Design of Reinforced and Prestressed Concrete Inverted T Beams for Bridge Structures

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Prefabricated concrete stringers with cast-in-place slab are frequently used to achieve economical and speedy bridge construction schemes. Beams constructed in the form of an inverted T possess on each side of the web a bracket or flange overhang that provides a convenient shelf or supporting surface for the precast stringers. Hence, cast-in-place, post-tensioned, prestressed concrete (PC) and reinforced concrete (RC) inverted T beams are frequently used in bridges as bentcap girders as indicated in Fig. 1.

Inverted T beams can be simply supported, cantilevered over simple supports, or they can be constructed monolithically with columns or piers. They reduce overall floor depth by avoiding deep cross members beneath prefabricated stringers, resulting in lower abutments and shorter approaches for the bridges.

Despite its frequent use for at least the past two decades, the inverted T shape remains as one of the least investigated cross sections. At present, no guidance for handling design problems specifically associated with the inverted T section is available in current North American standards.[1-3] Consequently, engineers have tended to rely on personal judgment and discretion for design of inverted T girders.

This paper summarizes design recommendations that are based on observations and analyses of cast-in-place normal weight, PC and RC laboratory test specimens reported in studies of inverted T beams.[4-7] The paper is directed toward the design office audience. Hence, only the highlights and conclusions from the reported research associated with design aspects of the inverted T beam bridge bentcaps are presented here.

Further information and details are fully available elsewhere.[4-8] It should be pointed out that these recommendations are being successfully used by the Texas State Department of Highways and Public Transportation for design of inverted T beams.

Synopsis

The structural behavior of reinforced and prestressed concrete inverted T Beams differs from that of conventional top-loaded beams. The loads that are introduced into the bottom rather than into the sides or the top of the web of an inverted T beam impose special problems, which are not dealt with in existing structural codes.

This paper provides recommendations for proportioning cross section dimensions and reinforcement of cast-in-place, post-tensioned and reinforced concrete inverted T beams used in bridge structures. A design example is included to elaborate the application of these recommendations.

PROBLEMS AND IMPLICATIONS

Stringer bearings on the top face of the flange of an inverted T beam impose vertical tensile forces (hanger tension) near the bottom of the web as indicated in Fig. 2. Such forces are not ordinarily encountered in conventional beams, where vertical forces are applied at the top of the web. Furthermore, the longitudinal and lateral bending of the flange of an inverted T beam produce a very complex stress distribution in the flange. Hence, the design of reinforcement for the web and for the flange of an inverted T section imposes special problems.

Loading conditions involving torsion on the inverted T section might create more severe requirements for proportioning reinforcement. Torsion of inverted T bentcap girders occurs with every passage of design vehicles across the bentcap. As traffic approaches the bentcap, stringer reactions cause twisting or torsion of the bentcap toward the approaching load. The direction of twist reverses after the passage of traffic imposes loads on stringers that react on the opposite flange overhang of the inverted T beam. Hence, the passage of traffic tends to make the twist a reversing phenomenon.

The state of combined stress in an inverted T beam cannot be obtained by superposition of simple stress cases. Concrete cracks at a nominal amount of tensile stress and the analytical description of mechanisms and material characteristics must change for each subsequent load stage. Hence, a general analytic solution for strength of an inverted T beam that is versatile enough for all possible load cases and simple enough for design office application does not seem to be within reach at the present time. The authors have, therefore, resorted to empiricism supported by a rational interpretation of test results in order to develop design criteria for inverted T beams.

TEST PROGRAM

Laboratory test specimens that formed the basis of design recommendations presented in this paper represented model bentcap girders at a scale of approximately one-third the size of the prototype members used by the Texas

State Department of Highways and Public Transportation. Strength of normal weight concrete, reinforcing steel and prestressing strand used were typical of those currently employed in the industry. The prestressing strands were straight and each strand was post-tensioned and grouted.

The principal variables for the specimens involved reinforcement details associated with the web and the flange. The observations included service load twist reversals to simulate the passage of traffic and subsequent loads that caused failure in a region of the test specimens. Hence, the typical load sequences involved three phases of observations:

First, loads simulating service dead load stringer reactions were applied and maintained.

Second, service live load forces were applied and removed to produce several cycles of simulated passage of traffic.

Third, loads were increased until failure occurred in a region of the test specimen.

Other details are reported elsewhere[4,5] and will not be repeated here.

RESULTS AND RECOMMENDATIONS

Six modes of failure plus at least one service load condition should be considered as a part of the design of in-

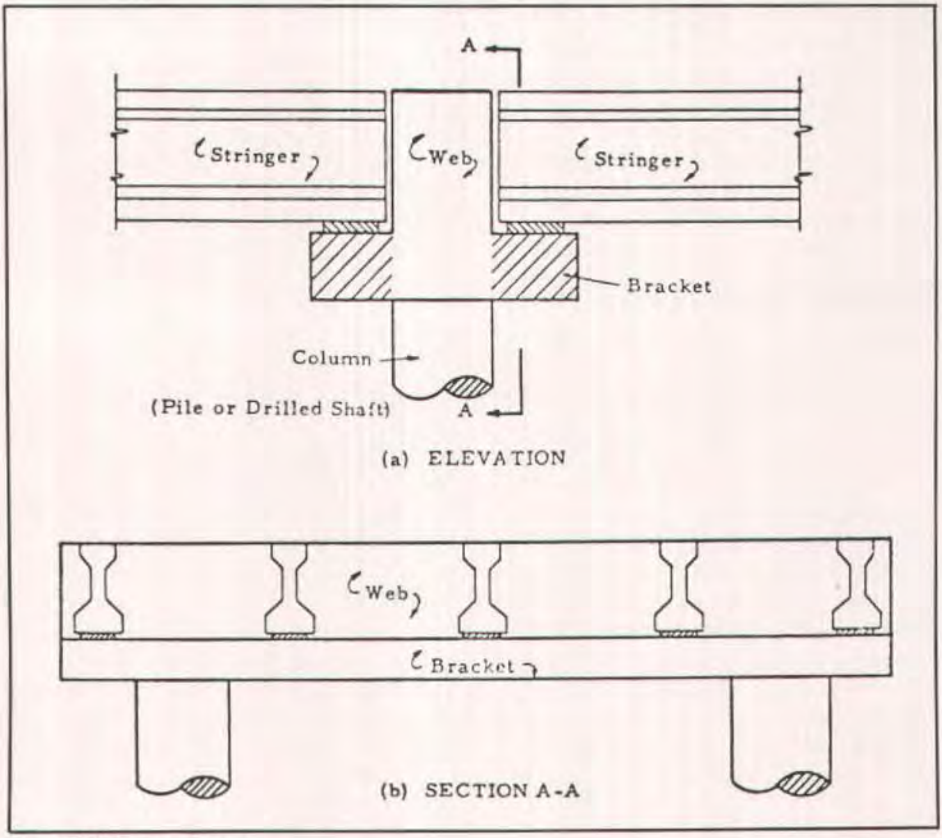

Fig. 1. Highway pier cap.

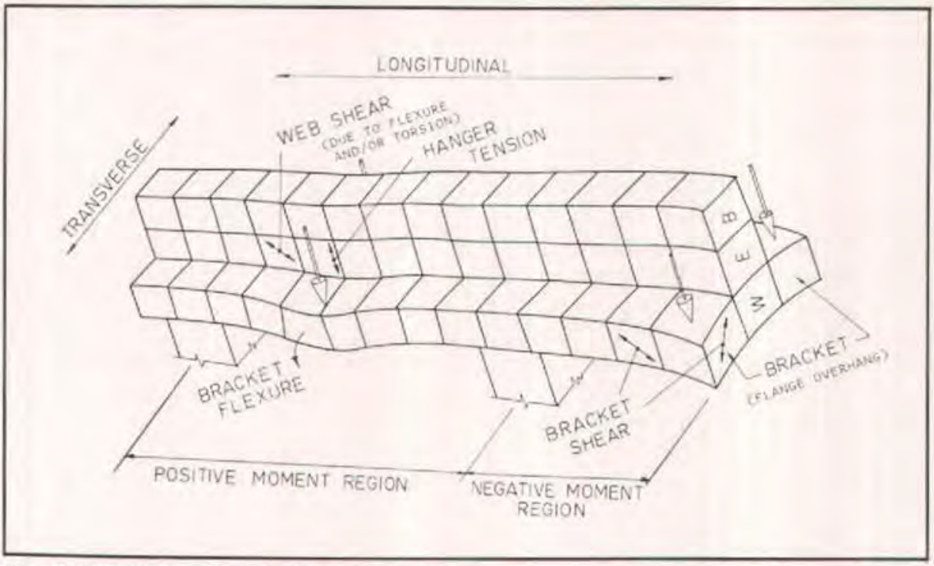

Fig. 2. Structural actions on inverted T beams.

verted T girders. The six modes of failure involved the possibility of failure due to:

(1) Flexure of the overall inverted T beam;
(2) Flexural shear acting on the overall inverted T beam;
(3) Torsional shear on the overall cross section;
(4) Hanger tension on web stirrups;
(5) Flange punching shear at stringer bearings; and
(6) Bracket type shear friction in flange at face of the web.

The service load condition involves the possible wide cracking at the interface of the web and the flange due to premature yielding of stirrups acting as hangers nearest the concentrated loads. Typical forces and stress types acting on inverted T beams are illustrated in Fig. 2.

The overall strength of an inverted T beam must be adequate to support ultimate flexure, flexural shear and torsional shear forces and any possible combination of such forces. The local strength of inverted T beam components must be adequate to support forces that are applied as concentrated loads on flange overhangs. Locally, the flange must be deep enough to avoid punching shear weakness, the transverse flange reinforcement must be strong enough to maintain shear friction resistance at the face of the web, and the web stirrups must be sufficient to act as hangers to transmit flange loads into the web.

Service load conditions of deflection and crack control may be more significant than strength requirements for some components of design. Decisions regarding the overall depth of web and the distribution of tensile reinforcement, both for flexure and for stirrups acting as hangers, may involve service load conditions of behavior.

The height of the web above the top face of the flange of an inverted T beam is determined by the required depth of the stringer to be supported on the flange. A minimum depth of the flange itself can be derived from punching shear requirements, but additional depth may be appropriate to provide enough flexural stiffness for the overall member. The width of the web can be selected for adequate strength in shear alone and in combined shear and torsion, or it may be determined by placement requirements of flexural reinforcement.

The length of the flange overhang is controlled by the size of the bearing pads used to support stringers and should not exceed the flange thickness. In addition to accommodating the bearing pad width, the flange overhang has to provide for the edge distance, the sweep tolerance of the T beam, the length tolerance of the stringers and the erection and placing tolerances.

The design of an inverted T beam can be divided into three parts:

(1) Design of flange;

(2) Design of web stirrups acting as hangers to deliver flange forces into the web; and

(3) Overall design of the beam itself.

Hence, the design recommendations are presented in three parts as well. Note that these recommendations apply to cast-in-place, post-tensioned concrete and reinforced concrete inverted T beams employed in bridge construction.

Design of Flange

The strength of the flange should be adequate to sustain the punching shear action of the stringer loads applied on the flange. In addition, the flange should be able to resist the shear friction forces at the face of the web caused by the bracket action of the flange.

Punching shear in flange — The flange should be deep enough to prevent punching shear failure. This can be achieved by satisfying the following equation for effective flange depth (d_f) from the top face of the flange to the top of the bottom layer of transverse reinforcement in the flange:

$$4\phi\sqrt{f_c'}\,(B_p + 2d_f)\,d_f \geq P_u \qquad (1a)$$

or

$$d_f \geq \frac{B_p}{4}\left[\sqrt{1 + \frac{2P_u}{\phi\, B_p^2\,\sqrt{f_c'}}} - 1\right] \qquad (1b)$$

in which

d_f = effective depth of flange (see Fig. 3 and definitions in Notation section)

$B_p = B + 2B_w$

B = length of bearing pad along edge of flange

B_w = width of bearing pad perpendicular to beam axis

P_u = ultimate concentrated load acting on one bearing pad

ϕ = capacity reduction factor for shear and equals 0.85

Eq. (1b) is a cumbersome expression. Fortunately, the equation can be applied readily as a graph with B_p versus P_u for various values of d_f. Such graphs for specified concrete strength f_c' taken as 4000 and 5000 psi (27.6 and 34.5 MPa) are shown in Figs. 3 and 4, respectively.

Similar graphs can be prepared for other concrete strengths. It may be pointed out that the most common strengths of concrete used for bentcap girders are 4000 and 5000 psi (27.6 and 34.5 MPa) for reinforced and post-tensioned prestressed concrete construction, respectively.

Eq. (1b) is based on a tensile strength of concrete equal to $4\sqrt{f_c'}$ acting on the surface of a truncated pyramid under a bearing pad and is supported by test results.[4] Stirrups that intersect a face of the truncated pyramid can help support the concentrated load if the anchorage of the stirrups can be developed above and below the face of the truncated pyramid. However, no such help from the stirrups was included in Eq. (1b) because this would require cumbersome checks on design and detailing of stirrups.

The surfaces of truncated pyramids resisting ultimate punching shear under adjacent stringers should not overlap. This can be achieved by providing enough longitudinal and transverse

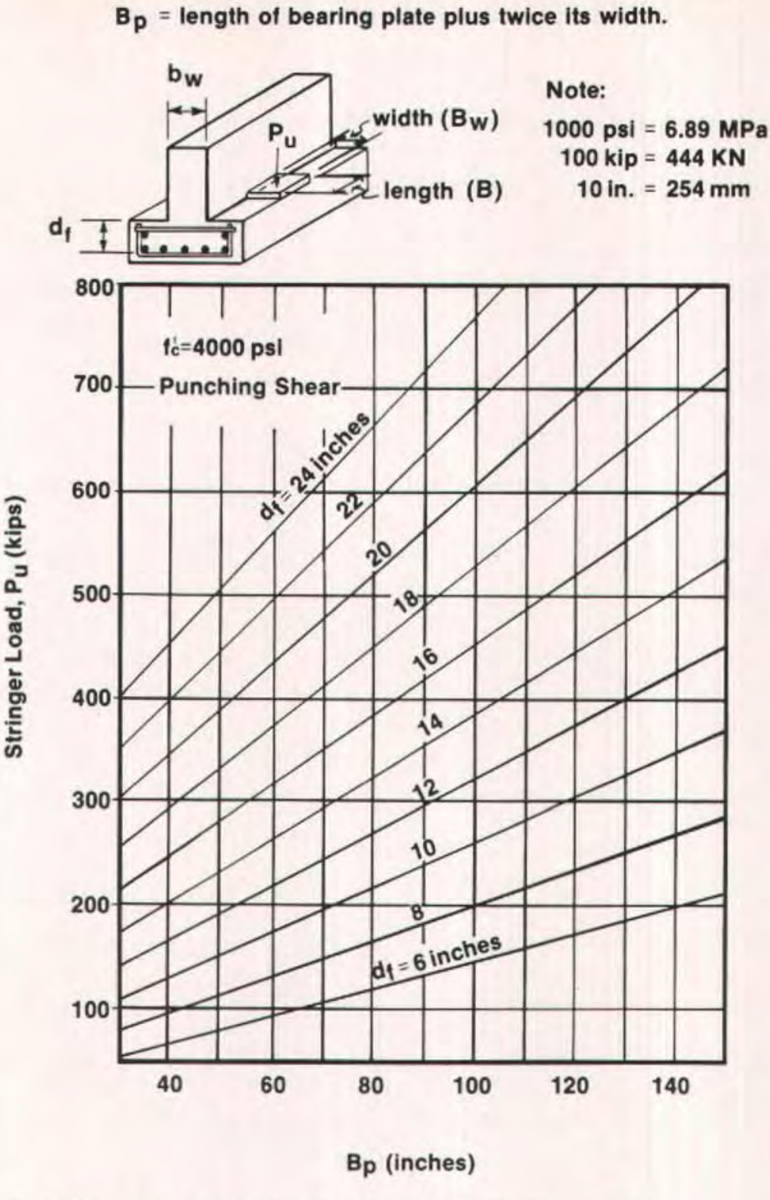

Fig. 3. Punching shear capacity of flange of an inverted T beam; $f_c' = 4000$ psi (27.6 MPa).

distance between stringers. Hence, the web width b_w should be such that the center-to-center transverse distance between the two stringer reactions acting on opposite sides of the web [$(2a + b_w)$ in Fig. 5a] at least equals $2d_f + B_w$, where B_w is the width of the bearing pad perpendicular to the beam axis.

Furthermore, stringers along the beam axis on each side of the web should be placed at a center-to-center spacing (shown as S in Fig. 5b) that exceeds $2d_f + B$, where B is the length of bearing pad along the edge of the flange.

For an end stringer, the distance from the edge of the bearing pad to the longitudinal end of the inverted T beam (shown as d_e in Fig. 5b) should be a minimum of $d_f + B_w$. This ensures the development of flange punching strength at end stringers at least as great as that developed at interior stringers.

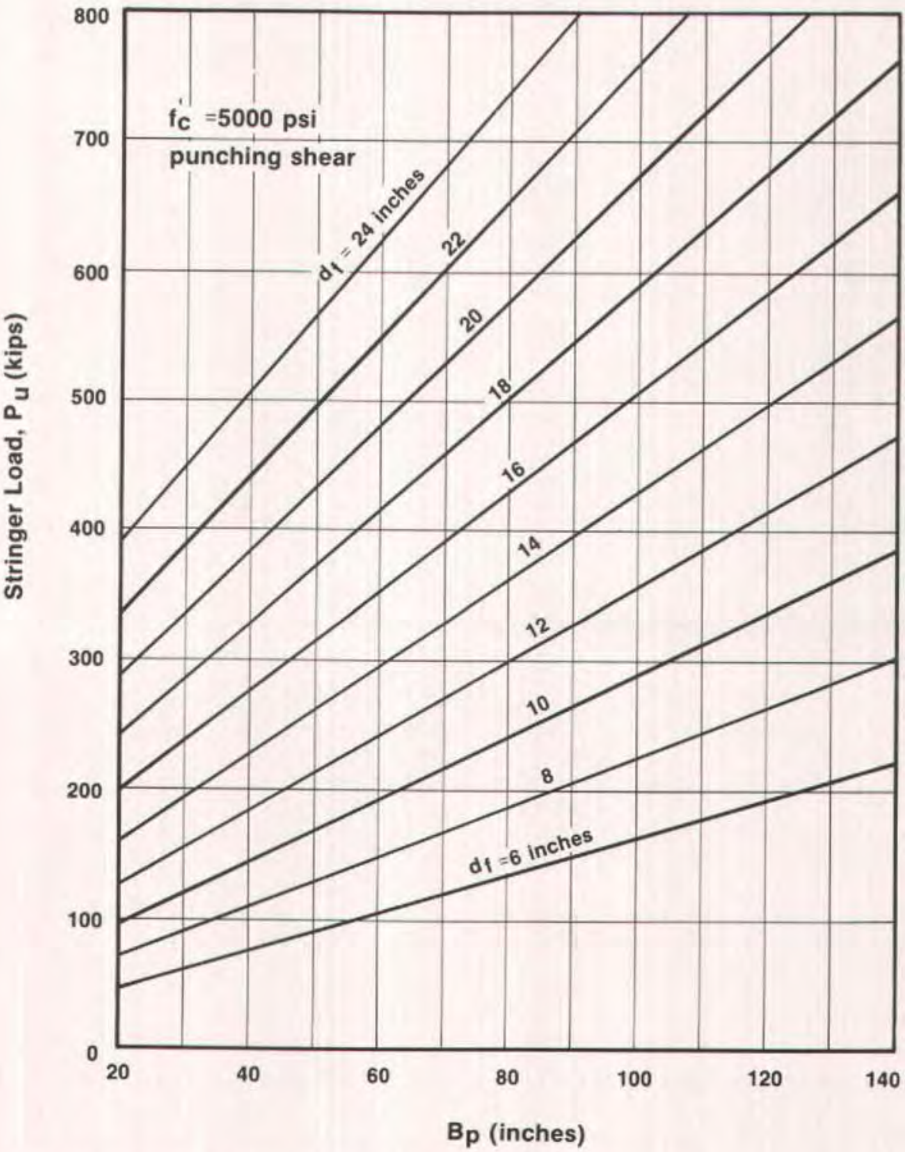

Fig. 4. Punching shear capacity of flange of an inverted T beam; $f_c' = 5000$ psi (34.5 MPa).

If the reaction from the end stringer is less than that from an interior stringer, an end distance smaller than $d_f + B_w$ may be provided and can be calculated fom the following expression:

$$d_e \geq \left[\frac{P_u}{4\phi\sqrt{f_c'}\, d_f} - (B + B_w + d_f) \right] \quad (2)$$

in which $\phi = 0.85$. The distance d_e should always be greater than zero in order to accommodate the bearing pad placed near the end of the inverted T beam.

Bracket-type shear friction in flange — The effective depth of flange (d_f) from centroid of top layer of flange transverse reinforcement to the bottom of flange shown in Fig. 6a and required to fulfill shear friction requirements should satisfy the following equation:

$$0.2\ \phi f_c' d_f\ (B + 4a) \geq P_u \quad (3a)$$

or

$$d_f \geq \frac{6 P_u}{f_c'\ (B + 4a)} \quad (3b)$$

in which

P_u = ultimate concentrated load acting on bearing pad
B = length of bearing pad along edge of flange
a = distance from face of web to center of bearing pad
$0.2 f_c'$ = shear strength of concrete resisting shear friction[1,2]
ϕ = 0.85

Use f_c' = 4000 psi for $f_c' \geq 4000$ psi (27.6 MPa) in computing d_f from Eq. (3a). This upper limit on f_c' is specified to limit the shear strength of concrete, because Eq. (4) used later for computing A_{vf} will become unconservative for higher values of f_c'.[9] Since shear friction seldom controls the flange depth of inverted T beams, this limit on f_c' will not affect most practical cases.

In Eq. (3b), the effective flange length resisting shear friction has been taken as $B + 4a$. In most cases, the stringer spacing along the beam axis will be large enough to permit the full effective flange length. However, if the longitudinal spacing of stringers is less than $B + 4a$, the stringer spacing should be used in place of $B + 4a$ in computing d_f from Eq. (3b).

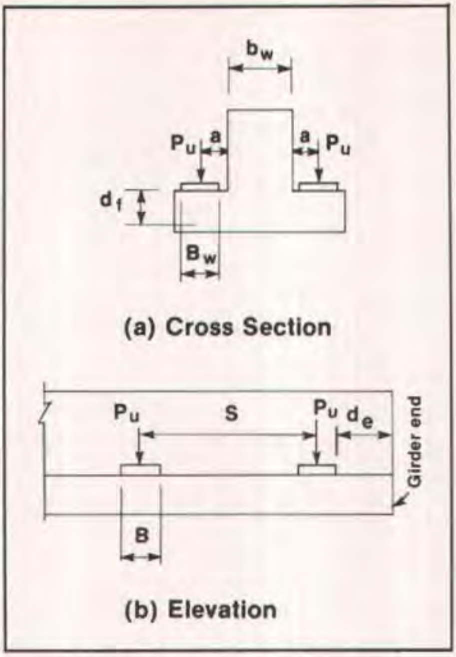

Fig. 5. Stringer spacings required for punching shear.

For a stringer placed near the longitudinal end of an inverted T beam, the stringer spacing for use in Eq. (3b) should be taken as twice the distance from the center of the bearing pad to the end of the inverted T beam or as the longitudinal distances between two adjacent stringers, whichever is smaller.

The transverse reinforcement should be placed perpendicular to the web near the top of the flange to resist flexural tension and to ensure enough pressure for sustaining shear friction force as indicated in Fig. 6a. The transverse reinforcement required to satisfy shear friction should be placed within a distance

2a each side from the edge of a bearing pad as shown in Fig. 6b. The area of cross section of such reinforcement (A_{vf}) should satisfy the following expression:[1,2]

$$\phi\mu \ A_{vf} f_y \geq P_u \qquad (4a)$$

or

$$A_{vf} \geq \frac{P_u}{1.2 f_y} \qquad (4b)$$

in which

- μ = coefficient of sliding friction taken as 1.4 for normal weight concrete cast monolithically[1,2]
- f_y = specified yield strength of reinforcement
- ϕ = 0.85

This reinforcement should be placed in two or more layers in the top half of the flange thickness; the area of reinforcement in the top layer should equal $2A_{vf}/3$ as indicated in Fig. 6b.

The flange transverse reinforcement placed in the top layer also resists flexural tension, which is caused by the cantilever action of the flange, and should satisfy the following equation:

$$0.8 \ \phi \, d_f A_{sf} f_y \geq P_u \, a \qquad (5a)$$

or

$$A_{sf} \geq \frac{1.4 P_u \, a}{f_y \, d_f} \qquad (5b)$$

in which

- A_{sf} = cross section area of transverse reinforcement required to resist flexural tension
- $0.8\, d_f$ = effective distance between centroid of compression and centroid of tension ($j\, d_f$) for calculating flexural reinforcement in flange
- ϕ = 0.9

The value of jd_f in a deep cantilever is expected to be smaller than that of an ordinary depth, shallower beam. The suggested value of $0.8d_f$ for bracket design is based on a finite element analysis.[8] All transverse reinforcement within a longitudinal distance 2.5 a each side of the bearing pad can be taken as A_{sf} as shown in Fig. 6c.

Thus, the reinforcement placed in the top layer should at least equal A_{sf} or $2A_{vf}/3$, whichever is greater, as indicated in Fig. 6a. If $a \leq 0.4\, d_f$, A_{sf} will not control the design. A_{vf} and distribution of shear friction reinforcement should always be checked through Eq. (4b).

The use of the term $B + 5a$ as the effective length of bracket for the distribution of flexural steel in the top of a bracket is based on a finite element analysis and is supported by test results.[8] The same holds for the use of the term $B + 4a$ as the effective length of bracket suggested for shear friction.

The suggested effective flange length ($B + 4a$ for A_{vf} and $B + 5a$ for A_{sf}) should not overlap for adjacent stringers. If the distance c between the center of a concentrated load and longitudinal end of the girder is less than one-half the effective flange length illustrated in Fig. 6b or 6c, the effective flange length should be taken as $2\,c$ or as the spacing of stringers, whichever is smaller.

The longitudinal forces due to sudden braking of a vehicle are transmitted from its wheels to the deck of the bridge. The magnitude of this longitudinal force depends on the weight, the velocity, and the braking time of the vehicle. The AASHTO specifications[1] call for a longitudinal force of 5 percent of the live load in all lanes carrying traffic headed in the same direction. This longitudinal force will add little stress to the deck and the stringers but may be important for the design of stringer bearings and the supporting brackets of the inverted T beams.

Consequently, a 5 to 10 percent longitudinal component of live load stringer reactions could be included for the design of flexural steel in the top of the bracket. The rigidity of the bridge

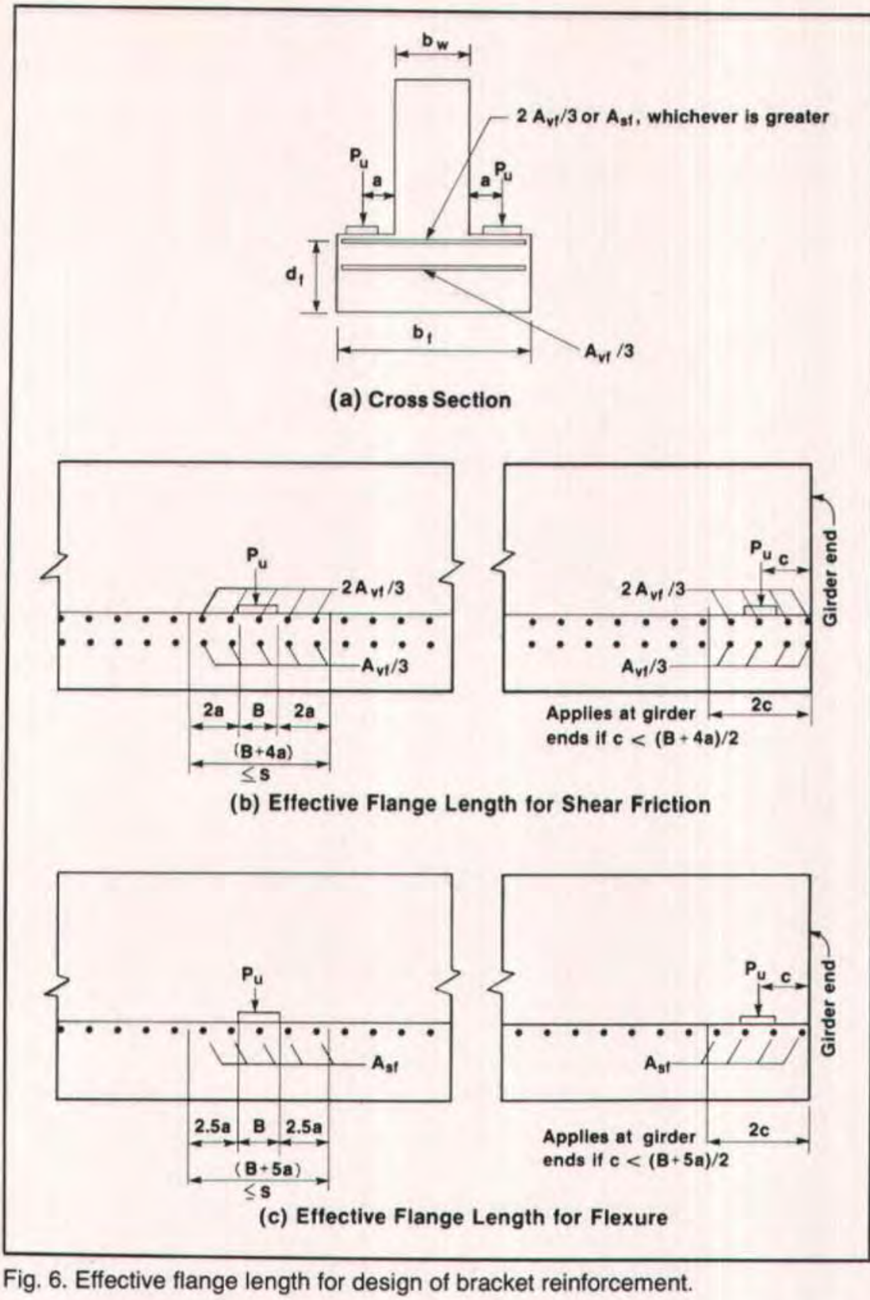

Fig. 6. Effective flange length for design of bracket reinforcement.

deck tends to spread such loads among all stringers, and the approximations used in estimating the effective flange length for flexure hardly justify the superposition of tension due to longitudinal component of live load stringer reactions and flexure unless stringers are spaced more closely

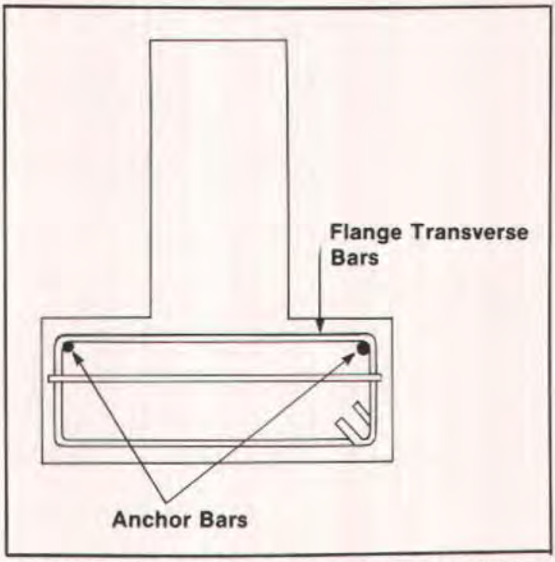

Fig. 7. Anchorage of flange transverse bars without the use of welding.

than $5a$ plus the length of the bearing pad.

The AASHTO specifications[1] also require that an additional longitudinal force due to friction at expansion bearings be included in design. The magnitude of this frictional force depends on the type of bearing used. Unless special provisions are made in the design of stringer bearings to avoid the frictional force, this force should be computed and additional reinforcement should be added to the flexural steel near the top of the bracket to resist this force.

In place of an exact computation of the longitudinal forces due to friction, the ACI Code[2] provisions for bracket design may be used. These clauses require the use of a longitudinal force that at least equals 20 percent of the stringer reaction due to dead plus live load. These provisions also regard the longitudinal force acting on a bracket as a live load even when this force results from creep, shrinkage or temperature change.

The anchorage of the flange transverse bars may impose a problem, because the flange overhang is usually too short to accommodate the development length of reinforcing bars of usual sizes employed in bridge construction. The detail shown in Fig. 7 is recommended wherever the reinforcement size would permit the development of yield strength in the flange transverse bars.

However, it may be necessary in some cases to weld the ends of transverse bars to an anchor bar at the exterior face of the flange and perpendicular to the transverse bars.

A welding detail used by the Texas State Department of Highways and Public Transportation is illustrated in Fig. 8. The development of flange transverse bars can also be achieved by furnishing a continuous steel angle (or plate) along the top corners of the flange and connecting these bars to the angle (or plate). The welding should conform to the American Welding Society D1.4 Code for reinforcing steel.[10]

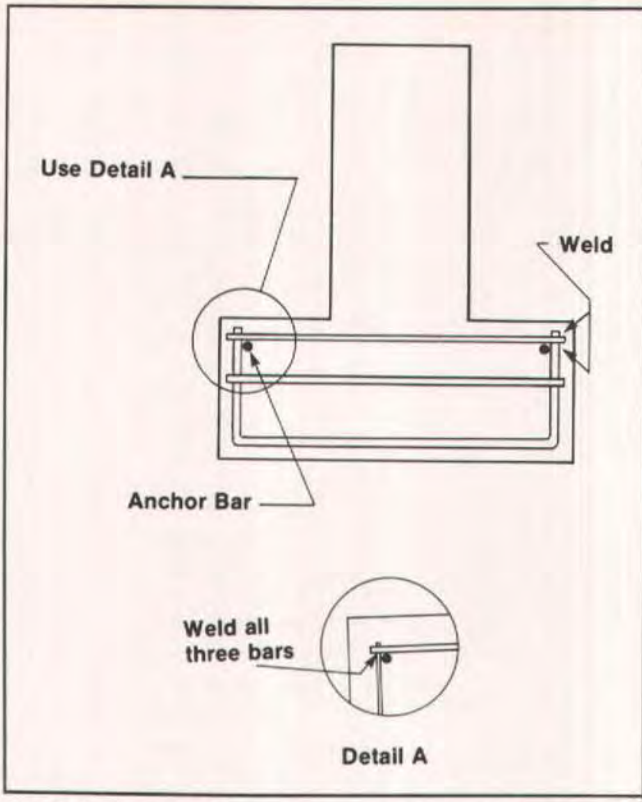

Fig. 8. Anchorage of flange transverse bars with the use of welding.

Design of Web Stirrups Acting as Hangers

Stirrups in the web of an inverted T beam act as hangers to deliver the concentrated loads applied on the flange into the body of the web. The maximum hanger stresses at ultimate load exist when both sides of the web are subjected to maximum live loads simultaneously.

The longitudinal distance over which hanger forces can be distributed, defined as the effective hanger distance in Fig. 9, is limited either by the shear capacity of concrete in the flange on each side of a bearing pad or by the longitudinal center-to-center spacing of stringers. Hence, to achieve safe delivery of flange loads into the web, the following strength relationships [Eq. (6b) and Eq. (7b)] should be satisfied:

$$\phi A_v f_y [(B + 2 d_f)/s] \geq [2 P_u - 2 (2 \phi \sqrt{f'_c} b_f d_f)] \quad (6a)$$

or

$$\frac{A_v}{s} \geq \frac{\frac{2 P_u}{\phi} - 4 \sqrt{f'_c} b_f d_f}{f_y (B + 2 d_f)} \quad (6b)$$

and

$$\phi A_v f_y (S/s) \geq 2 P_u \quad (7a)$$

or

$$\frac{A_v}{s} \geq \frac{2 P_u}{\phi f_y S} \quad (7b)$$

In the above equations:

b_f = overall flange width (Fig. 9)

d_f = flange depth from top of flange to center of bottom longitudinal reinforcement as indicated in Fig. 9a

S = stringer spacing along beam axis

P_u = ultimate concentrated load acting on one bearing pad

A_v/s = cross section area of both legs of a web stirrup divided by the spacing of stirrups

ϕ = 0.85

For a stringer load placed near the end of an inverted T beam, the distance S in Eq. (7b) should be taken as twice the distance from the center of the bearing pad to the longitudinal end of the inverted T beam or as the longitudinal distance between two adjacent stringers, whichever is smaller.

Note that Eq. (6b) controls the design of hangers in cases where the stringer spacing is large enough to permit a failure mode in which flange strength becomes effective in resisting hanger forces as shown in Fig. 9a. For these cases, the hanger reinforcement should be provided for twice the stringer load minus the shear strength of the flange on each side of the stringer as indicated by Eq. (6b).

If the stringers are too closely spaced, hanger failure will take place by separation of the flange from the web over the entire loaded length of the beam. Hence, hangers should be designed for full stringer loads as indicated by Eq. (7b). However, both equations should be satisfied to insure the safe transfer of hanger forces.

The premature yielding of hangers nearest the concentrated loads applied to the flange and the size of cracks that form at the junction of the web and the flange when these hangers reach high stress levels should be controlled at service load conditions. This can be achieved by satisfying the following equation:

$$A_v f_s [(B + 3a)/s] \geq 2P_s \quad (8a)$$

or

$$\frac{A_v}{s} \geq \frac{3P_s}{f_y (B + 3a)} \quad (8b)$$

in which

P_s = concentrated service load acting on a bearing pad

f_s = hanger stress at service loads limited to a maximum value of $2f_y/3$

In Eq. (8b) the effective distance over which hanger forces can be distributed at service loads has been taken as $B + 3a$. If $B + 3a$ exceeds the longitudinal stringer spacing, the stringer spacing S should be used in place of $B + 3a$ in computing A_v/s from Eq. (8b). For end stringers S should be taken the same as that defined for Eq. (7b). Note that Eq. (8b) is based on test results and is documented in Ref. 5.

The largest value of A_v/s from Eqs. (6b), (7b), and (8b) should be used. Only vertical stirrups anchored to develop by bond their tension yield forces above and below the top surface of the flange should be considered to carry hanger forces. Since most flanges have a depth inadequate for developing stirrup bar yield forces, hangers should be closed across the bottom of the inverted T beam as indicated in Fig. 9b.

It is not necessary to superimpose loads on stirrups acting as hangers and loads on stirrups acting as shear (and torsion) reinforcement. This will be discussed further in a subsequent section.

Overall Design of Inverted T Beam

The overall strength of an inverted T beam should be adequate to support ultimate flexure, flexural shear, and torsional shear forces and any possible combination of such forces.

Design for maximum flexural moment and shear — The most likely design

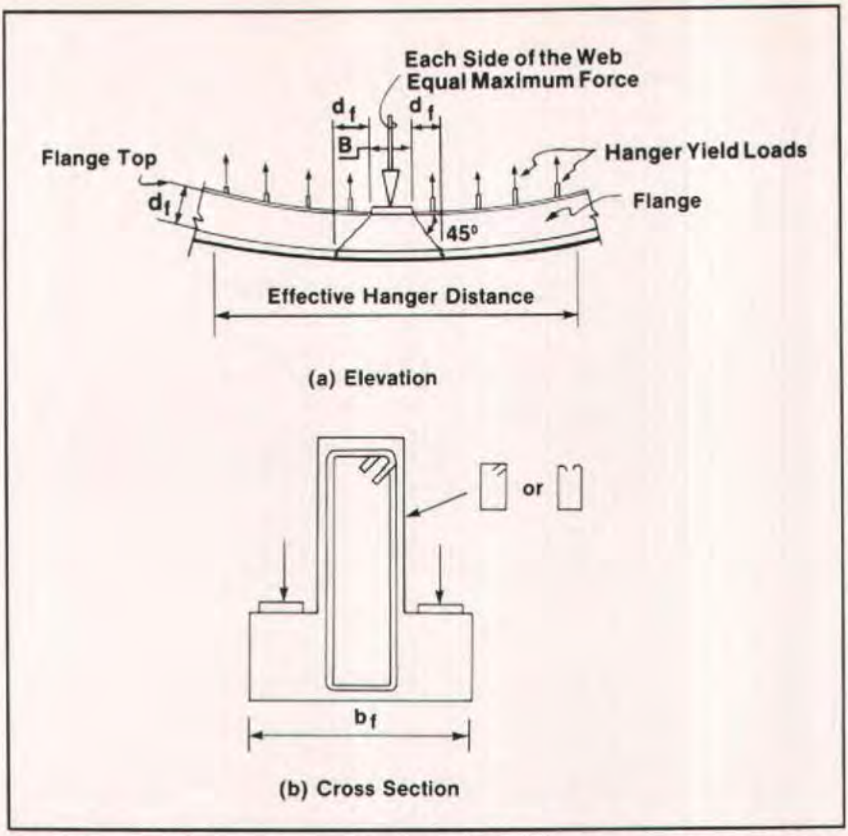

Fig. 9. Hanger forces in response to flange loads shown on the beam elevation and details of hanger reinforcement shown in the beam cross section.

condition for bridge bentcap girders involves flexural moments and flexural shear forces that are largest when torsion is absent, because traffic loads stringers fully on both sides of the girder web. Consequently, a logical procedure for overall design begins with the proportioning of the cross section and reinforcement solely on the basis of maximum flexural moment and maximum flexural shear force.

Requirements of flexural reinforcement for the overall design of the beam are not altered by the location of the T beam flange. The ultimate strength and serviceability design requirements of the ACI Code[2] and the AASHTO Specifications[1] are quite safe and adequate for flexure. This applies to both post-tensioned prestressed concrete and reinforced concrete inverted T beams.

The ACI Code[2] permits the designer to consider the maximum end shear as that occurring at a distance d from the face of the support for nonprestressed members and at a distance $h/2$ from the face of the support for prestressed members, where d and h are, respectively, the effective depth and overall thickness of the member. Of course, this is allowed only if no stringer load is placed between the face of the support and the critical section at a distance d or $h/2$ from the face of the support.

While this criterion is reasonable for conventional beams, this is not appro-

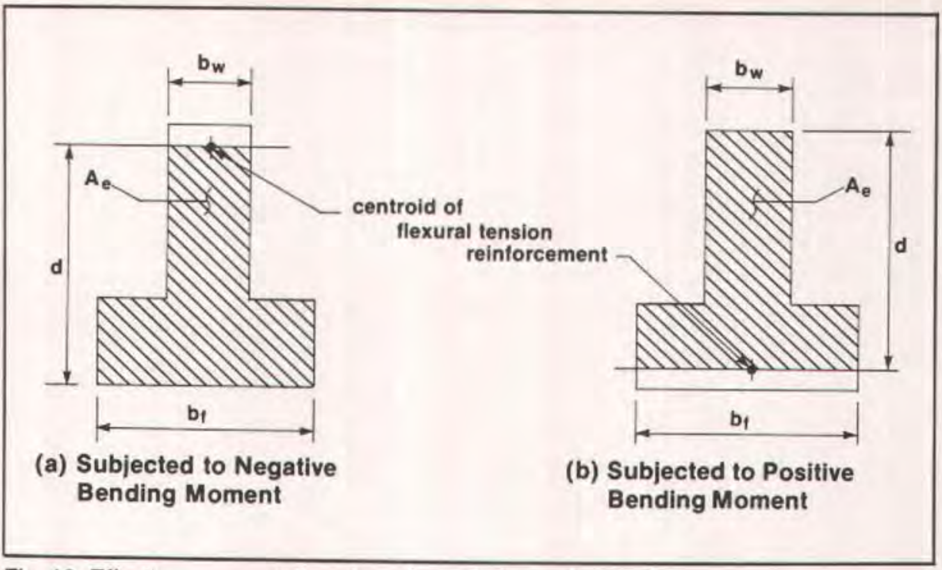

Fig. 10. Effective area resisting shear force A_e (cross-hatched).

priate for inverted T beams unless the terms d_f and $h_f/2$ related to the flange are substituted in place of d and $h/2$, respectively. To simplify the shear calculations, however, the critical section for the maximum end shear in inverted T beams may be taken at the face of the support. This simplification will not cause a significant loss of accuracy in shear design of bridge bentcap girders.

With a flange overhang to thickness ratio equal to or less than 1.0, the flange of an inverted T beam is expected to be stiff enough to fully participate in retention of the shear force. This seems particularly valid for a cross section subjected to negative moment, creating flexural compression in the flange. The tests on inverted T beams support this hypothesis.[4,8]

Hence, the flexural shear strength in absence of torsion can be based on modified versions of the ACI Code equations:[4]

For reinforced concrete:

$$V_o = \phi \left[2\sqrt{f'_c}\,A_e + A_v f_v\left(\frac{d}{s}\right)\right] \qquad (9)$$

For prestressed concrete:

$$V_o = \phi \left[n\sqrt{f'_c}\,A_e + A_v f_v\left(\frac{d}{s}\right)\right] \qquad (10)$$

In both equations, $A_v f_y\, d/s \leq 8\sqrt{f'_c}\, b_w d$; $n = 5$ for $M_u/V_u d = 1$, decreasing linearly to 2 as $M_u/V_u d$ increases from 1 to 5; A_e = area of all concrete between compression face and centroid of flexural tension (or prestressing) reinforcement as indicated in Fig. 10; $\phi = 0.85$; and d is defined in Fig. 10.

The suggested approximation for n applies only to the beams with $f'_c \leq$ 6400 psi (44 MPa) and having an effective prestressing force greater than or equal to 0.4 times the tensile strength of flexural reinforcement. Stirrups should be designed to resist all applied ultimate shear force above that resisted by the concrete section illustrated in Fig. 10. In addition, all the other shear provisions of the Code[1,2] should be satisfied.

The tests on inverted T beams have shown that the hanger failure cracks occur at the junction of the web and the flange, whereas the flexural shear failure cracks occur in the web above the flange.[5] Consequently, the yielding of stirrups acting as hanger reinforcement and as shear reinforcement takes place at different locations in the stirrups. The stirrups designed as hangers can then be used as part of the web reinforcement resisting flexural shear.

This can be further justified by the following argument. The failure of an inverted T beam due to flexural shear is an overall failure, whereas the hanger failure is a local failure of stirrups acting as hangers nearest an applied concentrated load to transmit flange forces into the web.

Hence, the hanger forces delivered to the web are carried to the beam supports by the web as flexural shear and it is not necessary to superimpose loads on stirrups acting as hangers and loads on stirrups acting as shear reinforcement. However, the web reinforcement in the stem of an inverted T beam should be proportioned on the basis of hanger requirement or shear strength requirement, whichever is greater.

Design for combined effect of flexural shear and torsion — When torsion is at maximum, traffic loads stringers on only one side of the web and flexural shear is less than the maximum value. Hence, stirrups also serve as vertical reinforcement of the web that is subjected to combined flexural and torsional shear. The local hanger forces need not be superimposed on the web shear forces for designing stirrups as explained earlier, but the cross section should be designed for the more critical of the two forces.

For the cross section to be adequate when the combined torsional and flexural shear acts, the following interaction expression should be satisfied:

$$\left(\frac{V_u}{V_o}\right)^2 + \left(\frac{T_u}{T_o}\right)^2 \le 1.0 \qquad (11)$$

in which V_u and T_u are applied ultimate shear force and applied ultimate torque, respectively.

The flexural shear capacity V_o can be taken from Eq. (9) or (10) and the ultimate pure torsion strength T_o for both reinforced and prestressed concrete members can be calculated from an adaptation[4] of the pure torsion strength equation recommended by the ACI Code:[2]

$$T_o = \phi \left[4\sqrt{f_c'}\ \frac{\Sigma x^2 y}{3} + A_t f_y \left(\frac{\alpha_t x_1 y_1}{s}\right)\right] \le 18\,\phi\sqrt{f_c'}\ \frac{\Sigma x^2 y}{3} \qquad (12)$$

in which

A_t = area of one leg of a web stirrup
s = space of web stirrups
$\alpha_t = [0.66 + 0.33\,(y_1/x_1)] \le 1.5$
$\phi = 0.85$
x_1 and y_1 = shorter and longer center-to-center dimension, respectively, of closed web stirrups

The contribution to torsion strength of the cross section of each component rectangle should be computed separately using the smaller dimension x and the larger dimension y for the rectangle under consideration as indicated in Fig. 11. Only closed rectangular web stirrups should be considered effective in resisting torsion. This assumption of neglecting the contribution of flange transverse reinforcement to torsional strength may lead to a slightly conservative design for the combined effect of flexural shear and torsion, but will greatly simplify the calculations.

Although the ACI Code[2] permits a value of $2.4\sqrt{f_c'}$ for the torsional shear strength of concrete, a value of $4\sqrt{f_c'}$ is used in Eq. (12). This value seems to be justified for inverted T beams and is documented in Ref. 4.

Alternately, the combined effect of flexural shear and torsional shear on the cross section can be satisfied by using the following equations:

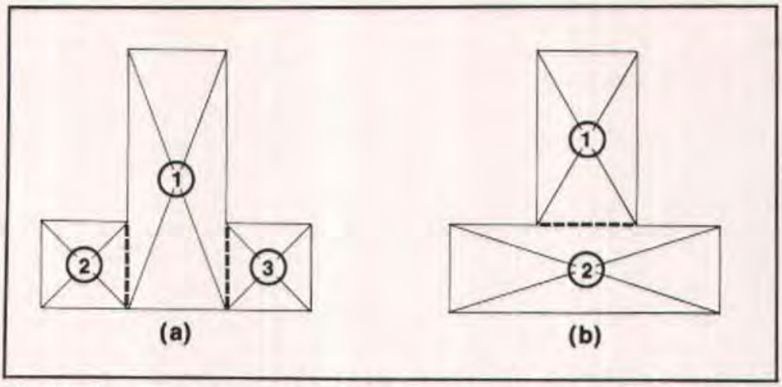

Fig. 11. Component rectangles of inverted T section for torsion analysis [use larger of the two values of Σx^2y from (a) and (b)].

$$6\sqrt{f_c'}\,\Sigma x^2 y \geq \frac{T_u}{\beta} \quad (13)$$

and

$$\frac{A_v}{s} \geq \frac{2}{\alpha_t x_1 y_1 f_y}\left[\frac{T_u}{\beta} - 1.33\sqrt{f_c'}\,\Sigma x^2 y\right] \quad (14)$$

In which $\beta = \phi\sqrt{[1 - (V_u/V_o)^2]}$ and can be easily determined from the plot shown in Fig. 12.

Eqs. (13) and (14) were derived by solving Eq. (11) for T_o and equating it to Eq. (12). The expressions can be used in place of Eqs. (11) and (12). Eq. (13) ensures yielding of stirrups before crushing of concrete takes place. Hence, the cross section should be revised if Eq. (13) is not satisfied.

Eq. (14) determines A_v/s, the area of cross section of both legs of a closed rectangular web stirrup divided by the stirrup spacing, required for the combined effect of shear and torsion. Note that only one set of equations for the combined effect of flexural shear and torsion needs to be checked: either Eqs. (13) and (14) or Eqs. (11) and (12).

If the area of transverse reinforcement in the web is controlled by the requirements of maximum flexural shear, there is apparently no need to check for longitudinal reinforcement required for torsion. The supplemental longitudinal reinforcement should be provided to help flexural reinforcement resist torsion for cases in which stirrup design is controlled by the combined effect of torsion and flexural shear.

If the area of web stirrups is increased to satisfy Eq. (14) or Eq. (11), supplemental longitudinal steel (A_l) with a volume at least equal to the volume of extra web transverse reinforcement (A_t') should be provided.

When the yield strength of A_l and A_t' is the same:

$$s\,A_l = 2(x_1 + y_1)A_t' = A_v'\,(x_1 + y_1)$$

and

$$A_l = \frac{A_v'}{s}\,(x_1 + y_1) \quad (15)$$

in which $A_v'/s = A_v/s$ required for the combined effect of flexural shear and torsion minus A_v/s required for maximum flexural shear acting alone.

The area of longitudinal reinforcement A_l should be distributed among the four corners of the web plus the four corners of the flange, and it should be added to the flexural reinforcement both for prestressed and nonprestressed concrete members. Note that all other relevant shear and torsion provisions of the Code[1,2] should be satisfied.

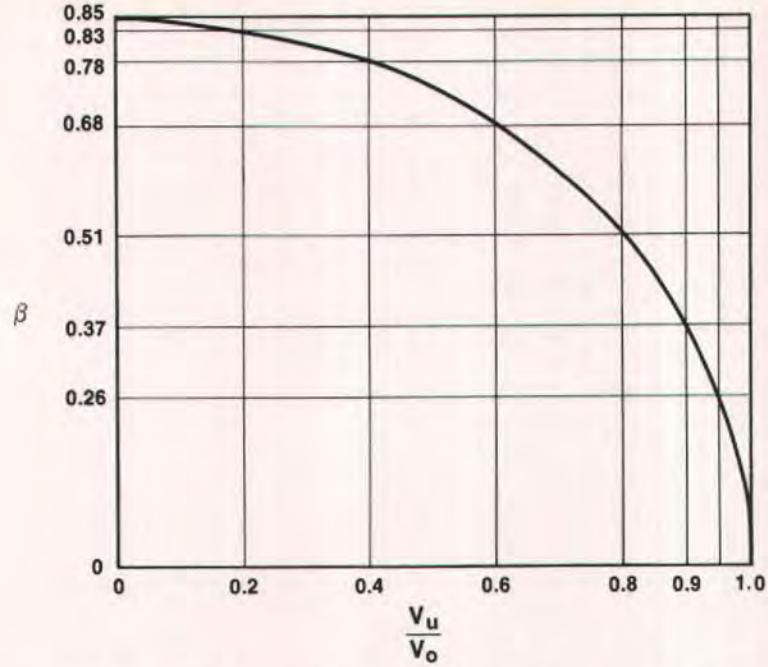

Fig. 12. Value of β for different ratios of V_u/V_o (linear interpolation of β may be conservatively used between plotted values of V_u/V_o).

DESIGN PROCEDURE

The design procedure for inverted T beam bentcap girders based on the criteria proposed in this paper can be summarized as follows:

1. Compute the flange thickness for punching shear requirements using Eq.(1b) (or Figs. 3 and 4) and select a value that at least equals the flange overhang. Establish the web width, so that the center-to-center transverse distance between the two stringer reactions acting on the opposite sides of the web $(2a+b_w)$ at least equals twice the flange depth plus the width of the bearing pad $(2d_f+B_w)$.

Note that this web width must be able to accommodate the longitudinal reinforcement required for negative bending moment. Check the requirements for minimum stringer spacings and end distances controlled by punching shear.

The height of the web above the top of the flange is determined by the depth of the stringers to be supported on the flange. The overall depth of the beam can then be computed and must provide the required flexural stiffness. The nominal span-to-depth ratios for inverted T beam bentcap girders appear to be from 2 to 4 for cantilevered spans, and 4 to 8 for spans supported at each end.[7]

2. Using Eq. (3b), compute the flange thickness required to resist shear friction and revise the furnished value if needed. Eqs. (4b) and (5b), respectively, calculate the flange transverse reinforcement required for shear friction (A_{vf}) and that required for flexural tension (A_{sf}) in the flange. Place A_{sf} or $2A_{vf}/3$, whichever is greater, near the top of the flange and $A_{vf}/3$ in one or more layers below the top layer within the top half of the flange thickness.

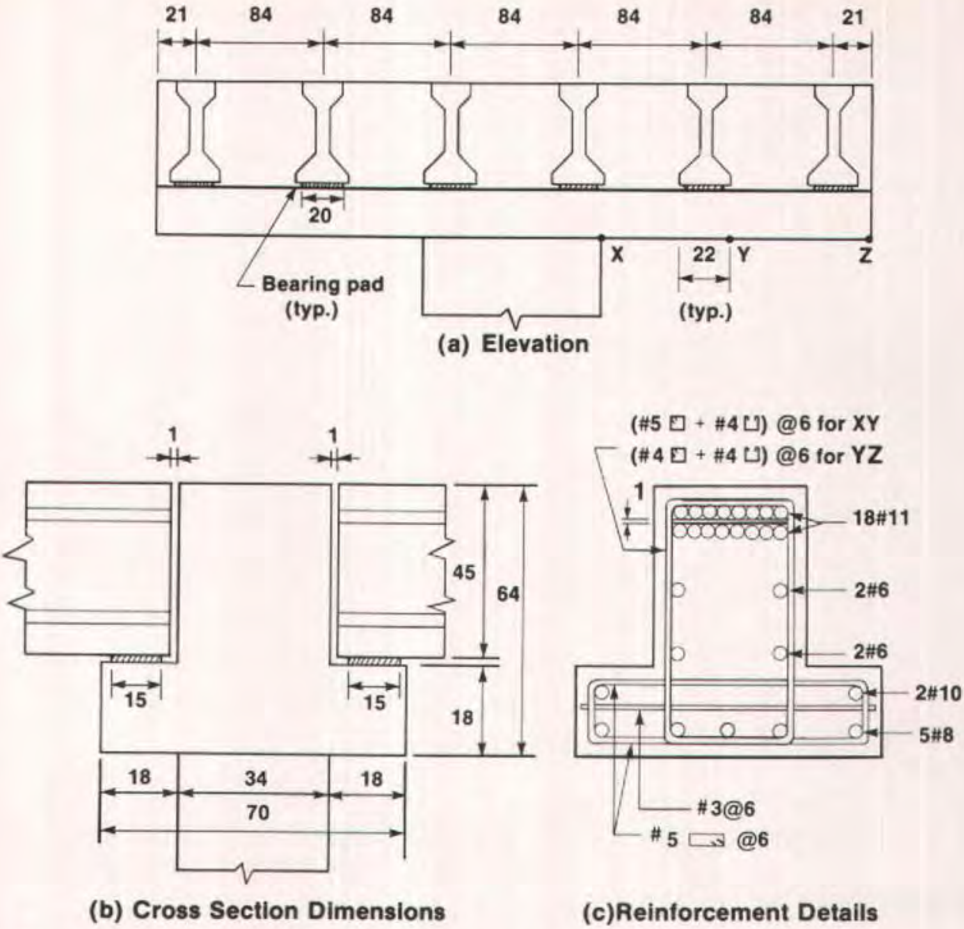

Fig. 13. Details of an inverted T beam designed in the example (all dimensions are in inches; 1 in. = 25.4 mm).

Include the effect of longitudinal forces at stringer bearings due to friction in the design of top transverse reinforcement in the flange unless provisions are made to avoid the frictional forces at stringer bearings.

The flange transverse reinforcement in the top layer is also subjected to a longitudinal component of stringer reactions due to braking of live and impact loads. The superposition of flange transverse reinforcement for tension due to this longitudinal force and that for tension due to flange flexure is not needed unless the stringers are spaced closer than $B + 5a$. Anchor the flange transverse bars as in Fig. 7 or 8.

3. Provide stirrups in the web for the most critical effect from hanger tension, maximum flexural shear, and maximum torque plus corresponding flexural shear:

(a) Determine the area of stirrup reinforcement required to resist hanger forces as the largest value of A_v/s obtained from Eqs. (6b), (7b), and (8b). Note that the maximum stresses due to hanger action act when both sides of the web are subjected to maximum live loads simultaneously.

(b) The maximum stresses due to flexural shear acting alone occur when traffic loads the stringers fully on both sides of the web. Determine the required area of stirrups in terms of A_v/s from Eq. (9) or (10). Note that the critical section for the maximum end shear in an inverted T beam may be taken at the face of the support.

(c) For maximum torsion to act on an inverted T beam, traffic loads the stringers on only one side of the web and the corresponding flexural shear will be less than its maximum value. Again, the critical section near the end of the member may be taken at the face of the support. Satisfy Eq. (13) to ensure yielding of stirrups prior to crushing of concrete under ultimate loads. Note that a larger cross section will be needed to resist combined shear and torsion if Eq. (13) is not satisfied. Calculate the required area of stirrups (A_v/s) using Eq. (14). The term β used in Eqs. (13) and (14) may be determined from Fig. 12.

The superposition of stirrup reinforcement (A_v/s) required for Cases (a), (b) and (c) is not needed. However, design stirrups for the maximum value of the three effects at all critical sections. Provide either closed rectangular stirrups or stirrups that are closed at least across the bottom of the beam as indicated in Fig. 9b. Use only closed rectangular stirrups to resist forces due to torsion.

4. The most critical section for flexure occurs at the face of the support and the maximum bending moment on an inverted T beam acts when full live loads are applied on both sides of the web. Determine the longitudinal reinforcement required to resist flexural tension at all critical sections. If the web reinforcement (A_v/s) computed for maximum flexural shear acting alone is greater than that calculated for the combined effect of torsion and flexural shear, no check on longitudinal reinforcement for torsion is necessary.

Provide supplemental longitudinal reinforcement as per Eq. (15) to help flexural reinforcement resist torsion in cases where stirrup design is controlled by the combined effect of torsion and flexural shear. Distribute the supplemental longitudinal reinforcement along the perimeter of the cross section, particularly at the corners of the web and flange.

SUMMARY

Reinforced concrete and post-tensioned prestressed concrete inverted T beams are frequently used for bridges. The structural behavior of inverted T beams differs from that of conventional top-loaded beams, because the loads are introduced into the bottom rather than into the sides or the top of the web. The application of loads near the bottom of the web in inverted T beams imposes special problems, which are not addressed to in the current North American structural codes.

This paper provides recommendations for proportioning cross section dimensions and reinforcement of cast-in-place normal weight concrete inverted T beams employed in bridge structures. These beams should be designed to have adequate strength against possible failure due to flexure, flexural shear, torsion and any possible combination of these forces.

Reinforcement details for the flanges of inverted T beams should accommodate flexure, shear friction, and punching shear on the short cantilevered shelf. The transverse reinforcement in the webs of inverted T beams should resist hanger tension forces caused by loads applied to the lower part of the web. The step-by-step procedure based on the proposed criteria summarizes in the previous section the design of inverted T beam bentcap girders. A design example given in the Appendix elaborates upon the application of the proposed criteria.

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APPENDIX A — NOTATION

A_c = area of cross section of all concrete between compression face and centroid of flexural tension (or prestressing) reinforcement

A_l = total area of cross section of supplemental longitudinal reinforcement required to resist torsion

A_{sf} = area of cross section of transverse reinforcement required to resist flexural tension in a flange overhang

A_t = area of cross section of one leg of a web stirrup

A_v = area of cross section of both legs of a web stirrup

A_{vf} = area of cross section of transverse reinforcement required to resist shear friction in a flange overhang

A_v'/s = A_v/s required for combined effect of flexural shear and torsion minus A_v/s required for maximum flexural shear acting alone

a = distance from face of web to center of bearing pad taken perpendicular to beam axis

B = length of bearing pad along edge of flange

B_p = $B + 2B_w$

B_w = width of bearing pad perpendicular to beam axis

b_f = overall width of flange of inverted T beam

b_w = width of web

d = effective depth of inverted T beam between compression face and centroid of flexural tension (or prestressing) steel

d_e = distance from edge of bearing

pad to longitudinal end of inverted T beam

d_f = effective depth of flange as defined in Eqs. (1b), (3b), and (6b)

f'_c = specified strength of concrete

f_s = service load stress limited to a maximum of $2f_y/3$ in stirrups acting as hangers

f_y = specified yield strength of reinforcement

h = overall thickness of inverted T beam

h_f = overall thickness of flange of inverted T beam

$j\,d_f$ = effective distance between centroid of compression and centroid of tension for calculating flexural reinforcement in flange overhang

M_u = applied ultimate bending moment acting on overall cross section of inverted T beam

n = coefficient defined after Eq. (10)

P_s = concentrated service load acting on one bearing pad

P_u = concentrated ultimate load acting on one bearing pad

S = stringer spacing along the beam axis

s = spacing of web stirrups

T_o = ultimate pure torsion strength of overall cross section of inverted T beam

T_u = applied ultimate torque acting on overall cross section of inverted T beam

V_o = ultimate flexural shear strength (in absence of torsion) of overall cross section of inverted T beam

V_u = applied ultimate flexural shear force acting on overall cross section of inverted T beam

x = smaller dimension of rectangle

x_1 = shorter center-to-center dimension of closed web stirrup

y = larger dimension of a rectangle

y_1 = longer center-to-center dimension of closed web stirrup

α_t = $[0.66 + 0.33\,(y_1/x_1)] \leq 1.5$

β = $\phi\sqrt{1 - (V_u/V_o)^2}$

μ = coefficient of sliding friction taken as 1.4 for normal weight concrete cast monolithically

ϕ = strength reduction factor taken as 0.9 for flexure and 0.85 for shear and torsion

APPENDIX B — DESIGN EXAMPLE

Consider a reinforced concrete inverted T beam shown in Fig. 13a. The beam acts as a bentcap girder in a bridge superstructure to support four interior and two exterior precast prestressed concrete stringers placed on each side of the web. Each interior stringer exerts a total force of 130,000 and 221,000 lbs (580 and 985 kN) at service load and ultimate load conditions, respectively.

The corresponding reactions from each of the exterior stringers are 90,000 and 143,000 lbs (400 and 635 kN). The ultimate live load plus impact reactions included in the foregoing total loads are 130,000 lbs (580 kN) for each of the interior stringers and 65,000 lbs (290 kN) for each of the exterior stringers.

All stringers are placed on 20 x 15 in. (508 x 381 mm) bearing pads spaced at 84 in. (2134 mm) on centers with an end distance of 21 in. (533 mm) as indicated in Fig. 13a. Assume that the frictional forces produced at the stringer bearings are negligible.

Specified concrete strength (f'_c) and specified yield strength of reinforcement (f_y) are 4000 and 60,000 psi (27.6 and 414 MPa), respectively. A minimum clear cover of 2 in. (51 mm) is used for all reinforcement and the maximum aggregate size is 1½ in. (38 mm).

Flange Design for Punching Shear

Since B_p equals $(20 + 2 \times 15) = 50$ in., $P_u = 221{,}000$ lbs, and $f'_c = 4000$ psi, the required value of d_f is 13.4 in. from Eq. (1b) or Fig. 3. This gives a flange thickness $(= 13.4 + 0.625 + 2.0) = 16.1$ in. Use a flange thickness of 18 in. with d_f furnished = 15.4 in.

These values are acceptable because the length of the flange overhang (= width of bearing pad plus 1 + 2 in. = 18 in.) is not greater than the flange thickness.

The required minimum transverse distance between stringers on the opposite sides of the web is equal to $2\,d_f + B_w$ $(= 2 \times 15.4 + 15) = 45.8$ in. and a (= ½ of the bearing pad width plus 2 in.) = 9.5 in. Hence, the required web width equals $(45.8 - 2 \times 9.5 =)$ 26.8 in. Use b_w = 34 in.

This width will be required to accommodate the longitudinal reinforcement near the top of the web. The required minimum longitudinal spacing of stringers is $(2\,d_f + B = 2 \times 15.4 + 20 =)$ 50.8 in., which is smaller than the spacing furnished.

At exterior stringers, P_u is equal to 143,000 lbs. Hence, the minimum end distance required for punching shear and calculated from Eq. (2) is zero, which is less than the furnished value $(= 21 - 10) = 11$ in.

Flange Design for Bracket Shear

$B + 4\,a$ equals $(20 + 4 \times 9.5 =)$ 58 in., which is less than $S = 84$ in. for interior stringers. Hence, use an effective flange length of 58 in. in computing d_f from Eq. (3b). From Eq. (3b), the required d_f is calculated as 5.7 in., which is less than the actual $d_f (= 18 - 2 - 0.625/2) = 15.6$ in. For end stringers, the effective flange length equals $(2 \times 21 =)$ 42 in. and the required $d_f = 5.1$ in., which is again less than the actual d_f.

From Eq. (4b), the required A_{vf} equals 3.1 and 2.0 in.² for interior and exterior stringers, respectively. The effective flange length associated with Eq. (4b) is 58 in. for interior stringers and 42 in. for exterior stringers. This makes the required A_{vf} = 0.65 and 0.57 in.²/ft for the interior and exterior stringers, respectively.

Note that two-thirds of this reinforcement $(2\,A_{vf}/3 = 0.43$ or 0.38 in.²/ft) is required for the top layer.

From Eq. (5b), the required A_{sf} is calculated to be 3.2 and 2.0 in.² for interior and exterior stringers, respectively. Considering the effective flange lengths associated with Eq. (5b) (67.5 in. for interior and 42 in. for exterior stringers), A_{sf} is equal to 0.57 in.²/ft for both the interior and exterior stringers.

The flange transverse reinforcement placed in the top layer equals $2\,A_{vf}/3$ or A_{sf}, whichever is greater. Use #5 bars at 6 in. on centers in the top layer over the entire length of the beam. This provides a steel area of 0.62 in.²/ft, which satisfies all the requirements for reinforcement in the top layer.

In order to develop the yield strength of flange transverse reinforcement, these bars will be furnished in the shape of closed rectangular stirrups as shown in Figs. 7 and 13c. These bars will also provide support for longitudinal reinforcement in the flange.

The flange transverse steel required in other layers below the top layer equals $(A_{vf}/3 =)$ 0.22 and 0.19 in.²/ft for interior and exterior stringers, respectively. Provide #3 bars at 6 in. on centers in a layer that is placed 4 in. below the top layer over the entire length of the beam. This satisfies the requirements on vertical spacing and area of reinforcement for these bars.

The longitudinal components of the live load stringer reactions were not included for the design of top reinforcement in the flange. Since the stringer spacing exceeds $B + 5\,a$, there is sufficient reserve strength in the flange to resist such forces.

Similarly, the effect of longitudinal forces due to friction at stringer bearings were not considered in the design since these forces are given as negligible.

Web Design for Hanger Action

Since b_f equals 70 in., d_f (= 18 − 2 − 0.625 − 0.5) = 14.9 in., P_u = 221,000 lbs with S = 84 in. for interior stringers, and B = 20 in., calculated A_v/s for strength requirements is 0.086 in. and 0.103 in. from Eqs. (6b) and (7b), respectively.

P_s equals 130,000 lbs and $B + 3a$ (= 20 + 3 x 9.5) = 48.5 in., which is less than S = 84 in. Hence, A_v/s required for serviceability considerations and calculated from Eq. (8b) is 0.134 in. The largest value of A_v/s obtained from Eqs. (6b), (7b), and (8b) is 0.134 in. and will be used in design.

At exterior stringers, P_u equals 143,000 lbs, P_s = 90,000 lbs, and S (= 2 x 21) = 42 in. The largest value of A_v/s obtained from Eqs. (6b), (7b), and (8b) is again 0.134 in. Hence, A_v/s = 0.134 in. is required over the length XZ of the beam for the design of hangers. The final selection of stirrups acting as hangers will be delayed until the design for shear and that for shear plus torsion has been completed.

Web Design for Maximum Flexural Shear

The critical sections occur at X and Y as indicated in Fig. 13a. Since d equals (64 − 2 − 0.625 − 1.41 − 0.5 =) 59.5 in., A_e = 2670 in.^{2}, and the applied ultimate shear force (V_u) at the face of the support or Section X [= 2 x (221,000 + 143,000) plus (57,000 for the self weight of the beam)] = 785,000 lbs, the required value of $A_v f_y d/s$ is calculated to be 585,000 lbs from Eq. (9).

Because this value is less than $8\sqrt{f'_c}\, b_w d$ (= 1,020,000 lbs), the stirrups can be provided to resist the required force and the cross section need not be revised. The required A_v/s for shear force at X is then calculated as 0.164 in., which will control the design of stirrups required for shear force in distance XY.

The applied ultimate shear force at Section Y is [2 x 143,000 plus (30,000 for the self weight) =] 316,000 lbs and the calculated A_v/s from Eq. (9) is 0.01 in. This is lower than the minimum value required. Hence, A_v/s equals $50\, b_w/f_y$ (= 50 x 34/60,000) = 0.03 in., which will control the design of stirrups for shear force in distance YZ. The final selection of stirrups will be delayed until the check for shear plus torsion has been done.

Web Design for Combined Flexural Shear and Torsion

The most critical section for torsion design occurs at X, where $\Sigma x^2 y$ = 85,650 in.^{3}, x_1 = 29.4 in., y_1 = 59.4 in., and α_t [= 0.66 + 0.33 x (59.4/29.4)] = 1.33.

The applied ultimate torque (T_u) at Section X [= (130,000 + 65,000) x (9.5 + 34/2)] = 5,168,000 lb-in.

The applied ultimate shear force (V_u) at Section X [= 785,000 − (130,000 + 65,000)] = 590,000 lbs, and the flexural shear capacity of the cross section (V_o) at X will at least equal the applied maximum ultimate shear force acting alone which is 785,000 lbs.

From Fig. 12, β is calculated as 0.56 which satisfies Eq. (13) and the cross section need not be revised. The A_v/s required over distance XY for the effect of flexural shear plus torsion is then computed from Eq. (14) as 0.03 in.

Since the applied ultimate torque at critical section Y is very small, only minimum reinforcement is required. Hence, A_v/s equals $50\, b_w/f_y$ = 0.03 in., which will control the design of stirrups for the combined effect of flexural shear and torsion in distance YZ.

Action on Web	Distance XY	Distance YZ
(a) Hanger tension	0.134 in.	0.134 in. (controls)
(b) Maximum flexural shear	0.164 in. (controls)	0.03 in.
(c) Torsion plus flexural shear	0.03 in.	0.03 in.

Selection of Web Stirrups

The table above shows the summary of A_v/s required for the web under different actions.

The stirrups required for hanger action or flexural shear alone will be closed at least across the bottom of the beam. Closed rectangular stirrups are required for resisting torsional forces. Use (1# 5 closed + 1# 4) stirrups at 6 in. on centers in distance XY and (1# 4 closed + 1# 4) stirrups at 6 in. on centers in distances YZ as shown in Fig. 13c. These stirrups will satisfy the requirements for all actions summarized above.

Longitudinal Reinforcement

The most critical section for flexure is at the face of the support and the maximum bending moment occurs when full live loads act on both sides of the web. Applied ultimate bending moment (M_u) at X equals ($2 \times 221{,}000 \times 6.1 + 2 \times 143{,}000 \times 13.1 + 3800 \times 14.8^2/2 =$) 6,859,000 lb-ft and $d = 59.5$ in.

This gives the required area of steel = 27.2 in.^{2} Use 18 #11 bars placed in two layers near the top of the beam as indicated in Fig. 13c. Other longitudinal bars shown in Fig. 13c are required to provide stiffness to the steel cage for handling purposes and to resist the longitudinal forces that occur when torsion acts.

A further check on longitudinal reinforcement required for torsion is not necessary since the design of web stirrups is not controlled by torsion plus flexural shear. The longitudinal bars in the top corners of the flange also act as anchor bars for flange transverse reinforcement.

The intent of this example was merely to elaborate the design requirements specifically associated with inverted T beams and recommended in the body of this paper. In addition to these requirements, all code provisions,[1,2] especially those for spacing and development of reinforcement, should be satisfied. Another area of major consideration is the connection of the pier and the inverted T beam.

SI Conversion Factors

1 ft = 12 in. = 305 mm
1 in.^{2} = 645 mm^2
1 lb = 4.45 N
1000 psi = 6.9 MPa
1000 lb-ft = 12,000 lb-in.
= 1356 N-m

* * *

NOTE: Discussion of this paper is invited. Please submit your comments to PCI Headquarters by March 1, 1986.