Design Charts for Proportioning Rectangular Prestressed Concrete Columns

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With the increased use of precast prestressed concrete comes the challenge to the engineer to more efficiently design such products. If the product is a prestressed column the design engineer has few options. Some references include load versus moment interaction curves for these columns, but most curves are for a specific size column and a specific prestressing steel percentage. If the designer cannot design within these limitations, the alternatives are to calculate and draw specific interaction diagrams for the particular column under consideration or use the computer to evaluate the column capacity. In either case, the process can be time consuming and costly.

The design aids presented here describe many different steel percentages and prestressing levels in a few simple curves. These curves are applicable to symmetrically prestressed short columns (no length effects are included) for both preliminary and final design.

Nondimensional load versus moment interaction curves and associated equations for rectangular columns with 270 ksi (1861.7 N/mm^2) prestressing steel in all four faces have been developed. These curves are based on particular concrete strengths, effective stress in prestressing steel after losses, and the distance between centers of steel in the tension and compressive faces. Within each chart there are curves which represent different percentages of prestressing steel that can be present in the column. With an additional equation these charts can be used to obtain a modified effective stress in prestressing steel after losses. The generalized interaction curves are presented in the Appendix as well as design examples illustrating how the interaction curves and the stress modification equations should be used.

Synopsis

A series of nondimensional ultimate load-moment interaction curves for short, rectangular concrete columns with axial prestressing are presented. Several design examples are included to show the application of the design charts.

General nondimensional equations are given which are based on nonlinear approximations for both the concrete and prestressing steel stress-strain curves, linear strains across the cross section, strain compatibility and equilibrium. The steel location is generalized by replacement of individual strands with a rectangular prestressed steel tube.

The series of curves presented are for a steel strength of 270 ksi (1861.7 N/mm^2), concrete strengths of 5, 6, 7, and 8 ksi (34.48, 41.37, 48.26, and 55.16 N/mm^2), and geometric parameters corresponding to the more commonly used columns. In addition, full and partial stressing of the prestressing steel were considered.

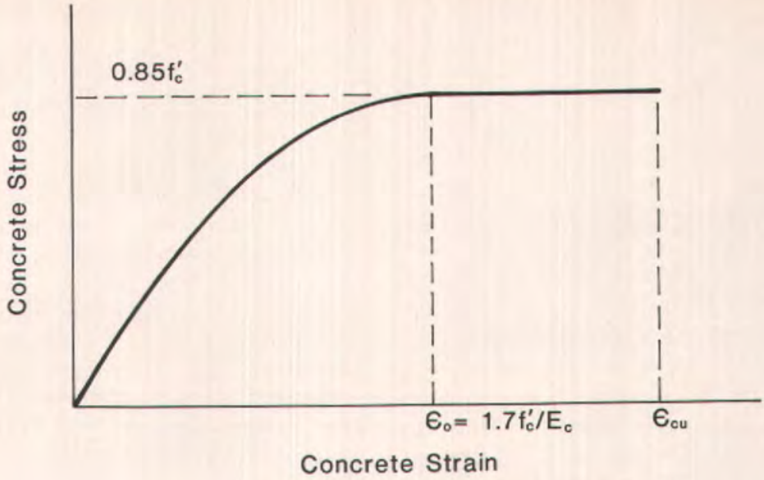
Governing Equations

The derivation of equations generated for a general prestressed concrete column parallels the development of those for load-moment interaction relationships for any specific column. For the general case, the equations depend upon the assuming of a neutral axis location and satisfying the compatibility of strains. The concrete compression force and steel forces are calculated from the strain distribution. These forces are nondimensionalized and, when used in the appropriate equations of equilibrium, produce a single point on the generalized load-moment curve.[1] The entire interaction curve is obtained by varying the neutral axis location within an appropriate range.

The concrete compression force is calculated from a concrete stress-strain relationship which accounts for a portion of the nonlinear region of the curve. This solution is more accurate over the entire range of the interaction curve than one based on the rectangular stress block approximation. The general equations are designed so that any concrete stress-strain curve can easily be substituted for the one used for the solution presented.

The steel forces are calculated in terms of a steel percentage rather than for a particular number of strands. For analysis, the prestressing strands have been replaced by a thin rectangular tube of prestressing steel with an equivalent area of steel. The total strain in the tube consists of the steel strain due to prestressing, precompression of the concrete and bending of the member. The force in the steel tube is calculated from these strains using an actual stress-strain relationship for prestressing strand.

The stress stage in the steel is analyzed using three specific cases: Case 1 is appropriate when all the steel strains are in the linear range of the stress-strain curve, Case 3 is used when all steel strains are in the nonlinear range, Case 2 is a combination of Cases 1 and 3 and deals with steel strains in both the linear and nonlinear portions of the stress-strain curve. Other pre-



For $0 < \epsilon_c \leq \epsilon_o$: $\quad f_c = 0.85f'_c\left[2\left(\frac{\epsilon_c}{\epsilon_o}\right) - \left(\frac{\epsilon_c}{\epsilon_o}\right)^2\right]$

For $\epsilon_o < \epsilon_c \leq \epsilon_{cu}$: $\quad f_c = 0.85f'_c$

Fig. 1. Assumed approximation for the concrete stress-strain relationship.

stressing steel stress-strain relationships could be used in the analysis presented here; however, the transformation would be more involved than that necessary for the replacement of the concrete stress-strain relationship.

NONDIMENSIONAL PARAMETERS

The governing equations are used to generate nondimensional load versus moment interaction curves for prestressed concrete columns. The evaluation of the equations is accomplished by incrementing the depth of the compressed concrete area (k_ut) which is measured from the extreme fiber on the compression face. The solution of each increment of k_ut produces nondimensional loads (K) and corresponding nondimensional moments (R) which represent a point on an interaction curve. Each of the two nondimensional terms can be considered in two parts which correspond to the contribution of the concrete and steel respectively.

These nondimensional terms K and R are defined as:

$$K = \frac{P_u}{f'_c bt} = K_c + K_s$$

$$R = \frac{M_u}{f'_c bt^2} = R_c + R_s$$

where

- P_u = ultimate axial load acting on column
- M_u = ultimate moment acting on column
- f'_c = concrete strength
- b = width of cross section
- t = depth of cross section
- K_c = portion of K due to compressive force in concrete
- R_c = portion of R due to compressive force in concrete
- K_s = portion of K due to steel forces
- R_s = portion of R due to steel forces

CONCRETE COMPRESSION PARAMETERS

The parameters K_c and R_c are dependent upon the compressive force of the concrete (C_c) and the moment (M_c) of this force about the bending axis of the

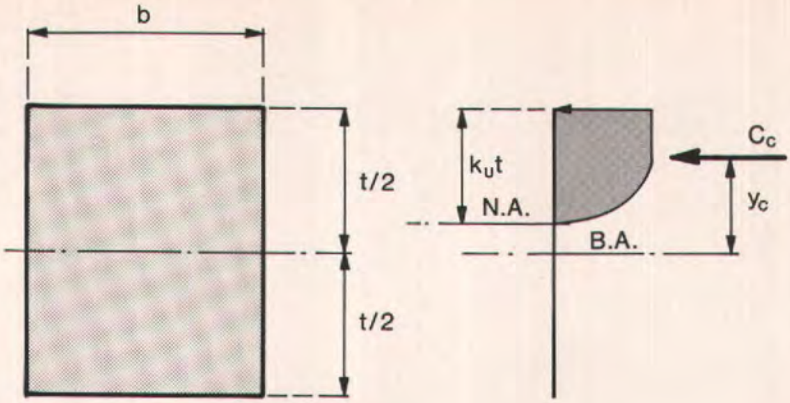


Fig. 2 Concrete stress distribution

column cross section. These quantities are derived based on the following assumptions and approximations.

1. The concrete area displaced by the prestressing steel in the compression zone has been neglected.
2. The tensile strength of the concrete has been neglected.
3. The approximation for the stress-strain relationship for concrete is shown in Fig. 1.[2]

A rectangular column cross section without the prestressing steel is shown in Fig. 2 where the concrete stress distribution and the depth of the neutral axis $(k_u t)$ are also illustrated.

The following equations can be written from Fig. 2.

$$C_c = f_{ave}\, b k_u t \tag{1}$$

$$M_c = y_c C_c \tag{2}$$

where

f_{ave} = average stress

y_c = distance between centroid of stress distribution and bending axis

In order to obtain K_c, both sides of the equation for C_c are divided by $f_c' bt$ and to obtain R_c both sides of the equation for M_c are divided by $f_c' bt^2$. The resulting expressions are:

$$K_c = \frac{C_c}{f_c' bt} \tag{3}$$

$$R_c = \frac{M_c}{f_c' bt^2} \tag{4}$$

STEEL FORCE PARAMETERS

The dimensionless expressions $(K_s$ and $R_s)$ for the prestressing steel are based on the following assumptions and approximations:

1. The bending strain in the prestressing steel and in the concrete are compatible.
2. Complete bond exists between the prestressing steel and the concrete.
3. The prestressing steel is assumed to be a thin rectangular tube as shown in Fig. 3. One-fourth of the total steel area is in each side of the rectangular tube.

The total strain in the steel at any location is evaluated in terms of three components; the strain in the steel due to prestressing (ϵ_{ps}), the strain in the concrete due to prestressing (ϵ_{ce}), the strain due to bending (ϵ_B). The strains ϵ_{ps} and ϵ_{ce} are defined as follows:[3]

$$\epsilon_{ce} = \frac{\rho_p f_{se}}{E_c} \quad \text{and} \quad \epsilon_{ps} = \frac{f_{se}}{E_{ps}}$$

where

f_{se} = effective stress in prestressing steel after losses

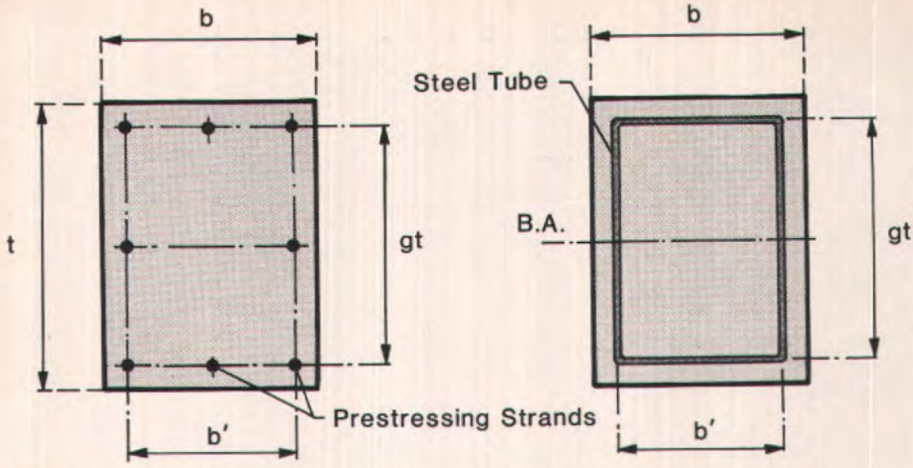


Fig. 3. Steel distributed as a thin tube.

E_{ps} = modulus of elasticity of prestressing steel

E_c = modulus of elasticity of concrete

ρ_p = $A_{ps}/(bt)$ = total steel percentage

A_{ps} = total area of prestressing steel

The strain due to bending is that which exists in the column concrete as a result of a failure strain at the compression face of the column. The dimensions and variables required to calculate the steel strains due to bending are shown in Fig. 4.

From the linear strain distribution, it is possible to calculate the bending strain in the top steel (ϵ_{B4}) and the bottom steel (ϵ_{B1}):

$$\epsilon_{B1} = \frac{d_1 - k_u t}{k_u t}\,\epsilon_{cu} = \frac{1 + g - 2k_u}{2k_u}\,\epsilon_{cu}$$

$$\epsilon_{B4} = \frac{k_u t - d_4}{k_u t}\,\epsilon_{cu} = \frac{1 - g - 2k_u}{2k_u}\,\epsilon_{cu}$$

The total strain in the top steel (ϵ_{p4}) and bottom steel (ϵ_{p1}), respectively, is:

$$\epsilon_{p1} = \frac{f_{se}}{E_{ps}} + \frac{\rho_p f_{se}}{E_c} + \frac{1 + g - 2k_u}{2\,k_u}\,\epsilon_{cu} \quad (5)$$

$$\epsilon_{p4} = \frac{f_{se}}{E_{ps}} + \frac{\rho_p f_{se}}{E_c} + \frac{1 - g - 2k_u}{2k_u}\,\epsilon_{cu} \quad (6)$$

Since the stress-strain curve for prestressing steel becomes nonlinear at higher strains, three possibilities of stress-strain relationships occur in the cross section. All, part, or none of the steel strains can be in the linear range of the stress-strain curve. Each of these possibilities is examined in the following three cases.

Case 1 — Steel Stresses Within the Linear Range

The first case considers a rectangular section in which all of the prestressing steel stresses are within the linear portion of the stress-strain curve.

$$\epsilon_{p1} \leq \epsilon_{NL} \quad \epsilon_{p4} < \epsilon_{NL}$$

where ϵ_{NL} is the strain at which the stress-strain curve becomes nonlinear (see Fig. 5).

For this case the nondimensional force and moment due to steel (K_s, R_s) can be calculated by the following expressions:

$$K_s = \frac{-\rho_p}{2f_c'} E_{ps}(\epsilon_{p1} + \epsilon_{p4}) \quad (7)$$

$$R_s = \frac{g\rho_p}{6f_c'} E_{ps}(\epsilon_{p1} - \epsilon_{p4}) \quad (8)$$

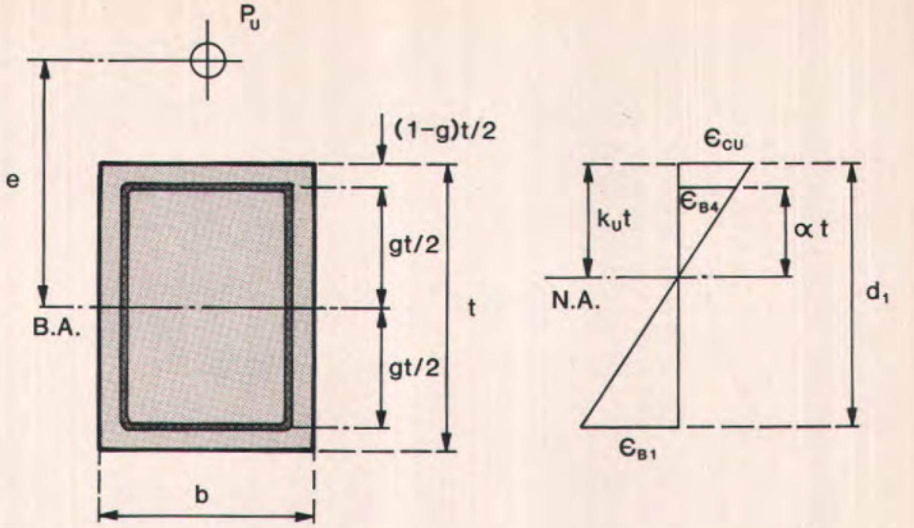


Fig. 4. Bending strain diagram.

Case 2 — Steel Stresses in Both Linear and Nonlinear Ranges

The second case considers a rectangular section in which the prestressing steel strains are in both the linear and nonlinear portions of the stress-strain curve:

$$\epsilon_{p1} > \epsilon_{NL} \qquad \epsilon_{p4} < \epsilon_{NL}$$

It now becomes essential to adopt a stress-strain relationship for the prestressing steel. The stress-strain curve shown in Fig. 5 has been used in these derivations.[4] The following equations describe this curve.

With $\epsilon_p < \epsilon_{NL} = 0.008$:

$$f_p = \epsilon_p E_{ps} \tag{9}$$

With $\epsilon_p > \epsilon_{NL} = 0.008$:

$$f_p = 268 - \frac{0.075}{\epsilon_p - 0.0065} < 0.98 f_{pu} \tag{10}$$

where f_p is the stress in the prestressing steel.

For this case, the nondimensional force and moment due to the prestressing steel (K_s, R_s) can be calculated by the following expressions:

$$K_s = \frac{-\rho_p E_{ps}}{4 f'_c}\left[\frac{f_{p1}}{E_{ps}} + \epsilon_{p4} + \frac{(\epsilon_{NL}^2 - \epsilon_{p4}^2)}{(\epsilon_{p1} - \epsilon_{p4})} - \frac{2(\epsilon_{NL} - \epsilon_{p1})}{(\epsilon_{p1} - \epsilon_{p4})}\,\frac{f_{p7}}{E_{ps}}\right] \tag{11}$$

$$R_s = \frac{g\rho_p}{8 f'_c} E_{ps}\left[\frac{f_{p1}}{E_{ps}} - \epsilon_{p4} + 2\epsilon_{p4} \times \frac{(\epsilon_{NL} - \epsilon_{p1})(\epsilon_{NL} - \epsilon_{p4})}{(\epsilon_{p1} - \epsilon_{p4})^2} + \frac{(4\epsilon_{NL} - \epsilon_{p4} - 3\epsilon_{p1})}{3(\epsilon_{p1} - \epsilon_{p4})^2}(\epsilon_{NL} - \epsilon_{p4})^2 - \frac{2 f_{p7}}{E_{ps}} \times \frac{(2\bar{\epsilon}_{p7} - \epsilon_{p1} - \epsilon_{p4})(\epsilon_{NL} - \epsilon_{p1})}{(\epsilon_{p1} - \epsilon_{p4})^2}\right] \tag{12}$$

where

f_{p1} is calculated from Eq. (10).

f_{p7} is the average stress between strains of ϵ_{NL} and ϵ_{p1} (corresponds to $\bar{\epsilon}_{p7}$, see Fig. 5)

$\bar{\epsilon}_{p7}$ is the strain at the centroid of the area under the stress-strain curve between ϵ_{NL} and $\epsilon_{p1} (A_7)$.

Case 3 — Steel Stresses Within the Nonlinear Range

The third case considers a rectangu-

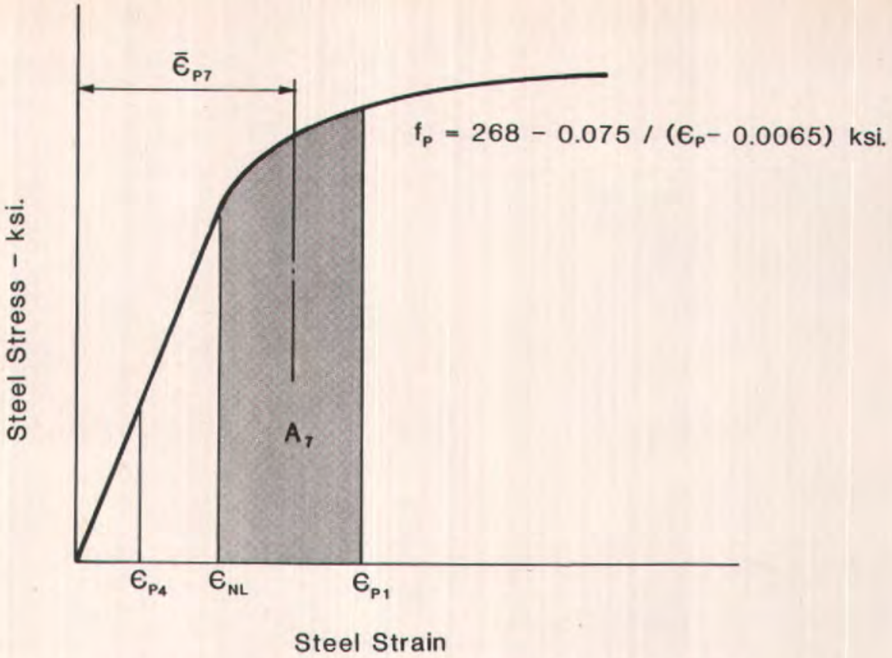


Fig. 5. Stress-strain curve for 270 ksi (1861.7 N/mm²) prestressing steel.

lar section in which all the prestressing steel stresses are within the nonlinear portion of the stress-strain curve.

$$\epsilon_{p1} > \epsilon_{NL} \qquad \epsilon_{p4} > \epsilon_{NL}$$

The nondimensional force and moment due to steel (K_s, R_s) can be calculated for this case with the following expressions:

$$K_s = \frac{-\rho_p}{4 f_c'} \left(f_{p1} + f_{p4} + 2 f_{p7}\right) \qquad (13)$$

$$R_s = \frac{g \rho_p}{8 f_c'} \left[f_{p1} - f_{p4} + \left(\frac{2\epsilon_{p7} - \epsilon_{p4} - \epsilon_{p1}}{\epsilon_{p1} - \epsilon_{p4}} \right) f_{p7} \right] \qquad (14)$$

The quantity f_{p7} is the average stress between strains of ϵ_{p4} and ϵ_{p1} ; $\bar{\epsilon}_{p7}$ is the strain at the centroid of the area under the stress-strain curve between ϵ_{p4} and ϵ_{p1}.

The equations presented here correspond directly to those used to develop load versus moment interaction curves for a specific column. Points on an interaction curve are calculated by assuming a neutral axis location, satisfying the compatibility of strains, calculating the concrete compression force and steel forces, and solving the equations of equilibrium. For general use, the concrete compression force and the steel forces were nondimensionalized. With this one deviation a general design aid was developed for rectangular prestressed concrete columns.

Modification for Partial Prestressing

For all points above the balance points on the interaction curves, the prestressing steel strain was found to be in the linear portion of the stress-strain relationship. Therefore, a change in the effective prestressing level causes a vertical translation of the interaction curve in the compression control region. The amount the curve translates (ΔK) for a change in the effective prestressing level (Δf_{se}) can be found by modifying the basic nondimensional equation for the nondimensional load K:

$$K = K_c + K_s$$

Since all prestressing steel strains are in the linear portion of the stress-strain curve, equations for modification to $K(\Delta K)$ were developed in terms of the change in prestressing (Δf_{se}).

$$\Delta K = \frac{\rho_p \, \Delta f_{se}}{f_c'} \left[1 + \rho_p \frac{E_{ps}}{E_c} \right] \quad (15)$$

This equation can be solved for Δf_{se}:

$$\Delta f_{se} = \frac{f_c' \, \Delta K}{\rho_p \left[1 + \rho_p \frac{E_{ps}}{E_c} \right]} \quad (16)$$

With Eqs. (15) and (16) and the interaction curves, a prestressed concrete column can be designed with any level of effective prestressing provided all the strands are stressed to the same level. Complete derivations of all equations are available in a previous publication.[5]

Effects of Derivation Approximations

Concrete Stress-Strain Relationship

A nonlinear approximation for the concrete stress-strain relationship is used in order to produce more accurate results regardless of the location of the neutral axis. This capability is important since the development of an interaction diagram requires the depth to the neutral axis to vary from zero to infinity. The details of the concrete stress-strain approximation are shown in Fig. 1.

The concrete stress-strain approximation is especially useful as the depth to the neutral axis becomes larger than the depth of the section. This allows a smooth reliable curve between the balance point and the load axis. However, at higher steel percentages and lower g values, the concrete stress-strain relationship used in this derivation also shows a slight inconsistency. This is caused by the inability of the stress-strain approximation to give accurate results for the concrete force and its location as the nonlinear portion of the curve moves off the cross section. At higher steel percentages and lower g values, the concrete compression force and its location is a major determinant of the load and moment capacity of the column. The result of this inconsistency is a small discontinuity in the curves near the load axis.

Prestressed Steel Tube Approximation

The replacement of the prestressing strand with a prestressed steel tube is the primary approximation in the derivation of the equations used to generate the load vs. moment interaction curves. The accuracy of the solutions obtained with this approximation was checked by calculating points on the interaction curves for specific columns. These specific columns consisted of various steel percentages, numbers of strand, t/b values, and values of g. The values obtained were then nondimensionalized and checked with the general interaction curves.

For the case in which all the prestressing steel strains are in the linear portion of the stress-strain curve, the errors were in the range of 2 percent. However, certain factors determine the magnitude of the error. For a given steel percentage, the column with the largest number of strands had the least error. Increasing the g value or the t/b value increased the error produced by the prestressed steel tube. Increasing the steel percentage increased the magnitude of the error.

For the case in which all the prestressing steel strains are in the nonlinear portion of the stress-strain approximation, the errors were on the order of 4 percent. The increase in error is due to the inability of the prestressed steel tube to accurately evaluate the discrete forces through the nonlinear portion of the stress-strain relationship. The maximum error occurs at large steel percentages and the higher values of g and t/b. The errors decrease as the value of g, t/b, and the steel percentage decreases.

The case in which the prestressing steel strains are in both the linear and nonlinear portion of the stress-strain curve resulted in errors on the order of 3 percent. The magnitude of the errors follow the same pattern mentioned in the two previous cases.

* * *

CONCLUDING REMARKS

The purpose of this study was to provide the engineer with a versatile design aid for rectangular prestressed concrete columns without accounting for length effects. This was accomplished by the development of a general formulation for a prestressed column analysis and of nondimensional load versus moment interaction curves.

The input information necessary in order to use these design aids is only the concrete and steel strengths and the ultimate load and moment acting on the column. From these parameters various column sizes, steel percentages and effective prestressing levels can be evaluated until a combination is found which satisfies the design requirements. This method offers efficiency and versatility in either preliminary or final design of rectangular prestressed concrete columns.

* * *

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2. *Notes on ACI 318-71 Building Code Requirements With Design Applications,* Portland Cement Association, 1972, Chapter 10, p. 26.
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4. *PCI Design Handbook – Precast Prestressed Concrete,* 2nd Edition, Prestressed Concrete Institute, Chicago, Illinois, 1978, Chapter 8, p. 17.
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APPENDIX A — DEVELOPMENT OF DESIGN CHARTS

The following charts were formulated using the numbered equations in the previous presentation. Two variables f'_c and g are considered in various combinations for rectangular columns with equal prestressing steel in each face. The steel percentage ρ_p, is based on the gross cross section of the column.

The following data were used in the development of the design charts:

ϵ_{cu} = 0.003 in./in. (0.003 cm/cm)
$E_c = 57.5\sqrt{f'_c\text{ (psi)}}$ (ksi)
f_{pu} = 270 ksi (1861.7 N/mm²)
f_{se} = 154.9 ksi (1068.0 N/mm²)
E_{ps} = 27,500 ksi (189.61 kN/mm²).

The charts were developed without accounting for column length effects. Also, the workmanship factor, ϕ, has not been applied to the curves. Therefore, the first step in design would be to adjust the loads for slenderness effects by using a moment magnifier approach and applying the workmanship factor, ϕ. The appropriate design charts can then be used as illustrated in the design examples.

The design curves for prestressed concrete columns are similar to those for regular reinforced columns in shape. The prestressed concrete column curves exhibit a less defined balanced point but do have a concrete compression control region and a tension control region for each steel percentage (see Fig. B2, Appendix B).

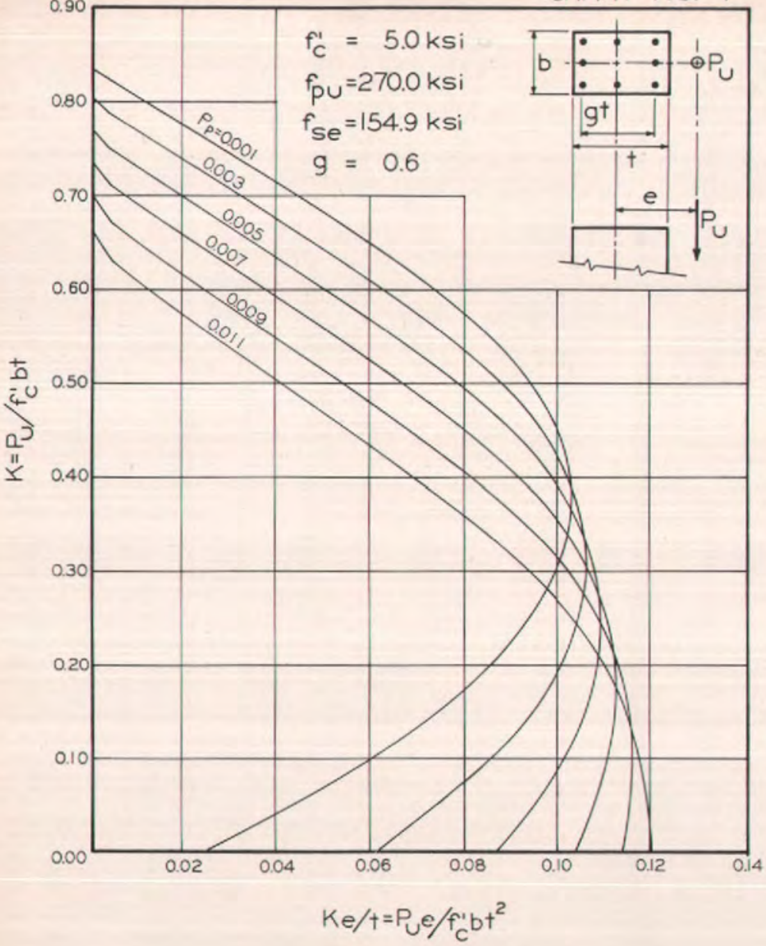
An interesting feature of the charts presented here is that the curves for prestressing steel percentages cross each other. This occurs because in the compression control region, the higher the prestressing steel percentage, the lower the column capacity. Hence, in the compression control region it is desirable to use less prestressing steel. However, in the tension control region the higher the percentage of prestressing steel, the higher the moment carrying capacity of the column.

* * *

FULL-SIZED DESIGN CHARTS

Because of the PCI JOURNAL size and the necessity to save expensive PCI JOURNAL space, the sixteen design charts in this article were reduced 50 percent. Readers interested in securing (at cost of reproduction and handling at time of request) photostats of the original charts should contact PCI Headquarters.

CHART NO. 1



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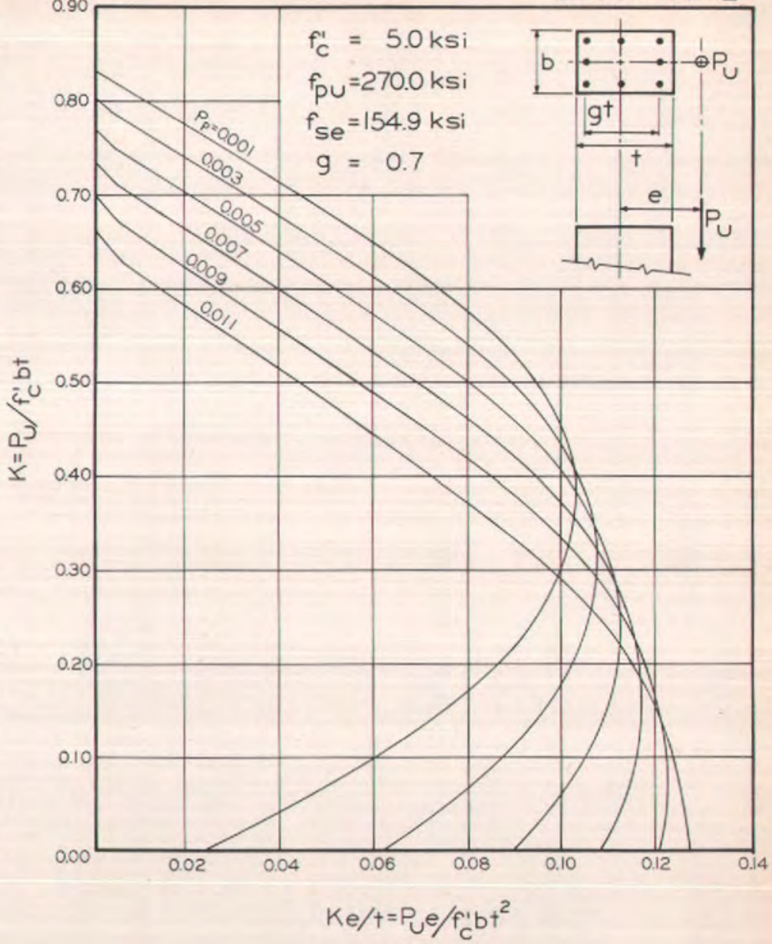

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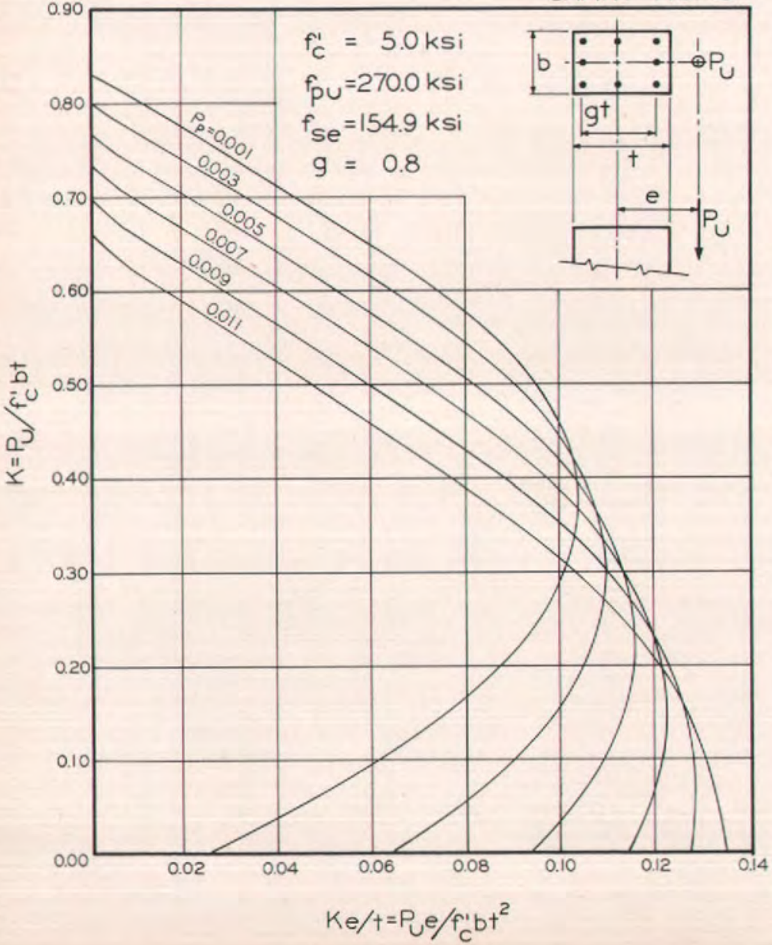

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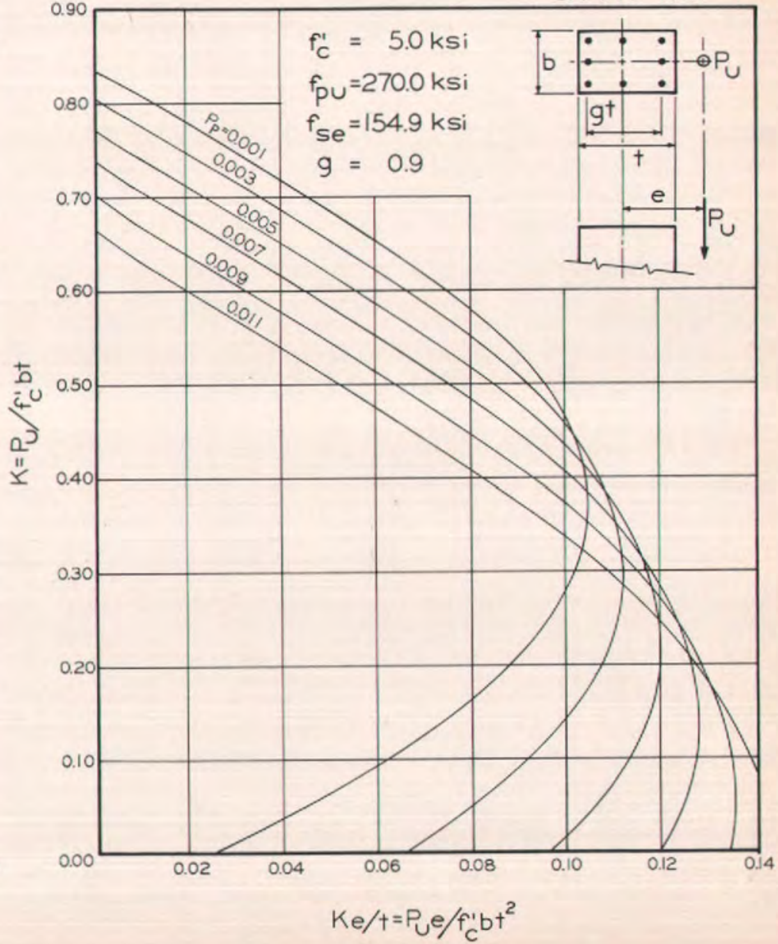

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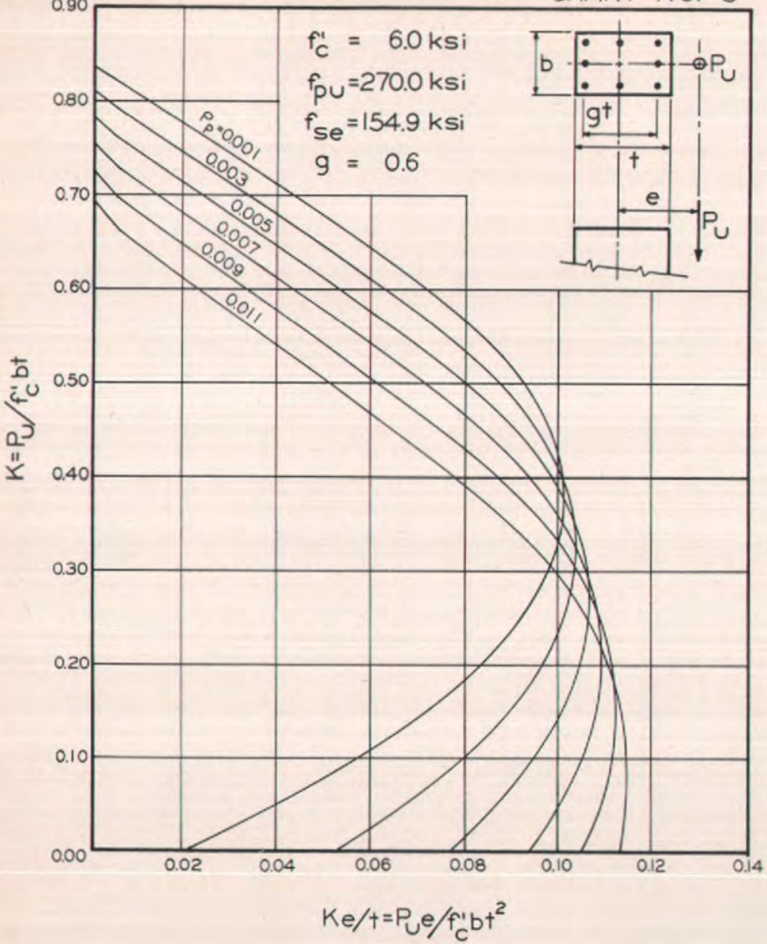

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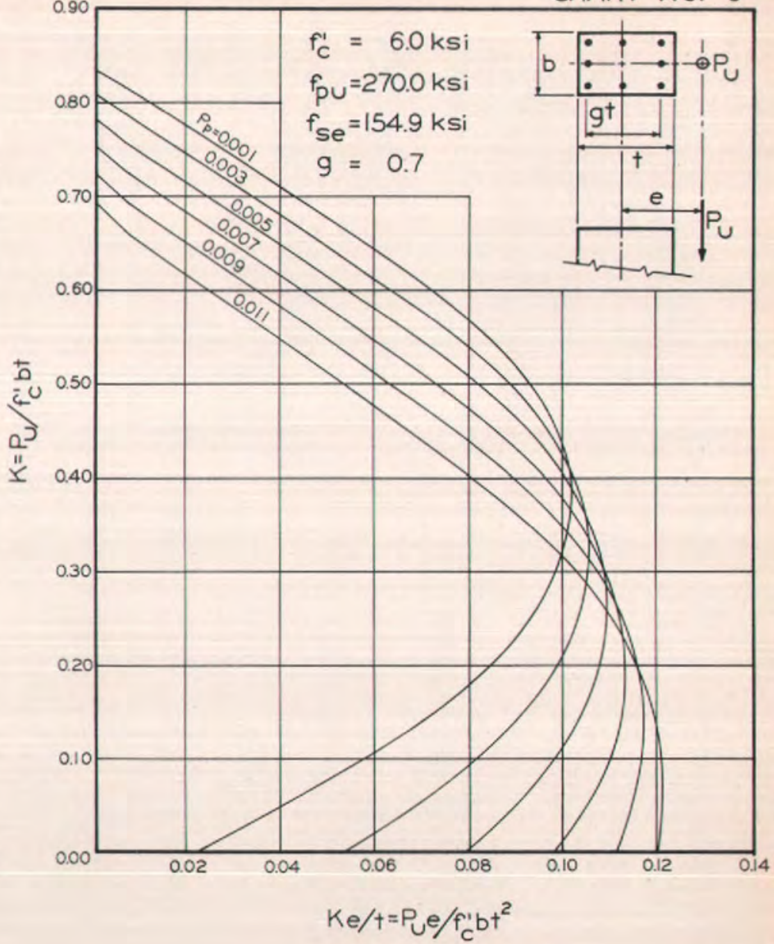

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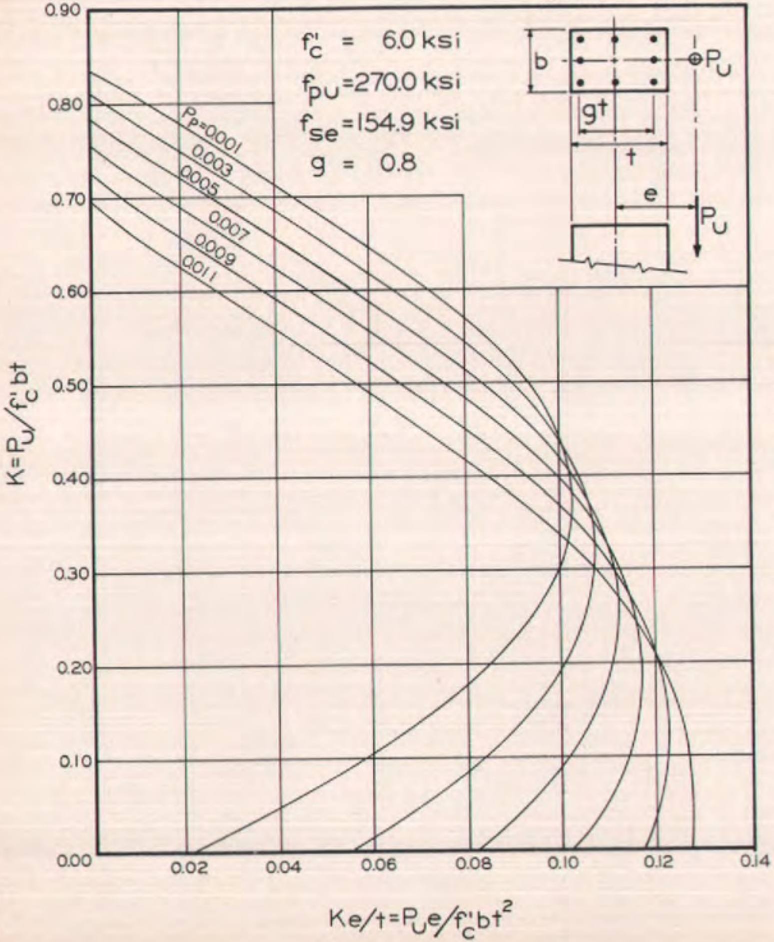

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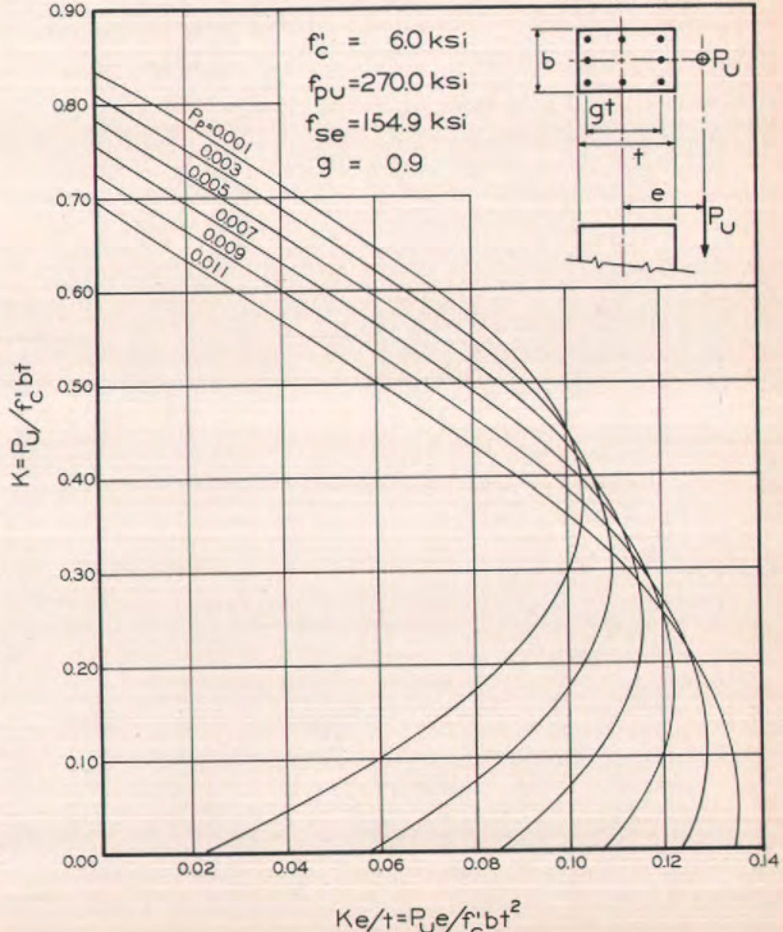

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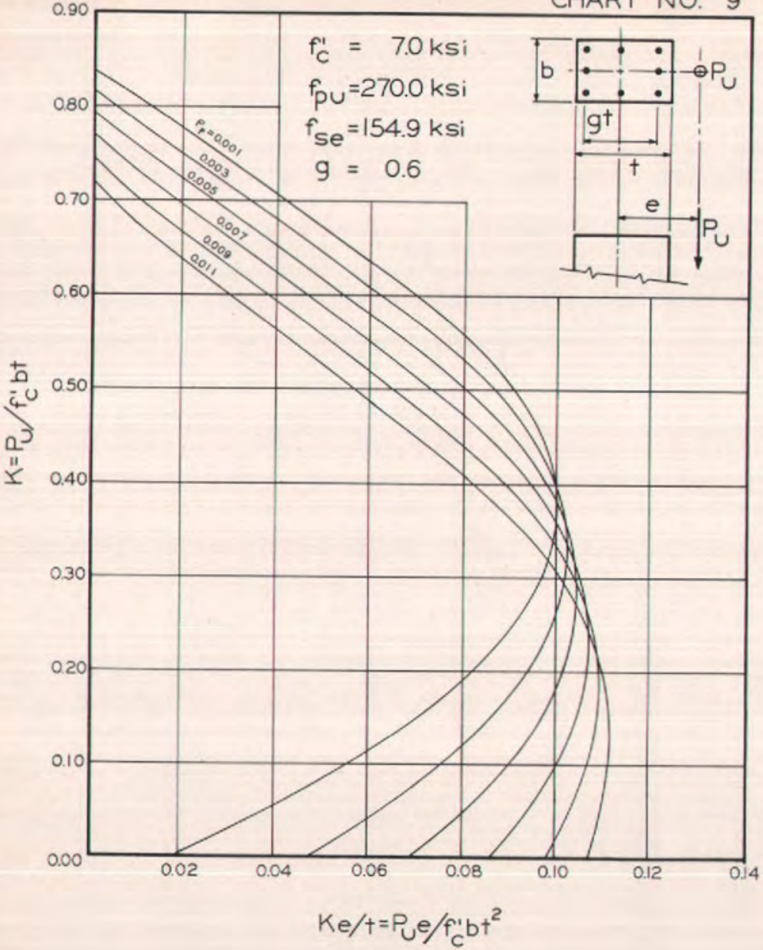

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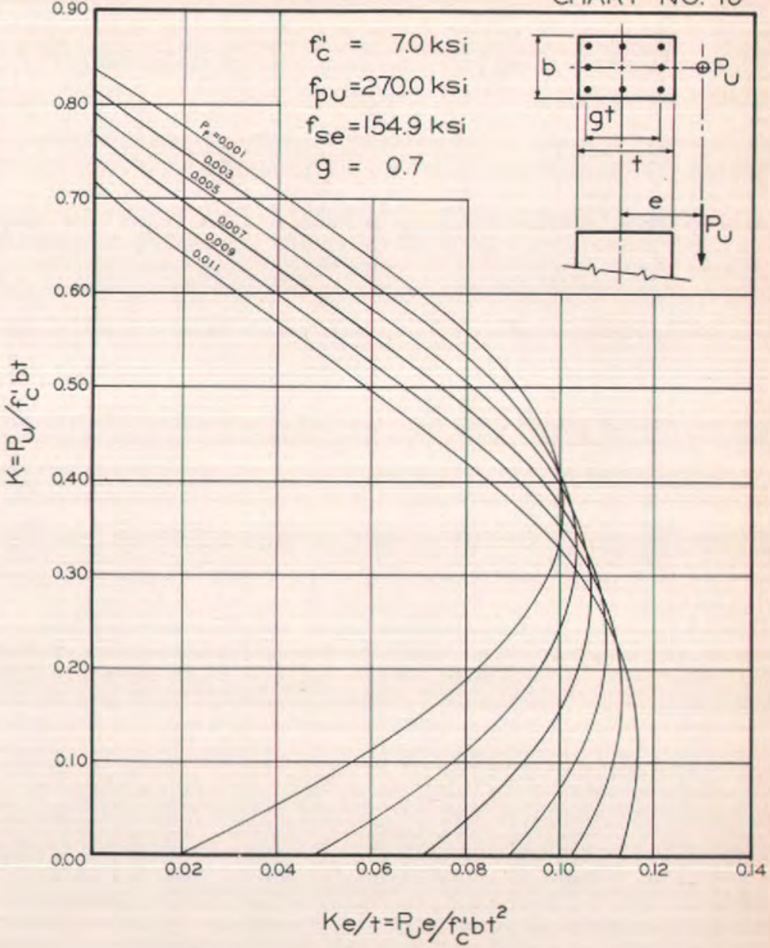

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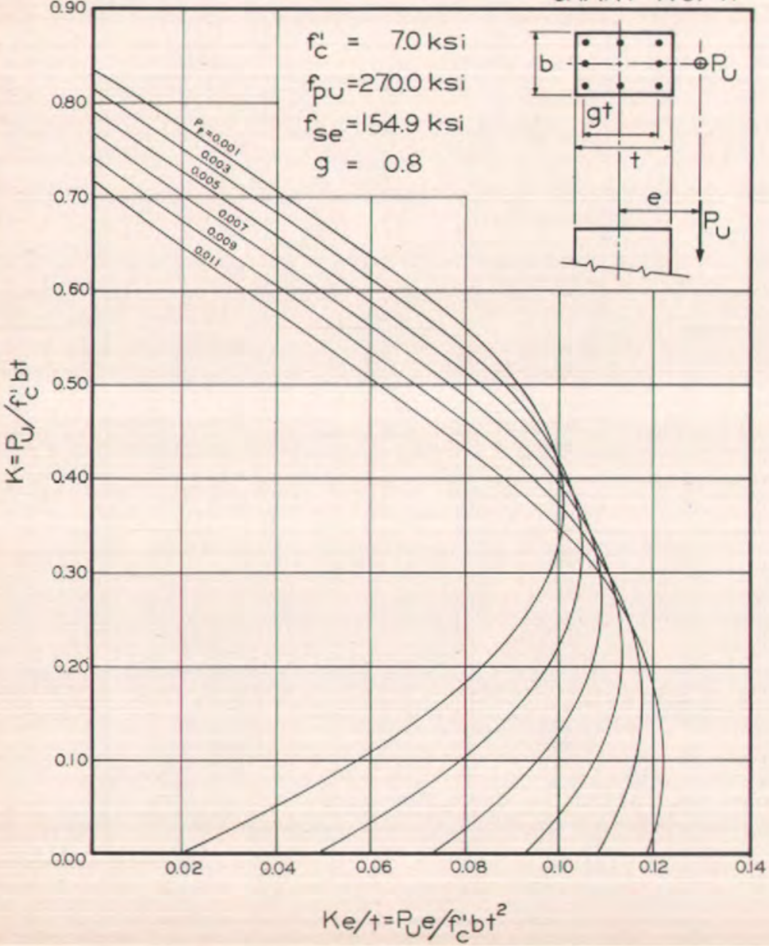

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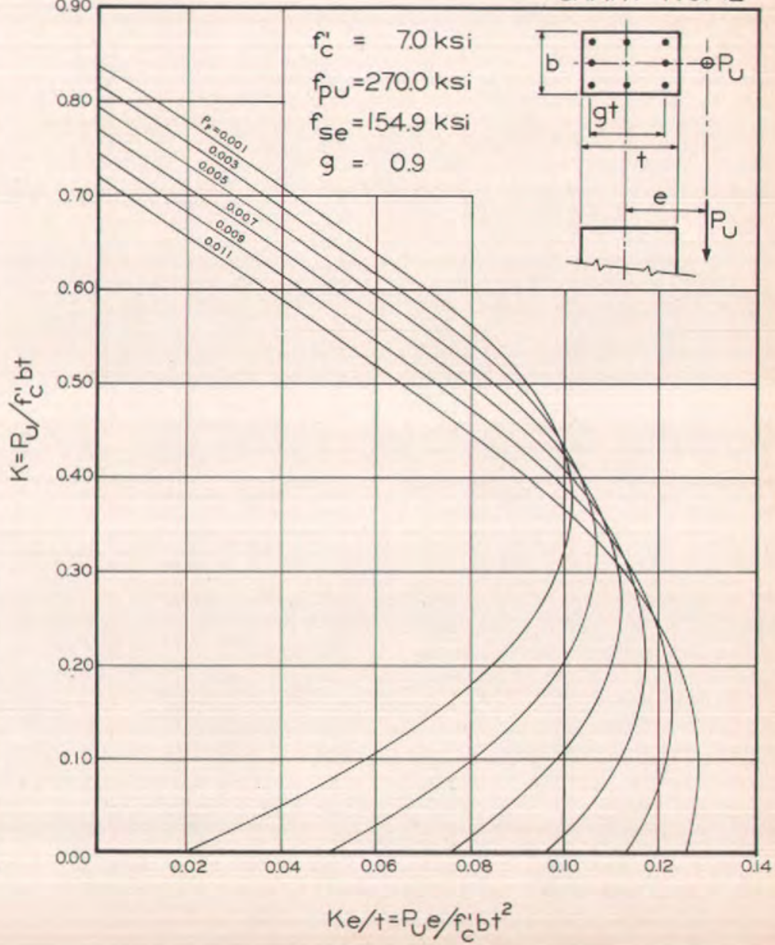

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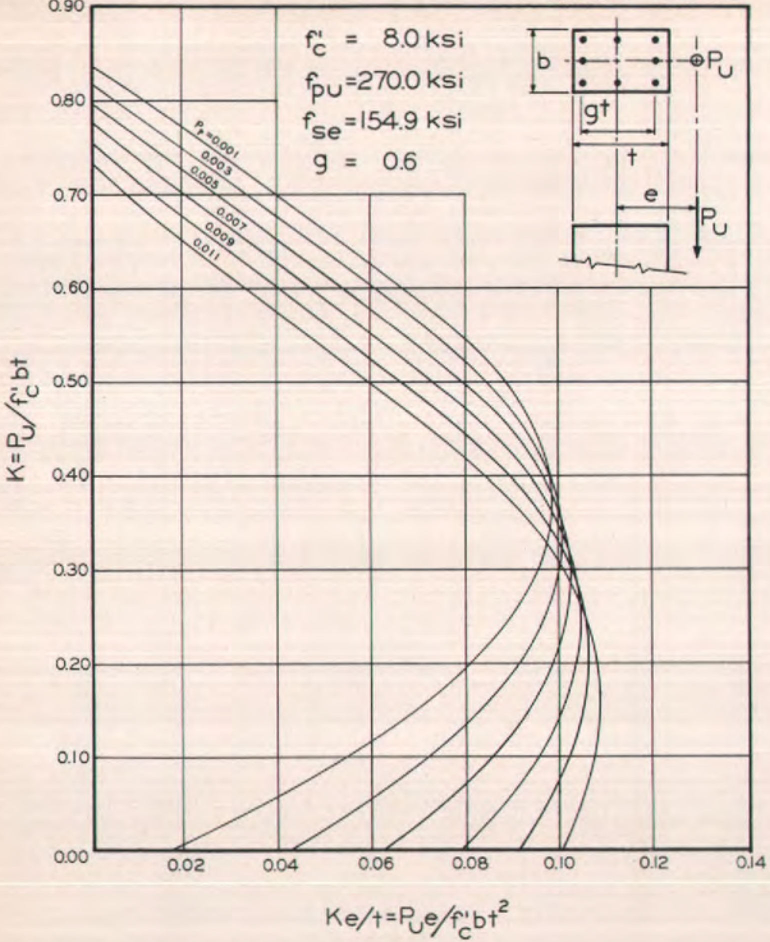

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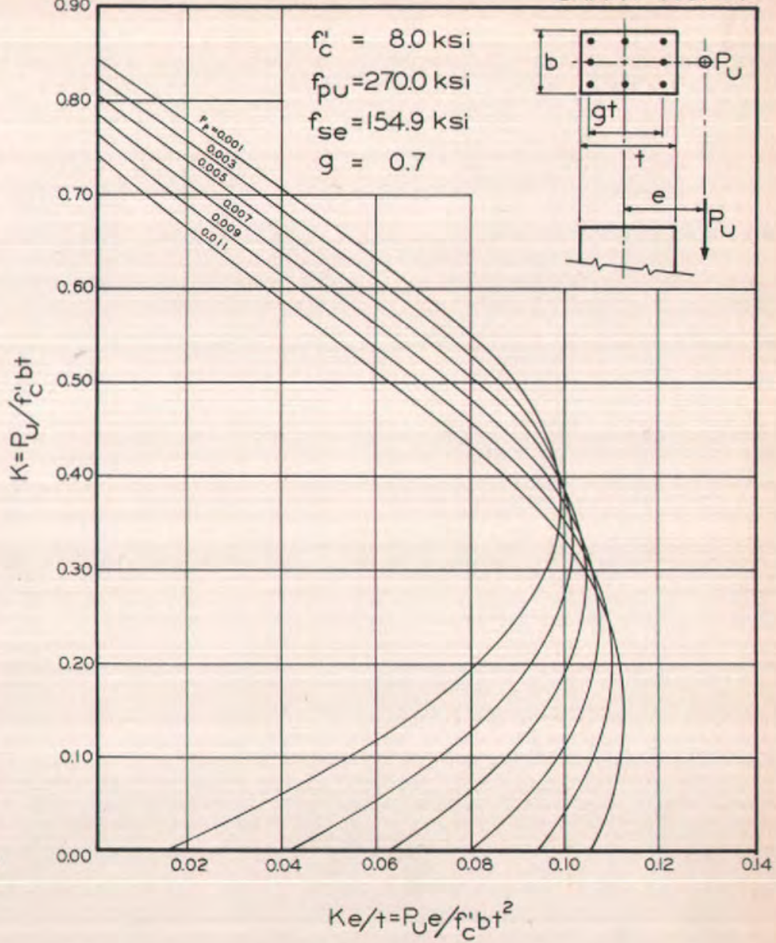

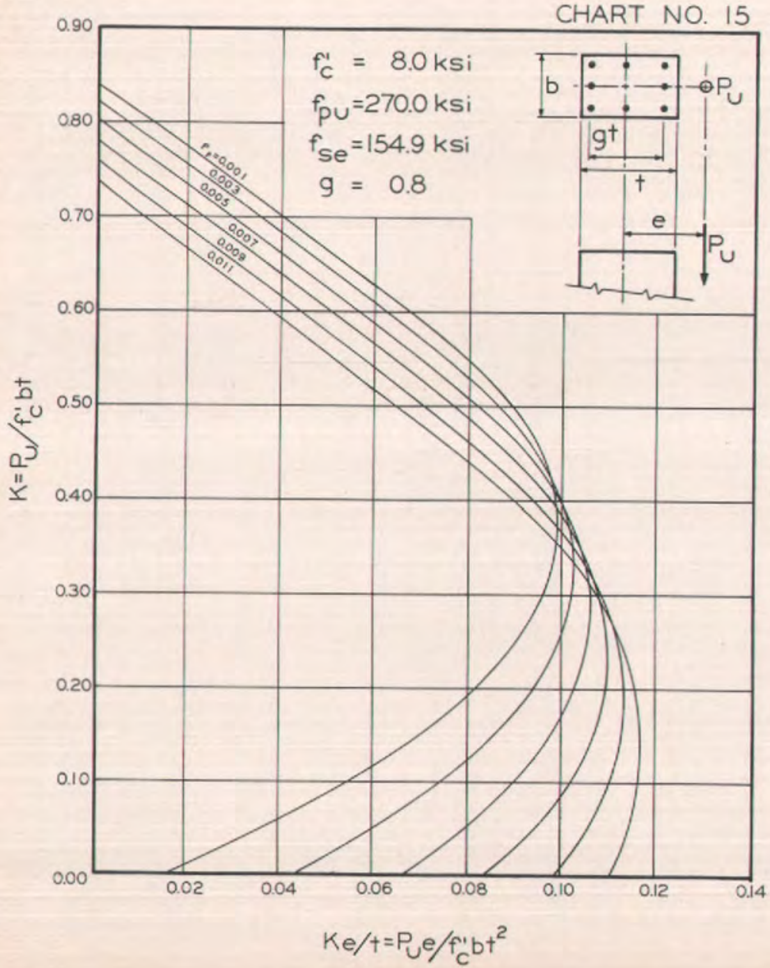
CHART NO. 15
f'c = 8.0 ksi
fpu = 270.0 ksi
fse = 154.9 ksi
g = 0.8
ρp = 0.001
0.003
0.005
0.007
0.009
0.011
b
gt
t
e
Pu
K = Pu/f'c bt
Ke/t = Pu e/f'c bt²
0.00
0.10
0.20
0.30
0.40
0.50
0.60
0.70
0.80
0.90
0.02
0.04
0.06
0.08
0.10
0.12
0.14

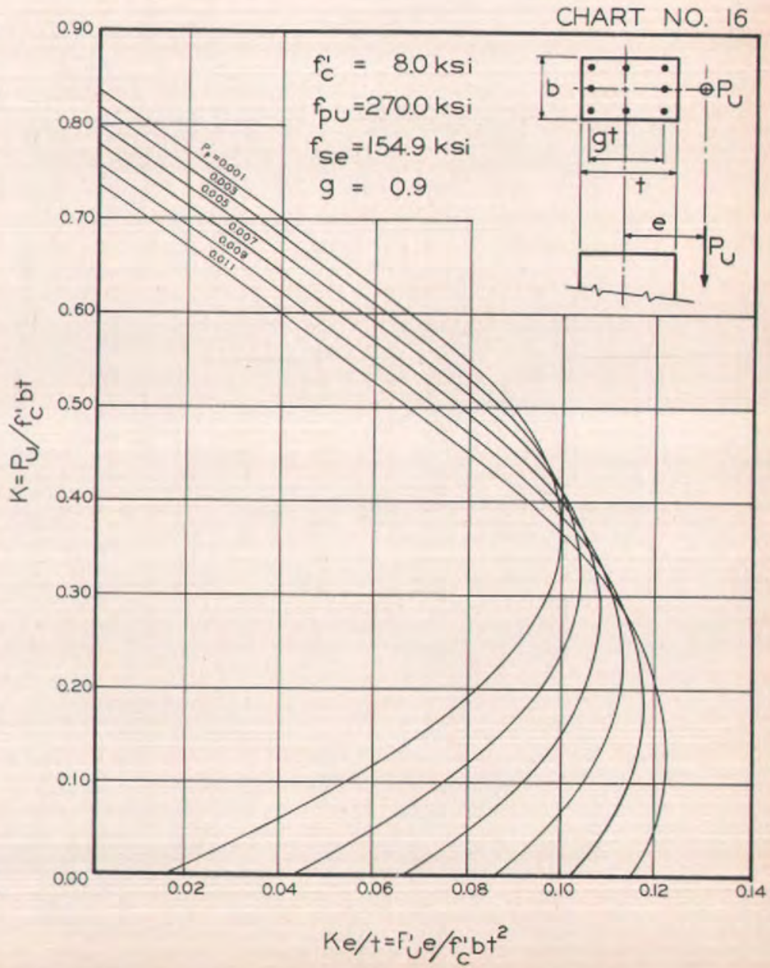
CHART NO. 16
f'c = 8.0 ksi
fpu = 270.0 ksi
fse = 154.9 ksi
g = 0.9
ρp = 0.001
0.003
0.005
0.007
0.009
0.011
b
gt
t
e
Pu
K = Pu/f'c bt
Ke/t = Pu e/f'c bt²
0.00
0.10
0.20
0.30
0.40
0.50
0.60
0.70
0.80
0.90
0.02
0.04
0.06
0.08
0.10
0.12
0.14

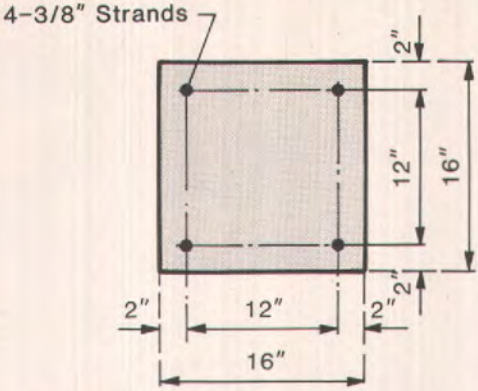

Fig. B1. Trial design 1.

DESIGN EXAMPLE 1

A square prestressed concrete column in the concrete compression control region is to be designed using the following data:

P_u = 550 kips; f_{pu} = 270 ksi
M_u = 875 kips; f_{se} = 154.9 ksi
f'_c = 6 ksi; ϕ = 0.7

The minimum cover to center of prestressing steel is 2 in.

Trial Design 1

By initially assuming that $b = t = 16$ in. and taking the design parameters into account the following parameters can be calculated.

$$K = \frac{P_u}{\phi f'_c bt} = \frac{550}{0.7(6)16(16)} = 0.512$$

$$R = K\frac{e}{t} = \frac{M_u}{\phi f'_c bt^2}$$

$$= \frac{875}{0.7(6)16(16)^2} = 0.051$$

gt = (16 in.) − 2(2 in.) = 12 in.

Therefore, g = 12/16 = 0.75.

The ϕ factor is applied to the parameters K and R since this workmanship factor has not been applied to the interaction curves.

Enter Chart No. 6 with $K = 0.512$, $K(e/t) = 0.051$ and find that a steel percentage of 0.001 is adequate (see Fig. B2). This steel percentage is also adequate when checked with Chart No. 7 with a g of 0.8. For a steel percentage of 0.001, the area of prestressing required (A_{ps}) is given by the following equation.

$$A_{ps} = \rho_p bt = 0.001(16)16 = 0.256 \text{ in.}^2$$

This requires four ⅜-in. diameter strands (A_{ps} = 0.34 in.²). The results of trial design 1 are shown in Fig. B1.

Analysis of Trial Design I

The interaction curves can be used to find the load and moment capacity of a column. For trial design 1 the following parameters are calculated:

$$K = 0.512 \qquad \frac{e}{t} = \frac{R}{K} = 0.01$$

$$R = 0.051$$

$$g = 0.75 \qquad \rho_p = \frac{A_{ps}}{bt} = \frac{0.34}{16(16)} = 0.0013$$

A line is drawn from the origin, through the point $K = 0.512$, $R = 0.051$ to a steel percentage of 0.0013 (see Fig. B2). The coordinates corresponding to this end point are as follows:

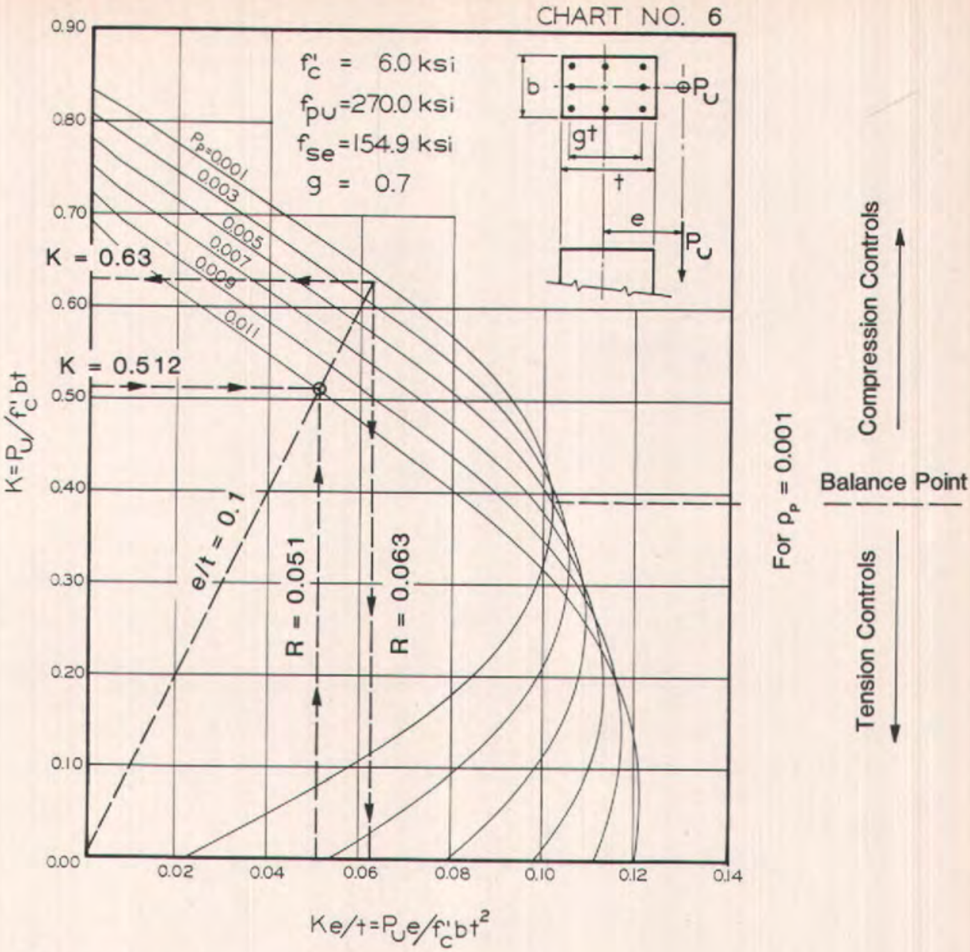

Fig. B2. Design chart for Example 1.

$K = 0.63$ and $K(e/t) = 0.063$

The actual design load capacity of the column is given by the following equations:

$$P_u = \phi K f'_c\, bt = 0.7(0.63)6(16)16 = 677 \text{ kips} > 550 \text{ kips}$$

$$M_u = \phi(K\frac{e}{t}) f'_c\, b(t)^2 = 0.7(0.063)6(16)(16)^2 = 1083 \text{ in.-kips} > 875 \text{ in.-kips}$$

The 16 x 16-in. column with four ⅜-in. diameter strands is adequate.

DESIGN EXAMPLE 2

Design a 14 x 14-in. prestressed concrete column with four ½-in. diameter strands with the following data:

P_u = 425 kips	f_{pu} = 270 ksi
M_u = 700 in.-kips	f_{se} = ?
f'_c = 5 ksi	ϕ = 0.7

First calculate K, $K(e/t)$, g and ρ_p.

$$K = \frac{P_u}{\phi f'_c\, bt} = \frac{425}{0.7(5)14(14)} = 0.62$$

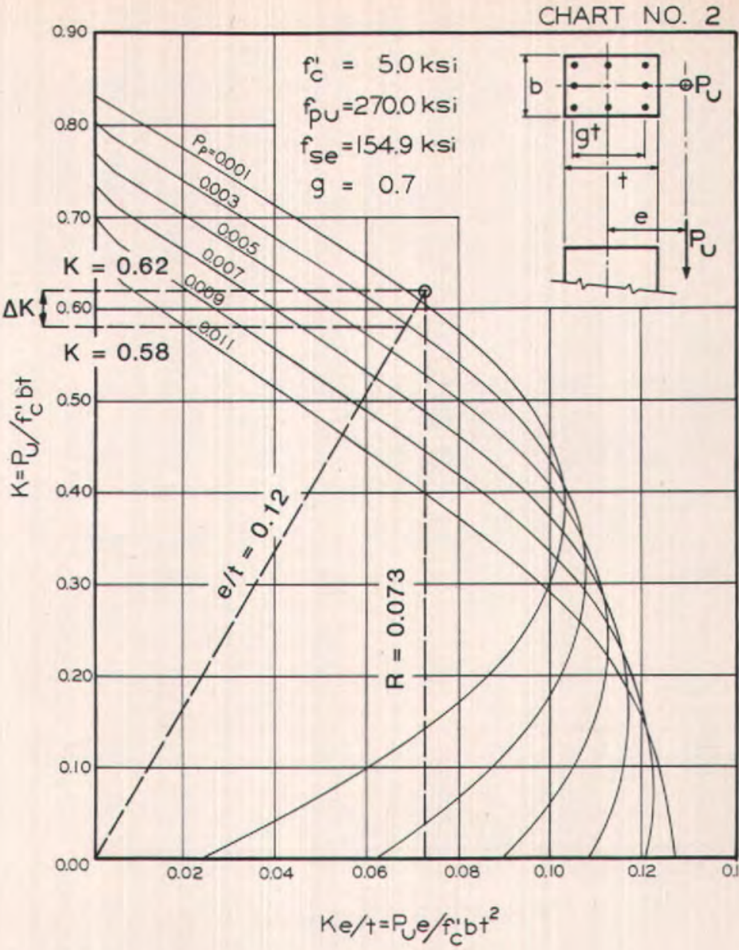

Fig. B3. Design chart for Example 2.

$$K\,\frac{e}{t} = \frac{M_u}{\phi f'_c\, b^2}$$

$$= \frac{700}{0.7(5)(14)(14)^2}$$

$$= 0.073$$

$$gt \quad = (14 \text{ in.}) - 2(2 \text{ in.}) = 10 \text{ in.}$$

$$g \quad = \frac{10}{14} = 0.71$$

$$\rho_p = \frac{4(0.153)}{14(14)} = 0.0031$$

The column will be checked for the fully prestressed case. Enter Chart No. 2 with $K = 0.62$ and $K(e/t) = 0.073$ (see Fig. B3). This point lies outside the $\rho_p = 0.0031$ interaction curve, hence the column will not work when fully prestressed ($f_{se} = 154.9$ ksi).

To calculate the required level of prestressing, locate the intersection of $e/t = 0.12$ line and the $\rho_p = 0.0031$ interaction curve. This point is the lower limit of ΔK as shown in Fig. B3.

$\Delta K = 0.62 - 0.58 = 0.04$

The differential stress Δf_{se} can be calculated from Eq. (16):

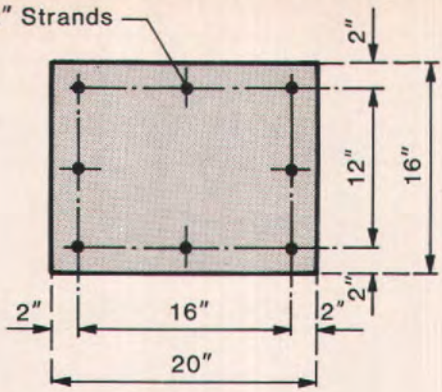

Fig. B4. Trial Design 1.

$$\Delta f_{se} = \frac{5(0.04)}{0.0031\left[1 + 0.0031\left(\frac{27500}{57.5\sqrt{5000}}\right)\right]}$$

$= 63.2$ ksi

The required prestressing level is:

$f_{se} = 154.9 - 63.2 = 91.7$ ksi

The 14 x 14-in. column with four ½-in. diameter strand will work with $f_{se} \leqslant 91.7$ ksi.

DESIGN EXAMPLE 3

Design a rectangular column using the following data:

Assume strong axis bending.

$P_u = 140$ kips $\quad M_u = 2250$ in.-kips

$f'_c = 5$ ksi $\quad f_{pu} = 270$ ksi

$f_{se} = 154.9$ ksi $\quad b/t \approx 0.8 \quad \phi = 0.7$

Trial Design 1

Initially assume that $b = 16$ in. and $t = 20$ in. and calculate the following parameters:

$$K = \frac{P_u}{\phi f'_c bt} = \frac{140}{0.7(5)(16)20} = 0.125$$

$$R = K\frac{e}{t} = \frac{M_u}{\phi f'_c bt^2} = \frac{2250}{0.7(5)(15)20)^2} = 0.100$$

$gt = (20 \text{ in.}) - 2(2 \text{ in.})$

$g = 16/20 = 0.8$

Enter Chart No. 3 with $K = 0.125$ and $R = 0.100$; the required prestressing steel percentage is 0.0035. The required area of prestressing (A_{ps}) is calculated as follows:

$A_{ps} = \rho_p bt = 0.0035\,(16)20 = 1.12$ in.^{2}

This relates to eight ½-in. diameter strands with a total prestressing steel

METRIC (SI) UNIT EQUIVALENTS

1 in.	= 25.4 mm
1 in.^{2}	= 645.6 mm^2
1 ft	= 0.3048 m
1 ft^2	= 0.0929 m^2
1 psf	= 47.99 N/m^2
1 psf	= 47.88 Pa
1 psi	= 0.006895 MPa
1 psi	= 0.006895 N/mm^2
1 pcf	= 16.02 kg/m^2
1 kip	= 4.448 kN
1 kip/ft	= 14.594 kN/m
1 in.-kip	= 113.0 N•m
1 ft-kip	= 1.356 kN•m

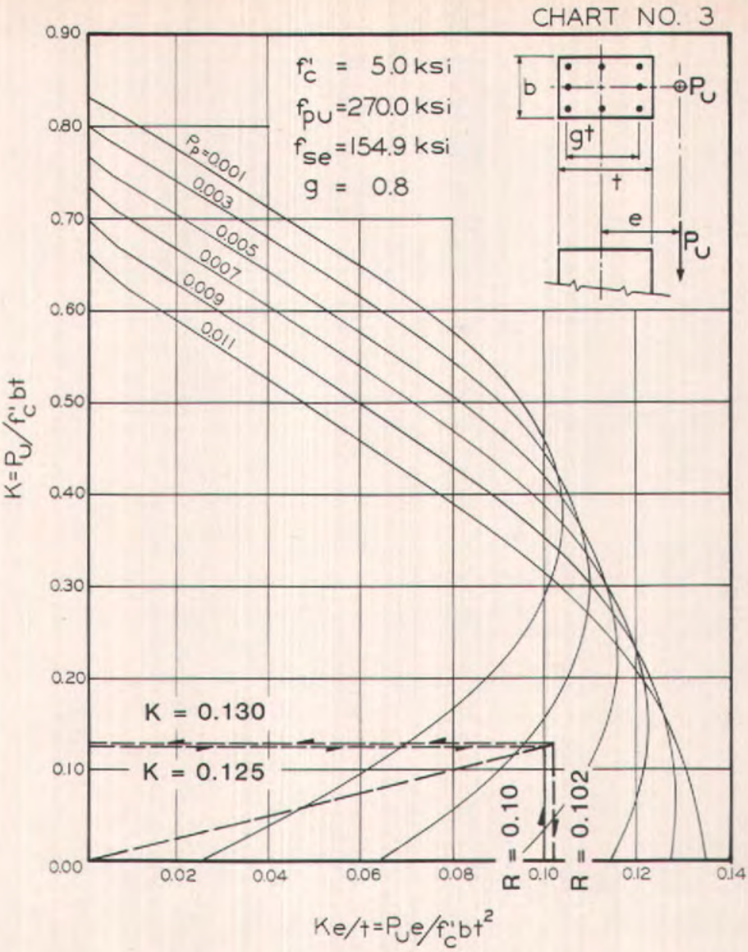

Fig. B5. Design chart for Example 3.

area of 1.224 in.^{2} The results of trial design 1 are shown in Fig. B4.

Analysis of Trial Design 1

The interaction curves can be used to find the load and moment capacity of the column by using the following parameters:

$K = 0.125 \quad R = 0.100 \quad g = 0.8$

$$\rho_p = \frac{A_{ps}}{bt} = \frac{1.224}{16(20)} = 0.0038$$

A line is drawn from the origin on Chart No. 3 through the point $K = 0.125$, $R = 0.100$ to a steel percentage of 0.0038 (see Fig. B5).

The coordinates corresponding to this end point are as follows:

$K = 0.13$ and $K(e/t) = 0.012$

The actual design load capacity of the column is given by the following equations:

$$P_u = 0.7\,(0.13)\,(5)\,(16)\,20 = 145.6 \text{ kips} > 140 \text{ kips}$$

$$M_u = 0.7\,(0.102)\,(5)\,(16)\,(20)^2 = 2285 \text{ in.-kips} > 2250 \text{ in.-kips}$$

The 16 x 20-in. column with eight ½-in. diameter strands is adequate.

APPENDIX C — NOTATION

A_{ps} = total area of prestressing steel
b = column width
B.A. = bending axis
C_c = compressive concrete force
$d_{(subscript)}$ = distance between extreme compression fiber and subscript location
E_c = modulus of elasticity of concrete
E_{ps} = modulus of elasticity of prestressing steel
e = eccentricity of ultimate load
$\epsilon_{B(subscript)}$ = prestressing steel bending strain at subscript location
ϵ_{ce} = strain in concrete due to prestressing
ϵ_{cu} = ultimate concrete strain
ϵ_{NL} = strain at which prestressing steel stress-strain curve becomes nonlinear
ϵ_o = concrete strain at which concrete stress-strain curves become linear
$\epsilon_{p(subscript)}$ = prestressing steel strain at subscript location
$\bar{\epsilon}_{p(subscript)}$ = strain at centroid of nonlinear portion at prestressing steel stress-strain curve
f_c = concrete stress
f'_c = compressive strength of concrete
$f_{p(subscript)}$ = prestressing steel stress at subscript location
f_{se} = effective stress in prestressing steel after losses
gt = distance between top and bottom steel layer
K = nondimensional load
K_c = portion of nondimensional load due to concrete
K_s = portion of nondimensional load due to prestressing steel
k_{ut} = depth of concrete compression block
M_c = moment due to concrete compression force
M_u = ultimate moment acting on column
N.A. = neutral axis
P_u = ultimate axial load acting on column
R = nondimensional moment
R_c = portion of nondimensional moment due to concrete
R_s = portion of nondimensional moment due to steel
ρ_p = prestressing steel percentage
t = column depth
y_c = distance between centroid of concrete stress distribution and bending axis

* * *

NOTE: Discussion of this paper is invited. Please submit your discussion to PCI Headquarters by September 1, 1982.