Structural Properties of High Strength Concrete and its Implications for Precast Prestressed Concrete

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High strength concrete with a uniaxial compressive strength, f'_c greater than 6000 psi (42 MPa), is experiencing increased use and acceptance by designers and contractors for both reinforced and prestressed concrete construction.[1,2,3] Currently, it is possible to produce concrete with strengths in excess of 12,000 psi (84 MPa). However, since not enough information is available on the structural properties of high strength concretes, discussion in this paper is restricted to concretes with strengths of up to 12,000 psi (84 MPa).

Initial use of high strength concrete, f'_c = 7000 psi (49 MPa), for buildings occurred in 1965 during construction of Lake Point Tower in Chicago, Illinois. Two years later, this durable building material was used to construct the Willows Bridge in Toronto, Canada, marking the first time high strength concrete was used for a bridge project. A summary of buildings and bridges in which concretes of higher than normal strengths have been used is presented in a recent ACI committee report.[4]

The principal advantage of high strength concretes is their relatively greater compressive strength to unit cost, unit weight and unit volume ratios as compared to normal strength concretes. High strength concrete, with its greater compressive strength per unit cost, is often the least expensive means of carrying compressive forces. In addition, its greater compressive strength per unit weight and unit volume allows lighter and more slender members.

Other advantages of high strength concrete include increased modulus of elasticity and increased tensile strength.

Increased stiffness is advantageous when deflections or stability govern the design, while increased tensile strength is advantageous for service load design in prestressed concrete.

Current ultimate strength design practice is based on experimental information obtained from concretes with compressive strength in the range of 3000 to 6000 psi (21 to 42 MPa). For developing a satisfactory procedure for the design of structures using higher strength concretes, additional considerations, validation or modification of existing strength design methods may be necessary.

In this paper, experimental data on high strength concrete obtained by the authors are reported. Based on these data as well as those reported by other investigators, the authors have proposed empirical expressions to substitute for some of the currently used relationships. Note that the details of the experiments are presented elsewhere. In this paper, the emphasis is on the results, comparison with normal strength concrete, development of empirical formulas and some discussion on structural design implications.

Synopsis

Experimental data on the structural properties of high strength concrete [f_c' greater than 6000 psi (42 MPa)] are reported. Based on these findings, as well as data on normal strength concrete, empirical expressions are proposed.

The implications of such parameters as compressive strength, compressive stress-strain curve, modulus of elasticity, tensile strength, shear strength, Poisson's ratio, ductility, lateral reinforcement, as well as economic considerations for the structural design of prestressed concrete are studied and design recommendations are made.

STRESS-STRAIN RELATION IN UNIAXIAL COMPRESSION

Several experimental investigations[5-10] have been undertaken to obtain the stress-strain curves of high strength concrete in compression. It is generally recognized that for high strength concretes, the shape of the ascending part of the curve is more linear and steeper, the strain at maximum stress is slightly higher, and the slope of the descending part is steeper, as compared to normal strength concrete.

To obtain the descending part of the stress-strain curve, it is generally necessary to avoid the specimen testing system interaction. A simple method of obtaining a stable descending part of the stress-strain curve is to load the concrete cylinders in parallel with a larger diameter, hardened steel tube with a thickness such that the total load exerted by the testing machine is always increasing. This approach can be used with most conventional testing machines.[6]

An alternative approach is to use a closed-loop testing machine so that specimens can be loaded to maintain a constant rate of strain increase to avoid unstable failure. The choice of feedback signal for the closed-loop operation is important and governs the occurrence of stable or unstable post-peak behavior. The difficulties of experimentally obtaining the post-peak behavior of concrete in uniaxial compression and methods of overcoming these difficulties are described in a study by Ahmad and Shah.[11] For very high strength concretes, it may be necessary to use the lateral strains as a feedback signal rather than the axial strains.[12]

For the present study, a closed-loop, servo controlled testing machine was used to obtain complete stress-strain curves. The testing was done under

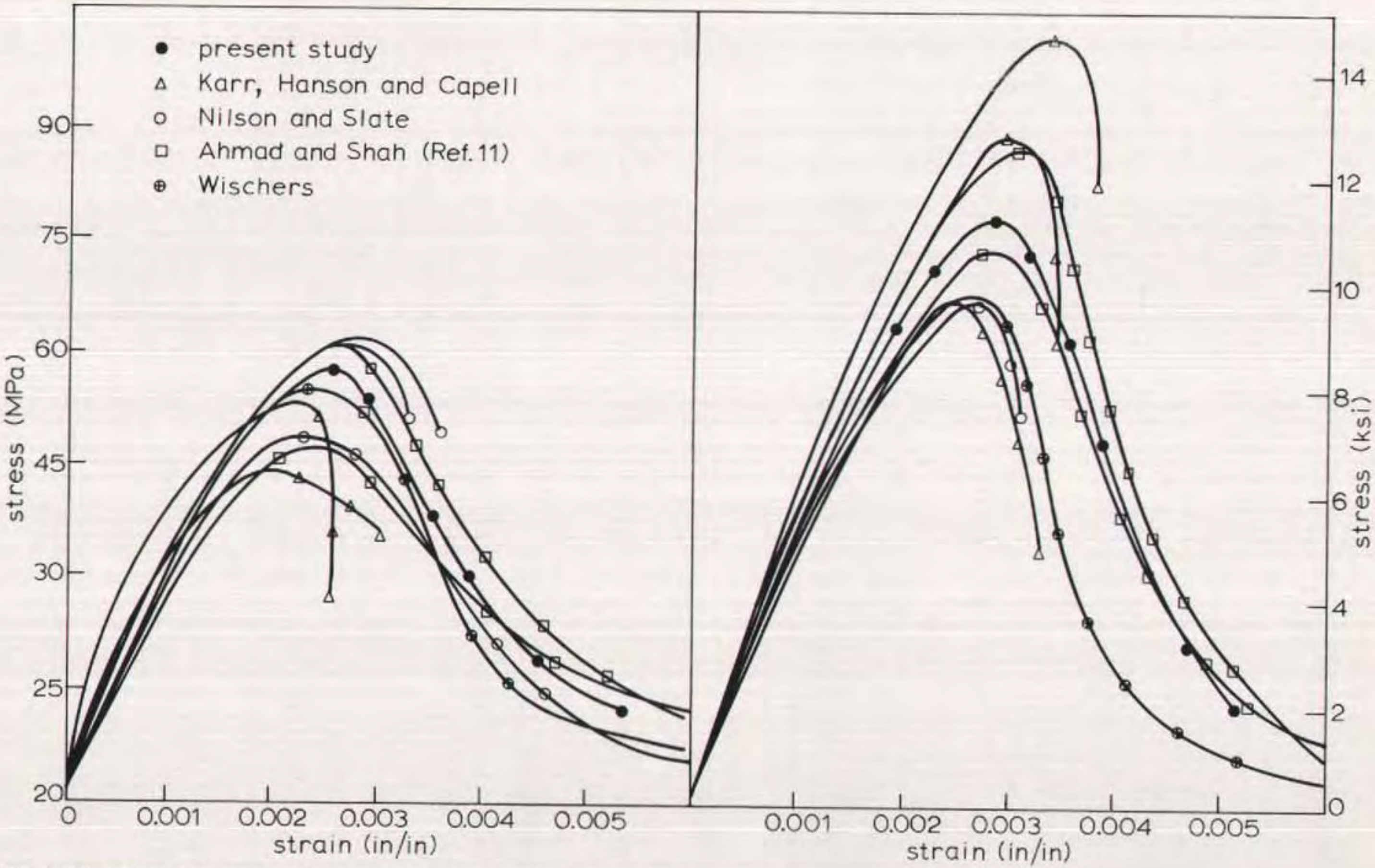


Fig. 1. Stress-strain curves of high strength concrete under uniaxial compression.

strain controlled conditions and a constant rate of increase of axial strain was maintained throughout the test. Fig. 1 shows the results of the present investigation along with other available experimental data. From Fig. 1 it can be seen that the slope of the curve in the post maximum stress range increases as the strength of concrete increases.

The stress-strain curve in uniaxial compression can be mathematically represented by a fractional equation.[6,13,14]

$$f = f_c' \frac{A\,(\epsilon/\epsilon_o) + (B-1)\,(\epsilon/\epsilon_o)}{1 + (A-2)\,(\epsilon/\epsilon_o) + B\,(\epsilon/\epsilon_o)^2} \quad (1)$$

$$f \nless 0.1 f_c'$$

for post peak region

or by a combination of power and exponential equation:[10]

$$f = f_c' \left[1 - \left(1 - \frac{\epsilon}{\epsilon_o} \right)^A \right] \quad (2a)$$

for ascending part

$$f = f_c' \exp\left[-k\,(\epsilon - \epsilon_o)^{1.15} \right] \quad (2b)$$

for descending part

and where f is the stress at strain (ϵ), f_c' and ϵ_o are the maximum stress and the corresponding strain, and A, B, and K are the parameters which determine the shape of the curve in the ascending and descending parts, respectively.

The value of the parameters A, B and K are determined by:

$$A = E_c \frac{\epsilon_o}{f_c'} \quad (3)$$

$$B = 0.88087 - 0.57 \times 10^{-4}\,(f_c') \quad (4)$$

$$K = 0.17 f_c' \quad (5)$$

$$\epsilon_o = 0.001648 + 1.14 \times 10^{-7}\,(f_c') \quad (6)$$

$$E_c = 27.55\,W^{1.5}\,\sqrt{f_c'} \quad (7)$$

where f_c' is the compressive strength in psi and W is the unit weight in lbs per cu ft.

Eqs. (3) to (6) were determined from the statistical analysis of the experimental data on 3 x 6 in. (75 x 152 mm) concrete cylinders.[8,10] These cylinders were tested in a closed-loop testing machine under strain controlled conditions and had a compressive strength varying from 3000 to 11,000 psi (20 to 75 MPa).

SECANT MODULUS OF ELASTICITY

The secant modulus of elasticity is defined as a the secant slope of the uniaxial stress-strain curve at a stress level of 45 percent of the maximum stress. A comparison of experimentally determined values[4] of the secant modulus of elasticity with those predicted by the expression recommended by ACI 318, Section 8.5,[15] based on a dry unit weight, W, of 145 lb per cu ft is given in Fig. 2. Also shown is the proposed equation for estimating the secant modulus of elasticity for low as well as high strength concretes which is:

$$E_c = W^{2.5}\,(\sqrt{f_c'})^{0.65} = W^{2.5}\,(f_c')^{0.325} \quad (7a)$$

Note that Eq. (7a) goes through the origin and is comparable to the ACI equation for low and normal strength concrete, but it is more accurate for high strength concrete. Other empirical equations proposed for predicting the elastic secant modulus are:[4,8,17]

$$E_c = 40{,}000\,\sqrt{f_c'} + 1.0 \times 10^6 \text{ psi}$$

$$\text{for } (3000 \text{ psi} \le f_c' \le 12{,}000 \text{ psi}) \quad (7b)$$

$$E_c = 26\,W^{1.5}\,\sqrt{f_c'} \quad (7c)$$

$$E_c = 27.55\,W^{1.5}\,\sqrt{f_c'} \quad (7d)$$

The values of the experimentally determined secant modulus of elasticity depend on the properties and proportions of the coarse aggregate (for example, with the same consistency and water-cement ratio, the larger the maximum size of aggregate and the

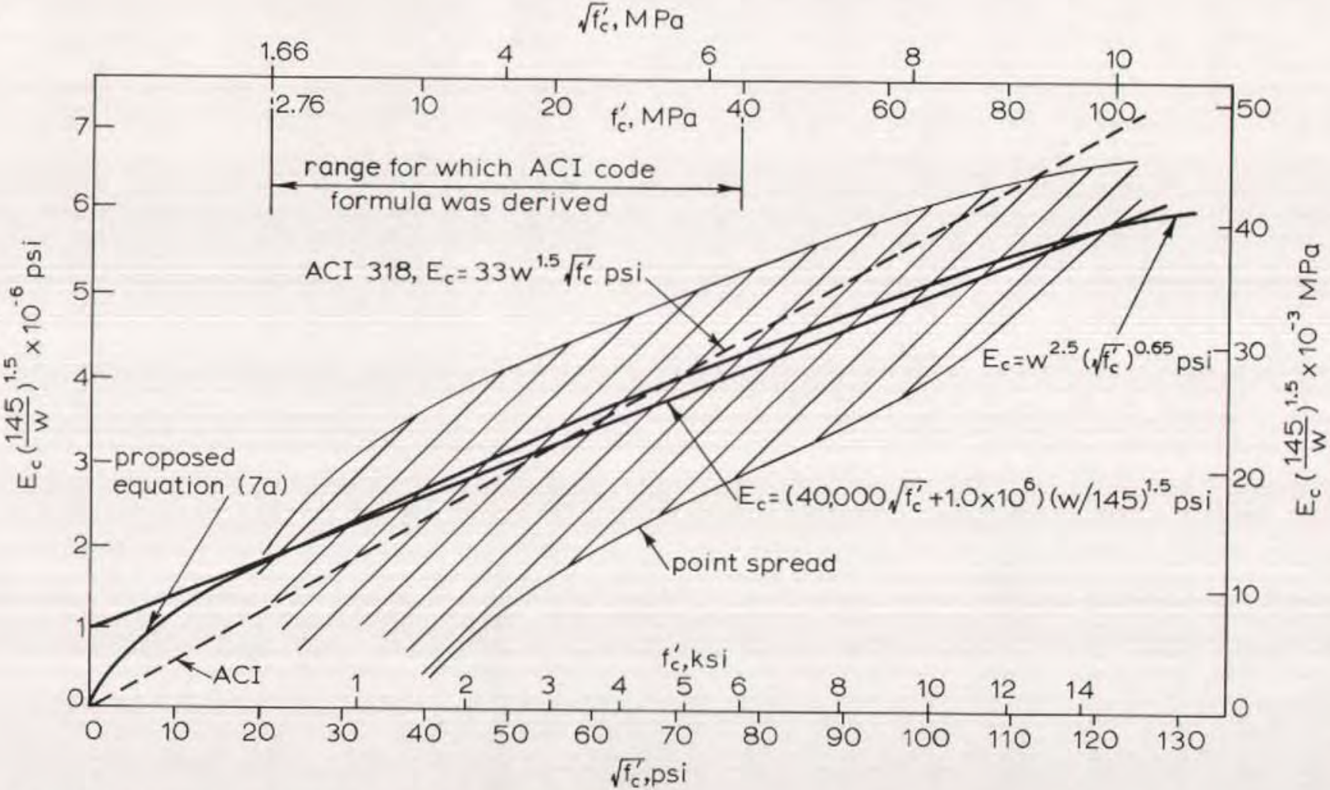


Fig. 2. Secant modulus of elasticity versus concrete strength.

coarser the grading, the higher the modulus of elasticity); the wetness or dryness of the concrete at the time of test (the drier the concrete at the time of the test the lower the modulus of elasticity—wet concrete is stiffer although often weaker); and the method of obtaining the deformations (strain gage, mechanical compressometer, transducers, etc.).

In view of the possible variability of experimental data on modulus, it is likely that any of the above equations can be used for estimating the secant modulus of elasticity and the development of a more accurate equation is perhaps unwarranted.

TENSILE STRENGTH

The tensile strength of concrete can be experimentally determined in three different ways: (1) uniaxial tensile test; (2) split cylinder test; and (3) beam test in flexure. The first method of obtaining the tensile strength may be referred to as "direct," and the second and third methods may be referred to as "indirect."

In the direct test for tensile strength, the specimen is gripped at its ends and pulled apart in tension; tensile strength is the failure load divided by the area experiencing tension.

In the splitting tension test, a cylinder is loaded in compression on two diametrically opposite sides, and the specimen fails in tension on the plane between the loaded sides.

In the beam flexure test (modulus of rupture test), a rectangular beam is loaded at the center or third points and fails in bending; the computed tensile stress at failure load is called modulus of rupture.

Many engineers assume that the direct tensile strength of concrete is about 10 percent of its compressive strength; splitting tensile strength is about the same, or perhaps 1 percent stronger; and modulus of rupture is about 15 percent of compressive strength. Based on the available experimental data of split cylinder and beam flexure tests on concretes of low, medium[19,20,21] and high strengths,[16,22,23] empirical equations to predict the average split-tensile strength (f'_{sp}) and modulus of rupture (f_r) for concretes of strengths up to 12,000 psi (84 MPa) are proposed as follows:

$$f'_{sp} = 4.34\,(f'_c)^{0.55} \tag{8}$$

$$f_r = 2.30\,(f'_c)^{2/3} \tag{9}$$

where f'_c is the compressive strength of concrete in psi.

Note that Eq. (9) is the same expression proposed by Jerome,[24] which was developed on the basis of data for concretes of strengths up to 8000 psi (56 MPa). Figs. 3a and 3b show the plot of the experimental data and the proposed equations for predicting the split cylinder strength (f'_{sp}) and modulus of rupture (f_r) of concretes with strengths up to 12,000 psi (84 MPa).

Also shown in Figs. 3a and 3b are the equations proposed by ACI Committee 363[4] which appear to overestimate the values of tensile strengths as compared to Eqs. (8) and (9). However, these equations[4,16] have the same functional form as currently used by the ACI Code (also shown in Fig. 3b). For design purposes, the equations proposed to predict the average results may be unsatisfactory. Design equations which are lower bound for the experimental data are also shown in Figs. 3a and 3b.

The complete stress-strain curve of concrete in tension is difficult to obtain, primarily because of the inability to correctly monitor the strains after tensile cracking. Due to the difficulties in testing concrete in direct tension, only limited and often conflicting data are available. Recent work at Northwestern University[25] points out that due to the localized nature of post-peak deformations, no unique tensile stress-strain relationships exist.

According to this study:[25]

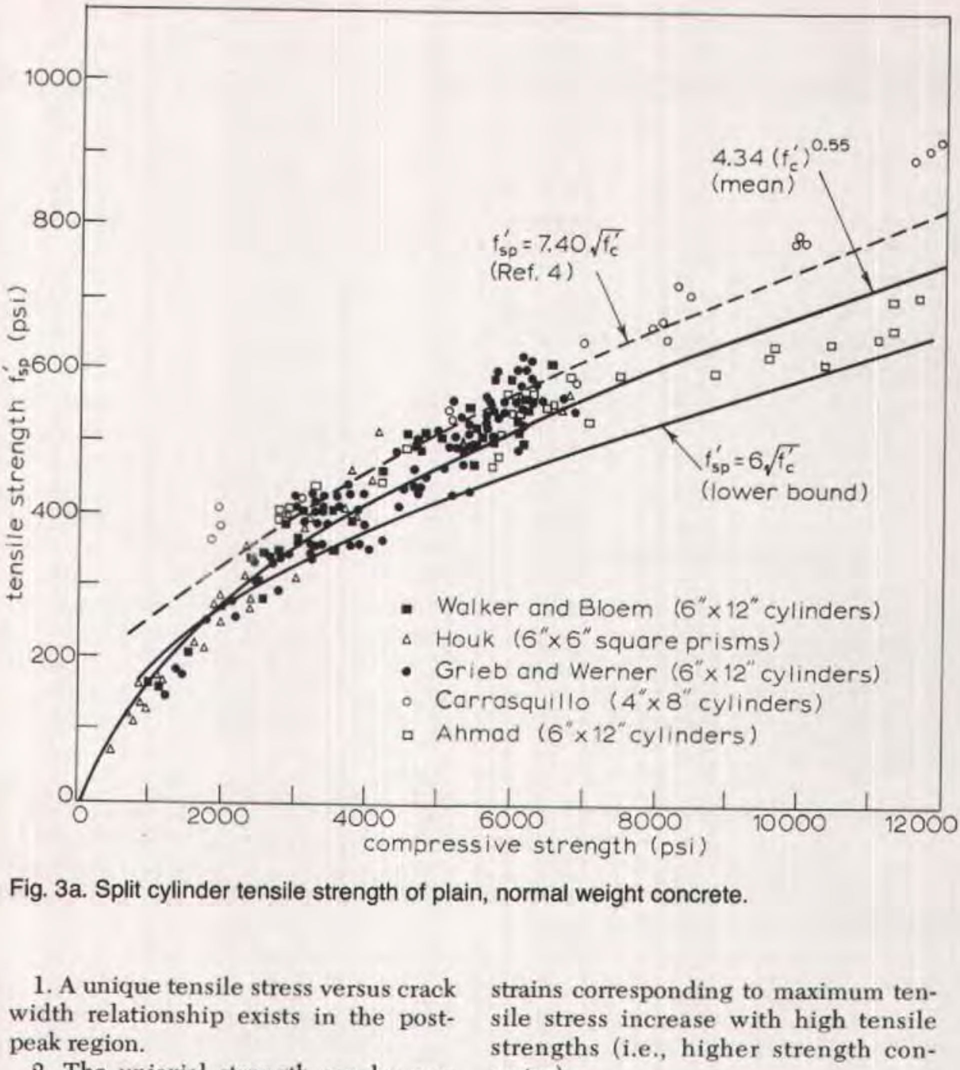


Fig. 3a. Split cylinder tensile strength of plain, normal weight concrete.

1. A unique tensile stress versus crack width relationship exists in the post-peak region.

2. The uniaxial strength can be predicted by the expression, $6.5\sqrt{f'_c}$, where f'_c is the uniaxial compressive strength in psi.

3. The tangent modulus of elasticity is identical in tension and compression.

4. The prepeak stress-strain curve in tension is relatively less nonlinear than in compression.

No data in uniaxial tension is reported for higher strength concretes. However, some unpublished data at North Carolina State University on tensile stress-strain curves, as obtained from split cylinder tests, indicate that tensile strains corresponding to maximum tensile stress increase with high tensile strengths (i.e., higher strength concretes).

POISSON'S RATIO

Poisson's ratio under uniaxial conditions is defined as the ratio of lateral strain to strain in the direction of loading. In the inelastic range due to volume dilation resulting from internal microcracking, the apparent Poisson's ratio is not constant but is an increasing function of axial strain. However, experimental data on the values of Poisson's ratio for high strength concrete are very limited.[16,27]

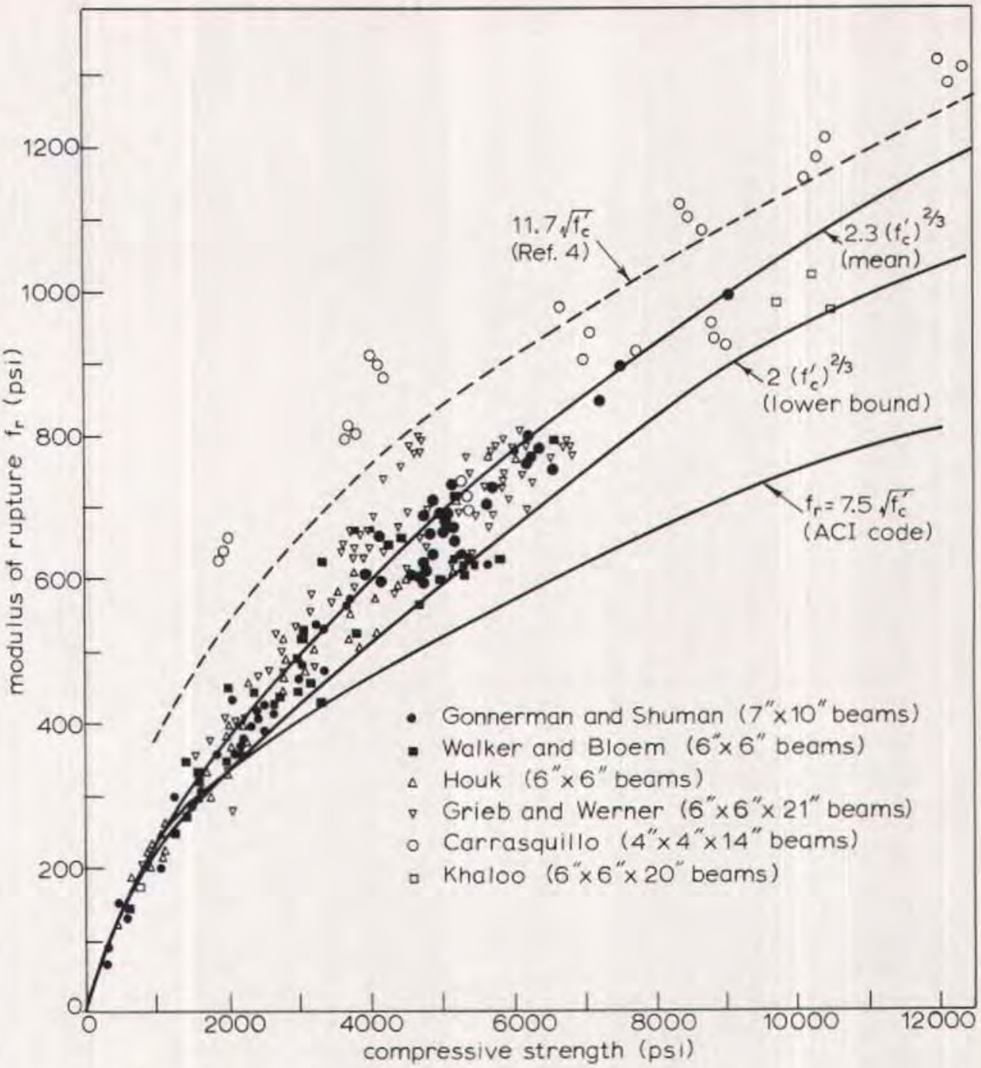


Fig. 3b. Beam flexural tensile strength of plain, normal weight concrete.

Based on the available experimental information, Poisson's ratio of higher strength concretes in the elastic range appears comparable to the expected range of values for lower strength concretes. In the inelastic range, the relative increase in lateral strains is less for higher strength concretes as compared to concretes of lower strengths.[14] That is, higher strength concretes exhibit less volume dilation than lower strength concrete (Fig. 4). This implies less internal microcracking for concretes of higher strengths.[26]

The lower relative expansion during the inelastic range may mean that the effects of triaxial stresses will be proportionally different for higher strength concretes. For example, the effectiveness of hoop confinement is reported to be less for higher strength concretes.[14]

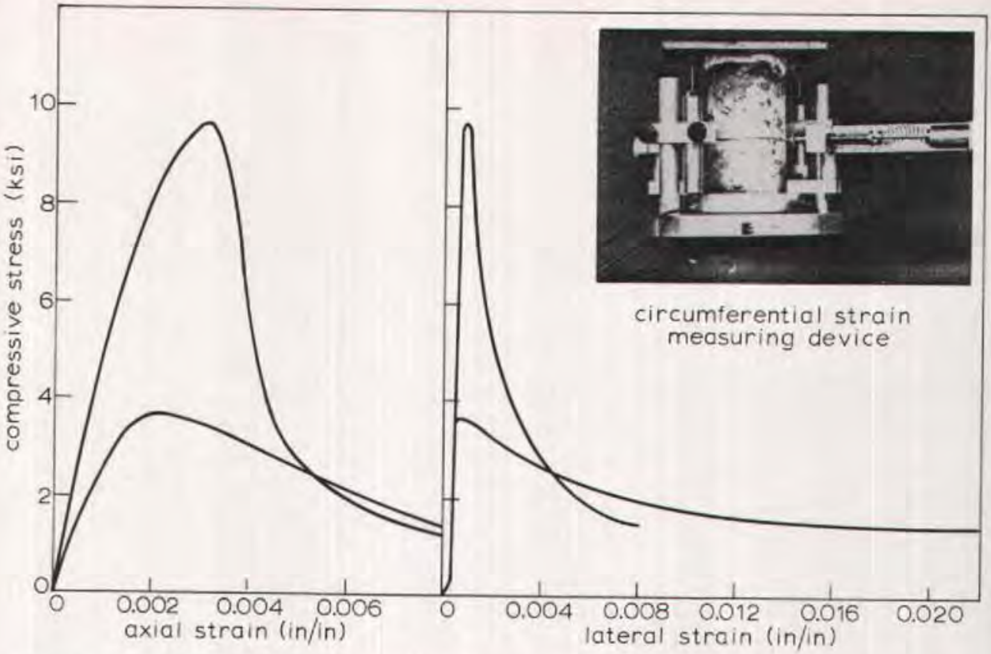


Fig. 4. Axial stress versus axial strain and lateral strain for plain, normal weight concrete.

MULTIAXIAL STRESSES

Experimental data on the behavior of high strength concrete under multiaxial stresses are not yet available. In a recent paper,[28] an orthotropic model for predicting the behavior of concrete under uni, bi and triaxial stresses has been proposed. This model incorporates the lower volume dilation of high strength concrete in the inelastic range.

EFFECT OF STRAIN RATE

Some experimental information is available on the effect of strain rate on the behavior of concretes with strengths in the range of 2000 to 5500 psi (14 to 39 MPa).[29-34] Very little information is available on the behavior of concretes of higher strengths under high strain rates (such as those that would be experienced during earthquakes). Ahmad and Shah[35] have tested concretes with strengths up to 7000 psi (49 MPa) under high strain rates (up to 30,000 microstrains per sec).

On the basis of experimental results[35] and the other available data,[29-34] empirical equations to predict the secant modulus of elasticity, maximum strength and the corresponding strain under high strain rates are proposed: The secant modulus of elasticity (at $0.45\, f_c'$) under fast strain rates is given by:

$$(E_c)_{\dot{\epsilon}} = (E_c)_s \left[0.962 + 0.038 \frac{\log \dot{\epsilon}}{\log \dot{\epsilon}_s} \right] \tag{10a}$$

where $(E_c)_s = 27.55, \sqrt{f_c'}$ and $\log \dot{\epsilon}_s = \log$ (32 microstrains per sec) are the values at the usual static loading rate, and $(E_c)_{\dot{\epsilon}}$ is the corresponding secant modulus of elasticity at a desired strain rate. Compressive strength under fast strain rates (for $\epsilon > 16$ microstrains/sec) is:

$$(f_c')_{\dot{\epsilon}} = f_c' \left[0.95 + 0.27 \frac{\log \dot{\epsilon}}{f_c'} \right] \alpha \tag{10b}$$

where f_c' is the compressive strength measured at the usual static rate and α is the shape factor to account for the different shapes. The shape factor is given by:

$$\alpha = 0.85 + 0.09\,(d) - 0.02\,(h) \text{ for } \frac{h}{d} \leq 5 \quad (10c)$$

where

d = diameter or least lateral dimension (in.)
h = height (in.)

and

$$(\epsilon_o)_i = [938.46 + 11.138\,(\dot{\epsilon}) + 0.272 f_c']\,\beta \quad (10d)$$

where f_c' is in psi and β is the shape factor given by:

$$\beta = 0.80 + 0.143\,(d) - 0.033\,(h) \text{ for } \frac{h}{d} \leq 5 \quad (10e)$$

From these equations, it can be seen that (1) the secant modulus of elasticity increases with increase in strain rate; (2) the strength enhancement (increase) due to higher strain rates is less for concretes of higher strengths as compared to normal strength concretes; and (3) the strain corresponding to the maximum stress increases with the increase in strain rate.

It should be noted that the study[35] is limited in scope and more research is needed in this area to quantify the effects of very fast strain rates on high strength concretes. Such information is currently being obtained by using an instrumented impact testing system at Northwestern University.[36]

MATERIAL AND SECTIONAL DUCTILITY

It is generally accepted that high strength concrete is less ductile than normal strength concrete. It is not possible to express the relative ductility (or brittleness) in a quantitative manner since no rational method of measuring this quantity currently exists. Attempts using nonlinear fracture mechanics to define fracture toughness are being made.[37,38]

Ductility can be quantitatively expressed, in a crude manner, from the slope of the post-peak response of concrete subjected to uniaxial compression; for example, if the slope is zero, then the material is perfectly plastic, while for perfectly brittle material, the slope is infinity. From Fig. 1 it can be seen that high strength concrete has a greater slope than that for normal strength concrete.

According to the above definition, unreinforced high strength concretes are more brittle than normal strength concrete; however, the same is not necessarily true for reinforced high strength concrete structural elements. Consider, for example, a typical under-reinforced concrete beam moment versus midspan deflection relationship shown in Fig. 5a. If ductility is defined as the ratio of the deflection at ultimate to that at yielding of the tensile steel, then this ratio depends not only on the compressive stress-strain curve of concrete but also on the amount of longitudinal reinforcement, shape of the beam cross section and the loading conditions (third point loading versus single central point loading, presence of axial loads, as well as many other factors).

Moment versus midspan deflection curves of the beam shown in Fig. 5a were theoretically calculated for three reinforcement ratios and five compressive strengths. The amount of longitudinal steel was varied such that the ratio between the actual steel content, p, and the balanced steel content, p_b (defined and calculated according to the ACI Code[15]) remained essentially the same for beams with five different concrete strengths.

The moment-curvature relationship for a section was calculated assuming that plane sections remain plane, using Eq. (1) for the stress-strain curve of concrete, while the stress-strain curve of the steel was as shown in Fig. 5a and was analytically expressed as outlined by Wang et al.[39] Note that the tensile strength contribution of concrete was ignored.

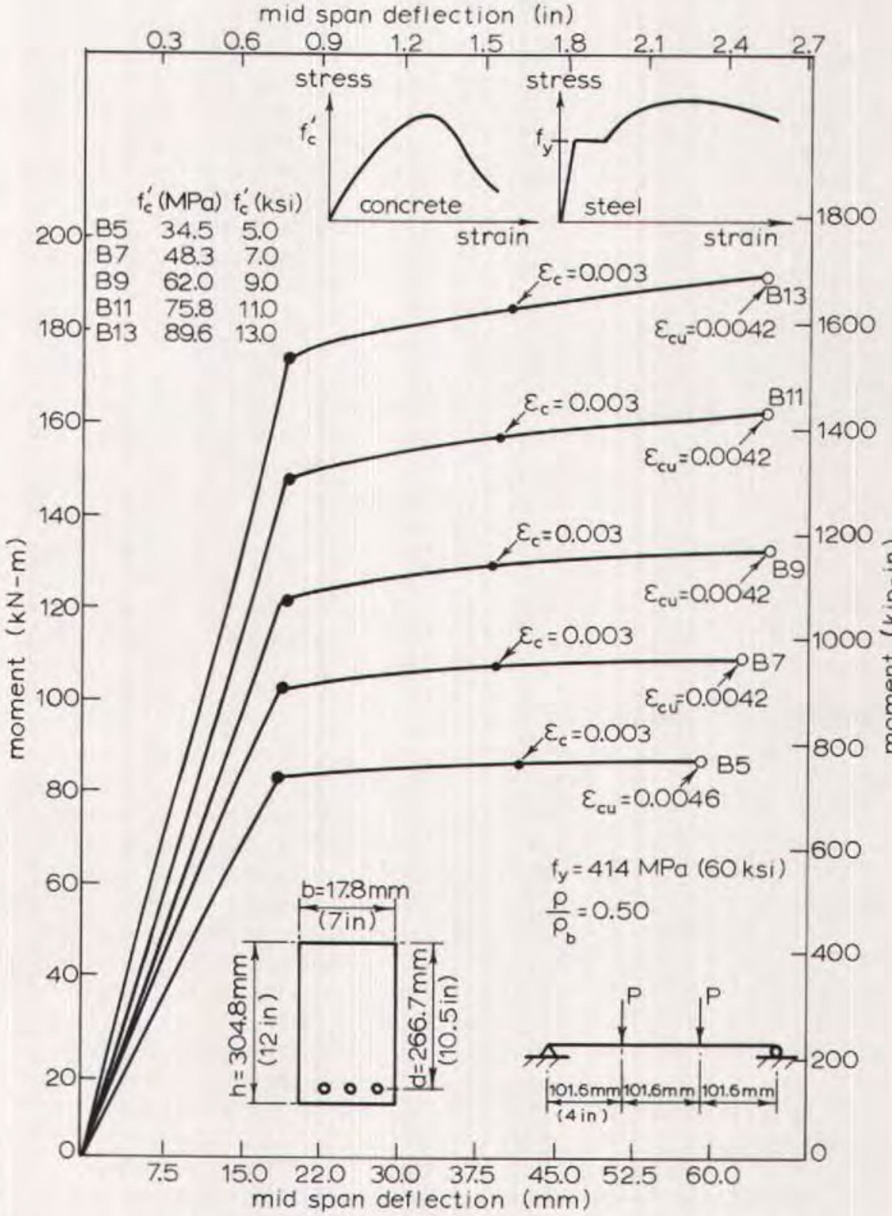


Fig. 5a. Analytical moment versus midspan deflection for a singly reinforced beam with different concrete.

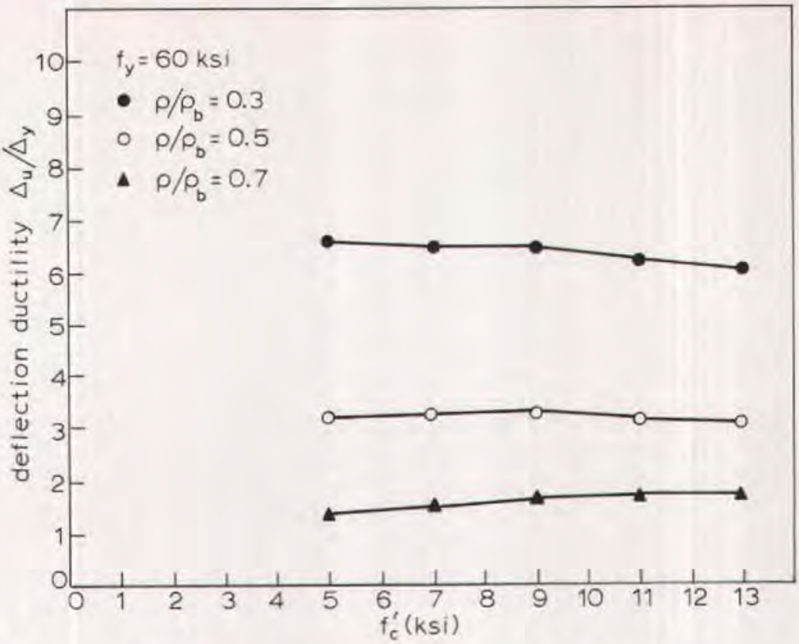


Fig. 5b. Effect of concrete compressive strength on the deflection ductility of a singly reinforced beam under third point loading.

The moment-deflection relationships were calculated from knowing the moment field and integrating the curvature along the beam. This procedure assumes that there are no discontinuities in the distribution of the curvatures. This may be a correct assumption for closely spaced narrow cracks. For wider cracks, curvatures may be computed by either discrete deformation summation or by using nonlinear strain distribution across the depth of the member.

The curves shown in Fig. 5a are for beams made with five different compressive strengths and reinforced such that they all had the same p/p_b. It can be seen that the ductility ratio is the same regardless of compressive strength. This is also true for other values of p/p_b as can be seen in Fig. 5b. From the theoretical results (Fig. 5b), it can be seen that the ductility ratio is essentially independent of the compressive strength of concrete, if the ratio of p/p_b is kept constant.

Table 1 compares the results of the theoretical predictions with the experimental results of research conducted at Cornell University and reported in the ACI report.[4] The dimensions of the singly reinforced beams tested[4] are the same as shown in Fig. 5a and yield strength of steel was 60,000 psi (414 MPa). It is seen that except for Beam A3 in Table 1, the theoretical prediction is close to the experimentally observed values. Note that in the testing of the beams, the shear failure was avoided by using the stirrups in the shear span.

SHEAR STRENGTH

The shear strength of concrete has been experimentally studied in two ways: by testing solid or hollow concrete cylinders in pure torsion and by testing beams under third point loading and studying the shear and diagonal tension strength.

Table 1. Comparison of analytical and experimental deflection ductility ratios Δ_u / Δ_y.

Beam No.	f'_c (ksi)	p/p_b	p	p'/p	Δ_u Δ_y Analytical	Δ_u / Δ_y Exp. (Ref. 4)
A1	3.7	0.51	0.0135	0	3.96	3.54
A2	6.5	0.52	0.0219	0	2.16	2.84
A3	8.5	0.29	0.0145	0	6.31	2.53
A4	8.5	0.64	0.0321	0	1.91	1.75
A5	9.3	0.87	0.0481	0	1.35	1.14
A6	8.8	1.11	0.0565	0	1.02	1.07

Note: 1 ksi = 6.895 MPa.

Fig. 6 shows the shear stress-shear strain and shear stress-axial tensile strain curves for concretes of different compressive strengths. These curves were obtained by testing solid 3 x 9 in. (76.2 x 228.6 mm) cylindrical concrete specimens under pure torsion. The shear strains were simultaneously obtained by the strain gages on the surface of the concrete and by measuring the change in arc length (as shown schematically in the subset of Fig. 6) with the help of a very sensitive, linear voltage direct transducer (LVDT). The results obtained from these methods were very comparable to each other.

In these tests, the lateral loads (to generate the torsion) were applied through a pair of horizontal jacks placed 24 in. (610 mm) apart. The axial tensile extension induced because of shear was also recorded through a LVDT placed between the top of the test specimen and the platten of the machine. The relationship between the shear stress (calculated by using the elastic torsion formula) and axial tensile strain is a measure of the shear dilation phenomenon in concrete (note that for metals this dilation is assumed to be zero).

The relatively lower axial tensile strain observed for high strength concrete may indicate that microcracks in high strength concrete are less rough. This may influence the so-called shear-aggregate-interlock phenomenon.[41]

The current shear design philosophy is to provide the total shear resistance in excess of shear imposed (required) by conditions using factored loads. The total shear resistance is made up of two parts: V_c provided by the concrete and V_s provided by the shear reinforcement. The value of V_c recommended by the ACI Code[15] includes the contributions of the uncracked concrete at the head of a hypothetical crack, the resistance provided by the aggregate interlock along the diagonal crack face, and the dowel resistance provided by the main reinforcing steel.

In a recent paper, Frantz[42] reported that the current ACI formulas for calculating V_c are applicable to high strength concrete. However, unpublished data by Nilson[4] indicates that current design methods are not conservative for higher strength concretes.

Recently, fifty-four singly reinforced beams were tested at North Carolina State University[43] to study the flexure-shear interaction of high strength concrete beams. All the beams were without web reinforcement and were 5 in. wide x 10 in. deep (127 x 254 mm). The beams were tested under third point loading with different shear span to depth (a/d) ratios. Some of the beams were designed to fail in flexure and others were designed to fail in shear. Only the results of beams which failed in shear are presented in Figs. 7a-7b.

The load which produced the first diagonal crack was defined as the diagonal

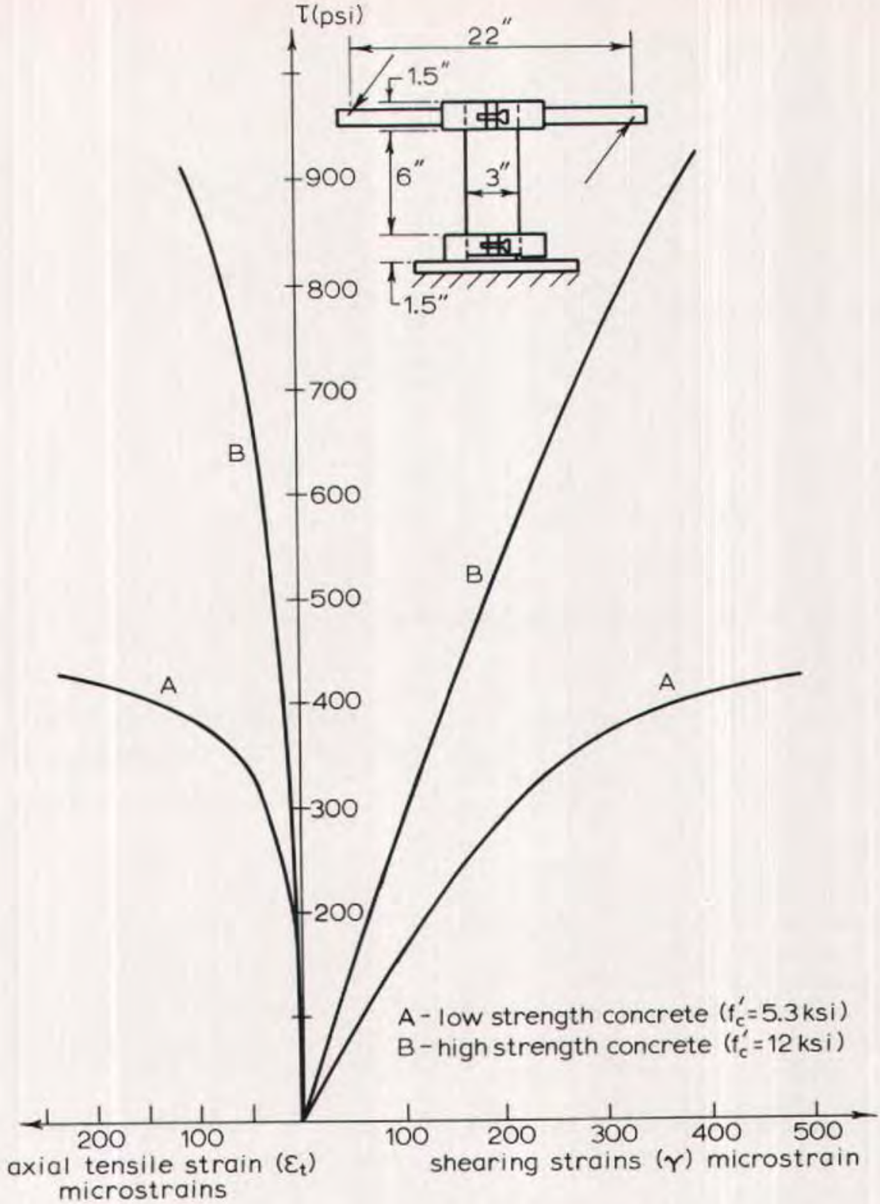


Fig. 6. Shear stress versus shear strain and axial tensile strain for plain, normal weight concrete.

cracking load and was used to calculate the shear stress at diagonal cracking (v_{cr}). Note that the magnitude of the cracking load (and thus the cracking stress) is sensitive to both the actual location of the initiating flexural crack and to the observer's judgment.

The ultimate shear stress (v_u) was calculated by dividing the failure (maximum) load by the cross-sectional

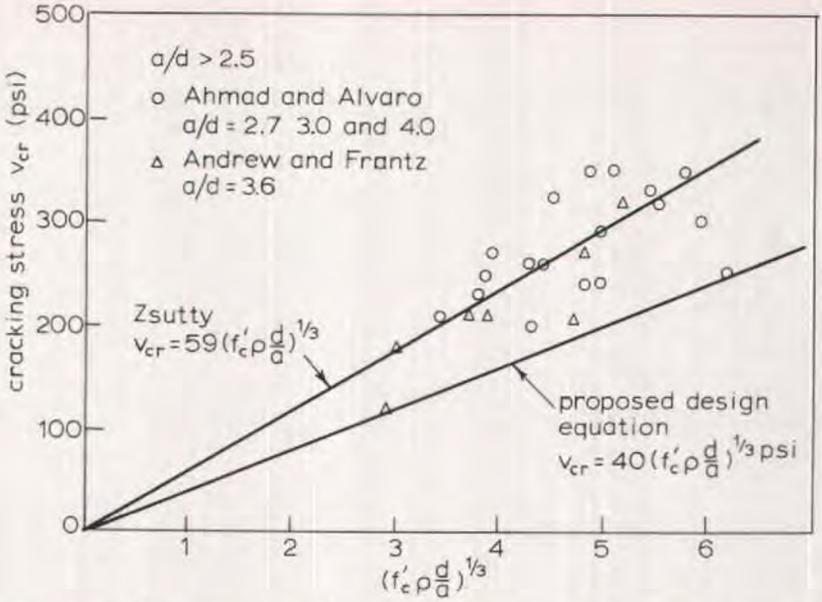


Fig. 7a. Cracking shear stress of slender beams without web reinforcement.

area of the beam. Figs. 7a and 7b show the experimental data for beams without web reinforcement[42,43] along with the equation recommended by Zsutty[44] for low strength concretes.

From these figures it appears that Zsutty's equation gives a good average estimate for the cracking and ultimate shear stress. However, it may prove to be unconservative for design purposes. On the basis of experimental data, design equations which are lower bound for the experimental data are proposed. The proposed design equations to estimate cracking and the ultimate shear stress are:

$$v_{cr} = 40\,(f'_c\,pd/a)^{1/3} \qquad (11a)$$

$$v_u = 50\,(f'_c\,pd/a)^{1/3} \qquad (11b)$$

where

v_{cr} = cracking shear stress
v_{cu} = ultimate shear stress
p = longitudinal steel content
d = effective depth of the beam
a = shear span

BENEFICIAL EFFECT OF LATERAL CONFINEMENT

In compression dominant structural elements like columns, it is advantageous to confine the concrete by providing lateral steel in the form of continuous spirals or ties. The beneficial effects of lateral confinement of concrete on column behavior are:

1. It increases the strength of the core concrete inside the spiral by confining the core against lateral expansion underload.

2. It increases the axial strain capacity of concrete, thereby permitting a more gradual and ductile failure.

Currently, no research data are available regarding the behavior of high strength concrete confined by rectangular ties. Recently, three research reports[14,45,46] on the beneficial effects of continuous spirals for low and high strength concretes have been published. From these reports, it can be observed that for high strength and lightweight aggregate concretes, the beneficial ef-

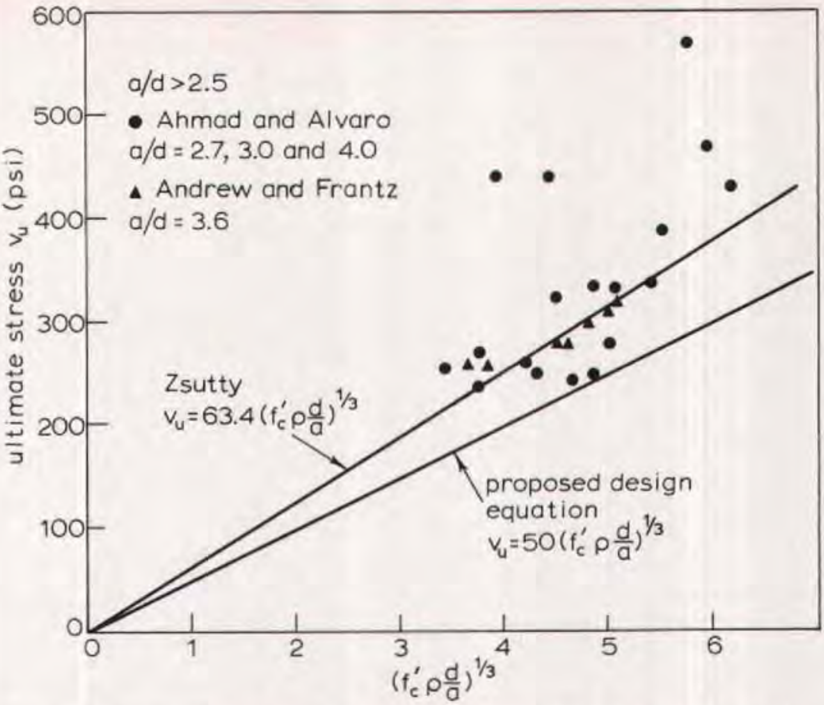


Fig. 7b. Ultimate shear stress of slender beams without web reinforcement.

fects from lateral confinement are different than those for normal strength concretes. This difference can be attributed to the different (less) volume dilation in the inelastic range for higher strength concretes[14] (Fig. 4).

Using the constitutive properties of concrete and the stress-strain relationships of the confining steel, an analytical model was proposed by Ahmad and Shah[14] to predict the beneficial effects of hoop confinement for low as well as high strength concrete. This work showed that adequate ductility can be obtained for high strength concrete by increasing the amount of confining reinforcement or by increasing the yield strength of hoop reinforcement. Similar conclusions have been reported from experiments with high strength, normal weight concrete conducted by Japanese researchers[47] and for lightweight, high strength concrete.[46]

This conclusion was also reached in a recent study at Northwestern University.[48] Moment-curvature relationships were calculated for confined concrete columns subjected to a constant axial load and increasing amount of lateral load. The current practice of providing confinement as suggested by ACI[15] for round columns is given by:

$$p_s = 0.45\,(A_g/A_c - 1)\,f_c'/f_{yh} \qquad (12a)$$

$$p_s = 0.12\,f_c'/f_{yh} \qquad (12b)$$

where

p_s = ratio of spiral reinforcement
A_g = gross area of the cross section
A_c = area of core of spirally reinforced column measured to outside diameter of the spiral
f_{yh} = yield stress of the hoop steel

Note that the higher the compressive strength, the higher the amount of confining reinforcement required by the

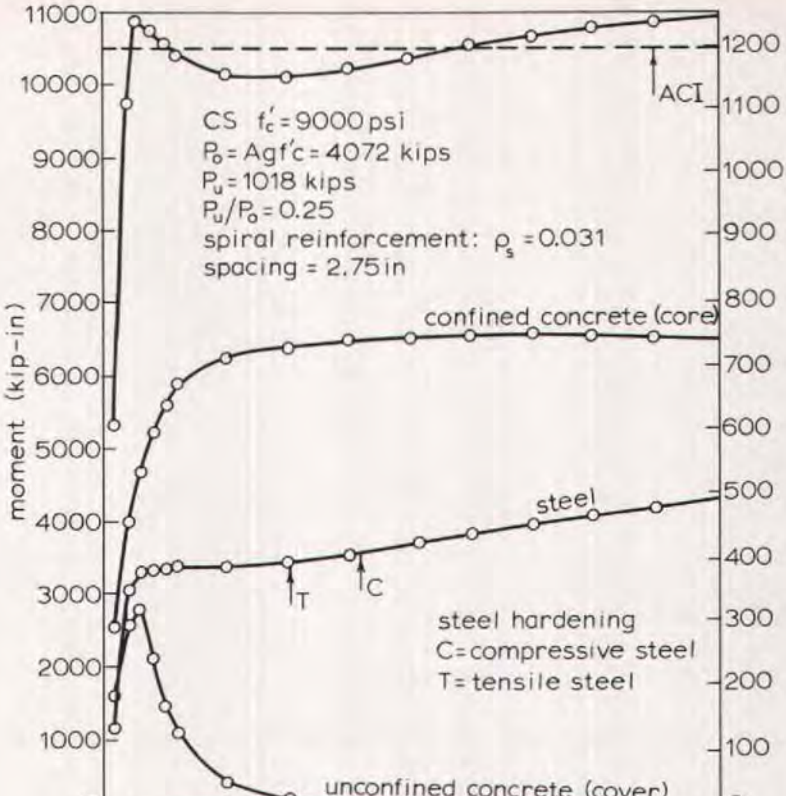


Fig. 8. Diagram of moment versus maximum core compressive strain.

ACI Code. It was observed[48] that Eqs. (12a) and (12b) adequately compensate for the inherently poor efficiency of the unit confinement for high strength concrete by increasing the confinement for increased compressive strength.

This can be seen in Fig. 8 where the theoretically calculated moment versus maximum core compressive strain for a round, high strength column[48] is shown. The column was subjected to increasing bending moment and a relatively high constant axial load. It was confined using the ACI Code requirement.

Even for a relatively high value of axial strain, the column is seen to maintain the ACI predicted value of the maximum load. As shown in Fig. 8, the contribution of confined core and the longitudinal steel compensates for the loss of cover capacity. The theoretically calculated curves could not be compared with the experimental results of high strength concrete columns since no data are available. However, a satisfactory comparison was obtained with the available results for normal strength concrete.[48]

ECONOMICS OF HIGH STRENGTH CONCRETE

To examine the possible savings in engineering costs of using high strength concrete, a 79-story high rise building similar to Water Tower Place in

Table 2. Cost comparison of using normal strength concrete and high strength concrete for a 79-story building (Ref. 49).

Materials	Compressive strength	
	Up to 12,000 psi	4,000 psi
Cost per 25 x 25 ft panel		
Concrete	\$ 45,035	\$ 88,836
Forms	35,729	54,606
Longitudinal steel	34,449	87,161
Spirals	1,441	1,930
TOTAL	\$116,654	\$232,533
Total cost for 33 columns =	\$3,849,582	\$7,673,589

Note: 1 ft = 0.3048 m; 1 psi = 0.006895 MPa.

Table 3. Unit cost of materials and placing for various levels of concrete compressive strength.

Materials and placing	Compressive strength		
	4,000 psi	9,000 psi	12,000 psi
Concrete per cu yd	\$50.00	\$68.62	\$96.60
Placing per cu yd	16.00	16.00	16.00
Forms per sq ft	2.8	2.8	2.8
Steel in place per lb	0.38	0.38	0.38

Note: 1 cu yd = 0.77 m^3; 1 sq ft = 0.093 m^2; 1 psi = 0.006895 MPa.

Chicago, Illinois was examined by Shah et al.[49] The total cost of constructing columns using high strength concrete with compressive strengths of up to 12,000 psi (84 MPa) was compared with that using concrete with a compressive strength of 4000 psi (28 MPa).

With the high strength concrete, column dimensions were kept constant and were calculated so that the lowest story columns can be made with a 12,000 psi (84 MPa) concrete and 1 percent longitudinal steel. The dimension of the column and the percentage of the longitudinal steel was maintained constant for all 79 stories. Note that, in general, the smaller the percentage of steel, the lower the column cost per unit load carrying capacity.[50] The advantage of keeping constant dimension for the entire height of the building is that the same forms can be used repeatedly for all stories.

For the computations a typical interior column was considered. Columns were designed for only axial loads and no moments were considered, since only a preliminary estimate was attempted. For the high strength concrete, the top 29 floors were designed with 4000 psi (28 MPa), the next 31 floors with 9000 psi (63 MPa), while the bottom 19 floors were designed with 12,000 psi (84 MPa).

For normal strength concrete, all floors had concrete with a compressive strength of 4000 psi (28 MPa). However, to maintain a 1 percent ratio of the longitudinal steel, the dimensions of the designed circular columns were increased from about 55 in. (1400 mm) at

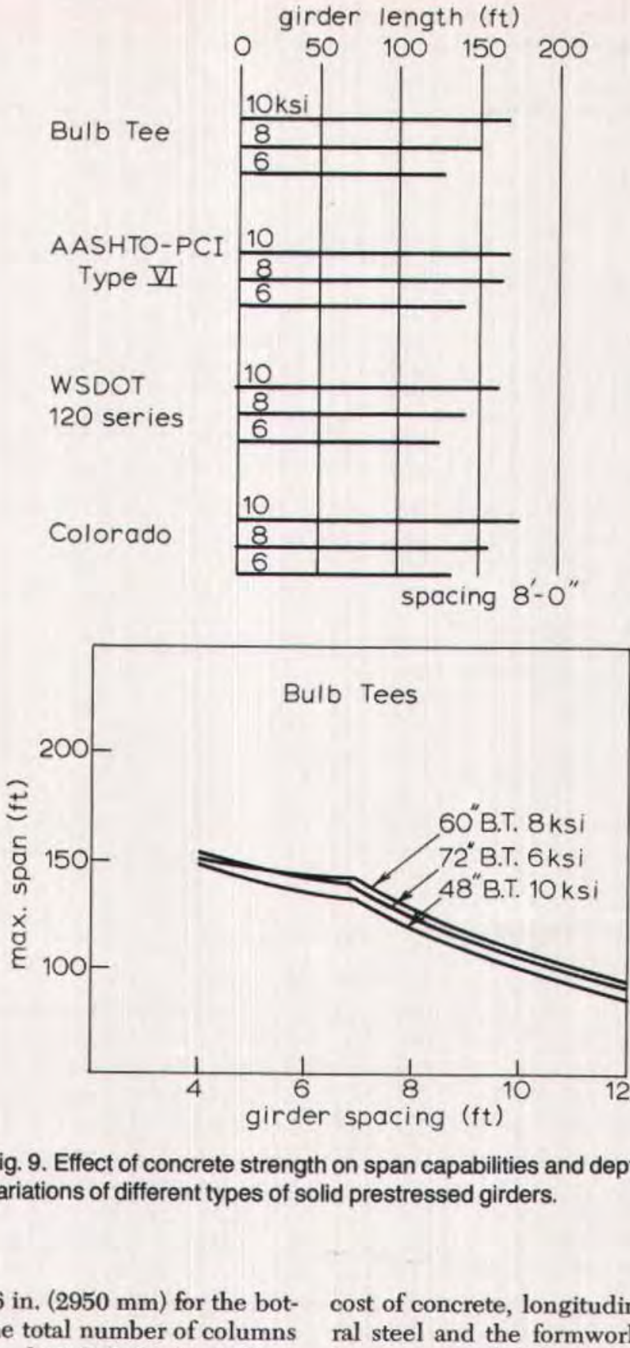


Fig. 9. Effect of concrete strength on span capabilities and depth variations of different types of solid prestressed girders.

the top to 116 in. (2950 mm) for the bottom story. The total number of columns for a spacing of 25 ft (7.6 m) and floor plan dimensions of 94 ft x 220 ft (28.6 x 67 m) was 33.

A cost comparison of these two design alternatives is shown in Table 2. The cost of concrete, longitudinal steel, spiral steel and the formwork were taken from the 1983 Chicago area cost estimate and are shown in Table 3.

A total savings of $3,824,007 is obtained when using a high strength concrete option for the columns. This

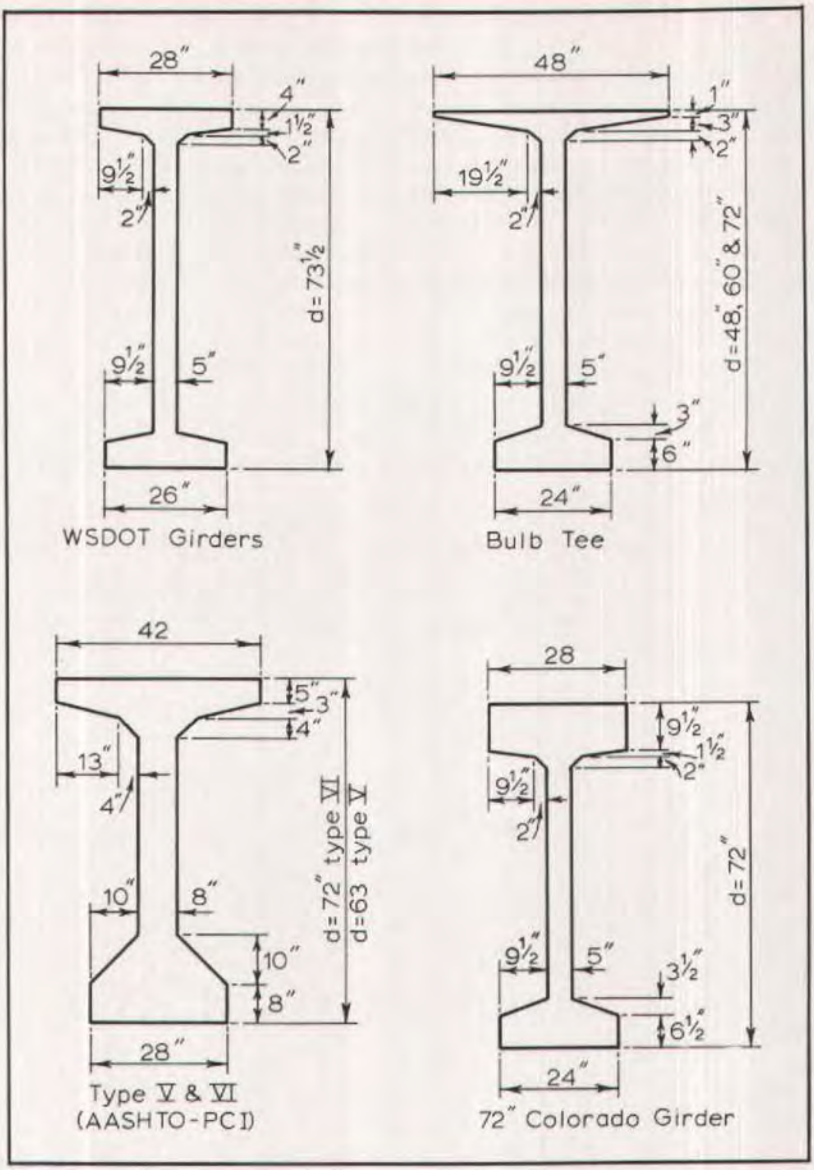


Fig. 10. Cross section of solid section girders.

amount is very approximate; the actual savings may be less. On the other hand, this amount does not include the savings due to an increase in rental space of 66,000 sq ft (6131 m²). The 1983 rental cost in downtown Chicago was approximately \$20 sq ft (1.86 m²) per year.

IMPLICATIONS FOR PRESTRESSED CONCRETE

1. The compressive strength in uniaxial compression does not substantially influence the resisting capacity of flexural systems because of the desir-

ability of the under-reinforced conditions in design. The location and the amount of steel are predominant in determining the ultimate capacity of such systems. For prestressed flexural systems the use of high strength concrete may not produce cost effective benefits in terms of ultimate capacity. However, if the design is governed by serviceability limit states, then high strength concrete can be beneficial. This was illustrated by Jobse and Moustafa.[17]

For cast-in-place decks the benefits of high strength concrete in increasing the span capabilities of four types of girders (see Fig. 9) are shown in Fig. 10. It can be seen from this figure that for the AASHTO-PCI Type VI girder of 72 in. (1830 mm) depth, the increase of concrete strength from 6000 to 10,000 psi (42 to 70 MPa) increases the span capability approximately from 140 to 165 ft (43 to 50 m)—an increase of 18 percent.

The potential for using shallow members with increasing concrete strengths is also shown in Fig. 10. For cast-in-place decks, the potential for reducing the depth from 72 to 48 in. (1830 to 1220 mm) and increasing concrete strengths from 6000 to 10,000 psi (42 to 70 MPa) can be realized for all girder spacings.

2. The use of high strength concrete

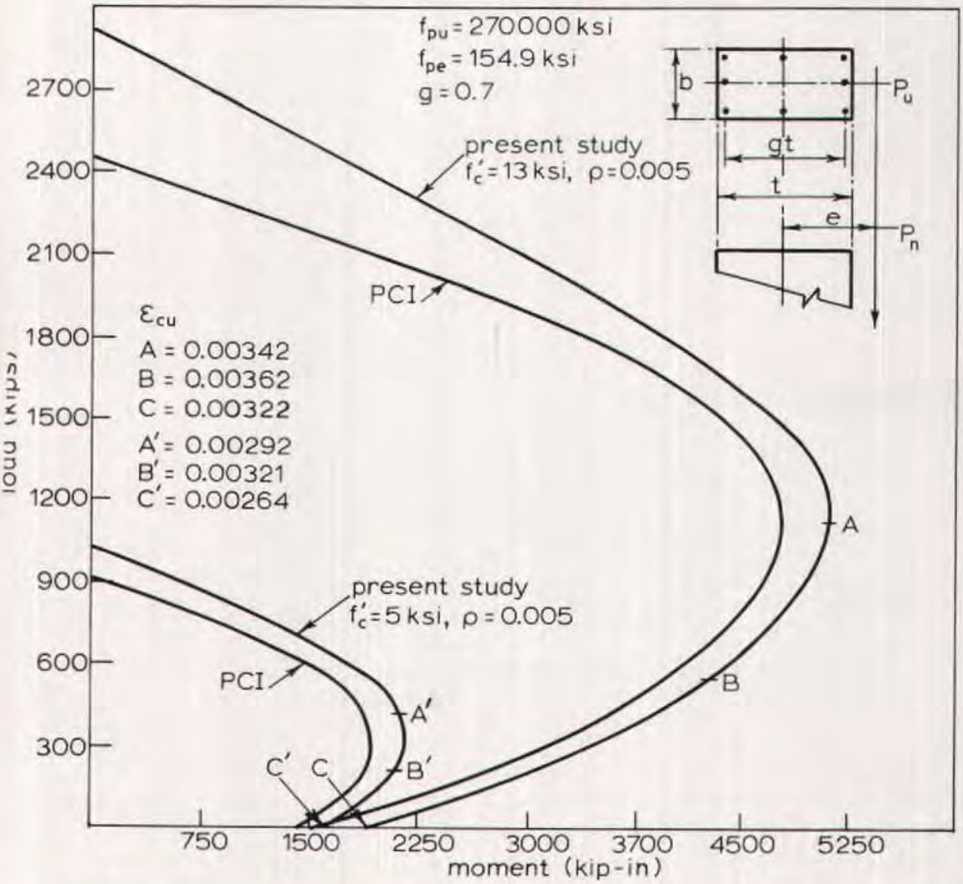


Fig. 11. Effect of concrete strength on the load-moment interaction curve of prestressed concrete beam-column element.

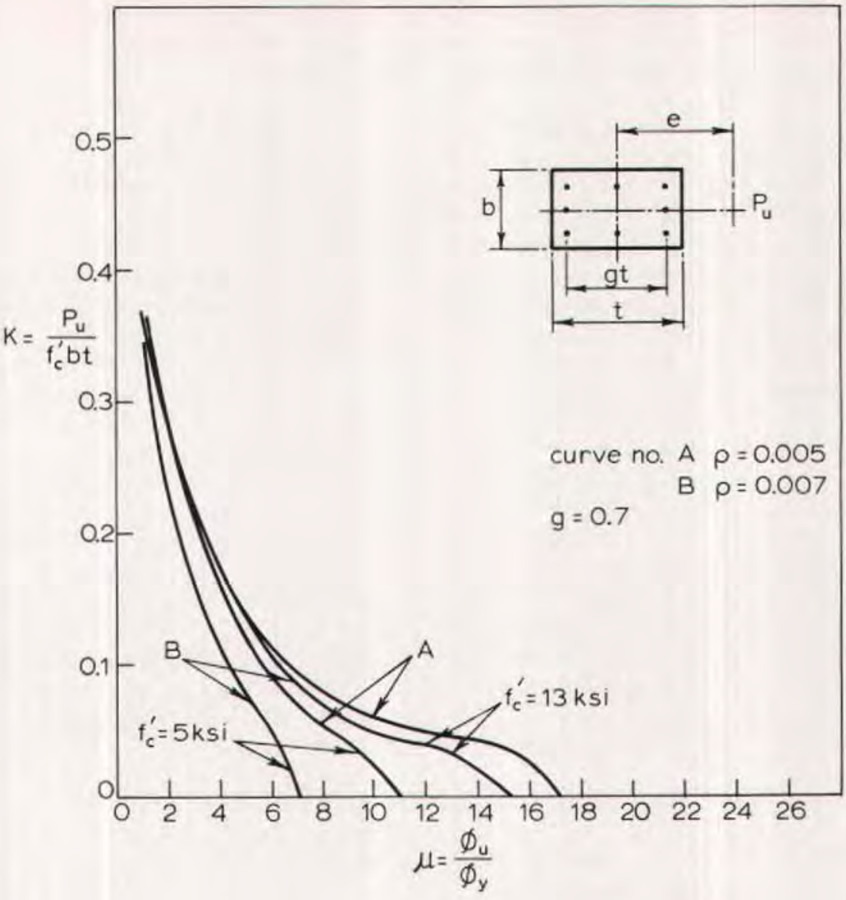


Fig. 12. Effect of concrete strength and level of axial load and amount of prestressing reinforcement on the sectional ductility of prestressed concrete beam-column element.

can, in general, speed-up construction time. Since given strength is attained earlier, post-tensioning and stress transferring operations can be performed earlier.

3. The use of high strength concrete shows a definite advantage in structural elements which predominantly carry compressive forces. The effect of higher strength concrete on the load-moment interaction, and the comparison of results obtained by using the PCI[51] procedure and a nonlinear computerized procedure[52] is shown in Fig. 11. The nonlinear computerized procedure assumes plane sections remain plane, uses Eq. (1) for the stress-strain curve of concrete and uses a polynomial equation to express the stress-strain relationship of a 270,000 psi (1890 MPa) seven wire prestressing strand. This comparison indicates that the current PCI method for strength computations is appropriate for beam-column members of higher strength concretes.

The PCI method is sufficiently accurate for high strength concrete, despite the approximate rectangular stress block, a constant value of ultimate strain (ϵ_{cu}), and an approximate equation for the stress (f_{ps}) in prestressing steel at the ultimate condition.

In Fig. 11 the values of the compressive strain in concrete at the maximum resistance of the section (ϵ_{cu}) are also shown and they are not constant. A similar conclusion was reached[39] for reinforced concrete. The analysis results presented in Fig. 11 do not include the long term effects due to lack of information on time effects on high strength concrete columns.

4. The modulus of elasticity of concrete is an important consideration when calculating the cambers and deflections of prestressed concrete members. The ACI equation for elastic modulus overestimates by as much as 20 percent the modulus for concretes with strengths of about 12,000 psi (84 MPa). The modulus of concrete is also an important parameter in computating prestress losses and buckling of slender, compression-dominant members such as columns. The reported creep and shrinkage of high strength concretes[4] are low, therefore, prestress losses will be reduced for high strength concrete elements.

5. A large number of design parameters in current practice are implicitly related to the tensile strength of concrete, such as development length, minimum reinforcement for flexure, shear and torsion, and maximum stress for shear and torsion. Whether these design parameters are applicable to high strength concrete remains to be examined.

The tensile strength of concrete is often relied upon in working stress design. The ACI Code's permissible extreme fiber tensile stress in the precompressed tensile zone for prestressed flexural elements, $6\sqrt{f'_c}$, can be used with an acceptable degree of conservatism for concretes of higher strengths if split cylinder tests are considered to be representative of the tension in the bottom flange of a prestressed beam. However, from beam flexural tests the results of flexural modulus indicate that $6\sqrt{f'_c}$ may be too conservative.

6. Poisson's ratio of normal and high strength concrete is comparable in the elastic range; hence, there should not be a difference in the behavior of biaxially loaded members such as slabs and triaxially loaded members such as piles and columns, under service load conditions.

7. Although at a material level, high strength concrete is relatively more brittle than normal strength concrete, the same is not the case for sectional ductility. Fig. 12 shows the variation of curvature ductility (ϕ_u/ϕ_y) with the level of axial load, the amount of longitudinal prestressing and the compressive strength of concrete. The computations were carried out using the strain compatibility and force equilibrium equations, assuming plane sections remain plane and using Eq. (1) for stress-strain curve of concrete. The stress-strain curve of 270,000 psi (1890 MPa) for the seven wire prestressing strand was expressed through a polynomial equation.

Prestressed beam-column members of higher strength concrete show similar sectional ductility capability in the region of loads below balanced condition. For low axial loads ($k < 0.1$) the section with higher strength concrete shows relatively more ductility as compared to normal strength concrete sections. Fig. 12 shows that if the amount of prestressing steel is kept constant, then increasing the strength of concrete increases the ductility ratio especially at low values of axial loads.

This analytical observation of increased sectional ductility for beam-column members of high strength concrete (for the same amount of longitudinal steel) should be substantiated with experimental results. The comparison of analytical results for strain at ultimate resistance of the section and the computed curvature ductilities for low and high strength concrete with differing amounts of prestressing steel is presented in Table 4.

Table 4. Analytical results of strain at ultimate and curvature ductility for low and high strength concrete.

f'_c	p	K^*	$K'^\dagger$	ϵ_{cu}	ϕ_u	ϕ_y	$\frac{\phi_u}{\phi_y}$	$\frac{\phi_u}{\phi_y}$ (at ϵ_{cu} = 0.003)
5,000 psi	0.007	0.000	0.118	0.00328	0.617	0.0875	7.051	6.350
		0.085	0.128	0.00335	0.5108	0.1121	4.557	3.979
		0.170	0.131	0.00298	0.3663	0.1360	2.694	—
		0.255	0.132	0.00323	0.3447	0.1677	2.056	1.875
		0.340	0.127	0.00284	0.2528	0.2046	1.236	1.319
	0.009	0.000	0.131	0.00277	0.4227	0.0918	4.606	—
		0.080	0.140	0.00348	0.4640	0.1146	4.048	3.383
		0.161	0.140	0.00343	0.3920	0.1473	2.662	2.626
		0.241	0.136	0.00298	0.2886	0.1712	1.686	1.702
		0.322	0.129	0.00312	0.2698	0.2227	1.211	1.147
13,000 psi	0.007	0.000	0.041	0.00312	1.1086	0.0777	15.255	14.212
		0.097	0.076	0.00340	0.7253	0.1169	6.204	5.132
		0.194	0.100	0.00344	0.5236	0.1626	3.221	2.550
		0.291	0.111	0.00358	0.4245	0.2069	2.052	1.552
		0.388	0.162	0.00347	0.3268	0.2555	1.2789	1.017
	0.009	0.000	0.072	0.00324	0.9425	0.0812	11.610	10.241
		0.940	0.096	0.00333	0.6272	0.1207	5.198	4.469
		0.188	0.112	0.00375	0.5455	0.1639	3.329	2.369
		0.282	0.119	0.00346	0.3820	0.2084	1.833	1.427
		0.376	0.119	0.00349	0.3193	0.2565	1.245	0.982

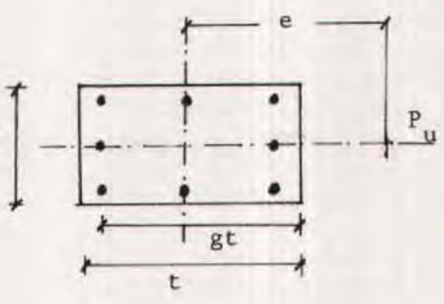
* $K = \frac{P_u}{f'_c\, b\, t}$

† $K' = \frac{Ke}{t}$

Note 1. The above values are for:

f_{pu} = 270,000 psi f_{pe} = 154.9 ksi, ϵ_y = 0.006 and g = 0.7.

Note 2. 1 psi = 0.006895 MPa; 1 ksi = 6.895 MPa.



8. The beneficial effects of the confining reinforcement on the stress-strain curve of concrete depends on the strength of concrete. The effect of the lateral confining reinforcement becomes predominant only after sufficient lateral dilation has taken place; for example, after the concrete has undergone large strain in the most compressed direction. In the inelastic range the lateral dilation of higher strength concrete is relatively less. Thus, to ensure adequate sectional ductility, additional lateral confining reinforcement will be necessary.

9. Additional considerations for use of high strength concrete for precast and prestressed concrete applications are detailed in a recent ACI Special Publication.[53] For examples, see papers by Aswad and Hester, Moksnes and Jakobsen, and Fafitis and Shah in the ACI publication.[53] Further information is given in the list of references.

CONCLUSIONS AND RECOMMENDATIONS

On the basis of the results of this work, the following conclusions can be drawn:

1. There are significant differences in the compressive stress-strain curves of normal and high strength concretes. The curve for higher strength concrete is much more linear to a much higher fraction of the compressive strength. The slope of the post maximum stress range increases as the strength increases.

2. The ACI equation for estimating the secant modulus of elasticity, $E_c = 33W^{1.5}\sqrt{f'_c}$, predicts values as much as 20 percent too high for concretes with compressive strengths in the vicinity of 12,000 psi (84 MPa).

3. The split cylinder strength for low and high strength can be conservatively represented by the expression, $f'_{sp} = 6\sqrt{f'_c}$.

4. The ACI Code's current expression for modulus of rupture, $f_r = 7.5\sqrt{f'_c}$ may be too conservative for high strength concrete and an alternate expression, $f_r = 2\ (f'_c)^{2/3}$, appears to be more representative of the test data.

5. In the inelastic range, high strength concrete exhibits less volume dilation, therefore, the effectiveness of confining lateral reinforcement is relatively less compared to normal strength concrete.

6. The effect of high strain rate on the strength increase is less for higher strength concretes.

7. The current PCI procedure for strength computation is adequate for beam-column members using high strength concrete.

8. At material level, high strength concrete is less ductile than normal strength concrete, but at the sectional level for reinforced concrete elements, if the ratio p/p_b is kept constant, the deflection ductility is essentially independent of the strength of concrete. For prestressed concrete beam-column members, the analytical results indicate that for high level axial loads there is no loss in the curvature ductility with the use of high strength concrete. For low axial load levels (i.e., predominantly flexural behavior) the curvature ductility of high strength concrete prestressed elements is superior to that of normal strength concrete prestressed beam-column members.

9. The test results of solid torsional cylinders and reinforced concrete beams subjected to shear suggest that shear strength appears to be related to compressive strength through a 0.333 power.

10. The use of high strength concrete can increase the span capabilities of prestressed concrete bridge girders and may also reduce the overall depth of the girders.

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NOTE: Discussion of this paper is invited. Please submit your comments to PCI Headquarters by July 1, 1986.

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APPENDIX — NOTATION

f = stress
ϵ = strain
f'_c = uniaxial compressive strength (peak stress)
ϵ_o = strain corresponding to peak stress
A, B, K = calibrating constant
$E_c, (E_c)_s$ = secant modulus of elasticity at $0.45 f'_c$ under static strain rate
W = unit weight in lb per cu ft
f'_{sp} = split cylinder strength
f_r = modulus of rupture of concrete
$(E_c)_{\dot\epsilon}$ = secant modulus of elasticity at strain rate $\dot\epsilon$
$\dot\epsilon$ = strain rate
$\dot\epsilon_s$ = static strain rate = 32 microstrains per sec
$(f'_c)_{\dot\epsilon}$ = compressive strength at strain rate $\dot\epsilon$
α, β = shape factors
$(\epsilon_o)_{\dot\epsilon}$ = peak strain at strain rate $\dot\epsilon$
a = shear span
v_{cr} = shear stress at diagonal cracking
v_u = ultimate shear stress
p = longitudinal steel ratio
d = effective depth, i.e., distance from extreme compressive fiber to center of gravity of tensile reinforcement
A_g = gross area of section
A_c = area of core of spirally reinforced column measured to outside diameter of spiral
f_{yh} = yield stress of hoop steel
p_b = reinforcement ratio producing balanced strain condition
ρ_s = ratio of spiral reinforcement