Computer Models for Nonlinear Analysis of Reinforced and Prestressed Concrete Structures

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Prestressed concrete designs have been widely used for buildings, bridges, tanks, offshore oil platforms, nuclear containment vessels, and many other structures. The design of these structures must satisfy the requirements of safety and serviceability. While this can be accomplished in most cases by following approximate or empirical procedures prescribed in codes or recommended practices, it is desirable to have refined analytical models and methods available which can trace the structural response of these structures throughout their service load history and under increasing loads through their elastic, cracking, inelastic, and ultimate ranges.

Such refined analytical methods, after having been verified by selected experimental results, may be used to study the effects of important parameters in a systematic way to provide a firmer basis for the codes and specifications on which usual designs are based, or they may be used directly in the analysis and design of unusual and complex structures.

The purpose of this paper is to describe a unified numerical procedure for the material and geometric nonlinear analysis of various types of reinforced and prestressed concrete structures. Time-dependent effects due to load history, temperature history, creep, shrinkage and aging of the concrete, and

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relaxation of the prestressing steel may also be included in the analysis.

A finite element tangent stiffness formulation, coupled with a time step integration scheme, is described which traces the quasi-static response of the structures throughout their load history up to ultimate failure. An updated Lagrangian formulation is used to take into account the effects of changing structural geometry.

Any combination of nonprestressed and prestressed reinforcement may be specified so that partial and fully prestressed design cases may be analyzed. The prestressing may be provided by either pretensioned tendons or bonded and unbonded post-tensioned tendons, in which friction is taken into account.

Synopsis

Analytical models and an efficient numerical procedure are described for the material and geometric nonlinear analysis of reinforced and prestressed concrete rigid frames, slabs, panels, thin shells, and three-dimensional solids. Time-dependent effects due to load history; temperature history; creep, shrinkage and aging of the concrete; and relaxation of the prestressing steel are included in the analysis.

Any combination of nonprestressed and prestressed reinforcement may be specified so that partial and fully prestressed cases may be analyzed. The prestressing may be provided by either pretensioned or bonded and unbonded post-tensioned tendons having general geometric layouts. Loss of prestress due to various effects, including friction, are taken into account.

Brief descriptions of four computer programs developed at the University of California, which are capable of performing the above analyses, are given in the Apendix.

ANALYTICAL MODELS

For the purposes of structural analysis, a prestressed concrete structure can be analytically modeled as a system of joints (nodal points) interconnected by discrete structural elements made up of concrete, reinforcing steel, and prestressing steel. The objective of the analysis may be stated simply: given the joint loading, prestressing and environmental history, the geometry of the structure (location of the joints) and the stiffness properties of the structural elements; find the resulting joint displacements and the internal stresses in the concrete, reinforcing steel, and prestressing steel within each structural element at any time up to ultimate failure.

In general, a prestressed concrete structure may be idealized by one or more of the following types of structural systems (Fig. 1):

1. Planar or three-dimensional rigid frames made up of one-dimensional (1D) elements;

2. Panels or slabs made up of two-dimensional (2D) triangular or quadrilateral flat finite elements;

3. Thin shells made up of two-dimensional (2D) flat or curved finite elements or axisymmetric thin shell elements; and

4. Solids made up of three-dimensional (3D) solid finite elements or axisymmetric solid elements.

Numerous general purpose computer programs have been developed and are available to perform linear elastic analyses for the above systems. However, only in recent years have nonlinear analysis methods and programs been developed for reinforced concrete systems and, to a much lesser extent, for pre-

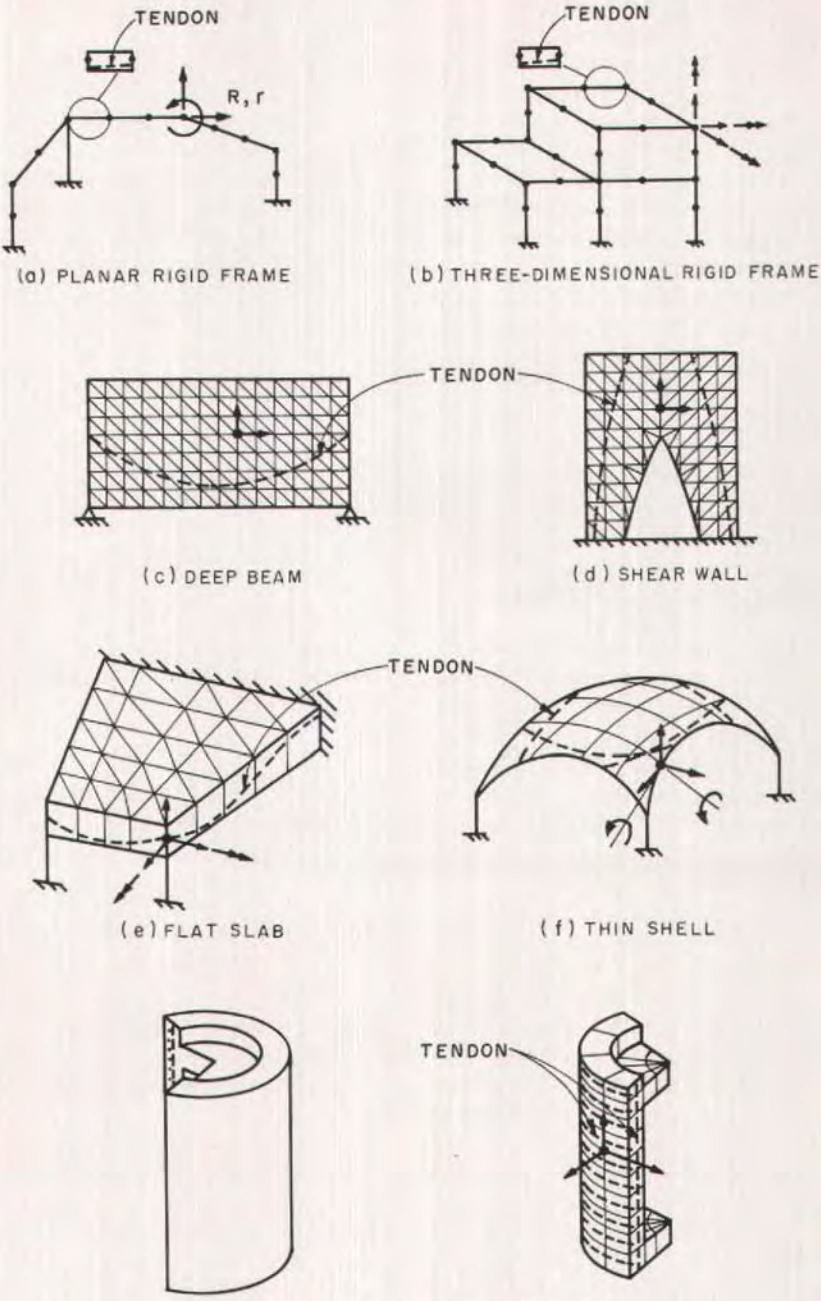

Fig. 1. Idealized analytical models.

stressed concrete systems. An excellent recent (1982) state-of-the-art report[1] on this subject presents a complete review of the factors which should be considered in such an analysis.

A unified approach, which can be applied to the nonlinear analysis of any of the above prestressed concrete systems, will be presented first for the most common type encountered, namely, a planar rigid frame loaded in its own plane.

PLANAR RIGID FRAME

The following brief summary is based on a comprehensive PhD research investigation completed in 1977 by Kang[2,3] at the University of California at Berkeley.

Analytical Model

A planar reinforced and prestressed concrete frame (Figs. 1a and 2) consists of joints with three degrees of displacement freedom (3DOF) interconnected by one-dimensional (1D) elements. Each element is assumed to have a prismatic cross section which has an axis of symmetry, but the shape of the cross section may differ element by element (Fig. 2).

The local coordinate system xyz for each element is defined by a longitudinal reference axis x, which need not be the centroidal axis. Each element is divided into a discrete number of concrete and reinforcing steel layers of given properties. Each of the prestressing steel tendons has a given profile, initial tensioning force, and a constant cross-sectional area along its length.

Each tendon is divided into a number of prestressing steel segments, each of which is straight, spans a single element, and is assumed to have a constant force. The locations of the two end points of a prestressing steel segment are defined by the element and eccentricities e_i and e_j (Fig. 2d).

Material Properties

Each concrete or reinforcing steel layer in the cross section or prestressing steel segment within an element is assumed to be in a state of uniaxial stress. It is assumed that plane sections remain plane and the deformations due to shearing strains are neglected.

The material properties of the concrete and steel at any time or under increasing load depend on the strain state of the material due to the effects of the nonlinear stress-strain relationship, cracking, yielding, and crushing of the concrete and yielding of the steel.

The element stiffness matrix or internal forces at any time are evaluated by integrating over the volume of the element the contributions of each concrete and reinforcing steel layer over the depth and of each discrete prestressing steel segment within the element.

For the concrete under a uniaxial stress state, the following basic material properties are needed: (1) short-time deformation and stress-strain relationship including load reversals; and (2) time-dependent relationships for creep, shrinkage, aging, and temperature history. If experimental data are not available for time-dependent effects, empirical design formulas as recommended by ACI or CEB-FIP may be used instead.

The short-time uniaxial stress-strain curve adopted for the concrete is based on the widely used relationship in compression suggested by Hognestad,[4] together with cracking in tension at the tensile strength. Using additional assumptions for load reversals, a total of eleven possible material states are obtained (Fig. 3a).

For the reinforcing steel, a bilinear stress-strain relationship is assumed with load reversals, making four different material states possible (Fig. 3b).

Besides a large difference in the magnitude of the ultimate tensile strength, the stress-strain curve for the prestressing steel differs from that of the rein-

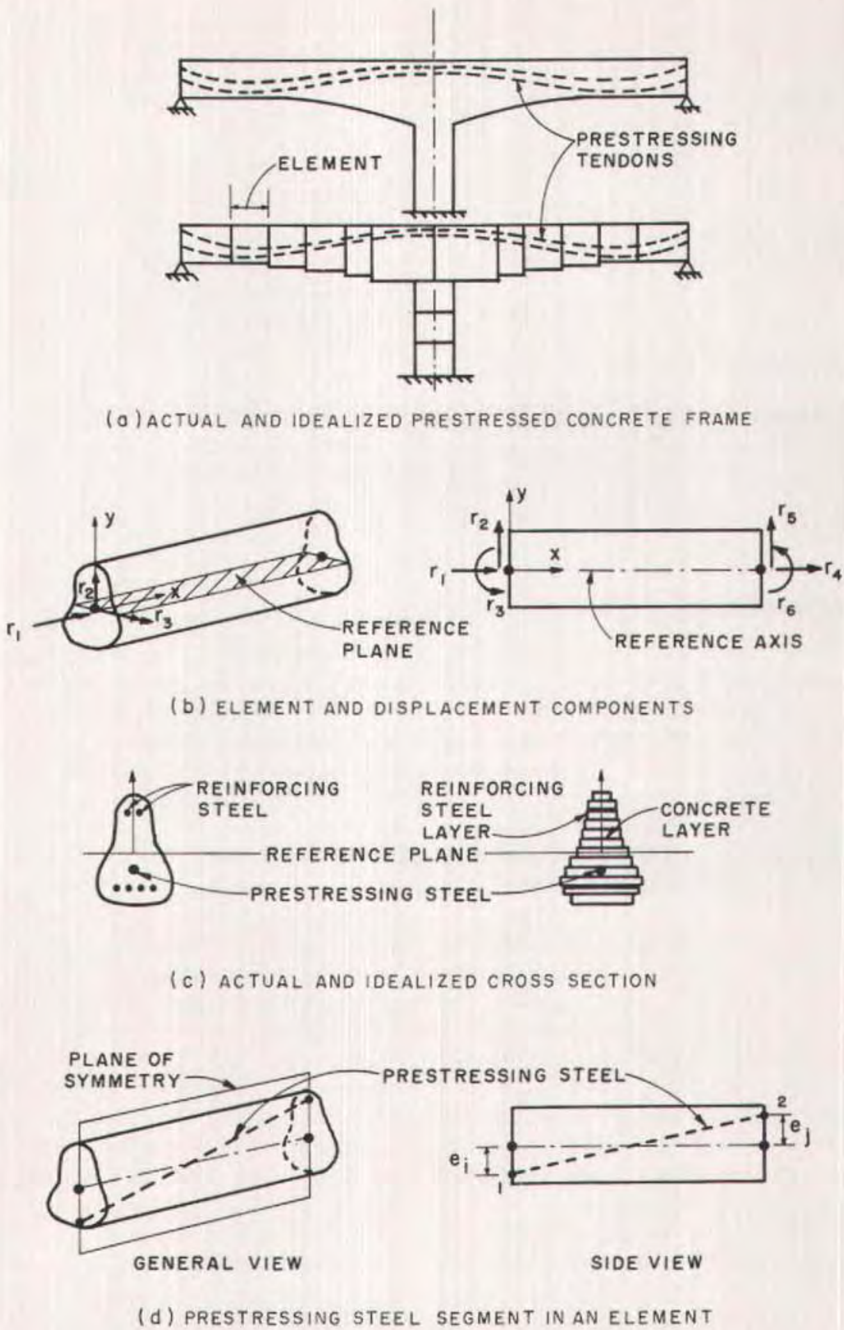

Fig. 2. Analytical model for planar rigid frame.

forcing steel in that it has no definite yield plateau. Thus, a multilinear stress-strain curve (Fig. 3c) is adopted for the prestressing steel. In addition, the usual empirical formulas are used for stress relaxation and friction properties of the prestressing steel.

Age and Temperature Dependent Integral Formulation for Creep

A numerical formulation, developed by Kabir,[5,6] is used for creep of the concrete in which the principle of superposition is assumed to be valid for evaluating the creep strain $\epsilon^c(t)$ at any time t as expressed by the following superposition integral:

$$\epsilon^c(t) = \int_0^t C(\tau, t-\tau, T)\, \frac{\partial \sigma(\tau)}{\partial \tau} d\tau \qquad (1)$$

in which $C(\tau, t-\tau, T)$ is the specific creep function dependent on age of loading τ and temperature variation T, and $\sigma(\tau)$ is the stress applied at time τ.

The total creep strain at any time t (Fig. 4c) can be found as the sum of the independent creep strains produced by the stress changes at different ages with different durations of time up to t.

For the effect of temperature variations on creep (Fig. 4a), the concrete is assumed to obey the time shift principle as described by Mukaddam and Bresler.[7]

Stress and temperature changes are assumed to occur only at distinct time steps $t_n; n = 1, 2, \ldots, N$ (Fig. 4a, b). And, for the calculation of the creep strain increment during a given time interval, the stress and the temperature are assumed to remain constant.

It has been found that certain forms of mathematical approximations for the specific creep function, while representing experimental or empirical creep curves accurately, overcome the necessity of storing all the stress increments of previous time steps. Such a form is adopted as follows:

$$C(\tau, t-\tau, T) = \sum_{i=1}^{m} a_i(\tau)\left[1 - e^{-\lambda_i \phi(T)(t-\tau)}\right] \qquad (2)$$

in which m, $a_i(\tau)$, λ_i, $\phi(T)$ are determined by a least squares fit to experimental or empirical creep curves.

An efficient numerical procedure for evaluating creep strains (Fig. 4) can be developed using the following definitions for incremental quantities of time steps, stresses, and creep strains:

$$\Delta t_n = t_n - t_{n-1} \qquad (3)$$

$$\Delta\sigma_n = \sigma_n - \sigma_{n-1} \equiv \sigma(t_n) - \sigma(t_{n-1}) \qquad (4)$$

$$\Delta\epsilon_n^c = \epsilon_n^c - \epsilon_{n-1}^c \equiv \epsilon^c(t_n) - \epsilon^c(t_{n-1}) \qquad (5)$$

Combining Eq. (1) to (5), after some extensive algebraic manipulations, the recursive relations necessary for calculating the increment of creep strain $\Delta\epsilon_n^c$ at a time step t_n are as follows:

$$\Delta\epsilon_n^c = \sum_{i=1}^{m} A_{i,n}\left[1 - e^{-\lambda_i \phi(T_{n-1})\Delta t_n}\right] \qquad (6)$$

$$A_{i,n} = A_{i,n-1}\left[e^{-\lambda_i \phi(T_{n-2})\Delta t_{n-1}}\right] + \Delta\sigma_{n-1} a_i(t_{n-1}) \qquad (7)$$

$$A_{i,2} = \Delta\sigma_1 a_i(t_1) \qquad (8)$$

A very important advantage of the above formulation is that the computation for each new creep strain increment requires only the stress history of the last time step and not the total stress history.

Effect of Prestressing

Kang[2,3] has described in detail how to treat pretensioned or bonded or unbonded post-tensioned tendons. Only the general case of post-tensioned bonded tendons will be discussed here, in which a prestress loss takes place during the tensioning operation due to

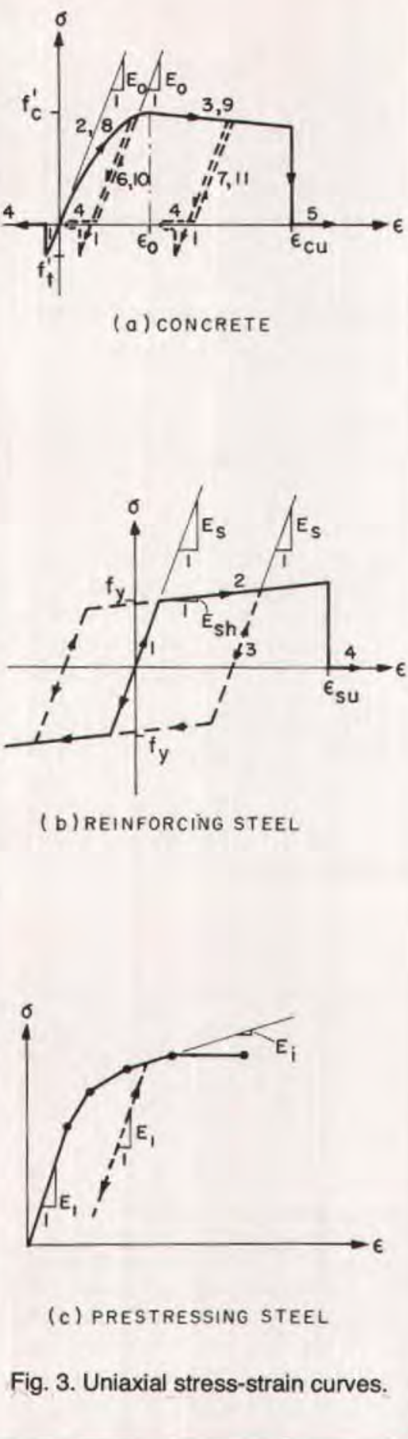

Fig. 3. Uniaxial stress-strain curves.

friction and anchorage slip; and after the transfer of prestress due to the creep and shrinkage of the concrete, the relaxation of the prestressing steel, and the effects of load and temperature history.

At the transfer of prestress the decrease in the prestressing steel force due to friction can be calculated by the well-known formula:

$$P_2 = P_1 e^{(\mu\theta + KL)} \tag{9}$$

in which P_1 and P_2 are the prestressing forces at Points 1 and 2, L is the length between Points 1 and 2, θ is the angle change in radians between Points 1 and 2, and μ and K are, respectively, the curvature and wobble friction coefficients.

Since each prestressing steel segment within a structural element along the length (Fig. 2) is assumed to be straight with known global (X, Y) end coordinates, it can be treated as a vector in space for the purposes of calculating L and θ to any point along the total length of the tendon. The force in a specific segment is assumed constant and is taken as the average of the forces at the two end points of the segment.

When there is an anchorage slip at the tensioning end, a simplified assumption of a uniform decrease in tendon strain and stress over the entire tendon length, which reduces the tendon segment forces, may be used or a more elaborate procedure accounting for friction as described by Van Greunen[8] may be adopted.

With the prestressing force in each tendon segment known, a simple vector addition and transformation to the reference axis (Fig. 2) of the contributing forces from two adjacent elements can be used to obtain the equivalent joint loads due to prestress. The analysis of the reinforced and/or prestressed concrete frame at transfer, in which the stiffness of the prestressing steel is not included, can then be made for the joint loads due to prestress and the dead load.

After the transfer of prestress, the prestressing steel in the duct is grouted and thus the displacement field is assumed continuous in each composite structural element made up of concrete, reinforcing steel, and bonded prestressing steel. For element stiffness calculations, the prestressing steel segment is assumed as a steel layer parallel to local x-axis and located at the tendon depth at the midlength of the segment (Fig. 2d). However, the prestressing steel strains, stresses, and forces for additional loads and time-dependent effects after transfer are calculated using the current length of each prestressing steel segment at any time as determined from the current global coordinates (X, Y) found in the incremental and iterative process used in the nonlinear analysis of the structure.

The following steps are used:

1. Calculate the strain increment $\Delta\epsilon = (L_c - L_p)/L_o$, where L_c is the segment length for the current iteration, L_p is the previous length and L_o is the original length;

2. Add $\Delta\epsilon$ to the previous total to obtain the current total prestressing steel strain ϵ;

3. Determine the stress σ corresponding to ϵ from the nonlinear stress-strain curve shown in Fig. 3c; and

4. Subtract the stress-relaxation calculated by a given empirical formula to obtain the current prestressing steel stress σ.

Nonlinear Analysis Procedure for the Frame

The external loads R are assumed to be applied only at the joints (Fig. 1a). The prestressing, external joint load history, temperature history, short-time stress-strain curves (Figs. 3, 4), time-dependent material properties, and boundary conditions are given. The unknown joint displacements r and the internal strains ϵ and stress σ in the concrete, reinforcing steel, and prestressing

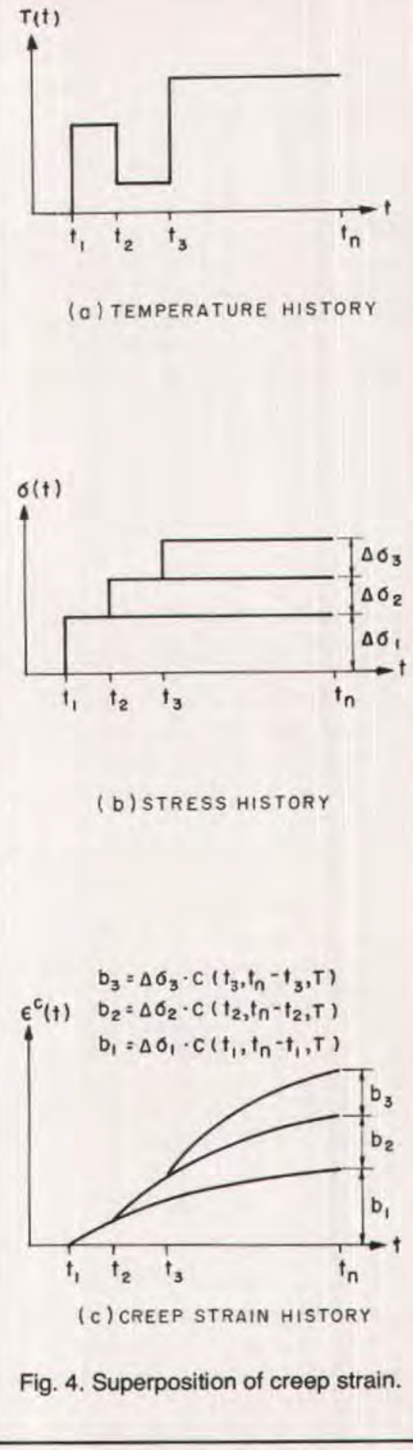

Fig. 4. Superposition of creep strain.

steel at any point in the structure are to be found for any instant of time. The resulting load-displacement (R vs. r) relationship will be nonlinear because of possible nonlinear material, geometric, or time-dependent effects.

To incorporate time-dependent nonlinearities, the time domain is divided into a discrete number of intervals, and a step forward integration is performed in which increments of displacements and strains are successively added to the previous totals as the solution marches forward in the time domain.

At each time step, a direct stiffness analysis based on a displacement formulation is performed in the space domain, in which the resulting equilibrium equations will be nonlinear to be valid for the current state of material properties and geometry.

To account for geometric nonlinearity, an updated Lagrangian formulation[2,3] is used, in which the direction of the local coordinate system (x, y) is continuously updated as the structure deforms. Internal forces and stiffnesses are calculated in the local coordinate system for each element and transformed to the fixed global coordinate system (X, Y) where the equilibrium equations for the entire structure are assembled by the direct stiffness method and solved. Thus, the continuously changing displacement transformation matrix for each element takes into account the effect of geometric nonlinearity along with the nonlinear form of the strain-displacement ($\epsilon - r$) relationship.

The total, Eq. (10a), or tangential, Eq. (10b), equilibrium equations in matrix form are:

$$K\,r = R \tag{10a}$$

$$K_t dr = dR \tag{10b}$$

in which the stiffness matrices K or K_t are functions of displacement r and material properties. Eqs. (10a) or (10b) are solved in the computer using an incremental load method with iterations within each load increment.

At a typical time t_{n-1}, all the joint displacements r, total strains ϵ, total nonmechanical strains ϵ^{nm}, and stresses σ at every point in the structure are known. The increments of nonmechanical strains $\Delta\epsilon^{nm}$ are evaluated due to creep and shrinkage in the concrete, relaxation in the prestressing steel, and temperature changes occurring during the time step from t_{n-1} to t_n. The equivalent joint load increments ΔR^{nm} at time t_n, summed for the concrete, reinforcing steel, and prestressing steel, are then calculated from their respective nonmechanical strain increments $\Delta\epsilon_n^{nm}$.

Thus, at time t_n, the load increment ΔR_n to be applied to the structure is obtained by adding the external joint load increment ΔR_n^j and the unbalanced load R_{n-1}^u left over from time t_{n-1} to the equivalent joint load increments ΔR_n^{nm} due to nonmechanical strains:

$$\Delta R_n = \Delta R_n^j + \Delta R_n^{nm} + R_{n-1}^u \tag{11}$$

If desired, ΔR_n may be further subdivided into several smaller load increments ΔR for incremental load analysis and unbalanced load iteration which is performed using the following steps:

1. Form the tangent stiffness in local coordinates for each element based on current geometry and material properties. Assemble the structure tangent stiffness K_t in global coordinates using the current displacement transformation matrix for each element.

2. Solve Eq. (10b) for displacement increments Δr and transform to local coordinates for the computation of strain increments $\Delta\epsilon$ using the nonlinear incremental strain-displacement relationships. Add $\Delta\epsilon$ to previous totals to obtain current total strains ϵ in the concrete, reinforcing steel, and prestressing steel.

3. Add displacements Δr to previous total to get current total joint displacements. Update geometry, element local axes and lengths, and displacement transformation matrix.

4. Subtract current total nonmechanical strains ϵ^{nm} from current total strains ϵ

to obtain current total mechanical strains ϵ^m and compute current total stress σ in concrete, reinforcing steel, and prestressing steel from the nonlinear stress-strain curves (Fig. 3).

5. Compute internal element end forces by integrating current total stresses for each element in local coordinates and transform into global coordinates using the updated displacement transformation matrices and assemble the internal resisting joint loads R^i.

6. Subtract the internal resisting joint loads R^i from the current total external joint loads R^j to obtain the unbalanced loads R^u:

$$R^u = R^j - R^i \tag{12}$$

7. Set $\Delta R = R^u$ and go back to Step 1. Steps 1 to 7 are repeated until the un-

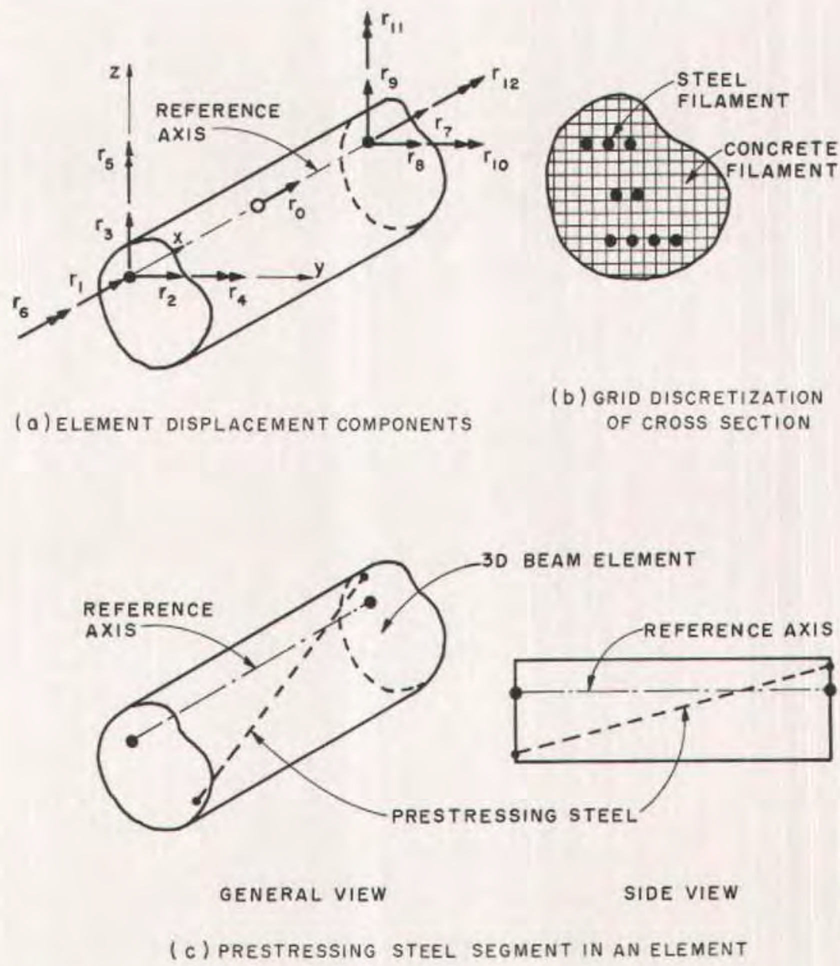

Fig. 5. One-dimensional element for three-dimensional rigid frames.

balanced loads R^u are within allowable tolerances. At this point, the current unbalanced loads R^u are added to the load increment ΔR for the next load step, and the iterative procedure (Steps 1 to 7) is performed again.

At the end of the final load step for time T_n proceed to the next time step t_{n+1} and repeat until final time is reached or ultimate failure occurs.

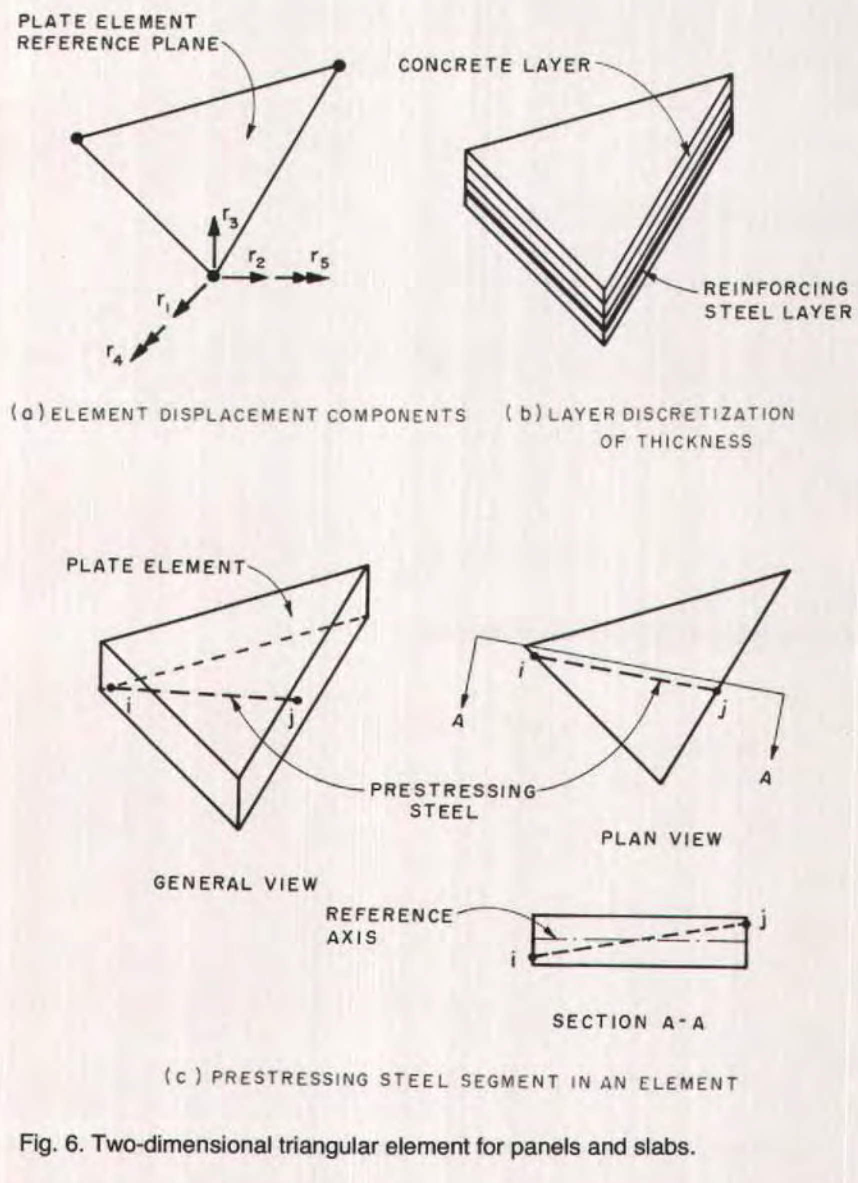

Fig. 6. Two-dimensional triangular element for panels and slabs.

A computer program PCFRAME has been developed by Kang[2,3] to perform the entire analysis described for planar rigid frames.

EXTENSION TO OTHER STRUCTURAL SYSTEMS

The general procedure just presented has been or can be extended to a number of other structural systems. These will only be described briefly here.

Three-Dimensional Frame

For this case (Fig. 1b), the frame consists of joints with six degrees of displacement freedom (6DOF) interconnected by one-dimensional (1D) elements. Each 1D element (Fig. 5) is prismatic with an arbitrary cross section, which is divided by a grid into a discrete number of concrete areas, longitudinal reinforcing steel areas, and a straight prestressing steel segment defined by a vector in space by its element end coordinates.

The concrete and the steel are again assumed to be in uniaxial states of stress, so that the stress-strain curves of Fig. 3 would apply. Numerical integration over the width and depth of cross-sectional grid can be performed in this case to evaluate an internal axial force and two bending moments about reference axes at each end of the element. Element end shears can be found from statics, but if torsion exists only approximate empirical assumptions may be used to incorporate its nonlinear contribution into the solution.

Chan[11] assumes the torsion to be uncoupled to other effects and uses an empirical trilinear torque-twist relationship to account for its material nonlinearity. He treats only reinforced concrete members, but the addition of prestressing has recently (1984) been implemented by Mari[10] into a computer program PCF3D.

Panels or Slabs

A comprehensive PhD study for this case (Figs. 1c, d, e) was completed in 1979 by Van Greunen.[8-9] The panel or slab consists of joints with five degrees of displacement freedom (5DOF) interconnected by two-dimensional (2D) flat triangular finite elements. Each element has three corner nodes and possesses in-plane membrane stiffness and plate bending stiffness (Fig. 6). The element thickness is divided into concrete and reinforcing layers (Fig. 6b) and each composite layer is assumed to be under a two-dimensional stress state.

The total panel or slab may be prestressed by panel tendons and/or slab tendons. A panel tendon lies in a plane parallel to and at the mid-thickness of the panel and may have any geometric profile along its length in that plane. A slab tendon lies in any plane normal to the slab and may have any geometric profile along its length in that plane. Within each finite element, the prestressing steel segment for either the panel or slab tendons is straight and can be defined by a vector in space by its element end coordinates (Fig. 6c).

A procedure similar to that described for the 1D elements of the planar frames is used to determine the effect of the prestressing tendons at Points i and j of the 2D triangular elements. The only difference is that the equivalent loads found at i and j (Fig. 6c) are then transferred to the element nodes on either side of i or j in proportion to the distance from the nodes. Also, after transfer for element stiffness calculations the bonded tendon is treated as an equivalent "smeared" layer of steel.

The uniaxial stress-strain curves (Figs. 3b, c) may still be used for the reinforcing and the prestressing steel, but because each concrete layer is under a two-dimensional plane stress state, a nonlinear constitutive relationship and failure theory which takes this into account must be used in the solution. An extensive discussion of these is given in Reference 1. Principal stresses must be calculated in each layer and cracking and nonlinear material response is traced layer by layer. While this complicates the numerical procedure, conceptually, it remains the same as described before for planar frames.

For panels with panel tendons only and subjected only to external in-plane loads, such as for a deep beam or a wall (Fig. 1c, d), only in-plane membrane stresses uniform across the thickness will be produced. In this case, only one concrete layer is needed in the solution and only the two in-plane translational DOF and the in-plane membrane stiffnesses are needed at each joint (Fig. 6a).

For slabs with slab tendons only and subjected only to external loads normal to the slab such as in a prestressed flat slab, both membrane and plate bending action occurs.

A layer discretization through the thickness of the element and all five DOF at each joint are required (Fig. 6a). Layer integration over the volume of the element is performed to determine the element stiffnesses and the internal resisting nodal forces for this case.

Van Greunen[8] has incorporated all of these effects into a computer program NOPARC for the nonlinear geometric, material and time-dependent analysis of reinforced and prestressed concrete slabs and panels.

Thin Shells

A comprehensive PhD study of the nonlinear geometric, material and time-dependent analysis of reinforced concrete shells with edge beams (Fig. 1f) was completed in 1982 by Chan.[11] Prestressing remains to be incorporated into the solution.

The extension from slabs to curved thin shells requires the selection of an appropriate layered finite element. The 2D flat triangular element described for slabs could be used, however, after an extensive study of a number of different elements, Chan[11] found that a nine node, 2D curved isoparametric element with five DOF at each node (Fig. 7a) gave the best solution with the least computational effort. Three translations, u, v, w in the global X, Y, Z directions and two rotations, α_1 and α_2 about the surface vectors V_1 and V_2 are used at each node.

Layer discretization of the concrete and reinforcing steel (Fig. 7b) and the treatment of a prestressing steel segment (Fig. 7c) within an element is similar to that used for slabs. Reinforcing steel is treated as uniaxial, equivalent curved "smeared" layers, the amounts and directions of which can vary at each integration point within the element. The prestressing steel segment can be assumed to be straight or curved between the boundaries of the element. Displacements, strains, and stresses in the concrete and reinforcing steel layers are monitored at each element integration point and at the integration points along the prestressing steel segment.

By adding two of the straight 1D elements shown in Fig. 5 to the side of a curved shell element, thin shells with edge beams can be analyzed. Chan[11] has incorporated all of the above effects, except for prestressing, into a computer program NASHL. Numerical studies with this program have shown that for some types of reinforced concrete shells, drastic reductions in the calculated ultimate load occur when nonlinear material, geometric and time-dependent effects are taken into account.

Three-Dimensional Solids

A strong impetus to the development of solutions for axisymmetric and general three-dimensional solids has been in connection with reinforced and prestressed concrete nuclear reactor vessels (PCRV), containment vessels, and also offshore oil concrete platforms. These structures involve large expenditures of

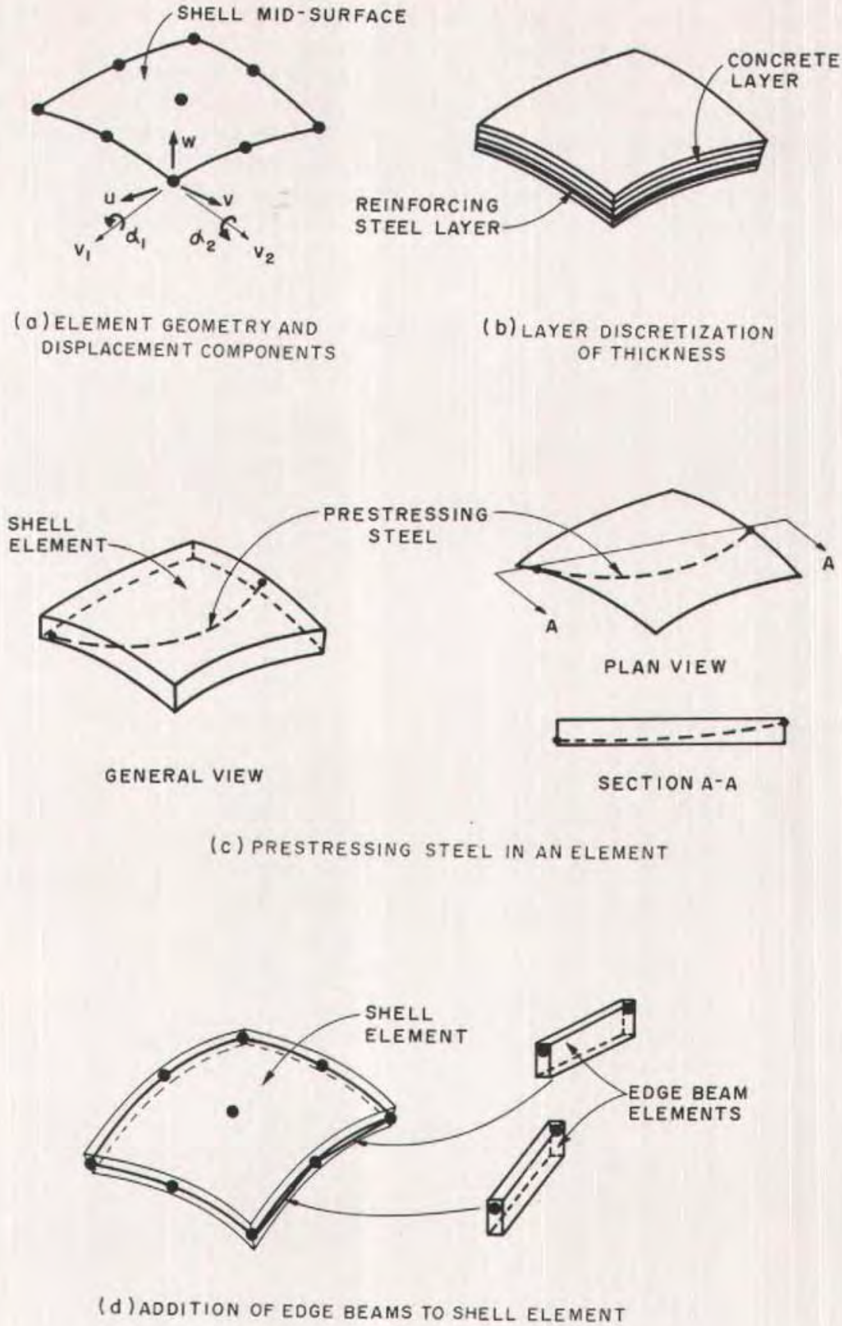

Fig. 7. Nine node curved isoparametric elements for thin shells.

money and must be designed under the most stringent design condition. General solutions to these problems have been developed by Argyris, et al.[12] and by Connor and Sarne.[13,14]

The family of elements developed by Argyris, et al.[12] for axisymmetric and three-dimensional PCRV analyses are shown in Figs. 8a, b. They include curved solid elements for the concrete; thin shell membrane elements for the steel liner and smeared layers of reinforcing or prestressing steel; discrete curved prestressing cables; and discrete linkage elements to model bond slip.

Connor and Sarne[13,14] have developed a somewhat similar family of elements for three-dimensional PCRV analyses. Three-dimensional constitutive relationships for the concrete are used

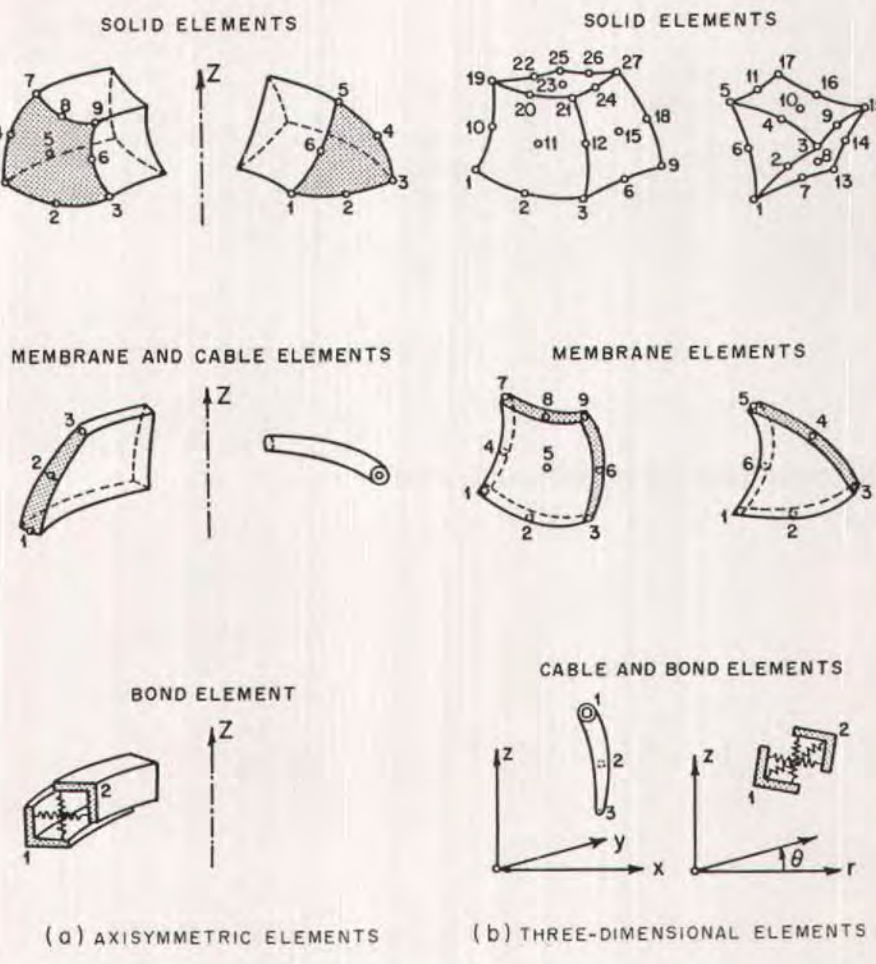

Fig. 8. Elements for the analysis of prestressed concrete reactor vessels (Ref. 12).

which consider nonlinearity under compressive stress, multi-dimensional cracking, time-dependent shrinkage, and stress/temperature induced creep of the concrete. In addition, the elastic-plastic behavior of the steel liner and the reinforcing steel as well as the loss of prestress due to various effects, including friction, are accounted for.

SUMMARY AND CONCLUSIONS

A unified approach for the nonlinear geometric, material and time-dependent analysis of reinforced and prestressed concrete structures of various types has been described. For general application, these analyses must be implemented into general computer programs. Details of the input-output capabilities and the availability of such programs are described in the Appendix to this paper.

It should be emphasized that considerable skill is needed to select the proper analytical model to input into the computer program for a given structure and a thorough understanding of structural behavior is essential to interpret the output of the results correctly. These are often time consuming and expensive jobs.

In conclusion, it should be remembered that the ultimate goal of all structural analysis is to aid the engineer in producing a satisfactory design in terms of serviceability and ultimate strength of a structure, which is designed to perform a given function. There is no question that new methods and computers will continue to improve our analysis capability. However, the selection of the correct structure to analyze, which satisfies the aesthetic, functional, and structural efficiency requirements, will still be dependent basically upon the conceptual and creative ability of the engineering designer.

ACKNOWLEDGMENTS

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NOTE: A list of References and details of the available computer programs appear on the following pages.

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APPENDIX—COMPUTER PROGRAMS FOR NONLINEAR ANALYSIS OF REINFORCED AND PRESTRESSED CONCRETE STRUCTURES

Four computer programs, developed at the University of California at Berkeley since 1977 are listed below. For each computer program a short identifying name of five to seven letters is given followed by a brief description of the program features.

Figures in parentheses indicate the year the program was first used and superscripts indicate references, where a detailed description of the program and its theoretical basis may be found.

These references also contain the input-output formats for the programs and numerical results for several examples obtained using the programs, which are compared with other analytical and/or experimental results for purposes of verifying the programs.

Those interested in any of the programs listed should first read the detailed references indicated. If they wish to obtain a copy of any of the research reports referred to or the computer program itself, they should write for further information to:

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All four of the computer programs listed below are written in Fortran IV language. Each of the programs provides a capability for the nonlinear geometric, material and time-dependent analysis of a different type of a prestressed and/or a reinforced concrete structural system. All of the programs utilize slightly varying versions of a finite element tangent stiffness formulation, coupled with a time step integration scheme, to trace the structural response of these systems through their service load history as well as throughout their elastic, cracking, inelastic and ultimate load ranges. Incremental and iterative solutions are used to achieve equilibrium at each time step.

General input into all of the programs consists of: (1) geometry of the structure (joint coordinates); (2) dimensions and material properties of the elements made up of concrete, prestessing steel, and/or reinforcing steel; and (3) loading, prestressing, and environmental history.

Basic output consists of: (1) joint displacements; and (2) internal stresses and strains in the concrete, prestressing steel and/or reinforcing steel within each element at any time up to failure. Features of the four programs are summarized below.

PCFRAME (1977)[2,3]

1. **Structural system**—Reinforced and prestressed concrete planar two-dimensional frames, beams, and columns.

2. **Joint degrees of freedom**—Three.

3. **Finite elements**—One-dimensional straight prismatic elements with a cross section having an axis of symmetry; section is divided into a discrete number of concrete and reinforcing steel layers of arbitrary width; external boundary spring elements can also be used.

4. **Concrete**—Uniaxial stress state; parabolic-linear Hognestad stress-strain law; time-dependent effects of creep, shrinkage and temperature included; no tension stiffening.

5. **Reinforcing steel**—Uniaxial stress state; bilinear stress-strain law for steel layers.

6. **Prestressing steel and tendons**—Uniaxial stress state; multilinear stress-strain law; any planar tendons idealized by a series of straight segments; pre-

tensioned, post-tensioned bonded or unbonded tendons; friction, anchor slip and stress relaxation included; equivalent joint loads due to prestressing calculated internally in the program.

7. **Loadings**—At each time step, external joint load increments as well as temperature and shrinkage increments can be specified.

NOPARC Program (1979)[8,9]

1. **Structural system**—Reinforced and prestressed concrete panels and/or slabs of arbitrary boundary shape.

2. **Joint degrees of freedom**—Five.

3. **Finite elements**—Two-dimensional flat triangular elements possessing both in-plane membrane and plate bending stiffnesses; thickness is divided into a discrete number of concrete and reinforcing steel layers; external boundary spring elements can also be used.

4. **Concrete**—Biaxial stress-state; Darwin-Pecknold biaxial stress-strain law; time-dependent effects of creep, shrinkage and temperature included; tension stiffening between cracks included.

5. **Reinforcing steel**—Uniaxial stress state for each layer, which can be any specified direction; several layers can be included; bilinear stress-strain law for each layer.

6. **Prestressing steel and tendons**—Uniaxial stress state; multilinear stress-strain law; two types of tendons, panel and/or slab, which are made up of straight segments; panel tendon lies in a plane parallel to and at the midthickness of the panel; slab tendon lies in any plane normal to the slab; pretensioned, post-tensioned bonded or unbonded tendons; friction, anchor slip and stress relaxation included; equivalent joint loads due to prestressing calculated internally in the program.

7. **Loadings**—At each time step, external gravity, surface and joint load increments as well as temperature and shrinkage increments can be specified.

PCF3D Program (1984)[10]

1. **Structural system**—Reinforced and prestressed concrete three-dimensional frames; beams; and columns.

2. **Joint degrees of freedom**—Six.

3. **Finite elements**—One-dimensional straight prismatic elements with an arbitrary cross section; section is divided into a discrete number of longitudinal concrete and reinforcing steel filaments; external boundary spring elements can also be used.

4. **Concrete**—Uniaxial stress state; parabolic-linear Hognestad stress-strain law; time-dependent effects of creep, shrinkage and temperature included; no tension stiffening.

5. **Reinforcing steel**—Uniaxial stress state; bilinear stress-strain law for steel filaments.

6. **Prestressing steel and tendons**—Uniaxial stress state; multilinear stress-strain law; any three-dimensional tendon idealized by a series of straight segments; post-tensioned bonded tendons only; friction, anchor slip and stress-relaxation included; equivalent joint loads due to prestressing calculated internally in the program.

7. **Torsion**—Assumed to be uncoupled to the other effects is accounted for by an empirical, trilinear torque-twist relationship.

8. **Loadings**—At each time step, external joint load or joint displacement increments as well as temperature and shrinkage increments can be specified.

NASHL Program (1982)[11]

1. **Structural system**—Reinforced concrete shell of arbitrary shape with edge beams.

2. **Joint degrees of freedoms**—Five on shell surface; six at shell-beam connecting joints.

3. **Finite elements**—For the shell, two-dimensional curved isoparametric elements with both in-plane membrane and plate bending stiffnesses; thickness is divided into a discrete number of con-

crete and reinforcing steel layers; for the edge beams, one-dimensional straight prismatic elements with rectangular cross sections divided into longitudinal concrete and reinforcing steel filaments; external boundary spring elements can also be used.

4. **Concrete**—Biaxial stress state; Darwin-Pecknold biaxial stress-strain law; time-dependent effects of creep, shrinkage and temperature included; tension stiffening between the cracks included for both shell and beam elements.

5. **Reinforcing steel**—Uniaxial stress-strain with bilinear stress-strain law; in shell elements each layer can have arbitrary specified direction; in beam elements longitudinal filaments can be at any point in cross section.

6. **Prestressing steel and tendons**—These have not yet been incorporated into NASHL.

7. **Loadings**—At each time step, external gravity, surface, pressure and joint load increments as well as displacement increments can be specified.

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NOTE: Discussion of this paper is invited. Please submit your comments to PCI Headqurters by July 1, 1985.