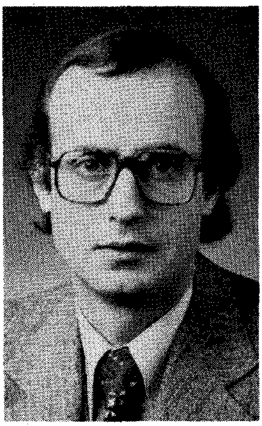
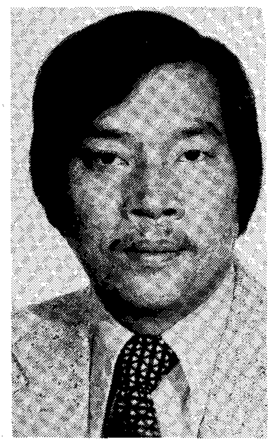
Reliability of Partially Prestressed Beams at Serviceability Limit States



Antoine E. Naaman
Professor of Structural Design
Department of Civil Engineering
Mechanics and Metallurgy
University of Illinois at Chicago
Chicago, Illinois

Amnuayporn Siriaksorn
Project Engineer
SRISUTEP
Engineering Company, Ltd.
Thailand



Conventional design methods require that a structure be designed so that its predicted capacity is larger than the specified design loads. In this approach (called Level I Design) it is assumed that all variables and factors are deterministic and their values exactly known. It is generally accepted, however, that the basic variables used in the prediction equations such as materials properties, section geometry, and the like are random variables and thus are characterized by a probability distribution function.

The concepts of reliability theory are being increasingly applied in structural engineering not only to assess the probability of survival of a structure but also to determine rational values of load and resistance factors prescribed by various building codes.[1-19] Two design methods have been so far proposed to account for the probabilistic nature of the variables at hand. One of them requires the knowledge of the entire distributions of resistance and loads (Level III approach) and is, therefore, quite complex and tedious to apply; the other one (Level II), called the first order second moment (FOSM) method, has been proven to apply with little difficulty to any type of structural member.[6,8,10,11,14,15,16,18]

The Level II approach requires the knowledge of only the mean and variance of the distributions of the resistance and load, and it has been shown to lead to results not significantly different from the more exact Level III approach. In the FOSM method a safety index or reliability index, β, is needed as an input variable specified by the code authority.

Although in a number of previous studies the reliability index β has been obtained for concrete or steel structures at the ultimate limit state,[7,9,13,17,18,20] little has been done to evaluate β at serviceability limit states. In this investigation the reliability index β of partially prestressed concrete beams at six serviceability limit states is evaluated. The reliability index β is directly related to the probability of failure or unserviceability where failure does not necessarily refer to collapse but to violation of any code specified limit; hence it implies abnormal levels of maintenance and repair during service life.

The six states considered are: maximum crack width, short- and long-term deflection and fatigue stress ranges in the concrete, the reinforcing steel and the prestressing steel. In order to provide a basis of comparison, β was also partly evaluated at the ultimate flexural strength limit state where it is related to the probability of failure or collapse.

In order to determine the β values for typical design situations, the method called Code Calibration[12,21] is used. Resistance prediction equations at each serviceability limit state are developed in terms of moments. A Monte Carlo simulation model is used to compute the statistical properties of the resistance at each limit state. The statistical properties of the load are collected from available literature[22-29] and used in the evaluation of the reliability index.

The effects of many parameters on the observed values of the reliability index β are analyzed. They include the effect of various types of beam cross sections (representative of those used in the American precast prestressed industry), span length, magnitude of live loads, amount of non-prestressed reinforcement and some important materials properties such as concrete compressive strength. A systematic comparison with the boundary cases of fully reinforced or fully prestressed beams is also presented. A summary of the method used and results obtained is given next. Detailed information can be found in Ref. 30.

Synopsis

A systematic investigation of the reliability of partially prestressed concrete beams at six serviceability limit states is described. The evaluation of reliability is based on the first order, second moment (FOSM) method which requires the knowledge of only the mean and variance of the distributions of the resistance and load.

The code calibration method was used to determine the reliability index (or safety index) β for 64 different beam designs representative of current practice in the precast prestressed concrete industry in the United States.

Many parameters were analyzed including the effects of cross-sectional shape, span length, magnitude of live load and degree of partial prestressing. The information derived should help code authorities to set consistent values of resistance and load factors in future codes.

Determination of Reliability Index

In classical reliability theory the total load (S) and the resistance (R) are considered random variables; thus they can be characterized by their probability distribution functions. Assuming S and R are dimensionally consistent, the failure event is described by the relation:

$$R < S \quad (1)$$

or

$$R - S < 0 \quad (2)$$

A new random variable m termed safety margin or reliability margin can be defined [see Fig. 1 (top)] as:

$$m = R - S \quad (3)$$

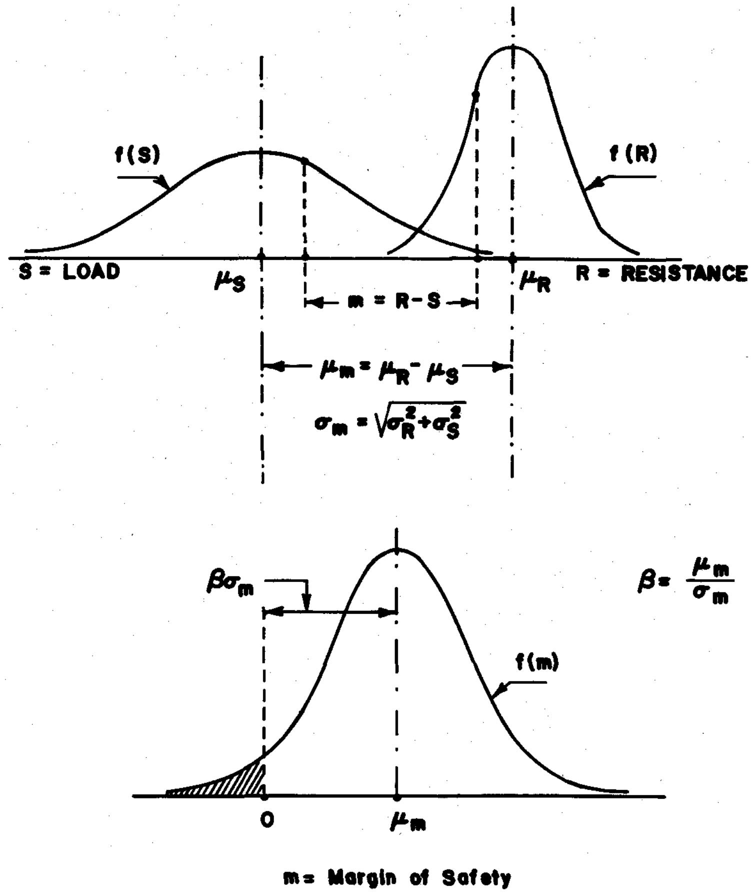


Fig. 1. Illustration of concept of reliability index.

Thus, failure is associated with $m < 0$ and survival is associated with $m \geqslant 0$. The mean value of m and its standard deviation can be derived from those of R and S as follows:

$$\mu_m = \mu_R - \mu_S \tag{4}$$

$$\sigma_m = \sqrt{\sigma_R^2 + \sigma_S^2} \tag{5}$$

In order to maintain a prescribed value of the probability of survival (or reliability), that is $m \geqslant 0$, it may be required that the mean value of m be larger than or equal to a coefficient β times its standard deviation, that is:

$$\mu_m \geqslant \beta\sigma_m \tag{6}$$

or

$$\mu_m - \beta\sigma_m \geqslant 0 \tag{7}$$

The above equation at equality combined with Eqs. (4) and (5) can be used to determine β:

$$\beta = \frac{\mu_m}{\sigma_m} = \frac{\mu_R - \mu_S}{\sqrt{\sigma_R^2 + \sigma_S^2}} \tag{8}$$

The coefficient β is called the "reliability index" or safety index. It is the distance from the mean of the probability distribution function of n to the origin in standard deviation units [see Fig. 1 (bottom)].

In the above treatment "failure" does not necessarily mean collapse (or the

Table 1. Serviceability Limit States Considered and Their Specified Limitations.

Limit states			Specified limitation*
No.	Symbol	Description	
1	W_{max}	Maximum crack width	0.016 in.
2	f_{cr}	Concrete fatigue stress range	$0.4f'_c - \frac{f_{min}}{2}$
3	f_{rr}	Non-prestressed steel fatigue stress range	20.0 ksi
4	f_{tr}	Prestressing steel fatigue stress range	$0.1 f_{pu}$
5	$\bar{\Delta}_{LL}$	Immediate live load deflection	$\frac{l}{180}$
6	$\bar{\Delta}_{add}$	Additional long-term deflection	$\frac{l}{240}$

*Note: 1 in. = 25.4 mm; 1 ksi ≈ 6.9 MPa.

attainment of an ultimate limit state) but may also imply unserviceability or violation of a serviceability limit state such as a maximum deflection limitation or a maximum crack width limitation. The serviceability limit states considered in this study are described in the next section.

The following steps were followed to evaluate the reliability index β of reinforced, prestressed and partially prestressed beams at each serviceability limit state:

1. The resistance prediction equations of the partially prestressed concrete member at any serviceability limit state are derived so that the resistance has the same unit as the applied loads (i.e., moment as a function of crack width or deflection limit). The method and equations developed in Ref. 31 were used. The resistance at each limit state is expressed in terms of basic variables and allowable maximum limitation at that limit state.

2. Statistical data on the variables used in the equations developed in Step 1 are collected. The statistical properties (μ, σ) of these variables are defined.

3. The value of each limitation for each limit state is selected from current codes or recommended practices, e.g., the allowable maximum crack width is taken as 0.016 in. (0.41 mm) for maximum crack width limit state as per the ACI Code (see Table 1).

4. Using Steps 1, 2 and 3, and applying a Monte Carlo simulation method, the statistical properties of the resistance (μ_R, σ_R) of the beam at each serviceability limit state are computed.

5. Statistical data on live loads are collected from available literature. Their statistics (μ_L, σ_L) are determined. Similarly, the statistical properties of dead load (μ_D, σ_D) are determined.

6. From Steps 4 and 5, the reliability index, β, of the beam at each limit state is calculated using Eq. (8).

A detailed description of the above procedure is given in Ref. 30. The last three steps are clarified further in the following sections.

Limit States

"When a structure or structural element becomes unfit for its intended purpose it is said to have reached a limit state."[16] Limit states are generally divided into two categories, ultimate limit states and serviceability limit states.

Ultimate limit states are related to a structural collapse of part or all of the structure. They are points that lie on the failure surface of the structure, such as ultimate moment, ultimate shear, ultimate ductility, etc. Serviceability limit states are related to disruption of the functional use of the structure, and/or its damage or deterioration during service. Examples include excessive cracking, leakage, and deflections which lead to abnormal maintenance and/or repair levels during service.

In this study the reliability of partially prestressed beams was evaluated at six serviceability limit states. They are summarized in Table 1 where the limitations used in the parametric analysis for each limit state are also given. These limitations are in accordance with the ACI Building Code,[32] or various ACI recommended practices. Also, in a limited number of cases and for comparative purposes the reliability index at ultimate flexural strength limit state was evaluated.

A note is in order here and is related to the manner in which the limit state fatigue was handled. Fatigue limit state can be classified either as: (1) an "ultimate limit state" because failure by fatigue results in the collapse of the member or (2) a "serviceability limit state" because it occurs during service loads and may have secondary effects on other serviceability limit states such as increases in crack widths, deflections and debonding. Here the second approach was used. The meaning of β in that case is clarified below in the section under "RESULTS."

Statistical Properties of Resistance

In general if a random variable can be expressed in function of other (basic) variables, its mean value and COV (coefficient of variation) can be estimated by expanding the function in a Taylor series about the mean values of the other (basic) variables, truncating higher order terms, and taking the expectations of that function. Thus, its mean and COV can be expressed in terms of the means and COV's of the basic variables which appear in the function.

This approximation is reasonably accurate, provided that the COV's of the basic variables are not too large and the equation is not highly nonlinear.[11,13] However, the above approach could not be applied to compute the mean and the COV of the resistance of partially prestressed concrete beams at each serviceability limit state since the prediction equation of the resistance cannot be always expressed in closed form.

In this study, relationships between the resistance of partially prestressed beams at serviceability limit states and corresponding loading conditions were first developed and expressed in terms of moments. The statistical properties of the basic variables present in these relationships were collected from available technical literature.[7,11,13,20,33-37]

Selected properties used in the reliability analysis are summarized in Table 2. A Monte Carlo simulation where values of the basic variables are selected at random was used to determine the mean and the coefficient of variation of the resistance. Each simulation comprised 200 iterations.

Since the coefficient of variation (δ_R) of the resistance obtained from the Monte Carlo simulation fails to include sources (such as incomplete information, insufficient data, inaccuracy of prediction equations, etc.) that contribute to the total variability (Ω_R) of the re-

Table 2. Summary of Statistical Properties of Variables Used in the Reliability analysis.

Variables	Distribution used	Mean, μ	COV., Ω	Remarks
b, b_1, b_w	Normal	$\left[0 \text{ to } + \frac{5}{32}\right] + b_n$	0.0 – 0.045	
h, h_{f1}, h_{f2}	Normal	$h_n - \frac{1}{8}$ to $h_n + \frac{1}{8}$	$\frac{1}{6.4\,\mu} - 0.045$	
d_p, d_s, e_1	Normal	d_{pn} to $d_{pn} + \frac{1}{16}$	$0.04 - \frac{0.68}{h_n}$	
l, a	Normal	l_n, a_n	$\frac{11}{32\,\mu} - \frac{11}{16\,\mu}$	
C_{E_c}	Normal	33.6	0.1217	nominal = 33 $E_c = C_{E_c} (\gamma_c)^{1.5} \sqrt{f'_c}$
C_{f_r}	Normal	9.374	0.0938	nominal = 7.5 $f_r = C_{f_r} \sqrt{f'_c}$
f'_c	Normal	$0.67 f'_{cn}$ to $1.17 f'_{cn}$	0.1 – 0.25	
γ_c	Normal	γ_{cn}	0.03	
$C_{f_{ci}}$	Normal	0.6445	0.073	nominal = 0.8 $f_{ci} = C_{f_{ci}} f'_c$
f_y	Beta	$1.07 f_{yn} - 1.19 f_{yn}$	0.09 – 0.15	
A_s	Normal	$0.9 A_{sn} - 1.01 A_{sn}$	0.015 – 0.04	
E_s	Normal	E_{sn}	0.024 – 0.033	
A_{ps}	Normal	$1.01176 A_{psn}$	0.0125	A_{psn} = 0.153 in.^{2}
f_{pu}	Normal	$1.0387 f_{pun}$	0.0142	f_{pun} = 270 ksi
f_{py}	Normal	$1.027 f_{pyn}$	0.022	f_{pyn} = 240 ksi
E_{ps}	Normal	$1.011 E_{psn}$	0.01	E_{psn} = 29000 ksi
$C_{f_{si}}$	Normal	$C_{f_{sin}}$	0.08	$C_{f_{sin}} = 0.7$ $f_{si} = C_{f_{si}} f_{pu}$
$C_{f_{se}}$	Normal	$C_{f_{sen}}$	0.08	$C_{f_{sen}} = 0.83$ $f_{se} = C_{f_{se}} f_{si}$
ΣO	Normal	ΣO_n	0.03	
d_{ss}	Normal	d_{ssn}	0.07	

sistance, a prediction error (ϵ_R) was considered. The prediction error, ϵ_R, for each limit state can be estimated as the coefficient of variation of the ratio of experimentally observed to theoretically predicted resistance values.

Prediction errors assumed in this study are given in Table 3. Two limits are shown for the limit states "crack width" and "additional long-term deflection" reflecting various experimental results analyzed. The total variabil-

Table 3. Prediction Errors Assumed for Each Limit State.

Limit State	ϵ_R
W_{max}	0.1 or 0.2
f_{cr}	0.05
f_{rr}	0.05
f_{tr}	0.05
$\bar{\Delta}_{LL}$	0.05
$\bar{\Delta}_{add}$	0.1 or 0.2

ity, Ω_R, of the resistance was then obtained from:

$$\Omega_R^2 = \delta_R^2 + \epsilon_R^2 \quad (9)$$

where

δ_R = value of coefficient of variation obtained from the Monte Carlo simulation. It is affected by inherent randomness and variabilities of the basic variables

ϵ_R = prediction error

Statistical Properties of Loading

Loads can be considered random variables and are generally divided into two groups termed "dead load" and "live load."

The "dead load" comprises the weight of the structure itself and the weight of non-structural elements (often called superimposed dead load) attached to the structure such as partitions, curtain walls and the like. A number of studies have suggested values of mean and coefficient of variation of "dead load."[11,12,14,15,21] These are summarized in Ref. 30.

In this study, however, a Monte Carlo simulation was used to generate from the basic random variables the statistics of the weight of the structure. A normal distribution with a coefficient of variation of 0.1 was assumed for the superimposed dead load, if any. In a treatment similar to that of the resistance, the mean μ_D and the coefficient of variation δ_D of dead load were obtained from the simulation; a prediction error ϵ_D = 0.1 was considered leading to a total coefficient of variation of dead load given by:

$$\Omega_D^2 = \delta_D^2 + \epsilon_D^2 \quad (10)$$

Live loads are generally associated with moving or movable loads such as occupants and furniture. The intensity of maximum live load depends on the type of occupancy, the tributary area and the projected service life of the structure. Live load intensity is in general measured as the equivalent uniformly distributed load which is the load that will produce the same load effect as the actual loads. Based on live load survey data,[23,26] probabilistic models of live loads and/or their statistical characteristics were developed.[20,21,22,24,27]

Ravindra, et al,[12] expressed the mean μ_L and variance (or coefficient of variation Ω_L) of lifetime maximum live load as functions of tributary area and number of tenancies (Table 4). A prediction error ϵ_L = 0.1 was included in the evaluation of the coefficient of variation Ω_L. Their proposed values of μ_L and Ω_L were adopted in this study and led to the consideration of sixteen sets of values of (μ_L,Ω_L) depending on tributary areas and number of tenancies.

Parametric Analysis

In order to determine the reliability index β of partially prestressed concrete beams, the following input parameters were studied (see Table 5 and Fig. 2): Eight different beam cross sections that are representative of sections widely used by the precast prestressed concrete industry in the United States; for

Table 4. Mean and COV. of Lifetime Maximum Office Live Load, μ_L (psf), Ω_L*

Number of tenancies	μ_L or Ω_L	Tributary area, sq ft			
		50	350	800	1200
1	μ_L	13.5	12.7	12.5	12.3
	Ω_L	0.8360	0.5787	0.5295	0.4900
2	μ_L	19.6	16.8	16.1	15.6
	Ω_L	0.4800	0.3736	0.3544	0.3350
5	μ_L	27.6	21.5	20.5	19.6
	Ω_L	0.3640	0.3160	0.2970	0.2880
10	μ_L	33.4	25.4	24.1	22.6
	Ω_L	0.3544	0.2970	0.2880	0.2880

*See Ref. 45. Note: 1 sq ft = 0.093m^2.

each beam three values of partial prestressing ratio (PPR) were explored, namely, PPR = 0 (corresponding to fully reinforced concrete), PPR = 1 (corresponding to fully prestressed concrete) and PPR = optimum (corresponding to partially prestressed concrete with one limit state binding;[29] various span lengths and representative live loads were also considered. In all, 64 different beam designs were analyzed each for six different serviceability limit states. In some cases the reliability index at ultimate flexural strength limit state was also determined.

A note is in order to understand the way in which the results of the parametric analysis were gathered.

In collecting from a multitude of sources,[7,11,13,20,33-37] information on the mean and COV (coefficient of variation defined as σ/μ) of the basic variables, it was generally found that a range between two extreme values could be identified for each variable. In this study a Monte Carlo simulation run of the resistance was used for each of the following six combinations of statistics of the variables: the lowest, mean and highest values of the mean, and the lowest and highest value of the COV.

These led for each beam and each limit state to six sets of values of mean (μ_R) and coefficient of variation (Ω_R) of the resistance. In evaluating the reliability index β, only the lowest and highest values of both μ_R and Ω_R were used leading essentially to four combinations of μ_R and Ω_R for each applied loading (μ_S and Ω_S).

The mean value (μ_S) and coefficient of variation (Ω_S) of the load S were determined from corresponding statistics of dead load and live load as:

$$\mu_S = \mu_D + \mu_L \tag{11}$$

$$\sigma_S^2 = \sigma_D^2 + \sigma_L^2 \tag{12}$$

$$\Omega_S = \frac{\sigma_S}{\mu_S} \tag{13}$$

where σ stands for standard deviation.

The statistics of dead load (μ_D, Ω_D) were obtained from a Monte Carlo simulation in a treatment similar to that used for the resistance R (see preceding section). In evaluating μ_S, Ω_S, and β four sets of (μ_D, Ω_D) were considered

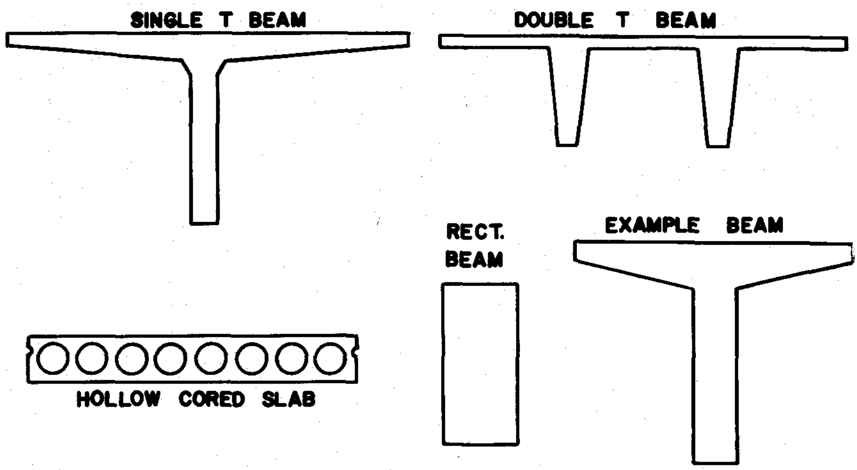


Fig. 2. Typical cross sections of beams analyzed.

Table 5. Design Cases Studied in the Reliability Analysis.

Beam No.	Type of beam	Live load* lb per ft	PPR	Span length, l (ft)*					Remarks
				30	40	50	60	70	
Ex.	Single-T	400	0	X	X	X	X	X	
			Optimum		X	X	X	X	
			1	X	X	X	X	X	
1	4 DT 14	200	0						X
			Optimum						X L = 35 ft
			1						X
2	8 DT 20	400	0	X	X	X	X		
			Optimum	X	X	X	X		
			1	X	X	X	X		
3	8 DT 24	400	0				X		
			Optimum				X		
			1				X		
4	8 ST 36	400	0					X	
			Optimum					X	
			1					X	
5	10 ST 48	500	0						X
			Optimum						X L = 100 ft
			1						X
6	Rectangular slab	50	0	X	X	X	X	X	
			Optimum		X	X	X	X	
			1	X	X	X	X	X	
7	Hollow-core slab	200	0	X	X	X	X		
			Optimum	X	X	X	X		
			1	X	X	X	X		

Note: 1 ft = 0.305 m, 1 lb per ft = 14.59 N/m.

Table 6. Average values of Reliability Index β for All Beams.

Limit states	ϵ_R	β		
		Average maximum	Average minimum	Average mean
Maximum crack width, W_{max}	0.1	5.6754	1.1114	4.1097
	0.2	3.4204	0.7729	2.5265
Concrete fatigue stress range, f_{cr}	0.05	3.8512	0.9890	2.6466
Non-prestressed steel fatigue stress range, f_{rr}	0.05	2.3917	1.2137	1.5480
Prestressing steel fatigue stress range, f_{tr}	0.05	3.3612	1.9779	2.4116
Immediate live load deflection, $\bar{\Delta}_{LL}$	0.05	5.9008	3.4526	4.7303
Additional long-term deflection, $\bar{\Delta}_{add}$	0.1	3.7395	0.4549	2.7722
	0.2	2.3348	0.3953	1.8322

corresponding to the combination of lowest and highest μ_D with lowest and highest Ω_D.

The intensity of maximum live load depends on the type of occupancy, the tributary area and the projected service life of the structure. To account for these parameters, the 16 different sets of values of (μ_L, Ω_L) recommended in Ref. 12 were considered in evaluating μ_S and Ω_S (Table 4). Hence, in computing the mean and coefficient of variation of load (μ_S, Ω_S) from Eqs. (11-13), 64 combinations of dead and live load statistics (4 x 16 = 64) were considered for each beam and each limit state.

To evaluate the reliability index β, the 64 combinations of statistics of load S were associated with the four combinations of statistics of resistance R. For each beam and each limit state, the minimum, maximum, and mean value of β were recorded. These were then averaged for all beams leading to an average minimum, average maximum, and average mean of β (Table 6).

RESULTS

The results reported in Table 6 represent in a way, at each serviceability limit state, average ranges of reliability indexes β for the precast prestressed concrete industry as a whole, assuming the current ACI Code is used. They are plotted in Fig. 3 and compared to similar data generated for one beam (the example beam) at ultimate flexural strength limit state.

It can be observed that (1) for each serviceability limit state the reliability index spans a relatively wide range of values reflecting uncertainties in the data and/or the prediction equations, (2)

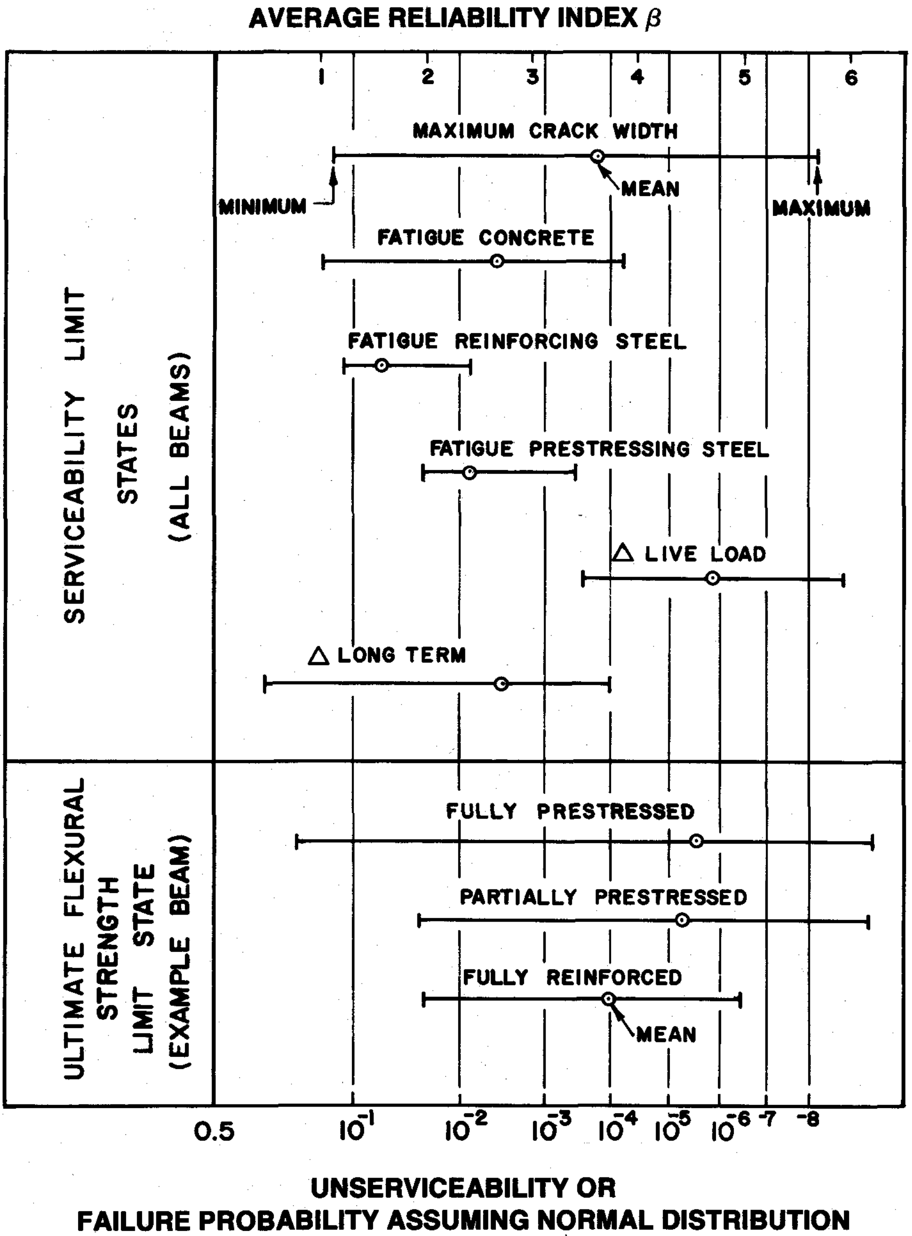


Fig. 3. Reliability index values representative of precast prestressed and partially prestressed concrete beams.

serviceability limit states are on the average more critical than ultimate flexural strength limit states, and (3) on the average, among various serviceability limit states, fatigue in the steel (reinforcing on prestressing steel) is most critical.

While there is little ambiguity about the meaning of β at ultimate flexural stength limit state which implies failure or collapse, the meaning of β at serviceability limit states is more subtle. This is particularly true for fatigue since fatigue can be classified either as a serviceability limit state or as an ultimate limit state.

Let us consider, for instance, the average mean value of $\beta = 1.548$ obtained in Table 6 at the non-prestressed steel fatigue stress range limit state for which a code stress range limit of 20 ksi (138 MPa) was used. Assuming a normal distribution for the safety margin leads to a probability of 6 percent of exceeding the code limit each time the random variable live load is applied. As the code limit of 20 ksi (138 MPa) does not represent the real fatigue resistance of the steel material for 2 million cycles of load repetition (assumed representative of service life), the above probability does not necessarily imply failure during service life or 2 million cycles.

Let us assume that the non-prestressed steel fatigue stress range limit of 20 ksi (138 MPa) is changed to 30 ksi (207 MPa) to reflect the actual resistance of the material up to 2 million cycles. The corresponding mean value of β becomes 2.32 (see Ref. 30) and the probability of exceeding the stress range limit for each load application is about 1 percent; it also implies that there is a 1 percent chance of failure by fatigue of the steel during service life or prior to 2 million cycles of load application.

Note that the values of β obtained at the three fatigue limit states (Table 6 and Fig. 3) are relatively low. This may be due to either one or a combination of the following causes: (1) fatigue is a critical design condition and should be controlled better, (2) the design limitations are too stringent, and (3) the quality control and workmanship associated with the influencing variables are poor.

To better focus on the value of β for a fatigue limit state, the COV of influencing variables can be reduced by improving their quality control and workmanship (such as achieved in the precast prestressed concrete industry) and the fatigue stress range limit can be relaxed to reflect actual experimental data instead of a code limitation.

Reaching a serviceability limit state does not imply collapse of the structure; however, it may lead to "extensive damage" and for all practical purposes "failure." For instance, exceeding the maximum allowable crack width in a liquid retaining structure may lead to extensive leakage and practically failure of the structure to perform the service it is intended for. Similarly, exceeding an allowable deflection does not lead to the collapse of a beam but may lead to the failure of non-structural elements attached to it. Hence, the meaning of reliability at any serviceability limit state should be examined with the proper perspective.

Typical results of the variation of the reliability index β with each of the parameters studied (PPR, span length, live load) are shown in Figs. 4 to 7 and are explained in the conclusions. More detailed information can be obtained from Ref. 30.

CONCLUSIONS

The following conclusions were made (see Ref. 30) in relation to the reliability index β of partially prestressed beams; note that not all these conclusions are supported in the figures of this paper.

1. The serviceability limit state which controls the design in the deterministic procedure is not necessarily

the one that controls in the reliability approach. This is because in the probabilistic approach, the uncertainties in the values of the basic variables are taken into account.

2. The β values for rectangular beams are much lower than those for typical T-beams in most of the serviceability limit states considered. Thus, the probability of failure or unservice-

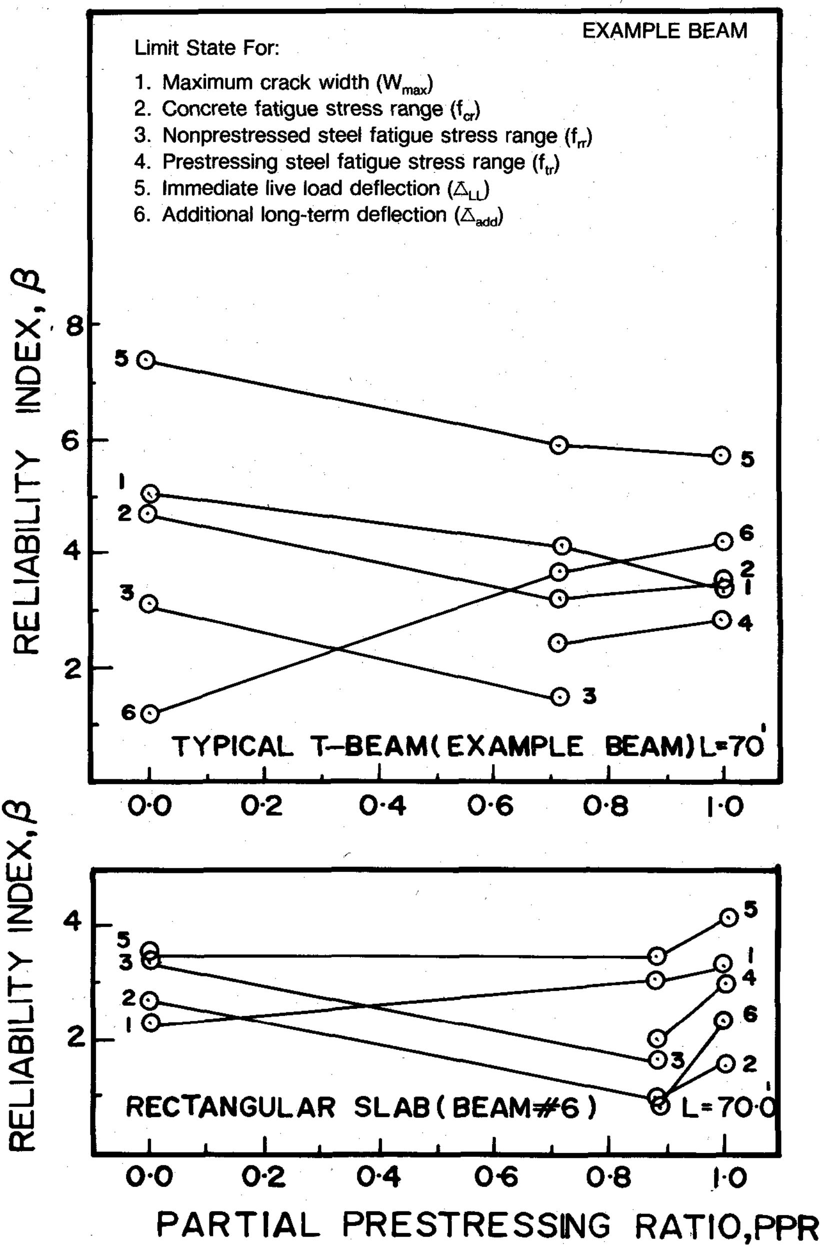


Fig. 4. Typical variation of the reliability index with the partial prestressing ratio.

ability of rectangular beams (or one-way slabs) is generally greater than the probability of failure or unserviceability of typical T-beams (Fig. 4).

3. For typical T-beams and for rectangular beams, the β values at "additional long-term deflection" limit state decreases substantially when the partial prestressing ratio (PPR) decreases. Smaller variations in the β values are observed at other limit states (Fig. 4).

4. For hollow-core slabs, the β values at both "additional long-term deflection" limit state and "maximum crack width" limit state decrease substantially when PPR decreases.

5. Everything else being equal, it appears that when the span length increases, the β values increase at the "maximum crack width," "non-prestressed steel fatigue stress range" and "prestressing steel fatigue stress range," limit states and decrease at the "concrete fatigue stress range," "im-

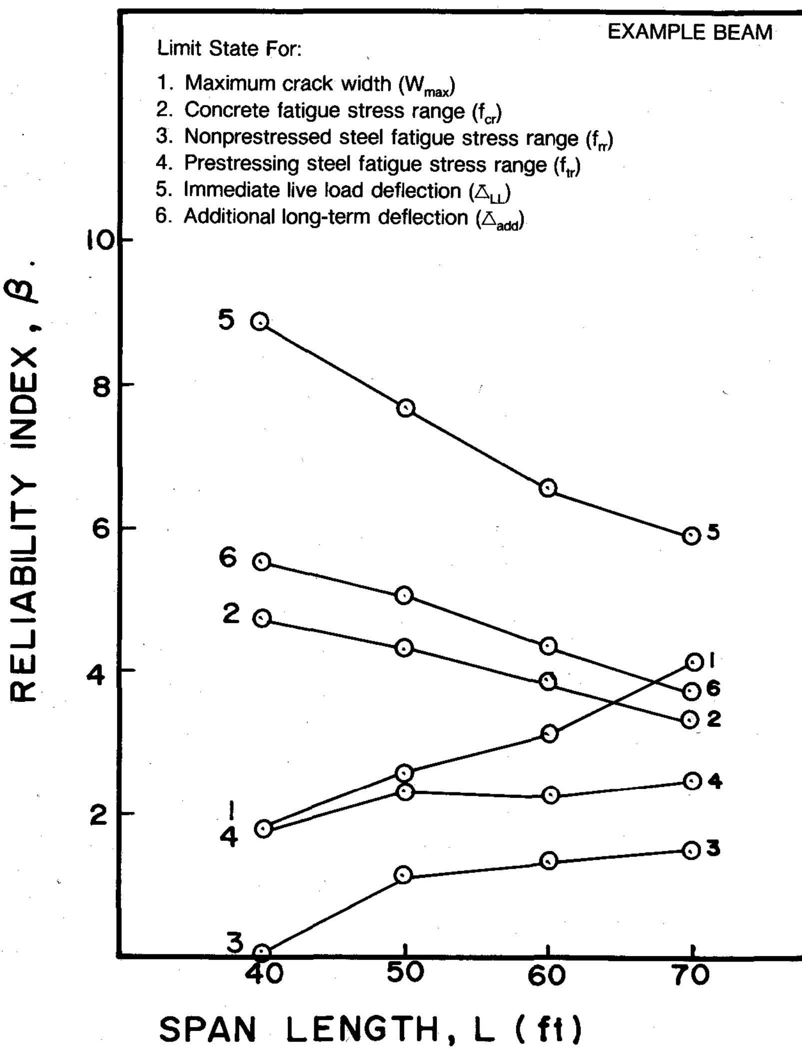


Fig. 5. Typical variation of the reliability index with span length.

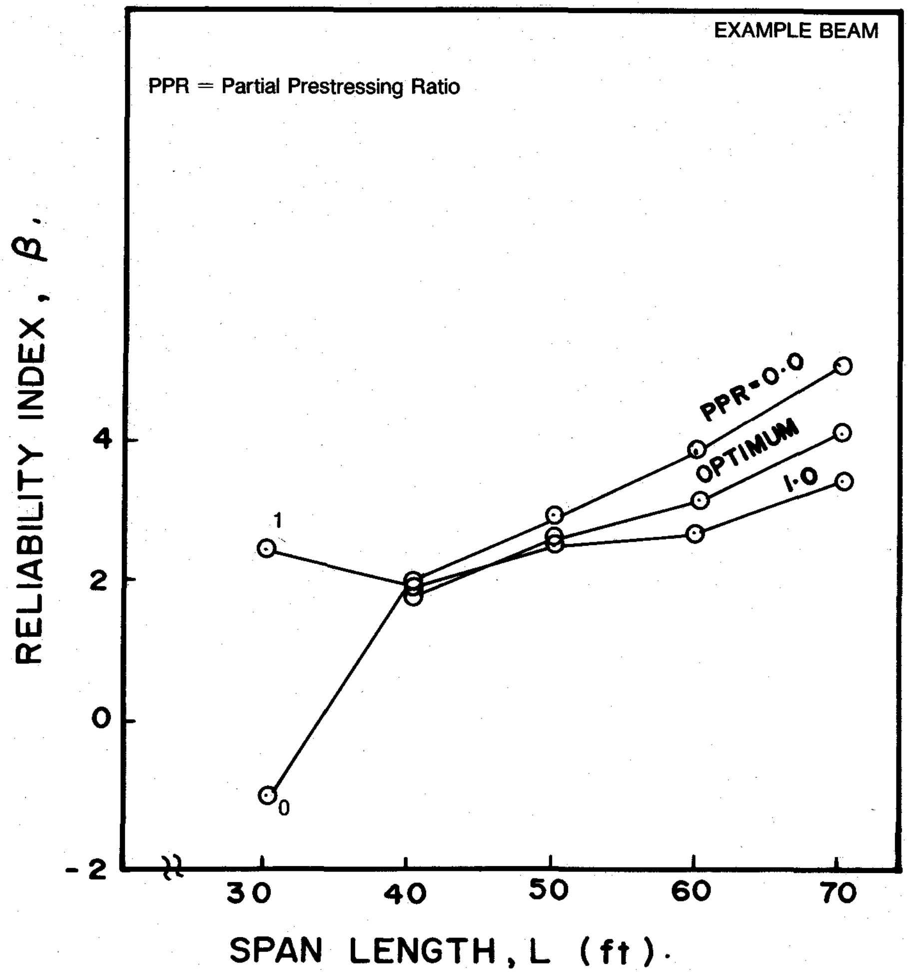


Fig. 6. Reliability index at maximum crack width limit state for various PPR (example beam).

mediate live load deflection" and "additional long-term deflection" limit states (Figs. 5 and 6).

6. Everything else being equal, it appears that when the nominal live load increases, the β values increase at "maximum crack width" and "prestressing steel fatigue stress range" limit states and decrease at "concrete fatigue stress range" and "additional long-term deflection" limit states (Fig. 7).

7. The average β values for all partially prestressed beams (at optimum PPR) and the corresponding probability of failure assuming a normal distribution for the safety margin are (see Table 6), respectively, 4.11 and 2 x 10^{-5} at "maximum crack width" limit state; 2.65 and 400 x 10^{-5} at "concrete fatigue stress range"; 1.55 and 6100 x 10^{-5} at "non-prestressed steel fatigue stress range" limit state; 2.41 and 800 x 10^{-5} at "prestressing steel fatigue stress range" limit state; 4.73 and 0.1 x 10^{-5} at "immediate live load deflection" limit state; and 2.77 and 280 x 10^{-5} at "additional long-term deflection" limit state.

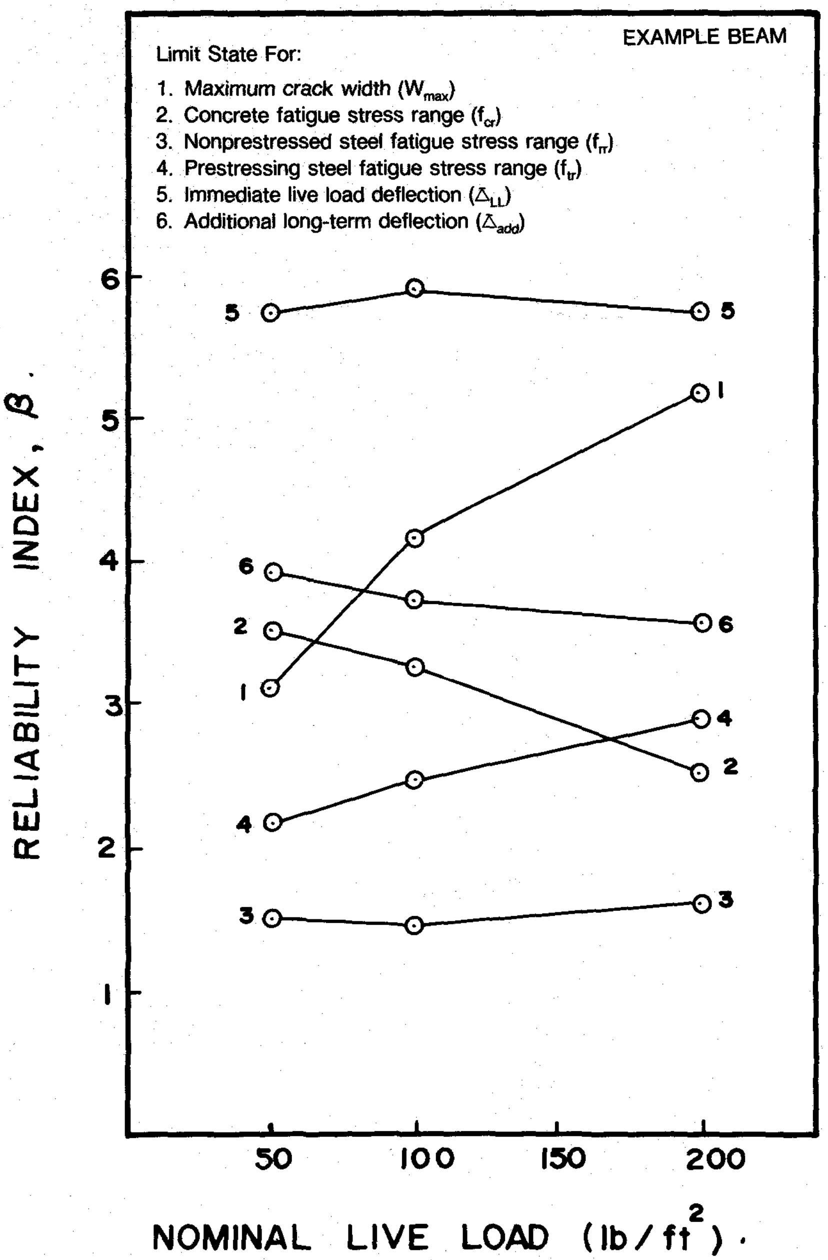


Fig. 7. Typical variation of the reliability index with nominal live load.

8. Serviceability limit states in reinforced, prestressed and partially prestressed concrete beams are more critical than their ultimate flexural strength limit state. Violating serviceability limit states generally leads to abnormal maintenance and repair levels during service and may lead, for all practical purposes, to failure.

It is finally observed that reliability-based design (Level II approach) can be successfully applied to prestressed and partially prestressed concrete beams provided an appropriate reliability index, β, is recommended by the code for each limit state and provided the mean and the COV values of resistance and loads are known.

RECOMMENDATIONS FOR FURTHER STUDY

The limitations at various serviceability limit states used in this investigation were taken from the 1977 ACI Code. In general, these limitations are lower bound values to actual data. It appears that a more accurate estimate of reliability at these limit states would be achieved if actual data were used instead of code limitations.

There is a need to develop a method to incorporate in the reliability analysis the often great uncertainty associated with a prediction equation. This is particularly true for crack width and long-term deflection.

In the analysis of stresses static short-term loading was assumed. The effect of time on redistribution of stresses was not considered. Substantial changes in stresses and crack widths occur when creep and shrinkage of concrete are considered in the analysis of the section.

There is a need to develop a procedure where the cumulative damage due to fatigue under the repetitive application of random levels of live loads can be accounted for in the reliability analysis. Such cumulative damage should affect not only the fatigue life of the materials involved but also other serviceability limit states such as crack widths, debonding and deflection.

ACKNOWLEDGMENTS

This study is based on the PhD thesis of A. Siriaksorn prepared under the direction of the senior author. It was supported in part by a Research Fellowship Award from the Prestressed Concrete Institute and by the University of Illinois at Chicago. Their support is gratefully acknowledged. The authors are also indebted to Dr. James G. MacGregor of the University of Alberta for his careful review of and constructive comments on the first draft of this paper.

NOTE: Discussion of this paper is invited. Please submit your discussion to PCI Headquarters by July 1, 1983.

REFERENCES

1. Freudenthal, T. V., Garrelts, J., and Shinozuka, M., "The Analysis of Structural Safety," *Journal of the Structural Division,* ASCE, V. 92, No. ST1, Feb. 1966, pp. 267-325.
2. Warner, R. F., and Rabaila, A. P., "Monte Carlo Study of Structural Safety," *Journal of the Structural Division,* ASCE, V. 94, No. ST12, Proc. Paper 6275, Dec. 1968, pp. 2847-2859.
3. Shah, H. C., "The Rational Probabilistic Code Format," *ACI Journal,* Proceedings, V. 66, No. 9, Sept. 1969, pp. 689-697.
4. Sexsmith, R. B., and Nelson, M. F., "Limitations in Application of Probabilistic Concepts," *ACI Journal,* Proceedings, V. 66, No. 10, Oct. 1969, pp. 823-828.
5. Benjamin, J. R., and Lind, N. C., "A Probabilistic Basis for a Deterministic Code," *ACI Journal,* Proceedings, V. 66, No. 11, Nov. 1969, pp. 857-865.
6. Cornell, C. A., "A Probability-Based Structural Code," *ACI Journal,* Proceedings, V. 66, No. 12, Dec. 1969, pp. 974-985.
7. Allen, D. C., "Probabilistic Study of Reinforced Concrete in Bending," *ACI Journal,* V. 67, No. 12, Dec. 1970, pp. 989-993.
8. Ang, A. H.-S., "Structural Risk Analysis and Reliability-Based Design," *Journal of the Structural Division,* ASCE, V. 99, No. ST9, Proc. Paper 10011, Sept. 1973, pp. 1891-1910.
9. Galambos, T. V., and Ravindra, M. K., "Tentative Load and Resistance Factor Design Criteria for Steel Buildings," Structural Division Research Report No. 18, Washington University, St. Louis, Missouri, Sept. 1973.
10. Ang, A. H.-S. and Cornell, C. A., "Reliability Bases of Structural Safety and Design," *Journal of the Structural Division,* ASCE, V. 100, No. ST9, Proc. Paper 10777, Sept. 1974, pp. 1755-1769.
11. Ellingwood, B., and Ang, A. H.-S., "Risk Based Evaluation of Design Criteria," *Journal of the Structural Division,* ASCE, V. 100, No. ST9, Proc. Paper 10778, Sept. 1974, pp. 1771-1788.
12. Ravindra, M. K., Lind, N. C., and Siu, W., "Illustrations of Reliability-Based Design," *Journal of the Structural Division,* ASCE, V. 100, No. ST9, Sept. 1974, pp. 1789-1811.
13. Ellingwood, B., "Statistical Analysis of RC Beam Column Interaction," *Journal of the Structural Division,* ASCE, V. 103, No. ST7, Proc. Paper 13061, July 1977, pp. 1377-1388.
14. Ellingwood, B., "Reliability-Based Criteria for Reinforced Concrete Design," *Journal of the Structural Division,* ASCE, V. 105, No. ST4, Proc. Paper 14480, April 1979, pp. 713-727.
15. Ellingwood, B., "Reliability of Current Reinforced Concrete Designs," *Journal of the Structural Division,* ASCE, V. 105, No. ST4, Proc. Paper 14479, April 1979, pp. 699-712.
16. Ellingwood, B., Galambos, T. V., MacGregor, J. G., and Cornell, C. A., "Development of a Probability-Based Load Criterion for American National Standard A58," National Bureau of Standards Special Publication 577, U.S. Department of Commerce, Washington, D.C., June 1980, 228 pp.
17. Shinozuka, M., and Yao, J. T. P. (Editors), "Probabilistic Methods in Structural Engineering," Proceedings of the ASCE Symposium, St. Louis, Oct. 1981, American Society of Civil Engineers, 1981, 409 pp.
18. Galambos, T. V., Ellingwood, B., MacGregor, J. G., and Cornell, C. A., "Probability-Based Load Criteria: Assessment of Current Design Practice," *Journal of the Structural Division,* ASCE, V. 108, No. ST5, May 1982, pp. 959-977.
19. Ellingwood, B., MacGregor, J. G., Galambos, T. V., and Cornell, C. A., "Probability-Based Load Criteria: Load Factors and Load Combinations," *Journal of the Structural Division,* ASCE, V. 108, No. ST5, May 1982, pp. 978-997.
20. Grant, L. H., Mirza, S. A., and MacGregor, J. G., "Monte Carlo Study of Strength of Concrete Columns," *ACI Journal,* V. 75, No. 8, Aug. 1978, pp. 348-358.
21. Siu, W. W. C., Parimi, S. R. and Lind, N. C., "Practical Approach to Code Calibration," *Journal of the Structural Division,* ASCE, V. 101, No. ST7, Proc. Paper 11404, July 1975, pp. 1469-1480.

22. Peir, J.-C. and Cornell, A. C., "Spatial and Temporal Variability of Live Loads," *Journal of the Structural Division*, ASCE, V. 99, No. ST5, Proc. Paper 9747, May 1973, pp. 903-922.
23. Siu, W. W. C., and Lind, N. C., "Some Aspects of Office Design Live Loads," *Journal of the Structural Division*, ASCE, V. 99, No. ST11, Nov. 1973, pp. 2245-2258.
24. McGuire, R. K., and Cornell, C. A., "Live Load Effects in Office Buildings," *Journal of the Structural Division*, ASCE, V. 100, No. ST7, Proc. Paper 10660, July 1974, pp. 1351-1366.
25. Culver, C. G., "Live Load Survey Results for Office Buildings," *Journal of the Structural Division*, ASCE, V. 102, No. ST12, Dec. 1976, pp. 2269-2284.
26. Ellingwood, B., and Culver, C., "Analysis of Live Loads in Office Buildings," *Journal of the Structural Division*, ASCE, V. 103, No. ST8, Aug. 1977, pp. 1551-1560.
27. Corotis, R. B., and Jaria, A. V., "Stochastic Nature of Building Live Loads," *Journal of the Structural Division*, ASCE, V. 105, No. ST3, Proc. Paper 14441, March 1979, pp. 493-510.
28. Dunham, J. W., "Design Live Load in Buildings," Transactions, ASCE, V. 112, Paper No. 2311, 1947, pp. 725-744.
29. "Minimum Design Loads in Buildings and Other Structures," American National Standards Institute, A58.1, 1972.
30. Siriaksorn, A., "Serviceability and Reliability Analysis of Partially Prestressed Concrete Beams," Thesis presented to the University of Illinois at Chicago Circle in partial fulfillment of the requirements for the degree of Doctor of Philosophy, Chicago, Illinois, 1980. See also Report No. 80-1 by A. Siriaksorn and A. E. Naaman, Department of Materials Engineering, June 1980, 126 pp.
31. Naaman, A. E., and Siriaksorn, A., "Serviceability-Based Design of Partially Prestressed Beams," Part 1: PCI JOURNAL, V. 24, No. 2, March-April 1979, pp. 64-89; Part 2: PCI JOURNAL, V. 24, No. 3, May-June 1979, pp. 40-60.
32. ACI Committee 318, "Building Code Requirements for Reinforced Concrete (ACI 318-77)," American Concrete Institute, Detroit, Michigan, 1977.
33. Ellingwood, B., "Reliability Bases of Load and Resistance Factors for Reinforced Concrete Design," National Bureau of Standards Building Science Series 110, Washington, D.C., 1978.
34. Mirza, S. A., and MacGregor, J. G., "Variations in Dimensions of Reinforced Concrete Members," *Journal of Structural Division*, ASCE, V. 105, No. ST4, Proc. Paper 14495, April 1979, pp. 751-766.
35. Mirza, S. A., and MacGregor, J. G., "Variability of Mechanical Properties of Reinforcing Bars," *Journal of the Structural Division*, ASCE, V. 105, No. ST5, Proc. Paper 14590, May 1979, pp. 921-937.
36. Pauw, A., "Static Modulus of Elasticity of Concrete as Affected by Density," *ACI Journal*, V. 32, No. 6, Dec. 1960, pp. 679-687.
37. Price, W. H., "Factors Influencing Concrete Strength," *ACI Journal*, V. 22, No. 6, Feb. 1951, pp. 417-432.

* * *

APPENDIX — NOTATION

a = distance from draping point of steel profile to support
A_{ps} = area of prestressing steel in tension zone
A_s = area of non-prestressed tension reinforcement
b = width of upper flange of a flanged member
b_w = web width of flanged member
b_l = lower flange width of a flanged member
C_{E_c} = coefficient associated with modulus of elasticity of concrete (see Table 2)
$C_{f_{ci}}$ = coefficient associated with compressive strength of concrete at transfer (Table 2)
C_{f_r} = coefficient associated with modulus of rupture of concrete (see Table 2)
$C_{f_{se}}$ = coefficient associated with effective prestress in steel (see Table 2)
$C_{f_{si}}$ = coefficient associated with initial prestress
COV = coefficient of variation
d_p = distance from extreme compression fiber to centroid of prestressing steel
d_s = distance from extreme compression fiber to centroid of non-prestressed tension reinforcement
d_{ss} = depth of area of concrete tensile zone associated with crack width prediction equation
D = dead load
e_o = eccentricity of prestressing force with respect to centroid of section at midspan
e_1 = eccentricity of prestressing force with respect to centroid of section at supports
E_c = modulus of elasticity of concrete
E_{ps} = modulus of elasticity of prestressing steel
E_s = modulus of elasticity of non-prestressed tension steel
f'_c = specified compressive strength of concrete
$f(x)$ = probability distribution function of x
f_{cr} = allowable stress range under repeated service load in concrete
f_{min} = minimum stress in concrete
f_{pu} = ultimate strength of prestressing steel
f_{py} = specified yield strength of prestressing steel
f_r = modulus of rupture of concrete
f_{rr} = stress range under repeated service load in non-prestressed tension steel
f_{se} = effective stress in prestressing steel, after losses
f_{si} = initial stress in the prestressing steel immediately after transfer
f_{tr} = stress range under repeated service load in prestressing steel
f_y = specified yield strength of non-prestressed steel
h = overall thickness or depth of member
h_{f1} = thickness of upper flange of flanged member
h_{f2} = thickness of lower flange of flanged member
l = clear span length of member
L = live load
m = margin of safety
n = subscript for nominal value
PPR = partial prestressing ratio
R = resistance
S = total load
W_{max} = maximum crack width
β = reliability index
γ_c = unit weight of concrete
δ = inherent coefficient of variation
$\bar{\Delta}_{add}$ = additional long-term deflection
$\bar{\Delta}_{LL}$ = immediate live load deflection
ϵ = prediction error
μ = mean value
Ω = total coefficient of variation
σ = standard deviation