The merits of partially prestressed concrete as opposed to fully prestressed concrete have been debated since the early days of prestressed concrete construction. On the one hand, the followers of Eugene Freyssinet believed very strongly that full prestressing produced a crack-free material in which flexural tensile stresses should not be allowed under full working load.

On the other hand, some felt that with partial prestressing, tensile stresses could be permitted. In this manner, partially prestressed concrete could be regarded similar to conventional reinforced concrete in which cracking is permitted in the flexural tensile zone. A good historical account of this controversy is given in Ref. 1.

The main advocate of partial prestressing was Dr. Paul Abeles who in the 1940's conducted research with partially prestressed concrete and after World War II designed several partially prestressed structures for British Railways. During the next decade, there was little activity in partial prestressing.

The major breakthrough in the implementation of partial prestressing came in 1968. Switzerland became the first nation in Europe to adopt partial prestressing in its national code, SIA Standard 162. In the last 15 years Swiss engineers have successfully designed and built numerous partially prestressed structures following their code. There is general satisfaction with the provisions in the Swiss Code and the experiences gained during the interim using it.

Unfortunately, the progress in designing and building partially prestressed structures in other parts of the world has been very limited. Reports delivered at the FIP Symposia in Bucharest (1980), the University of Waterloo Conference (1983), and the
recent NATO-sponsored Workshop in Paris (June 1984) reveal that there are not only many differing approaches to the design of partially prestressed concrete structures but also varying opinions on the definition of partial prestressing. In addition, experiences with partially prestressed structures actually built were reported to be scarce.

There are three major reasons why partial prestressing is not fully accepted today.

1. Many existing design procedures for analyzing cracked prestressed sections are considered either too complicated or the calculations are too long.

2. Most codes of practice do not encourage the use of partial prestressing.

3. Experience with actually built partially prestressed structures is lacking.

The purpose of this paper is to show that the design of partially prestressed structures is relatively simple and to present several design examples of actual structures. It should be mentioned that much of this work has been done in cooperation with Dr. Hugo Bachmann in Switzerland. In this manner it has been possible to unify the differing views on partial prestressing and to come up with a more general design procedure for designing such structures.

Experiences gained so far show that partially prestressed concrete structures can be designed and the structural dimensions determined as easily as those for reinforced or fully prestressed concrete structures. In many cases, partial prestressing offers a better solution with regard to simplification of the reinforcement and shape of the members, control of deflection and crack width, and favorable redistribution of bending moment by stiffness reduction of cracked sections.

The most obvious expression for the degree of prestress, $\kappa$, is the ratio of the applied partial prestressing force, $P_{\text{part}}$, and the prestressing force, $P_{\text{full}}$, which causes full prestress under maximum load, i.e., zero stress at the extreme fiber of a concrete member:

$$\kappa = \frac{P_{\text{part}}}{P_{\text{full}}}$$

The values of $P_{\text{part}}$ and $P_{\text{full}}$ are final forces after subtraction of all losses.

It should be noted that this definition of $\kappa$ is only correct when both prestressing forces have the same centroid. When this is not the case, as for example when fewer tendons are used in partial prestressing, the degree of prestress should be defined as the ratio:

$$\kappa = \frac{M_{\text{Dec}}}{M_{\text{max}}}$$

where

$M_{\text{Dec}}$ = moment which produces zero concrete stress at the extreme fiber of a section (nearest to the centroid of the prestressing force), when added to the action of the effective prestress alone.

Synopsis

The author presents a simplified approach for the design of partially prestressed concrete structures. It is shown that partial prestressing, together with supplemental nonprestressed reinforcement, offers a better solution (as compared to fully prestressed sections) with regard to dimensioning and shaping of members, simplification of reinforcement, deflection and crack control, and redistribution of stresses.

The proposed design procedure is illustrated with examples of box girder bridges; beams, double tees, and flat slabs for buildings; river tunnels; storage tanks; and other structures. Some of the examples pertain to precast pretensioned members.
Maximum moment caused by the total service load (dead load plus live load)

Note that this is the same definition used by Bachmann. It should be appreciated, however, that other definitions of the degree of prestress may be useful, as for example the ratio \( \lambda_r \), the force which can be resisted by the prestressing steel above to the force which can be resisted by the total amount of steel (prestressing steel plus nonprestressed steel):

\[
\lambda_r = \frac{A_p f_p}{A_p f_p + A_s f_y}
\]

Note that the ratio \( \lambda_r \) is independent of the magnitude of the actual prestressing force and refers to the portion of prestress in the ultimate limit state. On the other hand, the ratio \( \kappa \) refers to the serviceability state and is approximately the portion of the actual prestressing force to the full prestress.

**APPLICATIONS**

After presenting the philosophy for designing partially prestressed concrete structures, several different design examples are given (1) box girder bridge, (2) double tees, (3) roof beams, (4) submerged tunnel, (5) connection of tank wall to floor slab; (6) flat slabs. Note that some of the members in these examples are precast prestressed. The type of tendons also has been chosen and the number of tendons that can be placed in the section has been determined. In addition, the profile of the tendons and the location of the nonprestressed reinforcement are known.

With the member roughly designed and all loading provided, the bending moments for various load combinations can be determined. Knowing the profile and number of tendons, the prestressing force and the degree of prestress can be calculated. Next, the amount of nonprestressed reinforcement which has to be added to fulfill serviceability requirements can be determined, based on a ultimate limit design state. Then, from the crack width calculation, the bar diameters of the reinforcement are determined and, when necessary, the area is increased. In the event a section does not follow the above design requirements this design procedure must be repeated, but this is seldom necessary.

This method avoids choosing a degree of prestress beforehand including such impractical results as fractional numbers of tendons, tendons which cannot be contained in the section, and disruption of the regular stirrup arrangement. Therefore, the best approach is to first design the section and then choose the prestressing tendons and determine the amount of additional nonprestressed reinforcement. In this way a sound structure is assured.

**Design Philosophy**

In general, it is not necessary to choose the degree of prestress beforehand. In fact, in some cases it is even inadvisable. The majority of the following examples have been designed by starting with sound structural principles. The cross section and shape of the member have been chosen on the basis of simple formwork and reinforcement cage, as well as ease of casting, vibration, demolding, and curing. The type of
Fig. 1. Cross section of fully prestressed box girder in midspan with only nonprestressed reinforcement shown.
Fig. 2. Location of tendons in box girder at midspan (2) and oversupport (1). The three tendons per web, which are not necessary if partial prestressing ($\kappa = 0.68$) is applied, are shown hatched.

wall (taking into account losses of prestress due to shrinkage and creep of the concrete, relaxation of the prestressing steel, and frictional losses during tensioning of the tendons).

Fig. 2 shows the location of the prestressing tendons at midspan and over the support. In both parts of the structure, three tendons from the web are spread apart in the bottom and top flanges, respectively. This means that along the length of the girder the tendons from the bottom flange first have to be curved sideways and then upward into the web.

Three tendons from the web are similarly curved into the bridge deck over the support. This layout of the tendons is fairly complex and, therefore, not easy to carry out in the actual structure. In addition, the solution is uneconomical. If the prestressing tendons were not spread over the bottom flange and deck but rather kept only within the web zone, a total of 20 tendons would be required because of the smaller internal lever arm of the prestressing force.

Bending moment in a box girder causes almost pure tension (or compression) in the top flange. As is generally known, the required reinforcement percentage for pure tension is quite large (between 0.4 and 0.6 percent) since the reinforcement should not be allowed to
Table 1. Amount of prestressed and nonprestressed reinforcement in top and bottom flanges of box girder for two values of degree of prestress.

<table>
<thead>
<tr>
<th>Location</th>
<th>(\kappa = 1.02)</th>
<th>(\kappa = 0.68)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(A_p) mm(^2)</td>
<td>(A_s) mm(^2)</td>
</tr>
<tr>
<td>End span (bottom flange)</td>
<td>21870 (33.9)</td>
<td>5100 (7.9)</td>
</tr>
<tr>
<td>Intermediate support (top flange)</td>
<td>21870 (33.9)</td>
<td>12000 (18.6)</td>
</tr>
<tr>
<td>Center span (bottom flange)</td>
<td>21870 (33.9)</td>
<td>5100 (7.9)</td>
</tr>
</tbody>
</table>

yield if a tensile crack appears:

\[ A_s f_{yw} > A_e f_{ck} \]

where \(A_s\) is the area and \(f_{yw}\) is the yield strength of the nonprestressed reinforcement, respectively; \(A_e\) is the area of the deck or bottom flange and \(f_{ck}\) is the allowable tensile stress of the concrete (5 percent probability of having a lower value).

For example \(f_{yw} = 400\) N/mm\(^2\) (58 ksi); \(f_{ck} = 2\) N/mm\(^2\) (290 psi); \(A_e > 0.5\) percent of \(A_e\). The area of this reinforcement can be accounted for in calculating the ultimate bending moment \(M_u\). In addition, the nonprestressed reinforcement helps control the crack width.

If the box girder is partially prestressed, the degree of prestressing for the girder in the middle of the center span is found to be \(\kappa = 0.68\). Now 12 tendons are needed instead of 18 tendons. The tendon profiles have been kept as simple as possible, staying within the web zones and not spread into the deck (at the support) or into the bottom flange (at midspan) (see Fig. 2).

The nonprestressed reinforcement in the deck and bottom flange must now satisfy the following conditions:

1. Be at least equal to the minimum required cross-sectional area (top and bottom flange subjected to almost pure tension). In this case, the area of the prestressed reinforcement is not taken into account because it is situated only in or near the webs.

2. Together with the prestressing steel, provide adequate safety against structural failure.

3. Limit the crack width at the relevant sections.

4. Limit the stress variation \(\Delta \sigma_r\) in the prestressing steel and in the nonprestressed reinforcement due to maximum live load. Note that a value of \(\Delta \sigma_r \leq 140\) N/mm\(^2\) (20 ksi) has been adopted for both types of reinforcement.

The prestressed and nonprestressed reinforcement is shown in Figs. 1 and 2, and their areas are given in Table 1 for a fully prestressed (\(\kappa = 0.68\)) structure.

Table 1 shows that in the partially prestressed bridge, additional nonprestressed reinforcement is necessary only in the middle part of the center span. Here, the added nonprestressed reinforcement reduces the crack width and stress variations in the prestressed reinforcement and brings the safety factor to the required level (also see
Table 2. Mean concrete stresses and safety factors for two degrees of prestress for middle part of center span.

<table>
<thead>
<tr>
<th>Degree of prestress</th>
<th>Percentage of reinforcement</th>
<th>Mean concrete stress $\sigma_{c,max}$</th>
<th>Safety factor $\gamma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\kappa$</td>
<td>$\lambda_\gamma$</td>
<td>Nonprestressed $\rho_s^*$</td>
<td>Prestressed $\rho_p$</td>
</tr>
<tr>
<td>1.02 (0.95)</td>
<td></td>
<td>0.11</td>
<td>0.48</td>
</tr>
<tr>
<td>0.68 (0.83)</td>
<td></td>
<td>0.28</td>
<td>0.32</td>
</tr>
</tbody>
</table>

*Only the reinforcement in the tensile bottom flange is taken into account.

Table 2 for other pertinent data).

Calculations show that in the fully loaded, partially prestressed bridge no (or only very fine) cracks occur in the flexural tensile zones in the end span and over the intermediate support. In the end span the mean value of the concrete tensile stress in the bottom flange is 1.9 N/mm² (275 psi). The tensile stress above the intermediate support also is very low under maximum load.

The crack width in the center span is between 0.1 and 0.15 mm (0.004 and 0.006 in.) and the stress variation in the nonprestressed reinforcement is about 130 N/mm² (18.9 ksi). The degree of prestress (0.68) is related only to the bending moment of decompression, $M_{dec}$, in the tensile zone of the center span (see Table 2).

Changing from fully prestressed to partially prestressed concrete offers several advantages including fewer tendons, simpler tendon profiles, and a more appropriate safety factor (1.7 in the Dutch Code). Also, the area of the nonprestressed reinforcement in the center span increases locally (bars $\phi_k$ 19-200 instead of bars $\phi_A$ 12-200; or #6 bars at 8 in. instead of #4 bars at 8 in). Thus, cracks occur only in the center span box girder when the structure is fully loaded and close altogether under dead load.

2. Double Tees

Double tees are factory produced on a large scale in the precast prestressed concrete industry. The usual production

![Fig. 3. Cross section of a standard double tee.](image)
Table 3. Initial concrete stresses at bottom fiber and safety factor for two degrees of prestress.

<table>
<thead>
<tr>
<th>Degree of prestress</th>
<th>Percentage of reinforcement</th>
<th>Initial stress at bottom fiber $\sigma_{ei}$</th>
<th>Safety factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\kappa$</td>
<td>$\lambda_r$</td>
<td>Prestressed $\rho_p$</td>
<td>Nonprestressed $\rho_n$</td>
</tr>
<tr>
<td>1.0'</td>
<td>(1.0)</td>
<td>0.56</td>
<td>—</td>
</tr>
<tr>
<td>0.65</td>
<td>(0.85)</td>
<td>0.45</td>
<td>0.38</td>
</tr>
</tbody>
</table>

The method uses a prestressing bed with straight strands, although draped strands are also used. A cross section of a typical double tee (manufactured in the Netherlands) is given in Fig. 3.

The simply supported element has a span of 18.30 m (60 ft) and a depth of 0.60 m (2 ft). The double tee has draped tendons and the concrete quality is B 37.5 corresponding with a standard prism strength of 30 N/mm² (4.35 ksi) at 28 days.

The initial steel stress in the prestressing steel at transfer is 1300 N/mm² (189 ksi). The dead load of the double tee is 8.9 kN/m (0.62 kips/ft) and the live load is 10.6 kN/m (0.74 kips/ft), or 55 percent of the total load. In the case of full prestressing ($\kappa = 1$) in each rib of the double tee, ten seven-wire strands with a cross section of 100 mm² (0.155 in.²) each are needed.

After transfer of the prestress, the compressive stress in the bottom fiber at midspan is 23 N/mm² (3335 psi). The compressive stresses in the anchorage zones are also lower due to the use of draped tendons. Note that the Dutch Code allows only 18 N/mm² (2610 psi) permissible initial compressive stress in the bottom fiber for the given concrete quality.

From this example, it is clear that with the given span and depth, a fully prestressed double tee cannot be used. Therefore, one of the following solutions must be chosen: (1) increase the depth of the double tee or (2) decrease the initial prestressing force by either reducing the number of prestressing strands and adding mild steel reinforcement in the middle part of the span, or decreasing the initial stress in the strands in such a way that the initial compressive stress in the bottom fiber at midspan is 18 N/mm² (2610 psi) or less. In both cases, partially prestressed concrete is used, permitting cracks in the tensile zone at midspan.

Fig. 4 shows the distribution of strands and bars $\phi_{k12}$ over the sections at midspan. The use of mild steel rein-

![Fig. 4. Solution with partial prestressing.](image-url)
Corundum in the production of double tees might appear uneconomical in which case using a lower initial stress in the strands can be considered. However, one should also consider the problem of durability and the increase in steel stress due to cracking of the concrete.

Therefore, the preferred solution would be to reduce the number of prestressing strands and to add nonprestressed reinforcement at midspan.

The deflection of the double tee under permanent load is calculated to be 14 mm (0.55 in.). Under full load (taking into account loss of stiffness due to cracking of the concrete), the total deflection load is computed to be 37 mm (1.46 in.).

Table 3 shows that the initial compressive stress in the bottom fiber in the double tee is much less than in the case of $\kappa = 1$. The effect of reducing the initial prestressing force and adding mild steel reinforcement can now be seen. This illustrates one advantage of partial prestressing in factory produced prestressed concrete members.

In general, partial prestressing is advantageous if the ratio of the dead load to live load is high and some elements in a multistory building carry higher loads (e.g., archives) with no increase of construction depth allowed in that area.
3. Roof Beams

The roof beam considered here has a span of 30.4 m (100 ft) and is supported on 7 m (23 ft) high columns. The beam carries the actual roof structure consisting of roof slabs and joists. The loading on the beam consists of dead load \( q_d = 11.7 \text{ kN/m (0.80 kips/ft)} \) and live load \( q_l = 6.9 \text{ kN/m (0.47 kips/ft)} \). The dead weight of the beam is 12.8 kN/m (0.88 kips/ft). The cross section of the beam, designed as a fully prestressed member, is shown in Fig. 5.

The beam is prestressed by four tendons, BBR-B20, with 20 wires \( \phi_p \) 7 mm each. In addition, the beam also has nonprestressed longitudinal and stirrup reinforcement. The roof beam, designed as a partially prestressed concrete member, is shown in Fig. 6.

As a result of partial prestressing, the cross-sectional shape of the beam can be made simpler than with full prestressing. This can be explained as follows. With fully prestressed concrete the bottom flange has to be of considerable width (due to the large prestressing force) because the initial compressive stress in the bottom fiber is not allowed to exceed a certain maximum value, depending on the specified strength of concrete employed.

However, with partially prestressed
concrete the compressive force is significantly smaller because less prestress is needed. Therefore, a concrete section with a smaller bottom flange can be used (or in this case a section without a bottom flange). The beam is also easier to extract from its formwork because the side walls do not have to be removed.

In addition, partial prestressing reduces the number of prestressing tendons to three, as compared with four tendons in the case of full prestressing. Supplementary nonprestressed reinforcement has to be installed only at the tensile face in the midspan region of the beam. This reinforcement does not extend over the whole length of the beam. Detailing of the main longitudinal reinforcement is shown in Fig. 7.

When the beam is fully loaded, the calculated maximum crack width is less than 0.05 mm (0.002 in.). Some pertinent data of the two solutions are given in Table 4.

Although it will not be further analyzed here, it should be mentioned that this roof can also be produced as a precast concrete member with pretensioned steel. In that case, 22 strands of 12.4 mm (0.5 in.) nominal diameter steel grade FeP 1860 ($f_p = 298$ ksi) will be required together with 14 reinforcing bars of 12 mm nominal diameter (13-$\#4$ bars) in the midspan region.

4. Submerged Tunnel

Fig. 8 shows a typical cross section of a European submerged tunnel. The maximum load is 220 kN/m² (4.3 kips/ft²) or approximately 20 m (66 ft) water depth. An abnormal load of 265 kN/m² (5.2 kips/ft²) or about 24.5 m (81 ft) water depth is also considered, but the probability of this load occurring is only once in 10,000 years based on the water continuing to flow over existing river dikes.

Generally, submerged tunnels are...
Table 4. Mean concrete stresses and safety factor for two values of degree of prestress.

<table>
<thead>
<tr>
<th>Degree of prestress</th>
<th>Percentage of nonprestressed reinforcement</th>
<th>Percentage of prestressed reinforcement</th>
<th>Mean concrete stress, after losses $\sigma_{cm,a}$ (N/mm² (psi))</th>
<th>Safety factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\kappa$</td>
<td>$\lambda_r$</td>
<td>$\rho_s^*$</td>
<td>$\rho_p$</td>
<td>$\gamma$</td>
</tr>
<tr>
<td>0.90 (0.95)</td>
<td>0.15</td>
<td>0.60</td>
<td>$-6.1$ ($-885$)</td>
<td>2.0</td>
</tr>
<tr>
<td>0.68 (0.84)</td>
<td>0.42</td>
<td>0.47</td>
<td>$-4.9$ ($-710$)</td>
<td>1.7</td>
</tr>
</tbody>
</table>

*Pertains to midspan section.

composed of precast segments 100 to 150 m (325 to 500 ft) in length concreted in excavated dry dock. The completed segments are floated to their destination where they are sunk to the designated depth (in a trench dredged in advance) and joined to one another. The wall thickness of the segments is governed more by transporting and sinking requirements than by considerations of structural strength. Such tunnels are built mainly with reinforced concrete.

Because of the heavy water load on the structure, the bending moments to which it is subjected are very large, whereas the compressive stresses in the walls, roof, and bottom slab are relatively small. These conditions necessitate a large quantity of nonprestressed reinforcement, which can be accommodated only by the use of large diameter bars, often arranged in several layers. As a result, it is difficult to limit the crack widths in the tensile zones to low values and calculated crack widths of 0.4 mm (0.016 in.) or more are actually quite common.

Why, then, is prestressing not consid-

Fig. 8. Idealized cross section of submerged tunnel.
Curved prestressing cables can exert pressures of the same magnitude as the water pressure on the tunnel so that no tensile stresses need occur. However, additional prestressing must be applied while the tunnel segments are still in their construction dock and the loads acting on them are due only to their dead load weight and the reduction of the subgrade, which are both small in comparison with water pressure. Thus, full prestressing of the tunnel segments would cause large tensile stresses and failure of the structure would be very probable.

Nevertheless, prestressing of tunnel segments remains an attractive proposition. It will be apparent, however, that it will not be possible to apply full prestressing, only partial prestressing to a degree that no problems arise in stressing the tendons in the construction dock. Tensile stresses are allowed to occur, both in the dock and when the segment has been sunk into position, i.e., the combination of nonprestressed and prestressed reinforcement must ensure that the crack widths are suitably limited.

If partial prestressing is accepted, the application of a symmetrical nonprestressed reinforcement is attractive because of its simplicity. This means that the top and bottom reinforcement (e.g., in a roof or bottom section of the tunnel) have the same cross-sectional area over the entire width of the tunnel. The structure must then be designed so that the crack widths will be very nearly equal in the following cases: in the construction dock under initial prestress plus dead load, and under service conditions with effective prestress plus dead and live load.

In this context, note that a particular crack width in the dock is less serious than the same crack width under service conditions, with regard to the temporary character of the stay in the dock. Cracking also can be expected to occur in the

![Fig. 9. Prestressed and nonprestressed reinforcement of tunnel roof.](image-url)
tunnel roof. This will occur in the construction dock on both the upper face at midspan and on the lower (inner) face near the center support, as well as under service conditions on the lower face at midspan and on the upper face near the center support. The stress situation is just the opposite for the bottom slab of the tunnel.

The reinforcement and prestress arrangements for the tunnel roof are shown in Fig. 9. The area of the non-prestressed reinforcement at the upper as well as the lower face is approximately 0.27 percent of the concrete cross-sectional area.

The prestress is located about 0.11 m (3.6 ft) from both the upper face of the roof near the center support and at midspan from the lower face. The prestressing tendons are entirely within the reinforcement installed at the upper and lower faces. The prestressing system chosen for this tunnel is BBR-U3 with 12 strands of 12.9 mm diameter and initial prestressing force of 1680 kN (386 kips).

The calculations show that cracking can be expected only in that part of the tunnel roof which is adjacent to the center support. The crack widths (given in mm) were calculated with the formula given in the Dutch Code of Practice for concrete construction:

\[ w = (8) \times 10^{-6} \sigma_s \left( 2c + \frac{d_k}{40 \rho} \right) \]  

where

- \( \sigma_s \) = steel stress (N/mm²)
- \( c \) = concrete cover (mm)
- \( d_k \) = bar diameter (mm)
- \( \rho \) = \( A_s/A_c \)

The calculated crack widths near the inner support are:
- During stressing of the tendons: 0.11 mm on inner face
- Under dead plus live load: 0.16 mm on outer face
- Under abnormal load: 0.39 mm on outer face

A further analysis of cracking shows that the crack widths calculated in this way are sensitive to the amount of calculated effective prestress. If this prestress decreases to 90 percent of the value adopted, the crack width under dead plus live load will increase from 0.16 to 0.26 mm. This underlies the extreme importance of taking into account realistic prestress losses.

The tunnel roof at the above mentioned center support can be characterized as follows:
- Degree of prestress – \( \kappa = 0.73 \) (\( \lambda_r = 0.82 \))
- Percentage of nonprestressed reinforcement – \( \rho_n = 0.27 \) percent
- Percentage of prestressed reinforcement – \( \rho_p = 0.27 \) percent
- Mean concrete stress:
  - \( \sigma_{cm,n} = -3.3 \text{ N/mm}^2 \) (-480 psi) (prestress only)
  - \( \sigma_{cm,p} = -4.1 \text{ N/mm}^2 \) (-595 psi) (stresses by horizontal water pressure included)

It can be concluded that the tunnel roof and the bottom slab can be efficiently constructed with partially prestressed concrete. This will result in a simplified reinforcement arrangement and a relatively simple prestress profile. Moreover, crack widths can be very effectively controlled.

5. Connection of Tank Wall to Floor Slab

In many storage tank structures with horizontally prestressed concrete walls, the circular curved wall is designed to move freely in relation to the bottom or floor slab. The joint between wall and floor is detailed in such a way that the anticipated horizontal displacements can be properly accommodated and the joint remains liquid tight. A joint of this type is shown in Fig. 10.

A special case is the joint between the base slab and the safety wall around a tank containing liquified natural gas (LNG). This joint has to remain liquid
tight at very low temperatures to ensure that, in the event of an inner tank failure, the cold liquid will not flow out over a large area. Such a joint is complicated and expensive because the nonprestressed reinforcement, the horizontal tendons, and the anchorages of the vertical tendons are situated in the joint area. In addition, the joint has to be inspected regularly and maintained.

Separate connection details are needed for transmitting to the base slab the horizontal forces that may act upon the safety wall in the event of an external explosion or earthquake. Furthermore, in the occurrence of differential settlement of the relatively flexible slab, the joint will have to allow deformation in the vertical direction. Therefore, from a technical viewpoint, the construction of a wall-to-floor slab connection is quite complex.

In the tanks of sewage treatment plants and other such installations, a "free" joint between wall and base slab is a construction detail which requires much care and attention and which, after being put into service, will continue to need maintenance. If the wall-to-floor slab connection is of monolithic construction, many problems and difficulties are eliminated.

Such a joint does not require extra maintenance and the tank can be embedded easily in backfill soil. However, at the time of horizontally prestressing the wall, the deformation in the horizontal direction is restrained by the monolithic joint. This means that part of the prestress is transmitted to the floor slab, resulting in shear forces and bending moments (with respect to the horizontal plane) occurring in that part of the wall adjacent to the floor slab. This can result in local curvature of the floor slab (see Fig. 11).

The bending stiffness and bending moment at the wall-floor slab connection is reduced by cracking. Therefore, when partial prestressing is applied and horizontal cracks in the wall occur, the bending stiffness and the vertical
bending moment in the wall are reduced (Fig. 12). Research has shown that the bending moment at the wall-floor connection can be nearly halved and the shear forces reduced.

The mild steel reinforcement and prestressing steel arrangements for a connection of this type are shown in Fig. 13. This particular example concerns a safety tank for liquified petroleum gas (LPG) according to the C-IS system. This is an isolated steel tank, surrounded by a prestressed concrete safety tank. The circular space between the walls is filled with nitrogen gas. The tank wall has been designed to resist a water pressure corresponding to a depth of 26 m (85.5 ft) in the water test. The wall-to-floor connection was found to meet expectations entirely.

The following data are relevant to the pattern of forces acting at the connection:

1. **Ring prestress (tangential stresses) of tank wall:**
   - Free movement of wall in relation to the bottom: \( \sigma_c = -11.7 \text{ N/mm}^2 \) (\(-1695 \text{ psi}\)).
   - Monolithic connection, as in Fig. 13, analyzed on nonlinear elastic assumption: \( \sigma_c = -2.6 \text{ N/mm}^2 \) (\(-375 \text{ psi}\)).

2. **Bending moment at connection (Fig. 12a):**
   - Analyzed on linear elastic assumption: \( M = -500 \text{ kN/m} \) (\(-112 \text{ kip-ft/ft}\)).
   - Analyzed on nonlinear elastic assumption: \( M = -370 \text{ kN/m} \) (\(-85 \text{ kip-ft/ft}\)).
Fig. 12. Moment diagrams at wall-to-floor connection of tank. Left: after initial prestress. Right: water test and effective prestress. LE = linear elastic, NLE = nonlinear elastic.
(3) Water test, 26 m (86.5 ft):
(a) Tangential compressive stress at base of wall: \( \sigma_c = -0.9 \text{ N/mm}^2 \)  
\(-130 \text{ psi}\)
(b) Bending moment at connection, analyzed on nonlinear elastic assumption (Fig. 12b): \( M = +250 \)  
kNm/m (+52 kip-ft/ft)

It can be seen that the cracking of a partially prestressed structure has been used to reduce the bending moment to a value which can be resisted by the section.

6. Flat Slab Floors

In Europe, flat slab floors are usually designed by the support strip method. Such slabs are extremely suitable for partially prestressed concrete construction. The design should be based on a simple bottom reinforcement mesh, combined with simple mesh reinforcement at the upper face in the column strips. This mesh reinforcement must be supplemented with extra mild steel bars over the support. The prestress is

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Fig. 13. Monolithic wall-to-floor connection of LPG safety tank.
Fig. 14. Plan of middle panel. Unbonded tendons are concentrated in a zone of 31 in. (800 mm) around centerlines of columns.
applied in the column strips and functions as an artificially applied load which counteracts part of the load in the middle panel between the column strips (see Fig. 14).

As shown in Fig. 14, a load of 4.7 kN/m² (0.093 kips/ft²) has been adopted, excluding the dead weight of the concrete slab which is 0.22 m (9 in.) thick. The column strips contain ten prestressing tendons, each consisting of one 12.9 mm diameter strand. These tendons are installed in the two mutually perpendicular column strips which act as support strips.

Fig. 15 shows the reinforcement and prestressing arrangements at the slab-to-column connection. Studies indicate that in flat slab floors the effect of the normal force due to prestress upon the crack width is often negligible.

The middle panel under consideration has been analyzed for two cases, namely, prestress without bond and prestress with bond, both for $p_r = 0.27$ percent and a degree of prestress, $\kappa = 0.56$. The difference in $p_r$ of nonprestressed reinforcement is small: $p_r = 0.44$ percent for prestress without bond and $p_r = 0.41$ percent for prestress with bond. In both cases the safety factor $\gamma = 1.7$ and the mean concrete stress $\sigma_{cm} = -0.8$ N/mm² (~116 psi).

At the center of the panel, the approximate crack width under full load is 0.26 mm without bond and 0.11 mm with bond. It is apparent from these values that in the latter case the prestressing steel acts also as reinforcement and thus helps to reduce the crack width.

**INFLUENCE OF TIME-DEPENDENT EFFECTS**

In a reinforced concrete column subject to a large axial compressive load over a long period of time, a redistribution of stresses occurs. This stress redistribution is due to shrinkage and creep of the concrete and elastic shortening of the reinforcing steel. As a result of this time-dependent shortening of the concrete, the longitudinal reinforcement also shortens and carries a larger proportion of the column's load, thus relieving the concrete of part of the load.

For an axially loaded column the concrete stress at time $t = \infty$ can be calculated by various methods. Two of these are:

1. **Effective modulus method:**

   $$\sigma_{c_0} = \frac{1}{1 + n \rho (1 + \phi)} (\sigma_{c_0} - \rho E_t \epsilon_{c_0})$$

2. **Dischinger's method:**

   $$\sigma_{c_0} = \sigma_{c_0} + \frac{\epsilon_{c_0} E_t}{\phi (1 + n \rho)} (1 - e^{-\eta})$$

   where

   $$\eta = \frac{n \rho \phi}{1 + n \rho}$$

Although there are more advanced analytical methods than Dischinger's equations to determine the concrete stresses, it appears from the calculations in Ref. 10 that this method gives the best results when compared to test data.

Although the preceding discussion relates to a column, the theory also applies to the tensile zone of a prestressed concrete beam, except that the beam calculations are usually more complicated. The general behavior mode of a partially prestressed beam subject to sustained load is that the compressive stress in the extreme fiber of the concrete section decreases while the compressive stress in the reinforcing steel increases, i.e., the tensile stress in the prestressing steel decreases.

Using Dischinger's method, the relation between the concrete stress $\sigma_{c_0}$ in the bottom fiber, combinations of time-dependent effects ($\rho_r$; $\epsilon_{c_0}$), and the reinforcement cross section ($\rho_r$; $\rho_r$) have been plotted in Fig. 16.

The example chosen is a partially pre-
Fig. 15. Slab-to-column connection.
Fig. 16. Relation between concrete stress $\sigma_{c2}$ in bottom fiber and various combinations of creep factor $\phi_c$ and shrinkage value $\epsilon_{cs,\infty}$ for three values of $\rho_s$. 

- $\rho_s = 0$
- $\rho_s = 2 \rho_p$
- $\rho_s = 6 \rho_p$

Compressive stress (N/mm$^2$) in bottom fibre:

- $\sigma_{c2}$ vs $\rho_s$

Material properties:
- $M_{Dec} = 169$ kN/m
- $P_i = 573$ kN
- $\sigma_{p1} = 1100$ N/mm$^2$
- $A_p = 521$ mm$^2$
- $A_c = 180 \times 10^3$ mm$^2$
- $E_c = 35,000$ N/mm$^2$
- $E_s = 205,000$ N/mm$^2$
stressed concrete tee beam. Relaxation losses of the steel stresses were not taken into account. The calculation was based on the assumption that the bending moment \( M_0 \) due to the dead load does not change with time and at \( t = 0 \) the prestressing force is 573 kN (129 kips) regardless of the cross-sectional area of the nonprestressed steel.

Actually, the ultimate load carrying capacity of the member is increased when supplemental reinforcing steel is placed in the tensile zone. Directly after tendon stressing, the compressive stress in the concrete is reduced from \(-6.1 \text{ N/mm}^2\) (\(-885 \text{ psi}\)) for \( p_s = 0 \) to \(-5.3 \text{ N/mm}^2\) (\(-770 \text{ psi}\)) for \( p_s = 2 \rho_p \) or to \(-4.2 \text{ N/mm}^2\) (\(-610 \text{ psi}\)) for \( p_s = 6 \rho_p \). This is due to the influence of the nonprestressed steel.

In the case where \( p_s = 2 \) and \( \varepsilon_{cs, w} = -250 \times 10^{-6} \), the magnitude of the time-dependent shortening in the tensile zone is:

\[
\varepsilon_{c, t+} = \frac{2(-5.4)}{35000} - 250 \times 10^{-6} = -559 \times 10^{-6}
\]

Hence, the decrease in the steel stress is \( 559 \times 10^{-6} \times 205,000 = 114 \text{ N/mm}^2 \) (16.5 ksi).

For a bending moment:

\[
M_d = 169 \text{ kNm and } P_e = 521 (1100 - 114) \text{ N}, \text{ we now obtain } \sigma_{c, t} = -4.5 \text{ N/mm}^2 \text{ (-650 psi). For the case where } p_s = 0, \text{ Dischinger's method gives a value of } -4.7 \text{ N/mm}^2 \text{ (-680 psi).}
\]

This calculation turns out to be very simple and as Fig. 16 shows, reasonably accurate when \( p_s = 0 \). However, if supplemental reinforcing steel is present, the concrete compressive stress \( \sigma_{c, t} \) which develops over time is considerably smaller. Therefore, the compressive stress in the flexural tensile zone of a member can be significantly reduced by time-dependent effects when nonprestressed steel is added in the tensile zone of an originally fully prestressed section.

However, in a partially prestressed concrete member, part of the prestressing steel used to produce full prestress is replaced by nonprestressed steel. In this case, the time-dependent effects only moderately reduce (which can often be neglected) the compressive stress. Therefore, a question arises as to how the simplified analysis affects the magnitude of the decompression moment, \( M_{de,r} \), and the crack width, \( w_c \) associated with full load, \( M_{D,L} \).

To answer this question, a study was conducted at the Delft University of Technology using mathematical models in which the stress distributions in the cross sections of partially prestressed structures were determined. Using the simplified design method, several structures (double tees and box beam) were analyzed with different degrees of prestress.

**Example**

Suppose that immediately after stressing the tendons, the concrete stress at the level of the steel centroid is \(-6.1 \text{ N/mm}^2\) (\(-885 \text{ psi}\)). With \( \phi_a = 2 \) and \( \varepsilon_{cs, w} = -250 \times 10^{-6} \), the magnitude of the time-dependent shortening in the tensile zone is:

\[
\varepsilon_{c, t+} = \frac{2(-5.4)}{35000} - 250 \times 10^{-6} = -559 \times 10^{-6}
\]

Hence, the decrease in the steel stress is \( 559 \times 10^{-6} \times 205,000 = 114 \text{ N/mm}^2 \) (16.5 ksi).

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Fig. 17. Relation between decompression moment $M_{\text{dec}}$ and degree of prestress $\kappa$ for midspan section of double tee with span of 66 ft (20 m).

This simplified calculation procedure resulted in an almost linear relationship between the decompression moment $M_{\text{dec}}$ and the degree of prestress $\kappa$. The mathematical models also showed what the actual $M_{\text{dec}}$ versus $\kappa$ relationship is, based on a particular variation of creep and shrinkage of the concrete. The result is presented in Fig. 17.

It appears from Fig. 17 that $M_{\text{dec}}$ calculated for various degrees of prestress by the simplified method differs only slightly from $M_{\text{dec}}$ computed by Dirschinger's method. Also, variations in shrinkage and creep of the concrete have little effect on the magnitude of $M_{\text{dec}}$. The line terminating at 1175 kNm in the diagram was calculated by the simplified method. The line terminating at 1106 kNm relates to $\varepsilon_{\text{cs}, \infty} = 200 \times 10^{-6}$ and $\phi_\alpha = 1.5$. The next line relates to $\varepsilon_{\text{cs}, \infty} = 250 \times 10^{-6}$ and $\phi_\alpha = 2.0$, and the bottom line to $\varepsilon_{\text{cs}, \infty} = 300 \times 10^{-6}$ and $\phi_\alpha = 2.5$.

Although this study appears to contradict the results of the calculations presented in Fig. 16, this is not the case. To clearly show the effect of the steel cross section in the flexural tensile zone on the stress redistribution due to time-dependent effects (see Fig. 16), reinforcement was added to the section without the cross-sectional area of the prestressing steel being reduced in proportion. However, in Fig. 17, such a reduction was made in the calculations.

The magnitude of the prestressing force is reduced when the steel cross-sectional area is increased so that the ultimate moment $M_u$ of the section remains constant. Therefore, reduction of the prestressing force means a decrease in the magnitude of the compressive stress $\sigma_{\text{cs}}$, in addition to a decrease in the influence of time-dependent effects. As a result, the structure becomes stiffer and the concrete stresses undergo smaller increases per unit of bending moment than when there is little steel in the flexural tensile zone. This applies more particularly to the stage where the tensile zone is uncracked.
The Delft University of Technology study\textsuperscript{11} investigated the magnitude of the steel stress attains if a structure is designed and built with partially prestressed concrete. In the study, it was assumed that the initial stress in the prestressing steel was $\sigma_{pl} = 1395$ N/mm\textsuperscript{2} (202 ksi). This value is 75 percent of the characteristic value of the tensile strength (1860 N/mm\textsuperscript{2} or 298 ksi).

The analysis was based on complete bond of the prestressing steel to the concrete, such as occurs more particularly in prestressed concrete with pretensioned steel. As an example, the results of the calculations for a double tee with a span of 66 ft (20 m) are given in Fig. 18. Other calculations using other beams gave similar results.\textsuperscript{11}

The steel stresses occurring under three bending moments — $M_D = 695$ kNm (dead load), $M_{D+L} = 1175$ kNm (full load), and $M = 940$ kNm (half live load) — are shown in the diagram. The bending moment at which the tensile zone cracks is indicated as $M_{cr}$. It appears from the figure that the variation in shrinkage and creep of the concrete has little effect on the steel stress where the flexural tensile zone has cracked.

The diagram also shows that for low degrees of prestress, the steel stress under full load can become substantially higher than the initial steel stress, due to the effect of cracking in the tensile zone. Thus, for $\kappa = 0.6$ the value attained is approximately equal to the initial stress, and for $\kappa = 0.4$ it is higher, namely, 1450 N/mm\textsuperscript{2} (210 ksi). This means that the magnitude of the stress in the prestressing steel should be properly checked when low degrees of prestress are employed.

It is not possible to make definite assertions as to the maximum permissible steel stress under full load. However, this maximum should not be higher than the initial stress $\sigma_{pl}$, and preferably, it should be lower. The problem of the stress in the prestressing steel under full load and with a low degree of prestress is, however, not as serious as might be assumed.

Complete bond between the prestressing steel and the concrete only occurs when pretensioned tendons are used: partial prestressing with a low degree of prestress is seldom used. Should a low degree of prestress be considered, it will be necessary to limit the initial stress in the prestressing steel so that after cracking, a high value does not occur under full load. The steel stress at the crack can be calculated according to the method described in Ref. 11.

In addition, with prestressed concrete using post-tensioned tendons, there is never complete bond between the prestressing steel and the concrete. This is obvious when unbonded tendons are used. If the tendons are enclosed in sheaths which are grouted after tensioning, the role played by the tendon is different than in the case where each tendon is directly embedded in concrete. At a flexural tensile crack, the stress in the steel of a post-tensioned tendon does not increase as much as the stress in a fully bonded (embedded) bar of the reinforcement or a pretensioned tendon at such a crack. Due to the poor bond, the elongation is distributed over a greater length of the tendon.

In cases with bonded nonprestressed reinforcement in the tensile zone, the effect of the steel in grouted cables (post-tensioned tendons) can be restricted by the introduction of a reduction factor $c$ for the area $A_p$ of the prestressing steel:\textsuperscript{8}

$$c = \frac{1}{\sqrt{n}} \cdot \frac{\phi_x}{\phi_p} \leq 1$$  \hspace{1cm} (4)

where

- $n = \text{number of wires in a tendon}$;
- each wire in a strand to be counted separately
- $\phi_x = \text{diameter of a bar of the non-
prestressed reinforcement

\[ \phi_p = \text{diameter of prestressing steel, i.e., diameter of individual wires or bars or the wires of which a strand is composed} \]

To calculate the steel stress at the crack, it is necessary to take into account the same proportion of the cross section of prestressing steel, namely, the cross sectional area of the steel to be adopted as tensile reinforcement is equal to \( A_s \) plus \( cA_p \). For example, consider a prestressing cable comprising 20 strands (7 wires \( \phi_p = 4 \text{ mm} \)) in conjunction with reinforcing bars \( \phi_s = 12 \text{ mm} \). Then:

\[ c = \frac{1}{140} \times \frac{12}{4} = 0.25 \]

Therefore, when cracking occurs in the tensile zone of a member with only post-tensioned, grouted cables, there will be wide cracks at great distances. To distribute the cracks and limit the crack width, at least a minimum amount of nonprestressed reinforcement should be used in the tensile zone of post-tensioned beams or slabs in which cracking is not fully excluded. This also is advocated by Bachmann\(^6,7\) and others.

In prestressed concrete with post-tensioned tendons, the tendons are usually located at somewhat greater distances from the underside of the beam than the reinforcing bars are. For this reason, too, the increase in stress in the prestressing steel due to cracking will be less than that in the nonprestressed reinforcement.

The research at Delft University shows that the stress increase in the prestressing steel after cracking may become quite significant when low degrees of prestressing are employed. This is, however, particularly true of prestressed concrete with pretensioned steel.

![Fig. 18. Relation between degree of prestress \( \kappa \) and stress \( \sigma_p \) in prestressing steel under three different loads.](image-url)
LIMITING CRACK WIDTH UNDER FULL LOAD

The magnitude of the crack width is governed by the following factors:

1. The steel cross-sectional area that is joined to the concrete by bond in the tensile zone.
2. The quality (specified strength) of the concrete.
3. The diameter of the reinforcing bars and their surface profile (ribs, etc.).
4. The depth of the concrete cover and the location of the tendons within the section of the member.
5. The increase of the stresses in the prestressed and nonprestressed steel when the bending moment increases from $M_{pre}$ to $M_{max}$.

In applying the simplified method of analysis, the stress in the reinforcing steel or the increase of stress in the prestressing steel can be determined by considering the critical section as an eccentrically loaded section. The principle of this analysis is shown in Fig. 19.

The increase of strain in the nonprestressed reinforcement and in the prestressing steel will be calculated, as shown by Bachmann using the principle "bending with compressive force $P_e$." The section is conceived as being loaded by the effective prestressing force $P_e$ which acts at the centroid of the prestressing steel, and by a bending moment $M_{pre}$.

The eccentrically acting compressive force is resisted by the reinforced concrete section containing steel with a cross-sectional area $A_{st}$ in the compressive zone and $A_{st} + cA_p$ in the tensile zone. The compressive zone is subjected to a total compressive force:

$$ N = P_e + A_{st} \sigma_{et} + A_p \Delta \sigma_p $$

where $\Delta \sigma_p$ denotes the increase in tensile stress (over and above the effective stress) in the prestressing steel due to the cracking of the concrete.

For prestressing cables (post-tensioned tendons), a reduction factor $c$ is introduced. This compressive force is resisted by the concrete in the compressive zone (which has a depth $h_c$) and by the reinforcing steel $A_{st}$. The conditions of compatibility must be satisfied:

$$ \frac{\sigma_{et} \cdot \frac{1}{h_x}}{E_c} = \frac{\Delta \sigma_p}{E_p} \cdot \frac{1}{c} \cdot \frac{1}{(d_p - h_x)} $$

$$ = \frac{\sigma_{et}}{E_s} \cdot \frac{1}{(d_s - h_x)} \quad (5) $$

In the analysis, the magnitude of $h_x$ has to be determined by trial and error so this condition is satisfied. This approach is comparable to the one adopted in the modular ratio method for reinforced concrete except that in pure bending $h_x$ changes when the bending moment changes in magnitude. The tensile stress $\sigma_{et}$ in the reinforcing steel is obtained from this analysis. The crack width $w$ can then be determined with the aid of governing codes of practice for reinforced concrete.

Fig. 20 shows the relation between stress increase $\Delta \sigma_{et}$ in the prestressing steel due to cracking in the tensile zone and the action of both full load ($M = 1175 \text{ kNm}$) and dead load plus half the live load ($940 \text{ kNm}$). It can now be seen that by decreasing the degree of prestress the tensile zone increase becomes more pronounced and the effects of concrete shrinkage and creep become negligible.

In order to show the effects which govern crack widths, a reinforced concrete tension member subjected to elongation will briefly be considered. Such a member is comparable to the tensile flange of a box beam just after the cracking moment has been exceeded. The relation between crack width, bar diameter, and steel stress at the crack (here indicated as $\sigma_{ecr}$) is shown in Fig. 21. The reinforcing steel considered consists of ordinary deformed bars with a ratio $f_t$ between rib height and rib spacing of 0.065.

It is apparent from this diagram that in the case of elongation and an associated steel stress of, for example, $130 \text{ N/mm}^2$,
Fig. 19. Derivation of steel stresses under $M_{\text{max}}$. 
The crack width can be limited to 0.05 mm by using bars with \( \phi_k = 6 \) mm. This is, of course, not a suitable diameter for practical use. If bars with \( \phi_k = 10 \) mm are used, the crack width is limited to about 0.08 mm, while 16 mm and 25 mm bars limit the crack widths to about 0.10 mm and 0.15 mm, respectively.

Thus, crack width associated with elongations can be controlled by a suitable choice of deformed bar diameter, reinforcement cross-sectional area for the given concrete quality (specified strength) and location of the bars in the structural section. Raising the load above the value corresponding to the cracking moment causes the steel to increase, a fact that must be considered. This can be done by assuming that, at higher steel stress values, the crack width increases proportionately to the steel stress.

In the case of members loaded in bending, it is usually possible to apply the rules for crack width calculation which are given in various codes of practice.

**CONCLUSION**

This article briefly reviews research carried out at the Delft University of Technology with regard to the possibilities offered by partially prestressed concrete. On the basis of this research it is shown that a simplified approach to crack width calculation is permissible. This means that the stresses in the reinforcing steel under full load can be calculated by a procedure similar to the
Concrete cube strength is 22.5 N/mm$^2$ (cylinder strength is 18 N/mm$^2$ or 2.61 ksi).

If the stress in the reinforcing steel is known, the bar diameters of that steel can be chosen, within the limits used in actual practice, so that crack width limit requirements can be fulfilled. However, to do this, it may be necessary to adapt the reinforcement cross-sectional area to suit individual cases.

As a result of cracking in the flexural tensile zone, the stresses in the prestressing steel crossing the crack increase. By using the simplified method of analysis and design, this effect can be taken into account as well as bond behavior differences between post-tensioned prestressing cables and reinforcing steel anchored by bond.

In conclusion, it has been shown that the procedure for designing partially prestressed concrete structures is no more difficult than the design of fully prestressed or conventionally reinforced members. Partial prestressing offers many distinct advantages which can be used to produce more efficient and imaginative concrete structures.

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NOTE: Discussion of this paper is invited. Please submit your comments to PCI Headquarters by November 1, 1985.
APPENDIX — NOTATION

\( \begin{align*} 
A_c &= \text{cross-sectional area of concrete} \\
A_p &= \text{cross-sectional area of prestressing steel} \\
A_n &= \text{cross-sectional area of non-prestressed reinforcing steel} \\
E_c &= \text{modulus of elasticity of concrete} \\
E_p &= E_s = \text{modulus of elasticity of steel} \\
M_D &= \text{bending moment due to dead load} \\
M_{Dec} &= \text{bending moment of decompression} \\
M_{D+L} &= \text{maximum bending moment} \\
M_u &= \text{ultimate bending moment} \\
N &= \text{normal force} \\
P_i &= \text{initial prestressing force} \\
P_e &= \text{effective prestressing force} \\
c &= \left( \frac{1}{\sqrt{n}} \right) \left( \phi_k / \phi_p \right) = \text{factor that takes into account the reduced bond properties of grouted tendons; } (n = \text{number of wires of the tendon}) \\
F_{su} &= \text{yield strength of non-prestressed reinforcement} \\
f_{ck} &= \text{permissible tensile strength (upper value) of concrete} \\
j &= \text{internal lever arm} \\
n &= \text{modular ratio} \\
t &= \text{time} \\
w &= \text{crack width} \\
\gamma &= \text{ safety factor for ultimate load} \\
\epsilon_{ck,0} &= \text{ shrinkage strain from } t = 0 \text{ to } t = \infty \\
\eta &= \text{ coefficient } = n_p \phi_\infty / (1 + n_p) \\
\kappa &= \text{ degree of prestress } \kappa = M_{Dec} / M_{D+L} \\
\rho &= \text{ } A_{e+p}/A_e \\
\rho_s &= \text{ } A_s/A_e \\
\rho_p &= \text{ } A_p/A_e \\
\sigma_{ci} &= \text{ concrete stress at top fiber (compressive zone)} \\
\sigma_{ca} &= \text{ concrete stress at bottom fiber} \\
\sigma_{ce} &= \text{ concrete stress at } t = 0 \\
\sigma_{cm} &= \text{ concrete stress at } t = \infty \\
\sigma_{cm, m} &= \text{ concrete stress, mean value at } t = \infty (P_\infty + M_d) \\
\sigma_{pi} &= \text{ initial stress in prestressing steel at } t = 0 \\
\sigma_{st} &= \text{ steel stress in reinforcement in compressive zone} \\
\sigma_{ta} &= \text{ steel stress in reinforcement in tensile zone} \\
\sigma_{a,cr} &= \text{ steel stress in reinforcement at cracking} \\
\phi_\infty &= \text{ creep factor from } t = 0 \text{ to } t = \infty \\
\phi_k &= \text{ bar diameter of reinforcement} \\
\phi_p &= \text{ bar diameter of prestressing steel} \\
\Delta \sigma_p &= \text{ stress increase in prestressing steel due to cracking of concrete} \\
\Delta \sigma_{a,cr} &= \text{ stress increase in normal reinforcement due to cracking of concrete} \\
\end{align*} \)