}{f_{cei}} - 1\right) + \frac{M_i(x)}{A' (b h) f_{cei}}$$  \hspace{1cm} (6a)$$

Similarly, Eqs. (1b) to (1d) become:

$$e(x) \succeq \frac{S_t h}{A'} \left(1 - \frac{f_{ce}}{R f_{cei}}\right) + \frac{M_s(x)}{R A' (b h) f_{cei}}$$  \hspace{1cm} (6b)$$

$$e(x) \succeq \frac{S_t h}{A'} \left(1 - \frac{f_{ce}}{R f_{cei}}\right) + \frac{M_s(x)}{R A' (b h) f_{cei}}$$  \hspace{1cm} (6c)$$

$$e(x) \succeq \frac{S_b h}{A'} \left(\frac{f_{ce}}{R f_{cei}} - 1\right) + \frac{M_s(x)}{R A' (b h) f_{cei}}$$  \hspace{1cm} (6d)$$

The beam weight per unit length, $w_i$, may be written as:

$$w_i = A' (b h) w_c$$  \hspace{1cm} (7)$$

where $w_c$ equals the unit weight of concrete.

The load ratio, $W_R$, is defined as:

$$W_R = \frac{w_i}{(w_s + w_i)}$$  \hspace{1cm} (8)$$

in which $w_s$ is the sum of the uniform dead and live loads, $w_{DL}$ and $w_{LL}$, re-
respectively, and are usually expressed in units of pounds per foot.

Using the definitions from Eqs. (7) and (8), the moments as a function of \( x \) for a simply supported beam at initial and service conditions are:

\[
M_r(x) = \frac{A'(bh)w_c(Lx-x^2)}{2} \tag{9}
\]

\[
M_s(x) = \frac{A'(bh)w_c(Lx-x^2)}{2W_R} \tag{10}
\]

Substituting Eqs. (9) and (10) into Eqs. (6a) to (6d), the four basic design equations may be written as:

\[
\epsilon(x) \leq \frac{S_b'h}{A'} \left( \frac{f_{ci}}{f_{cci}} - 1 \right) + \frac{w_c(Lx-x^2)}{2f_{cci}} \tag{11a}
\]

\[
\epsilon(x) \leq \frac{S_i'h}{A'} \left( 1 - \frac{f_{ci}}{f_{cci}} \right) + \frac{w_c(Lx-x^2)}{2f_{cci}} \tag{11b}
\]
\[ e(x) = \frac{S'_i h}{A'} \left(1 - \frac{f_{cs}}{R f_{c_e}}\right) + \frac{w_c (L x - x^2)}{2 W_R R f_{c_e}} \]  
(11c)

\[ e(x) = \frac{S'_b h}{A'} \left(\frac{f_{ts}}{R f_{c_e}} - 1\right) + \frac{w_c (L x - x^2)}{2 W_R R f_{c_e}} \]  
(11d)

### DERIVATION OF A DESIGN COEFFICIENT

Dividing Eqs. (11a) to (11d) by \( c_b = c_b' h \), then multiplying by \( L^2 L^2 \), the following equations (now expressed as equalities in \( e_{\text{max}} \) and \( e_{\text{min}} \)) are obtained:

\[ \frac{e_{\text{max}}}{c_b} \left(\frac{x}{L}\right) = \frac{S'_i}{A' c_b'} \left(\frac{f_{cs}}{f_{c_e}} - 1\right) + \frac{w_c}{2 f_{c_e} c_b} \left[\frac{x}{L} - \left(\frac{x}{L}\right)^2\right] \left(\frac{L^2}{h}\right) \]  
(12a)

\[ \frac{e_{\text{max}}}{c_b} \left(\frac{x}{L}\right) = \frac{S'_i}{A' c_b'} \left(1 - \frac{f_{ts}}{f_{c_e}}\right) + \frac{w_c}{2 f_{c_e} c_b'} \left[\frac{x}{L} - \left(\frac{x}{L}\right)^2\right] \left(\frac{L^2}{h}\right) \]  
(12b)

\[ \frac{e_{\text{min}}}{c_b} \left(\frac{x}{L}\right) = \frac{S'_i}{A' c_b'} \left(1 - \frac{f_{cs}}{R f_{c_e}}\right) + \frac{w_c}{2 R f_{c_e} c_b'} \left[\frac{x}{L} - \left(\frac{x}{L}\right)^2\right] \left(\frac{L^2}{h W_R}\right) \]  
(12c)

\[ \frac{e_{\text{min}}}{c_b} \left(\frac{x}{L}\right) = \frac{S'_b}{A' c_b'} \left(\frac{f_{ts}}{R f_{c_e}} - 1\right) + \frac{w_c}{2 R f_{c_e} c_b'} \left[\frac{x}{L} - \left(\frac{x}{L}\right)^2\right] \left(\frac{L^2}{h W_R}\right) \]  
(12d)

Note that Eqs. (12a) to (12d) are dimensionless when written in this form. It is apparent that plotting Eqs. (12a) to (12d) would result in nondimensionalized design curves which define the maximum and minimum ratios of \( e/c_b \) at any point \( x \) along the span.

It is convenient here to define cross sections of the same group as those having the same dimensional ratios \((b/b_0, h/h_f, \text{ and } b_0/b)\), concrete strength, unit weight, and the same prestress loss ratio, \( R \). Thus, a closer look at Eqs. (12a) and (12b) reveals that within each group of cross sections, defined in this manner, the quantity \( e_{\text{max}}/c_b \) is only a function of \( L^2/h \).

Similarly, the quantity \( e_{\text{min}}/c_b \) is only a function of \( L^2/(h W_R) \). This leads to the definition of the eccentricity coefficient, \( C_e \), as:

\[ C_e = \frac{L^2}{h W_R} \]  
(13)

It can be stated that the nondimensionalized safe zone of eccentricity \( (e/c_b) \) for uniformly loaded, simply supported beams that are in the same group, varies only as \( C_e \) varies. Based on this observation, design charts utilizing the curves represented by Eqs. (12a) to (12d) can be generated for any group of cross sections, as defined above.

### DEVELOPMENT OF THE DESIGN CHARTS

Given a value of \( C_e \), the controlling function, \( e_{\text{min}}/c_b \), is chosen from Eqs. (12c) or (12d). This function defines the upper limit to the safe zone of eccentricity for this value of \( C_e \). Similarly, for a given value of:

\[ C_e W_R = \frac{L^2}{h} \]  
(14)

the controlling function, \( e_{\text{max}}/c_b \), is chosen from Eqs. (12a) or (12b). This function defines the lower limit of the safe zone of eccentricity for this value of \( C_e W_R \).

These curves of \( e/c_b \) versus \( x/L \) can be plotted for various values of \( C_e \) to obtain the design charts shown in Figs. 3 to 7. Intermediate values of \( C_e \) can be inter-
Fig. 3. Acceptable ranges of eccentricity for Example 1.
Fig. 4. Acceptable ranges of eccentricity for Section 1 of Example 2.
Fig. 5. Acceptable ranges of eccentricity for Section 2 of Example 2.
Fig. 6. Acceptable ranges of eccentricity for Section 3 of Example 2.

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Fig. 7. Acceptable ranges of eccentricity for Section 4 of Example 2.
polated to arrive at the required tendon profile for any section to which the chart pertains. As shown in the following example, it is possible to achieve a combination of maximum eccentricity and minimum prestressing force by narrowing the depth of the safe zone at midspan.

The following conventions are adopted for plotting purposes:

(a) For eccentricities below the centroidal axis, the values of \( e/c_e \) are plotted, while for eccentricities above the centroidal axis the corresponding values of \( e/c_e \) are plotted.

(b) A conventional \( x \)-\( y \) plotting routine is used which requires that eccentricities below the centroidal axis be plotted as negative.

The following design values have been selected to generate the sample charts. The allowable stress limits correspond to the current ACI Code. However, the allowable increase in tensile stresses at the end of the beam has not been included.

The value of the effectiveness ratio, \( R \), is taken as 0.80. The values of \( f'_{ci} \) and \( f'_{c} \) are 4000 and 5000 psi, respectively. The unit weight of concrete is taken as 150 pcf for normal weight concrete. Note that in calculating \( C_e \) for Eq. (13), the span length is given in feet and the beam height in inches.

The charts shown in Figs. 3 to 7 represent a small sample of those that have been generated. All of the computations and plotting of the curves are intended to be performed by a computer. Given the necessary software, charts for I-beams, T-beams, and box girders, using various combinations of \( f'_{ci}, f'_{c}, \) and \( R \) can be easily produced.

In addition, some of the aspects of the following design procedure could be computerized. For example, given the relative cost of placing the prestressing steel and concrete per unit volume and some initial architectural requirements, computer algorithms utilizing Eqs. (12a) to (12d) could be developed which would converge to an optimum cross section. Depending on the available computer facilities, this could be done in either an interactive or batch mode.

**DETERMINATION OF TENDON PROFILE**

In some cases the designer may wish to use the proposed design charts only to select a tendon profile. To do this, the designer first calculates the eccentricity coefficient, \( C_e \), from the known parameters of the design. The design chart, applicable to this cross section immediately describes the acceptable range of eccentricity along the entire span. This procedure is demonstrated by the following example.

**DESIGN EXAMPLES**

Two numerical design examples are presented to show the application of the proposed design method. The first example treats a symmetrical box beam while the second example covers an unsymmetrical I-beam.

**EXAMPLE 1**

Consider a symmetrical prestressed box beam. The tendon profile for the cross section of Fig. 8a will be determined by the use of the design charts. The dimensional ratios of this cross section are:

\[
\frac{b_w}{b} = 0.2 \quad \text{and} \quad \frac{h_f}{h} = 0.1
\]

The concrete strengths are specified as:

\[
f'_{ci} = 4000 \text{ psi and } f'_{c} = 5000 \text{ psi}
\]

Normal weight concrete is to be used, and a prestress loss ratio \( R = 0.80 \) is assumed. Based on this information, the design chart of Fig. 3 is appropriate for

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*Note: All units are expressed in U.S. foot and pound units. For metric conversion factors, see end of paper.
Fig. 8. Box beam of Example 1; (a) cross section; (b) elevation showing zone of acceptable eccentricity.
This cross section.

The following data are also specified for the design of the beam: the beam span \(L\) = 60 ft and the unfactored dead plus live load \((w_{DL} + w_{LL}) = 400\) psf. An acceptable tendon profile for this design is determined as follows.

**Solution**

The load ratio, \(W_R\), from Eq. (8) is equal to 0.252. This gives an eccentricity coefficient, \(C_e\), from Eq. (13) equal to 476. Given these values, the acceptable range of eccentricity is easily determined from the design chart of Fig. 3. The limits of eccentricity for this example are shown in Fig. 8b.

Once the designer has established the zone of acceptable eccentricity, the selection of the number and placement of the prestressing strands is a straightforward procedure. Any tendon profile for which the center of gravity of the steel (cgs) lies within this zone will satisfy the working stress requirements. A more comprehensive design example (Example 2), including the selection and layout of the prestressing strands, follows the proposed design procedure.

**PROPOSED DESIGN PROCEDURE**

For a given design problem, the values of \(L\), \(w_i\) (from \(w_{in}\) and \(w_{LL}\)), \(f'_c\), \(f'_t\), and \(R\) must be known or assumed. In many cases the values of \(b\) and/or \(h\) are known or predetermined by architectural requirements. Although both \(b\) and \(h\) may be varied throughout the design procedure, it is recommended that \(b\) remain constant. Then, if necessary, the procedure can be repeated using a refined value of \(b\). A flow chart of the design procedure is shown in Fig. 9, and is described in the following steps:

1. Select initial values of \(b\) and \(h\).

2. Select a number of design charts based on dimensional ratios. Although a working design can be determined using a single design chart, it is generally beneficial to obtain a number of feasible designs by using more than one chart. It may become apparent during the design iterations that one or more of the selected charts are inappropriate and are thus eliminated from consideration.

3. Tabulate the values of \(A\) [Eq. (2b)], \(w_i\) [Eq. (7)], \(W_R\) [Eq. (8)], and \(c_b\) [Eq. (2a)] for the chosen sections. Based on minimum cover requirements, determine a suitable value of \(elc_b\), usually around 0.8. By interpolation, at this value of \(elc_b\), obtain a preliminary \(C_e\) from each design chart.

4. Determine the minimum \(h\) as follows:

\[
\frac{L^2}{C_e W_R} \quad (15)
\]

The optimum value of \(h\), for each of the chosen sections, is between \(h_{min}\) and the initial value of \(h\) from Step 1.

5. If, in the judgment of the designer, \(h_{min}\) is not sufficiently close to the initial \(h\), select a new \(h\) and repeat Steps 3 and 4. In general, a value of \(h\) which is within 2 or 3 percent of \(h_{min}\) is a suitable solution. The reason that a value of \(h\) less than \(h_{min}\) may be acceptable is that the ratio \(elc_b\) chosen in Step 3 is an approximation and often conservative.

6. Using the final value of \(h\), calculate \(P_i\) and determine the required number of prestressing strands.

7. If the zone of acceptable eccentricity at midspan, based on the values of \(C_e\) and \(C_e W_R\) is too narrow to accommodate the required number of strands while maintaining minimum cover requirements, increase \(h\) and repeat Steps 3 through 6.

8. Select the tendon profile. Any profile which lies between the applicable \(C_e\) and \(C_e W_R\) curves is acceptable.

9. Satisfy ultimate strength and other established requirements.

It is important to note that a lower limit of eccentricity, as defined by \(C_e W_R\), which lies outside the beam cross section (i.e., \(elc_b > 1\)) indicates that the...
Fig. 9. Flow chart of proposed design procedure.
section selected is conservative. In this case, the designer may choose a different value of $b$ or an alternate design chart.

**EXAMPLE 2**

The proposed method will be demonstrated by designing an unsymmetrical I-beam given the following specifications:

- Beam span ($L$) = 50 ft
- Unfactored dead plus live load ($W_{DL} + W_{LL}$) = 275 psf
- $f'_{c} = 4000$ psi, $f'_{t} = 5000$ psi
- Prestress loss ratio ($R$) = 0.8
- Top flange width ($b$) = 5 ft

The following step-by-step solution parallels the proposed design procedure and the flow chart of Fig. 9.

**Solution**

**Step 1** — An $L/h$ ratio of 20 yields $h = 30$ in. This will be assumed as a reasonable initial value of $h$.

**Step 2** — The design charts of Figs. 4 to 7 are selected as having appropriate proportions for this design. The selection is based on the thicknesses of the flanges and the web that will result from the initial selection of $b$ and $h$.

**Step 3** — The results of the calculations of $A$, $w$, $W$, and $c_b$ for each of the design charts is given in Table 1. At midspan ($x/L = 0.5$), the minimum cover requirements are such that the tendon group eccentricity, $e$, minus $c_5$, would be a minimum of about 2.5 to 3 in. Therefore, a value of $c_5 - e = 2.75$ in. is used to determine $e/c_b$. By interpolation, the values of $C_e$ are taken from each design chart and shown in Table 1.

**Step 4** — Next, $h_{min}$ is calculated from Eq. (15). The values obtained for each cross section are included in Table 1.

**Step 5** — It is apparent that, according to the values of $h_{min}$, each of the four sections could be refined further, although Section 2, with $h = 30$ in. would appear to be a reasonably close design. Sections 1, 3, and 4, however, certainly warrant further iteration. For the purpose of this example, only the second iteration for Section 1 will be given.

**Step 3 (second iteration)** — An intermediate value of $h = 32$ in. is chosen for

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**Table 1. Steps 3 and 4 of Example 2.**

<table>
<thead>
<tr>
<th>Cross section</th>
<th>Design chart</th>
<th>Fig. 4</th>
<th>Fig. 5</th>
<th>Fig. 6</th>
<th>Fig. 7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Iteration</td>
<td>1st</td>
<td>2nd</td>
<td>1st</td>
<td>2nd</td>
<td>1st</td>
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<tr>
<td>$b$ (in.)</td>
<td>60</td>
<td>60</td>
<td>60</td>
<td>60</td>
<td>60</td>
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<tr>
<td>$h$ (in.)</td>
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<td>32</td>
<td>30</td>
<td>29.5</td>
<td>30</td>
</tr>
<tr>
<td>$A$ (in.$^2$)</td>
<td>378</td>
<td>403</td>
<td>450</td>
<td>443</td>
<td>414</td>
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<tr>
<td>$w$ (lb/ft)</td>
<td>394</td>
<td>420</td>
<td>469</td>
<td>461</td>
<td>431</td>
</tr>
<tr>
<td>$w_R$ (lb/ft)</td>
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<tr>
<td>$c_b$ (in.)</td>
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<tr>
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<tr>
<td>$C_e$</td>
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<td>340</td>
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<td>390</td>
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<tr>
<td>$h_{min}$ (in.)</td>
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<td>32.4</td>
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Table 2. Step 6 of Example 2.

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<th>3</th>
<th>4</th>
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<tbody>
<tr>
<td>Design chart</td>
<td>Fig. 4</td>
<td>Fig. 5</td>
<td>Fig. 6</td>
<td>Fig. 7</td>
</tr>
<tr>
<td>h (in.)</td>
<td>32</td>
<td>29.5</td>
<td>28</td>
<td>27</td>
</tr>
<tr>
<td>A (in.²)</td>
<td>403</td>
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<td>Pᵢ (kips)</td>
<td>289</td>
<td>339</td>
<td>330</td>
<td>389</td>
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<tr>
<td>Number of ½ in. diameter strands, fₚₗₘₜ = 270 ksi</td>
<td>10</td>
<td>12</td>
<td>12</td>
<td>14</td>
</tr>
<tr>
<td>Initial prestress, fₚᵢ (ksi)</td>
<td>189</td>
<td>185</td>
<td>180</td>
<td>182</td>
</tr>
</tbody>
</table>

Section 1. The values of A, wᵢ, Wᵦ, and cᵦ are given in Table 1. Minimum cover requirements indicate that a value of e/cᵦ = 0.87 is appropriate, which leads to an eccentricity coefficient (Cₑ) = 330.

**Step 4 (second iteration)** — The value of hₘᵢₜ₉ for Section 1 is 32.4 in., as given in Table 1. Values of hₘᵢₜ₉ for Sections 2, 3, and 4 are also given for their final iterations.

**Step 5 (second iteration)** — The assumed value of h = 32 in., for Section 1, is deemed adequate for continuation to Step 6. The final values of h = 29.5, h = 28, and h = 27 in. were determined to be appropriate for Sections 2, 3, and 4, respectively. Each of the four cross sections will be retained and carried over to Step 6.

**Step 6** — Having obtained the final values of h for the four possible sections, the values of Pᵢ are calculated from the simplified form of Eq. (3) found on the design charts. In order to compare the sections further, the required number of ½ in. diameter Grade 270 strands, at a maximum allowable stress of 189 ksi, is also calculated. These results are shown in Table 2.

At this point, the designer may decide
Fig. 11. Modified cross section used in Example 2 with strand pattern shown at (a) midspan and (b) ends.

to select a single cross section, or continue with any number of the four possible designs. For the purpose of this example, only one section will be chosen for Steps 7 to 9. Unless the beam depth is a controlling factor, Section 1,
which utilizes the maximum allowable initial prestress of 189 ksi, would appear to be the best choice for the final design. This section is shown in Fig. 10. Note that the flanges of the section may be tapered slightly without significant changes in the section properties. In this case, the flanges in the final section, shown in Fig. 11, are tapered from 2.5 to 4 in., resulting in an average thickness of 3.25 in., which is very close to the original flange thickness of 3.2 in. The changes in the distance to the centroidal axis and the moment of inertia as a result of this modification are less than ½ percent of the original values.

Step 7 — The acceptable region for the tendon profile is bounded by the curves representing $C_e = 334$ and $C_e W_k = 78$ as shown in Fig. 4. This range represents a 1.3 in. zone in which the cgs must be placed at midspan. Note that a straight tendon profile corresponding to a constant eccentricity is not possible given this optimized section. However, it would be a simple matter to find the minimum beam height required for any tendon profile.

Step 8 — The selected strand placement is shown in Fig. 11. For the center 20 percent (10 ft) of the span, the strands are placed such that the ratio $e/c_b$ is equal to 0.9. The end eccentricities are selected to give $e/c_b = 0.39$. The cgs is shown as a broken line in Fig. 3. It is clear that this strand profile results in ratios of $e/c_b$ which are positioned within the acceptable region for the entire length of the beam.

Step 9 — Check ultimate strength and satisfy other applicable requirements.

**PRACTICAL APPLICATIONS**

Through the use of the proposed design charts, a designer may quickly and easily arrive at a number of acceptable working stress design alternatives. The fact that the proposed charts define the safe zone of eccentricity for the entire span eliminates the need to check the actual stresses at critical points along the beam. As already noted, the values of $f_c$, $f_c'$, and $R$ may be varied, along with the dimensional ratios, to develop design charts for any group of cross sections. Additionally, the designer may develop charts using any values of the allowable working stresses (i.e., $f_{ct}$, $f_{ct}$, $f_{tu}$, and $f_{tu}$). Having generated the desired charts, considerable savings can be achieved by applying the same charts to numerous design situations.

This method may be of particular interest to the manufacturers of pre-stressed girders who generally utilize a limited number of cross sections. In such cases, the charts need only to be generated for the available cross sections. Regardless of the span length, design charts could be used for a quick determination of the arrangement and number of prestressing strands.

Another advantage of this method is its potential use as a teaching aid. The charts can be a useful tool in allowing the student to visualize the variation of acceptable tendon profile over the entire span. Finally, the method could be easily adapted to an interactive computer graphics format which would eliminate the need for preparation of the design charts in advance.

**SUMMARY AND CONCLUSIONS**

A set of dimensionless design curves has been developed which extends the applicability of Magnel's safe zone from a single point on the span to the entire length of the girder; the acceptable combinations of $P$ and $e$ are expressed for all points along the span. For ease of application, the curves have been plotted, and design charts for several types of cross sections have been included.

The design charts are dependent on the concrete strength, the prestress loss ratio, and the proportions of the cross
section. Therefore, a single chart can be applied to a number of cross sections all having the same relative proportions. In addition, the design charts are independent of the span length and load intensity. The two design examples demonstrate the use of the charts for determining tendon profiles, and the time-saving potential of the proposed design method.

ACKNOWLEDGMENT

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* * *

NOTE: Discussion of this paper is invited. Please submit your comments to PCI Headquarters by January 1, 1987.
REFERENCES

8. ACI Committee 318, “Building Code Requirements for Reinforced Concrete, (ACI 318-83),” American Concrete Institute, Detroit, Michigan, 1983.

* * *

**Metric (SI) Conversion Factors**

<table>
<thead>
<tr>
<th>Unit</th>
<th>Equivalent</th>
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<tbody>
<tr>
<td>1 ft</td>
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<tr>
<td>1 in.</td>
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<tr>
<td>1 in.²</td>
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<td>1 psi</td>
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<tr>
<td>1 lb/ft</td>
<td>14.59 N/m</td>
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APPENDIX — NOTATION

\( A \) = cross-sectional area of beam
\( A' \) = coefficient for cross-sectional area of beam
\( b \) = width of top flange of beam
\( b_2 \) = width of bottom flange of unsymmetric I-beam
\( b_w \) = width of beam web
\( C_e \) = eccentricity coefficient
\( c_b \) = distance from extreme bottom fiber to centroidal axis of beam
\( c_t \) = distance from extreme top fiber to centroidal axis of beam
\( c_b' \) = coefficient for distance from extreme bottom fiber to centroidal axis of beam
\( c_t' \) = coefficient for distance from extreme top fiber to centroidal axis of beam
\( CA \) = centroidal axis of beam
\( cgs \) = center of gravity of prestressing steel
\( e \) = tendon eccentricity
\( e_{max} \) = maximum allowable tendon eccentricity
\( e_{min} \) = minimum allowable tendon eccentricity
\( f_{ci} \) = allowable compressive stress in extreme fibers under initial loading conditions
\( f_{cs} \) = allowable compressive stress in extreme fibers under service loading conditions
\( f_{cci} \) = concrete centroidal stress under initial loading conditions
\( f_c' \) = compressive strength of concrete
\( f_{ci}' \) = initial compressive strength of concrete
\( f_{pu} \) = ultimate strength in prestressing steel
\( f_{si} \) = initial prestress in prestressing steel
\( f_{ti} \) = allowable tensile stress in extreme fibers under initial loading conditions
\( f_{ts} \) = allowable tensile stress in extreme fibers under service loading conditions
\( h \) = height of beam
\( h_f \) = height of beam flange
\( I \) = moment of inertia of beam about centroidal axis
\( I' \) = coefficient for moment of inertia of beam
\( k_b \) = kern distance measured from centroidal axis to top of beam
\( k_t \) = kern distance measured from centroidal axis to bottom of beam
\( L \) = beam span
\( M_i \) = bending moment in beam under initial loading conditions
\( M_s \) = bending moment in beam under service loading conditions
\( P \) = prestressing force
\( P_i \) = initial prestressing force
\( R \) = effectiveness or prestress loss ratio
\( S_b \) = section modulus relative to bottom fibers of beam cross section
\( S_t \) = section modulus relative to top fibers of beam cross section
\( S_b' \) = coefficient for section modulus relative to bottom fiber of beam cross section
\( S_t' \) = coefficient for section modulus relative to top fiber of beam cross section
\( w_c \) = unit weight of beam
\( w_1 \) = self weight of beam per unit length
\( w_{DL} \) = uniform dead load per unit area
\( w_{LL} \) = uniform live load per unit area
\( w_k \) = uniform dead plus live load on beam per unit length
\( W_R \) = load ratio
\( x \) = horizontal distance from end of beam

* * *